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Essays on the Roles of Financial and Higher Education Institutions in Macroeconomics

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Essays on the Roles of Financial and Higher Education Institutions in Macroeconomics

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ABSTRACT

The thesis consists of three essays in macroeconomics. Chapter 1 analyzes the role of higher education in shaping income inequality and intergenerational mobility. I introduce a model where overlapping generations of heterogeneous households make college choices subject to a borrowing constraint and with heterogeneous colleges that maximize quality. First, in response to the observed rise in the returns to human capital in the U.S. since 1980, the model predicts an increase in income inequality, tuition, the dispersion of spending per-student across colleges, the exclusion of low-income students from top colleges, and the intergenerational elasticity of earnings (IGE), all consistent with the data. Second, I use the model to run counterfactuals. If all students received the same higher education, the income Gini and the IGE would decrease by up to 9% and 33%, respectively. Current government interventions—financial aid and transfers to colleges—decrease the Gini coefficient by 3% and the IGE by 12% compared to the *laissez-faire*.

Chapter 2 presents series on dispersion of expenditures per student, revenues and faculty per student across colleges from 1980 to 2016 in the United States using IPEDS data. Inequality across students have been slowly and steadily increasing over the period. The increase is half as large as the increase in annual household income inequality. The trend is driven by four-year colleges, both within and between non-profit private and public colleges. The average and the progressivity of government transfers to higher education institutions have sharply declined since 1980, especially at four-year institutions.

Chapter 3 develops a model where large financial intermediaries subject to systemic runs internalize the effect of their leverage on aggregate risk, returns and asset prices. Near the steady-state, they restrict leverage to avoid the risk of a run which gives rise to an accelerator effect. For large adverse shocks, the system enters a zone with high leverage and possibly runs. The length of time the system remains in this zone depends on the degree of concentration through a *franchise value*, *price-drop* and *recapitalization channels*. The speed of entry of new banks after a collapse has a stabilizing effect.

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CONTENTS

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Chapter I

The Great Gatsby Goes to College: Tuition, Inequality and Intergenerational Mobility in the U.S.

I Introduction

TO WHAT EXTENT DOES THE HIGHER EDUCATION SYSTEM in the U.S. accentuate or dampen income inequality and reduce or enhance intergenerational mobility? College has traditionally been viewed as one of the main pathways to upward mobility. However, access remains extremely selective and unequal, especially at top-quality universities. For example, [Chetty, Friedman, Saez, Turner, and Yagan \(2019\)](#) report that children whose parents are in the top 1% of the income distribution are 77 times more likely to attend an Ivy League college than those whose parents are in the bottom income quintile. What are the forces determining the sorting of students and financial resources across colleges of different quality? To what extent does parental income matter relative to ability? How does this sorting in turn shapes inequality at the next generation and intergenerational mobility?

These questions regarding the contributions of colleges to income inequality and intergenerational mobility are all the more important as trends over the past forty years show that (a) the market returns

to education and income inequality have increased (Piketty and Saez, 2003; Autor, Katz, and Kearney, 2008); (b) the dispersion of expenditures per students across colleges has increased (Capelle, 2019); (c) the share of students from the lowest income quintile in top colleges has stagnated (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2019); (d) tuition fees before financial aid have increased by a factor of four in real terms since 1980; and (e) the intergenerational elasticity of income (hereafter IGE) has slightly increased, corresponding to a decline in intergenerational mobility (Davis and Mazumder, 2017).

In this paper, I provide a framework to understand the interaction between the allocation of students and financial resources across heterogeneous colleges, income inequality and intergenerational mobility. I analyze how the higher education system—the endogenous response of colleges and government policies—accentuates or dampens income inequality and reduces or enhances intergenerational (im)mobility. I then offer a unified explanation for the stylized facts (a) to (e) described in the previous paragraph: an increase in the returns to human capital. Finally, I use the model to analyze how the higher education system propagates the increase in income inequality and run policy counterfactuals.

The household side of the model builds on a large theoretical literature that formalizes how human capital transmission across generations perpetuates inequality (e.g., Benabou (2002)). A continuum of heterogeneous households characterized by their human capital transmit—with some randomness—ability to their children and make an educational investment choice subject to a borrowing constraint. The supply side of the market for higher education is a continuum of colleges that differ in quality. Households face an equilibrium tuition schedule that depends on college quality, student ability and parental income.¹ After college, each child becomes an adult, sells their human capital—a combination of their ability, college quality and some labor market shock—in a competitive labor market and has a child.

A key novelty of my framework is to embed into this overlapping generation general equilibrium model a distribution of heterogeneous colleges that is determined endogenously. Colleges seek to maximize the quality they provide to their students. Their quality depends not only on the amount of educational resources spent per student but also on the average ability of the student body, what will be

¹In Benabou (2002), households buy an educational “good” traded at a constant unit price, independent on the households/students’ characteristics and there is no notion of quality ladder.

referred to as the “peer-effect.” Colleges have an incentive to attract high-ability students because of the peer-effect, as well as students from rich families who bring in additional resources to finance educational spending. This microfoundation of the college sector is borrowed from a literature that estimates equilibrium models of higher education (e.g., [Rothschild and White \(1995\)](#); [Epple, Romano, and Sieg \(2006\)](#); [Cai and Heathcote \(2018\)](#)). As in [Cai and Heathcote \(2018\)](#), colleges are price-takers and the tuition schedule clears each segment of the higher education market. Finally, I close the model with an educational sector that produces educational services and a government that implements non-linear merit and need-based financial aid to students and non-linear transfers to colleges.

The first contribution of the paper is to provide an analytical characterization of the equilibrium allocation and a unified explanation for the stylized facts (a) to (e): under weak conditions, an increase in the returns to education—a primitive of the model—is shown to lead to an increase in income inequality, an increase in the inequality of resources across colleges, a decrease in the share of low income students at top colleges, a decline in intergenerational mobility and an increase in tuition. Intuitively, the rising returns to education increase the dispersion of labor earnings for a given distribution of human capital, thereby increasing income inequality. This leads richer households to demand higher quality of education, incentivizing top colleges to raise tuition, increasing the dispersion of revenues and educational spending across colleges. This in turn feeds back into more inequality in human capital at the following generation. Individuals from low-income background are priced out of top colleges, hence the stagnation of their shares and the decline in mobility. Higher education thus contributes to the gradual shift of the U.S. society to the right side of the Great Gatsby curve.²

I then estimate a more general version of the model that I use to run policy counterfactuals as well as to quantify the above effects. I allow for some degree of intergenerational transfers of financial wealth and allow individuals to not go to college. The model is estimated using several microdata sources: (i) the National Longitudinal Survey of Youth of 1997, a representative panel of high-schoolers, with detailed information on parental background, the children’s abilities, their journey through the higher education system and their income in their early thirties; (ii) the NCES-NPSAS, a detailed student-level dataset

²The Great Gatsby curve is the negative empirical correlation between cross-sectional income inequality and intergenerational mobility. It has been documented in the cross-section of countries and over time in the U.S.

on net tuition and financial aid; and (iii) the NCES-IPEDS, a panel of the universe of colleges. The estimation proceeds in two steps: first I estimate the closed-form model which I show is, under some assumptions, exactly identified. I then use the resulting estimates as initial values in the estimation of the richer quantitative version. The closed-form expressions for the targeted moments in the first step make it transparent which moments are important to pin down each parameter. They also make this first step computationally quick, which allows me to estimate a large set of parameters.

The second contribution is to quantify the extent to which the higher education system shapes income inequality and intergenerational (im)mobility. I run six policy counterfactuals. The first one aims at quantifying the total contribution of higher education to inequality and mobility. It consists in randomly allocating students to colleges and equalizing spending across institutions. This leads to a decrease in the income Gini by 9% (4.3 p.p.) and a decrease in the IGE by 24%.³ With the second counterfactual, I am interested in isolating the contribution of the peer-effect. To do so, I increase the degree of progressivity of transfers to colleges to the point where spending is equalized across all colleges. In equilibrium, students perfectly sort across colleges by ability.⁴ This policy experiment leads to a decrease in the income Gini by 3% (1.4 p.p.) and in the IGE by 15%.

The third policy counterfactual studies the implications of removing current government interventions in higher education. In a *laissez-faire* allocation, where all government interventions are removed, I find that the Gini coefficient would be 3% higher, and the IGE 10% higher. Moreover the most powerful dimension of current policy is the subsidy to colleges, which contributes to a reduction of the income Gini by 2%, followed by need-based financial aid, which decreases the income Gini by 1%. Merit-based financial aid has virtually no effect, because it is very small in the *status quo*. I document elsewhere that government transfers to colleges have become significantly less redistributive over the past forty years (Capelle, 2019), bringing the system closer to a *laissez-faire* allocation. In the fourth counterfactual, I set the parameters of the transfers schedule to what they were at the beginning of the 1980s. I find that the

³ To give a sense of the magnitudes, the Gini coefficient of household gross income in the last decade has been around .45. And it has increased by 10 p.p. since 1980. Estimates for the IGE range from .36 to .55. My favorite estimate, which is also the value targeted in the calibration, is .4.

⁴ A very progressive need-based financial aid schedule would generate the same equalization of resources and assortative matching by abilities.

income Gini and the IGE would be 1% and 3.4% lower than in the *status quo*, respectively.

The last two counterfactuals analyze the implications of a sharp increase in the progressivity of need-based aid provided by colleges and of a version of “College for All”, two proposals widely discussed in policy circles. To implement the former, I modify the strength of the social objective of colleges to the point where the tuition schedule at a given institution and for a given child ability is linear in parental income. Like in the first two experiments, the effect of parental income on the sorting of students is neutralized. However college spending remains strongly increasing in college quality because child ability and parental income are strongly positively correlated. This leads to an increase in the income Gini by 3% and in GDP by 22% because of the increase in the positive assortative matching of students across colleges. The increase in the Gini occurs despite the decrease in the IGE by 6%. Finally I evaluate the implications of a version of the “College for All” proposal. I implement the latter by fixing the dispersion of spending across colleges to its level in the *status quo* and by sorting students across colleges by ability. Perfect assortative matching is not an assumption but an equilibrium outcome. Surprisingly, I find that the income Gini would increase by 2.5% and the IGE would decrease by 7%. GDP would increase by 2.8%, thanks to the improvement in the allocation of students.

The last contribution is to assess the quantitative effects of a rise in the returns to human capital and to decompose the rise in inequality into a direct effect and the endogenous propagation through the higher education sector. The increase in the returns to human capital is calibrated to match the increase in the returns to college. Following this increase, the model generates an increase in the income Gini coefficient by 13 p.p., which corresponds to 130% of the empirical change, an increase in the expenditure per student Gini by 5 p.p. corresponding to 100% of the empirical change and an increase in the IGE by 6%. In a counterfactual world in which the returns to human capital increase but do not propagate through the higher education sector, the increase in the Gini coefficient of income would have been 6% lower. There are two sources of amplification: the allocation of resources and the allocation of student ability across colleges. More than 100% of the total amplification is coming from the former. The latter actually dampens the increase in inequality because the positive assortative matching of students along the quality ladder of colleges worsens as relatively richer and less able children buy their way to top colleges.

1.1 Literature Review

The present paper relates most directly to the literature that models and quantifies the transmission of human capital, educational choice and inequality in an intergenerational framework (Benabou, 1996; Fernandez and Rogerson, 1996; Kotera and Seshadri, 2017; Caucutt and Lochner, 2017; Guerrieri and Fogli, 2017; Durlauf and Seshadri, 2018; Blandin, Herrington, et al., 2018; Lee and Seshadri, 2019; Eckert and Kleineberg, 2019). A subset of these papers focuses on the role of higher education: Restuccia and Urrutia (2004) shows that half of the intergenerational correlation of earnings is accounted for by parental investment in education with a dominant role for early education. In these papers, in contrast with mine, there is a representative college whose tuition fees is exogenously parametrized and households have to make a binary decision—enrolling or not into college.

In contrast, another stream of the literature models in detail and in static frameworks the admission and tuition setting decisions of colleges and the rich heterogeneity in colleges and student types, but focuses mainly on the impact of financial aid policies (Epple, Romano, and Sieg, 2006; Fillmore, 2016), of a change in the supply of seats in public colleges (Fu, 2014) and affirmative action policies (Kapor, 2015) on the sorting by itself, while my focus is the study of equilibrium inequality and mobility. More recently and more closely related, Cai and Heathcote (2018) shows that income inequality can fully account for the observed rise in average net tuition since 1990 in a static model where households choose a quality of college and price-taking colleges maximize profit. My model has a flavor of theirs. We differ in that (i) there is a continuum of student ability instead of two types, which—maybe surprisingly—simplifies the analysis; (ii) higher education is not purely a consumption good but does matter for the accumulation of human capital. Incidentally, the equilibrium allocation is fully efficient in their model; and (iii) more importantly, my paper is dynamic and is, to the best of my knowledge, the first one to embed a sorting problem of heterogeneous students across heterogeneous clubs into an intergenerational setting, and still remains tractable and lends itself to the study of the role of higher education in the transmission of economic status over generation. One implication of this tractability is the possibility to work in an environment in which existence and uniqueness of the equilibrium—two issues that have plagued the theoretical and quantitative literature on clubs—can be characterized. Another related stream of the literature stud-

ies the macroeconomic determinants of tuition and the effects of financial aid. [Lucca, Nadauld, and Shen \(2015\)](#) stresses the role of the expansion of credit supply, [Gordon and Hedlund \(2017\)](#) the importance of financial aid and [Jones and Yang \(2016\)](#) the rising cost of universities input and professors—the Baumol’s disease, a mechanism our model accommodates—implied by the rise in the skill premium, to explain the rise in tuition. In this paper, I stress the role of the increase in the returns to education to explain the rise in the average and the dispersion of tuition fees, due to the increase in demand by households for higher quality of higher education, especially at the top of the distribution. This mechanism is akin to the revenue theory of cost by [Bowen \(1980\)](#), but applied to a framework with a ladder of colleges, whereby universities raise all the money they can through tuition fees and then spend it on projects that enhance quality. [Martin, Hill, and Waters \(2017\)](#) have estimated that this mechanism accounts for two third of the increase in the average real tuition fees. Regarding financial aid policy, [Abbott, Gallipoli, Meghir, and Violante \(2013\)](#) shows that current financial aid policy improves efficiency and increases GDP. Although we share the same deep source of inefficiency—a borrowing constraint—I focus on the misallocation of heterogeneous students across heterogeneous colleges rather than on the enrollment rate. A large reduced-form literature provides evidence on the returns to college quality and/or selectivity. While most of them find significant returns on the labor and marriage markets as well as for children’s achievements ([Black and Smith, 2006](#); [Long, 2010](#); [Hoekstra, 2009](#); [Zimmerman, 2014](#); [Bleemer, 2019](#)), some influential papers have cast doubt on these findings and the debate is still on-going ([Dale and Krueger, 2011](#); [Hickman and Mountjoy, 2019](#)). My results suggest moderate amplification effects of higher education. Another literature has shown that parental background matters a lot for achievements and access in top colleges ([Bailey and Dynarski, 2011](#); [Chetty, Friedman, Saez, Turner, and Yagan, 2019](#); [Hoxby and Turner, 2019](#)) and that financial aid policy has a significant impact on college decisions ([Dynarski, 2003](#); [Angrist, Autor, Hudson, and Pallais, 2016](#)).

The rest of the paper is organized as follows. Section 2 presents the model and section 3 explains the closed-form equilibrium expressions. Section 5 derives the key analytical comparative statics: an increase in the return to human capital generates facts (a) to (e). Section 2 explains the estimation procedure. Section 7 derives the quantitative results regarding the role of the higher education sector for the ampli-

fication of inequality and the reproduction of economic status over generation. Section 8 concludes.

2 Human Capital Transmission with a Hierarchy of Colleges

The economy is populated by two types of agents: dynastic households and colleges. At each generation, households imperfectly transmit human capital to their child and decide which college to send them to after high school. Colleges choose their pool of students as well as educational spending to maximize the quality they deliver.

2.1 Households

There is an infinity of dynasties, indexed by $i \in \mathcal{I}$. Individuals live for two periods: one as a child and one as an adult. Each adult has one child. A generation $t \in \mathbb{N}$ household of dynasty i is characterized by its level of human capital h_{it} and the child's human capital at the end of high school z_{it} . They choose consumption c_{it} , labor supply ℓ_{it} and college quality q_{it} for their child. When no confusion results, I drop the generation and dynasty subscripts and denote the state variables of the next generation with a prime. The current generation value $\mathcal{U}(b, z)$ is solution to

$$\mathcal{U}(b, z) = \max_{c, \ell, q} \left\{ (1 - \beta) [\ln c - \ell^\eta] + \beta E [\mathcal{U}(b', z')] \right\} \quad (1.1)$$

where β denotes the intergenerational discount factor. A child's human capital at the end of high school is modeled as a log-linear combination of parents' human capital b and the birth shock ξ_b , capturing the randomness of the transmission process.

$$z = (\xi_b b)^{\alpha_1} \quad \text{Child's High School Ability} \quad (1.2)$$

A household's lifetime earnings denoted y is a function of their level of human capital b , their supply of raw labor ℓ and the tax schedule. I describe the earning function in section 2.3.

Households are subject to a lifetime budget constraint. Their income y can be spent on consumption

and on tuition fees. The tuition schedule is an equilibrium object which depends on college quality q , household income y and the child ability z . Normalizing the price of the final good to one, it is given by

$$y = c + e(q, y, z) \quad \text{Household Lifetime Budget Constraint.} \quad (1.3)$$

This budget constraint implies that households face an intergenerational borrowing constraint, *i.e.* the current adults cannot leave bequest or pass-on debt along to their offspring. This assumption draws on a large set of evidence that borrowing constraints do matter for college choices. [Lochner and Monge-Naranjo \(2012\)](#) review the evidence on borrowing constraint in education. Although this specification rules out net financial transfers across generation, the quantitative version I introduce later partially relaxes this assumption.⁵ The adulthood human capital of the child after college is a log-linear combination of its pre-college ability, the quality of the college they went to and a labor market shock.⁶ It is given by

$$b' = zq^{\alpha_2}\xi_y \quad \text{Child's Post-College Human Capital} \quad (1.4)$$

There are two sources of randomness in the accumulation process of human capital. The birth shock ξ_b is known before the college quality decision has to be made, while the labor market shock ξ_y is realized once the child enters the labor market. It is assumed that the birth and labor market shocks are i.i.d across generations and households and log-normally distributed.⁷

$$\ln \xi_b \sim \text{i.i.d.} \mathcal{N}(\mu_b, \sigma_b^2) \quad \text{Birth Shock} \quad (1.5)$$

$$\ln \xi_y \sim \text{i.i.d.} \mathcal{N}(\mu_y, \sigma_y^2) \quad \text{Labor Market Shock} \quad (1.6)$$

⁵Ruling out net financial transfers across generations doesn't prevent gross flows to occur within a lifetime. For example, children are allowed to borrow from their parents early in life and repay later. It doesn't rule out student loans as long as they are exactly offset by a parental transfer of the same amount, *i.e.* student debt is possible as long as it is paid by parents.

⁶There is empirical evidence that the law of accumulation of human capital is characterized by complementarities between pre-college ability and college quality. [Dillon and Smith \(2018\)](#) finds evidence of such complementarities for long-term earnings. [Lee and Seshadri \(2019\)](#) estimate that the elasticity of substitution across periods of the human capital accumulation process is one, which amounts to a Cobb-Douglas functional form.

⁷This formulation of the household problem draws from and extends [Benabou \(2002\)](#). The latter can be seen as the case where there is no birth shock, $\sigma_b^2 = 0$ and a constant unitary price for education $e(q, y, z) = q$.

2.2 Colleges

Technology. There is a mass one of *ex-ante* identical colleges indexed by $j \in [0, 1]$. They are all of a fixed size, $\mathfrak{c}$.⁸ A college is a technology that delivers to its students a quality that depends on educational services per student I_j and the average of student ability $\bar{z}_j$, which will be referred to as the “peer effect.” Furthermore, I assume that quality depends negatively on the degree of dispersion of abilities and parental income within the college, $\sigma_{u,j}^2$, which I define later. The production function of quality is given by

$$\ln q_j = \ln I_j^{\omega_1} \bar{z}_j^{\omega_2} - \sigma_{u,j}^2$$

where $\omega_1, \omega_2 > 0$.

Colleges are clubs because who belongs to the college matters for the quality delivered to all members, through $\bar{z}$. There is empirical evidence that peers enter the production function of college quality. For example, [Sacerdote \(2011\)](#), [Smith and Stange \(2016\)](#) and [Mehta, Stinebrickner, and Stinebrickner \(2018\)](#) find evidence of peer-effects, especially from roommates, for achievements while in college. [Zimmerman \(2019\)](#) finds evidence that the network and social capital built while in college matters for labor market outcomes. Peer effects are also supported by the fact that colleges compete for the best students and seek to maximize $\bar{z}$ ([Hoxby, 2009, 2013](#)).

I make two assumptions about the negative impact of student heterogeneity on quality. First I assume that the peer-effects are a geometric average of student abilities which therefore punishes heterogeneity relative to an arithmetic average:

$$\ln \bar{z}_j = E_{\varphi_j(\cdot)}[\ln(z)]$$

where $\varphi_j(\cdot)$ denotes the distribution of student abilities within college j . Secondly, through $\sigma_{u,j}$, I explicitly assume that the more heterogeneous the class in terms of student ability and economic background the more difficult it is for a college to deliver a given quality to its students. I define $\sigma_{u,j}^2$ as the within-

⁸As I explained in appendix A.1.7, it is natural to set the size of a college to the cardinality of the continuum $\mathfrak{c} = \aleph_1$ as the paper analyzes equilibria in which there is a continuum of heterogeneous students in each college.

college variance of a weighted average of (log) ability and parental background, $\log z^{\frac{\omega_2}{\omega_1}} y^{-\frac{\omega_3}{\omega_1}}$:

$$\sigma_{u,j}^2 = \frac{\omega_1}{2} V_{\varphi(\cdot)} \left(\log z^{\frac{\omega_2}{\omega_1}} y^{-\frac{\omega_3}{\omega_1}} \right) \quad (1.7)$$

Defining $\sigma_{u,j}^2$ in this manner ensures tractability by making $I_j \times e^{-\sigma_{u,j}^2}$ a geometric average of tuition fees. The solution to this problem would therefore be the same if colleges maximized a weighted geometric average of tuition and student ability⁹

Educational services I_j are financed through the collection of tuition fees from all students. If p_I denotes the price of educational services, the static budget constraint of a college is

$$p_I I_j = E_{\varphi_j(\cdot)} [e_u(q, z, y)]$$

Objective and Problem. Taking the tuition schedule $e(q, z, y)$ and the price of educational services p_I as given, a college chooses the amount of educational services per student I_j and the composition of the student body $\varphi_j(z, y)$ —a density over (z, y) , which determines the average student ability $\bar{z}_j$ —to maximize the quality q_j they deliver to their students. For simplicity, asymmetries of information are assumed away and clubs have perfect information about the type of applicants (z, y) . Dropping the college subscript when no confusion results, the problem of a college is:

$$\max_{I, \bar{z}, \varphi(\cdot)} \mathcal{V} = q \quad (1.8)$$

$$\text{subject to: } \ln q = \ln I^{\omega_1} \bar{z}^{\omega_2} - \sigma_u^2 \quad \text{College Technology} \quad (1.9)$$

$$p_I I = E_{\varphi(\cdot)} [e_u(q, z, y)] \quad \text{Budget Constraint} \quad (1.10)$$

$$\ln \bar{z} = E_{\varphi(\cdot)} [\ln(z)] \quad \text{Average Student Ability} \quad (1.11)$$

The formulation of the college problem follows the literature that studies the behavior of universities (Fu, 2014; Epple, Romano, and Sieg, 2006). I depart from it by assuming that they behave competitively as in Cai and Heathcote (2018). While the latter paper assume colleges maximize profits, I maintain the

⁹From this perspective, the college's problem has a flavor of Fu (2014), where colleges maximize a weighted average of average student ability and a quadratic function of net tuition.

assumption that colleges maximize quality. It can be shown that the dual problem of maximizing profit subject to a constraint on $\mathcal{V}$ leads to the same first order conditions. However, when I allow colleges to have a social objective and implement need-based aid to student, it is not easy to interpret the nature of such problem.

Entry and Positioning Game. At each generation, before operating, colleges play a positioning game on the line of qualities.¹⁰ Taking the position of all other colleges as given, each college sequentially chooses which quality to offer, $q \in \mathbb{R}_+$. The order in which they choose is exogenous. Since colleges are otherwise identical, the order is arbitrary and inconsequential. The payoff for operating a given quality is given by (1.8) and is assumed to be $\mathcal{V} = 0$ if the college is not operating. A subgame perfect Nash equilibrium of this positioning game is a mapping from the set of colleges $j \in [0, 1]$ to the set of qualities $\mathbb{R}^+$ such that given the positioning of all other colleges, no college wants to change its position.

This structure for entry ensures that all positive qualities are offered in equilibrium. The assumption that a non-operating college gets $\mathcal{V} = 0$ sets the lower bound of the support of qualities offered in equilibrium, $q = 0$. The assumption that all colleges must be of size ϵ ensures that colleges do not agglomerate at the highest quality level with each one of them operating with an infinitely small mass of students.^{11,12}

2.3 Technologies for the Final Good and Educational Services

Apart from the college sector, there are two other sectors in the economy: the consumption good sector and the educational services sector. The consumption good is produced by households who operate their own production function and who sell their output on a competitive market at a price normalized to 1.

¹⁰In appendix A.1.7 I give a game-theoretic formalization of the positioning game.

¹¹Intuitively, if there were no lower bound to their size, all colleges would locate at the highest quality level and operate with virtually no students. In other words, all colleges would like to be Harvard but there is only one Harvard. To formalize this notion, I assume that colleges can't be too small and can't steal students from colleges that are already located. This forces colleges lower in the order to locate at qualities for which there are available students. In equilibrium this will imply that colleges will locate in decreasing order on the line of quality, with the first mover operating at the highest quality. More details are given A.1.7.

¹²One can see the positioning game with quality-maximizing colleges as the equivalent of the free-entry/non-profit condition with profit-maximizing colleges, like in Cai and Heathcote (2018). A key difference, however, is that contrary to a free-entry condition that equalizes payoff for all colleges, in a subgame perfect Nash equilibrium of the positioning game in my setup, colleges receive heterogeneous payoffs if (and only if) they offer different qualities. Again, all colleges would like to be Harvard, but there is room for only one Harvard (or maybe Princeton?).

The market earnings function is

$$y_m = Ab^\lambda \ell^\mu \quad \text{Household Market Income} \quad (1.12)$$

where A is an aggregate constant and $\lambda, \mu > 0$. The elasticity of income to human capital, λ will be called the “returns to human capital.” This parameter plays an important role in the rest of the paper. I argue in section 5 that an increase in λ is able to rationalize the trends observed in higher education and explained in introduction.

Although simple, this functional form is also the reduced-form expression of a more sophisticated production function with physical capital and/or the payoff to a household involved in an aggregate production process with some degree of complementarity across heterogeneous tasks. Educational services are produced using the final good as input. Each unit of final good gives A_I/A units of educational services: $y_I = \frac{A_I}{A} y_m$. Colleges buy services from the educational sector at relative price p_I . In equilibrium, it will be the case that $p_I = A/A_I$. Given that higher education is intensive in its use of high skilled labor, it is of interest to extend the analysis to this case. This would make the price of educational services endogenous to economy-wide conditions. In particular it would imply a positive relationship between the returns to human capital, λ , and the price of education services p_I , through the increase in the relative wages of faculty. I provide a tractable generalization along these lines in appendix A.1.3.¹³

2.4 Equilibrium

An equilibrium path is a sequence of tuition schedules, prices of educational services, household’s policy functions, colleges’ policy functions, a sorting rule, a distribution of human capital

$\{e_t(q, z, y), p_{I,t}, c_t(b, \xi_b), \ell_t(b, \xi_b), \varphi_t(q, y, z), I_t(q), q_t(j), q_t(b, \xi_b), f_t(b)\}_{t=0}^\infty$ such that i) given the sequence of prices, the household’s policy functions $c_t(b, \xi_b), \ell_t(b, \xi_b)$ are solution to (1.1), ii) the college’s policy functions $\varphi_t(q, y, z), I_t(q)$ are solution to (1.8), and the allocation of colleges along the quality line $q_t(j)$

¹³In this generalization, the production function of educational services is given by $y_I = A_I b^{\bar{\lambda}} \ell^\mu$ with $\bar{\lambda} \geq \lambda$, so that high human capital individuals have a comparative advantage in the educational sector. None of the key mechanisms emphasized in this paper depends on this generalization. And all key results are derived with the generalized version of the educational sector.

is a subgame perfect Nash equilibrium of the positioning game, iii) the demand for quality q from students of type (z, y) is matched by a supply for this type at that quality, iv) the final good market as well as the educational services market clear, v) the evolution of the distribution of human capital, $f_t(h)$, is consistent with the intergenerational law of motion of human capital and the sorting rule, $q_t(h, \xi_b)$.

3 Properties of the Decentralized Equilibrium

I construct an equilibrium in which the distribution of human capital is log-normal. A necessary and sufficient condition for this distribution to remain log-normal over generations is for the tuition schedule to be a log-linear function of college quality q , student ability z and parental income y . Given the assumptions laid out in the previous section, the unique tuition schedule compatible with the equilibrium conditions and colleges being in an interior solution is log-linear. These two restrictions—log-normality of human capital and interior solutions for colleges—ensure the tractability of the equilibrium expressions.¹⁴

3.1 Equilibrium Tuition Schedule

Consider a college that decides to supply quality q . It then has to choose the optimal combination of inputs—educational services I_j and the distribution of students' quality that are consistent with q . Given the substitutability between educational resources and student ability, a college will trade off lower tuition for higher student ability. The first-order conditions with respect to the density over student types and to the level of spending in the college's problem reflect this trade-off. The following proposition gives the unique equilibrium tuition schedule that is compatible with all colleges being at an interior solution. It takes a log-linear form and, incidentally, implies that all colleges are indifferent between all student types.¹⁵

¹⁴I cannot however rule out the existence of equilibria outside of this class.

¹⁵Although it is natural to focus on interior solutions, I cannot rule out the existence of other equilibria where tuition fees deviate from this log-linear expression and some colleges are at corner solutions for some student types. The real world tuition schedule does display kinks and a log-linear tuition schedule should be interpreted as a smooth approximation of reality. Although looking at a more general class of equilibria is potentially interesting, it is beyond the scope of this analysis and would defeat a key purpose of this paper, as all tractability would be lost.

Proposition 3.1. *The equilibrium before-financial-aid tuition schedule is given by*

$$e_{u,t}(q, z) = p_I q^{\frac{1}{\omega_1}} z^{-\frac{\omega_2}{\omega_1}} \quad (1.13)$$

and all colleges are indifferent between all types.

Tuition fees are increasing in quality q and decreasing in student ability z with respective elasticities of $\frac{1}{\omega_1}, \frac{\omega_2}{\omega_1}$. These elasticities are intuitive. Colleges of higher quality need to finance higher expenses, hence require higher tuition. If educational services are important for the production of college quality, ω_1 is high hence $\frac{1}{\omega_1}$ is low, tuition will not be very elastic to quality, because a small increase in revenues implies a large increase in quality. The elasticity $-\frac{\omega_2}{\omega_1}$ captures the importance of the peer-effect relative to educational spending: if peers significantly matter, colleges have strong incentives to subsidize high ability students to attract them.

3.2 Household Policy Functions

Given the equilibrium tuition schedule (1.26), households choose where to send their offspring. Since the tuition schedule is monotonic in q , this decision amounts to choosing how much of their income to spend on higher education. Define the spending rate of a household of type (z, y) going to college of quality q :

$$s_t(q, z, y) = \frac{e_t(q, z)}{y}.$$

An attractive feature of the class of models with unitary elasticity of intergenerational substitution, log-normal innovations and log-linear technologies is the possibility to obtain analytic expressions for the optimal spending rate and labor supply. The following proposition characterizes the solution to the F.O.Cs associated with the households' problem.

Proposition 3.2. *Defining $U = \frac{\partial \ln \mathcal{U}}{\partial \ln b}$, the elasticity of the value function to human capital, one has that, in equilibrium, for all households, the households' spending rate, labor supply and marginal value of human*

capital U are given by:

$$s_t = \frac{\beta \alpha_2 \omega_1 U_{t+1}}{1 - \beta + \beta \alpha_2 \omega_1 U_{t+1}} \quad (1.14)$$

$$\ell_t = \left[\frac{\mu}{\eta} \left(1 + \frac{\beta}{1 - \beta} \alpha_2 \omega_1 U_{t+1} \right) \right]^{\frac{1}{\eta}} \quad (1.15)$$

$$\text{with } U_t = (1 - \beta) \sum_{k=0}^{\infty} \beta^k \lambda_{t+k} \prod_{m=0}^{k-1} \alpha_{b,t+m} \quad (1.16)$$

$$\text{and } \alpha_{b,t} = \alpha_1 + \alpha_2 [\omega_2 \alpha_1 + \omega_1 \lambda_t]$$

where $\alpha_{b,t}$ is the Intergenerational Elasticity (IGE) of human capital at generation t . The spending rate and labor supply are independent of the household type and depends positively on U_{t+1} which is also common to all households. The latter depends positively on all current and future α_b 's, which is the IGE. The higher the future IGEs the more incentive the current generation has to invest in human capital and work. Importantly V_t —thus s_t —is also increasing in both the current and future returns to education— λ_t . It will play a key role in the dynamics of human capital afterwards.

3.3 Equilibrium Sorting Rule

By combining the equilibrium tuition schedule and the equilibrium positioning of colleges on the quality line—the “supply side”—with the household spending rule—the “demand side”—one obtains the equilibrium sorting rule, a mapping from the set of household and student types into the set of qualities of higher education.

Proposition 3.3. *In equilibrium, the sorting rule is given by*

$$q_t(y, z) = \left(\frac{s_t y}{p_I} \right)^{\omega_1} z^{\omega_2} \quad (1.17)$$

Equation (1.17) tells us which quality of higher education a student from family background y and ability z gets. The elasticity of quality to income and ability capture the strength of what I call, respectively, the income-sorting and ability-sorting channel. The income-sorting channel captures the fact that

richer households are able to buy a higher quality of college, not only because they are richer but also because colleges are ready to trade-off financial resources for ability. The ability-sorting channel captures the desire of colleges to attract high ability students because of the peer effect.

3.4 Educational Sector and Market Clearing

In the simple case considered here, the price of educational services is pinned down by the relative productivity parameter $p_I = A/A_I$. Market clearing then simply pins down the share of final good allocated to consumption and to the production of educational services. The more general and realistic case with a fully microfounded educational sector is described in detail in A.1.3.

3.5 Law of Motion of Human Capital

Having described the static equilibrium conditions, I now derive the law of motion for the distribution of human capital. Since the first two moments of this distribution are the only aggregate states, it also describes the dynamics of the aggregate economy. I start with the law of motion of human capital at the individual level.

Intergenerational Transmission of Status. Plugging the expression for the equilibrium sorting rule (1.17) into the law of accumulation of human capital (1.4) and gathering all terms in $\ln h$ gives the following intergenerational law of motion of human capital: $\ln h_{t+1} = \alpha_{b,t} \ln h_t + \ln \xi_y + (\alpha_1 + \alpha_2 \omega_2) \ln \xi_b + \ln \kappa + \alpha_2 x_t$ with $\alpha_{b,t}$ the intergenerational elasticity of human capital (IGE) and $x_t = \omega_1 \ln \left(\frac{s_t A_t \ell^2}{p_{I,t}} \right)$.

The IGE is a linear combination of the before, during and after college transmission of human capital. This paper focuses on and opens the box of the transmission of economic status through college. The transmission during college decomposes itself into the two sub-channels introduced in the previous paragraph: the income-sorting channel that emphasizes the role of parental income and the ability-sorting

channel that emphasizes the role of ability in the sorting of students across the ladder of college quality.

$$\alpha_{b,t} = \underbrace{\alpha_1}_{\text{Before Coll.}} + \underbrace{\alpha_2 \left(\underbrace{\alpha_1 \omega_2}_{\text{Ability-Sorting Channel}} + \underbrace{\omega_1 \lambda_t}_{\text{Income-Sorting Channel}} \right)}_{\text{College}}$$

Direct Transmission

Aggregate Law of Motion of Human Capital. Using the assumption of log-normality of both shocks, (1.5) and (1.6), if the economy starts from a log-normal distribution then human capital stays log-normally distributed along the equilibrium path:

Proposition 3.4. *If $\ln h_t \sim \mathcal{N}(m_{b,t}, \Sigma_{b,t}^2)$ then*

$$\ln h_{t+1} \sim \mathcal{N}(m_{b,t+1}, \Sigma_{b,t+1}^2) \quad (1.18)$$

$$m_{b,t+1} = \rho_t m_{b,t} + X_{1,t} \quad \text{Mean of (log) Human Capital} \quad (1.19)$$

$$\Sigma_{b,t+1}^2 = \alpha_{b,t}^2 \Sigma_{b,t}^2 + X_2 \quad \text{Variance of (log) Human Capital} \quad (1.20)$$

$$\text{where } \rho_t = \alpha_1 + \alpha_1 \alpha_2 \omega_2 + \alpha_2 \omega_1 \lambda_t$$

$$X_{1,t} = -\frac{\sigma_y^2}{2} + \ln \kappa - \alpha_1 (\alpha_2 \omega_2 + 1) \frac{\sigma_b^2}{2} + \alpha_2 \omega_1 \ln \left(A \ell_t^u \frac{s_t}{p_I} \right)$$

$$X_2 = \sigma_y^2 + (\alpha_1 [1 + \alpha_2 \omega_2])^2 \sigma_b^2.$$

It is intuitive that the shifter in the law of motion of the mean of the distribution (1.29) is increasing in the spending rate s_t , labor supply ℓ_t , but decreasing in the price of educational services p_I . The law of motion of the variance (1.30) is the mathematical expression of the Great Gatsby curve: the positive relationship between the level of inequality Σ_b and the strength of the intergenerational transmission of status, α_b .

The law of motion of Σ_b , given by (1.30), is an auto-regressive process of order 1. The expression (1.29) is also auto-regressive of order 1 since the paths of s and ℓ are only functions of the parameters and all the future λ 's. The full system, (1.29) and (1.30), is therefore recursive which allows us to characterize the existence and uniqueness of the equilibrium path.

Proposition 3.5 (Existence and Uniqueness of Equilibrium Path). *Within the class of equilibria with an initial log-normal distribution of human capital, there exists a unique globally stable steady-state and a unique equilibrium path.*

3.6 Distribution of Students along the Quality Ladder and Within-College Distribution of Students

Recall facts (b) and (c) noted in introduction: the dispersion of expenditures per student across colleges has increased and the share of low-income students at top colleges has stagnated. One can actually derive analytical expressions for the distribution of students across college qualities (and the implied distribution of expenditures) and for the within-college distributions of parental income and student ability. These closed-form solutions enable us to shed light on the forces that determine these two objects and will prove useful for the derivation of comparative statics in the next section. These three distributions are log-normal and their first and second moments depend on the aggregate states, directly and indirectly through the income-sorting and ability-sorting elasticities,

$$\varepsilon_{I,t} = \omega_1 \lambda_t \quad \text{and} \quad \varepsilon_A = \omega_2 \alpha_1.$$

As the following proposition establishes, the dispersion of qualities is an increasing function of both of these variables. But the dispersion of parental income and ability within a college is a function of their ratio. The former is increasing with the ratio $\varepsilon_A/\varepsilon_I$ while the latter is decreasing: the more students sort into colleges based on parental income, the less economic diversity there is in a college and the more students sort into colleges based on abilities, the lower the dispersion of abilities.

Proposition 3.6. *I. The distribution of college quality is given by*

$$\ln q \sim \mathcal{N} \left(\mu_{1,t}(m_{b,t}, \Sigma_{b,t}), \sigma_{1,t}^2(\Sigma_{b,t}, \varepsilon_{I,t}, \varepsilon_A) \right)$$

and $\sigma_{1,t}$ is increasing in ε_A , $\varepsilon_{I,t}$ and $\Sigma_{b,t}$.

2. *Within a college of quality q , the distribution of parents' (log) income is given by:*

$$\ln b|q \sim \mathcal{N}\left(\mu_{2,t}(m_{b,t}, \Sigma_{b,t}), \sigma_{2,t}^2(\Sigma_{b,t}, \varepsilon_{I,t}, \varepsilon_A)\right)$$

with μ_2 increasing in q and $\sigma_{2,t}$ increasing in ε_A and $\Sigma_{b,t}$ but decreasing in $\varepsilon_{I,t}$.

3. *And the distribution of students' (log) abilities is given by*

$$\ln z|q \sim \mathcal{N}\left(\mu_{3,t}(m_{b,t}, \Sigma_{b,t}), \sigma_{3,t}^2(\Sigma_{b,t}, \varepsilon_{I,t}, \varepsilon_A)\right)$$

with $\mu_{3,t}$ increasing in q and $\sigma_{3,t}$ increasing in $\varepsilon_{I,t}$ and $\Sigma_{b,t}$ but decreasing in ε_A .

4 Taxes, Transfers and Financial Aid in Higher Education

In this section, I introduce a government which implements non-linear transfers of income across households and provides merit and need-based financial aid to students as well as subsidies to colleges. I also allow colleges to provide need-based aid by assuming they have a social objective. I use log-linear tax and transfer schedules as introduced by [Persson \(1983\)](#) and [Benabou \(2002\)](#) and estimated more recently by [Heathcote, Storesletten, and Violante \(2017\)](#). They fit well the empirical schedules and they preserve the tractability of the framework introduced in the previous section.

4.1 Government

The government implements four kind of taxes: two are specific to higher education (non-linear merit-based and need-based financial aid to college students and non-linear transfers to colleges) and two that are more standard (a linear consumption tax and a progressive income tax).

Progressive Income Tax Schedule The household labor income is subject to a progressive tax schedule with a_y the average tax rate and τ_y its progressivity. The after-tax and transfers lifetime earnings is given

by

$$y = (1 - a_y)y_m^{1-\tau_y} T_y \quad \text{Household After-Tax Income} \quad (1.21)$$

where T_y is a normalizing aggregate endogenous factor ensuring that a_y parametrizes the average income tax rate. The non-linear schedules for financial aid and the college subsidy are in the same spirit as this income tax schedule.

Merit and Need-Based Financial Aid Financial aid is allowed to be progressive with income and regressive with abilities:

$$e(q, z, y) = T_e z^{-\tau_m} y^{\tau_n} \frac{e_u(q, z, y)}{(1 + a_b)} \quad \text{Net Tuition} \quad (1.22)$$

where $e(q, z, y)$ is the after financial net tuition faced by households, as specified in (1.3) and $e_u(q, z, y)$ is the before financial aid price, commonly referred to as the sticker price. τ_m is the rate of progressivity (or rather regressivity) of the merit-based subsidy, τ_n is the rate of progressivity of the need-based subsidy and T_e ensures that a_b is the average financial aid to students.

Transfers to Colleges Financial transfers to colleges by states and the federal government are large and highly progressive, as is documented in a companion paper [Capelle \(2019\)](#). Colleges that spend less per student receive relatively more subsidies.¹⁶ This progressivity is closely related to the location of public and private colleges in the distribution of quality. Most papers modeling the higher education sector differentiate between public and private colleges. In contrast, I do not specify any *ex ante* differences across colleges.¹⁷ In my model, the bottom and middle of the distribution of qualities, *i.e.* the colleges

¹⁶The notion of progressivity used here doesn't refer to the progressivity with respect to the average students parental income that populate these colleges. Even if students from richer families are more likely to be in high revenue colleges, it might be that overall these transfers are regressive since many children from low-income background do not enroll in college—a mechanism I abstract from in this version of the model but allow for in the quantitative version of the model presented in section 7.

¹⁷There is little agreement in the literature about what really differentiates public colleges' objectives and constraints from non-profit private ones, apart from the fact that the former receive public subsidies but not the latter. Most papers assume that tuition fees at public universities are subject to specific constraints. For example, [Epple, Romano, and Sieg \(2006\)](#) and [Cai and Heathcote \(2018\)](#) assume that tuition fees at public colleges are exogenous. This corresponds to the notion that tuition fees

that receive relatively more transfers from the government, can be interpreted as public colleges. This way of modeling government transfers allows me to keep the model tractable while capturing most of the heterogeneity in government transfers along the quality distribution. Taking into account these transfers, the budget constraint of a college is:

$$p_I I = T_u(1 + a_u) \left(E_{z,y}[e_u(q, z, y)] \right)^{1-\tau_u} \quad \text{Colleges After-Transfer Revenues} \quad (1.23)$$

where τ_u is the degree of progressivity of subsidies to universities and T_u ensures that a_u is the average amount of transfers per student received by colleges. The budget constraint presented in the college problem, (1.10), is the special case when $\tau_u = 0$. I show in [Capelle \(2019\)](#) that the functional form assumption in (1.23) is a good approximation of the data.

Government Budget Constraints. There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any period. The other three constraints pin down T_u , T_y , T_e such that a_y , a_b , a_u are respectively the average rate of income tax, financial aid and transfers to college. I give more details in appendix A.1.3.

4.2 College Need-based Aid and Social Objective

I now allow colleges to give need-based aid. There is indeed evidence that colleges do discriminate tuition fees based on parental income ([Epple, Romano, and Sieg, 2006](#)). To do so, I assume that colleges have a social objective. There is direct evidence for this social objective: the claimed and (growing) public effort of private and public universities to recruit low-income students. The social objective is modeled by a

are fixed by States' legislatures. But most States have been decentralizing and deregulating tuition policies. Public colleges now have significant autonomy in their tuition and hiring policies ([Mc Guinness, 2011](#)). (And even in the States where legislatures still have a lot of power, it is not clear that their objective would be radically different than maximizing the quality delivered.) For-profit colleges do display different behavior, but they make up a very small part of total enrollment.

penalty that increases in the (geometric) average of parental incomes, $\bar{y}_j$ and parametrized by ω_3 ¹⁸

$$\ln \mathcal{V}_j = \ln q_j - \omega_3 \ln \bar{y}_j \quad (1.24)$$

where the $\bar{y}_j$ is the geometric average parental income of students:

$$\ln \bar{y}_j = E_{\varphi_j(\cdot)}[\ln(\gamma)] \quad \text{Average Parental Income} \quad (1.25)$$

A college maximizes (1.24) subject to the technology for quality (1.9), the definition of the peer-effect (1.11), the after-subsidy budget constraint (1.23) and the definition of average parental income (1.25).

4.3 Properties of the Decentralized Equilibrium

Equilibrium Tuition Schedule. In this generalized framework, the log-linear form of the tuition schedule is preserved.

Proposition 4.1. *The equilibrium before-financial-aid tuition schedule is given by*

$$e_{u,t}(q, z, \gamma) = \left(\frac{p_{I,t}}{(1 + a_{u,t})T_{u,t}} q^{\frac{1}{\varepsilon_{1,t}}} z^{-\frac{\varepsilon_{2,t}}{\varepsilon_{1,t}}} \left(\frac{\gamma}{\kappa_{2,t}} \right)^{\frac{\varepsilon_{3,t}}{\varepsilon_{1,t}}} \right)^{\frac{1}{1-\tau_{u,t}}} \quad (1.26)$$

$$\text{where } \varepsilon_{l,t} = \frac{\omega_l}{1 - v_t(\Sigma_{b,t})\omega_3} \quad \forall l = 1, 2, 3$$

with $v_t(\Sigma_t)$ the elasticity of mean parental income within a college to quality

$$\bar{y}_t(q) = \kappa_{2,t} q^{v_t(\Sigma_{b,t})}$$

and all colleges are indifferent between all types.

There are three new elements relative to the previous section. First, tuition are increasing in parental income because of the social objective. The elasticity $\frac{\varepsilon_3}{\varepsilon_1(1-\tau_u)} = \frac{\omega_3}{\omega_1(1-\tau_u)}$, depends on the strength of the

¹⁸Colleges give need-based to students in [Epple, Romano, and Sieg \(2006\)](#) not because of a social objective but because of parents with higher income are less elastic to prices and therefore higher mark-up. Colleges do not discriminate by parental income in [Cai and Heathcote \(2018\)](#).

social objective: the larger ω_3 , the more progressive tuition fees are.¹⁹ Secondly, tuition decreases with the average subsidies to colleges a_u , and the elasticity of tuition fees with quality is increasing in the degree of progressivity of the college subsidy schedule, τ_u .

Thirdly, the elasticities of tuition with respect to quality, student ability and parental income— $\varepsilon_{1,t}$, $\varepsilon_{2,t}$, $\varepsilon_{3,t}$ —are equilibrium objects that depend on current aggregate states, in particular the dispersion of human capital in the economy Σ_b , and the policy parameters. Mathematical expressions for ν_t and $\kappa_{2,t}$ are given in appendix A.1.4. $\kappa_{2,t}$ depends not only on current states but also on all future states through the labor supply decision ℓ_t . The notation $\nu_t(\Sigma_{b,t})$ makes explicit that the elasticity of mean parental income to quality depends on the dispersion of human capital in the economy. As I show in appendix A.1.9, it is increasing in the latter. It also depends on the current policy parameters and λ the returns to human capital. Note that when colleges have no social objective and only maximize quality, $\omega_3 = 0$, then $\varepsilon_l = \omega_l$ for all $l = 1, 2, 3$ and the ω 's are independent of the state of the economy.

The term $\frac{1}{1-\nu_t(\Sigma_{b,t})\omega_3}$ that transforms ω_1 into ε_1 reflects the cross-subsidization from high-income to low-income families within a college implied by the social objective. Tuition fees for a family with a given income y increase with a lower elasticity with respect to quality when colleges have a social objective. This family becomes poorer and poorer relative to the within-college mean parental income as one climbs the college quality ladder (since parental income increases in equilibrium with quality). This effect is all the more pronounced as the social objective parameter ω_3 and the equilibrium elasticity of parental income to college quality ν are large.²⁰

Household Policy Functions and Sorting Rule For conciseness, and because it is very similar to its expression in the previous section, the equilibrium spending rate of households is given by equation (A.4) in appendix A.1.1. Combining the household decision with the equilibrium tuition schedule gives the equilibrium sorting rule.

¹⁹The equilibrium tuition function turns out to be similar to the one in [Epple, Romano, and Sieg \(2006\)](#). While the progressivity of tuition fees with parental income originates from market power in their framework, it comes from the social objective in this paper.

²⁰If inequality increases for exogenous reasons—as will be the case in our comparative statics with respect to the returns to education λ —the endogenous increase in ν provides a partial mitigating force by making colleges willing to endogenously redistribute more across students, provided $\omega_3 > 0$.

Proposition 4.2. *In equilibrium, the sorting rule is given by*

$$q_t = \left(\frac{s_t y_t^{1-\tau_n, t} z_t^{\tau_m, t} (1 + a_{b, t})}{T_{e, t}} \right)^{\varepsilon_{1, t} (1 - \tau_{u, t})} \left(\frac{(1 + a_{u, t}) T_{u, t}}{p_{I, t}} \right)^{\varepsilon_{1, t}} z_t^{\varepsilon_{2, t}} \left(\frac{y_t}{\kappa_{2, t}} \right)^{-\varepsilon_{3, t}} \quad (1.27)$$

The elasticity of quality to income and ability which capture the strength of the income-sorting and ability-sorting channel now captures the progressivity of taxes and financial aid:

$$\begin{aligned} \frac{\varepsilon_{I, t}}{(1 - \tau_y) \lambda_t} &= \varepsilon_{1, t} (1 - \tau_u) (1 - \tau_n) - \varepsilon_{3, t} && \text{Elasticity to Income} \\ \frac{\varepsilon_{A, t}}{\alpha_1} &= \varepsilon_{2, t} + \tau_m (1 - \tau_u) \varepsilon_{1, t} && \text{Elasticity to Ability} \end{aligned}$$

Relative to the framework without government intervention and a social objective for college, the income-sorting channel is tempered by government subsidies to colleges that are progressive with slope $(1 - \tau_u)$, by need-based financial aid that are progressive with slope τ_n , financial aid by colleges that is progressive with slope ω_3 . In theory, this elasticity, ε_I , could be negative, if the social objective parameter, ω_3 , was large enough, such that $\varepsilon_{1, t} (1 - \tau_u) (1 - \tau_n) < \varepsilon_{3, t}$. The elasticity with respect to ability—associated with the ability-sorting channel—is amplified by the merit-based subsidy of the government, τ_m .

Intergenerational Transmission of Status. The IGE is now given by

$$\underbrace{\underbrace{\alpha_1}_{\text{Before Coll.}} + \underbrace{\alpha_3}_{\text{After Coll.}}}_{\text{Direct Transmission}} + \underbrace{\alpha_2 \left(\underbrace{\alpha_1 (\varepsilon_{2, t} + \varepsilon_{1, t} (1 - \tau_{u, t}) \tau_{m, t})}_{\text{Ability-Sorting Channel}} + \underbrace{(\varepsilon_{1, t} (1 - \tau_{u, t}) (1 - \tau_{n, t}) - \varepsilon_{3, t}) (1 - \tau_{y, t}) \lambda_t}_{\text{Income-Sorting Channel}} \right)}_{\text{College}}$$

Aggregate Law of Motion of Human Capital. Like in the simpler version of the model, human capital remains log-normally distributed over the equilibrium path:

Proposition 4.3. *If $\ln h_t \sim \mathcal{N}(m_{b,t}, \Sigma_{b,t}^2)$ then*

$$\ln h_{t+1} \sim \mathcal{N}(m_{b,t+1}, \Sigma_{b,t+1}^2) \quad (1.28)$$

$$m_{b,t+1} = \rho_t m_{b,t} + X_1(m_{b,t}, \{\Sigma_b\}_{s=t}^\infty) \quad \text{Mean of (log) Human Capital} \quad (1.29)$$

$$\Sigma_{b,t+1}^2 = (\alpha_{b,t}(\Sigma_{b,t}))^2 \Sigma_{b,t}^2 + X_{2,t}(\Sigma_{b,t}) \quad \text{Variance of (log) Human Capital} \quad (1.30)$$

where ρ_t has the same expression as in the previous section. Expressions for $X_1(m_{b,t}, \{\Sigma_b\}_{s=t}^\infty)$ and $X_{2,t}(\Sigma_{b,t})$ can be found in appendix A.1.5.

In general the expression (1.29) is not a linear recursive formulation for the law of motion of m_b because s and ℓ are forward looking variables that depend on all the future Σ_b 's via the ε 's. In contrast, the law of motion of Σ_b , given by (1.30), is still recursive—although in general not linear since both the autoregressive coefficient and the shifter depend on Σ_b . The full system, (1.29) and (1.30), is therefore block-recursive which allows us to characterize the existence and uniqueness of the equilibrium path after the exposition of the government budget constraints.

Government Budget, Educational Sector and Market Clearing The aggregate government budget constraint (A.8) imposes, in all periods, a restriction on the path of the consumption tax rate $a_{c,t}$ given an exogenous path of income tax $a_{y,t}$, higher education subsidies $a_{b,t}$, $a_{u,t}$ and endogenous spending rate s_t . Analytical expressions for this constraint as well as for equations (A.9), (A.10) and (A.11) defining respectively $T_{y,t}$, $T_{e,t}$ and $T_{u,t}$ are derived in appendix A.1.3.

Existence and Uniqueness of the Equilibrium Path Existence and uniqueness of the steady-state and of the macroeconomic equilibrium path are slightly harder to obtain than in the previous section. Although existence and local stability is obtained under an intuitive sufficient condition, a sufficient condition for global stability is that ω_3 be small enough.

Proposition 4.4. *Existence and Uniqueness of Equilibrium Path*

- *If $\lim_{\Sigma_b \rightarrow \infty} \alpha_b(\Sigma_b) < 1$, there exists at least one locally stable steady-state.*

- For ω_3 small enough, there exists a unique globally stable steady-state and a unique equilibrium path.

with $\lim_{\Sigma_b^2 \rightarrow \infty} \alpha_b(\Sigma_b) = \alpha_1 + \alpha_1 \alpha_2 (\omega_2 + \tau_m(1 - \tau_u)\omega_1) + \alpha_2 [\omega_1(1 - \tau_u)(1 - \tau_n) - \omega_3](1 - \tau_y)\lambda$

A high ω_3 might lead to multiple equilibria by making inequality Σ_b potentially grow too fast in some parts of the state-space, *i.e.* by making the derivative of the right-hand-side of (1.30) higher than 1, thus failing to meet the crucial defining feature of a contraction mapping. This stems from the fact that ν is increasing in Σ_b , hence that ε_l for $l = 1, 2, 3$, α_b and X_2 are increasing in Σ_b .

5 Rationalizing Recent Trends in Higher Education

This section derives what I consider to be the most important analytical result of the paper. The increase in the market returns to education λ is able to generate (a) the increase in income inequality (Piketty and Saez, 2003; Autor, Katz, and Kearney, 2008); (b) the increase in the dispersion of expenditures per students across colleges (Capelle, 2019); (c) the stagnation of the share of students from the lowest income quintile in top colleges despite the increase in financial aid (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2019); (d) the increase in real terms of tuition fees before and after financial aid; and (e) the slight increase in the intergenerational elasticity of income mobility (Davis and Mazumder, 2017). It is natural to focus on the increase in the returns to education as it is widely recognized to be one of the main sources of the increase in inequality (Katz and Murphy, 1992; Autor, Katz, and Kearney, 2008).²¹ I also show that a decline in the progressivity of government subsidies to colleges—an observed feature of the data—is able to rationalize the same facts. However, the quantitative analysis in section 7 suggests that this change would fail to match the large increase in income inequality observed in the data.

5.1 An Increase in the Returns to Human Capital

The following proposition formally states the key comparative static result.

²¹ I do not take a stand on the exact source of increase in the returns to human capital. Many factors have contributed to this rise: a skill-biased technological change, capital-skill complementarity, an improvement in the assortative matching of workers and firms, an increase in assortative mating and in the number of single households and an increase in the substitutability across skills due to international trade or due to better communication technology.

Proposition 5.1. *Assume the economy starts from a steady-state at $t = 0$. Consider a weakly increasing sequence $\{\lambda_t\}_0^{+\infty}$. If $\omega_1(1 - \tau_n)(1 - \tau_u) > \omega_3$, along the equilibrium path,*

- a) The Gini coefficient of human capital and income increase.*
- b) The Gini coefficient of colleges' (log) expenditures per student (and quality) increase.*
- c) The ratio of variance of (log) income within a college over variance of (log) income in economy decreases.*
- d) The intergenerational elasticity increases.*
- e) The average expenditure for college as a share of income increases.*

The formal proof of this proposition is contained in appendix A.1.9. Here I present intuition for the stated effects.

Intuitively, when the returns to human capital, λ , increase, the dispersion of households' income rises for a given distribution of human capital [fact (a)]. Given that households all spend the same share of their income for the higher education of their child, it implies an increase in the dispersion of desires to pay for college. Following this change on the demand side of the higher education market, colleges react: top colleges take advantage of the rising willingness to pay of their pool of students by increasing their fees and their spending relative to colleges at the bottom. Inequality of revenues and spending across colleges rise [fact (b)].

Poor but high ability students get priced out of top colleges for two reasons. First tuition fees at top colleges have increased relative to lower ranked colleges. Second their parents' income have decreased relative to the average parental income. More generally this rise in the dispersion of tuition for colleges implies that parental income matters even more to access a higher quality college than it used to, relative to ability. It corresponds to an increase in the elasticity of college quality to income, ε_I , what I described earlier as a strengthening of the income-sorting channel.

Consequently, top colleges become less diverse in terms of economic background because poor students are priced out and students from rich families are able buy their way to the top. More generally,

colleges become more segregated and homogeneous in terms of parental income [fact (c)]. Another implication is that intergenerational mobility decreases, as parental income becomes increasingly determinant for the opportunities of children [fact (e)]. This is a direct manifestation of the Great Gatsby curve (Corak, 2013), whereby an increase in income inequality leads to a strengthening of the transmission of economic status, here through access to better quality higher education, which feeds back into higher inequality.

Overtime, the initial increase in inequality gets amplified through the higher education system. Students from richer backgrounds get relatively higher quality of higher education, which increases the dispersion of human capital and therefore of income once their generation become adults. The shock propagates over generations as this increased dispersion of human capital translates into a higher dispersion of children abilities which gets amplified by the increasingly unequal distribution of college quality.²²

The amplification of the initial increase in the returns to human capital, λ , through colleges happens through two channels: the reallocation of resources and the reallocation of students. As I have argued above, financial resources and expenditures become increasingly concentrated at the top of the college distribution. In contrast, high ability students become slightly less concentrated at the top of the college ladder, partially mitigating the amplification.

Why do colleges accommodate the increased dispersion in desires to pay for colleges? They are led to do so by their desire to maximize the quality they provide, despite their social objective. Even if an individual college at the top of the distribution didn't raise its tuition fees relative to, say, the median college, another college would fill up this gap, offering higher quality for higher tuition fees. This mechanism is akin to the revenue theory of cost by Bowen (1980), but now applied to a hierarchy of colleges.²³

Finally, average tuition fees and the share of total income devoted to higher education increase because

²²The quantitative section provides a structural decomposition of the increase in the Gini coefficient and in the IGE into the direct impact and the amplification through the higher education system.

²³Bowen summarizes his theory page 19:

1) The dominant goals of institutions are educational excellence, prestige, and influence. 2) In quest of excellence, prestige, and influence, there is virtually no limit to the amount of money an institution could spend for seemingly fruitful educational ends. 3) Each institution raises all the money it can. 4) Each institution spends all it raises. 5) The cumulative effect of the preceding four laws is toward ever increasing expenditure.

higher returns to human capital gives stronger incentives to households to accumulate human capital which drives their demand for higher education up [fact (d)]. It is therefore the same demand-driven mechanism that drives both the average increase in tuition and the rise in inequality across colleges.

5.2 A Decrease in Public Transfers Progressivity across Colleges

In a companion paper [Capelle \(2019\)](#), I have documented the large decline in the average rate of government subsidy to colleges, a_u , as well as in its progressivity, τ_u . In this section, I explain intuitively why a weakly decreasing sequence $\{\tau_{u,t}\}_0^\infty$ has the same qualitative effects as an increasing sequence $\{\lambda_t\}_0^\infty$ as stated in proposition 5.1.²⁴

For a given distribution of tuition fees across colleges, a decrease in the progressivity of public subsidies, τ_u , leads to an increase in the dispersion of financial resources and therefore of quality across colleges [fact (b)]. Mechanically, because of this decline in the progressivity of public subsidies, the college quality ladder becomes steeper and the sensitivity of the quality of higher education received by a student to the income of their parents increases. Moreover, the decline in τ_u implies that, from the point of view of colleges, the marginal productivity of tuition fees in terms of quality has increased relative to students ability. This gives incentives to colleges to target students with a higher desire to pay, at the expenses of high ability students. Overall, this implies a strengthening of the income-sorting channel.²⁵

Like in the case of an increase in λ , top colleges become less diverse in terms of family background and colleges become more segregated by parental income [fact (c)]. Intergenerational mobility decreases, as parental income become increasingly determinant for the opportunities of children [fact (d)]. The initial change in the allocation of students across colleges gets amplified over generations: because they experienced a more unequal distribution of higher education, the next generation of households is more unequal in terms of human capital and therefore income. This translates into higher inequality of abilities of their children, and so on and so forth... The economy gradually shifts rightwards (higher inequality, lower mobility) along the Great Gatsby curve.

²⁴The formal proposition and proof are given in appendix A.1.9.

²⁵Formally, it is easy to see that it also strengthens the ability-sorting channel—by increasing the impact of merit-based subsidy.

Finally, the decrease in τ_u incentives households to invest more in higher education, since the elasticity of quality to tuition has risen, which leads to an increase in the average spending rate s and in average tuition.

6 Taking the Model to the Data

In the previous sections I developed a tractable model of human capital accumulation with a ladder of colleges which allows for a sharp analytical characterization of the equilibrium sorting of students across colleges, income inequality, intergenerational mobility and aggregate production. The next two sections are devoted to the quantitative assessment of the role of higher education in shaping inequality and intergenerational mobility. This section has two goals. First I relax some of the assumptions I adopted for tractability in the previous sections and extend the model to a richer quantitative environment. Second I explain how I estimate this richer model, which I then use in section 7 to assess the quantitative relevance of the higher education system.

6.1 Quantitative Extension

I extend the model in two dimensions. First the restrictions on intergenerational financial transfers are partially relaxed: negative transfers up to a limit are allowed (student debt not offset by parental transfers) as well as positive transfers (bequests):

$$\mathcal{U}(b, z, a) = \max_{c, \ell, q, a'} \{ (1 - \beta) [\ln c - \ell^\eta] + \beta E(\mathcal{U}(b', z', a')) \} \quad (1.31)$$

$$y + e^{rH}a = c(1 + a_c) + e(q, y, z) + a' \quad (1.32)$$

$$a' \geq \underline{a} \quad (1.33)$$

where e^{rH} , r , H denote respectively the “generational” rate of return, the annual rate of interest and generation length and $\underline{a}$ is the exogenous borrowing limit.

Households therefore face a portfolio problem: they have to decide upon the optimal combination

of bequest and higher education for their offspring. High ability children from a poor background will take up loans and rich families with low ability children will choose to transmit financial wealth instead of buying a high quality college for their kid. Overall, allowing for financial transfers should weaken the link between parental income and the child's position on the college ladder.

Secondly, there is an outside option delivering $\underline{q}$ for free:

$$e(\underline{q}, y, z) = 0 \quad \forall (z, y) \quad (1.34)$$

Some individuals will find it optimal not to go to college and take up the free outside option. This gives rise to a meaningful enrollment decision that was absent from the previous framework where all individuals got at least some arbitrarily low quality of higher education. A direct implication of equation (1.34) is that, if $\underline{q} > 0$, in equilibrium no individual ever chooses $q < \underline{q}$ and there is a Dirac peak at $\underline{q}$. It is natural to define the enrollment rate as the share of individuals with $q > \underline{q}$.

The set of technological constraints faced by the household is otherwise similar to the original problem described in section 2. Formally, the problem of the household consists in maximizing (1.31) subject to (1.12), (1.21), (1.2)-(1.6) and the new constraints (1.32)-(1.34). The rest of the model remains the same.²⁶ The original problem is the special case when $a' = \underline{q} = 0$.

For future reference, I call M1 the model studied in sections 2 and 3 and M2 the augmented model described in this section. In the remainder of the paper, and unless otherwise stated, the expression “income Gini coefficient” refers to the “labor earnings Gini coefficient”, the key object of interest of our analysis.

6.2 Data

The core dataset is the restricted-use version of the NLSY-1997, a representative panel of individuals who were 12 to 17 years-old in 1997, whom I follow every year up to now. It features data on parental income, abilities measured by a common comprehensive test-score, the Armed Services Vocational Aptitude Bat-

²⁶I explain how the problem of the colleges is kept tractable in this more complicated framework in appendix A.2.1.

tery or ASVAB, a detailed description of their journey through the higher education system—each college they attended, the time spent and the degree obtained—and their labor earnings.

To estimate the parameters related to financial aid, I use the restricted-use NCES-NPSAS in 2000, which is the closest survey to the average year when individuals in the NLSY go to college. It is a representative survey of students that features detailed information about parental income, out-of-pocket college costs and financial aid disaggregated by source—federal government, state, private and institutional.

The publicly available NCES-IPEDS annual surveys provide college-level information on expenditures, revenues, enrollment and the distribution of test scores within each college. I use the 2000 to 2004 surveys. Finally I complement these data with statistics on enrollments from the NCES and measures of aggregate spending for higher education from the OECD.

6.3 External Calibration

The full list of the nineteen parameters that need to be calibrated is given in the first column of table 1.1. Seven of them I set without solving the model while the remaining twelve are calibrated solving it. I provide here a quick overview of the procedure for the former. See appendix A.2.2 provides more details.

The income tax schedule parameters a_y, τ_y are informed by the average income tax rate and the slope of the income tax schedule estimated by [Heathcote, Storesletten, and Violante \(2017\)](#). In a companion paper, I estimate the average per-student state transfers to college, a_u , and the degree of progressivity of these transfers τ_u ([Capelle, 2019](#)). I use the average financial aid received by students to calibrate a_b .

I use estimates of the Frisch elasticity of labor supply from the literature to calibrate η ([Chetty, Guren, Manoli, and Weber, 2011](#)). The elasticity of substitution in the educational services sector is set to $\bar{\lambda} = \lambda$ so that the price of educational services is given by $p_I = \frac{1}{A_I}$. The generation length is set to $H = 30$ years. The lower limit, $\underline{a}$ is set to match the official borrowing limit for student loan.

6.4 Internal Calibration

The algorithm used to estimate the parameters is akin to a Simulated Method of Moments. The results are reported in the fourth column of table 1.1.

I now make a heuristic identification argument that justifies the choice of moments used in the estimation. Although no parameter can be identified out of a single moment, I stress in this section which moment is important for each parameter. Thanks to the closed-form expressions of these moments in terms of structural parameters in M1, it is possible to formalize this argument, which I do in appendix A.2.3.²⁷

Assume for a moment that one perfectly observes child ability and college quality $\{z_i, q_i\}_i$. I first estimate the financial aid schedule (1.22), and use the elasticity of government financial aid to parental income and to students ability to inform the slopes of the financial aid schedule, which pins down τ_n and τ_m . I estimate the tuition schedule by running a regression of before-government-aid tuition fees on a college fixed-effect, ability and parental income. I use the elasticity with respect to parental income to inform the social objective parameter ω_3 . I then estimate the sorting rule by running a regression of college quality on students ability and parental income. The elasticity of college quality to students ability has a first-order effect on the peer-effect parameter ω_2 .

I then estimate the human capital accumulation function (1.4), and the market earnings function (1.12), to recover $\alpha_1, \alpha_2, \alpha_3$ and the returns to human capital λ . I thus run a regression of (log) child earning on (log) ability, (log) college quality and (log) parental income. The elasticity of a child's income to their ability identifies λ . Conditional on λ , the elasticity of child's income to college quality (resp. parental income) identifies α_2 (resp. α_3). Similarly, the elasticity of child's income to parental income identifies α_3 . Finally, I use the IGE of income to inform α_1 .

I have assumed so far that child ability and college quality were observable. However, they are not. I need to construct them by combining observable data and restrictions implied by the model. I first explain how I construct student ability. All children in the NLSY take the same test in high school. I assume that the resulting test scores are ranked in the same order as ability, z . Conditional on a (α_1, λ) it is possible to show that the correlation between the rank of the test scores and parental income $\text{corr}(\text{rank}(z), y)$ identifies the variance of the birth shock, σ_b^2 . I then generate model-consistent abilities that have the

²⁷ Another advantage of the closed-form expressions in M1 is the ability to investigate the invertibility of the model, given a set of targeted moments. It is possible to show that if abilities are directly observable and that there is no social objective $\omega_3 = 0$, then the parameters are exactly identified. Although child's ability z are non-observable, it helps build confidence in the identification of the parameters in my procedure. The proposition and more details are provided in appendix A.2.4.

following properties (i) they preserve the ranking of test scores, (ii) they are compatible with the distributional assumption for the birth shock (1.5) where σ_b^2 has just been estimated and (iii) they are compatible with the functional form for the transmission process (1.2). The construction of college quality is more direct. I use the information about which college each child has attended, average test scores and educational spending in each college and the assumed production function for quality (1.9) to construct the model-consistent variable q_i .

The Gini coefficient of income is used to inform the variance of labor market shocks, σ_y^2 .²⁸ The intergenerational rate of preference, β , is strongly related to the share of private spending in higher education in GDP. The outside option to college, $\underline{q}$, is directly related to the enrollment rate, the lower $\underline{q}$, the stronger the incentives to go to college. Recall that my model takes r as exogenous. Changing r has an impact on the incentives to accumulate the financial asset and, in steady-state, on the mass of households close to the borrowing constraint. r has consequently a first order effect on the elasticity of college quality to parental income.

²⁸I assume that the economy is at steady-state in the 2000s. More details in appendix .

Table 1.1: Parameters and Moments

Parameter	Description	Value		Target/Source	Moments		
		M1	M2		Data	M1	M2
η	(Inv.) elast. labor	2	<i>id.</i>	Chetty, Guren, Manoli, and Weber (2011), Own Comput.			
τ_y	Income Tax Slope	.23	<i>id.</i>	Heathcote, Storesletten, and Violante (2017), Own Comput.			
λ	Return to human capital	.67	<i>id.</i>	Own Comput.1			
a_u	Av. Transfer to College	.4	<i>id.</i>	Av. Transfer to College	.4	.4	<i>id.</i>
a_y	Av. Income Tax Rate	.2	<i>id.</i>	Av. Income Tax Rate	.2	.2	<i>id.</i>
a_b	Av. Financial Aid	.2	<i>id.</i>	Av. Financial Aid	.1	.1	<i>id.</i>
τ_u	Elas. Transfers to Coll.	.35	<i>id.</i>	Elas. Transfers to Coll. (Capelle, 2019)	.35	.35	<i>id.</i>
$\underline{a}$	Borrowing Limit	(0)	.03	Borrowing Limit	.03	(0)	.03
$\bar{a}$	Bequest Limit	(0)	$+\infty$	Bequest Limit	$+\infty$	(0)	$+\infty$
τ_n	Elas. Gov. Fin. Aid to y	.195	<i>id.</i>	Elas. Gov. Fin. Aid to y_m	.195	.195	<i>id.</i>
τ_m	Elas. Gov. Financial Aid to z	.07	<i>id.</i>	Elas. Gov. Financial Aid to z	.07	.07	<i>id.</i>
ω_3	Social Obj. Param. of Coll.	0	<i>id.</i>	Elas. Tuition to y	.13	.13	<i>id.</i>
ω_2	Elas. q to Average Ability	.84		Elas. q to z in sorting rule	.94	.94	.96
ω_1	Elas. q to I	1	<i>id.</i>	Normalization	-	-	
σ_b^2	Var. birth shock	6.6	<i>id.</i>	$\rho(y_{m,i}, \text{rank}(z_i))$	.43	.43	<i>id.</i>
α_1	Elas. b' to z	.21	<i>id.</i>	InterGen. Elas. (Mazumder, 2015)	.5	.5	<i>id.</i>
α_2	Elas. b' to q	.2	<i>id.</i>	Elas. y'_m to q	.13	.13	<i>id.</i>
α_3	Elas. b' to b	.2	<i>id.</i>	Elas. y'_m to y_m	.2	.2	<i>id.</i>
σ_y^2	Var. Lab. Mkt. shock	.74	<i>id.</i>	Income Gini Coef. (Kopczuk, Saez, and Song, 2010)	.45	.45	<i>id.</i>
β	Intergen. Preference	.235	.27	% Priv. Spend. High. Ed. in GDP (OECD)	1.3%	1.3%	<i>id.</i>
$\underline{q}$	Outside Option	(0)	.0278	Enrollment Rate (NCES)	70%	(100%)	70%
r	Interest Rate	-	3.5%	Elas. q to y in sorting rule	.2	(.4)	.21

See appendix A.2 for more details.

7 The Role of Higher Education: Quantitative Results

In this section I use the calibrated model from the previous section to assess the quantitative relevance of the higher education system in shaping inequality, intergenerational mobility and the efficiency of the accumulation of human capital. I first present a set of policy experiments that shed light on different aspects of the system and/or are of a specific political or historical interest. Second I assess the quantitative effects of a rise in the return to human capital and decompose the rise in inequality into a direct effect and the endogenous amplification through the higher education system.

7.1 Policy Experiments in Higher Education

In table 1.2, I gather the results of the six policy experiments discussed below. I provide the percentage change from the status quo steady-state to counterfactual steady-state of the Gini coefficient of labor earnings, expenditures per students, the intergenerational elasticity, GDP and a measure of welfare. The social welfare function is a generalized mean of households values with constant elasticity of substitution across households, σ , in the range $[\frac{1}{2}, 1]$.²⁹

How would the equilibrium outcome change if everyone got the exact same higher education? The first policy experiment consists in randomly allocating students across colleges. This leads to an equalization of spending per student and average student ability across all colleges. It is therefore a natural way to quantify the total effect that higher education has on income inequality, intergenerational persistence and GDP. Formally, I provide everyone with the same college quality, $\bar{q}$, where the latter is compatible with the production function of quality (1.9), the average children ability in society and average government transfers per student.³⁰ Because households optimal spending rate for higher education drops to zero, all the resources spent in the higher education system have to be financed through taxes and transfers to colleges. One therefore needs to take a stand on the level of government subsidies. I assume that there are

²⁹Recall that the cases $\sigma \rightarrow 0$, $\sigma = 1$ correspond to a Rawlsian and a utilitarian social welfare functions, respectively and the case $\sigma \rightarrow +\infty$ to a social welfare function that is a monotonic transformation of GDP. In the context of my model, with missing insurance markets for birth and labor market shocks, a concern for equity also captures a concern for insurance against these shocks.

³⁰The random allocation of students not only equalizes college experiences among college-goers but implies that everyone goes to college. It thus neutralizes both the extensive (going or not) and the intensive (quality) margin. More details in appendix A.3.

such that the share of GDP going to higher education remains the same as in the *status quo* allocation.³¹ I find that doing so would reduce the income Gini by 8.5% and the IGE by 24.3% (see line no. 1 in table 1.2). To get a sense of the magnitude, a reduction by 8.5% of the income Gini corresponds to a reduction of 4 p.p., which is half of the total increase since 1980. It is therefore a sizable effect. GDP however drops by 7% because of the increase in the misallocation of students and resources across colleges.

With the second policy experiment I am interested in isolating the contribution of the peer-effect. To do so, I equalize resources across all colleges. The most direct way to implement such a distribution of educational services is to redistribute and equalize resources across all colleges ($\tau_u = 1$). But one could also obtain the same allocation with a very progressive need-based financial aid schedule ($\tau_n = 1$). Incidentally such policies neutralize the effect that parental income has on the sorting of students across colleges conditional on child ability. The distribution of student abilities across colleges changes in the counterfactual. In particular, colleges become more homogeneous in terms of ability.³² Like in the previous policy experiment, I assume that government policies exactly offset the decrease in average private spending, so that the aggregate spending rate in higher education remains constant in the two steady-states. I find that in the counterfactual allocation, the income Gini is reduced by 2.5% (or 1.2 p.p.) and the IGE by 14.7% (line no 2). On the one hand the mismatch of student abilities is reduced with the elimination of the income-sorting channel, but the equilibrium equalization of spending across colleges leads to a less efficient accumulation of human capital. Overall, GDP falls by 1.8%.

The third experiment requires colleges to adopt progressive need-based aid. It is an interesting experiment because it is widely discussed in policy circles and, increasingly, colleges advertize their need-based financial aid program. To implement such progressive institutional aid, I raise the social objective param-

³¹ The choice for the level of subsidies does not influence inequality or mobility, but it does have a first order effect on the aggregate level of production. This assumption allows to focus on the effect of misallocation on aggregate production. In practice it means that the level of government subsidies should increase to offset the decline in private spending for higher education.

³² One could therefore argue that this counterfactual isn't capturing only the effect of equalizing resources across colleges. For example, Chetty, Friedman, Saez, Turner, and Yagan (2019) do a counterfactual exercise in which they reallocate students across colleges so that the distribution of abilities remain unchanged but so that conditional on ability, the allocation becomes independent of parental income. Like in our counterfactual, they neutralize the role of parental income on the allocation of students across colleges. However such a counterfactual is not compatible with a general equilibrium allocation of students and resources. Reallocating students entails a reallocation of financial resources, and therefore of value-added. The advantage of the counterfactual I propose is its implementability and compatibility with realistic policy tools and a decentralized equilibrium.

eter of colleges, ω_3 , to the point where $\omega_3 = \omega_1(1 - \tau_n)(1 - \tau_u)$ so that all households pay the same share of their income to get into a given college conditional on ability. Recall that in M1, it perfectly shuts down the income-sorting channel. But in contrast with the previous policy experiment, it doesn't lead to an equalization of resources and spending across colleges. The segregation of colleges by abilities and the positive correlation between abilities and parental income leads to a positive sorting of resources across colleges. Colleges with higher ability students have more resources and spend more. This policy has, maybe surprisingly, a positive effect on income inequality (+3%) because it improves the matching of students, like the previous policy, but increases (by 13%) rather than reduces the dispersion of expenditures across colleges. For the same reason, the IGE falls by less than in the previous experiment (-6.4%). The perfect positive assortative matching of students and the positive sorting of resources lead to a large increase in GDP (+22%). As a result, the welfare gains of this policy are very large, around 8%, irrespective of the strength of the concern for equity, σ .

The fourth policy experiment consists in eliminating all current government interventions in higher education. It is aimed at quantifying the extent to which the current government interventions affect inequality and mobility. Formally, a *laissez-faire* equilibrium corresponds to the case $\tau_u = \tau_n = \tau_m = a_u = a_b = 0$. I find that doing so leads to an increase in the income Gini by 2%, an increase in the IGE by 12% and an increase in the Gini coefficient of college expenditures by 70% (line no. 4a). Across all measures, two-third of these changes are due to the transfers to colleges (line no. 4b) and one-third to need-based financial aid (line no. 4c). Merit-based aid plays virtually no role (line no. 4d). While most of the policy debates have been, in recent decades, centered around the issue of federal financial aid and income tax credit and as transfers to colleges have been significantly cut, these results highlights the importance, in the current system, of transfers to colleges. The fifth policy experiment evaluates a (conservative) version of the recent proposal by democratic candidates to make college free for all. Although not fully specified as of now, the plan envisions (i) setting tuition to a minimum fee at public institutions (ii) offsetting the implied revenue losses with federal and state subsidies to colleges.³³ The proposal states that on aggregate the loss of tuition will be offset by subsidies to colleges, but it doesn't specify how much

³³ See [here](#) for a detailed version of the Act.

redistribution of resources across colleges shall occur. The most progressive option, where resources are fully equalized across colleges, corresponds to the second counterfactual (line no. 2) derived earlier. In contrast, I present here the most conservative option, where government transfers exactly offset the loss of tuition revenues at the college level. Formally, I assume (i) $\tau_n = 1$, (ii) a_b and a_u are such that the share of GDP going to higher education remains the same as in the benchmark allocation—as in counterfactual 1 and 2—and (iii) τ_m is such that the Gini of expenditures per students remain constant. The equilibrium allocation therefore features perfect stratification of colleges by student ability and unchanged sorting of expenditures per students across colleges. Such a policy would lead to an increase in the income Gini by 2.4% and an decrease in the IGE by 7.4% (line no. 5). GDP would increase by 2.8% thanks to the improvement in the matching of students.

The sixth policy experiment sets the level and progressivity of government transfers to colleges to what they were in 1980. It aims at quantifying the impact on inequality and mobility of the sharp decline in both the average and progressivity of government transfers to colleges over the past forty years, documented in [Capelle \(2019\)](#). I have shown in section 5 that qualitatively such policy changes have very likely contributed to the trends (a)-(e). Quantitatively, I find that setting the parameters of the subsidies to colleges schedule to what they used to be in 1980 implies a decrease in the income Gini by .6% and a decrease in the IGE by 3.4% (line no. 5). The decline in public transfers to colleges can quantitatively account for a very small share (2.7%) of the total increase in the income Gini but a large share of the total increase in the Gini of expenditures per student (90%). Although qualitatively consistent with fact (a)-(e), the latter finding makes the decline in public transfers a less compelling explanation than the increase in the returns to education, to which I now turn.

7.2 Increase in the Returns to Education and Propagation through Higher Education

I now quantify the extent to which a reasonably parametrized increase in the returns to education, λ , can explain the stylized facts (a)-(e) presented in the introduction. Let's denote λ_{1980} and λ_{2010} the value of the returns to education in the original steady-state (1980) and in the final steady-state (2010). The

Table 1.2: Policy Counterfactuals

		% Change from Status quo				
	Policy	Gini Earnings	Gini Exp./Stud.	Intergen. Elas.	GDP	Welfare $\sigma = [.2, 1]$
1	Random Admission	-8.5	-100	-24.3	-7.1	[-9.6,-0.8]
2	Equal Resources	-2.5	-100	-14.7	-1.8	[4.1,0.5]
3	Progressive Aid by College	3.0	13	-6.4	22.0	[8.3,7.9]
4a	Laissez-faire	2.0	70.3	12	2.9	[-6.1,-1.9]
4b	No Transfer to College	1.5	48.2	7.9	.6	[-2.5,0.2]
4c	No Need-based Aid	.5	18.2	3.0	2.4	[-2.7,-1.2]
4d	No Merit-based Aid	-.02	-3.1	-0.03	.2	[0.0,2.1]
5	College for All (conservative)	2.4	0	-7.4	2.8	[-2.2,0.8]
6	Transfers to College, 1980	-6	-21.6	-3.4	-3.0	[-1.0,-1.1]
	<i>Status quo Levels</i>	.45	.22	.4	-	-

value in 2010, λ_{2010} , is the one estimated in the previous section. In order to calibrate the value in the old steady-state, λ_{1980} , I target the change in the college premium across the two periods, keeping all other parameters constant.³⁴ Comparing the two steady-states corresponding to the two values, $\lambda_{1980} < \lambda_{2010}$, I find that the model generates an increase in the income Gini coefficient by 13 p.p., which corresponds to 130% of the empirical change, an increase in the expenditure per student Gini by 5 p.p. corresponding to 100% of the empirical change and an increase in the IGE by 6%.

I then isolate the contribution of the higher education system to the increase in income inequality. I compute the Gini coefficient of a counterfactual distribution of labor incomes where (i) the underlying distribution of human capital is the one in the 1980 steady-state, (ii) the returns to human capital parameter, λ , is set at its 2010 level, λ_{2010} , and (iii) the labor supply policy function is the one in the 2010-steady-state. This gives the level of inequality if there were no propagation through higher education. I provide more details in appendix A.3. I find that the higher education sector accounts for 6% of the seven percentage points increase generated by the model, which corresponds to a little bit more than half a percentage point increase in the Gini coefficient of income.³⁵

³⁴As I discuss in the previous section, the model overestimates the level of the college premium because of the fat tail of income in the data. It is therefore natural to target the relative increase in the college premium rather than its level in 1980. I use the value provided by Autor, Katz, and Kearney (2008) for the college premium: $\Delta \log\left(\frac{w_{college}}{w_{HS}}\right) = .65 - .45 = .2$, see figure 2 in their paper.

³⁵In M1, the version of the model without intergenerational transfers and without outside option to colleges, the endogenous amplification through the higher education is even lower, 3% of the total increase in inequality. The additional amplifica-

I now decompose the propagation through the higher education system into two channels: the reallocation of resources and the reallocation of students quality across clubs. Formally, I compute a counterfactual steady-state in which (i) the tuition schedule and the expenditures per student by college rank is fixed at what they were in the initial steady-state, in 1980, (ii) the returns to education parameter λ is set at its 2010 value. Notice that this counterfactual is not compatible with a decentralized equilibrium, because colleges are no longer on their F.O.C. and their budget constraint don't hold, so that some have deficits and some have surpluses. See appendix A.3 for more details. I find that more than 100% of the total effect stems from the increased dispersion of financial resources. The reallocation of students across the quality ladder of colleges dampens very slightly the effect.³⁶

Intuitively, in the counterfactual where expenditures have been fixed, top colleges display surpluses, because the willingness to pay of their equilibrium pool of students has increased with the rise of inequality pushing tuition fees up, while bottom colleges have deficits. The dispersion of average student ability however declines as rich but not so smart children manage to buy their way to top colleges while smart but poor children are being priced-out. From the counterfactual allocation to the final steady-state in the year 2010, expenditures per students adjust according to revenues and tuition fees, thus keeping up with the willingness to spend of their respective pool of families, which increases revenues and thus quality at the top and decreases it at the bottom. In the counterfactual, the Gini coefficient of income barely moves and if anything slightly declines. This allows us to conclude that it is really the increase in the dispersion of revenues and therefore expenditures per students across colleges that is the root of the increase in the amplification by colleges. One natural test of this mechanism is to look at the evolution of the dispersion of tuition fees. [Davies and Zarifa \(2012\)](#) indeed find that the Gini coefficient for tuition fees has increased over the period 1971-2006.

tion in M2 comes from the extensive margin: the fact that more students get enrolled into college when the returns to human capital increases reinforces the feedback mechanism of higher education on inequality. That the extensive margin plays an amplifying role is not a qualitative feature of the model but depends on the initial enrollment rate.

³⁶This dampening effect is stronger in the short-run. In the long-run, low ability kids from rich family "catch up."

8 Conclusion

This paper studies the extent to which the higher education system shapes economic inequality and intergenerational mobility in the U.S., how it has propagated and amplified macroeconomic shocks such as an increase in the returns to human capital and how government policies may affect these responses. An increase in the market returns to human capital increase inequalities directly but also indirectly over time through a more unequal sorting of resources across colleges and a (more unequal) accumulation of human capital.

The tractability of the framework goes with strong and simplifying assumptions. I partially relaxed two of them in the quantitative section (intergenerational financial transfers, and outside option to college,). Three assumptions would need further investigations: i) the allocation of students across colleges in the model works through a system of clearing markets, while the real world displays a mix of price mechanism and quantity restrictions, ii) beyond the accumulation of human capital and labor market returns, higher education has non-pecuniary returns to education and there is evidence that households get a direct consumption value from going to college. The implications for welfare and policy analysis are likely to be far-reaching, iii) the paper has focused on the role played by tuition for the shaping of inequality in higher education. The analysis should be extended to take into account donations and endowments as it is likely to provide an additional source of inequality.

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Chapter 2

Forty Years of Inequality and State Subsidies across U.S. Colleges

I Introduction

FINANCIAL RESOURCES AVAILABLE PER STUDENT VARY ENORMOUSLY ACROSS COLLEGES in the U.S. When this translates into large differences in educational inputs per student such as faculty time and quality, it has implications for the dispersion of quality across institutions of higher education. In this short paper, I document some patterns of inequality in expenditures and in faculty per student across different institutions of higher education, with a particular focus on trends in the past forty years. These facts motivate the model and the structural analysis presented in the first chapter of this dissertation.

The main results are as follows. First, inequality in resources per student across colleges are substantial and about the same order of magnitude as the dispersion of household annual income after tax and transfers—with a Gini of about .42 at the end of the sample. Second, inequality across students have slowly and steadily increased since 1980: the Gini coefficient for spending per student has increased by 5 p.p. which corresponds to half the increase in the Gini coefficient of household annual income over the same period. Third, the trend is entirely driven by four-year colleges, with an increase in inequality within both public and non-profit private institutions, and a slight increase in the inequality between the

two groups.

The series are constructed using the Integrated Postsecondary Education Data System (IPEDS), which is a set of surveys conducted annually since 1980 by the National Center for Education Statistics. These surveys gather information from every college about finances, enrollment and other variables related to the composition of the student body and college administration. Four measures of cross-sectional dispersion are considered: the Theil T index, the Gini coefficient, and the top 10% and top 5% shares. While the last three are the most standard measures of inequality in the literature, the first one allows us to decompose inequality across institutions into "within" and "between" components. Finally, three measures of resources per student are considered: expenditures per student, revenues per students and faculty over student ratio.

The increase in income inequality across households in the U.S. since 1980 and the decrease of state subsidies to public colleges may be the two main reasons behind these trends in the distribution of resources across colleges. While the former has been well documented ([Piketty and Saez, 2003](#); [Heathcote et al., 2010](#)), this paper's second contribution is to document the latter. Both the average transfers across colleges and the progressivity of transfers have declined sharply over the past forty years. In a related work, I show how an increase in household income inequality—originating in an increase in the labor market returns to human capital—can lead in general equilibrium to an increase in the dispersion of resources across colleges, through tuition fees and through donations and endowments ([Capelle, 2019](#)). In the same paper, I show that the decline in the progressivity of state subsidies to colleges also leads to an increase in inequality among colleges.

This rise in inequality of resources per student across colleges has efficiency and equity implications. Expenditures per student and the faculty-student ratio are important for student achievement and labor market outcome ([Jackson et al., 2016](#); [Deming and Walters, 2017](#); [Lafortune et al., 2018](#); [Hickman and Mountjoy, 2019](#); [Biasi, 2019](#)). Unequal resources have implications for the accumulation of human capital and the dispersion of labor earnings, and also potentially for access to top executive positions and political power. From an efficiency point of view, the optimal distribution of expenditures across colleges depends on the distribution of students across colleges and on the complementarities between student

ability and college spending. A perfectly egalitarian distribution of expenditures is an efficient allocation of resources only in the absence of complementarities or if students are randomly allocated to colleges. If strong complementarities exist and students sort according to their abilities, then it is no longer efficient to maintain an egalitarian distribution of resources.

I have shown in [Capelle \(2019\)](#) that the observed increase in the dispersion of resources across colleges may have led to less positive assortative matching of students along the quality ladder of colleges. Beyond efficiency, one might also care about inequality in college consumption across individuals, for equity reasons. Finally, from an intergenerational perspective, because the transmission of human capital from parents to children is strong and because both ability and parental income matter for how students sort across colleges, the unequal distribution of resources across colleges amplifies the intergenerational transmission of status.

Some papers have documented the large dispersion of resources and expenditures among colleges ([Bowen, 1980](#); [Desrochers and Wellman, 2011](#)). There is a disagreement about the evolution in the past four decades. I argue that part of this disagreement can be attributed to the time window as well as the measure of inequality used. Several papers have found that inequality has been stable. [Lau and Rosen \(2016\)](#)—who focus on expenditures, revenues, and endowments—and [Halffman and Leydesdorff \(2010\)](#)—who focus on publications—find that measures of inequality have remained flat over the period 2001-2010. In his book, [Clotfelter \(2017\)](#) document the widening gap in endowment and compensation for professors since 1970. Over the same period, [Davies and Zarifa \(2012\)](#) shows that the Gini coefficient of expenditures and revenue have increased among US colleges and universities¹. My results confirm the findings of the latter. The reason why our conclusion differs from the ones of [Lau and Rosen \(2016\)](#) is mainly the sample period and the length of the sample: inequality have increased steadily in the eighties and nineties. Indeed, the increase in inequality in the years 2001-2010 is small compared to the changes observed in the previous two decades.

¹ Most papers in this literature use the Gini as a measure of inequality. It is a measure of relative inequality, as opposed to a measure of absolute inequality, like the variance. If every agent in the economy sees their income grow at the same rate, the Gini coefficient remains constant, even though the variance might increase. [Clotfelter \(2017\)](#) insists for example that inequality in endowment has grown only in absolute terms, not in relative terms. The exact measures of inequality used should therefore be kept in mind when comparing conclusions of several papers.

This paper extends the literature in two directions. First I extend the analysis to the recent period 2006-2016, I show that the findings also hold for other popular measures of inequality, such as the share of the top 5 and top 10% and I offer decompositions by subsector of higher education (public/non-profit private/for-profit private; two/four years). Second, I offer an explanation for this evolution drawing from structural works done in the first chapter of this dissertation: the rise in the inequality of household income on the one hand and the decrease in the overall progressivity of government transfers to institutions of higher education on the other.

The remainder of this paper is organized as follows. Section 2 describes the data, construction of variables and methodology. Section 3 presents the main results for the trend in inequality as well as the decomposition by subgroup of higher education institutions. Section 4 presents trends in governments subsidies, both in terms of averages and in terms of progressivity. Section 5 concludes.

2 Data and Methodology

2.1 IPEDS Data and the Delta Cost Project Database

The Integrated Postsecondary Education Data System (IPEDS) is a set of surveys conducted annually by the National Center for Education Statistics gathering information from every college participating in the student financial aid programs, commonly called Title IV institutions. Almost all institutions participate so that the coverage of IPEDS can be thought as virtually universal. Institutions report data on enrollments, finances, faculty and staff among others. There are about 7500 institutions completing IPEDS surveys each year, including research universities, state colleges and universities, private religious and liberal arts colleges, for-profit institutions, community and technical colleges and non-degree granting institutions². It started collecting data in 1980 after the creation of the U.S. Department of Education in 1979. The set of variables and institutions covered have expanded over time.

The Delta Cost Project Database has compiled a large set of variables from the original IPEDS surveys for a large set of years—1987-2015—in order to construct a readily usable longitudinal dataset. The

²NCES

dataset has another appealing feature for researcher: data series have been adjusted to harmonize changes in financial reporting standards that occurred over time and they have imputed some of the missing data. The Delta Cost project has two limits: it doesn't compile data for 1980-1986 and it drops the less than two-year for-profit institutions from the sample. It is also unclear if they will continue to update the database in the future.

Most of our results use the Delta Cost Project database. In appendix B.1.4 I investigate whether using the original IPEDS data gives different results. Using the original data also allows us to go back further in time—1980 instead of 1986. For this robustness check exercise, I collected myself the relevant surveys, compiled them for a large set of years—the dataset covers half the years between 1980 and 2016—and constructed the variables to consistent with the ones in the Delta Cost Project.

2.2 Definition of Variables

The measure of expenditures used in this paper is constructed as the sum of expenditures for instructions, research and core activities. This corresponds to the sum of items 1 to 6 in table 2.1, which displays a simplified classification of the expenditures of colleges. It includes instruction, research, academic support, student service and institutional support. Total expenditures includes core educational spending, operation maintenance, student aid and others (hospitals, auxiliary services, etc...). Public service includes expenses for activities established primarily to provide non-instructional services beneficial to individuals and groups external to the institution (conference, institute for example). Academic support includes expenses of activities and services that support the primary missions of instruction, research and public service (libraries, museums, information technology, for example). Student services include expenses for admissions, registrar activities and activities whose primary purpose is to contribute to students emotional and physical well-being and to their intellectual, cultural, and social development outside the context of the formal instructional program (student organization, cultural events, supplemental instruction outside of normal academic program for example). Institutional support includes expenses for the day-to-day operational support of the institution (general administrative services, central executive-level

activities, employee personnel and records, for example)³.

Total Expenditure	Total Core Education	1	Instruction
		2	Research
		3	Public Service
		4	Academic Support
		5	Student Service
		6	Institutional Support
	Total Non-Core	7	Operation Maintenance
		8	Aid
		9	Others

Table 2.1: Classification of Expenditures

In the same way, the measure of revenues used in this paper is a measure of core revenues, constructed as the sum of items 1 to 7 in table 2.2, which displays a simplified classification of revenues of colleges. It includes tuition and fees, federal, state and local appropriations, grants and contracts, private gifts and contracts, earnings from endowment and investments and revenues from sales and services of educational activities. All items are quite self-explanatory except the last one which includes the sales of goods or services that are incidental to the conduct of instruction, research or public service (film rentals, sales of scientific and literary publications, testing services, university presses, data processing services, etc...). This measure of revenues excludes hospitals which is for some institutions a non-negligible part of their budget and auxiliary enterprises (residence halls, food services, student health services, movie theaters etc..).

I also consider time-adjusted version of these two variables that take into account the length of study at these institutions. I multiply expenditures per student by four (two and one resp.) at four(two and one resp.)-year institutions. Not all students starting in a two or four-year institution will finish, but the availability of the graduation rates variables for a large number of colleges in the dataset is too recent for a more refined analysis to be possible. I do not report these adjusted measures as they show very similar patterns to the non-adjusted ones.

Finally, I also consider the faculty over student ratio, a common measure of educational quality and a much more narrow measure than core educational spending. I divide the sum of full time faculty plus half

³Delta Cost Project Database

Total Revenues	Core Revenues	1	Tuition and fees
		2	Federal Appropriations, Grants and Contracts
		3	State Appropriations, Grants and Contracts
		4	Local Appropriations, Grants and Contracts
		5	Private Gifts, Grants and Contracts
		6	Investment and Endowment Returns
		7	Sales of Educational Activities
	Total Non-Core	8	Revenue from Auxiliary Enterprises
		9	Revenue from Hospitals
		10	Independent Operations and Others

Table 2.2: Classification of Revenues

the sum of part-time faculty by the full-time-equivalent number of students (FTE enrollment thereafter). The data for this variable has been less systematically reported by institutions in IPEDS, therefore the quality is not as good as balance-sheet data. This is the reason why I focus primarily on core educational expenditures.

2.3 Measures of Inequality

I consider four measures of inequality, the Gini coefficient, the Theil T index and the shares of expenditures, revenues and faculty going to the top 10% and the top 5% of students.

Importantly, all measures are FTE-enrollment weighted and are measures of inequality at the student level and not at the college level. Denote the set of college $\mathcal{J}$, $\mathbf{n}$ the vector of FTE-enrollment and $\mathbf{I}$ the vector of spending per FTE-student, where n_j and I_j are the entries corresponding to college j . The measures of inequality are computed by weighting the vector $\mathbf{I}$ with $\mathbf{n}$, i.e. where entry I_j counts n_j times.

The Theil T index is a special case of the generalized entropy index. Using the same notations, it writes:

$$T_T = \frac{1}{N} \sum_{j \in \mathcal{J}} n_j \frac{I_j}{\mu} \ln \left(\frac{I_j}{\mu} \right)$$

$$\text{with } \mu = \frac{1}{N} \sum_{j \in \mathcal{J}} n_j I_j$$

$$\text{and } N = \sum_{j \in \mathcal{J}} n_j$$

For specific distributions, the Gini coefficient and the Theil T index are analytically related. For example, if the distribution is log-normal, denoting Δ^2 the variance of the associated normal distribution, the Gini and the Theil T are given by

$$\begin{aligned} \text{Gini} &= 2\Phi\left(\sqrt{\frac{\Delta^2}{2}}\right) - 1 \\ T_T &= \frac{\Delta^2}{2} \end{aligned}$$

where $\Phi(\cdot)$ is the cdf of a standard normal. I use the Theil index because of its decomposability, a property that the Gini coefficient does not offer. For example, let's divide colleges into subgroups indexed by $m \in \mathcal{M}$. This implies a similar divide across students. Denoting $\bar{I}_m$, the (FTE-enrollment-weighted) mean expenditures in group m and N_m the number of FTE-students in group m , the Theil T index can be written:

$$\begin{aligned} T_T &= \sum_{m \in \mathcal{M}} s_m T_{T,m} + \sum_{m \in \mathcal{M}} s_m \ln\left(\frac{\bar{I}_m}{\mu}\right) \\ \text{with } s_m &= \frac{N_m \bar{I}_m}{N \mu} \end{aligned}$$

where the first term is the within component and the second term is the between group component of inequality.

2.4 Data Transformation and Quasi-Panel

I apply some transformations to the data for robustness purposes. Zeros are counted as missing. All observations above the top .5% are censored at the .5% percentile to avoid outliers to drive our results. All series displayed in this paper have been smoothed.

Despite their small share in total enrollment, one might be worried that the exclusion of less than two-year institutions in the Delta Cost Project dataset might affect our findings. I check the robustness of our result to this exclusion by constructing the same series from the raw data in appendix B.1.4.

Even though the Delta Cost Project imputes some of the missing data, their interpolation method is conservative and one can be worried that institutions non-random missing values might bias our results. To address that issue I construct a quasi-panel of colleges by excluding institution for which I do not observe at least one observations per decade. I compare the results with what one would obtain by working of the full and unrestricted sample in appendix B.1.5.

2.5 A Snapshot at the U.S. Higher Education System

Overall, inequality are large and especially concentrated at the top. Figure 2.3 reports summary statistics for the distribution of educational expenditures per FTE-Student in 2015 in hundreds of 2015-dollars. The first column reports for the entire sample, FTE-enrollment weighted, the second column reports the same statistics without weighting by enrollment—one college counts for one observation—, the next two columns restrict the sample to public 2-year and 4-year institutions respectively, the next two do the same for private non-profit and the last two for private for-profit. The first line report the mean, while the next lines report selected percentiles. The last two lines report the number of institutions and the FTE-enrollment (in thousands). For space reason, I do not report statistics on less than two years institutions.

In the overall distribution, the 90th student gets 2.6 as much resources as the median. But the 99th student gets 3.4 as much as the 90th student. This fat right tail is driven by the private non-profit colleges. In general inequalities are larger within 4-year than 2-year institutions. In contrast with the public and non-profit sector where inequality within 4-year on the one hand and between 2-year and 4-year institutions on the other are large, inequality within the for-profit sector are rather small along both dimensions.

3 Inequality Across U.S. Colleges, 1980-2016

3.1 General Trend

Figure 2.1 shows a clear upward trend in the four measures of inequality over the period 1987-2015. The Gini coefficient has gone up from .35 to .41, the Theil index from .23 to .31, the top 10% from 29% to 33%, and the top 5% from 19% to 22%. It means that the amount of educational resource students have

Table 2.3: Summary Statistics on the Distribution of Expenditures per FTE Student, in 2015

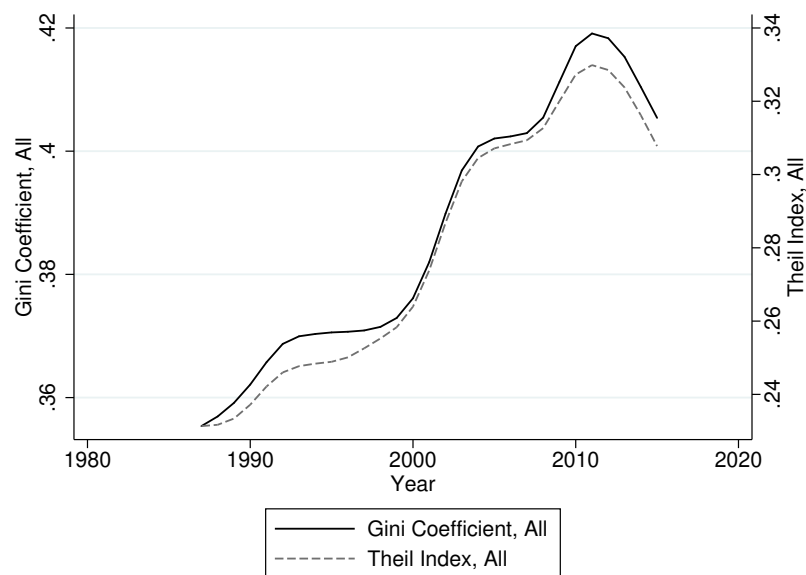
Expenditures per FTE Student								
Stat.	All		Public		Private Non-Profit		Private For-Profit	
	Weighted	Unweighted	2-yr	4-yr	2-yr	4-yr	2-yr	4-yr
Mean	222	200	103	243	149	337	133	133
50 th	155	153	96	184	124	221	135	125
75 th	255	223	114	324	164	343	171	174
90 th	405	344	139	435	236	729	211	219
95 th	585	474	160	554	283	1295	234	267
99 th	1374	1201	206	926	449	1728	381	368
No. Colleges	4868	4868	902	591	155	1572	738	618
FTE Students	15457	15457	3565	7036	34	3465	321	989
Weighted	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes

Legend: Expenditures are in hundreds of 2015-dollars. FTE Students is in thousands. Weights refer to FTE enrollment.

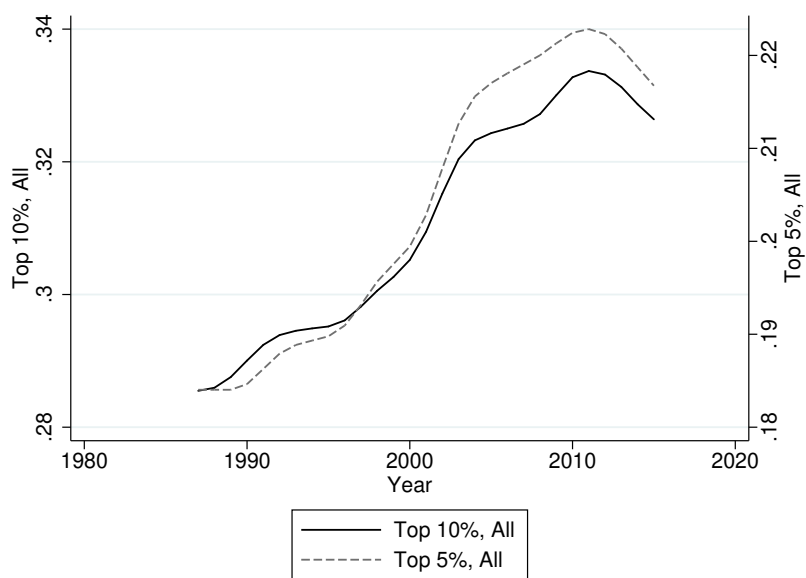
Source: Delta Cost Project.

been exposed to have been increasingly unequal over time. For example, at the end of the period, the top 10% of students—all measures of inequality are weighted by FTE enrollment—enjoys a third of all spending in higher education.

For comparison, the market income and the after tax and transfers Gini coefficients in the U.S. reported in appendix on figure B.3 have increased from .47 to .6 and from .35 to .44 respectively over the period 1979-2015. The top 10% and 5% shares of income have increased from 33% to 47% and from 21% to 33% respectively over the same period. In levels, inequality among expenditures per student are slightly lower but comparable to the level of inequality in the after tax and transfers income distribution. Regarding trends, *the increase in inequality among expenditures per student has been about half as large as the the increase in income inequality.*



(a) Gini and Theil Indexes



(b) Top 10 and 5 shares

Figure 2.1: Inequality in Educational and Research Expenditures

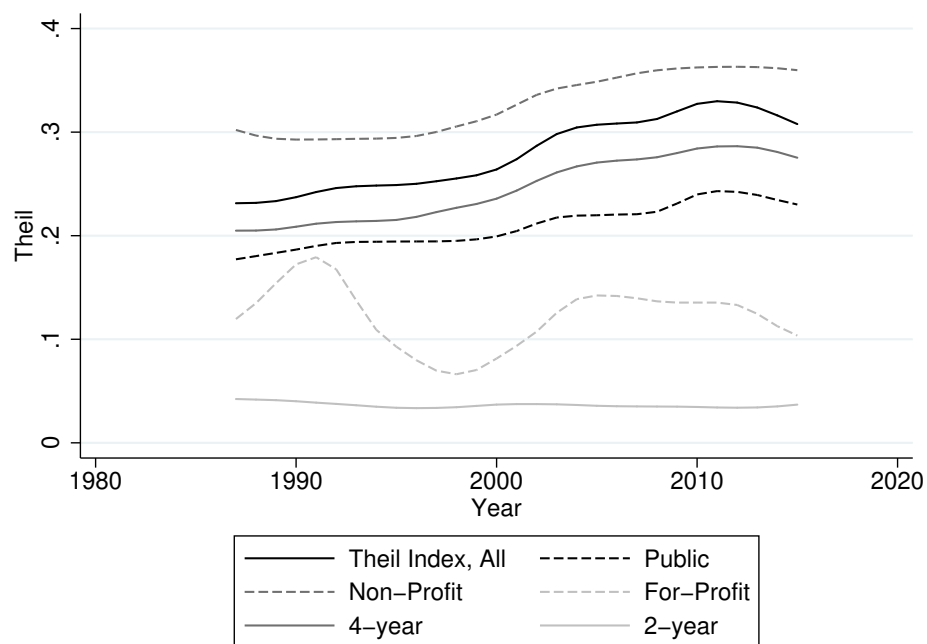
3.2 Revenue and Faculty per FTE Students

Although more noisy, figure B.1 shows that the increase in inequality in revenues has been similar to the one documented for educational spending.

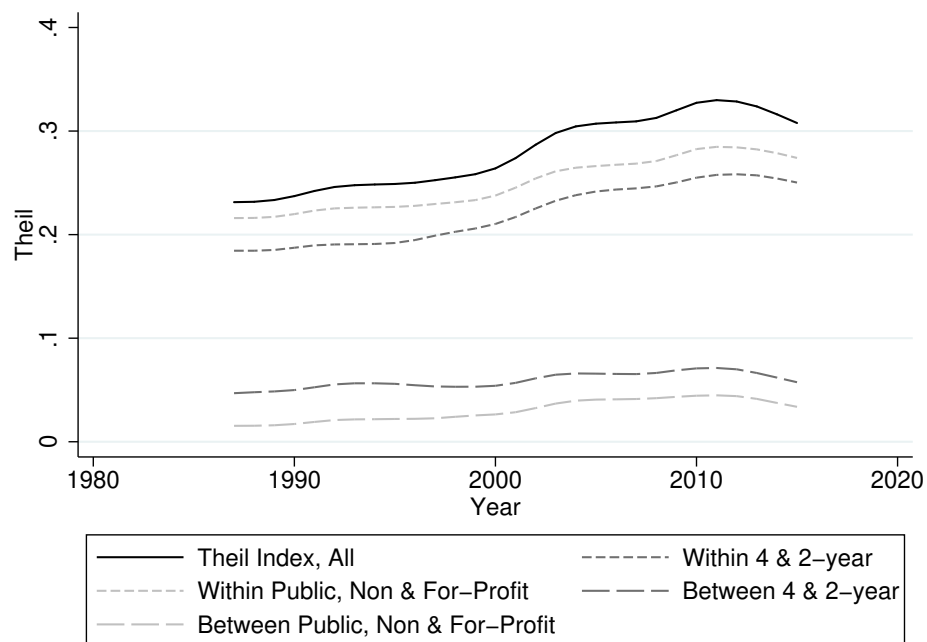
Has the increase in inequality in educational expenditures and total revenues per student translated into inequality in the number of faculty per student? Figure B.2 shows that it indeed has. The 10 percent students with the highest faculty-student ratio have seen their shares of the total stock of faculty going from 20% to 23% percent from 1987 to 2015. One should however be careful with those measures given that the data are of lesser quality than the measures of expenditures per student—number of faculty were not as systematically reported by colleges in the early years.

3.3 Decomposing the Trends in Inequality

What part of the higher education system is driving the overall increase in inequality among colleges? The decomposability of the Theil T index allows us to separate the trend in inequality into a within and a between group components. I consider two different grouping: 2-year *vs* 4-year colleges on the one hand and public, private non-profit and private for-profit on the other hand. The first graph of figure 2.2 reports the Theil Index for the five different subsamples while the second reports the within and between Theil measures for both decompositions.



(a) By Type of Control and Length of Study



(b) Within/Between Decomposition

Figure 2.2: Theil T Index and Subgroup Decompositions

The decomposition in the two/four-year dimension suggests that the increase in inequality is driven neither by an increase in inequality within 2 year colleges nor by an increase between 2-year and 4-year colleges, but that *it is mainly due to an increase in inequality within 4-year colleges*. The decomposition by type of control shows that the increase in inequality happened within public and within non-profit private colleges to a similar extent and that *the between public-private colleges component of inequality seems to have also contributed to the general trend*. Finally the within for-profit series doesn't display any clear trend.

3.4 Total-Enrollment Adjusted Gini

Not every individual of a given cohort goes to college. What are the implications for the measure of inequality if one takes into account the total enrollment margin into account by assigning a value of 0 to the share of individuals who do not go to college? I focus on the Gini coefficient because there exists a simple mapping between the enrollment rate e , the share of people who do go to college, and $\text{Gini}_{\text{Goers}}$, the Gini coefficient of the individuals going to college. The Gini index for the entire population can be written as:

$$\text{Gini} = e \times \text{Gini}_{\text{Goers}} + (1 - e)$$

The adjusted Gini—reported in figure B.6—declined from the middle of the 1980s to the year 2000 because the enrollment rate was increasing faster than the Gini within the set of goers but has stabilized since then as the enrollment rate, e , has been growing at a slower rate.

4 The Rise in Income Inequality and Decline in Government Transfers to Colleges

What are the roots of this increase in the dispersion of educational spending across colleges? I present here two hypotheses: increase in household income inequality and decrease in the progressivity of government

funding.

Increase in Income Inequality. Income inequality across households have dramatically increased in the U.S. since 1980 as reported in figure B.3, (Piketty and Saez, 2003). In Capelle (2019), I show that an increase in the market returns to human capital, a well-documented fact, is very likely to lead to an increase in a) household income inequalities, b) in college tuition and importantly c) in the dispersion of resources across colleges. The underlying model features overlapping generations of heterogeneous households subject to a borrowing constraint who choose which college to send their kids to and in which heterogeneous colleges maximize the quality they deliver. The mechanism at play in the model is akin to the revenue theory of cost by Bowen (1980), but applied to a framework with a ladder of colleges, whereby revenue and spending are dictated by how much tuition each college can raise, and ultimately by the households' willingness to pay. Bowen summarizes his theory:

...at any given time, the unit cost of education is determined by the amount of revenues currently available for education relative to enrollment. The statement is more than a tautology, as it expresses the fundamental fact that unit cost [i.e., the cost of education] is determined by hard dollars of revenue and only indirectly and distantly by considerations of need, technology, efficiency, and market wages and prices. (p. 19)

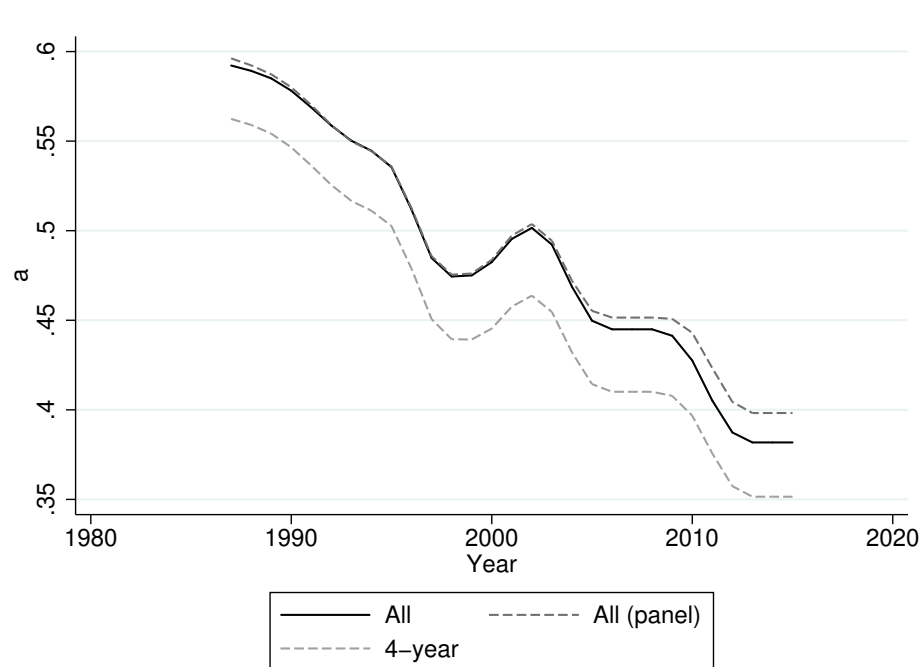
In the model, the production technology of education quality is constant returns to scale, hence the equilibrium level of quality in each college is demand-determined. When household income inequality increases, the amount of resources going to top colleges increases relatively to lower quality colleges because the former draw—in equilibrium—from a richer pool of students than the latter.

Decline in Government Transfers to Colleges. It is well-known that states' transfers to public institutions of higher education have declined, especially at four-year colleges. Figure 2.3a plots the ratio of the sum of government transfers at all institutions over the sum of total revenue at all institutions:

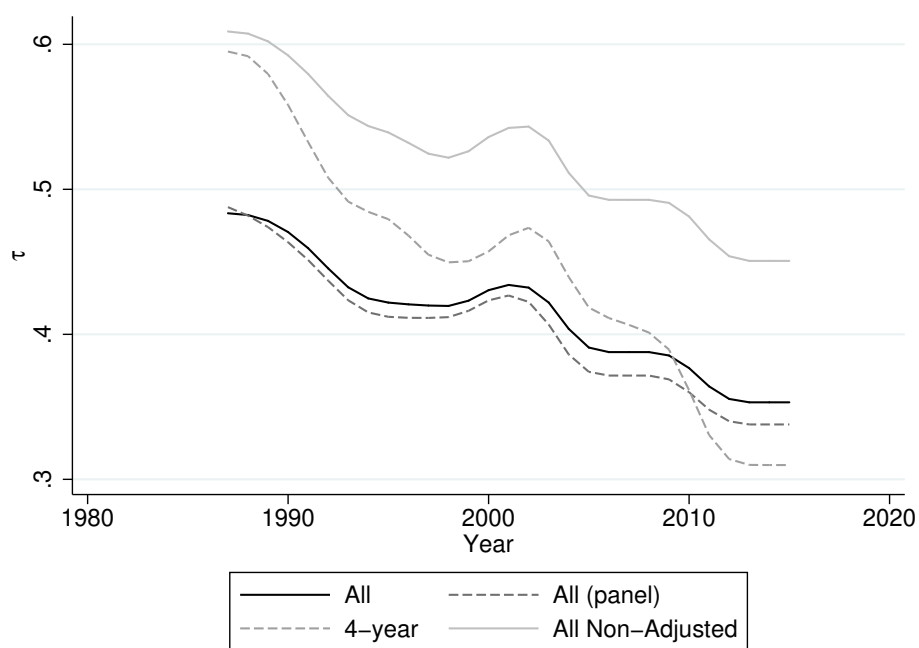
$$a_u = \frac{\sum_{j \in \mathcal{J}} T_j}{\sum_{j \in \mathcal{J}} R_j}$$

where T_j is the sum of items 2,3 and 4 in table 2.2 and R_j is defined as in the previous section, as the sum of items 1 to 7⁴. The downward trend is very striking: while 60% of all revenues in higher education was government funded in the middle of the 1980s, it has declined to 38% in 2015. The light dashed line plots the same series for 4-year colleges only and shows that the overall decline is really led by 4-year colleges. The dark dashed line reports the same series for a restricted sample where we observe every college every year, in case one would be worried about college entering or exiting the sample.

⁴Although Pell-Grants are usually channeled from the federal budget to students through colleges and do appear in the balance-sheet of colleges, I exclude them from the measure of revenues and government transfers since they are ultimately student financial aid.



(a) Government Subsidies in % of Total Revenues



(b) Progressivity of Government Subsidies

Figure 2.3: Decline in Average and Progressivity of Government Subsidies

But what really matters for the inequality of resources across colleges is the progressivity of transfers, as I show in (Capelle, 2019). The decrease in state subsidies to public colleges leads to an increase in inequality among colleges if it implies a decrease in the progressivity of the government transfers schedule. Figure 2.3b plots a measure of the progressivity of government transfers. If we define $\tilde{R}_j$ the total revenues net of government transfers at college j

$$\tilde{R}_j = R_j - T_j$$

and n_j FTE enrollment at college j , the progressivity is given by the coefficient τ_u estimated by running the following regression⁵

$$\log \left(\frac{x_j R_j}{n_j} \right) = \alpha + (1 - \tau_u) \log \left(\frac{x_j \tilde{R}_j}{n_j} \right)$$

where x_j is an adjustment factor equal to 4 (2 and 1 resp.) for four (two and less than two resp.)-year colleges. This adjustment aims at taking into account the fact that individuals at four-year institutions are expected to spend twice as much time in the institution than individuals at 2 year colleges, and four times as much time as those at 1-year colleges. The gray solid line shows the same estimated coefficient with $x_j = 1$ for all colleges.

Three observations stem from figure 2.3b. First of all, the decline in the progressivity of the transfer schedule is very large, from .5 in the middle of the 1980s to .35 after 2010. For comparison, the progressivity of the income tax schedule has been estimated to be around .2 and has remained fairly stable (Heathcote et al., 2017)⁶. Secondly, the decline has been particularly dramatic at four-year institutions, in line with the earlier finding that the increase in inequality across colleges is concentrated in this sector of the higher education system. Finally, looking at the non-adjusted line reveals that adjusting for the expected time spent in the institution significantly lower the measured degree of progressivity of govern-

⁵This functional form is used and explained in Capelle (2019) and is inspired by the progressive income tax schedule introduced by Benabou (2002). Notice that a progressive transfer schedule requires $\tau_u \in [0, 1]$ and $\tau_u = 1$ means complete redistribution and full equality of resources.

⁶Piketty and Saez (2003) show however that the progressivity of the income tax schedule has sharply declined for very top income.

ment transfers.

5 Conclusion

The paper presents series on dispersion of expenditures per student, revenues and faculty per student across colleges from 1980 to 2016 in the United States using IPEDS data. Inequality across students have been slowly and steadily increasing over the period. All measures of inequality suggest that the increase is half as large as the increase in annual income inequality. The trend are entirely driven by four-year colleges, with an increase in inequality within-public, within non-profit private institutions and between non-profit private and public colleges.

The paper also shows that the average and the progressivity of government transfers to higher education institutions have sharply declined over the period, especially at four-year institutions. This, together with the rise of household income inequality, may explain the rising inequality across colleges, as I have argued in a companion paper [Capelle \(2019\)](#).

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Chapter 3

Competition *vs.* Stability: Oligopolistic Banking System with Run Risk

I Introduction

DOES THE DEGREE OF COMPETITION AMONG FINANCIAL INTERMEDIARIES INCREASE INSTABILITY?

Compared to the U.S. banking system, the stability of the Canadian system whose three largest banks total three fourth of total bank asset is often cited as the example that an oligopolistic banking system enhances financial stability. This paper revisits this question in the context of the shadow banking system.¹ As the Great Recession has shown, it is much more subject to the risk of systemic runs than traditional banks, for three main reasons: i) shadow banks rely heavily on wholesale short-term funding (Pozsar et al., 2010), ii) their assets are continuously and freely traded on financial markets which makes shadow banks' assets more liquid but also their value more volatile, and iii) they operate in a different regulatory framework and are not covered by public deposit insurance. How does the level of competition among unregulated financial intermediaries shape the likelihood of systemic runs and what are the implications

¹The unregulated financial intermediaries total more than a third of total intermediation in the US (Board, 2017). Herfindhal indexes range from .05 to .5 across different segments of financial markets, corresponding to an average of 20 to 2 actors (Cetorelli et al., 2007).

for macroeconomic stability and welfare?

In this paper, I build a stylized macroeconomic stochastic general equilibrium model featuring atomistic households and a finite number of large leveraged financial intermediaries investing in a freely traded asset. At any instant, households can decide to withdraw their deposits from banks, which makes the system subject to run in some part of the state-space. Multiple equilibria exist whenever otherwise solvent banks are insolvent at the run price, where the run price is a macroeconomic endogenous object. In contrast with a framework in which banks are atomistic, when choosing their capital investment, the composition of their liability, and their exposure to risk, large banks internalize their impact on the aggregate likelihood of a run, asset prices and excess returns.

The competition *vs.* stability debate has a long history, but most works have been done in static and partial equilibrium framework. This literature is vast and the following review is very selective. Building on the traditional bank run framework (Diamond and Dybvig, 1983; Morris and Shin, 1998), papers have studied the role of non-competitive profits in either increasing abilities of banks to repay running depositors *ex post* (Vives, 2016) - which share some similarities with what I will call the *recapitalization channel* - or giving them incentives to lower their risk exposure *ex ante* (Matutes and Vives, 2000), what is known in the literature as the charter or *franchise value channel*. In contrast, recent studies have shown mechanisms through which less competition increases risk-taking through the adverse selection of borrowers implied by higher loan rate (Nicolo and Boyd, 2003; Corbae and D’Erasmus, 2014) or through cheaper source of funding implied by higher franchise value, two mechanisms from which I abstract. Freixas and Ma (2015) also revisits the concentration-stability in banking. Carletti and Hartmann (2003); Vives (2016) provide an overview of this literature.

I revisit the competition-stability debate in a dynamic and general equilibrium framework by drawing on the recent development in the macrofinance literature. At the core of these models, there are two kinds of agents, a continuum of households who consume and invest in financial intermediaries’ (called banks) short-term debt (called deposits) - as well as in the productive asset (called capital) and atomistic banks that intermediate that issue deposits to invest in capital. Banks exist because they are experts at managing capital. Claims on capital are traded and priced on a centralized financial market. I depart

from this literature in two ways. First I assume a finite number of large banks that internalize their impact on aggregate risk, returns and asset prices and therefore that compete *à la* Cournot. Second, apart from the cost of managing capital on the side of households, there is no other friction. For banks' net worth to matter for their lending behavior, [Mendoza \(2010\)](#); [Gertler and Kiyotaki \(2015\)](#) (hereafter GK), and [Gertler et al. \(2017\)](#) for a more quantitative version, use a collateral constraint originating in a moral hazard friction; [Brunnermeier and Sannikov \(2014\)](#) (hereafter BS) assumes a fixed amount of equity and therefore occasionally binding constraint; [Adrian and Shin \(2014\)](#); [Coimbra and Rey \(2017\)](#) use Value-at-Risk constraints. I do not make any of these assumptions: restrictions to leverage arise endogenously from the desire of banks to limit their exposure to the risk of run. While the main focus of the last two strands of the literature has been about how financial risk builds up over time, I rather focus on the time of crisis itself and the recovery that follows, which is when run happen.

My first contribution is the analytical study of the equilibrium path of my economy. I show that in the neighborhood of the steady-state, when banks are well-capitalized, they optimally choose to restrict their level of capital investment to a multiplier of their net worth in order to avoid the risk of run, a part of the state-space I call the safe zone. In this zone, the system is characterized by a classical relationship between banks' net worth, aggregate asset price and capital investment where leverage is a function of the drop in prices should a run occur. In the aftermath of an adverse aggregate productivity shock, banks experience a drop in net worth. The economy lands in the run (or risky) zone where banks choose to take on high leverage, by investing in the interior solution of the Cournot game and where runs might happen with positive probability. After finite time, banks have accumulated enough net worth and the system enters the safe zone. In this safe zone bank continue to accumulate net worth until they reach the steady-state where they have enough net worth for the unconstrained investment to be safe. The endogenous amplification of the initial adverse macroeconomic shock is therefore of a very different nature in the run and in the safe zone: in the former, it comes from the possibility of a run, while in the latter it comes from the restriction on intermediation.

My second contribution is to shed light on the mechanisms through which concentration as well as the speed at which new entrants replace banks that went bankrupt shape stability. Stability depends crucially

on how much time the system spends in the run zone. It therefore depends on the size of the zone but also on the speed at which the system travels through it. I show that there are three channels through which the *level* of competition shapes stability: firstly the *franchise value channel*, already present in the older static and partial equilibrium literature, makes large firms less willing to take on the risk of run because they have more to lose from bankruptcy, thereby reducing the size of the run zone. Secondly, the *price-drop channel*, probably the most insightful of the three, is the focus of the next paragraph and will also influence the length of the run zone. Thirdly the *recapitalization channel* makes more concentrated market more stable by increasing the speed at which they can accumulate net worth and escape the run zone. Finally, the speed of entry influences the stability of the system: the faster the entry of new banks after a run, the less likely a run happens *ex ante*, what I label the *entry channel*. In the limit, where entry happens infinitely fast, there is no run in equilibrium. This is because the expectation of new entrants raises the run price to the point that annihilates the possibility of a run.

The *price-drop channel*—the effect that competition has on stability through the drop in asset prices conditional on a run occurring—is probably the most insightful and controversial of my findings. The size of the price drop is a measure of financial market liquidity and when institutions are large, the latter depends on how many institutions are operating. I find that more concentration leads to smaller price drops which stems from the fact that asset prices are initially (in “normal times”) depressed because of the oligopoly distortion. Because the price-drop is smaller, it requires less net worth for a bank to insure from the risk of a run, giving it incentives to reduce deleverage quickly, which reduces the size of the run zone. This finding is in sharp contrast with what happens when runs are not systemic. When runs are on individual banks, the failure of large institutions imply large price-drops which impair the balance-sheet of surviving banks paving the way for contagion. This Too-Big-To-Fail argument is reversed in the case of systemic runs.

A *paradox of safety* emerges in our setting. Internalizing the smaller amount of systemic risk, banks operating in a concentrated sector end up having lower net worth to asset ratio, which makes the system vulnerable to large exogenous shocks. Although, a more concentrated sector leaves more quickly the run zone, for a given initial level of net worth, it generally starts its post-shock transition with less net worth as

its net worth to asset ratio is smaller before the shock.² This counteracting force mitigates the stabilizing properties of concentration.³

Finally, I extend the model to a framework where shadow and traditional banks coexist and compete with each other.⁴ I use it to study the implications for stability of competition and macroprudential policies affecting the traditional banking sector. Even if regulated banks are insulated from the risk of run thanks to deposit insurance schemes and access to the discount window of the lender of last resort, changes in concentration and in macro-prudential policies affecting them have implications for stability through the macroeconomic equilibrium. Increasing concentration in the regulated sector has a positive effect on stability through the franchise value and recapitalization channels but a negative effect through the price-drop effect, the net effect being ambiguous. Tighter capital requirements on regulated banks have in general a stabilizing effect because regulated banks are better capitalized which implies that should the shadow banking sector disappear, their capital investment and therefore asset prices would drop by less. However, I also find that tightening capital requirements at the same time as an adverse shock hits might actually increase instability through the price-drop channel because the regulated banking system will need more time to recapitalize thereby depressing the run asset price.

A complete welfare analysis weighs the potential stabilizing advantages of lower competition with the output costs introduced by *market power*: more competition increases output because the system as a whole intermediates a larger proportion of the capital stock. A social planner should balance a potentially more efficient in the long-run with less stability. And this trade-off is all the more acute as entry takes time.

My qualitative findings are consistent with empirical evidence of the dynamics of the financial system in the aftermath of an adverse shock as well as with the findings of the effect of concentration on stability. Regarding the former, the equilibrium transition features a period of low franchise value, high leverage and deleveraging after an adverse macroeconomic shock, during which the risk of run is high. Regarding the relationship between concentration and stability, I do find an a priori ambiguous rela-

²Because productivity is time-varying, this statement says that the distribution of net worth to asset ratios in less concentrated sectors dominates the ones in more concentrated sectors in a first order stochastic dominance sense.

³This *paradox of safety* echoes the paradox of volatility in Brunnermeier and Sannikov (2014) where low fundamental volatility may actually lead to high endogenous volatility.

⁴This analysis can be found in appendix C.1.

tionship between concentration and risk, although more forces work in the sense of more concentration implying more stability. Most paper find indeed a negative relationship using many different methodologies and measure of risk and concentration (Beck et al., 2003, 2013; Keeley, 1990) and more recently Degl’Innocenti et al. (2018) for investment banks. Contestability is shown to have a positive effect on stability Beck et al. (2013) consistent with the *entry channel*. Table 3 in Freixas and Ma (2015) provides a summary of the empirical findings and Zigravova and Havranek (2016) provides a meta-analysis of the literature.

My analysis gives rise to several new testable predictions and further empirical research needs to be done. In light of my first contribution, I highlight the limits of empirical analyses that do not take into account the state-dependency of banks’ behaviors. A key prediction of my analysis is indeed that the optimal capital investment of banks remains an endogenous multiplier of net worth for small shocks around the steady-state but does not for large shock. One implication of this behavior is that after a large shock to net worth, run may occur in the short run and the decline in intermediation happens with a delay, once the banks have reached the safe zone. In light of my second contribution, I highlight the importance of two channels that have been overlooked by the current literature: the recapitalization channel according to which more concentrated markets have the ability to recapitalize more quickly and escape the run zone and the price-drop channels according to which concentration influences the size of the drop in asset prices when a run occur.

Plan. In section 2, I present the benchmark model with systemic run. In section 3, I describe the properties of the equilibrium and solve for the transition path without aggregate productivity shock. I discuss the different channels through which concentration shape stability as they arise in the analysis. In section 4, I illustrate the working of the model with numerical examples reintroducing. Section 5 concludes.

2 Oligopolistic Banks with Systemic Run

2.1 Environment

Time is continuous. There are two stages within an instant. In the first stage, the two types of agents, bankers and households, make their portfolio and accumulation decisions. In the second stage, a run game among households takes place.

There is a fixed supply of a real asset called capital, denoted $\bar{K}$, as in GK, freely tradable at price q on a centralized market. Each unit of capital provides a flow z of good per unit of time.

There are two sources of uncertainty. Productivity z can take two values, $\underline{z} < \bar{z}$. When it is low (resp. high), it switches to low (resp. high) with arrival rate $\zeta(\bar{z})$.⁵ I define the function ζ as follows $\zeta(z) = \underline{\zeta}$ if $z = \underline{z}$ and $\zeta(z) = \bar{\zeta}$ if $z = \bar{z}$. The second source of uncertainty is the endogenous risk of a run. In this section, I restrict attention to systemic run, which is a run on the entire financial system. I posit that in equilibrium systemic run occurs with arrival rate λ^* , which will be determined endogenously.⁶

Figure 3.1 summarizes the different actors, their balance-sheet and their financial relationships.

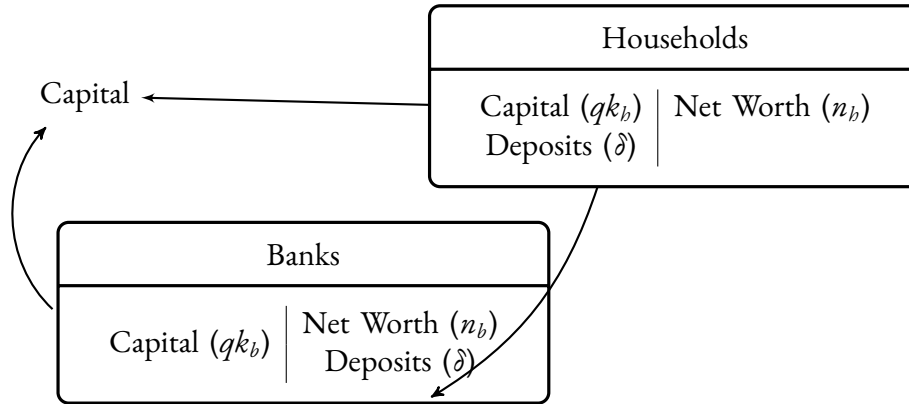


Figure 3.1: Schema of financial system

⁵Discrete jumps in TFP are needed to trigger discrete jump in asset prices and in banks net worth in order for the economy to experience runs. If the exogenous process were continuous in time, like most of the macrofinance literature assumes, banks would continuously adjust their net worth to avoid the crisis zone.

⁶I need to posit its existence to write down in a simple way the households' and banks' value function, and I show that λ is indeed well-defined under some conditions.

2.2 Households

There is a mass one of identical households. They are risk-neutral and maximize the discounted flow of utility from consumption, with instantaneous utility function $b(c)$, with $b'(\cdot) > 0$, $b''(\cdot) \leq 0$. They take the price of the asset q as well as its price in case of a run q^R as given. In the first stage of any instant, they can invest their net worth n_b in two kinds of assets: the real asset k_b and deposits at bank b , δ_b . I omit time subscripts when to confusion results.

$$n_b = qk_b + \sum_b \delta_b$$

Each unit of capital generates a stream of income z and possibly capital gains dq per unit of capital. Each unit of deposits generates interest income r_b . If the bank has to be liquidated because of a run, the household recovers a proportion ψ_b of its deposits at bank b , determined endogenously. Households are subject to a convex cost of managing capital in terms of good as in GK. The convex cost implies that it is increasingly costly at the margin for households to manage capital directly. This cost ensures that financial institutions have a meaningful role in the intermediation of funds without forbidding households from directly investing in real assets.

$$f(k_b) = \frac{\alpha}{2}(k_b)^2$$

Law of motion of net worth. When neither productivity nor run shocks occur, the net worth n_b of a household b evolves according to

$$\dot{n}_b = (\dot{q} + z)k_b + \sum_b \delta_b r_b - f(k_b) - c_b$$

where c_b denotes household consumption. In case of a discrete productivity change Δz , asset prices and net worth discontinuously jump. Denoting q^z the post z -shock asset price, $\Delta n_b = \Delta q^z k_b$ with $\Delta q^z = q^z - q$. And the post z -shock net worth is: $q^z k_b + \sum_b \delta_{b,b}$

When a systemic run occurs, households experience a change of net worth stemming from both the

change in asset prices and the loss on deposits at all banks:

$$\Delta n_b = \Delta q^R k_b + \sum_b \delta_{b,b}(\psi_b - 1)$$

where $\Delta q^R = q^R - q$ is the discontinuous drop in asset prices at the time of the run and ψ_b is the recovery rate per unit of deposit at bank b when a systemic run occurs.

Hamilton-Jacobi-Bellman equation. The aggregate state variables of this economy are the vector of banks net worth $\mathbf{n}$, the value of productivity z and the number of banks $\mathcal{B}$. Taking the asset price q , the intensity of run risk λ^* , and the aggregate state variables as given, the following HJB is solution to the household's problem

$$\begin{aligned} \rho H(n_b, \mathbf{n}, z, \mathcal{B}) = & \max_{c, \{\delta\}_{b=1}^{\mathcal{B}}, k_b} b(c) + H_n \left((\dot{q} + z)k_b + \sum_b \delta_b r_b - f(k^b) - c \right) \\ & + \lambda^*(\mathbf{n}, z, \mathcal{B}) \left[H \left(n_b + \Delta q^R k_b + \sum_b \delta_{b,b}(\psi_b - 1), 0, z, 0 \right) - H(n_b, \mathbf{n}, z, \mathcal{B}) \right] \\ & + \zeta(z) (H(n_b + \Delta q^z k_b, \mathbf{n} + \Delta q^z \mathbf{k}, z', \mathcal{B}) - H(n_b, \mathbf{n}, z, \mathcal{B})) \\ & + \sum_b H_{n_b} \dot{n}_b \\ & + \eta (H(n_b, \mathbf{n}_E, z, \mathcal{B}') - H(n_b, 0, z, 0))_{\mathcal{B}=0} \\ \text{with } \dot{n}^b = & q k^b + \sum_b \delta_b \end{aligned}$$

The first term is the flow utility of consumption. The second term corresponds to the deterministic drift in net worth. The third corresponds to the run shock where I posit that the arrival rate λ^* is a function of the aggregate state variables. In case of a run, all banks fail, hence $\mathcal{B} = 0$. I adopt the convention that when all banks fail, the vector of banks net worth is replaced by a 0. It is obviously unsequential. The fourth term corresponds to the productivity shock. The fifth to the deterministic drift in the vector of banks net worth. The last term corresponds to the entrance of new banks after a run. I postpone its explanation to a later section.

Run Game. In the second stage, all household play a run game. Taking the behavior of others as given,

a household decides whether to withdraw all of their deposits from all banks. Since I focus on symmetric equilibria where all households take the same action, I describe payoffs only for individual departures from the (symmetric) equilibrium behavior. Assume first that no one runs. An atomistic individual is able to withdraw their funds from all banks because the required sale of real assets by banks in order to honor their creditor will not move the asset price at all. Doing so has a cost for the individual: they have to give up the interest payment on their deposits. Assume now that everyone runs. Banks have to liquidate sufficient amount of real asset to reimburse their creditors and this moves the asset price. Either they can honor all of their deposits, despite the decline in the value of their asset caused by the drop in asset prices, in which case, not running has a benefit for households: it saves them the interest payments delivered at the next instant. Or banks can't honor their debt, in which case they are liquidated and I assume that the liquidation value of the banks is shared only among households who ran. A household who didn't run would lose all of their deposits.

2.3 Banks

There is a finite number $\mathcal{B}$ of banks indexed by $b \in \{1, \dots, \mathcal{B}\}$. They are owned by a banker, who maximizes the discounted flow utilities from dividends. Instantaneous utility is denoted $u_b(\cdot)$. It is assumed to be HARA with parameters γ, θ :⁷

$$u_b(d) = \xi \frac{(d - \gamma\theta)^{1-\gamma}}{1-\gamma} \quad \text{with} \quad d > 0, \gamma \in [0, 1), \xi > 0, \theta \geq 0 \quad (3.1)$$

The assumption that utility is concave is a modeling device aimed at capturing costly equity issuance. Concavity in dividends is indeed a necessary condition for net worth to matter, otherwise a bank would issue equity to jump directly to the steady-state of net worth. The assumption that $\gamma < 1$ makes sure that utility and the value function are always positive - which guarantees that the latter be always bounded from below, a required property I come back to when describing the HJB of a bank.

A bank chooses dividends d_b and investment in real asset k_b , thereby a path of net worth n_b and an

⁷ $\gamma \rightarrow +\infty$ gives the CARA case, $\theta = 0$ gives the usual CRRA case and $\theta > 0, \theta < +\infty$ is the decreasing relative risk aversion case.

issuance of deposits δ_b . They behave a la Cournot, *i.e.* they make their investment in capital and their accumulation of equity decisions taking others' capital and accumulation decisions as given but internalizing their impact on returns to capital, on asset prices and on the probability of a run. Contrary to households, they are financial experts and do not face capital management costs.

Law of motion of banks' net worth. Real asset holdings, qk_b are financed either by deposits, δ_b , or by net worth, n_b . I omit banks indexes when no confusion results.

$$n = qk - \delta$$

As for the household, the general law of motion of equity

$$dn = (dq + z)k - r\delta - d$$

can be rewritten separately for the case without shock and the cases with shock. If neither run nor productivity shocks occurs, the law of motion is deterministic

$$\dot{n} = (\dot{q} + z)k - r\delta - d$$

When there is a run, prices jump discretely and net worth evolves according to

$$\Delta n = k\Delta q^R$$

where Δq^R denotes the discrete jump in asset price when all banks experience a run. After a successful run - and all runs are successful in equilibrium -, all banks are liquidated. Each banker is forced to exit and gets v^R . I think of v^R as the value of becoming an atomistic household, which is essentially $v^R = 0$ from the point of a banker who is a large agent in the economy. In the calibration, I set it to 0.

Hamilton-Jacobi-Bellman equation. The state of the economy is summarized by the vectors of banks' net worth $\mathbf{n}$, the level of productivity z and the number of banks $\mathcal{B}$. Denote $\mathbf{n}_{-b}$, $\mathbf{k}_{-b}$, $\dot{\mathbf{n}}_{-b}$ the vector

of net worth, capital investment and change of net worth of all other banks except b . Taking these three vectors as given, a bank's b value function solves the following HJB equation

$$\begin{aligned} \rho v(n, \mathbf{n}_{-b}, z, \mathcal{B}) = & \max_{d,k} u(d) + v_n((\dot{q} + z)k - r\delta - d) + \sum_{j \neq b} v_{n_j} \dot{n}_{-j} \\ & + \lambda(k, \mathbf{k}_{-b}, n, \mathbf{n}_{-b}, z, \mathcal{B})(v^R - v(n, \mathbf{n}_{-b}, z, \mathcal{B})) \\ & + \zeta(z) [v(n + \Delta q^z k, \mathbf{n}_{-b} + \Delta q^z \mathbf{k}_{-b}, z', \mathcal{B}) - v(n, \mathbf{n}_{-b}, z, \mathcal{B})] \end{aligned}$$

The first term is the flow utility of dividends. The second term corresponds to the deterministic drift in net worth. When choosing dividends and capital investment, a bank will internalize the impact of their action on the returns $\dot{q}$ and the interest rates paid on deposits r . The third to the deterministic drift in the vector of other banks' net worth, which bank b takes as given. The fourth corresponds to the run shock where $\lambda(k, \mathbf{k}_{-b}, n, \mathbf{n}_{-b}, z, \mathcal{B})$ is the arrival rate of a systemic run. It depends on the aggregate state variables of the economy and on the banks' capital investments. $\lambda^*(n, \mathbf{n}_{-b}, z, \mathcal{B})$ which I used in the households problem, is the equilibrium arrival rate, once banks have chosen their optimal capital investment. In case of a run, a banker gets v^R . The assumption that utility is always positive and therefore that the value is bounded away from 0 is to ensure that $v^R - v(n, \mathbf{n}_{-b}, z, \mathcal{B}) < 0$ at every point in the state space. Intuitively it means that a banker would never want to voluntarily leave banking and get the outside value v^R . The fifth term corresponds to the productivity shock.

2.4 General Equilibrium Conditions

Recovery rate. The recovery rate of deposits at bank b is the amount that depositors get back after liquidation of the bank per unit of deposits. It is given by:

$$\psi_b = \max \left(\frac{q^R k_b}{\delta_b}, 1 \right)$$

Market clearing. The good, deposits and capital markets must clear. By Walras' law, it is sufficient that

the deposits and capital market clear:

$$\forall b \in \{1, \dots, \mathcal{B}\} \quad \int \delta_{b,b} dh = \delta_b \quad (3.2)$$

$$\sum_{b=1}^{\mathcal{B}} k_b + \int k_b dh = \bar{K} \quad (3.3)$$

Entry and run price. After a run has occurred, $\mathcal{B}'$ households are simultaneously and randomly drawn with intensity η per unit of time to become the new bankers. η and $\mathcal{B}'$ are parameters. Entry occurs only after a run. In normal times, the number of banks remains constant. I further assume that $\mathcal{B}' = \mathcal{B}$, *i.e.* the number of new banks randomly created at some point after a run is the same as the number of banks prevailing before the run.

I microfound neither the timing nor the number of entrants. Dealing with a free-entry condition turns out to be rather complicated in a dynamic setting with a finite number of banks because it would break the symmetry among banks on the equilibrium path that I need for tractability. More explicitly, my assumption amounts to saying that a) once $\mathcal{B}$ banks are operating, at no point of the state-space would an additional bank be willing to enter ; b) the cost of entry is such that $\mathcal{B}$ simultaneously entering is optimal for private agents ; c) opportunity to enter occurs randomly over time.

I call the "immediate entry", the case corresponding to $\eta \rightarrow +\infty$, in which the new banks enter immediately after the liquidation of the incumbents, and the "no entry", the case corresponding to $\eta = 0$, in which no bank ever enters after a run.

I assume that, whenever banks enter, they are exogenously endowed with a level of equity equal to n_E , with E stands for entry. I do not model the origin of this endowment and set it to 0 throughout the paper.

Finally, the run price should be consistent with the post-run macroeconomic conditions, which is:

$$q^R(\mathbf{n}, z, \mathcal{B}) = q(0, z, 0) \quad (3.4)$$

where I use the convention that banks' net worth is zero whenever there is no banks currently operating. In the aftermath of a run, and as long as the random draw of new bankers hasn't occurred, all the

capital stock is held by households.

2.5 Equilibrium Definition

An equilibrium is a collection of functions $H(n_b, \mathbf{n}, z, \mathcal{B})$, $\{\delta_{b,b}(n_b, \mathbf{n}, z, \mathcal{B})\}_{b=1}^B$, $c(n_b, \mathbf{n}, z, \mathcal{B})$, $k_b(n_b, \mathbf{n}, z, \mathcal{B})$, $v(n, \mathbf{n}_{-b}, z, \mathcal{B})$, $k(n_b, \mathbf{n}_{-b}, z, \mathcal{B})$, $\delta(n_b, \mathbf{n}_{-b}, z, \mathcal{B})$, $d(n_b, \mathbf{n}_{-b}, z, \mathcal{B})$, $\lambda^*(\mathbf{n}, z, \mathcal{B})$, $q(\mathbf{n}, z, \mathcal{B})$ such that i) taking prices as given, $H(n_b, \mathbf{n}, z, \mathcal{B})$, $c(n_b, \mathbf{n}, z, \mathcal{B})$, $k_b(n_b, \mathbf{n}, z, \mathcal{B})$, $\{\delta_{b,b}(n_b, \mathbf{n}, z, \mathcal{B})\}_{b=1}^B$ are solutions to the households' problem, ii) taking other bank's behavior as given, $v(n, \mathbf{n}_{-b}, z, \mathcal{B})$, $k(n_b, \mathbf{n}_{-b}, z, \mathcal{B})$, $d(n_b, \mathbf{n}_{-b}, z, \mathcal{B})$, $\delta(n_b, \mathbf{n}_{-b}, z, \mathcal{B})$ are solutions to the bank's problem, iii) the deposits, asset and goods markets clear, iv) the run price is given by 3.4 is compatible with the post-run equilibrium conditions and v) $\lambda^*(\mathbf{n}, z, \mathcal{B})$ is consistent with the equilibrium of the run game.

3 A Symmetric Equilibrium with Systemic Run

In this section, I solve for the equilibrium of the model described in the previous section in the special case where there is no productivity shock, $\zeta(z) = 0$, and banks have the same amount of net worth at every point in time. I describe the properties of a transition path with only run risk and come back to the case with productivity risk in the numerical examples. The restriction to symmetric path of net worth is essentially for tractability issue. It allows us to derive the main intuitions and mechanisms through which the degree of concentration influences the likelihood of bank runs. All proofs can be found in appendix.

3.1 Run Game and Arrival Rate of Run

I first solve for the equilibria of the run game happening in the second stage of an instant. The existence of multiple equilibria is a function of the aggregate level of equity should the entire system be liquidated. A run might occur whenever the system is solvent if there is no run and insolvent if there is a run. Denote Δ the sum of deposits of all banks, q^R , N^R , K^R and q , N , K the price of the asset, the sum of net worth of all banks, the sum of capital investment of all banks just after and before the systemic run. There are

multiple equilibria whenever

$$N^R \equiv N + (q^R - q)K < 0$$

Denoting Ψ the aggregate recovery rate, the second inequality becomes

$$\Psi = \frac{q^R K}{qK - N} = \frac{q^R K}{\Delta} < 1$$

The lower the equity over capital ratio, the more leveraged the system, the more likely the existence of multiple equilibria. Whenever there are multiple equilibria, I assume that a sunspot with constant arrival rate $\bar{\lambda}$ coordinates depositors on the run equilibrium.⁸ The unconditional arrival rate of a run is therefore given by $\lambda = \bar{\lambda}_{\Psi < 1}$.

From the point of view of an individual bank b taking others' bank net worth and capital as given, when all other banks are symmetric, this condition is:

$$\lambda(k, \mathbf{k}_{-b}, n, \mathbf{n}_{-b}) = \bar{\lambda}_{((B-1)n_{-b} + n_b + (q^R - q)((B-1)k_{-b} + k_b)) < 0} \quad (3.5)$$

where I have dropped the dependence of $\lambda(\cdot)$ on $\mathcal{B}$ and z .

3.2 Households Optimal Decisions and Pricing Equations

The dynamics of asset prices and the interest rate on deposits at bank b should be consistent with the households optimal portfolio and intertemporal allocation.

Pricing equations. Assuming that households are risk neutral and no productivity shock $\zeta(\cdot) = 0$,

⁸Should $\bar{\lambda}$ be related to fundamentals? There isn't a consensus in the literature regarding the best way to model the likelihood of runs arising from coordination failure among depositors. Some papers choose to make the sunspot probability independent of any endogenous variables, e.g. [Gertler et al. \(2016, 2017\)](#), while others make the probability of a run an increasing function of some measure of risk in an *ad-hoc* manner, usually leverage ([Gertler and Kiyotaki, 2015](#)). The former is obviously an extreme case of the latter, but it has the advantage of being transparent and simple and to avoid having to take a stand on the exogenous relationship between the probability of a run and the fundamentals of the system. In one of the numerical exercise, 4.5, I show that my results are qualitatively similar when the arrival rate is a smooth function of the fundamentals.

their F.O.C. write

$$\dot{q} + z - f'(k_b) = -\lambda \Delta q^R + \rho q \quad (3.6)$$

$$\forall b \in \{1, \dots, \mathcal{B}\} \quad r_b = \lambda(1 - \psi_b) + \rho \quad (3.7)$$

where the first condition is the asset pricing equation and the second is the traditional Euler equation for deposits adjusted for the probability of a run and the recovery rate. Through these equations, the bankers will internalize the impact of its choices on prices and returns.

3.3 Bank's Capital Investment and Deposit Decisions

In this section I solve for the optimal choice of capital investment and deposit of a bank. I fix the vectors of banks' net worth, $\mathbf{n}$, and solve for the static problem of a bank taking other banks capital investments as given.

Law of motion of a bank's net worth. Using the F.O.C. of the households 3.6 and 3.7, one can rewrite the law of motion of a bank in case of no run shock as

$$\dot{n} = (\lambda(\mathbf{k}, \mathbf{n}) + \rho)n + f' \left(\bar{K} - \sum_{j=1}^{\mathcal{B}} k_j \right) k_b - d \quad (3.8)$$

For brevity, I drop the dependence of all the endogenous variables on $\mathcal{B}$ and z when no confusion results.

This shows how the net excess return on capital depends on the choice of capital by the individual banker through the aggregate capital investment of banks $K = \sum_{j=1}^{\mathcal{B}} k_j$. λ also depends on k_b , in a discontinuous fashion, as shown in 3.5.

Capital investment decision. Ignoring the terms that do not depend on k , the optimal capital invest-

ment is solution to

$$\max_k v_n \left((\rho + \lambda(k, \mathbf{k}_{-b}, n, \mathbf{n}_{-b}))n + f' \left(\bar{K} - \sum_j^B k_j \right) k_b - d \right) + \lambda(k, \mathbf{k}_{-b}, n, \mathbf{n}_{-b})(v^R - v(n, \mathbf{n}_{-b})) \quad (3.9)$$

Given the non-linearities introduced by λ , the objective function of a bank at a given instant is piecewise concave with respect to capital, with a downward step at the point where λ jumps up. There are therefore two candidates for optimality. Figure 3.2 illustrates the optimal capital investment for two different levels of n . The optimal investment can be the interior point as in the gray curve or at the corner as in the black curve.

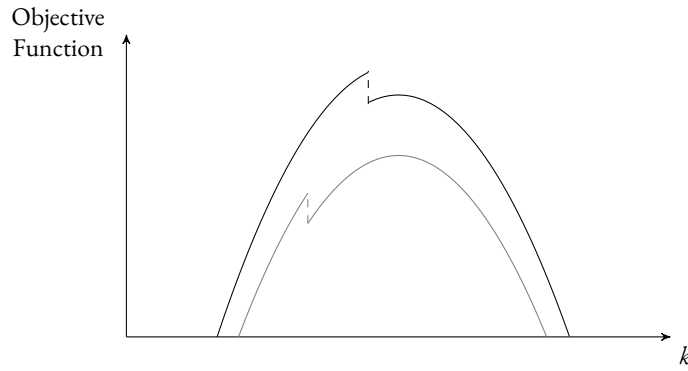


Figure 3.2: Stylized drawing of the objective function

Legend: The black line corresponds to a high net worth, the gray line to a low net worth

The interior solution, denoted k^I for interior, is solution to the following F.O.C.:

$$f' \left(\bar{K} - \sum_{j=1}^B k_j \right) - f'' \left(\bar{K} - \sum_{j=1}^B k_j \right) k_b = 0 \quad (3.10)$$

In a symmetric equilibrium of the Cournot game, and using the assumed functional form for f , 3.10 is

$$k^I = \frac{\bar{K}}{B+1}$$

Each bank holds a fraction $\frac{1}{B+1}$ of the capital stock and the remaining fraction is held by households. k^I is independent of the vector of banks' net worth. When I extend the model to allow for productivity shocks, k^I will depend on the vector of banks' net worth. The corner solution, denoted k^C , is given by the highest capital investment such that $\lambda = 0$. It is not only a function of one's net worth but also a function of the other banks' net worth and capital investment

$$k^C(n, \mathbf{n}_{-b}, \mathbf{k}_{-b}) = \frac{1}{q - q^R} \left(\sum_{j \neq b} n_j + n \right) - \sum_{j \neq b} k_j = \frac{N}{q - q^R} - \sum_{j \neq b} k_j$$

where N is the sum of banks' net worth as defined above. I denote this level of capital with superscript C for corner.

The corner investment is a decreasing function of the difference between the current price and the price in a run. In the limit where the recovery rate is complete $\Delta q^R \rightarrow 0$, capital investment and leverage can be infinite without the bank taking any run risk. The ratio $\frac{q^R}{q}$ therefore measures the degree of illiquidity of the financial system aggregate balance sheet. The closer it is to 1, the higher the aggregate liquidity of the financial sector and the less risky high leverage is.

Choosing between the interior and the corner strategy amounts to choosing the one that brings the highest flow of utility. As I will see in the next subsection, the optimal choice of dividends d depends only on the marginal value of net worth, which is independent of the investment strategy at the current instant. Denoting the net return on capital

$$R(k_b, \mathbf{k}_{-b}) = f' \left(\bar{K} - \sum_{j \neq b} k_j - k_b \right) \quad (3.11)$$

the global optimum for capital investment is solution to the following comparison

$$\max \left[\lambda(k, \mathbf{k}_{-b}, n, \mathbf{n}_{-b}, z, \mathcal{B}) \left(n - \frac{v(n, \mathbf{n}_{-b}) - v^R}{v_n(n, \mathbf{n}_{-b})} \right) + R(k^I, \mathbf{k}_{-b}) k^I, \right. \\ \left. R(k^C(n, \mathbf{n}_{-b}, \mathbf{k}_{-b}), \mathbf{k}_{-b}) k^C(n, \mathbf{n}_{-b}, \mathbf{k}_{-b}) \right] \quad (3.12)$$

The decision amounts to a comparison between higher profits and risk-taking. At the interior point, a bank enjoys - in the relevant part of the state-space - more profit

$$R(k^I, \mathbf{k}_{-b}) k^I > R(k^C(n, \mathbf{n}_{-b}, \mathbf{k}_{-b}), \mathbf{k}_{-b}) k^C(n, \mathbf{n}_{-b}, \mathbf{k}_{-b})$$

which gives it an incentive to take on more risks. But by doing so, it also runs the risk of losing their franchise value

$$\frac{v(n, \mathbf{n}_{-b}) - v^R}{v_n(n, \mathbf{n}_{-b})} > 0$$

Franchise value channel and concentration. Banks operating in a more concentrated sector have incentives to take on less risk in order to protect a higher franchise value. However, as I just discussed, the relationship between risk-taking and the degree of concentration is not as simple as it is usually assumed and for some parametrization and some part of the state-space more concentration could actually lead to more risk-taking because large banks make a lot of profit out of investing at the interior point. For my parametrization, the franchise value channel will always go the "right" way.

3.4 Symmetric Cournot Equilibria

I focus on symmetric Cournot equilibria, in which either all banks simultaneously play the corner solution or all play the interior solution. I also assume that banks cannot commit to future actions and therefore restrict attention to static strategies: banks can condition their current action only on current state variables.

By assumption of a symmetric equilibrium path, all banks have the same level of equity, n . Therefore n summarizes the state of the economy. I introduce new notations to denote the value function, the marginal value of net worth, the asset price and corner investment along the symmetric equilibrium path.

When for all $j \in 1, \dots, \mathcal{B}$, $n_j = n$, I define

$$\begin{aligned} v^*(n) &= v(n_b, \mathbf{n}_{-b}) & v_1^*(n) &= v_1(n_b, \mathbf{n}_{-b}) & F^*(n) &= \frac{v^*(n) - v^R}{v_1^*(n)} - n \\ q^*(n) &= q(n_b, \mathbf{n}_{-b}) & k^{C*}(n) &= k^C(n_b, \mathbf{n}_{-b}, \mathbf{k}_{-b}) \end{aligned}$$

Along a symmetric equilibrium path and in a symmetric Cournot equilibrium, the corner capital investment is:

$$k^{C*}(n) = \frac{n}{q^*(n) - q^R}$$

In the rest of section 3, I restrict attention to equilibrium paths that satisfy the following natural properties: (Equilibrium Restrictions) I restrict attention to equilibrium paths where the following is true

1. All banks have the same amount of net worth.
2. $F^*(n)$ is differentiable with $(F^*)' > 0$.
3. There exists $\varepsilon > 0$ such that $k^{C*}(n)$ is differentiable with $(k^{C*})' > \varepsilon$.
4. Banks' capital investment strategies are Markovian, symmetric and stationary.

The following proposition characterizes the sustainable symmetric Cournot equilibria along an equilibrium path satisfying the set of assumptions 3.4.

Proposition 3.1. *Under the set of assumptions 3.4, there exists a unique $\{n_1^C, n_1^I\}$ and n^{SS} with $n_1^C \leq n_1^I < n^{SS}$ such that*

- *If $0 \leq n < n_1^C$, the only symmetric Cournot equilibrium is one in which all banks play the interior solution, k^I .*
- *If $n_1^C \leq n < n_1^I$, all banks playing the interior investment, k^I , or all banks playing the corner solution, $k^{C*}(n)$, are two possible symmetric equilibria*

- If $n_1^I \leq n \leq n^{SS}$ the only symmetric Cournot equilibrium is one in which all banks play the corner solution, $k^{C*}(n)$

This proposition says that as the equity of banks increases, the set of sustainable symmetric equilibria goes from interior to corner, with some overlap for intermediate level of net worth. For low level of net worth, banks have a strong incentive to play the risky strategy while for high level of net worth they prefer to play safe. This stems from the fact that the franchise value is increasing in net worth: the higher its equity, the stronger the incentives to play safely and avoid the risk of run.

Figure 3.3 illustrates the best response functions of a bank when all other banks play the corner solution (black line) or the interior solution (gray line) for all values of net worth.

In between the two thresholds both equilibria are sustainable. If all banks play safe, it is optimal to play safe and if all banks play interior and risky it is optimal to play interior as well. This is an interesting form of complementarities among banks. It stems from the fact that the probability of run depends on the system-wide leverage and not on individual leverage. If all banks have low leverage, an individual bank can eliminate the risk of run without restricting its investment as much as it would have to should all other banks have high leverage.

For the sake of simplicity, I pick any $n_1 \in [n_1^C, n_1^I]$ as the threshold value below which all banks play interior and above which all banks play the corner solution.

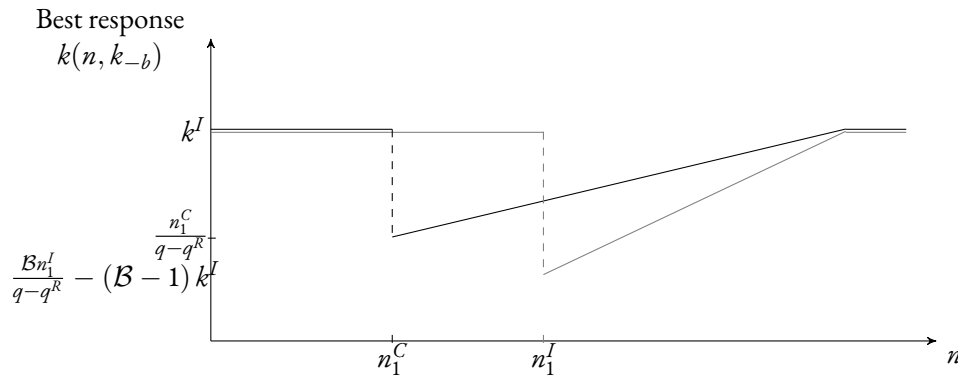


Figure 3.3: Stylized drawing best response functions

Legend: The black line displays the best response of a bank when all other banks play corner.

The gray line displays the best response of a bank when all other banks play interior.

Corollary 1. *Under assumption 3.4, there exists $n_1 \in [n_1^C, n_1^I]$ with $n_1 > 0$ such that the capital investment function $k^*(n)$ given by*

$$k^*(n) = \begin{cases} k^I & \text{if } n < n_1 \\ k^{C*}(n) & \text{if } n_1 \leq n < n^{SS} \\ k^I & \text{if } n^{SS} \leq n \end{cases}$$

is compatible with an outcome of a Cournot game at every point along the equilibrium path.

Furthermore, in such an equilibrium path, the unconditional arrival rate of systemic runs $\lambda^*(n)$ is given by

$$\lambda^*(n) = \begin{cases} \bar{\lambda} & \text{if } n < n_1 \\ 0 & \text{if } n \geq n_1 \end{cases}$$

In the rest of section 3, I restrict to equilibrium paths where capital investment is of the form given by corollary 1. I call the part of the state space below n_1 the run zone, between n_1 and n^{SS} the safe zone, and above n^{SS} the steady-state zone.

Coordination failure. The existence of multiple equilibria to the Cournot game played by banks for intermediate level of net worth implies that coordination failure might occur in equilibrium. Consider the implications for expected output of a small increase in the size of the run zone dn_1 . I show in section 4 that q exactly measures expected output. On the one hand it increases the time spent by the system in the run zone by $dt = \frac{dn_1}{\dot{n}(n_1)}$. Hence it increases the risk that the system experiences a run which implies the following expected output cost: $\bar{\lambda}(q^R - q^*(n_1))dt$ On the other, spending less time in the safe zone increases current output by raising the scale of intermediation by the financial system: $\frac{\alpha}{2} \left[\left(\frac{\bar{K}}{B+1} \right)^2 - (\bar{K} - k^{C*}(n_1))^2 \right] dt$.

It is *a priori* ambiguous which effect is stronger. Everything else equal, one would want to reduce the size of the run zone if runs occur with a high probability $\bar{\lambda}$, and if a systemic run is very disruptive for

expected output $q^R - q^*(n_1)$. Although this coordination failure is a feature of the equilibrium, it is independent of the level of competition and for that reason I do not discuss it further.

3.5 Steady-State, Dividend and Accumulation Policy

I now turn to the accumulation and dividends policies of banks. I first solve for the steady-state of the model.

Steady-State. In steady-state, banks invest at the optimal interior and their net worth is enough to make the optimal interior, k^I , safe.

Lemma 1. *The interior steady-state with positive net worth is given by*

$$\begin{aligned} k^{SS} &= \frac{\bar{K}}{\mathcal{B} + 1} & n^{SS} &= k^{SS}(q^{SS} - q^R) \\ q^{SS} &= \frac{z - f'(\bar{K} - \mathcal{B}k^{SS})}{\rho} & d^{SS} &= \rho n^{SS} + f'(\bar{K} - \mathcal{B}k^{SS}) k^{SS} \end{aligned} \quad (3.13)$$

Lemma 1 doesn't provide steady-state expressions as a function of primitives because q^R is an endogenous object, which has not been solved for yet. I specify that this steady-state is the one with positive net worth because there also exists an (unstable) steady-state with negative net worth.

The steady-state expression for capital investment makes clear that concentration has an efficiency cost due to market power: large firms restrict their capital investment which lowers output. Total output per unit of time net of management cost is

$$\bar{K}z - f(\bar{K} - \mathcal{B}k^{SS}) = \bar{K}z - \frac{\alpha}{2} \left(\frac{\bar{K}}{\mathcal{B} + 1} \right)^2$$

The cost is a decreasing function of the number of banks and an increasing function of α , which governs the convexity of the households' cost function. The more banks there are, the more competition and the less the distortion from market power. The higher α , the more convex it is at the margin for households to manage capital, the higher the output cost for a given amount of capital holding.

Dividends and saving policy. Denoting $k_1^*(n), q_1^*(n)$ the derivative of $k^C(n_b, \mathbf{n}_{-b}), q(n_b, \mathbf{n}_{-b})$ respec-

tively, with respect to their first component, along the equilibrium path, the following lemma characterizes the path of dividends and accumulation decisions

Lemma 2. *With utility 3.1, along the symmetric equilibrium path, if the path of capital investment, $k^*(n)$, is of the form described in corollary 1, dividends are given by*

- For all $n \in [0, \max(n_{-1}, 0)]$, $\dot{d} = 0$ and $d(n) = (\rho + \bar{\lambda})n + R(k^I)k^I$
- For all $n \in (\max(n_{-1}, 0), n_1)$, $\dot{d} = 0$ and $d(n) = d(n_1)$
- For all $n \in [n_1, n^{SS})$, d solves

$$\gamma \frac{\dot{d}}{d - \gamma\theta} = k_1^{C*}(n) \alpha (\bar{K} - (\mathcal{B} + 1)k^{C*}(n)) \quad (3.14)$$

$$\text{with } k^{C*}(n) = \frac{n}{q(n) - q^R}, \quad k_1^{C*}(n) = \frac{1 - k^{C*}(n)q_1^*(n)}{q(n) - q^R}$$

- For all $n \geq n^{SS}$, $d(n) = d^{SS}$
- For all n , the path of saving is given by

$$\dot{n} = (\rho + \lambda^*(n))n + \alpha (\bar{K} - \mathcal{B}k^*(n))k^*(n) - d$$

$$\text{where } n_{-1} = \frac{d(n_1) - R(k^I)k^I}{\rho + \bar{\lambda}}$$

Lemma 2 says that dividends are constant in the run zone, then increasing in the safe zone until the steady-state has been reached.

Dividends are increasing in the safe zone - $(\bar{K} - (\mathcal{B} + 1)k^{C*}(n)) > 0$ - because the marginal value of net worth is strictly positive: postponing dividends to future periods and building net worth allows the banks to increase its investment capacity in the future and eventually reach the level of net worth for which it is safe to invest at the interior Cournot capital investment. At this point dividends stop increasing.

The Euler equation for dividends 3.14 in the safe zone also displays the derivative of the corner capital investment with respect to a bank own net worth, which itself depends negatively on the size of the price

drop $q(n) - q^R$ as well as on the derivative of the asset price with respect to a bank own net worth. The larger the price drop, the smaller the amount of additional capital a marginal increase in net worth permits and the slower the increase in dividends over time should be. The presence of the derivative of asset prices is another manifestation, in dynamic, of the non-atomisticity of banks: they take into account the positive impact the accumulation of their net worth has on asset prices in equilibrium, which tempers the increase in capital investment that an additional unit of net worth can buy. This derivative has to be consistent with the equilibrium asset price function as I make clear in the section devoted to the analysis on the equilibrium transition path.

In the run zone, the bank understands that maintaining low dividends enables to accumulate net worth and finally reach the safe zone. But locally they do not have any incentives to tilt the time path of dividends either in an increasing or decreasing manner, because their current investment - the interior Cournot - is disconnected from their current level of net worth. That is why they keep a constant time path of dividends in the run zone. The level of dividends in this part of the state-space, $d(n_1)$ is pin-downed by the requirement that the path of dividends has to be continuous at n_1 by concavity of the utility function.

For low enough net worth - below n_{-1} - the constant level of dividends $d(n_1)$ is not sustainable without net worth decreasing, i.e. for $n < n_{-1}$

$$d(n_1) > (\rho + \bar{\lambda})n + R(\mathbf{k}^I)k^I$$

In that part of the state-space - which exists only if $n_{-1} \geq 0$ - banks will also decide to set their dividends constant, but they are not constrained to set it to $d(n_1)$ by the same continuity argument because it will turn out that in equilibrium, they will never reach n_1 . In that zone, banks set the level of dividends to keep net worth constant.

3.6 Transition Path

I analyze the transition path in two steps. I first analyze the dynamics in the safe zone, when $n \in [n_1, n^{SS}]$, gives an existence and uniqueness result for the transition path for an approximation around steady-state

and then characterize the full transition path, including the run zone. For simplicity, I assume that the arrival rate of new entry after a run goes to zero $\eta \rightarrow 0$. It implies that the run price is given by a combination of parameters. The intuitions go through in the more general case. I discuss the implications of $\eta > 0$ in the subsection 3.7.

Transition path in the safe zone. In the safe zone, when $n \in [n_1, n^{SS}]$, capital investment is given by $k^{C*}(n)$. The dynamics of the system is characterized by the following system of coupled differential equations for net worth, dividends and asset prices

$$\begin{aligned} \dot{n} &= \rho n + \alpha (\bar{K} - \mathcal{B}k^{C*}(n)) k^{C*}(n) - d \\ \dot{q} &= \rho q - z + \alpha (\bar{K} - \mathcal{B}k^{C*}(n)) \\ \gamma \frac{\dot{d}}{d - \gamma \theta} &= k_1^{C*}(n) \alpha (\bar{K} - (\mathcal{B} + 1)k^{C*}(n)) \\ \text{with } k^{C*}(n) &= \frac{n}{q - q^R}, \quad k_1^{C*}(n) = \frac{1 - k^{C*}(n)q_1^*(n)}{q - q^R}, \quad q_1^*(n) = \frac{\dot{q}}{\mathcal{B}\dot{n}} \end{aligned} \tag{3.15}$$

and initial condition $n(t_0) = n_1$ and terminal conditions

$$k^{SS} = \frac{\bar{K}}{\mathcal{B} + 1} \quad q^{SS} = \frac{z - \alpha (\bar{K} - \mathcal{B}k^{SS})}{\rho} \quad d^{SS} = \rho n^{SS} + \alpha (\bar{K} - \mathcal{B}k^{SS}) k^{SS}$$

The following lemma establishes that in the safe zone, dividends, prices, capital and net worth increase over time. It also establishes existence and uniqueness for the solution to the linearized model around the steady-state which gives us confidence for the existence and uniqueness of the non-linear solution. I finally give an expression for the speed of convergence whose dependence on the level of competition is discussed later.

- Lemma 3.** 1. *Along any transition path solution to the initial-terminal value problem 3.15, all variables, $(d(t), n(t), k^C(t), q(t))$, are increasing.*
2. *There exists a unique solution $(d(t), n(t), k(t), q(t))$ to the linear approximation of the system 3.15 around $(d^{SS}, n^{SS}, k^{SS}, q^{SS})$ and the (negative of the) rate of convergence of capital to its steady-state is*

given by:

$$\ell_- = \frac{z - \rho q^R - \sqrt{(z - \rho q^R)^2 + \frac{4}{\gamma} (1 - k^{SS} q_1^{SS}) (d^{SS} - \theta \gamma) (\mathcal{B} + 1) \alpha}}{2 (q^{SS} - q^R)}$$

Figure 3.4 represents the system 3.15 in the $(k, \tilde{q})$ plane with $\tilde{q} = q - q^R$. In black I draw the line corresponding to the locus of $(k, \tilde{q})$ compatible with $\dot{\tilde{q}} = 0$. The dotted black line represents the locus of $(k, \tilde{q})$ compatible with the initial condition. Capital is always increasing in the safe zone, so all horizontal arrows point to the right. Asset prices decrease if they start below the black line and increase if they start above it. The system has to start at some point on the dotted black line. It is immediate from the phase diagram why capital and asset prices should be increasing along any transition path. The grey line represents one possible path.

It is not straightforward at all to obtain general conditions over the parameter space such that the forward-backward system of differential equations 3.15 admits one and only one solution. Lemma 3 establishes existence and uniqueness only for the linear approximation of the system but not for the original non-linear system. I check numerically that for my parametrization, a solution to the non-linear system exists and is unique. I discuss further this issue in the proof of lemma 3.

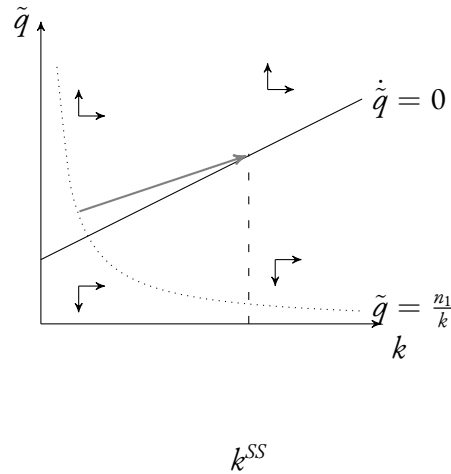


Figure 3.4: Phase diagram of system in safe zone

Full transition path. I am now ready to characterize the equilibrium path of the system.

Proposition 3.2. *Any transition path starting at $n_0 \geq 0$ and satisfying assumption 3.4 is characterized by:*

1. *(Run zone with certain death). If $n_0 \in [0, \max(n_{-1}, 0)]$, $k^*(n) = k^I$, the system operates in the run zone, $\lambda^*(n) = \bar{\lambda}$, n is constant over time, the system is in steady-state, banks eventually experience a run.*
2. *(Run zone with escape). If $n_0 \in (\max(n_{-1}, 0), n_1)$, $k^*(n) = k^I$, the system operates in the run zone, $\lambda^*(n) = \bar{\lambda}$, n increases and reaches in finite time n_1 , at which point the system enters the safe zone*
3. *(Safe zone). If $n_0 \in [n_1, n^{SS})$, then all variables increase and converge to their steady-state value: $k^*(n) = k^{C*}(n) \rightarrow k^{SS}$ and $n \rightarrow n^{SS}$, $q \rightarrow q^{SS}$ and the system is not subject to run.*
4. *(Safe Steady-State) If $n_0 \geq n^{SS}$, then $k = k^{SS}$ and $n = n_0$ forever. It is a steady-state.*

Moreover, all assumptions in 3.4 hold.

On the equilibrium path, the capital investment jumps at n_1 from k^{SS} to $k^{C*}(n_1) < k^{SS}$.

Accelerator effect in the safe zone. The accelerator effect describes the amplification coming from the decrease in lending by banks after a shock that depletes their net worth. There are three main ways the literature relates banks' net worth to their lending behavior. [Kiyotaki and Moore \(1997\)](#); [Gertler and Kiyotaki \(2015\)](#); [Mendoza \(2010\)](#) use a collateral constraint originating in moral hazard, [Brunnermeier and Sannikov \(2014\)](#); [He and Krishnamurthy \(2013\)](#) assumes a fixed amount of equity and therefore occasionally binding constraint and [Adrian and Shin \(2014\)](#); [Coimbra and Rey \(2017\)](#) use Value-at-Risk constraints. My model gives rise to a fourth one: the willingness of banks to optimally restrict their leverage to avoid the risk of a run when they are sufficiently capitalized implies the following rule for leverage:

$$k^{C*}(n) = \frac{n}{q - q^R} \iff \frac{qk^{C*}(n)}{n} = \frac{1}{1 - \frac{q^R}{q}}$$

While the multiplier of leverage in [Gertler and Kiyotaki \(2015\)](#) is constant over the entire state-space, mine depends on the endogenous difference between asset prices and run prices.

Around the steady-state, for small enough shocks to net worth, the economy remains within the safe zone and features a traditional accelerator effect. But a large adverse shock throws the economy into the run zone where this relationship between net worth and banks' investment endogenously breaks down. The amplification mechanism in the run zone is not coming from restriction on capital investment by banks but from the possibility that a run further lowers output. The endogenous amplification of the financial sector therefore takes two different forms over time and the traditional accelerator effect only kicks in with a delay, once the economy reaches the safe zone. This model provides a testable explanation for the fact that during the Great Recession, runs happened between 2007 and 2009 and restriction of lending happened later, in 2009-2010 (Gorton and Metrick, 2009; Kacperczyk and Schnabl, 2009; Covitz et al., 2013).

Recapitalization channel. Banks operating in more concentrated systems earn higher profits. One implication is that they can potentially recapitalize more quickly. Whether they will or not depends on their dividends policy and ultimately on the specification on the utility function.

In order to get a sense of how the strength of the recapitalization channel depends on the parameters of the bank's utility function, I look at the speed of convergence around the steady-state. It is therefore an analysis, not only limited to the safe zone, but also to a neighborhood of the steady-state. The speed of convergence given in lemma 3 indicates the speed at which capital approaches its steady-state value. It depends on the number of banks through 3 channels. To get more intuition, I consider the special case where $\eta = 0$ for which I have (quasi) analytical solutions. Denoting $\tilde{z} = z - \rho q^R$, the expression for the speed of convergence is given by:

$$\ell = \frac{\tilde{z} - \sqrt{\tilde{z}^2 + \frac{4}{\gamma} (1 - k^{SS} q_1^{SS}) (\bar{K}^2 \alpha - \theta \gamma (\mathcal{B} + 1)) \alpha}}{2 \frac{\alpha \mathcal{B} \bar{K}}{\rho (\mathcal{B} + 1)}}$$

Increasing the number of banks increases the denominator which slows down convergence. This is an implication of the price-drop effect which I discuss in the next section: because the price-drop is larger in more competitive markets, each additional unit of net worth buys less additional capital in the safe zone. Increasing $\mathcal{B}$ has an ambiguous effect on the numerator. On the one hand, the term $k^{SS} q_1^{SS}$ gives

more competitive banks more incentive to accumulate net worth - because they don't move asset prices so much - and therefore makes less concentrated market quicker to converge. On the other hand, the term $\theta\gamma(\mathcal{B} + 1)$ reflects the fact that more concentrated market makes higher profits and, if they do not distribute dividends "too early", converges more quickly.

The latter mechanism is what I label the recapitalization channel. As I mentioned, it depends crucially on the parameters of the utility function and governs the speed at which banks distribute dividends. In the case where banks have linear utility and a non-negativity constraint on dividends, dividends are zero until the steady-state is reached and the speed of convergence would vary one for one with the level of profits and therefore one for one with the level of competition. When banks want to smooth dividends, whether the recapitalization channel is strong, weak or negative, depends on their preferences. When $\theta = 0$, corresponding to a CRRA utility function, banks distribute dividends in proportion to their income and the recapitalization channel has no effect on the speed of convergence in the neighborhood of the steady-state. But in the general case, the higher θ - which governs how quickly the intertemporal elasticity (relative risk aversion) increases (decreases) with the level of dividends - the stronger the negative effect increasing the number of banks has on the speed of convergence.

How fast banks distribute dividends as they recapitalize is ultimately an empirical question. And it is an important question, as the recapitalization channel governs the speed at which the system escapes the run zone in which banks might experience run and governs the speed at which the system converges towards the steady-state once in the safe zone, and therefore the output cost along the path through the safe zone. It is unfortunately a question that hasn't received much attention in the empirical (and theoretical) literature.

3.7 Run Price, Entry and Price-Drop Channels

The run price is a key endogenous macroeconomic object because it governs the aggregate liquidity of the balance sheet of the banking system as a whole. It has strong implications for the size of the run zone and for the speed of convergence to the steady-state once in the safe zone. The smaller the difference between the price before a run and the price after a run - the price drop-, the smaller the run zone and the quicker

the convergence to the steady-state.

In this section, I start by showing how the speed of entry governed by η determines the size of the price drop and thereby influences the stability of the financial system, what I call the *entry channel*. Rather paradoxically, increasing the speed of entry of new banks raise the run-price, which stabilizes the system by shortening the run zone. In the limit of immediate entry, runs never happen in equilibrium.

I also show that the level of competition, $\mathcal{B}$, has sizable implications for the price drop, what I call the *price-drop channel*. Because more concentrated markets tend to have relatively depressed asset prices due to the output cost, they display a smaller drop in asset price upon run, which has a stabilizing effect.

Entry channel. In the extreme case of immediate entry where $\eta \rightarrow +\infty$, the equilibrium doesn't feature any run zone, and banks always play the interior solution without taking any risk.

Proposition 3.3. *If $\eta \rightarrow +\infty$, then $n_1, n^{ss} \rightarrow 0$ and the equilibrium is one in which for all $n \in [0, +\infty)$*

$$\lambda^*(n) = 0 \quad k^*(n) = k^{ss} \quad q^*(n) = q^{ss} \quad d(n) = \rho n + \alpha (k')^2$$

Expectations of very fast entry after a run raise the run price to the point that it annihilates the possibility of large price drop and therefore of runs. Banks always invest at the interior k^{ss} and asset prices remain stable at their steady-state level. This is a version of contestable markets in which the existence of potential entrants eliminates the inefficiency caused by a small number of actors. In my case potential entrants do not influence the pricing behavior of these actors but the inefficiency caused by runs on large institutions. I am not the first one to notice this effect, see for example [Gertler and Kiyotaki \(2015\)](#). This theoretical result is consistent with the finding of [Beck et al. \(2003\)](#) and [Beck et al. \(2013\)](#) that contestability increases stability.

The speed of entry, η , could also be reinterpreted in terms of availability of public fund for recapitalization of distressed institutions: by standing ready to recapitalize any institution experiencing a run, the government decreases the likelihood of runs by raising the run price.

I now give a heuristic proof for this result by showing that the size of the run zone has to shrink as $\eta \rightarrow +\infty$, i.e. as entry happens faster. Since after a run $\mathcal{B}$ banks enter after a run at some arrival rate η ,

the run price is given by:

$$q^R = \frac{\eta}{\rho + \eta} q^*(0) + \frac{z - \alpha \bar{K}}{\rho + \eta}$$

The price-drop and a first order approximation of the safe strategy around the point $n = 0$ is $q^*(0) - q^R = \frac{\rho}{\rho + \eta} q^*(0) - \frac{z - \alpha \bar{K}}{\rho + \eta}$ hence $\frac{n}{q^*(n) - q^R} \simeq_{n=0} \frac{n}{q^*(0) - q^R} = \frac{(\rho + \eta)n}{q^*(0)\rho - (z - \alpha \bar{K})}$.⁹

As η tends towards $+\infty$, the price-drop goes to zero and the interior strategy is below the corner strategy for lower and lower level of net worth.¹⁰ Banks can switch all the more quickly to the safe strategy. In the limit, I find the result that the run zone disappears and the interior strategy is always safe.

Price-drop channel. I now discuss the last mechanism through which the level of banks concentration influences stability. A more concentrated banking oligopoly implies smaller price drops in case of run because the level of asset prices in normal times decrease with the number of banks and post-run prices do not decrease as much. For example, consider the extreme case where the speed of entry goes to zero, $\eta \rightarrow 0$, in which case q^R is a parameter independent of the number of banks

$$q^R = \frac{z - \alpha \bar{K}}{\rho}$$

Consider further the case in which $n = n^{SS}$ for which I have an analytic expression for q . The following expression makes clear that more concentrated system are characterized by smaller price drop $q^{SS} - q^R = \alpha \frac{B\bar{K}}{(B+1)\rho}$. The size of the price-drop has strong implications for stability. In a concentrated system, the price-drop is small and banks require less net worth to reach a given level of capital investment and be safe, and therefore implies that playing safe will be more appealing to banks in a more concentrated system. This effect makes more concentrated system more stable by decreasing the size of the run zone. The price-drop also governs how much capital one additional unit of net worth can buy in the safe zone and therefore the speed of convergence to the steady-state, as explained in the previous section.

⁹The derivative of the safe strategy around $n = n_0$ is

$$\frac{\partial \frac{n}{q(n) - q^R}}{\partial n} = \frac{1}{q(n_0) - q^R} \left(1 - \frac{n_0 q_n(n_0)}{q(n_0) - q^R} \right)$$

¹⁰Although $q(0)$ also moves with η , it converges to some finite value $\frac{z - \alpha \frac{\bar{K}}{B+1}}{\rho}$ as η increases.

This last result is rather paradoxical. One would think that runs in more concentrated systems imply larger price-drop. But this intuition, based on the idea that large banks move a lot the market when they fail - which is also the basis for the famous "too big to fail" -, corresponds to the case of individual bank runs which I abstract from in this paper, and does not translate to the systemic run case.

3.8 Concentration and Stability: Summary of Channels

I summarize the different channels through which the degree of concentration shapes the trade-off between efficiency and stability. First note that there are two dimensions of competition: the level of competition as well as the speed of entry of new banks. I start with the second dimension.

Entry channel. I have seen in section 3.7 that the faster the entry of banks after a run - or the more available public funds for recapitalization of distressed banks - the smaller the price-drop, the smaller the run zone and the more stable the system. In the extreme case of immediate entry, the run zone disappears, the system never experiences runs and banks always invest at their interior optimum because potential entrants make large drop in asset prices impossible.

Franchise value channel. Larger and more profitable banks have a more valuable franchise value and are therefore willing to take on less risk. They however weigh the loss of a higher franchise value with the higher profit they could make by taking risk and investing in the interior as discussed in section ???. If the former effect is stronger than the latter - as will be the case for most parametrization-, this channel implies a shorter run zone.

Price-drop channel. Higher concentration in the banking sector implies smaller price drops in case of run because asset prices in normal times are more depressed. Consequently, banks find it more appealing to play safe because it requires less net worth to reach a given level of capital investment. By decreasing the size of the run zone, this effect makes more concentrated systems more stable.

Recapitalization channel. A more concentrated system recapitalizes more quickly because it makes higher profits. It can therefore not only escape the run zone more quickly making the system more stable but it also converges to the steady-state with interior investment faster which is good from an efficiency point of view since in the safe zone output is depressed. The strength - and even the sign - of this channel

is however dependent on the dividends policy of banks and ultimately the parameters characterizing the banks' preferences.

Market power channel. A more concentrated system will generate greater output loss because of its lower level of investment in capital, as discussed in section 3.5. This statement is true in both specifications of entry. Therefore, with immediate entry, since concentration has no effect on stability, there is no doubt that the welfare maximizing policy is to make entry easy to increase the number of banks in the financial system. However as entry happens more and more slowly, one has to weigh this efficiency argument with the stability implications of high competition.

4 Numerical Examples

In this section I provide some suggestive numerical examples to illustrate the workings of the model. Given that households have linear utility, a natural measure of welfare that puts zero weight on bankers, is expected discounted lifetime income. It turns out that the price of the asset is exactly the expected discounted sum of future flow of output. A direct corollary is that the asset price is a sufficient statistics for welfare. Regarding my measure of instability, I look at the cumulative probability of a run, which I define as the probability that a run happens by the end of the transition.

4.1 Parameter Choices

Table 3.1 lists the parameter's value in the benchmark calibration assigned to $\rho, \gamma, \theta, z, \bar{\lambda}, \bar{K}, \alpha, v^R$. I set a period to be a year. I set the discount rate, ρ to 5%, a standard value in the literature and γ the risk-aversion parameter of bankers to 0 in absence of clear estimates from the literature. I calibrate α to match a 2% interest margin. I set the benchmark number of banks to 5 to match a HHI of 2000: which is on the higher end of the range of HHI reported by [Cetorelli et al. \(2007\)](#), but consistent with my focus on concentrated markets. Incidentally 5 is also the number of the largest US investment banks before the crisis, the main broker-dealers of the US shadow banking ([Adrian and Shin, 2010](#)). I set v^R to 0 to capture the fact that if a banker is forced to liquidation and becomes an household, their payoff will be of an order

of magnitude smaller given that they go from being a large actor to being a minuscule one. I set $\bar{\lambda}$ to 3% to get a cumulative probability of run at the beginning of the transition path of 55% consistent with the finding of Valencia and Laeven (2008).

Parameters	ρ	γ	θ	$\bar{\lambda}$	α	$\bar{K}$	$\mathcal{B}$	v^R
Value	.05	.5	0	.03	.02	10	5	0

Table 3.1: Parameters

4.2 Increasing Concentration: $\mathcal{B}$

Policy rules. Figure C.7 displays the policy rule for individual bank capital investment, the implied total capital investment of all banks and saving together with the equilibrium asset price and implied leverage for the set of relevant level of net worth, for three different degrees of concentration, $\mathcal{B} = 1, 3, 5$. My benchmark is in black, and the lightest gray corresponds to the monopoly case. The three zones of the state-space are easiest to see on the capital investment figure, (a). For low level of net worth, the optimal capital investment is interior and high, and the bank is taking on the risk of run. In this zone leverage is high and decreasing. When n passes the threshold n_1 , capital investment optimally and discontinuously drops to a safe level. For high level of net worth, the interior investment becomes safe. The optimal saving policy is to save all profits until the steady-state has been reached. The discontinuity in the saving policy at the point n_1 corresponds to the drop in profits when the system enters the safe zone.

In this example, the threshold net worth n_1 at which the system switches to the safe zone is similar across different level of concentration. The fact that banks in less concentrated systems, despite having smaller a balance sheet, switch at the same level of net worth as banks in more concentrated systems stems from 1-the *price-drop effect* that makes it more costly in terms of net worth for a bank to reach the same level of capital investment 2-the *franchise value effect*. These two effects are enough to keep the net worth similar across degrees of concentration. This obviously has far reaching consequences for stability along the transition path.

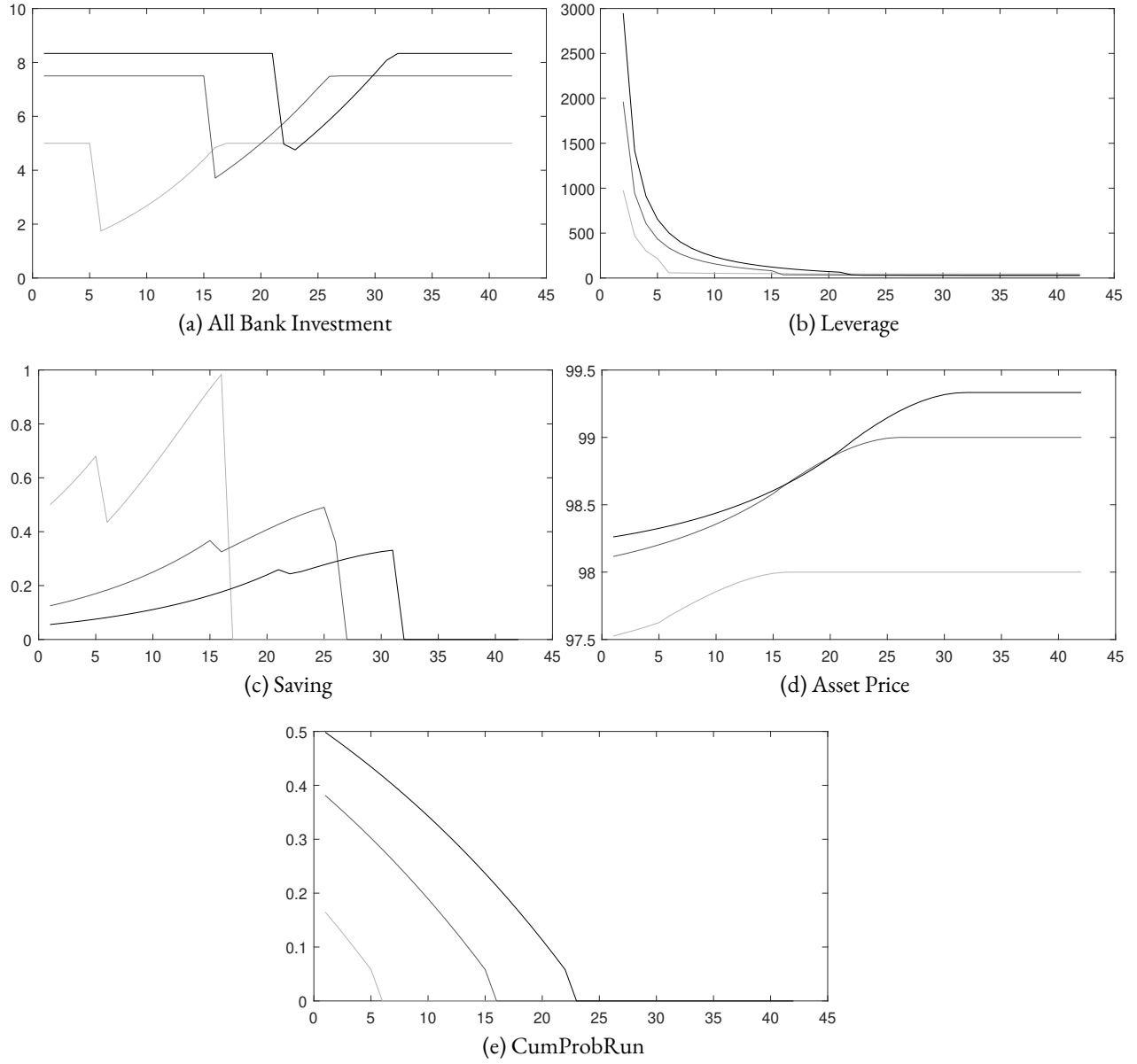


Figure 3.5: Transition path as a function of time for $B = 1, 3, 5$

Transition path. Figure 3.5 displays the transition path through time of the total amount of capital intermediated by banks, leverage, saving, asset prices and the cumulative probability of a run, defined as the probability that a run occurs by the end of time. Given that the threshold n_1 is similar across systems, the length of their respective run zone is comparable, hence the amount of time spent in it depends only on the speed at which banks travel through it, i.e. on *the recapitalization effect*. which is determined by their saving and in turn by their profits. A more concentrated system reaches the safe zone, and then

its steady-state, much faster than my benchmark specification. In this example, looking at 3.5c and 3.5e, going from $\mathcal{B} = 5$ to 1 implies a reduction of the time spent in the crisis by a factor 4 and of the cumulative probability of a run by a factor 3.

4.3 Increasing the Speed of Entry: η

In order to exemplify the effect of the speed of entry, figure 3.6 displays the transition path for the total amount of intermediated capital, asset prices and the cumulative probability of a run for three different values of η . The black line is again my benchmark $\eta = 0$, the middle gray is $\eta = .05$ while the lighter gray represents $\eta = .1$ which corresponds to an average time before entry of 10 years. To compute the transition path, I assume that the entry level of net worth is $n_E = 0$.

As the speed of entry η increases, the run price, given by

$$q^R = \frac{\eta}{\rho + \eta} q(0) + \frac{z - \alpha \bar{K}}{\rho + \eta}$$

increases, through a direct effect - the weight on $q(0)$ increases relative to the weight on $\frac{z - \alpha \bar{K}}{\rho + \eta}$ but also through an indirect effect coming from a higher $q(0)$, which can be seen on 3.6b. The latter effect stems from the fact that in general equilibrium the run zone is smaller, and the convergence to the safe zone and then to the steady-state occurs more quickly, which raises the $q(0)$. The higher run price, q^R implies a smaller price-drop, which makes bank willing to switch more quickly to the safe investment strategy, as displayed in 3.6a. The run zone becomes smaller as well as the safe zone: the system travels faster to the steady-state. Consequently, welfare 3.6b, increases with η , and instability 3.6c decreases.

4.4 Dividends Policy and the Recapitalization Channel: γ

In the example shown in 4.2, I had assumed that $\gamma = 0$ and $d > 0$ so that banks didn't distribute dividends until the marginal utility had reached ξ . Assuming $\gamma = 0$ implies that the recapitalization channel will be very strong because all profits—which increase with concentration—are being kept for the accumulation of equity. I show now that allowing for a higher γ doesn't change qualitatively the

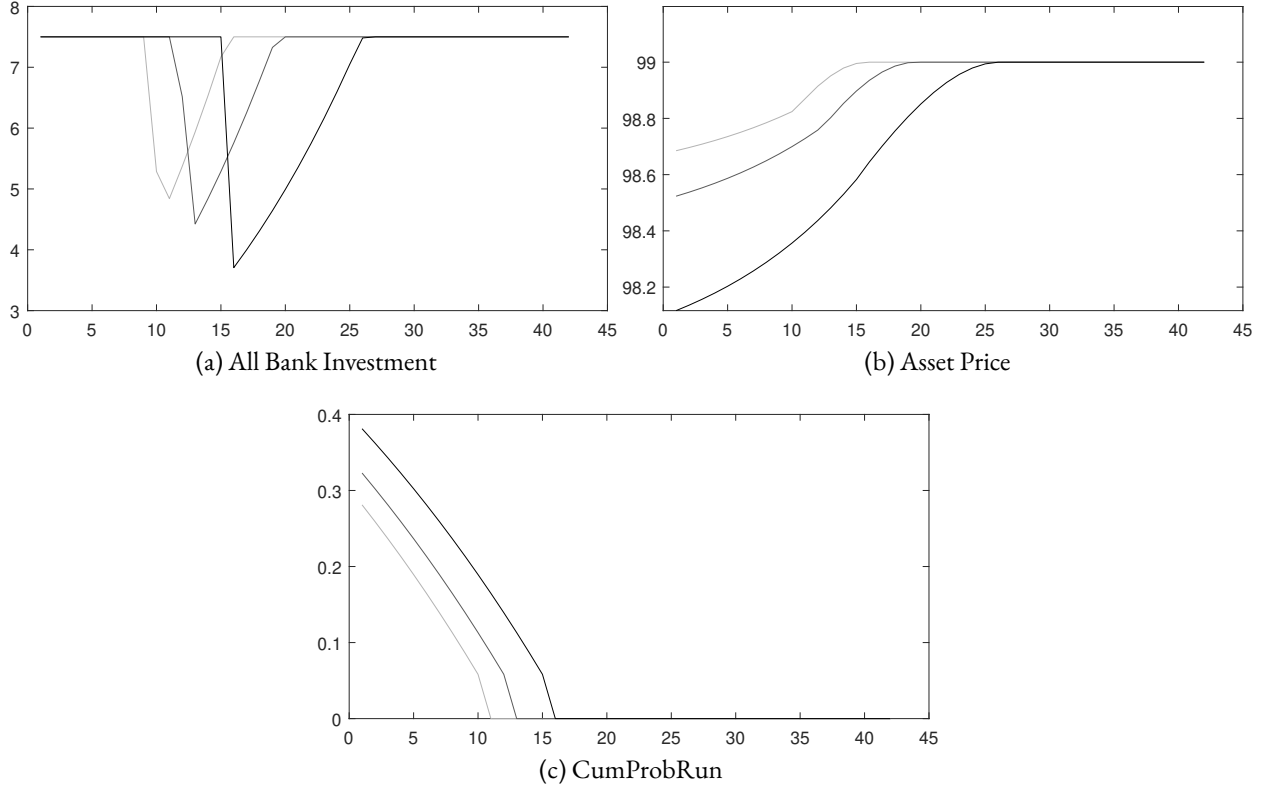


Figure 3.6: Transition path as a function of time for $\eta = 0, .05$, and $.1$

results but tempers the recapitalization channel. I set $\gamma = .1$.

Figure C.8 displays the transition path through time of the total amount of capital intermediated by banks, leverage, saving, asset prices, dividends and the cumulative probability of a run, defined as the probability that a run occurs by the end of time for three different degree of competition.

Compared to the transition path when $\gamma = 0$ displayed in 3.5, it takes much more time for all economies to escape the run zone, travel through the safe zone and reach their steady-state because banks distribute dividends along the transition path, C.8e, and do not save all their profits, C.8c. This affects dis-proportionally system with higher level of competition. At time 0, the probability of a run in the system with five banks increases from 50% to 80% while it remains unchanged for the monopoly, C.8f.

4.5 A Smooth Functional Form for λ

I show in this section how my results are affected by making λ a smooth function of fundamentals. The following functional form for the arrival rate proves tractable

$$\lambda = \frac{\bar{\lambda}}{2} \left(\max \left(\sum_b k_b - \frac{\sum_b n_b}{(q - q^R)}, 0 \right) \right)^2$$

with $\bar{\lambda} = .002$. $\lambda(\cdot)$ is (weakly) increasing in k and decreasing in n : $\frac{\partial \lambda}{\partial k} = \sqrt{2\lambda\bar{\lambda}} \geq 0$. Figure C.9 reports the transition path for three different values of $\mathcal{B} = 1, 3, 5$. Note that C.9f is not the cumulative probability of a run but the instantaneous probability of a run. It decreases over time, revealing the returns-safety trade-off that banks make for low level of net-worth. As they accumulate net worth, the franchise value increases and they play a safe strategy. The patterns for dividends, asset prices, saving and leverage are very similar to the case 3.2 except for investment for which there is no longer a downward jump at the entrance of the safe zone.

As can be seen on panel C.9f, a more concentrated system will tend to restrict more its investment and play more safely for low level of net worth and will exit more quickly the zone with high risk of runs. It is also interesting to see that more competitive sector have trouble bringing the risk of run to zero even in the long run since the probability of a run remains strictly positive (although decreasing over time). This reflects a force that I couldn't see in the case studied previously because I had no intensive margin to risk-taking: when deciding upon how much net worth to accumulate, a monopoly understands that the marginal dollar reduce the risk of a run by a greater extent than the marginal dollar accumulated by a bank in a more competitive sector since the risk depends on the aggregate amount of net worth. This free-rider problem becomes more and more acute as the number of banks increases.

4.6 Exogenous Risk: the Paradox of Safety

I now reintroduce exogenous productivity uncertainty and solve the model with two productivity levels. I calibrate $\underline{z}$, $\bar{z}$, $\underline{\zeta}$ and $\bar{\zeta}$ as follows. I set $\bar{z} = 5$ and $\underline{z} = 4.25$ for the GDP drop to be of 15% in recession. I choose the transition rates between productivity states, $\bar{\zeta} = .02$, $\underline{\zeta} = .08$ so that the average time spent

in the low state is 12 years and in the high state 50 years. This is consistent with the size and frequency of large and rare macroeconomic recessions.

The analysis of the stationary distributions of systems with different levels of concentration reveals the *paradox of safety*: although starting from the same initial level of net worth, a more concentrated sector is safer on the transition path, it will in general start the transition path at a lower initial level after a large shock to GDP hits. This is likely to be the case since banks operating in a concentrated sector have a lower ratio of net worth over capital, due to the lower price-drop; hence the *paradox of safety*, in reference to the paradox of volatility in Brunnermeier and Sannikov (2014) where low fundamental volatility may actually lead to high endogenous volatility. This counteracting force mitigates the stabilizing properties of concentration.

Figure 3.7 represents the stationary distribution of this economy where the x-axis represents the sum of banks' net worth for $\mathcal{B} = 1, 3$ and 5 where I keep the same color convention as before. The solid (resp. dotted) line represents the distribution in the high (resp. low) state. The plotted densities are conditional on having effectively banks operating on financial markets; after a run, the system will operate without banks for a while.

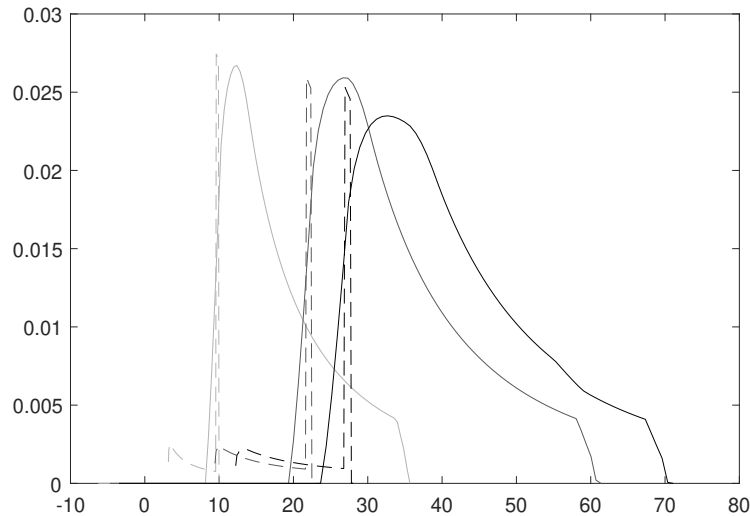


Figure 3.7: Stationary distributions as a function of net worth for $\mathcal{B} = 1, 3$ and 5

5 Conclusion

This paper has developed a dynamic general equilibrium model with a finite number of large financial intermediaries subject to endogenous - systemic and idiosyncratic - run risk and playing *à la* Cournot, internalizing the effect of their action on aggregate risk, returns and asset prices. In the neighborhood of the steady-state, they optimally restrict their investment to be a function of their net worth to avoid the risk of a run which gives rise to an accelerator effect. For a large adverse shock, the financial system enters a run zone characterized by high leverage, deleveraging over time and risk of run from which it may escape in finite time. The instability of the system is a function of how long it stays in this zone. This depends on the level of competition through three channels: the *franchise value channel*, the *price-drop channel* and the *recapitalization channel*. I also show that contestability - the speed of entry - has significant stabilizing effect, what I call the *entry channel*.

My qualitative findings are consistent with empirical evidence of the dynamics of the financial system in the aftermath of a adverse shock as well as with the findings of the effect of concentration on stability. My analysis gives rise to several new testable predictions and areas for further empirical research. I highlight the limits of empirical analyses that do not take into account the state-dependency of banks' behaviors. For small shocks, banks behavior is characterized by the a safe strategy while after a large shock, run may occur in the short run and the decline in intermediation happens with a delay, once the banks have reached the safe zone. I also highlight the importance of two channels that have been overlooked by the current literature: the recapitalization channel according to which more concentrated markets have the ability to recapitalize more quickly and escape the run zone and the price-drop channels according to which concentration influences the size of the drop in asset prices when a run occur.

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Appendix A

The Great Gatsby Goes to College: Tuition, Inequality and Intergenerational Mobility in the U.S.

A.1 Analytical Model - Details

I solve the model using a guess and verify. I guess that the tuition function before government financial aid are given respectively by:

$$e_u(q, z, y) = \left(\frac{p_I}{(1 + a_u)T_u} q^{\frac{1}{\varepsilon_1}} z^{-\frac{\varepsilon_2}{\varepsilon_1}} \left(\frac{y}{\kappa_2} \right)^{\frac{\varepsilon_3}{\varepsilon_1}} \right)^{\frac{1}{1-\tau_u}} \quad (\text{A.1})$$

A.1.1 Solution to the Household Problem

Using the guess (A.1) and the expression for financial aid (1.22), the problem of the Households is

$$\mathcal{U} = \max_{s, \ell} (1 - \beta) \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + \ln (Ab^\lambda \ell^\mu)^{1-\tau_y} - \ell^\eta \right] + \beta EU' \quad (\text{A.2})$$

$$\begin{aligned} \ln b' &= \ln \xi_y + \ln \kappa + \alpha_1(1 + \alpha_2(\varepsilon_2 + \tau_m(1 - \tau_u)\varepsilon_1)) \ln \xi_b + \alpha_b \ln b \\ &+ \alpha_2 \varepsilon_1 (\ln (s(1 + a_b)/T_e)^{1-\tau_u} (1 + a_u)T_u/p_I) + \alpha_2(\varepsilon_1(1 - \tau_u)(1 - \tau_{n,t}) - \varepsilon_3) \ln \ell^{(1-\tau_y)\mu} \\ &+ \alpha_2 (\varepsilon_1(1 - \tau_u)(1 - \tau_n) - \varepsilon_3) ((1 - \tau_{y,t}) \ln A(1 - a_y)T_y)) + \alpha_2 \varepsilon_3 \ln \kappa_2 \end{aligned} \quad (\text{A.3})$$

with $\alpha_b = \alpha_1 + \alpha_3 + \alpha_1 \alpha_2(\varepsilon_2 + \tau_m(1 - \tau_u)\varepsilon_1) + \alpha_2(\varepsilon_1(1 - \tau_u)(1 - \tau_n) - \varepsilon_3)(1 - \tau_y)\lambda$ and s the spending rate, i.e. the amount of spending for college over income. I then guess that $\mathcal{U}_t = U_t \ln b_t + Z_t \ln \xi_{b,t} + B_t$. Replacing this guess into (A.2), then using (A.3) to substitute for $\ln b_{t+1}$ and using (1.5) and (1.6)

$$\begin{aligned} U_t \ln b_t + Z_t \ln \xi_{b,t} + B_t &= \max_{s, \ell} (1 - \beta) \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + (1 - \tau_y) \ln b^\lambda \ell^\mu A - \ell^\eta \right] \\ &+ \beta \left[U_{t+1} \left(\mu_y + \ln \kappa + \alpha_1(1 + \alpha_2(\varepsilon_2 + \tau_{m,t}(1 - \tau_u)\varepsilon_1)) \ln \xi_{b,t} + \alpha_{b,t} \ln b_t \right. \right. \\ &+ \alpha_2 \varepsilon_1 (\ln (s(1 + a_b)/T_e)^{1-\tau_u} (1 + a_u)T_u/p_I) + \alpha_2(\varepsilon_1(1 - \tau_u)(1 - \tau_{n,t}) - \varepsilon_3) \ln \ell^{(1-\tau_y)\mu} \\ &\left. \left. + \alpha_2(\varepsilon_1(1 - \tau_u)(1 - \tau_{n,t}) - \varepsilon_3) ((1 - \tau_{y,t}) \ln A(1 - a_{y,t})T_{y,t}) + \alpha_2 \varepsilon_3 \ln \kappa_{2,t} \right) + Z_{t+1} \mu_b + B_{t+1} \right] \end{aligned}$$

Gathering all the terms in $\ln b_t$ one gets that U_t has to verify

$$U_t = (1 - \beta) \sum_{k=0}^{\infty} \beta^k (1 - \tau_{t+k}^y) \lambda_{t+k} \prod_{m=0}^{k-1} \alpha_{t+m}^b$$

$$\text{Gathering all the terms in } \ln \xi_{b,t}, \text{ one gets } Z_t = (U_t - (1 - \beta)(1 - \tau_{y,t})\lambda) \frac{\alpha_1(1 + \alpha_2(\varepsilon_2 + \varepsilon_1(1 - \tau_u)\tau_{m,t}))}{\alpha_t^b}.$$

Finally gathering the independent terms, the F.O.C for s and ℓ give

$$s_t = \frac{\beta \alpha_2 \varepsilon_1 (1 - \tau_u) U_{t+1}}{1 - \beta + \beta \alpha_2 \varepsilon_1 (1 - \tau_u) U_{t+1}} \quad (\text{A.4})$$

$$\ell = \left[(1 - \tau_{y,t}) \frac{\mu}{\eta} \left(1 + \frac{\beta}{1 - \beta} \alpha_2 (\varepsilon_1(1 - \tau_u)(1 - \tau_{n,t}) - \varepsilon_3) U_{t+1} \right) \right]^{\frac{1}{\eta}} \quad (\text{A.5})$$

A.1.2 University problem

I first provide a generalized definition of σ_u that takes into account government policies

$$\sigma_u^2 = \frac{\omega_1(1-\tau_u)}{2} E \left(\left(\ln \left(\bar{z}^{\frac{\omega_2}{\omega_1(1-\tau_u)}} \bar{y}^{-\frac{\omega_3}{\omega_1(1-\tau_u)}} \right) - \ln z^{\frac{\omega_2}{\omega_1(1-\tau_u)}} y^{-\frac{\omega_3}{\omega_1(1-\tau_u)}} \right)^2 \right) \quad (\text{A.6})$$

Using this definition and our guess for tuitions (A.1), one gets the following

$$\sigma_u^2 = \frac{\omega_1(1-\tau_u)}{2} E \left(\left(\ln e_u(z, y) - \ln \left(\frac{p_I \tilde{I}}{(1+a_u)T_u} \right)^{\frac{1}{1-\tau_u}} \right)^2 \right) = \frac{\omega_1(1-\tau_u)}{2} \tilde{\sigma}_u^2$$

where I define $\ln \tilde{I} = \ln I - \frac{\tilde{\sigma}_u^2}{2}$. I now show that $\tilde{\sigma}_u^2$ is the within-university variance of log tuition. I guess that tuition fees are log-normally distributed within the university. Denoting $\mu_{e,q}, \sigma_{e,q}$ the mean and standard deviation of log tuition within the university of quality q , the budget constraint of the university -given by (1.23)-becomes

$$\begin{aligned} p_I I &= T_u(1+a_u) \left(E_{z,y} [e_u(q, z, y)] \right)^{1-\tau_u} = T_u(1+a_u) e^{(1-\tau_u)\mu_{eq} + (1-\tau_u)\frac{\sigma_{eq}^2}{2}} \\ \iff \frac{1}{(1-\tau_u)} \ln \frac{p_I}{(1+a_u)T_u} \tilde{I} + \frac{\tilde{\sigma}_u^2}{2} - \frac{\sigma_{eq}^2}{2} &= \mu_{e,q} = E \ln e_u(z, y) \end{aligned}$$

Substituting this last line into the expression of σ_u^2 above gives

$$\begin{aligned} \tilde{\sigma}_u^2 &= \int \varphi(z, y) \left(\ln e_u(z, y) - E \ln e_u(z, y) + \frac{\sigma_{eq}^2 - \sigma_u^2}{2} \right)^2 dz dy \\ \iff \tilde{\sigma}_u^2 &= \sigma_{e,q}^2 + \left(\frac{\sigma_{eq}^2 - \sigma_u^2}{2} \right)^2 + 0 \Rightarrow \tilde{\sigma}_u^2 = \sigma_{e,q}^2 \quad \text{or} \quad \tilde{\sigma}_u^2 = \sigma_{e,q}^2 + 4 \end{aligned}$$

$\tilde{\sigma}_u = \sigma_{e,q}$ is a solution to the quadratic equation. This verifies our guess. $\mu_{e,q} = E \ln e_u(z, y) = \ln \left(\frac{p_I \tilde{I}}{(1+a_u)T_u} \right)^{\frac{1}{1-\tau_u}}$ and $\sigma_u^2 = \sigma_{eq}^2$ are respectively the mean and standard deviation of within-university

log tuitions. Therefore I can now rewrite the problem of the university replacing I with $\tilde{I}$

$$\begin{aligned} & \max_{\tilde{I}, \bar{z}, \bar{y}, r(\cdot)} \tilde{I}^{\omega_1} \bar{z}^{\omega_2} \bar{y}^{-\omega_3} \\ & \ln \tilde{I} \int_0^1 r_{z,y} dz dy = \int r_{z,y} ((1 - \tau_u) \ln(e_u)^i + \ln(1 + a_u) T_u / p_I) dz dy \\ & \ln \bar{z} \int_0^1 r_{z,y} dz dy = \int_0^1 r_{z,y} \ln z dz dy \quad \text{and} \quad \ln \bar{y} \int_0^1 r_{z,y} dz dy = \int_0^1 r_{z,y} \ln y dz dy \end{aligned} \tag{A.7}$$

where $r_{z,y}$ denotes the mass of individuals of type (z, y) .

$$\text{The F.O.Cs are} \quad \frac{\omega_1}{\tilde{I}} + \frac{\lambda_1}{\tilde{I}} = 0, \quad \frac{\omega_2}{\bar{z}} + \frac{\lambda_2}{\bar{z}} = 0 \quad \text{and} \quad -\frac{\omega_3}{\bar{y}} + \frac{\lambda_3}{\bar{y}} = 0$$

$$r_{z,y} = \begin{cases} 0 & \text{if } \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\omega_1}} \bar{z}^{-\frac{\omega_2}{\omega_1}} (\bar{y}/\bar{y})^{\frac{\omega_3}{\omega_1}} \right)^{\frac{1}{1-\tau_u}} < e_u(q, z, y) \\ c \in \mathbb{R} & \text{if equality} \\ +\infty & \text{if strictly larger} \end{cases}$$

where I have solved for the Lagrange multipliers. I guess that in equilibrium, $\bar{y} = \kappa_2 q^\nu$. Therefore whenever a college admits a certain student type, the tuition formula is:

$$\begin{aligned} e_u(q, z, y) &= \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1-\nu\tilde{\varepsilon}_3}{\omega_1}} \bar{z}^{-\frac{\omega_2}{\omega_1}} \bar{y}^{\frac{\omega_3}{\omega_1}} \kappa_2^{-\frac{\omega_3}{\omega_1}} \right)^{\frac{1}{1-\tau_u}} = \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_1}} \bar{z}^{-\frac{\varepsilon_2}{\varepsilon_1}} \bar{y}^{\frac{\varepsilon_3}{\varepsilon_1}} \kappa_2^{-\frac{\varepsilon_3}{\varepsilon_1}} \right)^{\frac{1}{1-\tau_u}} \\ \text{with } \varepsilon_1 &= \frac{\omega_1}{1-\nu\tilde{\varepsilon}_3} \quad \varepsilon_2 = \frac{\tilde{\varepsilon}_2}{1-\nu\tilde{\varepsilon}_3} \quad \varepsilon_3 = \frac{\tilde{\varepsilon}_3}{1-\nu\tilde{\varepsilon}_3} \end{aligned}$$

I can solve for ν and κ_2 using the equilibrium outcome given by the mean income in proposition A.1.4. I do this later in appendix A.1.4. This confirms the guess for tuition fees (A.1). Given this guess for tuition, a university is always at the interior solution, therefore always indifferent between all types.

A.1.3 Other Equilibrium Conditions

Government Budget Constraints

There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any

period.

$$\int_0^1 a_y y(i) + a_c c(i) + e(i) di = \int_0^1 e(i)(1 + a_u)(1 + a_b) di \quad (\text{A.8})$$

The other three constraints, (A.10), (A.9) and (A.11) pin down T_u , T_y , T_e such that a_y , a_b , a_u parametrize respectively the average rate of income tax, financial aid and transfers to college. Denoting f_q the mass of students in colleges of quality q

$$\int_0^1 y(i)^{1-\tau_y} T_y di = \int_0^1 y(i) di \quad (\text{A.9})$$

$$(1 + a_b) \int_0^1 e(i) di = \int_0^1 e_u(i) di \quad (\text{A.10})$$

$$\int E_{z,y}[e_u(q, z, y)] f_q dq = \int T_u (E_{z,y}[e_u(q, z, y)])^{1-\tau_u} f_q dq \quad (\text{A.11})$$

Lemma 4. *Along the equilibrium path, the government budget constraints (A.8), (A.9), (A.10) and (A.11) are given by*

$$\frac{a_{c,t}(1 - s_t)}{(1 + a_{c,t})} = s_t(1 + a_{u,t})(1 + a_{b,t}) - \frac{a_{y,t}}{1 - a_{y,t}} - s_t \quad (\text{A.12})$$

$$\ln T_y = \tau_y \ln A + \tau_y \mu \ln \ell + \tau_y \lambda m_b + \frac{\lambda^2}{2} (2 - \tau_y) \tau_y \Sigma_{b,t}^2 \quad (\text{A.13})$$

$$\ln T_e = (-\tau_n \lambda + \alpha_1 \tau_m) m_b + \frac{\alpha_1 \tau_m}{2} (\alpha_1 \tau_m - 1) \sigma_b^2 - \tau_n (\ln A \ell^\mu (1 - a_y)) \quad (\text{A.14})$$

$$+ [\lambda^2 (1 - \tau_y)^2 (\tau_n - 2) \tau_n + 2\lambda (1 - \tau_n) (1 - \tau_y) \tau_m \alpha_1 + (\alpha_1 \tau_m)^2 - \tau_n \lambda^2 (2 - \tau_y) \tau_y] \frac{\Sigma_b^2}{2} \quad (\text{A.15})$$

$$\ln T_u = \tau_u \left(\ln A \ell^\mu s (1 + a_b) (1 - a_y) + \lambda m_b + \lambda^2 \frac{\Sigma_b^2}{2} \right) + \frac{\tau_u}{1 - \tau_u} \frac{\sigma_I^2}{2} \quad (\text{A.16})$$

1. Solving for the aggregate state budget constraint is immediate
2. Solving for T_y . Using (A.9), and the expression for market income y_m , (1.21), and using the guess that $\ln b$ is normally distributed one gets:

$$\int_0^1 (A \ell^\mu b^\lambda)^{1-\tau_{y,t}} T_y di = \int_0^1 A \ell^\mu b^\lambda di \iff T_y e^{\lambda(1-\tau_y)m_b + \frac{(\lambda(1-\tau_y))^2}{2} \Sigma_b^2} = A^{\tau_y} \ell^{\tau_y} e^{\lambda m_b + \frac{\lambda^2}{2} \Sigma_b^2}$$

3. Solving for T_e . Using (A.10), one gets:

$$\begin{aligned} (1 + a_b) \int_0^1 e^i di &= \int_0^1 (e_u)^i di \iff (1 + a_b) \int_0^1 s y_I di = \int_0^1 \frac{s y (1 + a_b)}{T_e z^{-\tau_m} y^{\tau_n}} di \\ T_e (1 - a_y)^{\tau_n} (\mathcal{A} \ell)^{\tau_n (1 - \tau_y)} (T_y)^{\tau_n} e^{\lambda (1 - \tau_y) m_b + (\lambda (1 - \tau_y))^2 \frac{\Sigma_b^2}{2}} \\ &= e^{(\lambda (1 - \tau_y) (1 - \tau_n) + \tau_m \alpha_1) m_b - \alpha_1 \tau_m \frac{\sigma_b^2}{2} + (\lambda (1 - \tau_y) (1 - \tau_n) + \tau_m \alpha_1)^2 \frac{\Sigma_b^2}{2} + \frac{(\alpha_1 \tau_m)^2}{2} \sigma_b^2} \end{aligned}$$

4. Solving for T_u . Substituting (1.23) into (A.11), one gets

$$\begin{aligned} \int E_{z,y} [e_u(q, z, y)] f_q dq &= \int T_u (E_{z,y} [e_u(q, z, y)])^{1 - \tau_u} f_q dq \iff \int \left(\frac{p_I I_q}{(1 + a_u) T_u} \right)^{\frac{1}{1 - \tau_u}} f_q dq = \int \frac{p_I I_q}{(1 + a_u)} f_q dq \\ \left(\frac{p_I}{(1 + a_u) T_u} \right)^{\frac{1}{1 - \tau_u}} \int I_i^{\frac{1}{1 - \tau_u}} di &= \frac{p_I}{(1 + a_u)} \int I_i di \iff \left(\frac{p_I}{1 + a_u} \right)^{\frac{\tau_u}{1 - \tau_u}} \int I_i^{\frac{1}{1 - \tau_u}} di = (T_u)^{\frac{1}{1 - \tau_u}} \int I_i di \end{aligned}$$

where i indexes households. I then guess that I_i is log-normally distributed with mean μ_I and variance σ_I^2

- I give expression for these variables in appendix A.1.6):

$$\left(\frac{p_I}{1 + a_u} \right)^{\frac{\tau_u}{1 - \tau_u}} e^{\frac{\mu_I}{1 - \tau_u} + \frac{\sigma_I^2}{2(1 - \tau_u)^2}} = (T_u)^{\frac{1}{1 - \tau_u}} e^{\mu_I + \frac{\sigma_I^2}{2}} \Rightarrow \ln T_u = \tau_u \ln \left(\frac{p_I}{1 + a_u} \right) + \mu_I \tau_u + \frac{\sigma_I^2}{2} \frac{\tau_u (2 - \tau_u)}{(1 - \tau_u)}$$

Using the guess and from appendix A.1.3, one gets

$$\begin{aligned} \ln E(I) &= \mu_I + \frac{\sigma_I^2}{2} = \ln \frac{\mathcal{A} \ell^\mu s (1 + a_b) (1 + a_u) (1 - a_y)}{p_I} + \lambda m_b + \lambda^2 \frac{\Sigma_b^2}{2} \\ \text{Hence } \mu_I &= \ln \frac{\mathcal{A} \ell^\mu s (1 + a_b) (1 + a_u) (1 - a_y)}{p_I} + \lambda m_b + \lambda^2 \frac{\Sigma_b^2}{2} - \frac{\sigma_I^2}{2} \end{aligned}$$

Substituting back into the expression for T_u gives

$$\ln T_u = \tau_u \left(\ln \mathcal{A} \ell^\mu s (1 + a_b) (1 - a_y) + \lambda m_b + \lambda^2 \frac{\Sigma_b^2}{2} \right) + \frac{\tau_u}{1 - \tau_u} \frac{\sigma_I^2}{2}$$

I derive the expression for σ_I^2 in appendix A.1.6

Educational Sector: Generalization

In this section I present an extension of the very simple educational sector presented in the main text. I start by assuming that the production function for educational services is given by $y_I = A_I b^{\bar{\lambda}} \ell^\mu$ with $\bar{\lambda} \geq \lambda$. The latter assumption says that households with higher human capital have a comparative advantage in working in the educational services sector.

Colleges buy services at price p_I from the educational sector. The latter has two peculiarities. First of all it is made of one non-profit agency that aggregates heterogeneous inputs to produce an homogeneous educational service and whose objective is to minimize costs subject to a non-negative profit condition. Secondly it has full power in the setting of wages. I discuss these two assumptions below. The problem of the educational service agency willing to produce $\bar{I}$ is:

$$\min_{d_b} p_I \quad s.t. \quad \int A_I b^{\bar{\lambda}} \ell^\mu d_b db \geq \bar{I} \quad \text{and} \quad p_I \int A_I b^{\bar{\lambda}} \ell^\mu d_b db \geq \int b^\lambda \ell^\mu d_b db \quad (\text{A.17})$$

where the right hand side of the last line embeds the assumption of bilateral bargaining with full power to the agency and d_b is the indicator function equal to 1 if a household with human capital b is employed in the sector.¹

Educational Sector: Hiring Rule

From the assumption that $\bar{\lambda}_t > \lambda_t$ (individuals with high human capital have a comparative advantage working for the agency), the hiring rule takes the form of threshold rule where the most educated individuals work in the educational sector.

Proposition A.1.1. *Provided $a_{u,t}, a_{b,t}, \beta$ are not too high, there exists a unique $\underline{b}_t \in \mathbb{R}$ such that*

$$d_t(b) = 1 \iff b \geq \underline{b}_t \quad (\text{A.18})$$

¹The assumption that wages are set through a bilateral negotiation where the agency has full bargaining power implies that wages are determined by the marginal product in the final good sector. These assumptions are not particularly unrealistic and buy us some tractability. The assumption of minimizing the price of educational services will imply that the price of educational services will not reflect the marginal cost of producing these services but rather their average cost. The joint assumption of price-minimizing monopoly allows us to get rid of profit in this sector, that would otherwise need to be given back to either the household sector or the government.

The difference between the product and cost of hiring human capital b .

$$p_I A_I b^{\bar{\lambda}} - A b^{\lambda} \quad (\text{A.19})$$

Lemma 5. Equation (A.19) is increasing in b if

$$\frac{E(b_{\bar{\lambda}} | b_{\bar{\lambda}} \geq 1)}{E(b_{\lambda} | b_{\lambda} \geq 1)} = \frac{m\left(\frac{m_b - \log(\underline{b})}{\Sigma_b} - \bar{\lambda}\Sigma_b\right)}{m\left(\frac{m_b - \log(\underline{b})}{\Sigma_b} - \lambda\Sigma_b\right)} \leq \frac{\bar{\lambda}}{\lambda}$$

where $b_{\bar{\lambda}} = \left(\frac{b}{\underline{b}}\right)^{\bar{\lambda}}$ and $b_{\lambda} = \left(\frac{b}{\underline{b}}\right)^{\lambda}$ and $m(\cdot)$ denotes the Mills ratio.

Proof. We want to show that this is increasing in b so that the agency—willing to minimize the average cost—wants to hire first the individuals with highest human capital. The condition is: $p_I A_I \bar{\lambda} b^{\bar{\lambda}-1} > A \lambda b^{\lambda-1} \iff \frac{\bar{\lambda}}{\lambda} b^{\bar{\lambda}-\lambda} > \frac{A}{A_I p_I}$. It is sufficient to show that this is true for $b = \underline{b}$ since $\bar{\lambda} > \lambda$, i.e.

$$\frac{\bar{\lambda}}{\lambda} \underline{b}^{\bar{\lambda}-\lambda} > \frac{A}{A_I p_I} \iff \frac{\bar{\lambda}}{\lambda} > \frac{\int_{\underline{b}} \left(\frac{b}{\underline{b}}\right)^{\bar{\lambda}} db}{\int_{\underline{b}} \left(\frac{b}{\underline{b}}\right)^{\lambda} db} = \frac{E(b_{\bar{\lambda}} | b_{\bar{\lambda}} \geq 1)}{E(b_{\lambda} | b_{\lambda} \geq 1)}$$

where the last line uses the expression for the price p_I given by (A.21). Let's now show that the RHS of the last line is equal to the ratio of the Mills ratio. Taking the conditional expectation

$$\begin{aligned} E(b_{\bar{\lambda}} | b_{\bar{\lambda}} \geq 1) &= \frac{e^{\bar{\lambda}(m_b - \log \underline{b}) + \frac{\bar{\lambda}^2}{2} \Sigma_b^2} \Phi\left(\frac{\bar{\lambda}(m_b - \log \underline{b}) + \bar{\lambda}^2 \Sigma_b^2 - 0}{\bar{\lambda} \Sigma_b}\right)}{1 - \Phi\left(\frac{0 - \bar{\lambda}(m_b - \log \underline{b})}{\bar{\lambda} \Sigma_b}\right)} \\ \Rightarrow \frac{E(b_{\bar{\lambda}} | b_{\bar{\lambda}} \geq 1)}{E(b_{\lambda} | b_{\lambda} \geq 1)} &= \frac{e^{(\bar{\lambda}-\lambda)(m_b - \log \underline{b}) + \frac{\bar{\lambda}^2 - \lambda^2}{2} \Sigma_b^2} \Phi\left(\frac{m_b - \log \underline{b}}{\Sigma_b} + \bar{\lambda} \Sigma_b\right)}{\Phi\left(\frac{m_b - \log \underline{b}}{\Sigma_b} + \lambda \Sigma_b\right)} = \frac{m\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - \bar{\lambda} \Sigma_b\right)}{m\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - \lambda \Sigma_b\right)} \end{aligned}$$

□

Lemma 6. For $\underline{b}$ high enough, $\frac{m\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - \bar{\lambda} \Sigma_b\right)}{m\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - \lambda \Sigma_b\right)} \leq \frac{\bar{\lambda}}{\lambda}$

Proof. First denote $m\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - \bar{\lambda} \Sigma_b\right) = m_{\bar{\lambda}}$ and $m\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - \lambda \Sigma_b\right) = m_{\lambda}$. The Mills ratio

associated with a standard normal is decreasing and convex, which implies

$$\frac{m_{\bar{\lambda}}}{m_{\lambda}} > 1 \iff \bar{\lambda} > \lambda \quad \text{and} \quad \frac{\partial \frac{m_{\bar{\lambda}}}{m_{\lambda}}}{\partial \log \underline{b}} = \frac{m_{\lambda} \left(m'_{\bar{\lambda}} - \frac{m_{\bar{\lambda}}}{m_{\lambda}} m'_{\lambda} \right)}{m_{\lambda}^2} < 0$$

since $m'_{\bar{\lambda}} < m'_{\lambda}$ and $-m'_{\lambda} < -m'_{\lambda} \frac{m_{\bar{\lambda}}}{m_{\lambda}} \Rightarrow m'_{\bar{\lambda}} - \frac{m_{\bar{\lambda}}}{m_{\lambda}} m'_{\lambda} < 0$. I now show that the limit of the ratio of Mills ratio tends to $+\infty$ when $\underline{b}$ tends to 0. Denoting $X_{\lambda}(\underline{b}) = \frac{\log \underline{b} - m_b}{\Sigma_b} - \lambda \Sigma_b$ and similarly for $\bar{\lambda}$, one has

$$\begin{aligned} \lim_{\log \underline{b} \rightarrow -\infty} \frac{m(X_{\bar{\lambda}}(\underline{b}))}{m(X_{\lambda}(\underline{b}))} &= \lim_{\log \underline{b} \rightarrow +\infty} \frac{m_{\bar{\lambda}}(-X_{\bar{\lambda}}(\underline{b}))}{m(-X_{\lambda}(\underline{b}))} = \lim_{\log \underline{b} \rightarrow +\infty} \frac{\frac{1-\Phi(X_{\bar{\lambda}}(\underline{b}))}{\Phi(X_{\bar{\lambda}}(\underline{b}))} m(X_{\bar{\lambda}}(\underline{b}))}{\frac{1-\Phi(X_{\lambda}(\underline{b}))}{\Phi(X_{\lambda}(\underline{b}))} m(X_{\lambda}(\underline{b}))} \times 1 \\ &= \lim_{\log \underline{b} \rightarrow +\infty} \frac{-\varphi(X_{\bar{\lambda}}(\underline{b}))}{-\varphi(X_{\lambda}(\underline{b}))} = \lim_{\log \underline{b} \rightarrow +\infty} e^{\Sigma_b(\bar{\lambda}-\lambda)\left(\frac{\log \underline{b} - m_b}{\Sigma_b} - (\lambda + \bar{\lambda})\frac{\Sigma_b}{2}\right)} = +\infty \end{aligned}$$

where the second line uses the symmetry of the standard normal cdf and pdf, the third uses l'Hôpital's rule and the fourth that $\bar{\lambda} > \lambda$.

Hence the ratio of Mills ratios is decreasing, continuous, goes to $+\infty$ when $\underline{b}$ approaches 0 and to 1 when it goes to $+\infty$. Therefore there exists a unique threshold at which it crosses $\frac{\bar{\lambda}}{\lambda}$ and is below it for $\underline{b}$ higher. This finishes the proof. $\square$

Given the market clearing condition (A.20) pinning down $\log \underline{b}$, it is clear that a_u, a_b, s (and hence β) low enough ensures that $\log \underline{b}$ will be high enough in equilibrium. I have checked that in our calibration this property holds.

Market Clearing

It remains to be checked that the final good and the educational services market clear. By Walras' law, if the latter clears, since college admission markets already clear, the final good market should clear. The market clearing condition in the educational sector requires that the demand for educational services coming from colleges—the left-hand-side of equation (A.20)—be equal to the supply of human capital supplied by the agency. It implicitly pins down the threshold $\underline{b}_t$. The price of educational services is then given by the no-profit condition in (A.17) of the agency. The following proposition summarizes these

results.

Lemma 7. 1. The threshold to work in the education sector $\underline{h}$ is implicitly given by

$$(1 + a_{u,t})(1 + a_{b,t})s_t e^{\lambda_t m_{b,t} + \frac{\lambda_t^2}{2} \Sigma_{b,t}^2} (1 - a_{y,t}) = \underline{H}_t = e^{\lambda_t m_{b,t} + \frac{\lambda_t^2}{2} \Sigma_{b,t}^2} \frac{\Phi \left[\frac{m_{b,t} - \log \underline{h}_t}{\Sigma_{b,t}} + \lambda_t \Sigma_{b,t} \right]}{1 - \Phi \left[\frac{\log \underline{h}_t - m_{b,t}}{\Sigma_{b,t}} \right]} \quad (\text{A.20})$$

where $\Phi(\cdot)$ denotes the c.d.f. of a standard normal.

2. Defining $\underline{H}_t^I = \int_{\underline{h}_t}^{+\infty} b^{\bar{\lambda}} db$, the equilibrium price of education services is

$$p_{I,t} = \frac{A \int_{\underline{h}_t}^1 b^{\lambda_t} db}{A_I \int_{\underline{h}_t}^1 b^{\bar{\lambda}} db} = \frac{A}{A_I} e^{(\lambda_t - \bar{\lambda}) m_{b,t} + (\lambda_t^2 - \bar{\lambda}^2) \frac{\Sigma_{b,t}^2}{2}} \frac{\Phi \left[\frac{m_{b,t} - \log \underline{h}_t}{\Sigma_{b,t}} + \lambda_t \Sigma_{b,t} \right]}{\Phi \left[\frac{m_{b,t} - \log \underline{h}_t}{\Sigma_{b,t}} + \bar{\lambda} \Sigma_{b,t} \right]} \quad (\text{A.21})$$

Proof. Demand for educational services is given by $\frac{(1+a_u)(1+a_b)}{p_I} s \int_0^1 \gamma_I di$ and supply is given by $A_I \ell \int_{\underline{h}}^1 (b^i)^{\bar{\lambda}} db$

hence market clearing is:

$$\begin{aligned} \frac{(1 + a_u)(1 + a_b)}{p_I} s \int_0^1 \gamma_{I,t} di &= A_I \ell^\mu \int_{\underline{h}}^1 (b^i)^{\bar{\lambda}} db \iff (1 + a_u)(1 + a_b) s A \ell^\mu e^{\lambda m_b + \frac{\lambda^2}{2} \Sigma_b^2} (1 - a_y) = p_I A_I \ell^\mu \underline{H}^I \\ (1 + a_u)(1 + a_b) s A \ell^\mu e^{\lambda m_b + \frac{\lambda^2}{2} \Sigma_b^2} (1 - a_y) &= A \ell^\mu \underline{H} \iff (1 + a_u)(1 + a_b) s e^{\lambda m_b + \frac{\lambda^2}{2} \Sigma_b^2} (1 - a_y) = \underline{H} \end{aligned}$$

To derive the expression for the equilibrium price (A.21), I combine the no-profit condition in (A.17) with the threshold condition (A.18) and use the expression for conditional expectation of a log-normal distribution. $\square$

A.1.4 Quality distribution and within-college distributions

Parent's education and income. Taking the logarithm of (1.17): $\ln q = (\varepsilon_I + \varepsilon_A) \ln b + \varepsilon_A \ln \xi_b + x$ with

$x = \varepsilon_1 \left(\ln \left(s^{\frac{(1+a_b)}{T_c}} \right)^{1-\tau_u} \left(\frac{(1+a_u)T_u}{p_I} \right) \right) + (\varepsilon_1(1-\tau_u)(1-\tau_n) - \varepsilon_3) \ln(A \ell^\mu)^{1-\tau_y} T_y (1-a_y) + \varepsilon_3 \ln \kappa_2$, where $\varepsilon_I = (\varepsilon_1(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_3)(1-\tau_y)\lambda$ and $\varepsilon_A = \alpha_1(\varepsilon_2 + \tau_m(1-\tau_u)\varepsilon_1)$. All pairs (b, ξ_y) that verify this condition will go to a university with quality q . The distribution of parents human capital within a

given university of quality q can therefore be computed explicitly. The mass of individuals with $\ln b$ and going to $\ln q$ is given by:

$$\begin{aligned}
 f\left(\frac{1}{\varepsilon_A}(\ln q - x - (\varepsilon_I + \varepsilon_A) \ln b) \cap \ln b\right) &= f_{\xi_b}\left(\frac{1}{\varepsilon_A}(\ln q - x - (\varepsilon_I + \varepsilon_A) \ln b)\right) f_b(\ln b) \\
 &= \varphi\left(\frac{\ln q - x - (\varepsilon_I + \varepsilon_A) \ln b}{\varepsilon_A}, \mu_b, \sigma_b^2\right) \varphi(\ln b, m_b, \Sigma_b^2) \\
 &= \varphi\left(\ln b, \underbrace{\frac{\ln q - x - \varepsilon_A \mu_b}{\varepsilon_I + \varepsilon_A}}_{\mu_1^q}, \underbrace{\left(\frac{\varepsilon_A \sigma_b}{\varepsilon_A + \varepsilon_I}\right)^2}_{\sigma_1^2}\right) \varphi(\ln b, m_b, \Sigma_b^2) \\
 &= \varphi(\ln b, \mu_1^q, \sigma_1^2) \varphi(\ln b, m_b, \Sigma_b^2) = \varphi(\mu_1^q, m_b, \sigma_1^2 + \Sigma_b^2) \varphi(\ln b, \mu_2^q, \sigma_2^2)
 \end{aligned}$$

where the RHS is the mass of individuals going to quality q and the LHS is the density of people whose parents have human capital b conditional on college q .

$$f(\ln b|q) \sim \mathcal{N}\left(\frac{\Sigma_b^{-2} m_b + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^{-2} \sigma_b^{-2} \frac{(\ln q - x - \varepsilon_A \mu_b)}{\varepsilon_I + \varepsilon_A}}{\Sigma_b^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^{-2} \sigma_b^{-2}}, \frac{\Sigma_b^2 \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^2 \sigma_b^2}{\Sigma_b^2 + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^2 \sigma_b^2}\right) \sim \mathcal{N}(\mu_2^q, \sigma_2^2)$$

For future reference I introduce $\mu_2^q = \mu_{2,1} m_b + \mu_{2,2}(\ln q - x - \varepsilon_A \mu_b)$

$$\text{with } \mu_{2,1} = \frac{\Sigma_b^{-2}}{\Sigma_b^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^{-2} \sigma_b^{-2}} \quad \text{and} \quad \mu_{2,2} = \frac{\left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^{-2} \sigma_b^{-2}}{\left[\Sigma_b^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A}\right)^{-2} \sigma_b^{-2}\right] (\varepsilon_I + \varepsilon_A)}$$

where the second line stems from independence of b and ξ_b . The mass of individuals studying in college of quality q is $\varphi(\mu_1^q, m_b, \sigma_1^2 + \Sigma_b^2)$ and the density of $\ln b$ conditional on being in this college is $\varphi(\ln b, \mu_2^q, \sigma_2^2)$. From the distribution of parents' human capital within a college, the distribution of parents' income is $\ln y \sim \mathcal{N}(\ln(1 - a_y) + (1 - \tau_y) [\ln A + \lambda \mu_2^q + \mu \ln \ell] + \ln T_y, (1 - \tau_y)^2 \lambda^2 \sigma_2^2)$.

Distribution of college quality Since $\varphi(\mu_1^q, m_b, \sigma_1^2 + \Sigma_b^2)$ —with $\mu_1 = \frac{1}{\varepsilon_I + \varepsilon_A}(\ln q - x - \varepsilon_A \mu_b)$ and $\sigma_1^2 = \left(\frac{\varepsilon_A}{\varepsilon_A + \varepsilon_I}\right)^2 \sigma_b^2$ —is the mass of students in college of quality q , the distribution of quality is given by : $\ln q \sim \mathcal{N}((\varepsilon_I + \varepsilon_A) m_b + x + \varepsilon_A \mu_b, \varepsilon_A^2 \sigma_b^2 + (\varepsilon_I + \varepsilon_A)^2 \Sigma_b^2)$

Students' abilities From the definition of abilities $\ln z = \alpha_1 \ln b + \alpha_1 \ln \xi_b$ and the sorting rule used above $\ln q = (\varepsilon_I + \varepsilon_A) \ln b + \varepsilon_A \ln \xi_b + x$, one gets

$$\ln z = \frac{\alpha_1}{\varepsilon_A} (\ln q - \varepsilon_I \ln b - x) \Rightarrow \ln z | q \sim \mathcal{N} \left(\frac{\alpha_1}{\varepsilon_A} (\ln q - \varepsilon_I \mu_2^q - x), \left(\frac{\alpha_1 \varepsilon_I}{\varepsilon_A} \right)^2 \sigma_2^2 \right)$$

Solving for κ_2 and ν

The initial guess was that $\bar{y} = \kappa_2 q^\nu$. Recall that $\ln \bar{y}$ is the mean log (after tax) income within a college $\ln \bar{y} = \ln(1 - a_y) (A \ell^\mu)^{1-\tau_y} T_y + (1 - \tau_y) \lambda (\mu_{2,1} m_b + \mu_{2,2} (\ln q - x - \varepsilon_A \mu_b))$

Identifying coefficients with the guess $\ln \bar{y} = \ln \kappa_2 + \nu \ln q$, one gets:

$$\begin{aligned} \nu &= (1 - \tau_y) \lambda \mu_{2,2} = (1 - \tau_y) \lambda \frac{\left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^{-2} \sigma_b^{-2}}{\left[\Sigma_b^{-2} + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^{-2} \sigma_b^{-2} \right] (\varepsilon_I + \varepsilon_A)} \\ \Leftrightarrow \nu &= \frac{1}{\left[\Sigma_b^{-2} \left(\frac{\omega_A}{\omega_I + \omega_A} \right)^2 \sigma_b^2 + 1 \right] \left[(\omega_1 (1 - \tau_u) (1 - \tau_n) - \omega_3) + \frac{\omega_A}{(1 - \tau_y) \lambda} \right] + \omega_3} \end{aligned}$$

ν is therefore only a function of Σ_b^2 . Identifying the coefficient independent of $\ln q$, and recalling that x_t is a linear function of $\ln \kappa_2$, and defining $\tilde{x} = x - \varepsilon_3 \ln \kappa_2$, one gets:

$$\begin{aligned} \kappa_2 &= (1 - a_y) (A \ell^\mu)^{1-\tau_y} T_y e^{(1-\tau_y) \lambda (\mu_{2,1} m_b - \mu_{2,2} (x + \varepsilon_A \mu_b))} \\ &= \left((1 - a_y) (A \ell^\mu)^{1-\tau_y} T_y e^{(1-\tau_y) \lambda (\mu_{2,1} m_b - \mu_{2,2} (\tilde{x} + \varepsilon_A \mu_b))} \right)^{1-\nu \omega_3} \end{aligned}$$

A.1.5 Law of motion

Replacing κ_2 , T_y , T_e and T_u obtained above in the law of motion for human capital

$$\begin{aligned}
 \ln b' &= \ln \xi_y + \ln \kappa + \alpha_1(1 + \alpha_2(\varepsilon_2 + \tau_m(1 - \tau_u)\varepsilon_1)) \ln \xi_b + \alpha_b \ln b \\
 &+ \alpha_2 \omega_1 \left(\ln s + \ln(1 + a_b) + \ln \frac{(1 + a_u)}{p_I} + \tau_u \left(\ln A \ell^\mu (1 - a_y) + \lambda m_b + \lambda^2 \frac{\Sigma_b^2}{2} \right) + \frac{\tau_u}{1 - \tau_u} \frac{\sigma_I^2}{2} \right) \\
 &+ \alpha_2 \omega_1 (1 - \tau_u)(1 - \tau_n) \left(\ln A \ell^\mu + \ln(1 - a_y) + \tau_y \lambda m_b + \frac{\lambda^2}{2} (2 - \tau_y) \tau_y \Sigma_b^2 \right) \\
 &+ \alpha_2 \omega_3 \nu \varepsilon_A \frac{\sigma_b^2}{2} + \alpha_2 \omega_3 (1 - \tau_y) \lambda \mu_{2,1} m_b \\
 &+ \alpha_2 \omega_1 (1 - \tau_u) \left[(\tau_n \lambda - \alpha_1 \tau_m) m_b + \frac{\alpha_1 \tau_m}{2} (1 - \alpha_1 \tau_m) \sigma_b^2 + \tau_n (\ln A \ell^\mu (1 - a_y)) \right. \\
 &\left. - \left[\lambda^2 (1 - \tau_y)^2 (\tau_n - 2) \tau_n + 2 \lambda (1 - \tau_n) (1 - \tau_y) \tau_m \alpha_1 + (\alpha_1 \tau_m)^2 - \tau_n \lambda^2 (2 - \tau_y) \tau_y \right] \frac{\Sigma_b^2}{2} \right]
 \end{aligned}$$

I now take the expectation, I factorize out all the terms in m_b as well as all the terms in σ_b^2 . The next steps consist in simplifying the coefficient in front of σ_b^2 , of factorizing out all the terms in Σ_b^2 and in using the expression in (A.1.6) for σ_I^2 . I also use the fact that $\mu_{2,1} = 1 - \mu_{2,2}(\varepsilon_I + \varepsilon_A)$. One obtains

$$\begin{aligned}
 m'_b &= \rho m_b - \frac{\sigma_y^2}{2} + \ln \kappa \\
 &+ \left[\frac{\tau_u}{1 - \tau_u} \left(\frac{\alpha_1(1 - \tau_u)}{(1 - \nu \omega_3)} (\tau_m + \omega_2(1 - \tau_n)\nu) \right)^2 - \alpha_1 (\alpha_2 (\omega_2 + \omega_1(1 - \tau_u)(\tau_m)^2 \alpha_1) + 1) \right] \frac{\sigma_b^2}{2} \\
 &+ \alpha_2 \omega_1 (\ln A \ell^\mu (1 - a_y) s(1 + a_u)(1 + a_b) - \ln p_I) \\
 &+ \alpha_2 \omega_1 \left[\lambda^2 + \frac{\tau_u}{1 - \tau_u} \left(\frac{\alpha_1(1 - \tau_u)}{(1 - \nu \omega_3)} (\tau_m + \omega_2(1 - \tau_n)\nu) \right)^2 \left(\frac{\omega_I}{\omega_A} + 1 \right)^2 \right. \\
 &\quad \left. - (1 - \tau_u) [\lambda(1 - \tau_y)(1 - \tau_n) + (\alpha_1 \tau_m)]^2 \right] \frac{\Sigma_b^2}{2}
 \end{aligned}$$

with $\rho = \alpha_1 + \alpha_3 + \alpha_1 \alpha_2 \omega_2 + \alpha_2 \omega_1 \lambda$. Finally taking the variance gives the expression for the law of motion of Σ_b^2 : $\Sigma_b'^2 = \sigma_y^2 + (\alpha_1(1 + \alpha_2(\varepsilon_2 + \tau_m(1 - \tau_u)\varepsilon_1)))^2 \sigma_b^2 + (\alpha_b)^2 \Sigma_b^2$ Gathering all our results, the law of

motion of human capital is given by

$$\ln h_{t+1} \sim \mathcal{N}(m_{b,t+1}, \Sigma_{b,t+1}^2)$$

$$m_{b,t+1} = \rho_t m_{b,t} + X_1(m_{b,t}, \{\Sigma_b\}_{s=t}^\infty)$$

Mean of (log) Human Capital

$$\Sigma_{b,t+1}^2 = (\alpha_{b,t}(\Sigma_{b,t}))^2 \Sigma_{b,t}^2 + X_{2,t}(\Sigma_{b,t})$$

Variance of (log) Human Capital

$$\rho_t = \alpha_1 + \alpha_3 + \alpha_1 \alpha_2 \omega_2 + \alpha_2 \omega_1 \lambda_t$$

$$X_{1,t}(m_{b,t}, \{\Sigma_b\}_{s=t}^\infty) = -\frac{\sigma_y^2}{2} + \ln \kappa$$

$$+ \left[\frac{\tau_{u,t}}{1 - \tau_{u,t}} \left(\frac{\alpha_1(1 - \tau_{u,t})}{(1 - \nu_t(\Sigma_{b,t})\omega_3)} (\tau_{m,t} + \omega_2(1 - \tau_{n,t})\nu_t(\Sigma_{b,t})) \right)^2 - \alpha_1 (\alpha_2 (\omega_2 + \omega_1(1 - \tau_{u,t})(\tau_{m,t})^2 \alpha_1) + 1) \right] \frac{\sigma_b^2}{2}$$

$$+ \alpha_2 \omega_1 (\ln A \ell_t^\mu(\{\Sigma_{b,t}\}_{s=t}^\infty)(1 - a_{y,t})s_t(\{\Sigma_b\}_{s=t}^\infty)(1 + a_{u,t})(1 + a_{b,t}) - \ln p_{I,t}(m_{b,t}, \Sigma_{b,t}))$$

$$+ \alpha_2 \omega_1 \left[\lambda_t^2 + \frac{\tau_{u,t}}{1 - \tau_{u,t}} \left(\frac{\alpha_1(1 - \tau_{u,t})}{(1 - \nu_t(\Sigma_{b,t})\omega_3)} (\tau_{m,t} + \omega_2(1 - \tau_{n,t})\nu_t(\Sigma_{b,t})) \right)^2 \left(\frac{\omega_{I,t}}{\omega_{A,t}} + 1 \right)^2 \right. \\ \left. - (1 - \tau_{u,t}) [\lambda(1 - \tau_{y,t})(1 - \tau_{n,t}) + (\alpha_1 \tau_{m,t})]^2 \right] \frac{\Sigma_{b,t}^2}{2}$$

$$X_{2,t}(\Sigma_{b,t}) = \sigma_y^2 + (\alpha_1(1 + \alpha_2(\varepsilon_2 + \tau_m(1 - \tau_u)\varepsilon_1)))^2 \sigma_b^2$$

A.1.6 From the distribution of $\ln q$ to the distribution of $\ln I$.

Using the definition of q and the expression for $\bar{z}$ obtained earlier, $\ln q = \ln \tilde{I}^{\omega_1} \bar{z}^{\omega_2} = \omega_1 \ln \tilde{I} + \omega_2 \left(\frac{\alpha_1}{\varepsilon_A} (\ln q - \varepsilon_I \mu_2^q - x) \right)$ which implies $\ln \tilde{I} = \frac{1}{\omega_1} \left(\ln q \left(1 - \alpha_1 \frac{\omega_2}{\varepsilon_A} \right) + \alpha_1 \frac{\omega_2}{\varepsilon_A} (\varepsilon_I \mu_2 + x) \right)$.

Given the expression for $\mu_2^q \mu_2^q = \mu_{2,1} m_b + \mu_{2,2} (\ln q - x - \varepsilon_A \mu_b)$, one gets

$$\ln \tilde{I} = \frac{1}{\omega_1} \left(\ln q \left(1 - \alpha_1 \frac{\omega_2}{\varepsilon_A} + \alpha_1 \frac{\omega_2}{\varepsilon_A} \varepsilon_I \mu_{2,2} \right) + \alpha_1 \frac{\omega_2}{\varepsilon_A} \left(\varepsilon_I \mu_{2,1} m_b + (1 - \varepsilon_I \mu_{2,2}) x - \varepsilon_I \mu_{2,2} \varepsilon_A \mu_b \right) \right)$$

Hence from the distribution $\ln q$ I can recover the distribution of $\ln \tilde{I} \sim \mathcal{N}(\mu_{\tilde{I}}, \sigma_{\tilde{I}}^2)$ with

$$\mu_{\tilde{I}} = \frac{1}{\omega_1} \left(\mu_q \left(1 - \alpha_1 \frac{\omega_2}{\varepsilon_A} + \alpha_1 \frac{\omega_2}{\varepsilon_A} \varepsilon_I \mu_{2,2} \right) + \alpha_1 \frac{\omega_2}{\varepsilon_A} \left(\varepsilon_I \mu_{2,1} m_b + (1 - \varepsilon_I \mu_{2,2}) x - \varepsilon_I \mu_{2,2} \varepsilon_A \mu_b \right) \right)$$

$$\text{and } \sigma_{\tilde{I}}^2 = \left(\frac{\alpha_1(1 - \tau_n)}{(1 - \nu\omega_3)} (\tau_m + \omega_2(1 - \tau_n)\nu) \right)^2 \left(\sigma_b^2 + \left(\frac{\omega_I}{\omega_A} + 1 \right)^2 \Sigma_b^2 \right)$$

The last line stems from

$$\frac{1}{\omega_1} \left(1 - \alpha_1 \frac{\omega_2}{\varepsilon_A} + \alpha_1 \frac{\omega_2}{\varepsilon_A} \varepsilon_I \mu_{2,2} \right) = \frac{1}{\omega_1} \left(1 - \alpha_1 \frac{\omega_2}{\varepsilon_A} + \alpha_1 \frac{\omega_2}{\varepsilon_A} \varepsilon_I \mu_{2,2} \right) = \frac{\alpha_1(1 - \tau_u)}{\varepsilon_A(1 - \nu\omega_3)} (\tau_m + \omega_2(1 - \tau_n)\nu)$$

where I used $\nu = (1 - \tau_y)\lambda\mu_{2,2}$ and $\varepsilon_I = \frac{\omega_I}{1 - \nu\omega_3}$. Finally

$$\sigma_I^2 = \left(\frac{\alpha_1(1 - \tau_u)}{\varepsilon_A(1 - \nu\omega_3)} (\tau_m + \omega_2(1 - \tau_n)\nu) \right)^2 \sigma_q^2 = \left(\frac{\alpha_1(1 - \tau_u)}{(1 - \nu\omega_3)} (\tau_m + \omega_2(1 - \tau_n)\nu) \right)^2 \left(\sigma_b^2 + \left(\frac{\omega_I}{\omega_A} + 1 \right)^2 \Sigma_b^2 \right)$$

Since $\ln \tilde{I} = \ln I - (1 - \tau_u)\frac{\sigma_u^2}{2}$ and σ_u^2 is common across all colleges, I have $\ln I \sim \mathcal{N}(\mu_I, \sigma_I^2)$ with $\mu_I = \mu_{\tilde{I}} + (1 - \tau_u)\frac{\sigma_u^2}{2}$ and $\sigma_I^2 = \sigma_{\tilde{I}}^2$

Expression for σ_u^2 Given that all households save a fraction s of their disposable income and the selection equation into college, one gets

$$\ln e_u = \ln \frac{(1 + a_b)s}{T_e} + \tau_m \frac{\alpha_1}{\varepsilon_A} (\ln q - x) + \ln b^{(1 - \tau_n)(1 - \tau_y)\lambda - \tau_m \frac{\alpha_1}{\varepsilon_A} \varepsilon_I} + (1 - \tau_n) \ln T_y(1 - a_y) A^{1 - \tau_y} \ell^{(1 - \tau_y)\mu}$$

Hence the within-university variance of tuitions is given by:

$$\sigma_u^2 = \left((1 - \tau_n)(1 - \tau_y)\lambda - \tau_m \frac{\alpha_1}{\varepsilon_A} \varepsilon_I \right)^2 \sigma_2^2 = \left((1 - \tau_y)\lambda \frac{(1 - \tau_n)\omega_2 + \tau_m\omega_3}{\omega_2 + \omega_1(1 - \tau_u)\tau_m} \right)^2 \sigma_2^2$$

which is indeed constant across universities wince σ_2^2 is an aggregate constant.

A.1.7 Details on the Positioning Game

In this appendix I give a formal explanation of the positioning game as well as a characterization of the equilibrium. Recall the general environment. There is a continuum of colleges $j \in [0, 1]$. At each generation $t \in \mathbb{N}$, they play a positioning game. The games played at any two generations $t > t'$ are independent of each other.

At a given generation $t \in \mathbb{N}$, and before playing the positioning game, each college is given a real

number $o \in [0, 1]$. The positioning game is sequential and o is the order in which colleges play. Without loss of generality, since all colleges are identical, one can relabel colleges $j = o$ so that their label is also their order.² Colleges play sequentially in descending order: j plays before j' if and only if $j > j'$. Each college plays once.

All colleges have the same set of actions: the line of qualities $q \in \mathbb{R}_+$. The history of the sequential game up to college j 's turn is a (injective) function $\mathcal{H}_j^+ : (j, 1] \rightarrow \mathbb{R}_+$ that describes the colleges' actions up to j 's turn. A strategy for college j is a choice of quality $q \in \mathbb{R}_+$ whenever it is its turn; abusing notation I denote it $q_j(\mathcal{H}_j^+)$. Denoting $\mathcal{H}_j : [j, 1] \rightarrow \mathbb{R}_+$ the history including college j 's action, one has, for all $k > j$, $\mathcal{H}_j(k) = \mathcal{H}_j^+(k)$ and $\mathcal{H}_j(j) = q_j(\mathcal{H}_j^+)$. $\mathcal{H}_0$ denotes a terminal history.

I now introduce the notion of the *set of available students* at quality q at history $\mathcal{H}_j$. Denote $S(q) \subset \mathcal{I}$ the subset of students who demand quality q and $\text{card}(S(q))$ its cardinal, similarly denote $S(q, \mathcal{H}_j)$ the subset of students demanding quality q who are not in a college yet after history $\mathcal{H}_j$ (we call it the set of available students).

The cardinality of the set of available students at each quality to colleges that play later $j' \leq j$ is a function of the positions of colleges that have already played, $j' > j$, because when college j chooses quality q it takes a subset of these students, $S(q, \mathcal{H}_j) \subset S(q, \mathcal{H}_j^+)$. More specifically, I assume that college j picks a subset of students of cardinality $\aleph_1$ (its assumed size). I further assume that at any history $\mathcal{H}_j^+$, if $\text{card}(S(q), \mathcal{H}_j^+) \leq \aleph_1$ and j chooses q , then college j takes all the students at quality q and $\text{card}(S(q, \mathcal{H}_{j'})) = 0$ for all $j' \leq j$.³ If $\text{card}(S(q, \mathcal{H}_j^+)) > \aleph_1$, I assume that college j picks a subset of students of cardinality $\aleph_1$ which implies $\text{card}(S(q), \mathcal{H}_j) > \aleph_1$.⁴

Recall that the objective of the college is to deliver the highest quality possible. If they faced no con-

²This assumption of an order across colleges captures in a very direct way the notion that colleges do not start on an equal foot in the competition for prestige. In the real world, there are slow-moving state variables that gives an advantage to some colleges in this race, such as their reputation, their faculty member, their stock of publications, their endowment. Our assumption should be seen as a reduced-form expression of this *ex ante* hierarchy of advantages created by these state variables that this paper abstracts from.

³This is indeed a restriction, and not a tautology. It would be possible for a countable number of colleges to offer the same quality q and still respect the size constraint since a countable set of set of cardinal $\aleph_1$ is still of cardinal $\aleph_1$. It is however an inconsequential restriction which allows to associate one college with one quality since in equilibrium it is true that $\text{card}(S(q)) = \aleph_1$.

⁴Although this case might arise in some other version of the model, it doesn't happen in any equilibria analyzed in this paper.

straint, they would all choose to deliver the highest quality. All colleges would like to be Harvard (or Princeton), but there is room for only one. This notion is captured by the size constraint: colleges can't be too small. Specifically, if at history $\mathcal{H}_j^+$ the set of available students at q is lower than the cardinality of the continuum, $\text{card}(S(q, \mathcal{H}_j^+)) < \aleph_1$, the payoff of college j if it chooses q is 0, and I say that college j is *not operating*.⁵ If $\text{card}(S(q, \mathcal{H}_j^+)) \geq \aleph_1$, and college j chooses q , then its payoff is simply q and we say that it is *operating*.

This induces a preference relationship over the set of possible terminal histories. Consider any two terminal histories $\mathcal{H}_0, \mathcal{H}'_0$ in which college j is operating. College j prefers $\mathcal{H}_0$ to $\mathcal{H}'_0$, $\mathcal{H}_0 \succsim \mathcal{H}'_0$ if and only if $\mathcal{H}_0(j) = q_j \geq q'_j = \mathcal{H}'_0(j)$ with strict preference for strictly higher quality. A college always prefers a terminal history in which it is operating over one in which it is not.

Denote $q_{<j}^* = \{q_k^*(\mathcal{H}_k^+)\}_{k \in [0, j)}$ the strategy profile of colleges playing (strictly) after j and $\mathcal{H}_0(\mathcal{H}_j^+, q_j(\mathcal{H}_j^+), q_{<j}^*)$ the terminal history that follows history $\mathcal{H}_j^+$ and induced by the strategies of college j , $q_j(\mathcal{H}_j^+)$ and of the colleges playing afterwards $q_{<j}^*$. A subgame perfect Nash equilibrium of this game is a strategy profile $\{q_j^*(\mathcal{H}_j^+)\}_{j \in [0, 1]}$ such that for all j , given the strategies of the colleges playing next $q_{<j}^*$

$$\mathcal{H}_0(\mathcal{H}_j^+, q_j^*(\mathcal{H}_j^+), q_{<j}^*) \succsim \mathcal{H}_0(\mathcal{H}_j^+, q, q_{<j}^*)$$

for all $q \in \mathbb{R}_+$.

Detail on the index set of households, $\mathcal{I}$. To be consistent with the notion that there is a continuum of colleges and a continuum of students within each college, it has to be the case that the cardinality of the set of students be strictly higher than the cardinality of the set of colleges, i.e. $\text{card}(\mathcal{I}) > \text{card}([0, 1]) = \aleph_1$. It seems natural to consider the smallest such cardinal. Using the axiom of choice, such a cardinal is $\aleph_2$. To fix ideas, this corresponds for example to the index set $\mathcal{I} = [0, 1]^{[0, 1]}$.

⁵The size constraint is what makes the game strategic: the positioning decisions of higher-ranked colleges influence the payoffs of lower-ranked colleges.

Assumption 1. The cardinal of the set of households is the same as the continuum of continua

$$\text{card}(\mathcal{I}) = \aleph_2$$

Equilibrium Characterization. The following lemma says that the quality delivered by each college follows the same order as the order in which colleges play the game.

Lemma 8. *Assume the distribution of students over quality is continuous over $\mathbb{R}_+$. Then in equilibrium,*

$$q_j > q_{j'} \iff j > j'.$$

Proof. Since the distribution is continuous over $\mathbb{R}_+$ and there are a cardinal $\aleph_2$ of students, there must be a cardinal $\aleph_1$ of students demanding a given quality q , i.e. $\text{card}(S(q)) = \aleph_1$ for all $q \in \mathbb{R}_+$. (Otherwise there would be a mass point at some q , contradicting the assumption of a continuous distribution). Hence, by the assumption made earlier, whenever college j chooses a location q that is unoccupied $\text{card}(S(q, \mathcal{H}_j^+)) = \aleph_1$, it takes all of its students and no students is left for a college playing later, $\text{card}(S(q, \mathcal{H}_{j'})) = 0$ for all $j' \leq j$. This implies that if there exists $\underline{q}$ such that the history up to j is bounded on the left by $\underline{q}$: $\mathcal{H}_j^+((j, 1]) = (\underline{q}, +\infty)$, then a college j 's optimal location is $\underline{q}$: choosing strictly above $\underline{q}$ would mean not operating by the previous argument, and choosing exactly $\underline{q}$ rather than a strictly lower quality is strictly preferred. This shows that in any equilibrium in which the distribution for quality demanded is continuous over $\mathbb{R}_+$, for any $j > j'$, one has $q_{j'} < q_j$.

□

A.1.8 Existence and Uniqueness of Equilibrium Path

The set of equations defining an equilibrium path in proposition 4.3 is block-recursive. In particular, the law of motion of Σ_b , is independent and the path of all other variables are pinned-down by the path of Σ_b . It is therefore necessary and sufficient to focus on the existence and uniqueness of the path of Σ_b . I

first define new notations:

$$\Sigma_b'^2 = f(\Sigma_b^2) = \left[\alpha_1^2 + \left(\frac{A}{1 - \nu\omega_3} \right)^2 + \frac{2\alpha_1 A}{1 - \nu\omega_3} \right] \Sigma_b^2 + \sigma_y^2 + \left[\alpha_1^2 + \frac{B^2}{(1 - \nu\omega_3)^2} + \frac{2B\alpha_1}{1 - \nu\omega_3} \right] \sigma_b^2$$

$$\text{with } A = \alpha_1\alpha_2(\omega_2 + \tau_m(1 - \tau_u)\omega_1) + \alpha_2(\omega_1(1 - \tau_u)(1 - \tau_n) - \omega_3)(1 - \tau_y)\lambda$$

$$B = \alpha_1\alpha_2(\omega_2 + \tau_m(1 - \tau_m)\omega_1) \quad \nu = \frac{C}{E\Sigma_b^{-2} + (E + \omega_3)C}$$

$$C = \left(\frac{\omega_A}{\omega_I + \omega_A} \right)^{-2} \sigma_b^{-2} \quad E = (\omega_1(1 - \tau_u)(1 - \tau_n) - \omega_3) + \frac{\omega_A}{(1 - \tau_y)\lambda}$$

$f(\cdot)$ is differentiable for $\Sigma_b^2 \in (0, \infty)$ and $\lim_{\Sigma_b^2 \rightarrow 0} f(\Sigma_b^2) = \sigma_y^2 + [\alpha_1^2 + B^2 + 2B\alpha_1] \sigma_b^2 > 0$. The derivative

$f'(\cdot)$ is:

$$\begin{aligned} & \left[\alpha_1^2 + \left(\frac{A}{1 - \nu\omega_3} \right)^2 + \frac{2\alpha_1 A}{1 - \nu\omega_3} \right] + \left[\left[\left(\frac{A}{1 - \nu\omega_3} \right)^2 + \frac{\alpha_1 A}{1 - \nu\omega_3} \right] \Sigma_b^2 + \left[\frac{B^2}{(1 - \nu\omega_3)^2} + \frac{B\alpha_1}{1 - \nu\omega_3} \right] \sigma_b^2 \right] \frac{2\omega_3}{1 - \nu\omega_3} \frac{\partial \nu}{\partial \Sigma_b^2} \\ & \text{with } \frac{\partial \nu}{\partial \Sigma_b^2} = \frac{CE}{(E + \Sigma_b^2(E + \omega_3)C)^2} \\ & \text{Hence } \lim_{\Sigma_b^2 \rightarrow \infty} \frac{\partial f}{\partial \Sigma_b^2} = \left[\alpha_1^2 + \left(\frac{A}{1 - \frac{\omega_3}{E + \omega_3}} \right)^2 + \frac{2\alpha_1 A}{1 - \frac{\omega_3}{E + \omega_3}} \right] \\ & = [\alpha_1 + \alpha_1\alpha_2(\omega_2 + \tau_m(1 - \tau_m)\omega_1) + \alpha_2[\omega_1(1 - \tau_u)(1 - \tau_n) - \omega_3](1 - \tau_y)\lambda]^2 \end{aligned}$$

Therefore if $[\alpha_1 + \alpha_1\alpha_2(\omega_2 + \tau_m(1 - \tau_m)\omega_1) + \alpha_2[\omega_1(1 - \tau_u)(1 - \tau_n) - \omega_3](1 - \tau_y)\lambda]^2 < 1$, the equation $\Sigma_b^2 = f(\Sigma_b^2)$ has at least one solution since f is continuous and $\lim f(0) > 0$. Moreover, it has to be that an odd number of these solutions are characterized by $f'(\Sigma_b) < 1$, which guarantees local stability of the equilibrium path around these solutions.

Let's now show that the equilibrium path is unique for ω_3 small enough. A first order approximation of f in the neighborhood of $\omega_3 = 0$ is

$$\begin{aligned} f(\Sigma_b^2) & \simeq [\alpha_1^2 + A^2 + 2\alpha_1 A] \Sigma_b^2 + \sigma_y^2 + [\alpha_1^2 + B^2 + 2B\alpha_1] \sigma_b^2 + [[A^2 + \alpha_1 A] \Sigma_b^2 + [B^2 + \alpha_1 B] \sigma_b^2] 2\nu\omega_3 \\ & \simeq [\alpha_1^2 + A^2 + 2\alpha_1 A] + \underbrace{\left([[A^2 + \alpha_1 A] \Sigma_b^2 + [B^2 + \alpha_1 B] \sigma_b^2] \frac{E}{E + E C \Sigma_b^2} + [A^2 + \alpha_1 A] \Sigma_b^2 \right) \frac{C}{E + E C \Sigma_b^2}}_{F(\Sigma_b^2)} 2\omega_3 \end{aligned}$$

with $\nu = \frac{C}{E\Sigma_b^{-2} + EC}$. Since I have assumed that $[\alpha_1^2 + A^2 + 2\alpha_1 A] < 1$, and $F(\Sigma_b^2)$ is bounded for $\Sigma_b^2 \in (0, \infty)$, there exists an ω_3 small enough such that for all Σ_b^2 , $\frac{\partial f(\Sigma_b^2)}{\partial \Sigma_b^2} < 1$. This is sufficient for the existence and uniqueness of a globally stable steady-state.

A.1.9 Rise in returns to human capital

The total derivative of the IGE with respect to λ is given by

$$\left[\frac{\partial \nu}{\partial \lambda} + \frac{\partial \nu}{\partial \Sigma_b^2} \frac{\partial \Sigma_b^2}{\partial \lambda} \right] \left[\alpha_1 \alpha_2 \left(\frac{\partial \varepsilon_2}{\partial \nu} + \tau_m \frac{\partial \varepsilon_1}{\partial \nu} \right) + \alpha_2 \left(\frac{\partial \varepsilon_1}{\partial \nu} (1 - \tau_n) - \frac{\partial \varepsilon_3}{\partial \nu} \right) (1 - \tau_y) \lambda \right] + \alpha_2 (\varepsilon_1 (1 - \tau_n) - \varepsilon_3) (1 - \tau_y)$$

I then compute the derivatives:

$$\frac{\partial \varepsilon_l}{\partial \nu} = \varepsilon_l \varepsilon_3 > 0 \quad \frac{\partial \nu}{\partial \Sigma_b^2} = \frac{CE}{(E + (E + \omega_3)C\Sigma_b^2)^2} > 0$$

with C and E have been defined in the proof of existence and uniqueness.

$$\begin{aligned} \frac{\partial \nu}{\partial \lambda} &= \frac{2C \left(\frac{\omega_A}{\omega_A + \omega_I} \right) \frac{1}{\omega_I} [E\Sigma_b^{-2} + \omega_3 C] + C \frac{\omega_A}{(1 - \tau_y)\lambda^2}}{(E\Sigma_b^{-2} + (E + \omega_3)C)^2} > 0 \\ \frac{\partial X_2}{\partial \lambda} &= \sigma_b^2 \alpha_1 (1 + \alpha_2 (\varepsilon_2 + \tau_m \varepsilon_1)) \alpha_1 \alpha_2 \varepsilon_3 (\varepsilon_2 + \tau_m \varepsilon_1) \varepsilon_1 \frac{\partial \nu}{\partial \lambda} > 0 \\ \frac{\partial \Sigma_b^2}{\partial \lambda} &= \frac{\frac{\partial X_2}{\partial \lambda} + \Sigma_b^2 2 \frac{\partial \alpha_b}{\partial \lambda} \alpha_b}{1 - (\alpha_b)^2 - \Sigma_b^2 2 \frac{\partial \alpha_b}{\partial \Sigma_b^2} \alpha_b - \frac{\partial X_2}{\partial \Sigma_b^2}} > 0 \end{aligned}$$

where $\frac{\partial \alpha_b}{\partial \lambda}$ has to be understood as the partial derivative of α_b w.r.t. λ keeping Σ_b^2 constant. The last line stems from the fact that the steady-state is locally stable - which requires that $1 - (\alpha_b)^2 - \Sigma_b^2 2 \frac{\partial \alpha_b}{\partial \Sigma_b^2} \alpha_b - \frac{\partial X_2}{\partial \Sigma_b^2} = \frac{\partial (\Sigma_b^2)^2}{\partial (\Sigma_b^2)^2} > 0$. Hence, putting everything together yields

$$\frac{\partial \alpha_b}{\partial \lambda} = \underbrace{\left[\frac{\partial \nu}{\partial \lambda} + \frac{\partial \nu}{\partial \Sigma_b^2} \frac{\partial \Sigma_b^2}{\partial \lambda} \right]}_{>0} \varepsilon_3 \left[\alpha_1 \alpha_2 (\varepsilon_2 + \tau_m \varepsilon_1) + \alpha_2 (\varepsilon_1 (1 - \tau_n) - \varepsilon_3) (1 - \tau_y) \lambda \right] + \alpha_2 (\varepsilon_1 (1 - \tau_n) - \varepsilon_3) (1 - \tau_y) > 0.$$

This proves not only that the steady-state IGE is increasing in λ but that the variance of human capital in the economy is as well. Given that the variance of market income is given by $\lambda^2 \Sigma_b^2$ it is immediate that

it increases too. Turning to the private spending on higher education, given by s , it is immediate to see from the expressions (1.14) and (1.16) that it is increasing in the future path of λ . Let's now turn to the ratio of within college variance of (log) parental income over economy-wide variance of (log) income:

$$\begin{aligned} \frac{V(\ln y|q)}{V(\ln y)} &= \frac{1}{\lambda^2 \Sigma_b^2} \lambda^2 \frac{\Sigma_b^2 \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2}{\Sigma_b^2 + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2} = \frac{\left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2}{\Sigma_b^2 + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2} \\ \Rightarrow \frac{\partial \frac{V(\ln y|q)}{V(\ln y)}}{\partial \lambda} &= \frac{\sigma_b^2 \frac{\partial B}{\partial \lambda} \left[\Sigma_b^2 + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2 \right] - B \sigma_b^2 \left[\frac{\partial \Sigma_b^2}{\partial \lambda} + \sigma_b^2 \frac{\partial B}{\partial \lambda} \right]}{\left[\Sigma_b^2 + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2 \right]^2} = \frac{\sigma_b^2 \frac{\partial B}{\partial \lambda} \Sigma_b^2 - B \sigma_b^2 \frac{\partial \Sigma_b^2}{\partial \lambda}}{\left[\Sigma_b^2 + \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2 \sigma_b^2 \right]^2} < 0 \end{aligned}$$

with $B = \left(\frac{\varepsilon_A}{\varepsilon_I + \varepsilon_A} \right)^2$ and since $\frac{\partial \Sigma_b^2}{\partial \lambda} > 0$ and $\frac{\partial B}{\partial \lambda} < 0$.

Finally the variance of (log) college quality is given by $\varepsilon_A^2 \sigma_b^2 + (\varepsilon_I + \varepsilon_A)^2 \Sigma_b^2$. It is immediate that it increases with λ since $\varepsilon_I, \varepsilon_A, \Sigma_b^2$ increase with λ .

Monotonic transition path. From the law of motion of Σ_b^2 , in the first period the initial increase in λ raises α_b and triggers the initial increase in the dispersion of human capital. Since $X_2(\Sigma_b)$ and $\alpha_b(\Sigma_b)$ are both increasing in Σ_b it further increases Σ_b^2 at the following period and so on... This establishes that Σ_b^2 is strictly increasing over the transition path. This also establishes the monotonic increase in α_b and all ω 's.

Turning to the private spending on higher education, given by s , it is easy to see that it is increasing in the future path of λ , α_b and ε_1 . Since these three variables are increasing over the transition path, s also increases. The variance of log college quality is also increasing because $\varepsilon_I, \varepsilon_A, \Sigma_b^2$ are increasing over the transition path. The ratio of within college variance of (log) parental income over economy-wide variance of (log) income will decrease monotonically over the transition path because of the initial increase in λ , this is the first term in the derivative $\sigma_b^2 \frac{\partial B}{\partial \lambda} \Sigma_b^2$, and then decreases further as Σ_b increases, this is the second term $B \sigma_b^2 \frac{\partial \Sigma_b^2}{\partial \lambda}$.

A.2 Estimation - Details

A.2.1 The College Problem in the Quantitative Version

For future reference, I call M1 the benchmark model explored in section 2 and 3 and M2 the model with intergenerational financial transfers and enrollment decision.

In order to keep the college problem tractable despite the loss of closed-form expressions for the distribution of students within the college and equilibrium tuition, I assume that the problem of the college is still given by (A.7). While it is not possible to derive (A.7) from the primitive problem (1.8), the following re-interpretation gives support to the reduced-form formulation: there is a loss in the efficiency with which resources are used when the inequality of tuition fees among students rises. One can interpret it as a rise in human resources and administrative costs or as an increase in the sentiment of unfairness among students. The first order conditions for this problem are the same as in M1, see appendix A.1.2 for more details.

A.2.2 External Calibration

Income Tax Schedule a_y, τ_y . In order to calibrate τ_y , I take an average between the value estimated by [Heathcote, Storesletten, and Violante \(2017\)](#) and the ones needed to match the ratio between the market income and after tax and transfers Gini in the U.S in 2000, $\tau_y = .23$. The latter estimate τ_y directly using two different datasets over the period 2000-2005: using the data provided by the CBO—itsself based on the SOI and the CPS— they obtain $\tau_y = .2$ and using the PSID, they find $\tau_y = .18$. The value needed to rationalize the .12 difference in the Gini coefficient of households market income (.56) and after tax and transfers income (.44) in 2000-2005, within the log-normal framework of M1 is .26.⁶ The discrepancy is due on the one hand to the log-normal assumption and on the other to the slightly different different measure of income used—for example, [Heathcote, Storesletten, and Violante \(2017\)](#) exclude Medicare benefits from their measure of transfers. I calibrate a_y using average income tax rate provided by CBO: $a_y = .2$.

⁶CBO ([The Distribution of Household Income and Federal Taxes, 2013](#), CBO and own calculations

College Subsidies, a_b, a_u, τ_u . From the IPEDS, I compute a_u by dividing the total amount of public subsidies by the total revenues before public aid. From the NPSAS I obtain a_b by dividing the total amount of public financial aid by the sum of out of pocket payments. According to the specification for subsidies to university, τ_u can be estimated in a weighted least-square regression of (log) total revenues per student on (log) revenues before public transfers in the cross-section of colleges, where the weights are given by students enrollment. I run this regression in a companion paper [Capelle \(2019\)](#) and find $\tau_u = .35$ at the beginning of the 2000s.

Frisch Elasticity, η . η is set to match the Frisch elasticity of labor supply, $\varepsilon_{\ell,w} = \frac{1}{\frac{\eta}{\mu}-1}$. Empirical estimates of $\varepsilon_{\ell,w}$ range from .2 to .7 ([Chetty, Guren, Manoli, and Weber, 2011](#)). I explain in the next paragraph why my preferred estimate in the conservative value $\varepsilon_{\ell,w} = .2$. This implies $\eta = \mu \left(1 + \frac{1}{.2}\right) = 2$. Ideally we would have elasticities of lifetime household income to wages in order to be consistent with the model. Estimates however are at the individual and yearly level. I argue that they are likely upper bounds for two reasons. First they do not capture intra-household substitution. Second they do take into account the intertemporal substitution stemming from temporary fluctuations in wages.

Calibrating $\bar{\lambda}$, and μ . The returns to human capital in the educational services sector is set to $\bar{\lambda} = \lambda$ so that the price of educational services is given by $p_I = \frac{A_I}{A}$. I calibrate $\mu = 1$.

A.2.3 Internal Calibration

General Strategy. The algorithm used to estimate the parameters is akin to a Simulated Method of Moments. If $\bar{z}$ denotes the vector of the twelve parameters to be estimated, $M(\bar{z})$ the vector of model-generated moments, $\hat{M}(\bar{z})$ one such realization and $\hat{m}$ the vector of empirical moments, one seeks to find $\hat{\bar{z}}$ such that $\hat{\bar{z}} = \arg \min_{\bar{z}} \left[\hat{m} - \hat{M}(\bar{z}) \right]' W \left[\hat{m} - \hat{M}(\bar{z}) \right]$ with $E \left[\hat{M}(\bar{z}) \right] = M(\bar{z})$ and W a weighting matrix. I depart from this standard expression in two ways. First the empirical moments themselves $\hat{m}$ are function of a subset of parameters, $\bar{z}_1 = \{\alpha_1, \omega_2\}$. Many of the targeted moments are coefficients in a regression involving one of two variables that need to be constructed using $\bar{z}_1$: students ability z and college quality q . Secondly, as I have explained earlier, the procedure to generate the student ability variable, based on test scores, isn't a deterministic function of the observables and the pa-

rameters but involves some randomness. Our estimation procedure actually takes the following form: $\hat{\bar{z}} = \arg \min_{\bar{z}} \left[\hat{m}(\bar{z}_1, \varepsilon) - \hat{M}(\bar{z}) \right]' W \left[\hat{m}(\bar{z}_1, \varepsilon) - \hat{M}(\bar{z}) \right]$ where ε denotes the noise introduced in the process of constructing the student ability variable.

Despite the large dimension of the parameter space, the algorithm is quick and the global minimum to the loss function can be easily and with certainty found. I proceed in two steps. First I estimate M1, the version of the model without outside option and financial asset. The closed-form solutions to the model enables me to compute the exact value for the moments— $\hat{M}(\bar{z}) = M(\bar{z})$ —and to run the estimation on a fine and large grid. This in turn allows me to check numerically that the parameters are well identified, in the sense that the loss function is steep at the global minimum, which I do in A.2.4. The estimates are reported in the third column of table 1.1. I then estimate the augmented version M2, the version of the model with an outside option and a financial asset. When estimating the richer quantitative version, I use the estimates from the first step as initial values. The key assumption here is that the parameters estimated with M1 are not too far from the true parameters in M2. It turns out that almost all estimates but one, ω_2 , are identical to their counterpart in M1.

Assumption of Steady-State. I assume that the economy is at steady-state at the beginning of the 2000s. The assumption of steady-state really matters only for the identification of σ_y^2 . Moreover, the endogenous convergence speed is quite fast—the half life of the AR(1) for the variance of income is given by $-\ln(2)/\ln(\alpha_b^2) \simeq .5$, which corresponds to half a generation. But one could still be concerned that the movement of exogenous variables—especially persistent increase in λ —have not reached a steady-state. However, the future increase in λ feeds into the current allocation only through the aggregate investment rate into higher education—at least in the version of the model for which I have closed-forms—and through labor supply, so would not change the cross-sectional and inequality implications, on which most of the identification is based.

Identification of σ_b^2 and Construction of Model-Consistent Abilities. At this point of the procedure, I construct a grid on α_1 . Assume the relationship between children's high school abilities and parental human capital given by equation (A.22) is true. In NLSY97, I observe gross parental income $y_{m,i}$ and the ranking in test scores $\text{rank}(z_i)$. For a given (α_1, λ) , there exists a unique σ_b^2 that matches the correlation

between parental income and the rank of the child at the test, $\rho(\gamma_{m,i}, \text{rank}(z_i))$. This identifies $\sigma_b^2(\alpha_1)$.

I can then construct a model-consistent measure of abilities $\{\ln z_i\}(\alpha_1, \varepsilon)$ that is consistent with $(\alpha_1, \lambda, \sigma_b^2(\alpha_1))$ and $\text{rank}(z_i)$ to use in subsequent steps. This step implies a random draw that introduces some noise in the measure of ability, i.e. $\{\ln z_i\}$ are not deterministic function of $\alpha_1, \lambda, \sigma_b^2(\alpha_1)$ and $\text{rank}(z_i)$.

The Progressivity of Financial Aid, τ_n, τ_m and ω_3 . In M1, the government financial aid and the equilibrium tuition schedule are given by (A.23) and (A.24). In the NCES-NPSAS dataset, one observes parental income $\gamma_{m,i}$, test score, institutional aid, government aid as well as out of pocket payment. Regressing the (log) ratio of after-government aid payment on before-government aid payment over parental income and student ability gives τ_n and τ_m .

The second equation relates between before-government aid payment and college fixed effect, parental income and student ability. It tells us that the elasticity of before-government aid tuition to parental income $\varepsilon_{e, \gamma_m}(\alpha_1)$ identifies the progressivity of institutional financial aid ω_3 , what I called the social objective parameter. In the data I find that the fit of the second equation to the data is very high, $R^2 = 80\%$. In the model M2, I run the second regression on a simulated population.

Constructing a Measure of Quality, q_i . At this point of the procedure, I need to define a grid on $\frac{\omega_2}{\omega_1}$. At each point of the grid $(\alpha_1, \frac{\omega_2}{\omega_1})$, I construct a measure of annual quality delivered by all colleges, indexed by j . I collect average real spending per student by college using IPEDS and median test score within a college. The median test score is defined as the mean between the bottom and top quartile—the only available data in the IPEDS—which is exactly the median if the distribution is symmetric. When test scores data are not available for 2000, I either use years up to 2004, or impute them based on a regression of test scores on spending per student and other characteristics. This measure based on ACT or SAT is converted into a model-consistent measure of abilities using a scale based on publicly available quantiles of the distribution of these scores and using the guess on $\frac{\omega_2}{\omega_1}$. I construct the annual quality delivered by a college consistent with the functional form for the production function given by 1.9 and the guess for $\frac{\omega_2}{\omega_1}$.⁷

I then construct a measure of quality received by an individual i . q_i is a weighted average of the q_j where

⁷I do not need to take into account the within college heterogeneity, corresponding to σ_u^2 because, at least in the model, it is common to all colleges and will therefore factor out and leave our regression coefficients unchanged.

the weights depend on the time spent in each college and whether they have graduated at any point. I have checked the robustness of the results to alternative ways of aggregating annual college qualities.

Estimating the Elasticity of Quality to Ability, $\varepsilon_{q,z} \left(\alpha_1, \frac{\omega_2}{\omega_1} \right)$ and Identifying $\frac{\omega_2}{\omega_1}$.

According to the equilibrium sorting rule in M1, the elasticity of quality to ability identifies $\frac{\omega_2}{\omega_1}$ as can be seen from (A.25). Since the measure of quality is based on a guess on $\frac{\omega_2}{\omega_1}$, finding $\frac{\omega_2}{\omega_1}$ is a fixed point problem. Empirically, the elasticity of quality to ability doesn't change a lot with the guess on $\frac{\omega_2}{\omega_1}$, the latter is therefore tightly identified. Although this relationship should hold perfectly in the model, the R^2 in the data associated with this regression is 23%. Finally notice that even in M1, it is not possible to recover $\frac{\omega_2}{\omega_1}$ directly from the elasticity of quality to abilities without numerically solving the model, since it depends on the endogenous variable, $b(\Sigma_b)$.

The Human Capital Accumulation Function $\alpha_2, \alpha_3, \lambda$. According to the law of motion for human capital $\ln y'_{m,i} = c_y + \lambda \ln z_i + \alpha_2 \omega_1 \lambda \ln q_i + \alpha_3 \ln y_{m,i} + \ln \xi_{y,i}$ running a regression of children's income on parental income, abilities and quality identifies α_2 and α_3 and λ . I run the regression using the NLSY97, where I observe parental income, children ability, children college quality and children earnings. At this point, I gather all the moments that depend on $\left(\alpha_1, \frac{\omega_2}{\omega_1} \right), \left(\sigma_b^2 \quad \tau_n \quad \tau_m \quad \varepsilon_{e,y_m} \quad \varepsilon_{q,z} \quad \alpha_2 \quad \alpha_3 \quad \lambda \right)$

The remaining targeted moments are independent of $\left(\alpha_1, \frac{\omega_2}{\omega_1} \right)$.

Using the Gini Coefficient for Income to Identify σ_y^2 . I target the Gini coefficient for income. In M1, the Gini is given by equation (A.28) and (A.29). The best estimate for the Gini coefficient of lifetime labor earnings is from [Kopczuk, Saez, and Song \(2010\)](#) who have access to administrative data. There would be two issues with the NLSY97: first children labor earnings are observed only up to 2015, *i.e.* in their first years of labor market experience and a lot of them are not in a households yet. Secondly, top income are censored. [Kopczuk, Saez, and Song \(2010\)](#) finds that the eleven-year Gini coefficient is between .45 and .50. This is slightly lower than the annual Gini coefficient, which is between .49 and .57—depending on the exact measure of gross income used—in 2000 according to the [CBO](#), probably because of transitory income shock. I keep a Gini of lifetime labor earnings of .45 as a target.

The Intergenerational Elasticity identifies α_1 . From the steady-state equilibrium law of motion of human capital in M1, the children income elasticity to their parents income is the IGE: $\ln y'_{m,i} = c +$

$\alpha_b \ln y_{m,i} + \varepsilon_i$ The IGE directly informs α_1 . There is disagreement in the literature regarding the magnitude of the IGE, with estimates ranging from .3 to .6. Even the recent literature that uses administrative data is not immune to the short-panel and lifecycle biases.⁸ I take the intergenerational elasticity from Mazumder (2015) who provides the most robust estimates, $\alpha_b = .5$.

β and the Share of Private Spending for Higher Education in GDP. I calibrate the intergenerational discount factor, β , to match the average private spending on higher education in GDP. In M_1 , the latter is given by equation (A.30). The OECD reports that share of private spending for higher education in GDP in the U.S. over the period 2000-2004 is 1.3%.⁹

Calibrating $\underline{q}$ and the enrollment rate. To calibrate $\underline{q}$ —the outside option to college—it is natural to target the enrollment rate: the lower $\underline{q}$, the stronger the incentives to go to college. The immediate enrollment rate, provided by the NCES, in the U.S. in the 2000s is about 70%.

Calibrating limits to intergenerational financial transfers, $\underline{a}$, $\bar{a}$. There is no limit to how much individuals can bequeath, $\bar{a} = +\infty$. For $\underline{a}$, I target the official limit on student loans, as a percentage of lifetime GDP per capita, which amounts to 3%.

r and the income-sorting channel. First I consider a small open economy, and do not try to find the interest rate that ensures market clearing of the financial asset market. Instead, I target the elasticity of quality to parental income. It turns out that in M_1 , the elasticity resulting from the estimation is too high compared to what is in the data—.4 instead of .2. By increasing r , one gives incentives to individuals to avoid debt, which relaxes the borrowing constraint and decreases the dependence of college quality to parental income. I find a generational net interest rate of 180% which corresponds to an annual interest rate of 3.5% for a generation length $H = 30$ years.

A.2.4 Analytical Evidence of Identification in M_1

Lemma 9. *In M_1 , given our set of targets (detailed in column 4 of table (1.1)), if $\omega_3 = 0$ and $\{z_i\}$ are observables, there exists a unique set of parameters $\bar{z}_1$ consistent with the model-implied restrictions.*

⁸ Given that only the first years of the children earnings are observed while the parents are observed when they are already older, the estimate of the intergenerational elasticity from the NLSY97 is biased downward—I indeed find $\hat{\alpha}^b = .3$.

⁹ For reference, they also report that the share of public spending is 1%, making spending in higher education 2.3% of GDP.

From equations (A.23) and (A.24) one gets $\tau_n, \tau_m, \omega_3/\omega_1$. From the coefficient in front of z in (A.25), one identifies ω_3/ω_1 . From (A.26), one identifies $\alpha_2\omega_1$, and α_3 . The IGE is obtained with (A.27) (although I use another, more reliable source, as explained in the main text), which together with the previously obtained $\alpha_2\omega_1, \alpha_3, \omega_2/\omega_1, \omega_3/\omega_1, \tau_u, \tau_n, \tau_m, \tau_y, \lambda$ gives α_1 . From equation (A.22) one gets σ_b^2 . The computation of the steady-state Gini identifies σ_y^2 (equation (A.28) and (A.29)). Finally, targeting the LHS of (A.30) and computing the steady-state value of V gives us β .

The assumption $\omega_3 = 0$ implies $b(\Sigma_b) = b$ in equation (A.25). When ω_3 , the set of restrictions for the estimation is given by

$$\ln z_i = \ln(\xi_{b,i} b_i)^{\alpha_1} = \frac{\alpha_1}{\lambda} \ln y_{m,i} + \alpha_1 \ln \xi_{b,i} \quad \text{with } V(\ln \xi_{b,i}) = \sigma_b^2 \quad (\text{A.22})$$

$$\ln \frac{e_i^b}{e_i^u} = \tau_n(1 - \tau_y) \ln y_{m,i} - \tau_m \ln z_i + c_0 \quad (\text{A.23})$$

$$\ln e_{i,j}^b = \gamma_j + \left(\frac{\omega_3}{\omega_1}(1 - \tau_u) + \tau_n \right) (1 - \tau_y) \ln y_{m,i} - \left(\frac{\omega_2}{\omega_1(1 - \tau_u)} + \tau_m \right) \ln z_i + c_1 \quad (\text{A.24})$$

$$\ln q_i = c + b \left[\left((1 - \tau_u)(1 - \tau_n) - \frac{\omega_3}{\omega_1} \right) (1 - \tau_y) \ln y_{m,i} + \left(\frac{\omega_2}{\omega_1} + \tau_m(1 - \tau_u) \right) \ln z_i \right] \quad (\text{A.25})$$

$$\ln y'_{m,i} = c_y + \lambda \ln z_i + \alpha_2 \omega_1 \lambda \ln q_i + \alpha_3 \ln y_{m,i} + \ln \xi_{y,i} \quad (\text{A.26})$$

$$\ln y'_{m,i} = c + \alpha_b \ln y_{m,i} + \varepsilon_i \quad (\text{A.27})$$

$$\text{Gini}(y_m) = 2\Phi \left(\lambda \sqrt{\frac{(\Sigma_b^2)^{SS}}{2}} \right) - 1 \quad (\text{A.28})$$

$$(\Sigma_b^2)^{SS} = \frac{\sigma_y^2 + (\alpha_1[1 + \alpha_2(\varepsilon_2(\Sigma_b^{SS}) + \tau_m(1 - \tau_u)\varepsilon_1(\Sigma_b^{SS}))])^2 \sigma_b^2}{1 - (\alpha_b)^2} \quad (\text{A.29})$$

$$s(1 - a_y) = \frac{\beta \alpha_2 \omega_1 (1 - \tau_u) V(1 - a_y)}{1 - \beta + \beta \alpha_2 \omega_1 (1 - \tau_u) V} \quad (\text{A.30})$$

A.3 Counterfactuals: Details

Policy Counterfactual: Random Allocation In this appendix, I provide details regarding the computation of the counterfactual with random allocation of students. All individuals receive the same amount of college quality $\bar{q}$, so that the law of accumulation of human capital is: $b' = z\bar{q}^{\alpha_2} b^{\alpha_3}$ where

$\bar{q}$ is consistent with the average child ability in the economy and the level of spending per student: $\bar{q} = (\bar{s} \times \text{Disp Income})^{\omega_1} \bar{z}^{\omega_2}$ with $\ln \bar{z} = E[\ln z]$ where $\bar{s}$ denotes the aggregate spending rate for higher education and "Disp Income" is the average disposable income in the economy. Given that private agents stop spending on higher education in this counterfactual, all resources have to be financed through taxes and transfers to colleges. I set the proportion of disposable income going to higher education, $\bar{s}$, equal to what it was in the *status quo* equilibrium, *i.e.* $\bar{s} = s(1 + a_u)(1 + a_b)$. Although inconsequential for mobility and inequality, the choice of $\bar{s}$ has a first order effect on the steady-state level of GDP. One can easily show the laws of motion for the mean and the variance are given by $m' = \rho m + \ln \kappa - \frac{\sigma_y^2}{2} - \alpha_1(1 + \alpha_2\omega_2)\frac{\sigma_b^2}{2} + \alpha_2\omega_1 \left[\ln(\bar{s}A(1 - a_y))\ell^\mu + \frac{\lambda^2}{2}\Sigma_b^2 \right]$ and $\Sigma_b^{2'} = (\alpha_1 + \alpha_3)^2\Sigma_b^2 + \sigma_y^2 + \alpha_1^2\sigma_b^2$. These two moments are sufficient to compute the income Gini and GDP.

Counterfactuals with respect to the returns to education Changing λ in the first counterfactual changes not only the returns to education but also the level of output for a given distribution of human capital. Although it has no impact on inequality and mobility—our two measures of interest—in M1, this change in the level of GDP matters in M2 through the enrollment choice. A lower λ decreases GDP which results in a decrease in the enrollment rate for a given $\underline{q}$. To address this issue, I consider three possible assumptions: i) Keep $\underline{q}$ unchanged, ii) Adjusting $\underline{q}$ to target an enrollment rate of 50% which corresponds to its level in 1980, iii) Adjusting $\underline{q}$ to target an enrollment rate of 70% which corresponds to its level in the original calibration. The results for the main variables of interest are reported in table (A.1):

Table A.1: Change in s , Gini coefficient and IGE in counterfactuals

	Assumption	Enrollment	s	Gini y_m	IGE	Endog. Amplif. of Gini y_m
$\lambda = .67$		70%	1.3%	.45	.5	-
	1	5%	-92%	-23%	-14%	+ 35%
$\lambda = .55$	2	50%	-36%	-17%	-2.8%	+ 6.3%
	3	70%	-18%	-16%	-.6%	+ 2.4%

Legend: The three assumptions are as follows: 1- Keep $\underline{q}$ unchanged. 2- Adjusting $\underline{q}$ to target an enrollment rate of 50% which corresponds to its level in 1980. 3- Adjusting $\underline{q}$ to target an enrollment rate of 70% which corresponds to its level in the original calibration.

Fixing Tuition and Spending within Counterfactual 1. Start from the price schedule in equilibrium

faced by HH in 1980: $e(q, z, y) = z^{-\tau_m} y^{\tau_n \frac{T_e}{(1+a_b)}} \left(\frac{p_I}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_1}} z^{-\frac{\varepsilon_2}{\varepsilon_1}} \left(\frac{y}{\kappa_2} \right)^{\frac{\varepsilon_3}{\varepsilon_1}} \right)^{\frac{1}{1-\tau_u}}$. Consider the marginal distributions of high school ability and income, denoted $F_{z,1980}(z), F_{y,1980}(y)$. As well as the distribution of colleges quality $F_{q,1980}(q)$. Denote $F_{hs,1980}^{-1}(\cdot), F_{y,1980}^{-1}(\cdot), F_{q,1980}^{-1}(\cdot)$ the respective quantile function. The object I fix is the following function

$$e(\text{rk}_q, \text{rk}_z, \text{rk}_y) = C \left(F_{q,1980}^{-1}(\text{rk}_q) \right)^{\frac{1}{\varepsilon_1(1-\tau_u)}} \left(F_{z,1980}^{-1}(\text{rk}_z) \right)^{-\frac{\varepsilon_2}{\varepsilon_1(1-\tau_u)} - \tau_m} \left(F_{y,1980}^{-1}(\text{rk}_y) \right)^{\frac{\varepsilon_3}{\varepsilon_1(1-\tau_u)} + \tau_n}$$

where rk (rank) denotes the quantile in their respective distribution and $C = \frac{T_e}{(1+a_b)} \left(\frac{p_I}{(1+a_u)T_u} \left(\frac{1}{\kappa_2} \right)^{\frac{\varepsilon_3}{\varepsilon_1}} \right)^{\frac{1}{1-\tau_u}}$ as well as $\varepsilon_1, \varepsilon_2, \varepsilon_3, \tau_u$ are fixed at their 1980 value. In other words, I fix the rank of a college a household gets into.

From the constant spending rate across all households at a given time t , I get the following relationship:

$$\begin{aligned} sy &= C \left(F_{q,1980}^{-1}(\text{rk}_q) \right)^{\frac{1}{\varepsilon_1(1-\tau_u)}} \left(F_{z,1980}^{-1}(\text{rk}_z) \right)^{-\frac{\varepsilon_2}{\varepsilon_1(1-\tau_u)} - \tau_m} \left(F_{y,1980}^{-1}(\text{rk}_y) \right)^{\frac{\varepsilon_3}{\varepsilon_1(1-\tau_u)} + \tau_n} \\ \text{rk}_q &= F_{q,1980} \left[\left(\frac{sy}{C} \right)^{\varepsilon_1(1-\tau_u)} \left(F_{z,1980}^{-1}(\text{rk}(z)) \right)^{\varepsilon_2 + \tau_m \varepsilon_1(1-\tau_u)} \left(F_{y,1980}^{-1}(\text{rk}(y)) \right)^{-\varepsilon_3 - (1-\tau_u)\varepsilon_1\tau_n} \right] \end{aligned}$$

In the counter-factual I fix the spending rate to the one in the final steady-state of the benchmark economy. This identifies the partial effect of colleges' reactions to market forces keeping the policy rules of families constant. The next step consists in mapping rk_q to an actual quality. Here I do two experiments. In the first, the level of quality remains constant at what it was in 1980, and in the second I allow for change in peer effects but not in spending. For this, I create a grid on $\text{rk}_q = [0, 1]$, put people in bins according to their choice of rank, and take the geometric average. This gives us a mapping $\hat{z}(\text{rk}_q) : [0, 1] \rightarrow \mathbb{R}^+$. I then combine it with our fixed mapping of investment per student to get quality q .

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Appendix B

Forty Years of Inequality and State Subsidies across U.S. Colleges—Appendix

B.I Additional Figures

B.I.I Revenue per Student

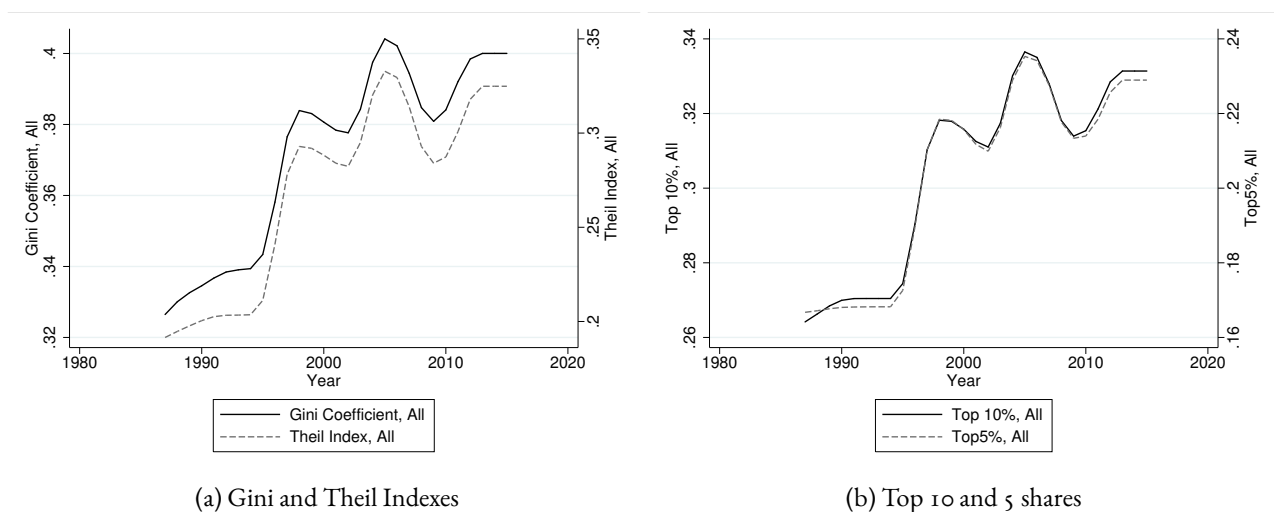


Figure B.1: Inequality in Total Revenues

B.1.2 Faculty per Student

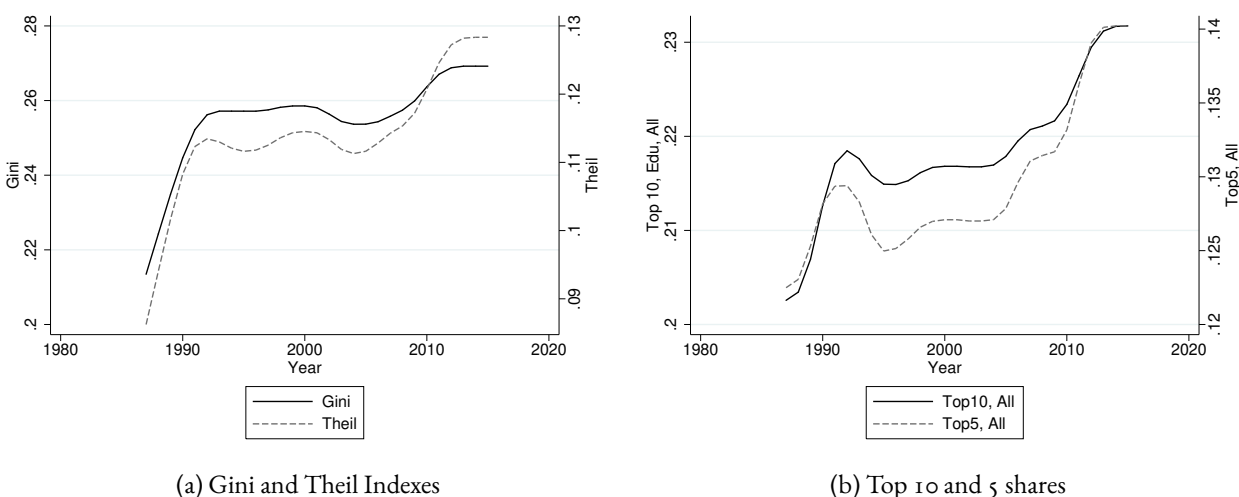


Figure B.2: Inequality in Faculty per FTE Students

B.1.3 Income Inequality

The data for the Gini coefficient are from the The Distribution of Household Income, 2015 released by the CBO and available [here](#). The CBO reports four different measures of Gini, from the one based on pure market-income to the one based on after tax and transfers income. I do not report the two intermediary measures. The data for the top income shares are from the updated excel file available [here](#) originally published in [Piketty and Saez \(2003\)](#). All series have been smoothed.

B.1.4 IPEDS *vs* Delta Cost Project

In this appendix are displayed the Gini and Theil Indexes as well as the top 10 and top 5% share computed in the original IPEDS dataset instead of the Delta Cost Project dataset. The data extended before 1987 confirms the results found with the Delta Cost Project Data.

Figure B.3: Income Inequality

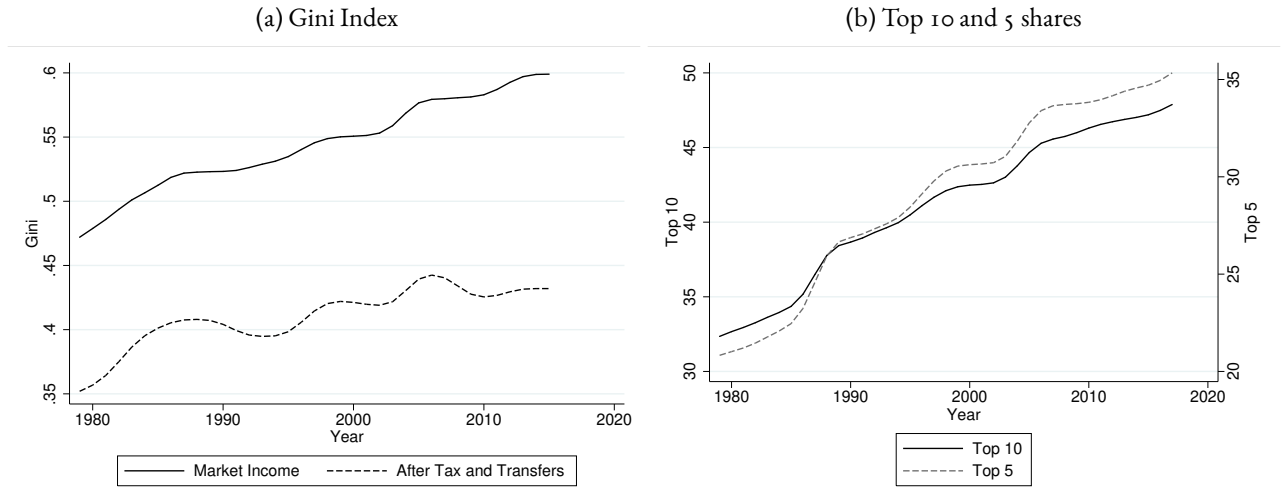
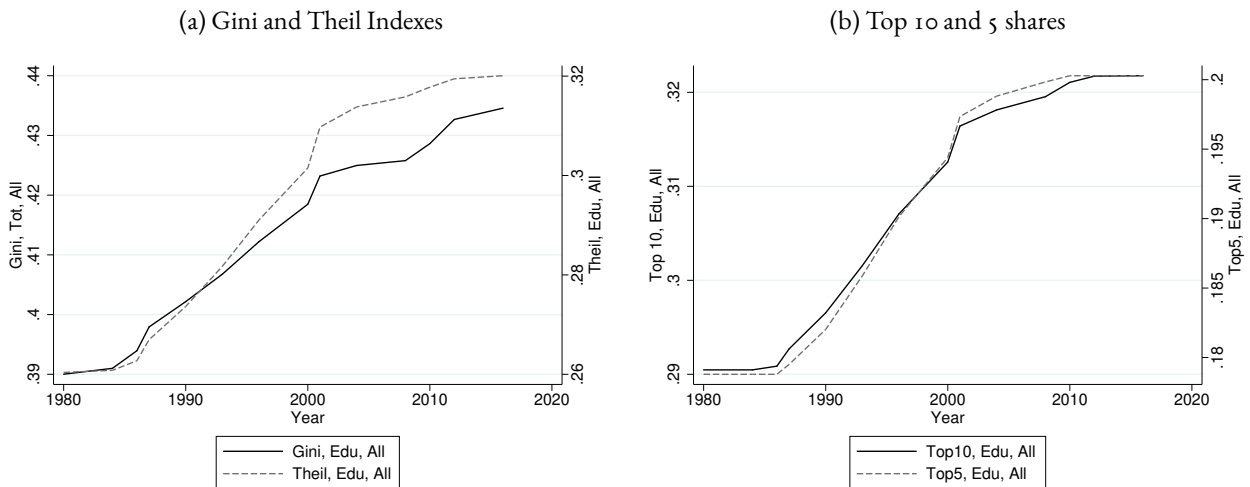


Figure B.4: Inequality in Educational and Research Expenditures in Original IPEDS



B.1.5 Quasi-Panel *vs* Full Sample

In this appendix are displayed the Gini and Theil Indexes as well as the top 10 and top 5% share computed over the full sample instead of the restricted quasi-panel. Our previous results remain almost unaffected when considered the unrestricted sample. The only noticeable difference is the sharper increase in all measure of inequality.

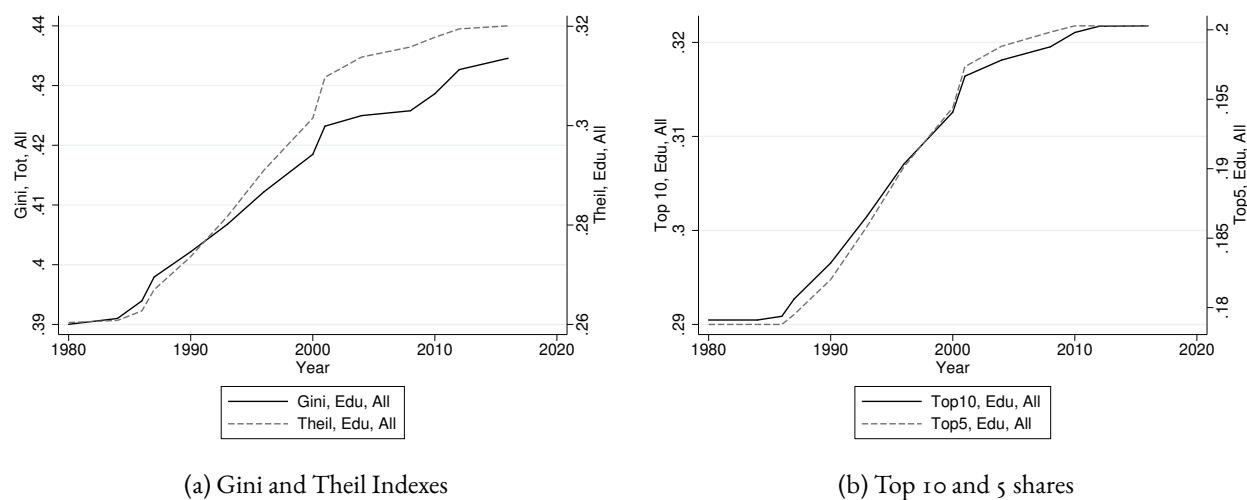


Figure B.5: Inequality in Educational and Research Expenditures, Full Sample

B.1.6 Total-Enrollment-Adjusted Gini

Figure B.6: Enrollment-Adjusted Gini



Note: The enrollment rate is the immediate enrollment rate, provided by the [NCES](#)

References

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Appendix C

Competition *vs.* Stability: Oligopolistic Banking System with Run Risk—Appendix

C.1 A Dual System of Regulated and Shadow Banks

In modern financial systems, most intermediaries benefit from public deposit insurance schemes and access to the Central Bank discount window, which—if not totally at least to a large extent—protect them from the risk of a run. In exchange from this public insurance, they have to abide by capital and liquidity requirements that impose constraints on the composition of their balance-sheets.

In the past three decades, the landscape of the shadow and traditional banking sector has experienced significant changes in terms of concentration and regulation. Firstly since the Great Recession, three of the big five investment banks have disappeared: two large investment banks have been absorbed by former competitors—Bear Stern by JP Morgan Chase and Merrill Lynch by Bank of America—and Lehman Brothers has been liquidated. Secondly, many of the remaining investment banks converted to traditional bank holding companies in order to benefit from TARP and are now therefore subject to stricter rules and supervision. Furthermore, the last three decades have seen a significant movement of concentration in the traditional banking industry as well as a toughening of capital requirements since the Great Recession.

In this section, I ask three questions. In a world in which only the shadow banking sector is subject to systemic run, how does the interactions with the traditional banking sector modify the previously established results? What are the implications of stronger regulations and higher level of concentration in the traditional banking sector for the overall stability of the financial system?

C.1.1 A Model with Regulated and Unregulated Banks

I set up a generalized version of the model in section 2 that includes both an unregulated shadow banking system subject to systemic run and a regulated traditional banking system that is subject to a leverage ratio rule and benefits from the public deposit insurance scheme or access to the discount window of a lender of last resort. Figure C.1 summarizes the different actors and financial relationships across parts of the financial system with shadow and regulated banks.

There is a finite number $\mathcal{R}$ (resp. $\mathcal{S}$) of regulated (resp. unregulated) banks. The total number of institutions is $\mathcal{S} + \mathcal{R} = \mathcal{B}$. Both types of banks invest in the same asset.

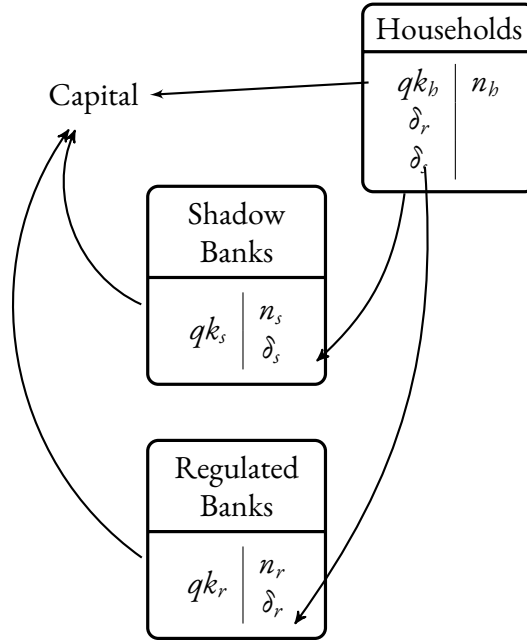


Figure C.1: Schema of Financial System

Regulated Banks. Regulated banks choose their capital investment, their stream of dividends and net worth a) subject to a capital requirement constraint C.1 and b) benefiting from deposit insurance either in the form of a federal deposit insurance scheme or because the depository institution has access to the discount window of the Lender of Last Resort.

$$k_r = \kappa n_r \quad (\text{C.1})$$

$$\lambda_r = 1 \quad (\text{C.2})$$

for all $r = 1 \dots \mathcal{R}$. I do not specify the capital requirement rule C.1 in terms of market value of asset, but instead adopt a specification more consistent with the Bales III rules in terms of the book value of the banks' asset.¹ Throughout this paper I will assume that $\kappa < \frac{1}{-\Delta q^R}$ so that if a run occurs, the net worth of regulated banks always remains positive. The relevant aggregate state variables of the economy are the vectors of net worth of regulated and unregulated banks, productivity and the number of each type of financial intermediaries. Because in equilibrium, $\mathcal{R}$ will remain fixed over time, I omit it when no confusion results. The problem of a regulated bank is:

Taking $\mathbf{k}_{-r}, \mathbf{k}_s, \dot{\mathbf{n}}_{-r}, \dot{\mathbf{n}}_s$ as given, a regulated bank r HJB equation is:

$$\begin{aligned} \rho v(n, \mathbf{n}_{-r}, \mathbf{n}_s, \mathcal{S}) = & \max_{d, k} u(d) + v_n((\dot{q} + z)k - r\delta - d) + \sum_{j \neq r}^{\mathcal{R}} v_{n_j} \dot{n}_j + \sum_s^{\mathcal{S}} v_{n_s} \dot{n}_s \\ & + \lambda(v(n + \Delta q^R k, \mathbf{n}_{-r} + \Delta q^R \mathbf{k}_{-r}, 0, 0)) - v(n, \mathbf{n}_{-r}, \mathbf{n}_s, \mathcal{S}) \\ \text{s.t. } & k_r = \kappa n_r \end{aligned}$$

¹Basel Committee on Banking Supervision, "Basel III leverage ratio framework and disclosure requirements"

Shadow banks. Shadow banks solve the same problem as before but now taking also the investment of regulated banks as given. Taking $\mathbf{k}_{-s}$, $\mathbf{k}_r$, $\dot{\mathbf{n}}_{-s}$, $\dot{\mathbf{n}}_r$ as given, a shadow bank s HJB equation is:

$$\begin{aligned} \rho v(n, \mathbf{n}_{-s}, \mathbf{n}_r, \mathcal{S}) = & \max_{d, k} u(d) + v_n ((\dot{q} + z)k - r\delta - d) \\ & + \sum_{j \neq s}^S v_{n_j} \dot{n}_j + \sum_r^{\mathcal{R}} v_{n_r} \dot{n}_r + \lambda(v^R - v(n, \mathbf{n}_{-b}, \mathbf{n}_r, \mathcal{S})) \end{aligned}$$

Market clearing. The good, deposits and capital markets must clear. By Walras law, it is sufficient that the deposit and capital markets clear. The market clearing condition for deposits is the same as previously. The market clearing for capital is:

$$\sum_{s=1}^S k_s + \sum_{r=1}^{\mathcal{R}} k_r + \int k_b db = \bar{K} \quad (\text{C.3})$$

Entry and run price. Given the results I have established earlier, I will focus here on the case with no entry—corresponding to $\eta = 0$ —where after a run on the shadow banking system, the system stays forever after only populated by regulated banks.

$$\mathcal{S}' = 0 \quad (\text{C.4})$$

$$\mathcal{R}' = \mathcal{R} \quad (\text{C.5})$$

Finally, the run price should be consistent with the post-run macroeconomic condition:

$$q^R(\mathbf{n}_s, \mathbf{n}_r, \mathcal{S}) = q(0, \mathbf{n}_r + \Delta q^R \mathbf{k}(\mathbf{n}_r), 0)$$

where $q(0, \mathbf{n}_r + \Delta q^R \mathbf{k}(\mathbf{n}_r), 0)$ denotes the price of the asset when no shadow banks is operating, all the capital stock is held by regulated banks and households and when no shadow banks may enter consistently with C.4.

C.1.2 Symmetric Equilibrium and Transition Dynamics

I assume symmetry among regulated banks on the one hand and among shadow banks on the other, but allow for asymmetry between the two groups.

Run game. During the second stage of an instant, depositors might run. The condition for a run on the shadow banking sector is

$$N_S + \Delta q^R K_S < 0 \quad (\text{C.6})$$

where $N_S = \mathcal{S}n_s$ and $K_S = \mathcal{S}k_s$. The condition is independent of the regulated banks' net worth and capital investment because regulated banks' deposits are insured and they cannot transfer liquidity to the shadow banking system in times of run because their leverage constraint is already binding. No pooling of liquidity is possible between the two parts of the financial system.²

Behavior of regulated and shadow banks. Investment by regulated banks is strictly tied to their net worth given by C.1, so that aggregate investment by regulated banks write

$$K_{\mathcal{R}} = \kappa N_{\mathcal{R}} = \kappa \mathcal{R} n_{\mathcal{R}}$$

They accumulate net worth until they reach the optimal Cournot interior:

$$k^{SS} = \frac{\bar{K}}{\mathcal{S} + \mathcal{R} + 1}$$

The behavior of shadow banks is the same as the one described in the equilibrium without regulated

²The following condition

$$N_S + N_{\mathcal{R}} + \Delta q^R (K_S + K_{\mathcal{R}}) < 0$$

would entail more than mere pooling of liquidity, and would require pooling of equity and would mean that when a run hits, the entire shadow banking system is acquired by the regulated sector. Although I focus on the case described in C.6, the failure of Lehman Brothers and the acquisition of Bear Sterns and Merrill Lynch by JP Morgan and Bank of America respectively suggests that there was partial pooling of equity during the Great Recession and that I am looking at an extreme case.

banks where $\bar{K}$ should be replaced by

$$\tilde{K}(n_{\mathcal{R}}) = \bar{K} - \mathcal{R} \min(\kappa n_{\mathcal{R}}, k^{SS})$$

C.1.3 Competition Policies in the Regulated Sector

To illustrate the impact of increasing the degree of competition in the shadow and in the regulated banking sector, I do two counterfactuals. In the first one, I decrease the number of regulated banks from 3 to 2, keeping the number of shadow banks to 2. In the second, I keep the former at 3 but decrease the number of shadow banks to 1. Figure C.2 plots the transition path of an economy unexpectedly hit by a shock that brings the net worth of shadow banks to 0. The transition path in the original economy is depicted in black, the first counterfactual in dark gray and the second in light gray. On (a) and (b), the solid line (dashed line) represents shadow banks (regulated banks). On (c), the solid line (dashed line) represents the asset price (run price).

The decrease in competition in the regulated sector has a slight stabilizing effect in my example by increasing profits of shadow banks. The franchise value channel and the recapitalization channels indeed favor less risk-taking and a faster travel speed through the crisis zone thanks to higher profits as may be seen on (b). But the price-drop effect severely dampens their impact: less regulated banks implied that the drop in prices is more severe after a run on the entire shadow banking system as may be seen on (c). This contributes to making the crisis zone larger. This larger drop is not only caused by the market power channel according to which less banks implies lower output and lower asset prices but also caused by the large drop in regulated banks' net worth caused by their relatively larger exposure.

Decreasing the number of shadow banks and keeping the number of regulated banks constant— $(\mathcal{R}, \mathcal{S}) = (3, 1)$ —has the already-described stabilizing effect through the franchise value channel, the recapitalization channel and the price-drop channel.

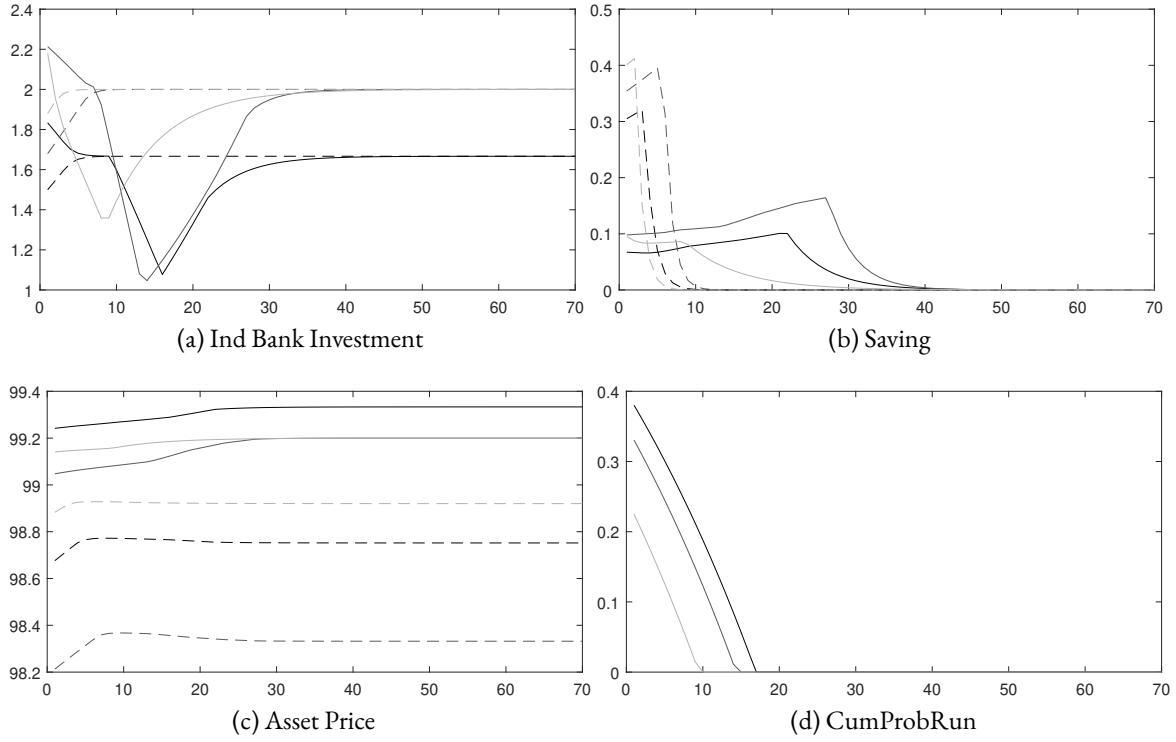


Figure C.2: Transition path as a function of time for $(\mathcal{R}, \mathcal{B}) = (3, 2), (2, 2), (3, 1)$

C.1.4 Macprudential Policies in the Regulated Sector

To illustrate the impact of making regulation tighter for regulated banks, I simulate the transition path of two economies, $\kappa = 1$ and $\kappa = .25$, unexpectedly hit by a shock that brings the net worth of shadow banks to 0. Because regulated banks are better capitalized, their starting level of net worth on the transition path is strictly positive. The black lines represent the benchmark case $\kappa = 1$.

In my example, a more tightly regulated banking sector implies a more stable overall financial system. The main reason is that regulated banks are better capitalized. Should a run occur and the shadow banks disappear, capital investment by financial intermediaries would drop by less. This maintains a higher run-price when $\kappa = .25$ as may be seen on (c).

I now do the same experiment but assuming that before the unexpected shock hit both economy were at the steady-state where the regulated sector was subject to the same capital requirement, $\kappa = .25$. After the shock hits, one economy experiences a tightening of its capital requirement to $\kappa = 1$. This counterfactual is closer to what happened in the aftermath of the Great Recession.

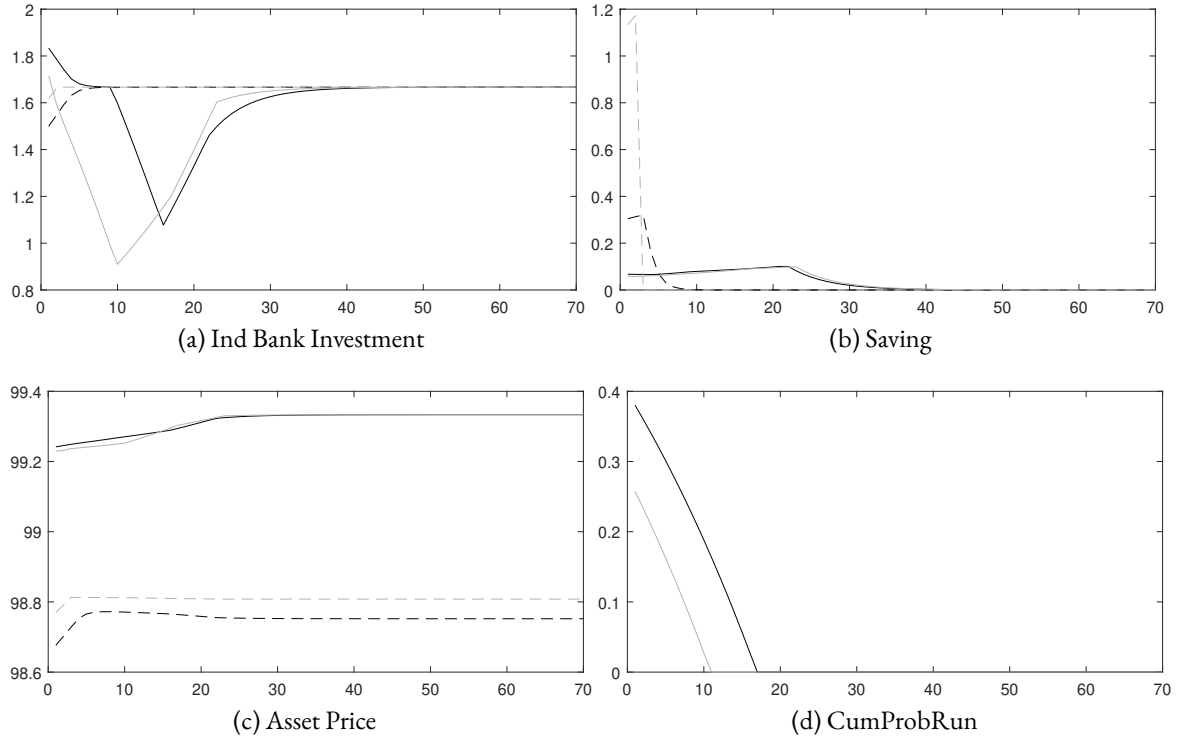


Figure C.3: Transition path as a function of time for $\kappa = 1$ and $.25$

Interestingly, the more tightly regulated system is characterized by a higher probability of a run. The effect described previously doesn't apply anymore because both economies start from the same point in the state-space after the productivity shock hits. There are two key mechanisms. The larger price-drop in the more tightly regulated system stems from the fact that it takes much longer for the regulated banking system to recapitalize which depresses asset price should they be the only ones left to intermediate funds. This makes the run zone with the more tightly regulated system larger. The recapitalization channel dampens this first effects: from the point of view of shadow banks, stricter capital requirements for regulated banks means less competition and therefore higher profits, higher savings and quicker recovery along the transition path. In this example, the first effect dominates.

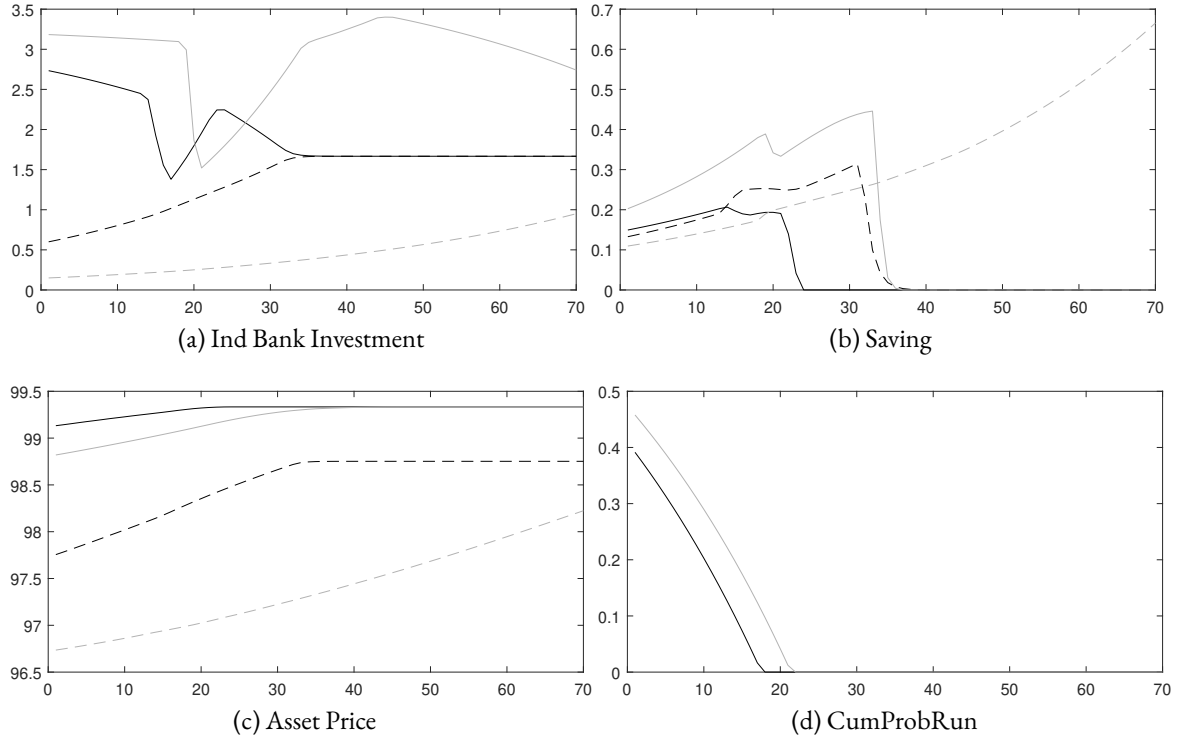


Figure C.4: Transition path as a function of time for $\kappa = 1, .25$ and common starting net worth

C.2 Proofs

C.2.1 F.O.C HH and pricing equations

It is immediate that the first order conditions when the households are risk-neutral gives the following no-arbitrage conditions $\dot{q} + z - f'(k_b) = -\lambda \Delta q^R - \zeta(z) \Delta q^z + \rho q$ and $\forall b \quad r_b = -\lambda(\psi_b - 1) + \rho$.

C.2.2 Derivation of the law of motion for a bank's net worth

After substituting out for deposits using the definition of net worth, one gets:

$$\dot{n}_b = r n_b + (\dot{q} + z - r_b q) k_b - d_b$$

where it appears clearly that the net excess return to an investment of one unit of capital if no run occurs is $\dot{q} + z - r_b q$. By the Household F.O.C together with the market clearing condition for the capital market, and denoting $K = \sum_j^B k_j$ the aggregate capital investment of banks, the net excess return

is equal to:

$$R(K) = R \left(\sum_j^B k_j \right) = \dot{q} + z - r_b q = f' \left(\beta - \sum_j^B k_j^b \right) - \lambda_b \Delta q_b^R + \lambda_b q (\psi_b - 1) - \zeta(z) \Delta q^z$$

which shows how the net return on capital depends on the choice of capital by the individual banker through the aggregate K . Replacing into the law of motion for net worth one gets

$$\begin{aligned} \dot{n} &= rn + (\dot{q} + z - r_b q) k - d \\ &= (\lambda_b + \rho - \lambda_b \psi_b) n + \left(f' \left(\beta - \sum_j^B k_j^b \right) - \lambda_b \Delta q_b^R + \lambda_b q (\psi_b - 1) - \zeta(z) \Delta q^z \right) k - d \end{aligned}$$

Finally, I use the definition of $\psi_b = \frac{q_b^R k_b}{\partial_b} = \frac{q_b^R k_b}{q k_b - n_b}$:

$$\begin{aligned} \dot{n} &= (\lambda_b + \rho - \lambda_b \psi_b) n + \left(f' \left(\beta - \sum_j^B k_j^b \right) - \lambda_b \Delta q_b^R + \lambda_b q (\psi_b - 1) - \zeta(z) \Delta q^z \right) k - d \\ &= (\lambda_b + \rho) n + \left(f' \left(\bar{K} - \sum_j^B k_j \right) - \zeta(z) \Delta q^z \right) k - d \end{aligned}$$

In the particular case without productivity shock, the law of motion becomes:

$$\dot{n} = (\lambda_b + \rho) n + \left(f' \left(\bar{K} - \sum_j^B k_j \right) \right) k - d$$

C.2.3 Proof of proposition 3.1

The rule for choosing between the interior vs the corner solution is:

$$\max \left[\lambda(\mathbf{k}, \mathbf{n}) \left(n - \frac{v(n, \mathbf{n}_{-b}) - v^R}{v_n(n, \mathbf{n}_{-b})} \right) + R(k^I, \mathbf{k}_{-b}) k^I, R(k^C(n, \mathbf{k}_{-b}, \mathbf{n}_{-b}), \mathbf{k}_{-b}) k^C(n, \mathbf{k}_{-b}, \mathbf{n}_{-b}) \right] \quad (\text{C.7})$$

Definitions of payoffs in equilibrium and deviations I are now ready to check for deviations. Define $P_i^j(k^C)$ is the profit of playing j (Interior or Corner) if the banks are supposed to play i and the corner solution is k^C .

$$\begin{aligned} k^I &= \frac{\bar{K}}{\mathcal{B} + 1} & k^{C*} &= \frac{n}{-\Delta q^R} & k^C &= \frac{\mathcal{B}n}{-\Delta q^R} - \sum_{j \neq b} k_j = \mathcal{B}k^{C*} - \sum_{j \neq b} k_j \\ P_C^C(k^{C*}) &= \alpha (\bar{K} - \mathcal{B}k^{C*}) k^{C*} & P_C^I(k^{C*}) &= \alpha \left(\frac{\bar{K} - (\mathcal{B} - 1)k^{C*}}{2} \right)^2 \\ P_I^I(k^{C*}) &= \alpha \left(\frac{\bar{K} - (\mathcal{B} - 1)k^I}{2} \right)^2 & P_I^C(k^{C*}) &= \alpha (\bar{K} - \mathcal{B}k^{C*}) (\mathcal{B}k^{C*} - (\mathcal{B} - 1)k^I) \\ F_I(k^{C*}) &= P_I^I(k^{C*}) - P_I^C(k^{C*}) & F_C(k^{C*}) &= P_C^I(k^{C*}) - P_C^C(k^{C*}) \end{aligned}$$

where the last two lines define the net payoff from deviating from the interior equilibrium and playing the safe one F_I , and the net payoff from the deviating from the safe equilibrium and playing the risky one F_C .

Existence of n_1

Lemma 10. 1. The four functions $P_I^I, P_C^I, P_I^C, P_C^C$ of k^C are parabolas

2. For all $k^C \setminus \{\frac{\bar{K}}{\mathcal{B}+1}\}$, $P_C^C(k^C) < P_C^I(k^C)$ and $P_I^C(k^C) < P_I^I(k^C)$
3. The functions F_I, F_C are strictly decreasing for $k^{C*} \in [0, k^I]$ and strictly increasing for $k^{C*} > k^I$. They are equal to 0 at $k^{C*} = k^I$
4. And at $k^C = \frac{\bar{K}}{\mathcal{B}+1}$, $P_C^C\left(\frac{\bar{K}}{\mathcal{B}+1}\right) = P_C^I\left(\frac{\bar{K}}{\mathcal{B}+1}\right)$ and $P_I^C\left(\frac{\bar{K}}{\mathcal{B}+1}\right) = P_I^I\left(\frac{\bar{K}}{\mathcal{B}+1}\right)$

5. $\forall k^{C*} \in [0, k^I] F_I(k^{C*}) > F_C(k^{C*})$ with equality for $k^{C*} = k^I$.

Proof. All proofs are immediate except the last one.

$$\begin{aligned}
 F_C(x) - F_I(x) &= \alpha (\bar{K} - \mathcal{B}k^{C*}) (\mathcal{B}k^{C*} - (\mathcal{B} - 1)k^I) - \alpha \left(\frac{\bar{K} - (\mathcal{B} - 1)k^I}{2} \right)^2 \\
 &\quad - \alpha (\bar{K} - \mathcal{B}k^{C*}) k^{C*} + \alpha \left(\frac{\bar{K} - (\mathcal{B} - 1)k^{C*}}{2} \right)^2 \\
 &= \alpha (\bar{K} - \mathcal{B}k^{C*}) (\mathcal{B} - 1)(k^{C*} - k^I) + \frac{\alpha}{2} (\mathcal{B} - 1)(k^I - k^{C*}) \left(\bar{K} - (\mathcal{B} - 1) \frac{k^I + k^{C*}}{2} \right) \\
 &= \frac{\alpha}{2} (\mathcal{B} - 1)(k^{C*} - k^I) \left(2(\mathcal{B} + 1)k^I - 2\mathcal{B}k^{C*} - \bar{K} + (\mathcal{B} - 1) \frac{k^I + k^{C*}}{2} \right) \\
 &= -\frac{\alpha}{2} (\mathcal{B} - 1)(k^{C*} - k^I)^2 \left(\mathcal{B} + 1 + \frac{\mathcal{B} - 1}{2} \right) \\
 &< 0
 \end{aligned}$$

□

I now redefine all functions in terms of net worth. Since I have assumed that $k^C(n)$ was a bijection and increasing in n with first derivatives bounded away from 0, there exists n^I such that k^C is increasing over $[0, n^I]$ with $k^C(0) = 0, k^C(n^I) = k^I$. I can also redefine the function $F_I(\cdot), F_C(\cdot)$ as functions of n over the interval $[0, n^I]$.

I now prove the existence and uniqueness of the two cutoff points. Define for $k = I, C, \tilde{P}_k^C(n) = P_k^C(n) + F^*(n)$. The following is true:

Lemma 11. *If $v^R = 0$ and F^* is differentiable with $(F^*)' > 0$, there exists a unique $\{n_1^I, n_1^C\} \in [0, n^I]$ such that*

$$\tilde{P}_C^C(n_1^C) = P_C^I(n_1^C) \quad \text{and} \quad \tilde{P}_I^C(n_1^I) = P_I^I(n_1^I)$$

Proof. Furthermore, I also have that $v^*(0) \geq 0$ for $n \geq 0$, hence $F^*(0) \geq 0$, which combined with $(F^*)' > 0$ gives $F^*(n) > 0$ for $n > 0$.

$$\text{Define now } H_I(n) = F_I(n) - F^*(n) \quad H_C(n) = F_C(n) - F^*(n)$$

Case 1: $H_I(0) > H_C(0) > 0$ Gathering all the properties I have established and the one I just assumed, I have that

1. $H_I(0) > H_C(0) > 0$
2. $H_I(n^I) = H_C(n^I) < 0$
3. H_I, H_C are continuous and strictly decreasing for $n \in [0, n^I]$
4. $H_I(n) > H_C(n)$ for $n \in [0, n^I)$

From the first three properties, there exists a unique pair $\{n_1^I, n_1^C\} \in [0, n^I]$ such that $H_I(n_1^I) = 0 = H_C(n_1^C)$ with $n_1^C < n_1^I$ where the latter comes from the third property.

Case 2: $H_I(0) > 0 > H_C(0)$ In this case I conventionally set $n_1^C = 0$.

Case 3: $0 > H_I(0) > H_C(0)$ In this case I conventionally set $n_1^C = n_1^I = 0$.

Proof of the Corollary I only need to establish that $n_1 > 0$. I show that in the equilibrium I am constructing, if $n_1 = 0$ then $v^*(0) = 0$ and that this is a contradiction.

For a contradiction assume $n_1 = 0$. Then all banks play safe for $n \geq 0$. For $n = 0$, the banks cannot invest at all if it plays the safe strategy and its profit will be zero, forever. Therefore $v^*(0) = 0$. But $v^*(0)$ implies that $H_C(0) > 0$, and hence that it is optimal to deviate from the safe strategy for a net worth. This is a contradiction with the assumed Cournot equilibrium at $n = 0$. Hence $n_1 > 0$.

□

C.2.4 Dividend policy and steady-state: lemma 1 and 2

When there is no aggregate shock, $\zeta(z) = 0$, assuming the solution for dividends is interior - and it will be under the CRRA assumption for the instantaneous utility - the F.O.C and envelope theorem in the safe

zone is

$$\begin{aligned}
 u_d(d) = v_n(n, \mathbf{n}_{-b}) &\Rightarrow -\gamma \frac{\dot{d}}{d} = \frac{v_{nn}(n, \mathbf{n}_{-b})\dot{n} + \sum_{j \neq b} v_{nn_j}(n, \mathbf{n}_{-b})\dot{n}_j}{v_n(n, \mathbf{n}_{-b})} \\
 \rho v_n(n, \mathbf{n}_{-b}) &= v_{nn}(n, \mathbf{n}_{-b})\dot{n} + \sum_{j \neq b} v_{nn_j}(n, \mathbf{n}_{-b})\dot{n}_j \\
 &\quad + v_n(n, \mathbf{n}_{-b}) (\rho + k_n^C(n, \mathbf{n}_{-b})\alpha (\bar{K} - (\mathcal{B} + 1)k^C(n, \mathbf{n}_{-b}))) \\
 \Rightarrow 0 &= \frac{v_{nn}(n, \mathbf{n}_{-b})\dot{n} + \sum_{j \neq b} v_{nn_j}(n, \mathbf{n}_{-b})\dot{n}_j}{v_n(n, \mathbf{n}_{-b})} + k_n^C(n, \mathbf{n}_{-b})\alpha (\bar{K} - (\mathcal{B} + 1)k^C(n, \mathbf{n}_{-b}))
 \end{aligned}$$

where the second line uses the CRRA assumption for utility and the third line uses the Cournot assumption that a banker takes $\dot{n}_{-b}$ as given. Combining both gives

$$\gamma \frac{\dot{d}}{d} = k_n^C(n, \mathbf{n}_{-b})\alpha (\bar{K} - (\mathcal{B} + 1)k^C(n, \mathbf{n}_{-b})) \quad (\text{C.8})$$

When the capital investment is interior, the solution is different because the marginal effect of net worth on capital is null:

$$\gamma \frac{\dot{d}}{d} = k_n^C(n, \mathbf{n}_{-b})\alpha (\bar{K} - (\mathcal{B} + 1)k) = 0$$

Moreover by continuity of the dividend policy rule, it has to be that $d(n) = d(n_1)$ for $n_{-1} < n < n_1$ where $n_{-1} = \frac{d(n_1) - R(\mathbf{k}^I)k^I}{\rho + \bar{\lambda}}$

is the level of net worth below which distributing the level of dividends $d(n_1)$ is no longer sustainable without making net worth decrease over time. Consequently, below n_{-1} - and this applies only if $n_{-1} > 0$ -, the only way to keep dividends constant is to set it such that net worth is constant over time: $d(n) = (\rho + \bar{\lambda})n + R^I k^I$

If dividends are set higher, then net worth will decrease and reach 0 in finite time. If dividends are set lower, net worth will increase and reach n_{-1} in finite time at which point dividends would have to discontinuously jump to $d(n_1)$ which contradicts the continuity of the dividends path and the concavity of the utility function.

Steady-state. Setting equation C.8 to 0, $\dot{d} = 0$ give the steady-state level of capital: $k^{SS} = \frac{\bar{K}}{B+1}$. Using the asset price equation 3.6 with $\dot{q} = 0$ solves for the steady-state asset price as a function of k^{SS} . Since the steady-state capital investment is also safe, it has to be that $n^{SS} = (q^{SS} - q^R)k^{SS}$ and finally one finds d^{SS} by equating $\dot{n} = 0$.

C.2.5 Proof of proposition 3.3

Suppose for a contradiction that with immediate entry, an equilibrium of the type described in proposition 3.2 with $0 \leq n_1 < +\infty$ exists. Then $\forall n \in [0, n_1)$, $\lambda(n) = \bar{\lambda}$ and $k^*(n) = k^I$

By the assumption that $\eta \rightarrow +\infty$, I have that the run price is the price when the system is at 0 net worth and B banks are operating, i.e. $q^R = q(0)$.

For $n = 0$, I have $q(n) = q(0)$, hence $q(n) = q^R$ and $\frac{n}{q(n)-q^R}$ is infinitely large and therefore larger than k^I , $\lim_{n \rightarrow 0^+} \frac{n}{q(n)-q^R} = +\infty$ therefore greater than k^I

Hence the corner investment $\frac{n}{q(n)-q^R}$ is larger than the interior solution k^I at 0. And by continuity it is larger in a neighborhood of zero. Hence the interior investment doesn't imply a risk of run and $\lambda(n) = 0$ for n in a neighborhood of 0. This contradicts the properties of the assumed equilibrium.

Let's now check that when $\eta \rightarrow +\infty$, $\forall n \geq 0$

$$\lambda(n) = 0 \quad k^*(n) = k^I \quad d(n) = \rho n + \alpha (k^I)^2 \quad q(n) = \frac{z - \alpha k^I}{\rho}$$

is an equilibrium.

First, I have that $q(n) - q^R = q(n) - q(0) = 0$ hence that $\frac{n}{q(n)-q^R}$ is infinitely large and therefore that k^I is always safe. Hence, the optimal capital investment of banks is always k^I . This implies $\lambda(n) = 0$. From the optimal conditions for dividends, I have that $\dot{d} = 0$, i.e. that $d(n) = \rho n + \alpha (k^I)^2$. Finally, given the optimal investment of banks $k^*(n) = k^I$ and $\lambda(n) = 0$, the P.D.E. for asset prices is $\dot{q} = \alpha k^I - z + \rho q$ with final condition $q(n) = q^SS$. This has only one stable solution, which is $q(n) = \frac{z - \alpha k^I}{\rho} = q^SS$ for all $n \geq 0$. The proof for the individual bank run case is exactly the same.

C.2.6 Proof of lemma 3

Global analysis For the first part of lemma 3, I provide a graphical proof. First I derive a simplified version of the system of differential equations 3.15 in terms of $(d, k, \tilde{q})$ where $\tilde{q} = q - q^R$. From $k = \frac{n}{q - q^R} = \frac{n}{\tilde{q}}$, one gets

$$\begin{aligned} \dot{k} &= \frac{1}{\tilde{q}} (\dot{n} - k\dot{\tilde{q}}) = \frac{1}{\tilde{q}} (\rho n + \alpha(\bar{K} - \mathcal{B}k)k - d - k[\rho\tilde{q} + \rho q^R - z + \alpha(\bar{K} - \mathcal{B}k)]) \\ &= \frac{1}{\tilde{q}} (k(z - \rho q^R) - d) \\ &= \frac{1}{\tilde{q}} (k\alpha\bar{K} - d) \quad \text{if } \eta = 0 \end{aligned}$$

Then I use the following system

$$\dot{k} = \frac{1}{\tilde{q}} (k(z - \rho q^R) - d) \quad \dot{q} = \rho\tilde{q} + \rho q^R - z + \alpha(\bar{K} - \mathcal{B}k) \quad \gamma \frac{\dot{d}}{d} = k_1 \alpha (\bar{K} - (\mathcal{B} + 1)k)$$

to draw the associated phase diagram C.6 in the $(d, k, \tilde{q})$ plane. The $\dot{k} = 0$ locus is strictly increasing because $\tilde{q}^R < \frac{z}{\rho}$. To gain intuition I first focus on the representation in the (d, k) -plane in figure C.5, fixing $\tilde{q}$ for the time being.

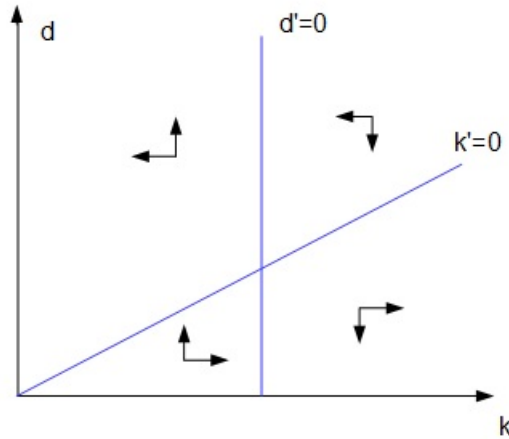


Figure C.5: Phase diagram in the (k, d) dimension

The steady-state, the $\dot{k} = \dot{d} = 0$ loci and, importantly, the direction of the arrows are independent of $\tilde{q}$. The steady-state is the point at which the two lines cross. Under the assumption that $k_1 > 0$, the

saddle path - if it exists - has to feature d and k moving in the same direction - either both increasing or both decreasing, depending on where the system starts. Since I will consider cases in which net worth starts below its steady-state value, k is necessarily lower than its steady-state value, it has to be that both are increasing over time. Since k is increasing over time until k^{ss} , profits are increasing over this range of k and d is increasing over time, $\dot{n}$ has to be increasing too. This confirms that in equilibrium $k_1 > 0$. Also note that if $k_1 < 0$ then all arrows in the d -dimension are reversed and no saddle path could exist.

I now draw the full phase diagram in three dimensions in figure C.6. The x -axis corresponds to capital while the y -axis corresponds to dividends. The third dimension, the z -axis is for $\tilde{q}$. The $\dot{q} = 0$ locus is a fully vertical plane - i.e. orthogonal to the d -dimension -, from the back-left of the cube to the front right. The $\dot{d} = 0$ plane is coming from the vertical line in the back plane towards us, orthogonal to both the q and the d -dimension. Finally the $\dot{k} = 0$ plane is the ramp starting on the q -axis and increasing from the bottom left to the top right. Their intersection - the red point - is the steady-state of the system. I have drawn the same arrows as in the previous graph and added arrows in the (q, k) -plane to indicate the direction for q .

From the first part of the proof, if a saddle-path exists, it must be to the left of the $\dot{d} = 0$ plane and below the $\dot{k} = 0$ plane. Furthermore, the saddle-path must lie in the zone where all arrows, including those in the $\tilde{q}$ dimension, point towards the steady-state. I have delimited and colored this zone: the figure with the hatched triangular basis, the blue triangular faces, the purple triangular back and the transparent roof has to contain the saddle-path.

Finally the green line lying in the $(k, \tilde{q})$ plane, is the line on which the system should start, as commanded by the initial condition, $k(t_0)\tilde{q}(t_0) = n_1$. This shows that $\tilde{q}$ has to be increasing over time along the equilibrium path.

Local determinacy I now prove that the linear approximation of the system 3.15 around the point (d^{ss}, n^{ss}, q^{ss}) admits one and only one solution satisfying the initial and terminal conditions. I linearize the following system

for it below. Denote the three eigenvalues of A , l_1, l_2, l_3 . From the properties of eigenvalues,

$$A = 2\rho + \frac{\alpha}{\tilde{q}^{SS}} (\bar{K} - \mathcal{B}k^{SS}) = l_1 + l_2 + l_3$$

$$\det A = -\rho \frac{(d^{SS} - \theta\gamma) k_1^*(n^{SS})(\mathcal{B} + 1)\alpha}{\gamma \tilde{q}^{SS}} = l_1 \times l_2 \times l_3$$

Hence one gets the following restrictions on the signs of eigenvalues: $l_1 + l_2 + l_3 > 0$ and $l_1 \times l_2 \times l_3 < 0$.

The last inequality implies that either 1 or 3 eigenvalues are negative. The first inequality implies that at least one eigenvalue is strictly positive. This establishes that 1 eigenvalue is negative and 2 are positive. Since my system has two jump variables with a terminal condition and one initial condition for n , this implies that the system admits one and only one saddle path solution.

Speed of convergence To compute the speed of convergence of capital around the steady-state, I use the same linearized model but in the (k, d) space.

$$\dot{k} = \frac{1}{\tilde{q}^{SS}} \left(\hat{k}(z - \rho q^R) - \hat{d} \right) \quad \dot{d} = \frac{k_1^{SS} d^{SS} \alpha}{\gamma} \left(\bar{K} - (\mathcal{B} + 1)\hat{k} \right)$$

In matrix form it is

$$\begin{pmatrix} \dot{k} \\ \dot{d} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{z - \rho q^R}{\tilde{q}^{SS}} & -\frac{1}{\tilde{q}^{SS}} \\ \frac{-(d^{SS} - \theta\gamma) k_1^{SS} \alpha}{\gamma} (\mathcal{B} + 1) & 0 \end{pmatrix}}_A \begin{pmatrix} \hat{k} \\ \hat{d} \end{pmatrix}$$

It is easy to check that the system admits one negative and one positive eigenvalue:

$$A = \frac{z - \rho q^R}{\tilde{q}^{SS}} = \ell_1 + \ell_2 > 0 \quad \det A = \frac{-(d^{SS} - \theta\gamma) k_1^{SS} \alpha}{\gamma \tilde{q}^{SS}} (\mathcal{B} + 1) = \ell_1 \ell_2 < 0$$

Denoting ℓ_- the negative root - the only that matters for the convergence rate - one has:

$$\ell_- = \frac{z - \rho q^R - \sqrt{(z - \rho q^R)^2 + \frac{4}{\gamma} (1 - k^{SS} q_1^{SS}) (d^{SS} - \theta\gamma) (\mathcal{B} + 1) \alpha}}{2 (q^{SS} - q^R)}$$

Expression for $k_1^*(n^{SS})$ In this paragraph, I show how to find the solution for $k_1^*(n^{SS})$. It is not directly needed to establish the previous results, but it shows how one should use the equilibrium conditions to derive the derivatives of asset prices with respect to a bank's own net worth.

From the expression $k_1^*(n) = \frac{1-k^{C*}(n)q_1^*(n)}{q(n)-q^R}$ I see that the challenge is indeed to compute $q_1^*(n^{SS})$. I start by using the equilibrium condition for the dynamics of asset prices

$$\begin{aligned}\dot{q}(n) &= \rho q(n) - z + \alpha (\bar{K} - \mathcal{B}k^C(n)) \\ \Rightarrow \mathcal{B}q_1(n)\dot{n} &= \rho q(n) - z + \alpha (\bar{K} - \mathcal{B}k^C(n)) \Rightarrow q_1(n) = \frac{\dot{q}(n)}{\mathcal{B}\dot{n}(n)}\end{aligned}$$

where the second line takes the total derivative of $\dot{q}$ with respect to the net worth of every bank and then uses the property of symmetry along the equilibrium path. The next step consists in using L'Hospital rule to find an expression for this ratio who takes an indeterminate form in the limit where $n \rightarrow n^{SS}$:

$$\lim_{n \rightarrow n^{SS}} q_1(n) = \lim_{n \rightarrow n^{SS}} \frac{\dot{q}(n)}{\mathcal{B}\dot{n}(n)} = \lim_{n \rightarrow n^{SS}} \frac{\rho \mathcal{B}q_1(n) - \alpha \mathcal{B} \frac{1-k^{C*}(n)\mathcal{B}q_1^*(n)}{q(n)-q^R}}{\mathcal{B} \left(\rho + \frac{1-k^{C*}(n)\mathcal{B}q_1^*(n)}{q(n)-q^R} \alpha (\bar{K} - 2\mathcal{B}k^{C*}(n)) - d_n(n) \right)}$$

The expression for the limit will depend on $d_n(n^{SS})$ the derivative of dividends with respect to net worth in equilibrium. In a similar fashion, one can use:

$$\lim_{n \rightarrow n^{SS}} d_n(n) = \lim_{n \rightarrow n^{SS}} \frac{\dot{d}(n)}{\dot{n}(n)} = \frac{d^{SS} \frac{1-k^{C*}(n)q_1(n)}{q(n)-q^R} \left(-(\mathcal{B}+1) \frac{1-k^{C*}(n)q_1(n)}{q(n)-q^R} \right)}{\rho + \frac{1-k^{C*}(n)\mathcal{B}q_1^*(n)}{q(n)-q^R} \alpha (\bar{K} - 2\mathcal{B}k^{C*}(n)) - d_n(n)}$$

These two equations have two unknowns $d_n(n^{SS})$, $q_1(n^{SS})$ that can be solved for numerically. In general, there is no closed-form solution. In the monopoly case $\mathcal{B} = 1$, one can simplify a little bit the expression. From the first and second equation, one gets:

$$q_1(n^{SS}) = \frac{1}{k^I \rho + d_n} \quad d_n(n^{SS})(\rho - d_n(n^{SS})) = -2d^{SS} \left(\frac{1 - k^I q_1(n^{SS})}{q(n^{SS}) - q^R} \right)^2$$

Using the first one to substitute for $k^I q_1$ in the second, gives

$$\begin{aligned} d_n(\rho - d_n) &= -2d^{SS} \left(\frac{d_n}{(q(n^{SS}) - q^R)(\rho + d_n)} \right)^2 \\ \frac{(\rho - d_n)(\rho + d_n)^2}{d_n} &= -2d^{SS} \left(\frac{1}{q(n^{SS}) - q^R} \right)^2 \end{aligned}$$

This polynomial of degree 3 in d_n has exactly one solution with $d_n > \rho$

Existence and uniqueness of solution to non-linear system It turns out to be rather difficult to prove existence and even more difficult to show uniqueness of a solution to the non linear system 3.15. Using the Cauchy-Peano theorem for existence and Picard-Lindelöf for uniqueness could give us restrictions such that a local solution around the steady-state exists and is unique. However, my problem is slightly different because I have both an initial condition for n and a terminal conditions for the other variables.

C.2.7 Proof of proposition 3.2

I start the proof with the middle zone $n_{-1} < n_0 < n_1$.

1. At $n_{-1} < n_0 < n_1$, I know it is optimal to invest at the optimal interior $k = k^I$ from corollary 1. I need to establish that the bank has strictly positive saving in the run zone.

I have established in lemma 3 that capital, prices, net worth and dividends were growing in the safe zone. So this means that the limit of dividends from the right of n_1 is low enough so that the bank is saving. Moreover at n_1 , it has to be the case that the limit of profits from the right is stricter smaller than the limit of profits from the left (there is a discontinuity). Since dividends are continuous at n_1 , it has to be that the bank that net worth is accumulating on the left limit of n_1 . This establishes only that the property holds locally. It might still be that n_{-1} is strictly positive, i.e. $n_{-1} \in (0, n_1)$ where I have defined n_{-1} such that $\rho n_{-1} + R^I k^I = d(n_1)$. I treat this case in the following bullet point.

2. If $n_{-1} \in (0, n_1)$, and $n \in [0, n_{-1})$ then by lemma 2, I know that dividends should be constant and set such that net worth is constant. Hence, the system is in steady-state and banks wait for a

certain death. Given that saving are strictly positive and not going to zero as the bank approaches n_1 , the bank reaches n_1 in finite time.

3. This is an implication of lemma 3.

4. Immediate

I now need to check that the assumptions in 3.4 hold in the equilibrium I have characterized. The function $k^C(n)$ is positive, continuous, strictly increasing as shown in lemma 3 with a derivative with respect to n bounded away from 0. I need to check that $q - q^R$ is always well-defined for $n > 0$, and strictly positive. q^R is given by $q^R = \frac{\eta}{\rho+\eta}q(0) + \frac{z-\alpha\bar{K}}{\rho+\eta}$. I know that in the safe zone, $q - q^R > 0$. I need to establish the same property in the run zone. Either q is increasing or decreasing in the run zone. If it is increasing, since

$$q(n) - q^R \geq q(0) - q^R = \frac{\rho}{\rho+\eta}q(0) - \frac{z-\alpha\bar{K}}{\rho+\eta} = \rho \frac{q(0) - \frac{(z-\alpha\bar{K})}{\rho}}{\rho+\eta} > 0$$

where the second line comes from the definition of $q(0)$ and the last line comes from the fact that $\frac{(z-\alpha\bar{K})}{\rho}$ is the smallest price possible, when no banks ever operate. If prices are decreasing in the run zone, then for $n \in [0, n_1]$, $q(n) - q^R \geq q(n_1) - q^R > 0$, where the second line stems from the property established in lemma 3 as well as the continuity of prices at n_1 . Hence $- \Delta q^R$ is well defined and strictly positive.

I finally establish that the assumptions regarding F^* and the value function in lemma 3.4 hold. I admit that the value is twice differentiable.

$$\begin{aligned} (F^*)' &= \frac{v_1^*(n) \sum_b v_{n_b}^*(\mathbf{n}) - (v^*(n) - v^R) \sum_b v_{1,n_b}^*(n)}{(v_1^*(n))^2} - 1 \\ (F^*)' &= \frac{v_1^*(n) \sum_{j \neq b} v_{n_b}^*(\mathbf{n}) - (v^*(n) - v^R) \sum_b v_{1,n_b}^*(n)}{(v_1^*(n))^2} \end{aligned}$$

From the point of view of a bank, the net worth of another bank increases the value because it allows to increase the pool of equity the system can rely on. And the returns of this equity are diminishing at the margin. Hence $v_{n_b}^*(\mathbf{n}) > 0$ and $v_{1,n_b}^*(n) < 0$ for all b . This proves that $(F^*)' > 0$.

C.3 Additional Figures

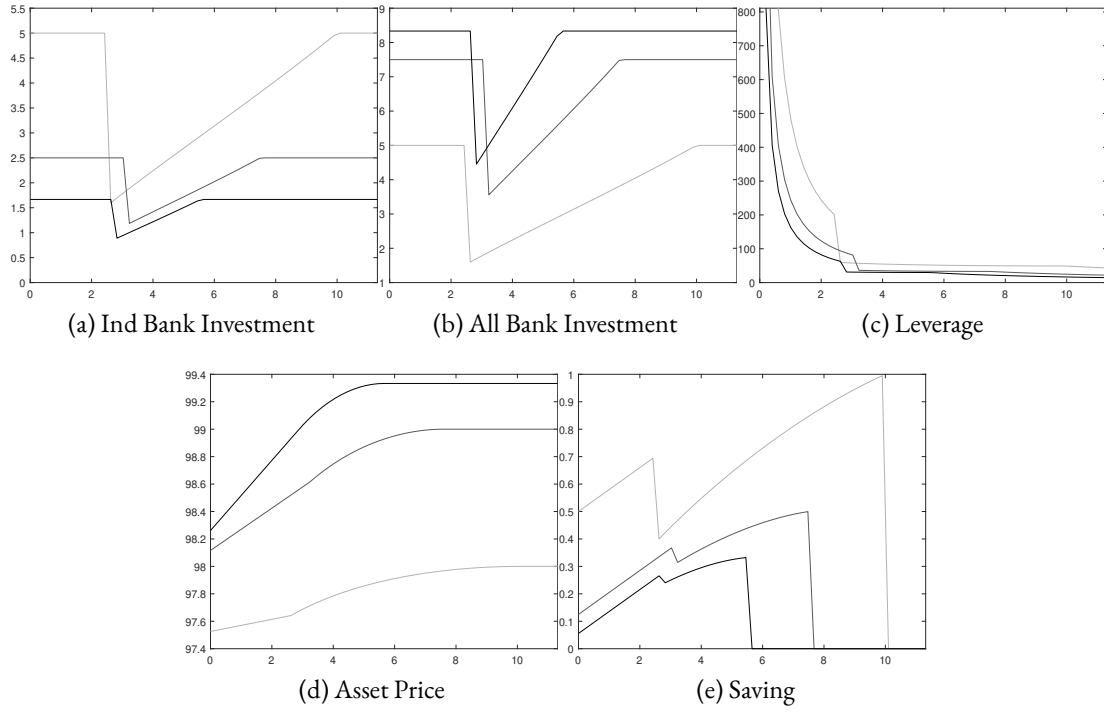


Figure C.7: Policy rules and asset price as a function of net worth in benchmark model for $B = 1, 3, 5$

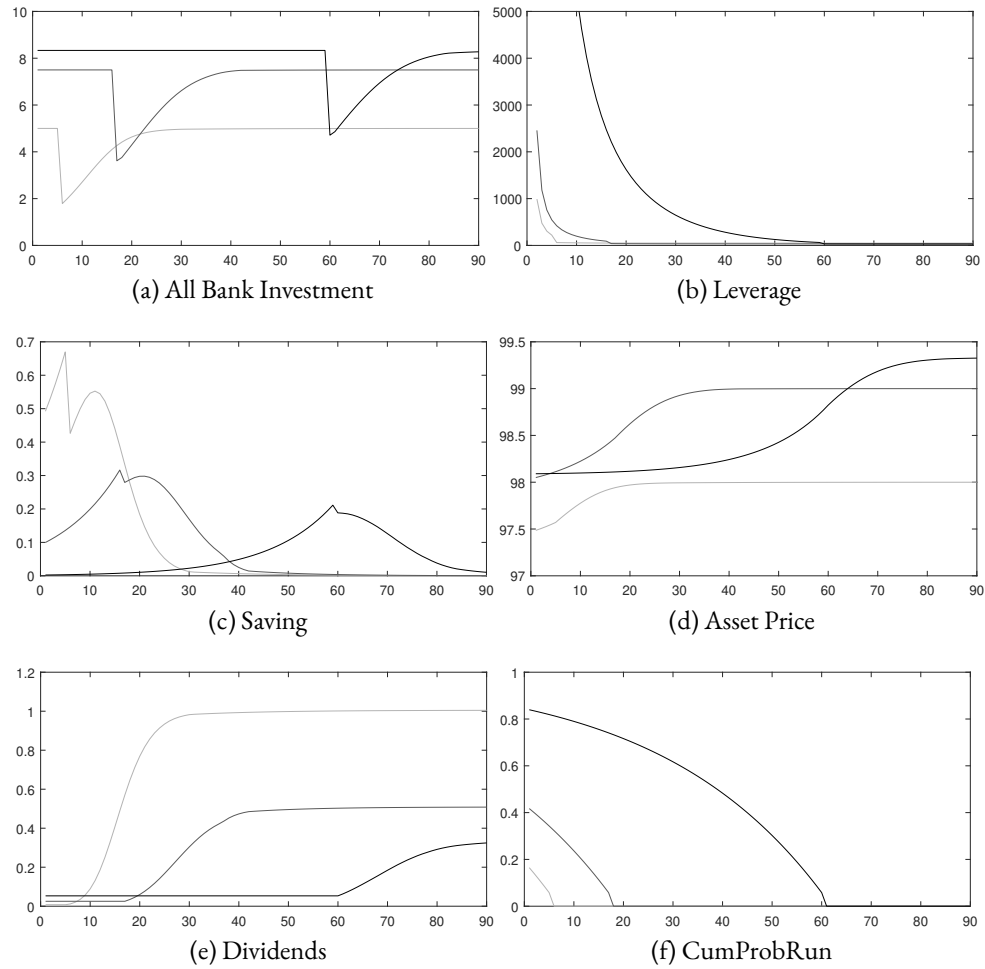


Figure C.8: Transition path as a function of time for $\mathcal{B} = 1, 3, 5$

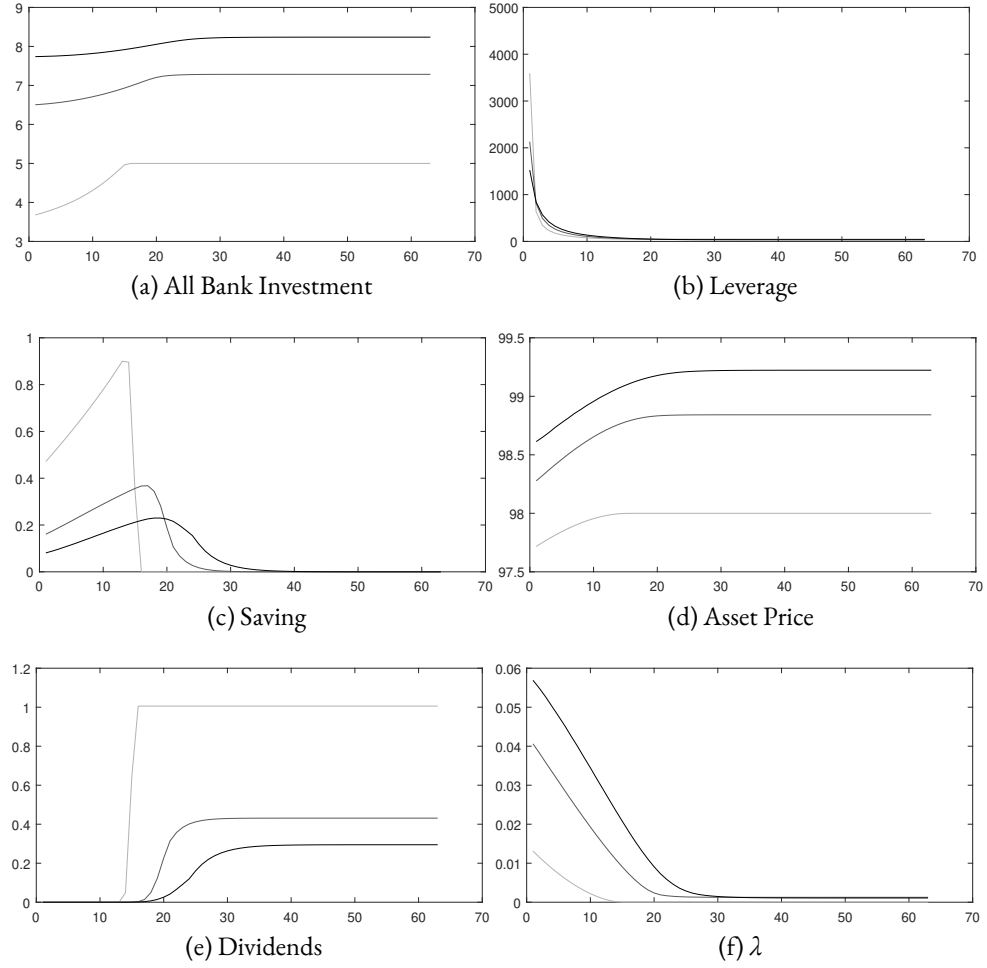


Figure C.9: Transition path as a function of time for $\mathcal{B} = 1, 3, 5$