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Doctoral Dissertations, 2011-2026 Economics

2017

# Essays on Monetary and Macro-prudential Policy

Van der Ghote, Alejandro Nicolas

Princeton, NJ : Princeton University

http://arks.princeton.edu/ark:/88435/dsp01vt150m86z

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# Essays on Monetary and Macro-prudential Policy

Alejandro Nicolas Van der Ghote

A Dissertation
Presented to the Faculty
of Princeton University
in Candidacy for the Degree
of Doctor of Philosophy

Recommended for Acceptance
by the Department of
Economics
Adviser: Nobuhiro Kiyotaki

June 2017

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# Abstract

This dissertation comprises three essays that investigate the transmission mechanism of monetary policy and the interaction between monetary policy and macro-prudential policy.

In Chapter 1, I examine the costs and benefits of coordinating monetary policy and macro-prudential policy. I obtain that the coordination between monetary and macro-prudential policies helps reducing the risk of entering into a financial crisis; helps also speeding up the exit from the crisis, if any; but implies further variability in inflation and in employment gap which is costly.

In Chapter 2, I explore the interaction between monetary policy and macro-prudential policy in economies in which the natural rate of return occasionally attains negative values. In those economies, the zero-lower-bound (ZLB) constraint on the nominal interest rate occasionally prevents monetary policy from conducting its conventional task of replicating the natural rate of return with the nominal rate. I obtain that tighter macro-prudential policies, that restrict intermediary leverage more severely, mitigate the aggregate fluctuations resulting from frictions in financial markets; lift the natural rate of return; and whence facilitate the conventional task of monetary policy.

In Chapter 3, I revisit the transmission mechanism of monetary policy in the context of a financially developed economy in which the provisions of settlement services and of financial intermediary services are highly interrelated. To this end, I develop a framework in which the joint provision of settlement and financial intermediary services creates a liquidity management problem at the intermediary level, and a corresponding demand for liquid assets. I analyze the real effects of unconventional monetary policies that target the width of the corridor between the discount window rate and interest rate on excess reserves. I obtain that the real effects of a narrower corridor in general depend on how liquid the financial intermediary system is.

# Acknowledgements

I would like to thank Nobuhiro Kiyotaki, Markus Brunnermeier, and Oleg Itskhoki for their invaluable guidance. I would also like to thank Mark Aguiar, SeHyoun Ahn, Cristian Alonso, Adrien Auclert, Martin Beraja, Qingqing Cao, Damien Capelle, Thomas Carter, Michael Dobrew, Mikhail Golosov, Zongbo Huang, Ryo Jinnai, Greg Kaplan, So Kubota, Simas Kucinskas, Stephan Luck, Caio Machado, Kazushige Matsuda, Falk Mazelis, Fernando Mendo Lopez, Sebastian Merkel, Benjamin Moll, Christian Moser, Ezra Oberfield, Jason Ravit, Richard Rogerson, Esteban Rossi-Hansberg, Sajan Saini, Andres Schneider, Donghwa Shin, Christopher Sims, Chang Sun, Christian Wolf, Wei Xiong, Motohiro Yogo, and Yu Zhang for useful discussions and comments. I would like to thank Laura Hedden, Jennifer Bello, Karen Neukirchen, and Julie Newton for their administrative support.

To my family and friends.

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# Chapter 1

# Coordinating Monetary and Financial Regulatory Policies

## 1.1 Introduction

A consensus in policy circles is that monetary policy and macro-prudential policy have primary objectives. The primary objective of monetary policy is generally considered to be to stabilize inflation and output gap. The primary objective of macro-prudential policy is generally considered to be to stabilize the financial system. Both monetary policy and macro-prudential policy in general are set contingent upon the aggregate state of the economy. The conventional tool of monetary policy is the reference short-term nominal interest rate. The tools of macro-prudential policy comprise a battery of taxes and of quantity restrictions on asset classes and/or on market participants. How to conduct macro-prudential policy? Should monetary policy be used for stabilizing the financial system along with macro-prudential policy? If it should, how should monetary policy and macro-prudential policy be coordinated?

To address these questions, we develop a continuous-time New Keynesian economy in which there is a financial intermediary sector that faces a leverage constraint.

We use a continuous-time framework similar to Brunnermeier and Sannikov (2014) to fully capture the non-linear dynamics associated with occasionally constrained agents. The model economy is populated by households, firms and financial intermediaries. Firms produce differentiated goods using labor and capital services as inputs. Firms hire labor and rent capital in competitive markets; reset their price occasionally; and accommodate their demand at the prevailing market prices (similar price-setting behavior to Calvo 1983). Financial intermediaries are good at allocating capital across firms: Relative to households, financial intermediaries can provide more capital services to firms per unit of physical capital (similar comparative advantage in financial intermediation to Brunnermeier and Sannikov 2014). Financial intermediaries are subject to a leverage constraint that limits their borrowing capacity and their capacity for intermediating capital by a multiple of net worth.[1] Households consume, supply labor and save. Households save in physical capital and/or in deposit contracts. Deposit contracts are short-term and non-contingent, and are provided by financial intermediaries.

The model economy captures the notion that instabilities in inflation and in financial markets are costly. Specifically, the staggered price adjustments of firms generate price dispersion on differentiated goods, and a misallocation problem of labor and of capital services across firms, when inflation deviates from its structural rate. Leverage constraints depress the aggregate supply of capital services to firms below its potential level, when financial intermediaries are undercapitalized and can intermediate only a fraction of the aggregate capital stock. Incomplete financial markets, together with leverage constraints, give rise to a pecuniary externality in financial markets that ex-

[1]The leverage constraint follows from a moral hazard problem that is similar to Gertler and Karadi (2011) and to Gertler and Kiyotaki (2010): Financial intermediaries can divert a fraction of their assets at the penalty of losing their franchise, and, as a consequence, they have to maintain a sufficiently low ratio of total assets to net worth to attract external funding.

acerbates the fluctuations on intermediary net worth, on the share of intermediated capital and on the aggregate supply of capital services to firms.[2]

To study how to conduct macro-prudential policy, we first examine a benchmark competitive economy in which monetary policy stabilizes inflation at its structural rate and there is no macro-prudential policy. In such economy, in normal situations, financial intermediaries have enough net worth relative to the aggregate wealth to intermediate the aggregate capital stock, leverage constraints are slack and the aggregate supply of capital services reaches its potential level. Following a series of negative shocks, however, financial intermediaries lose net worth, become financially constrained and fail to intermediate all of the aggregate capital stock. Moreover, a positive feedback loop between intermediary net worth losses, drops in the price of capital and drops in the share of intermediated capital, lead to a situation in which the aggregate supply of capital services falls significantly below its potential level (i.e. a financial crisis). Relative to the constrained efficient allocation, financial crises occur too frequently and are too intense, and the recovery from the crisis occurs too infrequently and is too slow.

The first main result in the paper is that, if monetary policy is only concerned with stabilizing inflation, macro-prudential policy should restrict the ratio of total assets to net worth in the financial intermediary sector when financial intermediaries are soft constrained (i.e. when the aggregate state is close to the threshold state at which financial intermediaries have just enough net worth to absorb the aggregate capital stock). Only mild restrictions are beneficial because macro-prudential policy is locally distortionary: restricting the leverage multiple of financial intermediaries below its natural level reduces the aggregate supply of capital services below its natural level as well. The benefits of macro-prudential policy come from boosting the spread between the return on capital and the return on deposits (i.e. the credit spread), and

[2]The pecuniary externality operates through the price of capital as in Lorenzoni (2008), Bianchi and Mendoza (2011), Jeanne and Korinek (2010) and Bianchi (2011).

from mitigating the excessive fluctuations in financial markets that the pecuniary externality generates. The former effect speeds up the recovery from the financial crisis ex-post as well as reduces the likelihood and the intensity of the financial crisis ex-ante. The latter effect reduces the likelihood of the financial crisis ex-ante.

The second main result in the paper is that monetary policy, instead of being exclusively concerned with stabilizing inflation, should coordinate with macro-prudential policy to further stabilize the share of intermediated capital. Specifically, we show that monetary policy should deviate from the natural interest rate (i.e. the interest rate required to stabilize inflation at its structural rate), and that macro-prudential policy should soften its restrictions on intermediary leverage relative to the case in which policies do not cooperate (i.e. the case described in the first main result). We show also that monetary policy should deviate both ex-ante and ex-post: During the financial crisis, monetary policy should target a policy rate below the natural interest rate; once the financial crisis is over, monetary policy should target a policy rate above the natural interest rate.

Monetary policy can help macro-prudential policy to speed up the recovery from the financial crisis because temporary low policy rates boost credit spreads temporarily. Helping macro-prudential policy however is costly as it requires variability in inflation and in employment gap. Monetary policy should deviate from the natural interest rate everywhere because temporary lower policy rates have stronger effects on credit spreads, and because the side-effects on inflation and on employment gap are fairly independent of the situation in financial markets (as well as strictly increasing and strictly convex on the deviations from the natural interest rate). Macro-prudential policy should soften, relative to the case in which policies do not cooperate, because restrictions on intermediary leverage are costly, and because monetary policy is already helping to stabilize the financial system.

Overall, using monetary policy to stabilize the financial system along with macro-prudential policy is beneficial because, when policies do not cooperate, monetary policy can stabilize inflation at its structural rate whereas macro-prudential policy cannot stabilize the share of intermediated capital at its potential level. Generating a small amount of variability in inflation and in employment gap therefore entails a second-order loss relative the first-order gain that follows from reducing the cost of financial disintermediation.

In the baseline calibration, we obtain that the welfare gains from coordinating monetary and macro-prudential policies amount to 0.21% in terms of annual consumption equivalent. We obtain also that the annual consumption equivalent gains from reducing the costs of financial disintermediation amount to 0.29%, and that the annual consumption equivalent losses from generating inflation variability and employment-gap variability amount to 0.07% and to 0.01%, respectively.

**Related Literature** This paper relates to the body of literature that studies the interactions between monetary policy and macro-prudential policy. Likewise De Paoli and Paustian (2013) and Carrillo et al. (2016), the objectives of monetary policy and of macro-prudential policy are grounded on the sources of inefficiency in the model economy. As opposed to those papers, we use global solution methods to solve for the equilibrium outcome instead of log-linearized solution methods around the steady state.

In the context of our application, global solution methods are valuable both in terms of the analytical solution and of the numerical solution. In terms of the analytical solution, global solution methods reveal that dynamic considerations are important for understanding how to conduct policy. For instance, we show that to understand how to conduct policy in normal times (i.e. when the aggregate state is close to the steady state), it is necessary to first understand how policy in normal times affects the equilibrium outcome in crisis times (i.e. when the aggregate state

is far away from the steady state). Global solution methods reveal also that the behavior of private agents (specifically, the leverage decisions of financial intermediaries and the price-setting decisions of firms), as well as the welfare losses associated with the sources of inefficiency, are highly non-linear on the state. In terms of the numerical solution, the aforementioned non-linearities imply that approximation methods of low orders in general perform poorly.

This paper also relates to the body of literature that studies the effect of monetary policy on financial stability in economies in which conventional monetary policy is the only policy instrument. As in Svensson (2014, 2016), temporary high policy rates in normal times hurt financial intermediaries (i.e. the debtors in our model economy) locally. As opposed to Svensson (2014, 2016), hiking the policy rate above the natural interest rate in normal times is beneficial. The difference on results follows from the insights that macro-prudential policy is costly and that macro-prudential policy should soften when monetary policy helps to stabilize the financial system. Overall, our analysis is consistent with the view that monetary policy should be the second line of defense against risks to financial stability being macro-prudential policy the first.

On methodological grounds, our model economy builds upon the works of Calvo (1983); Gertler and Karadi (2011) and Gertler and Kiyotaki (2010); and of Brunnermeier and Sannikov (2014). The main difference with respect to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) is that leverage constraints occasionally bind. The main difference with respect to Brunnermeier and Sannikov (2014) is that financial intermediaries are subject to a leverage constraint. Our model economy also relates to Drechsler, Savov and Schnabl (2015); Kaplan, Moll and Violante (2016); and to Brunnermeier and Sannikov (2016a). Likewise the models in the former two papers, our model is a continuous-time economy with nominal rigidities and financial frictions. As opposed to Drechsler, Savov and Schnabl (2015), nominal rigidities are

grounded on the staggered price-setting behavior of firms. As opposed to Kaplan, Moll and Violante (2016), aggregate shocks are locally stochastic. The main difference with respect to the model in Brunnermeier and Sannikov (2016a) is the role of money: In our model economy money serves the role of a unit of account whereas in Brunnermeier and Sannikov (2016a) money serves the role of a store of value.

**Layout** Section 1.2 describes and solves the model economy. Section 1.3 examines the costs and benefits of coordinating monetary policy and macro-prudential policy. Section 1.4 provides a quantitatively assess the aforementioned costs and benefits. Section 1.5 concludes.

## 1.2 The Model

The model is a standard New Keynesian economy in which there are financial intermediaries that face a leverage constraint. The model admits a representative agent as the occupational structure of the household is similar to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010).

**Setup** Time is continuous and runs forever. The model economy is populated by a continuum of identical households. Households are composed of a continuum of family members with unit measure. Family members share the same preferences for consumption and the same discount rate, $\rho > 0$. There is perfect consumption insurance within the household meaning that all family members always consume the same flow of goods, $c_t \geq 0$.

Family members serve different occupations. A fraction $f \in (0, 1)$ of them are financiers and the remaining fraction are either workers or producers, non-financiers for short. Family members switch their occupation stochastically according to Poisson processes with arrival rate $\tilde{\rho} > 0$ for financiers, and arrival rate $\tilde{\rho} f / (1 - f)$ for non-financiers.

Family members perform different tasks depending on their occupation. Workers run the household; producers manage the firms; and financiers manage the financial intermediaries.

### 1.2.1 Firms

There are two types of firms: intermediate and final firms.

**Final firms** Final firms produce a final consumption good $y_t$ out of a continuum of intermediate goods $y_{j,t}$, with $j \in [0,1]$. Final firms operate a CES production technology

$$y_t = \left[ \int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

with elasticity of substitution $\varepsilon \in (1, +\infty)$.

Let $p_t$ denote the price of the final good and let $p_{j,t}$ denote the price of the intermediate good $j$. Because final firms behave competitively, the input demand system is

$$y_{d,t}(p_{j,t}) = \left( \frac{p_{j,t}}{p_t} \right)^{-\varepsilon} y_t$$

and the price of the final good equals the marginal costs of producing the final good

$$p_t = \left[ \int_0^1 p_{j,t}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$$

We interpret the price of the final good as the aggregate price level.

**Intermediate firms** Intermediate firms produce the intermediate goods out of labor and of capital services. Each intermediate firm produces a single intermediate-good variety. Intermediate firms operate a Cobb-Douglas production technology

$$y_{j,t} = A_t l_{j,t}^{\alpha} k_{j,t}^{1-\alpha}$$

with labor share coefficient $\alpha \in (0,1)$. Here, $l_{j,t}$ and $k_{j,t}$ refer to the quantity of labor and of capital services employed in the production of the intermediate good $j$. Productivity $A_t$ is common to all intermediate goods and follows the Ito process

$$dA_t/A_t = \mu_{A,t}dt + \sigma_{A,t}dZ_t$$

with drift process $\mu_{A,t}$ and diffusion process $\sigma_{A,t}$. The process $\{Z_t \in \mathbb{R} : t \geq 0\}$ is a standard Wiener process defined on a filtered probability space $(\Omega, \mathcal{F}, P)$. We interpret $dZ_t$ as an aggregate supply shock. The Brownian shock $dZ_t$ is the only source of risk in the model economy.

Intermediate firms compete monopolistically and adjust their price only sporadically according to Calvo (1983) pricing.[3] When they have the opportunity to adjust their price, intermediate firms maximize the present discounted value of their profits flows

$$\max_{p_{j,t} \geq 0} E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} \left[ (1-\tau) \frac{p_{j,t} y_{d,s}(p_{j,t})}{p_s} - x_s \left[ y_{d,s}(p_{j,t}) \right] \right] ds$$

The parameter $\theta > 0$ is the arrival rate of the Poisson process that allows them to adjust their price. The coefficient $\tau < 0$ is an ad-valorem sales subsidy on intermediate firms. Intermediate firms hire labor and rent capital services on the spot in competitive markets; the real cost function $x_t(y_{j,t})$ therefore is

$$x_t(y_{j,t}) = \frac{1}{A_t} \frac{w_t^\alpha r_{k,t}^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} y_{j,t}$$

The real wage rate is $w_t$ and the real rental rate of capital services is $r_{k,t}$. Intermediate firms discount future profits flows with the Stochastic Discount Factor (SDF) of the

[3] Calvo (1983) pricing has three key features: (*i*) intermediate firms can adjust their price only when they are hit by a Poisson shock; (*ii*) the Poisson shock is idiosyncratic and iid across intermediate firms; and (*iii*) intermediate firms pay no cost for adjusting their price. Intermediate firms who cannot adjust their price accommodate their demand at the prevailing market prices.

household, $\Lambda_t$, weighted by the survival density function of the price chosen at time $t$, $\theta e^{-\theta(s-t)}$.

The solution to the price-setting problem of intermediate firms is $p_{*,t}$, with

$$p_{*,t}/p_t \equiv \underbrace{\frac{1}{1-\tau}\frac{\varepsilon}{\varepsilon-1}}_{=1} * \frac{E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} x_s \left[y_{d,s}\left(p_t\right)\right] ds}{E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} \frac{p_t y_{d,s}(p_t)}{p_s} ds}$$

Intermediate firms who can adjust their price set a same price because their problem is identical. The optimal real price $p_{*,t}/p_t$ is the product of two factors. The first factor is the product between a sales subsidy multiplier $1/\left(1-\tau\right)$ and a distortion coefficient from monopoly pricing $\varepsilon/\left(\varepsilon-1\right)$. We impose $\tau = -1/\left(\varepsilon-1\right)$ to eliminate the distortions from monopoly pricing.[4] The second factor is the ratio of the present discounted value of production costs to that of sales revenues (gross of sales subsidies), of an hypothetical firm who charges a price equal to the aggregate price level $p_t$. The second factor is forward-looking because intermediate firms can adjust their price only sporadically: if they could adjust their price continuously instead, i.e. $1/\theta \to 0$, the second factor would reduce to the spot marginal production costs $x_t\left(y_j\right)/y_j$.

**The Inflation Rate** The price-setting behavior of intermediate firms determines the cross-section distribution of intermediate-goods prices. Specifically, the latter is given by the survival density function of the past optimal prices.[5] The aggregate price level therefore satisfies

$$p_t = \left[\int_{-\infty}^t \theta e^{-\theta(t-s)} p_{*,s}^{1-\varepsilon} ds\right]^{\frac{1}{1-\varepsilon}}$$

[4]This is standard in the New Keynesian framework (see Woodford 2003 and Gali 2008). The rationale is that neither monetary policy nor macro-prudential regulation should be concerned with imperfect competition in goods markets.

[5]The reason is twofold. First, intermediate firms who adjust their price at any time $s < t$ set the same optimal price $p_{*,s}$. Second, a law of large numbers applies because the Poisson shock is idiosyncratic and iid across intermediate firms.

**Proposition 1** *The inflation rate is locally risk-free*

$$\frac{dp_t}{p_t} = \pi_t dt + 0 * dZ_t$$

*where* $\pi_t dt \equiv E_t \left[ dp_t / p_t \right]$ *is the locally expected inflation rate.*

**Proof.**[6] ■

The intuition behind Proposition 1 goes as follows. Calvo (1983) pricing allows only a small share of firms to adjust their price within a short time interval. As the length of the time interval shrinks, the share of firms who can adjust their price shrinks as well, so that in the limit the spot Brownian shock $dZ_t$ cannot affect the evolution of the aggregate price level on impact. Calvo (1983) pricing therefore generates an aggregate price level that is sticky: the path of the aggregate price level does not fluctuate at Brownian velocity, as opposed to the path of the Brownian shocks that hit the economy.

### 1.2.2 Households

Households take consumption, labor and portfolio decisions. The workforce of households comprises workers alone who supply labor hours $l_t \geq 0$ at the real wage rate $w_t$. The investment portfolio of households is composed of deposit contracts and of physical capital.

Deposit contracts are financial securities with a short-term maturity and a fixed payment structure: deposits issued at time $t$ mature at time $t + dt$ and yield the locally risk-free nominal interest rate $i_t \geq 0$. Deposit contracts are locally risk-free also in real terms because the real deposit rate $i_t - \pi_t$ is locally risk-free.

[6] Proof. The integral at the RHS of $p_t$ is a Riemann Integral.

Physical capital is a real asset in fixed supply.[7] The aggregate stock of physical capital is $\bar{k} > 0$. Physical capital is traded in fully liquid markets at the real spot price $q_t$.

Physical capital is the key input for producing the capital services that firms require. Financial intermediaries have a comparative advantage at managing physical capital relative to households. Specifically, out of one unit of physical capital, financial intermediaries can produce, and hence rent out to firms, one unit of capital services whereas households can produce only $u_{h,t} \in (0,1)$ units.[8]

The dividend return that households obtain out of physical capital is therefore $u_{h,t} * r_{k,t} < r_{k,t}$. The total real return that households obtain on physical capital is $dR_t$, with

$$dR_t \equiv u_{h,t} \frac{r_{k,t}}{q_t} dt + \frac{dq_t}{q_t} \tag{1.1}$$

The total real return is the sum of the dividend yield plus the capital gain/loss.

The net worth of households $n_{h,t} \geq 0$ evolves in accord with their consumption, labor and portfolio decisions, and with the net transfers that they receive from firms and from financial intermediaries

$$dn_{h,t} = dR_t q_t \bar{k}_{h,t} + (i_t - \pi_t)\left(n_{h,t} - q_t \bar{k}_{h,t}\right) dt + w_t l_t dt + \Pi_t dt + Div_t dt - c_t dt \tag{1.2}$$

The process $\bar{k}_{h,t} \geq 0$ is the position that households take on physical capital. Households cannot short-sell physical capital because the latter is a real asset. The process $\Pi_t$ is the transfers that households receive from firms net of the lump-sum taxes that

[7] Physical capital does not mature nor depreciate. Neither is there an investment technology that allows for the accumulation of physical capital across time.

[8] Intuitively, the productivity gap $1 - u_{h,t}$ captures the notion that financial intermediaries are good at allocating capital across firms. The productivity coefficient $u_{h,t}$ is time-varying as we allow for convex adjustment costs in the management of physical capital. Specifically, we assume that $u_{h,t}$ is inversely related with the share of the aggregate capital stock that households manage. This implies that individual households can produce more (less) units of capital services when on aggregate they manage a lower (higher) share of the aggregate capital stock. In the calibration (see section 4.1), we specify a linear functional form between $u_{h,t}$ and $\bar{k}_{h,t}/\bar{k}$, being $\bar{k}_{h,t}/\bar{k}$ the share of the aggregate capital stock that households manage.

households pay to finance the ad-valorem sales subsidy. $\Pi_t$ amounts to the aggregate profit flows of intermediate firms gross of sales subsidies, as households are the residual claimants of the profit flows that firms generate.[9] $Div_t$ is the net transfers that households receive from financial intermediaries.

The objective of households is to maximize the present discounted value of their utility flows

$$E_t \int_t^\infty e^{-\rho(s-t)} \left[ \ln c_s - \chi \frac{l_s^{1+\psi}}{1+\psi} \right] ds \tag{1.3}$$

The parameter $\psi \geq 0$ is the inverse of the Frish elasticity and $\chi > 0$ weights the labor disutility.

**Portfolio Problem** The problem of households is a standard portfolio problem with consumption and labor decisions. Specifically, households maximize (1.3) subject to $n_{h,t} \geq 0$, (1.1) and (1.2) . We conjecture that the price of capital follows an Ito process

$$\frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{q,t} dZ_t$$

with drift process $\mu_{q,t}$ and diffusion processes $\sigma_{q,t}$. Intuitively, the spot capital gains/losses $dq_t/q_t$ depend on the spot Brownian shock $dZ_t$ because future rental rates of capital services depend on future productivity levels. The drift process $\mu_{q,t}$ and the diffusion process $\sigma_{q,t}$ are endogenous objects to be determined in equilibrium.

**Proposition 2** *The optimality conditions in the portfolio problem of households are three:*

1. *The intra-temporal condition between consumption and labor*

$$\frac{1}{c_t} w_t = \chi l_t^\psi$$

[9] Final firms make zero profits because they behave competitively and because the CES production technology exhibits constant returns to scale.

*2. The asset pricing condition on deposits*

$$(i_t - \pi_t)\, dt = \rho dt + E_t\left[dc_t/c_t\right] - Var_t\left[dc_t/c_t\right]$$

*3. The asset pricing condition on capital*

$$E_t\left[dR_t\right] - (i_t - \pi_t)\, dt + Cov_t\left[-dc_t/c_t, dR_t\right] \leq 0$$

*with equality if* $\bar{k}_{h,t} > 0$.

**Proof.**[10]■

The optimality conditions are similar to those in the consumption-based capital asset pricing model [C-CAPM] (Rubinstein 1976, Breeden and Litzenberger 1978, Lucas 1978 and Breeden 1979). At the optimal solution, households are indifferent on the margin between consumption and labor (first condition) and between consumption and deposits (second condition). Households prefer deposits to capital depending on the risk-adjusted excess return on capital that they obtain over deposits (third condition). When they obtain a null risk-adjusted excess return, households are indifferent on the margin between capital and deposits, and $\bar{k}_{h,t} \geq 0$. When households obtain a negative risk-adjusted excess return, they strictly prefer on the margin deposits to capital, and $\bar{k}_{h,t} = 0$.[11] The risk premium that households demand on capital $Cov_t\left[dc_t/c_t, dR_t\right]$ is composed of consumption risk alone (third condition).

### 1.2.3 Financial Intermediaries

Financial intermediaries take portfolio decisions. The investment portfolio of financial intermediaries is composed of deposit contracts and of physical capital. The total real

[10] See Appendix A.1.

[11] Households cannot obtain a positive risk-adjusted excess return because they are not subject to portfolio constraints. If they were to obtain a positive risk-adjusted excess return, they would take unbounded levered positions on capital and markets would not clear.

return that financial intermediaries obtain on physical capital is $d\tilde{R}_t$, with

$$d\tilde{R}_t \equiv 1 * \frac{r_{k,t}}{q_t} dt + \frac{dq_t}{q_t} > dR_t \tag{1.4}$$

Financial intermediaries obtain a higher return on capital relative to households because they manage physical capital more productively.

The net worth of financial intermediaries $n_{f,t} \geq 0$ evolves in accord with their portfolio decisions

$$dn_{f,t} = d\tilde{R}_t q_t \bar{k}_{f,t} - (i_t - \pi_t) \left( q_t \bar{k}_{f,t} - n_{f,t} \right) dt \tag{1.5}$$

The process $\bar{k}_{f,t} \geq 0$ is the position that financial intermediaries take on physical capital. Because $d\tilde{R}_t > dR_t$, we conjecture that financial intermediaries take levered positions on capital (i.e. $q_t \bar{k}_{f,t} > n_{f,t}$). Taking levered positions on capital exposes financial intermediaries to a liquidity mismatch problem in their balance sheets.[12] The reason is that deposit contracts are locally risk-free whereas physical capital is locally risky.[13]

The objective of financial intermediaries is to maximize the present discounted value of their dividend payouts. Financial intermediaries pay out dividends either when $(i)$ they are forced to close down; or $(ii)$ the financier in office switches her occupational role. In either event, financial intermediaries transfer back to the household their entire net worth. Financial intermediaries discount future dividend payouts

[12] Equivalently, when they take levered positions on capital, financial intermediaries concentrate aggregate risk $\sigma_{q,t}$ in their balance sheets. Aggregate risk is measured with the diffusion process of total wealth ( He and Krishnamurthy 2013; Brunnermeier and Sannikov 2014, 2015). In the model economy, total wealth is given by $q_t \bar{k}$ because physical capital is the only real asset.

[13] A capital structure based on deposit contracts (short-term contracts with a fixed payment structure) is consistent with the empirical findings of Hanson, Shleifer, Stein and Vishny (2014) and Drechsler, Savov and Schnabl (2015). These papers document that the salient feature of financial institutions is to invest in long-term and risky assets funded with short-term instruments: traditional banks rely mainly on demand and/or savings deposits whereas shadow banks on repos, ABCP and CP.

with the SDF of the household, weighted by the probability of making the dividend payout.

Financial intermediaries are subject to a moral hazard problem. The moral hazard problem is similar to Gertler and Karadi (2011) and to Gertler and Kiyotaki (2010). Financial intermediaries can borrow only from households other than the household to which they belong. At every point in time, financial intermediaries have to choose between diverting funds or behaving properly. If they divert funds, financial intermediaries can walk-away with a positive fraction $1/\lambda$ of their assets but they are forced to close down immediately afterwards. The value of diverting funds is $\frac{1}{\lambda} q_t \bar{k}_{f,t}$. If they behave properly, financial intermediaries solve their portfolio problem, and pay out dividends eventually when the financier in office switches her occupational role. The value of behaving is given by the franchise value of financial intermediaries, $V_t$.

The moral hazard problem gives rise to a incentive-compatible constraint that limits capital positions accordingly

$$q_t \bar{k}_{f,t} \leq \lambda V_t \tag{1.6}$$

The incentive-compatible constraint guarantees that financial intermediaries never divert funds.

Financial intermediaries are subject also to macro-prudential regulation. Macro-prudential regulation limits their capital position by a multiple $\Phi_t \geq 1$ of net worth

$$q_t \bar{k}_{f,t} \leq \Phi_t n_{f,t} \tag{1.7}$$

**Portfolio Problem** The problem of financial intermediaries is a standard portfolio problem with portfolio constraints. Specifically, financial intermediaries

solve

$$V_t \equiv \max_{\bar{k}_{f,t}\geq 0} E_t \int_t^\infty \tilde{\rho} e^{-\tilde{\rho}(s-t)} \frac{\Lambda_s}{\Lambda_t} n_{f,s} ds \quad (1.8)$$
$$s.t. : n_{f,t} \geq 0, \ (1.4), \ (1.5), \ (1.6), \ (1.7)$$

where $\tilde{\rho} e^{-\tilde{\rho}(s-t)}$ is the probability density function of making the dividend payout (equivalently, it is the survival density function of the financier in office).

We conjecture that the franchise value of financial intermediaries is linear on net worth

$$V_t = v_t n_{f,t}$$

where $v_t \geq 1$ is the Tobin's Q. The value $V_t$ is proportional to net worth because the portfolio problem of financial intermediaries (1.8) is linear. The value $v_t$ is never below 1 because the Tobin's Q of an hypothetical financial intermediary who can invest only in deposits is always equal to 1 : such financial intermediary is worthless because households are indifferent on the margin between consumption and deposits. We conjecture that $v_t$ follows an Ito process.

Let $\phi_t \equiv q_t \bar{k}_{f,t} / n_{f,t}$ denote the leverage multiple of financial intermediaries. Financial intermediaries are subject to the leverage constraint $\phi_t \leq \min\{\lambda v_t, \Phi_t\}$.

**Proposition 3** *The optimality conditions in the portfolio problem of financial intermediaries are two:*

*1. The asset pricing condition on capital*

$$E_t\left[d\tilde{R}_t\right] - (i_t - \pi_t)\, dt + Cov_t\left[-dc_t/c_t + dv_t/v_t, d\tilde{R}_t\right] \geq 0$$

*with equality if* $\phi_t < \min\{\lambda v_t, \Phi_t\}$.

*2. The asset pricing condition on the Tobin's Q*

$$\tilde{E}_t\left[d\tilde{R}_{n,t}\right] + \frac{\tilde{\rho}}{v_t}dt + E_t\left[dv_t/v_t\right] - \tilde{\rho}dt + Cov_t\left[-dc_t/c_t, dv_t/v_t\right] = 0$$

*with*

$$\tilde{E}_t\left[d\tilde{R}_{n,t}\right] \equiv \underbrace{E_t\left[dn_{f,t}/n_{f,t}\right] - \left(i_t - \pi_t\right)dt + Cov_t\left[-dc_t/c_t + dv_t/v_t, dn_{f,t}/n_{f,t}\right]}_{risk-adjusted\ excess\ return\ on\ equity}$$

**Proof.**[14] ■

The asset pricing condition on capital is standard (first condition). When financial intermediaries obtain a null risk-adjusted excess return on capital, they are indifferent between capital and deposits, and leverage constraints are slack. When financial intermediaries obtain a positive risk-adjusted excess return, they strictly prefer capital to deposits, and leverage constraints bind.[15] Positive risk-adjusted excess returns on capital for financial intermediaries are feasible because (*i*) households become the marginal investors on capital when financial intermediaries become financially constrained; (*ii*) households have a lower valuation for capital than financial intermediaries;[16] and (*iii*) binding leverage constraints prevent financial intermediaries from fully exploiting positive risk-adjusted excess returns.

The asset pricing condition on the Tobin's Q determines the process for the marginal value of intermediary wealth $v_t$. The reason is that risk-adjusted excess return on equity enters as a dividend yield component in such condition. Intuitively, the Tobin's Q reflects the expected profit flows that financial intermediaries make out of one unit of net worth.

[14] See Appendix A.1.

[15] Financial intermediaries cannot obtain negative risk-adjusted excess returns because otherwise they would prefer to take no position on capital.

[16] The spread on the risk-adjusted excess return on capital between financial intermediaries and households is non-negative.

The Tobin's Q fluctuates iff leverage constraints bind and/or are expected to bind in the future: otherwise, $v_t = 1$ solves the asset pricing condition on the Tobin's Q (notice that the risk-adjusted excess return on equity is given by the product between the risk-adjusted excess return on capital and the leverage multiple). Equivalently, financial intermediaries make profits in expected value iff leverage constraints prevent them from fully exploiting positive risk-adjusted excess returns (otherwise, positive risk-adjusted excess returns would not materialize in the first place).

The risk premium that financial intermediaries demand on capital is composed of consumption risk and of financial risk. Financial intermediaries are concerned about the co-movement between the return on capital and their marginal value of wealth because they are subject to leverage constraints.

### 1.2.4 Monetary Policy and Macro-prudential Policy

Policy instruments comprise monetary policy and macro-prudential policy. Monetary policy sets the policy rate, i.e. the reference short-term nominal interest rate. The policy rate is perfectly arbitraged against the nominal deposit rate because the implementation mechanism of monetary policy is the same as in the New Keynesian framework.[17] Macro-prudential policy determines the quantity restriction $\Phi_t$ on leverage.

### 1.2.5 Competitive Equilibrium

The following definition of the competitive equilibrium takes monetary policy $i_t$ and macro-prudential policy $\Phi_t$ as given.

**Definition** A competitive equilibrium is a set of stochastic processes adapted to the filtration generated by $Z$ : the aggregate price level $\{p_t\}$ ; the real price of capital $\{q_t\}$ ; the real rental rate of capital services $\{r_{k,t}\}$ ; the real wage rate $\{w_t\}$ ; the

[17] See Clarida, Galí and Gertler (1999) for a reference.

inflation rate $\{\pi_t\}$; the quantity of the final consumption good $\{y_t\}$; the quantity of the intermediate goods $\{y_{j,t}\}$; labor hours $\{l_t\}$; the labor hours employed in the production of the intermediate good $\{l_{j,t}\}$; the units of capital services employed in the production of the intermediate goods $\{k_{j,t}\}$; the capital position of households $\left\{\bar{k}_{h,t}\right\}$; the capital position of financial intermediaries $\left\{\bar{k}_{f,t}\right\}$; the optimal price $\{p_{*,t}\}$; the leverage multiple $\{\phi_t\}$; and productivity $\{A_t\}$ such that:

1. Individual optimization conditions

   (a) $\left\{y_t, l_t, \bar{k}_{h,t}\right\}$ solve the problem of households

   (b) $\{y_t, y_{j,t}\}$ solve the problem of final firms

   (c) $\{p_{*,t}, y_{j,t}, l_{j,t}, k_{j,t}\}$ solve the problem of intermediate firms

   (d) $\{\phi_t\}$ solves the problem of financial intermediaries

2. Market clearing conditions

   (a) The labor market and the rental market for capital services clear

$$\int_0^1 l_{j,t}dj = l_t \quad \text{and} \quad \int_0^1 k_{j,t}dj = u_{h,t}\bar{k}_{h,t} + \bar{k}_{f,t}$$

   (b) The market for capital holdings clears

$$\bar{k}_{h,t} + \bar{k}_{f,t} = \bar{k}$$

The market for deposits automatically clears due to Walras Law.

### 1.2.6 Distortions, Wedges and The Sources of Inefficiency

**Sources of Inefficiency** There are three sources of inefficiency: staggered price adjustments, incentive-compatible constraints and incomplete financial markets (only

deposit contracts are admissible). We label the former source as nominal rigidities, and the latter two sources as frictions in financial markets, or, as financial frictions, for short.

**Distortions** Nominal rigidities and financial frictions together scale down aggregate Total Factor Productivity (TFP) by the endogenous factor $\zeta_t \in [0,1]$

$$y_t = \zeta_t A_t l_t^{\alpha} \bar{k}^{1-\alpha}$$

with $\zeta_t \equiv u_t^{1-\alpha}/\omega_t$. The process $\omega_t \in [1,+\infty)$ is a productivity wedge from nominal rigidities. The process $1/u_t \in [1, 1/u_{h,t}]$ is a capital services wedge from financial frictions. Nominal rigidities also generate a labor wedge $h_t \in (0,+\infty)$.

**Productivity Wedge.** The productivity wedge is the consequence of price dispersion on intermediate goods. Price dispersion is inefficient because the CES production technology exhibits decreasing marginal returns.

The productivity wedge is given by the measure of price dispersion that incorporates the substitution effect in the demand system

$$\omega_t \equiv \int_{-\infty}^{t} \theta e^{-\theta(t-s)} \left( \frac{p_{*,s}}{p_t} \right)^{-\varepsilon} ds$$

Intuitively, $\omega_t$ measures the total quantity of the final good that could have been produced relative to actual quantity $y_t$, if the aggregate quantity of intermediate goods $\omega_t y_t = \int_0^1 y_{j,t} dj$ would have been evenly allocated across intermediate-goods varieties. Jensen inequality implies that $\omega_t \geq 1$.

**Labor Wedge.** The labor wedge is the consequence of a distorted price system.[18] The labor wedge is given by the ratio between the marginal rate of substitution

[18]Distortions in the price system, specifically, distortions in the relative prices of intermediate goods (i.e. inefficient price dispersion), in general prevent the aggregate demand of inputs from reflecting the marginal productivity of inputs. Wedges between input prices and marginal productivities therefore arise. These wedges in turn affect the equilibrium quantity of labor since labor is not in fixed supply.

of consumption for leisure and the marginal product of labor

$$h_t \equiv \frac{y_t \chi l_t^{\psi}}{\alpha y_t / l_t}$$

The labor wedge departs from 1 when labor deviates from its efficient level, $l_{*,t} = (\alpha/\chi)^{\frac{1}{1+\psi}}$ .

**Capital Services Wedge.** The capital services wedge is the consequence of a misallocation problem of physical capital between financial intermediaries and households. The (inverse of the) capital services wedge is given by the ratio of the aggregate supply of capital services to the aggregate stock of physical capital

$$u_t \equiv \bar{k}_{h,t}/\bar{k} * u_{h,t} + \bar{k}_{f,t}/\bar{k}$$

Intuitively, $u_t$ is a weighted average between the productivity coefficient of households and that of financial intermediaries. The weights in $u_t$ are given by the individual capital shares.

**The Role of Nominal Rigidities** The productivity wedge and the labor wedge are ultimately the consequence of the staggered price-setting behavior of firms. When firms adjust their price continuously (i.e. $1/\theta \to 0$), there is no price dispersion nor productivity wedge because $(i)$ all firms can always adjust their price at every instant; and $(ii)$ price-setting problems are identical. Neither is there a labor wedge since prices are competitive. In contrast, when firms adjust their price only sporadically (i.e. $1/\theta \not\to 0$), price dispersion is a possibility because firms set prices contingent upon the realization of different shocks. A positive labor wedge is also a possibility because the aggregate demand of inputs responds sluggishly to the available productivity information.[19]

[19] Notice that for any $s < t$ there is a positive measure of firms $\theta e^{-\theta(t-s)}$ that responds only to the productivity information available up to time $s$.

**The Role of Financial Frictions** The capital services wedge is ultimately the consequence of leverage constraints that bind/occasionally bind. Let $\eta_t \equiv n_{f,t}/q_t\bar{k} \in [0,1]$ denote the wealth share of financial intermediaries. When leverage constraints bind, financial intermediaries lack enough borrowing capacity to absorb the aggregate capital stock (i.e. $\min\{\lambda v_t, \Phi_t\}\eta_t < 1$), and, as a result, households have to hold the share of the aggregate capital stock that financial intermediaries cannot absorb. Namely, $\phi_t = \min\{\lambda v_t, \Phi_t\} < 1/\eta_t$ and, as a result, $\bar{k}_{f,t}/\bar{k} < 1$ and $u_t < 1$. When leverage constraints are slack, financial intermediaries do have enough borrowing capacity to absorb the aggregate capital stock, and, as a consequence, financial intermediaries take on leverage until clearing the market for physical capital.[20] Namely, $\phi_t = 1/\eta_t$ and $\bar{k}_{f,t}/\bar{k} = u_t = 1$.

Leverage constraints that bind/occasionally bind, together with deposit contracts, furthermore, give rise to a pecuniary externality in financial markets that exacerbates the fluctuations on the capital services wedge. The pecuniary externality operates through the price of capital and follows from the self-centered behavior of individual financial intermediaries: Financial intermediaries do not internalize the cross-price effects that their own leverage decisions have on the others' leverage constraints because they take prices and aggregate variables as given (similar pecuniary externality to Lorenzoni 2008, Bianchi and Mendoza 2011, Jeanne and Korinek 2010 and Bianchi 2011). The pecuniary externality generates excessive fluctuations on the capital services wedge as it generates excessive fluctuations on financial net worth. A positive feedback loop between leverage, capital gains/losses, and the tightening of leverage constraints, in turn, amplifies the effect that the pecuniary externality has on the capital services wedge.

[20] In equilibrium, financial intermediaries always take the highest possible position on capital because they value physical capital more than households.

### 1.2.7 Policy Objectives

The objectives of and the strategic interactions between monetary policy and macro-prudential policy depend on whether policies cooperate or not. We say that policies cooperate if monetary policy and macro-prudential policy have a common objective and are set together. We say that policies do not cooperate if monetary policy and macro-prudential policy have separated objectives and are set separately.

Let partition the utility flows of households accordingly

$$\underbrace{\ln \frac{1}{\omega_t}}_{Price\ \ Dispersion} + \underbrace{\alpha \ln l_t - \chi \frac{l_t^{1+\psi}}{1+\psi}}_{Labor\ \ Misallocation} + \underbrace{(1-\alpha) \ln u_t}_{Fin.\ \ Disintermediation} + \underbrace{\ln A_t + (1-\alpha) \ln \bar{k}}_{Exogenous}$$

If policies do not cooperate, we assume that monetary policy minimizes the present discounted value of the distortions from nominal rigidities (price dispersion + labor misallocation), and that macro-prudential policy minimizes the present discounted value of the distortions from financial frictions (financial disintermediation). We assume also that monetary policy takes macro-prudential policy as given and vice versa. This separation of policy objectives is consistent with the intuitive notion of separation, between monetary policy and macro-prudential policy, typically used in policy circles (Smets 2013, Svensson 2014).

If policies cooperate, we assume that monetary policy and macro-prudential policy jointly maximize the present discounted value of the utility flows of households.

### 1.2.8 Markov Competitive Equilibrium

We conjecture that a Markov equilibrium exists. We conjecture furthermore that the state variables in the Markov equilibrium are $\{A, \omega, \eta\}$.[21]

[21] We restrict $\mu_{A,t}$ and $\sigma_{A,t}$ to depend only on $A$. Additionally, we restrict monetary policy and macro-prudential policy to depend only on $\{A, \omega, \eta\}$. To work with a Markov equilibrium that is scale invariant with respect to $A$, we restrict $\sigma_{A,t}$ to be constant.

The Markov equilibrium adds to the conditions of the competitive equilibrium the consistency conditions on the evolution of the endogenous states $\omega_t$ and $\eta_t$. The consistency conditions demand price dispersion and the wealth share of financial intermediaries to evolve in accord with the conditions of the competitive equilibrium.

## 1.3 Costs and Benefits from Coordination

### 1.3.1 Uncoordinated Policy Benchmark

The uncoordinated policy is the benchmark of comparison used to evaluate the costs and benefits from coordinating monetary policy and macro-prudential policy. The uncoordinated policy is interesting in its own sake as it shows how to conduct macro-prudential policy in the context of a flexible price economy (i.e. an economy in which firms adjust their price continuously).

#### Monetary Policy

Let $r_t$ denote the real interest rate and let hat variables denote variables in the flexible price economy. The natural interest rate is the real interest rate $\hat{r}_t$.

**Proposition 4** *If policies do not cooperate, monetary policy mimics the natural interest rate. Namely,* $i_t = \hat{r}_t$.

**Proof.**[22] ■

An informal discussion concerning the proof goes as follows. Mimicking the natural interest rate is optimal here because $i_t = \hat{r}_t$ stabilizes the employment gap on impact and eliminates price dispersion as fast as possible.[23] Equivalently, $i_t = \hat{r}_t$ implies that

[22] See Appendix A.3.

[23] Eliminating price dispersion as fast as possible is optimal because the utility losses from price dispersion are $\ln \omega_t$.

$g_t \equiv \ln l_t/l_{*,t} = 0$ on impact and that $\omega_t$ evolves accordingly[24]

$$\frac{d\omega_t}{\omega_t} = \min_{p_{*,t}/p_t} \left\{ \theta \left( \frac{p_{*,t}}{p_t} \right)^{-\varepsilon} \frac{1}{\omega_t} - \theta + \varepsilon \frac{\theta}{\varepsilon - 1} \left[ 1 - \left( \frac{p_{*,t}}{p_t} \right)^{-(\varepsilon-1)} \right] \right\} dt$$

Price dispersion shrinks as fast as possible because intermediate firms are willing to target a real price of $1/\omega_t$. The real price $p_{*,t}/p_t = 1/\omega_t$ minimizes the growth rate of price dispersion because at such price final firms balance their quantity demanded between goods with new and with fixed prices optimally: Notice that at such price the depreciation of the aggregate price level (equivalently, the depreciation of the marginal production costs of the final good) exclusively reflects the productivity gains from reducing the productivity wedge

$$\frac{p_{*,t}}{p_t} = \frac{1}{\omega_t} \Longleftrightarrow \frac{dp_t}{p_t} = \frac{d\omega_t}{\omega_t} = \frac{\theta}{\varepsilon - 1} \left( 1 - \omega_t^{\varepsilon-1} \right) dt \leq 0$$

The real price $p_{*,t}/p_t = 1/\omega_t$ is indeed consistent with the individual behavior of intermediate firms because at such price intermediate firms break-even gross of sales subsidies[25] (provided that $g_t = 0$ always)

$$\frac{p_{*,t} y_{d,s}\left(p_{*,t}\right)}{p_s} - x_s\left[y_{d,s}\left(p_{*,t}\right)\right] = \underbrace{\left[ \frac{1}{\omega_t} \frac{p_t}{p_s} - \frac{1}{\omega_s} \right]}_{markup=0} * y_{d,s}\left(p_{*,t}\right) = 0 \quad \forall s \geq t$$

Future sales markups are always null since fixed nominal prices appreciate in real terms at the same rate as average production costs.

[24] The law of motion of $\omega_t$ follows from differentiating the integral at the RHS of $\omega_t$ with respect to $t$. A corollary from Proposition 1 is that inflation is positively related with the optimal real price that intermediate firms target

$$\frac{dp_t}{p_t} = \frac{\theta}{\varepsilon - 1} \left[ 1 - \left( \frac{p_{*,t}}{p_t} \right)^{-(\varepsilon-1)} \right] dt$$

[25] The marginal production cost of the intermediate goods is $x_t\left(y_j\right)/y_j = h_t/\omega_t$ (see the Appendix A.2).

The employment gap remains stable at zero because households are willing to supply the efficient level of labor hours $l_{*,t}$ provided that $c_t \propto \hat{y}_t/\omega_t$ (see the asset pricing condition on deposits in the problem of households). Consumption effectively equals $\hat{y}_t/\omega_t$ because risk-adjusted excess returns, and whence portfolio decisions, remain the same as in the flexible economy (price dispersion is locally deterministic, and, as a consequence, the real interest rate and the return on capital co-move in tandem with respect to price dispersion).

It is worth noticing that if policies do not cooperate a weak version of the divine coincidence result holds (Blanchard and Gali 2007). As in the divine coincidence result, monetary policy faces no trade-off between employment-gap stabilization and inflation stabilization. As opposed to the divine coincidence result, inflation stabilization on impact (i.e. $\pi_t = 0$ on impact) is not always optimal. In the model economy, $\pi_t = 0$ is optimal only when there is no price dispersion. When there is price dispersion, $\pi_t = \frac{\theta}{\varepsilon - 1}\left(1 - \omega_t^{\varepsilon - 1}\right) < 0$ is optimal. The differences on results follows from the fact that we work with global solution methods whereas Blanchard and Gali (2007) work with log-linearized solution methods around the steady state.

### Macro-prudential Policy

**Proposition 5** *If policies do not cooperate, macro-prudential policy replicates the regulation of the flexible price economy.*

**Proof.** The regulation of the flexible price economy is feasible here because mimicking the natural interest rate does not restrict the choice set of macro-prudential policy relative to the flexible price economy. It is optimal because portfolio problems, and whence the costs of financial disintermediation, remain the same as in the flexible price economy.[26] ■

[26]Intuitively, through the lens of game theory, if policies do not cooperate, monetary policy and macro-prudential policy play a non-cooperative static game. The dominant strategy of monetary

**Flexible Price Economy** To understand the behavior of macro-prudential policy in the flexible price economy, it is convenient to first examine a benchmark economy in which there is no macro-prudential policy. The equilibrium outcome in the latter economy critically depends on the binding status of the leverage constraint $\phi_t \leq \lambda v_t$ (Figure 1.1). Additionally, the dynamics of the equilibrium outcome critically depend on the evolution of the wealth share of financial intermediaries (Figure 1.2).

**Equilibrium Outcome** In equilibrium, leverage constraints bind only when financial intermediaries lack enough borrowing capacity to absorb the aggregate capital stock (i.e. when $\lambda v_t \eta_t < 1$). Equivalently, leverage constraints bind only when financial intermediaries are poor-capitalized (i.e. when $\eta_t < \bar{\eta}$ with $\bar{\eta} \in (0, 1)$, Figure 1.1a).

**Financially Constrained Region.** When leverage constraints bind, households are the marginal investors on capital. The allocation of physical capital is inefficient as households hold the share of the aggregate capital stock that financial intermediaries cannot absorb (Figure 1.1a). The price of capital is temporary low despite the fact that dividend returns are temporary high (Figure 1.1c). Dividend returns are temporary high because the aggregate supply of capital services is temporary low. The price of capital is temporary low, though, because the negative valuation effect of households more than offsets the positive effect of temporary high dividend returns. The financial accelerator exists (Figure 1.1d) because binding leverage constraints, together with the liquidity mismatch problem, force financial intermediaries to partially liquidate their capital positions to households when a negative productivity shock hits the economy: Binding leverage constraints force financial intermediaries to scale down their capital positions when the latter suffer net worth losses. The liquidity mismatch problem gives rise to credit spreads $d\tilde{R}_t - r_t dt$ that positively

policy is to mimic the natural interest rate. The best response of macro-prudential policy to mimic the natural interest rate is to replicate the regulation of the flexible price economy. Mimicking the natural interest rate is always feasible for monetary policy as we abstract from an occasionally binding zero-lower-bound constraint on the nominal interest rate away.

co-move with productivity shocks. The pecuniary externality in financial markets amplifies the size of the financial accelerator: Following a negative (positive) productivity shock, joint fire-sales (purchases) of financial intermediaries generate further capital losses (gains), that lead to deeper net worth losses (gains), that in turn lead to more joint fire-sales (purchases), and so on ad infinitum. The liquidity mismatch problem, binding leverage constraints and the pecuniary externality together amplify fundamental risk $\sigma_A$ accordingly

$$\begin{aligned}\sigma_{q,t}/\sigma_A &= \frac{1}{1-(\phi_t-1)\,\varepsilon_{q,t}}\\ \\ \sigma_{y,t}/\sigma_A &= 1+\frac{1}{1-(\phi_t-1)\,\varepsilon_{q,t}}\,(\phi_t-1)\,\varepsilon_{y,t}\end{aligned}$$

where $\varepsilon_{q,t}$ and $\varepsilon_{y,t}$ are the elasticity of the price of capital and of aggregate output, respectively, with respect to $\eta$.[27]

All in all, when leverage constraints bind, the equilibrium outcome experiences episodes with financial disintermediation (i.e. a misallocation problem of physical capital between financial intermediaries and households), and with high instability on the price of capital and on aggregate risk. Financial disintermediation worsens as financial intermediaries become more undercapitalized. We interpret episodes of deep financial disintermediation as a costly financial crisis.

**Financially Unconstrained Region.** When leverage constraints are slack, financial intermediaries are the marginal investors on capital. The allocation of phys-

[27] Amplifications risk factors follow from Ito's Lemma (see Brunnermeier and Sannikov 2014, 2016b). Intuitively, $\sigma_{q,t}/\sigma_A$ is given by the infinite geometric sum over $(\phi_t-1)\,\varepsilon_{q,t}$ because the pecuniary externality generates a feedback loop with countably infinite many rounds. The term $(\phi_t-1)\,\varepsilon_{q,t}$ is the percentage change on the price of capital in each round of (the feedback loop of) the pecuniary externality

$$\sigma_{q,t}=\sigma_A+(\phi_t-1)\,\varepsilon_{q,t}\sigma_{q,t}$$

The amplification risk factor $\sigma_{y,t}/\sigma_A$ weights $\sigma_{q,t}/\sigma_A$ by $(\phi_t-1)\,\varepsilon_{y,t}$ because the term $(\phi_t-1)\,\varepsilon_{y,t}$ is the percentage change on aggregate output in each round of the pecuniary externality per unit of financial net worth.

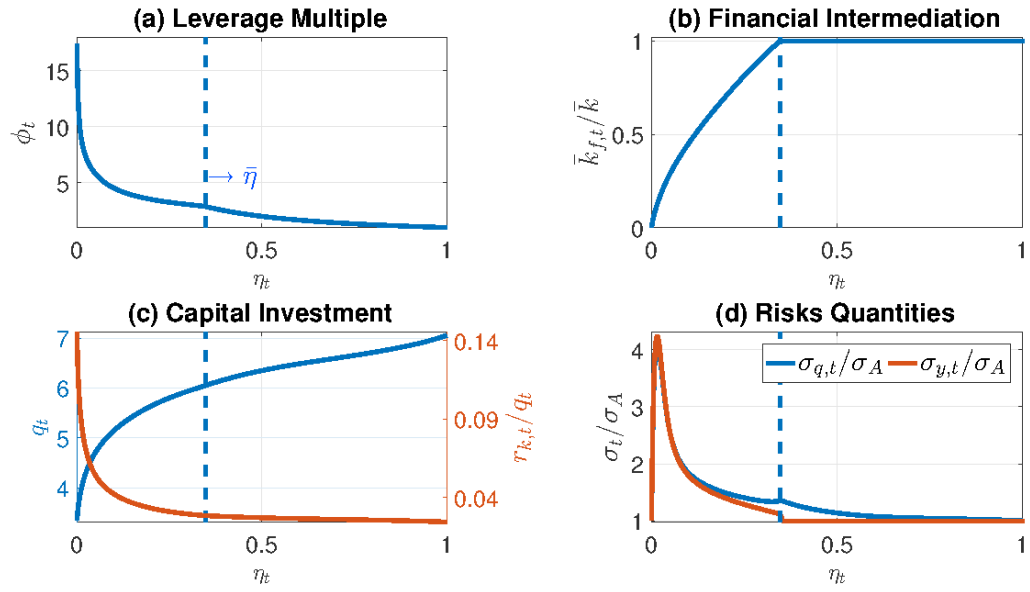



Figure 1.1: The Equilibrium of the Unregulated Economy

ical capital is efficient as financial intermediaries absorb the aggregate capital stock (Figure 1.1a). The price of capital is temporary high despite the fact that dividend returns are temporary low (the rationale is the same as when leverage constraints bind). The financial accelerator is latent (Figure 1.1d) because financial intermediaries use their idle borrowing capacity to maintain their initial capital positions (i.e. $\varepsilon_{y,t} = 0$ as $u_t = \phi_t \eta_t = 1$). The amplification risk factor $\sigma_{q,t}/\sigma_{A,t}$ is nonetheless positive as it reflects the likelihood of returning back to the region in which leverage constraints bind.

**The Dynamics of the Equilibrium Outcome** The wealth share of financial intermediaries is the state variable that governs the dynamics of the equilibrium outcome. The wealth share of financial intermediaries evolves accordingly[28]

$$\frac{d\eta_t}{\eta_t} = \left[\frac{r_{k,t}}{q_t} + \left[\frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) - \sigma_{q,t}^2\right](\phi_t - 1) + \frac{\kappa}{\eta_t} - \tilde{\rho}\right] dt + (\phi_t - 1)\,\sigma_{q,t} dZ_t$$

[28]The law of motion of $\eta_t$ follows from Ito quotient rule.

The first two terms in the drift process add up to the expected growth rate of the share of gross financial wealth. The first term is the dividend yield on internal leverage, i.e. capital positions funded with net worth. The second term is the risk-adjusted excess return on external leverage, i.e. capital positions funded with leverage.[29] Financial wealth here is gross of net transfers to households. The expected growth rate of the share of gross financial wealth is positive as financial intermediaries make non-negative profits in expected value.

The last two terms in the drift process account for the net transfer between financial intermediaries and households.[30] During an infinitesimal time interval $dt$, a share $\tilde{\rho}dt$ of financial intermediaries transfers their entire net worth back to the household in the form of dividend payouts. Immediately afterwards, each of these financial intermediaries receives a share $\kappa/\tilde{\rho}$ of the aggregate capital stock back from the household as initial endowment. Mechanically, we introduce transfers between financial intermediaries and households as otherwise leverage constraints would eventually remain slack forever after: Financial intermediaries would eventually accumulate enough net worth to the point where they always have enough borrowing capacity to absorb the aggregate capital stock.[31]

The diffusion process measures the aggregate risk that financial intermediaries concentrate in their balance sheets in excess of the aggregate risk in the economy.

The law of motion of $\eta_t$ reveals that the equilibrium outcome is prone to recover from financial crises (i.e. $E_t\left[d\eta_t/\eta_t\right] > 0$ when $\eta_t$ is sufficiently low) but, at the same time, that the equilibrium outcome is prone to instability in normal times (i.e. $E_t\left[d\eta_t/\eta_t\right] < 0$ when $\eta_t$ is sufficiently high) (Figure 1.2a). The law of motion of

[29] Leverage risk is formally defined as $Cov_t\left[d\tilde{R}_t, -\left(\phi_t - 1\right)d\tilde{R}_t\right]$.

[30] The net dividend payments that financial intermediaries on aggregate make to households are $Div_t = \tilde{\rho}n_{f,t} - \kappa q_t \bar{k}$.

[31] Financial intermediaries receive transfers back from households as otherwise they would eventually be left out of the economy. Financial intermediaries who have no net worth can take no positions on physical capital nor on deposits as $q_t\bar{k}_{f,t} \leq \min\left\{\lambda v_t, \Phi_t\right\} n_{f,t} = 0$.

$\eta_t$ reveals also that the equilibrium outcome fluctuates stochastically, and that the strength with which productivity shocks hit the economy fluctuates stochastically as well (Figure 1.2b). Specifically, concerning the latter result, the diffusion process $\sigma_{\eta,t}\eta_t$ reveals that productivity shocks have a stronger impact on the equilibrium outcome when $\eta_t$ is close to the threshold state $\bar{\eta}$. The rationale is that, around a neighborhood of $\bar{\eta}$, financial intermediaries are large enough to have significant effects on the aggregate level, but not so large to always have enough borrowing capacity to absorb the aggregate capital stock.

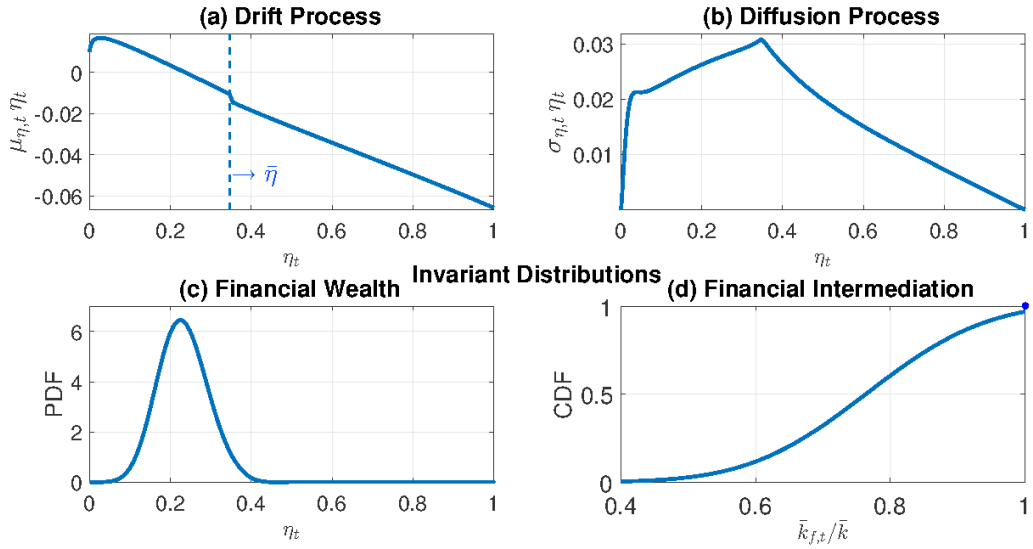



Figure 1.2: The Equilibrium Dynamics of the Unregulated Economy

**Macro-prudential Policy** Macro-prudential policy can affect the equilibrium outcome, as well as the dynamics of the equilibrium outcome, since it can restrict leverage below the natural upper bound $\min\{\lambda v_t, 1/\eta_t\}$. To simplify the technical analysis concerning the optimization problem of macro-prudential policy, we make the following two assumptions.

**Assumption 1**. First, we assume that macro-prudential policy minimizes the *unconditional* present discounted value of the distortions from leverage constraints

$$\max_{\Phi_t \geq 1} E_\eta \left[ (1-\alpha) E_t \int_t^\infty e^{-\rho(s-t)} \ln u_s ds \right]$$

The conditional expectation operator $E_t$ is interpreted here as the expectation operator conditional on the value of $\eta$ at time $t$. This interpretation is done without loss of generality as in the flexible price economy $\eta$ is the only relevant state variable: $\omega = 1$ and the equilibrium outcome is scale invariant with respect to $A$. The expectation operator $E_\eta$ is defined relative to the invariant probability distribution of $\eta$.

Intuitively, *unconditional* present discounted values represent "commitment" meaning that state-contingent policy schemes are designed just before the economy unravels and remain unchanged forever after.

**Assumption 2**. Second, we restrict macro-prudential policy to behave accordingly

$$\phi_t = \begin{cases} \lambda v_t & \text{if } \eta_t \leq \eta_L \\ a_1 * \eta_t + a_0 & \text{if } \eta_t \in (\eta_L, \eta_H) \\ 1/\eta_t & \text{if } \eta_t \geq \eta_H \end{cases}$$

where $a_1, a_0$ are constants. Intuitively, macro-prudential policy, if operative, restricts leverage below the natural upper bound only when financial intermediaries are average-capitalized (i.e. when $\eta_t \in (\eta_L, \eta_H)$). Macro-prudential policy restricts leverage continuously according to a linear function of the state $\eta$.[32]

Together the two simplifying assumptions reduce the problem of macro-prudential policy to a static problem with two choice variables $\{\eta_L, \eta_H\}$. We solve for the optimal $\{\eta_L, \eta_H\}$ numerically (see Appendix A.3).

[32]The constants $a_1, a_0$ are such that $(i)$ $\lambda v(\eta_L) = a_1 * \eta_L + a_0$; $(ii)$ $1/\eta_H = a_1 * \eta_H + a_0$; and $(iii)$ $\Phi_t = a_1 * \eta_t + a_0 < \min\{\lambda v_t, 1/\eta_t\}$ for $\eta_t \in (\eta_L, \eta_H)$.

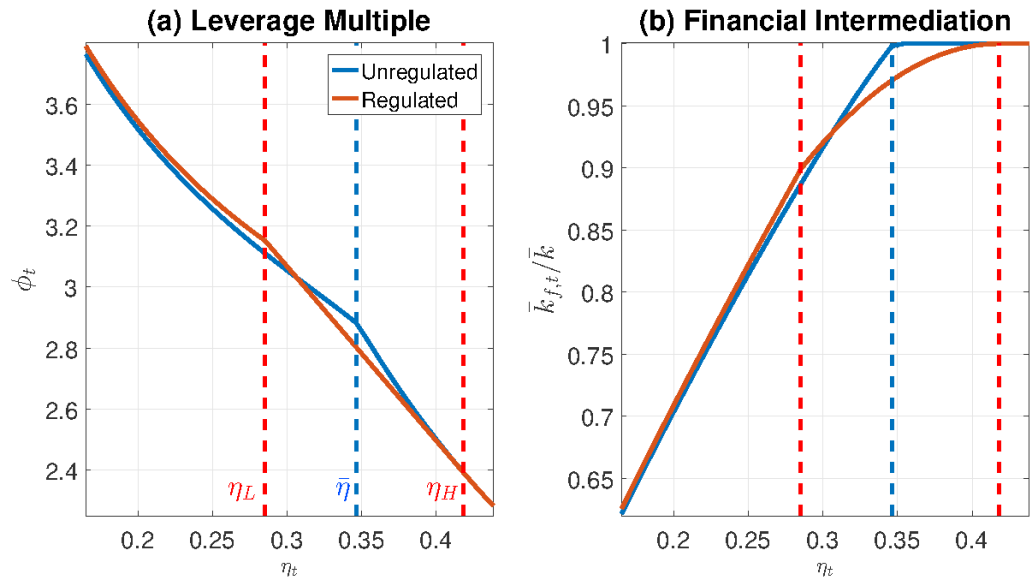



Figure 1.3: The Prudential Role of Macro-prudential Policy

**The Prudential Role of Macro-prudential Policy** In the flexible price economy, macro-prudential policy restricts leverage below the natural upper bound when financial intermediaries are locally soft constrained (namely, when $\eta_t$ is close to the threshold $\bar{\eta}$ of the economy without macro-prudential policy, Figure 1.3a).

Broadly speaking, the objective of macro-prudential policy is to stabilize the quantity of financial intermediation (i.e. the share of the aggregate capital stock that financial intermediaries manage) around the highest possible value. The reason is that utility flows are strictly increasing, and strictly concave, on the aggregate quantity supplied of capital services.

Given its objective, macro-prudential policy is effective depending on the capitalization of financial intermediaries. When financial intermediaries are poor-capitalized, macro-prudential policy is ineffective because restricting leverage below the natural upper bound reduces even more the already relatively small quantity of financial intermediation. Letting financial intermediaries hit their leverage constraint is therefore better.

When financial intermediaries are average-capitalized, macro-prudential policy becomes effective. Only mild restrictions on leverage (such as those in Figure 1.3a) are relevant because macro-prudential policy is locally distortionary: Restricting leverage below the natural upper bound reduces the quantity of financial intermediation below its potential capacity locally (Figure 1.3b). Mild restrictions on leverage are beneficial because they increase risk-adjusted excess returns everywhere (Figures 1.4c, 1.4d), and because they mitigate the amplification effect of the pecuniary externality primarily when financial intermediaries are average-capitalized (Figure 1.4b).

Risk-adjusted excess returns increase mainly because dividend yields $r_{k,t}/q_t$ increase (Figure 1.3a). Dividend returns $r_{k,t}$ increase as reductions in leverage depress the aggregate supply of capital services. The price of capital $q_t$ falls because households become the marginal investors on capital and/or take larger positions on capital when financial intermediaries are forced to deleverage.[33] The pecuniary externality softens primarily when financial intermediaries are average-capitalized because leverage falls in such region (the elasticity $(\phi_t - 1)\,\varepsilon_{q,t}$ therefore falls).

Higher risk-adjusted excess returns are beneficial because they increase the profitability of financial intermediation almost everywhere (Figure 1.5a), and because they boost leverage when financial intermediaries are poor-capitalized (Figure 1.3a). Leverage increases when financial intermediaries are locally hard constrained (i.e. when they hit their leverage constraint) since higher risk-adjusted excess returns, specifically, on equity, relax the moral hazard problem of financial intermediaries (moral hazard problems soften globally as financial intermediation is forward-looking). A softer pecuniary externality is beneficial because it mitigates the excessive fluctuations on the wealth share of financial intermediaries, and on the quantity of financial intermediation, (absent macro-prudential policy, the pecuniary externality is exces-

[33]The price of capital falls despite the increase in $r_{k,t}$ because the negative valuation effect of households more than offsets the positive effect of higher dividend returns. The price of capital falls globally because physical capital is a long-term asset: Its price is forward-looking.

sively powerful because financial intermediaries concentrate an excessive amount of aggregate risk in their balance sheets).

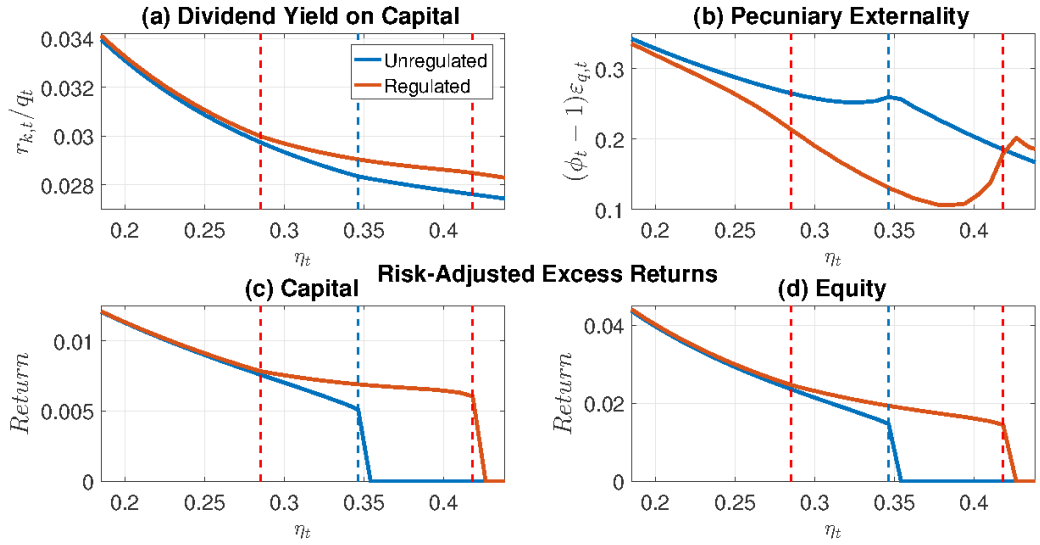



Figure 1.4: A Comparison of the Equilibrium between the Regulated and the Unregulated Economies

Macro-prudential policy ultimately boosts the average quantity of financial intermediation (Figure 1.5d) as well as reduces the frequency and the intensity of episodes with deep financial disintermediation (Figures 1.5c, 1.5d). Macro-prudential policy operates both state-by-state and through the invariant distribution. The quantity of financial intermediation becomes less pro-cyclical (i.e. positively smoother as a function of $\eta$) (Figure 1.3a) because leverage becomes more counter-cyclical (i.e. negatively steeper as function of $\eta$) (Figure 1.3b). The invariant distribution of $\eta$ shifts rightward (Figure 1.5c) since financial intermediation becomes more profitable (Figure 1.5a), and since amplification risk factors soften primarily when financial intermediaries are average- to rich-capitalized (Figure 1.5b). These effects together shift the average wealth share of financial intermediaries rightward, and help to stabilize the wealth share of financial intermediaries around relatively higher values.

All in all, macro-prudential policy serves a prudential role consistent with Jeanne and Korinek (2010); Bianchi (2011); and Hanson, Kashyap and Stein (2011). Ex-post,

macro-prudential policy speeds up the recovery from the crisis. Ex-ante, it reduces the likelihood and the intensity of financial crises. The downside of macro-prudential policy is that it depresses the aggregate supply of capital services when financial intermediaries are average-capitalized.

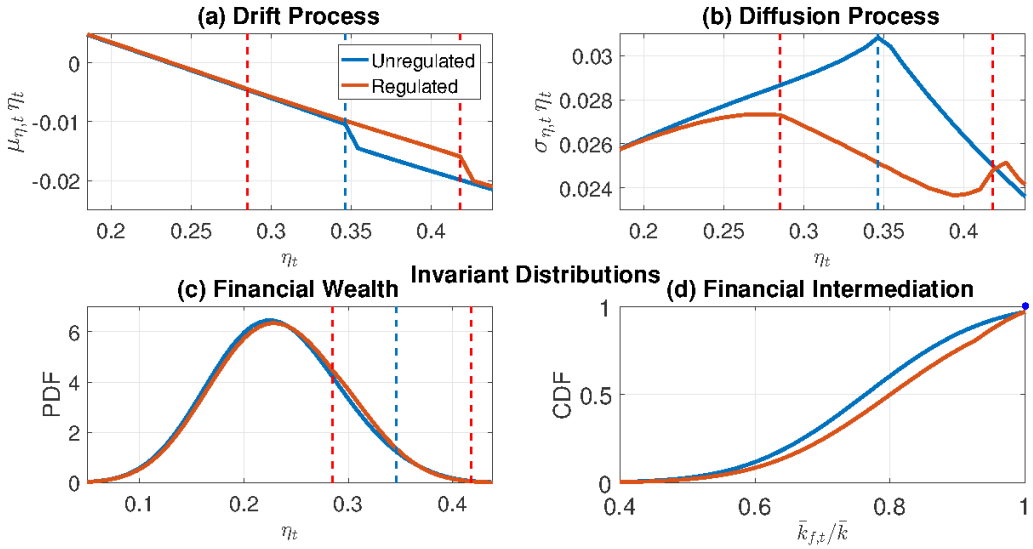



Figure 1.5: A Comparison of the Equilibrium Dynamics between the Regulated and the Unregulated Economies

### 1.3.2 Coordinated Policy

To simplify the technical analysis regarding the choice of the coordinated policy, as well as to conduct a relevant comparison between the coordinated and the uncoordinated policies, we make the following three assumptions.

**Assumption 1**. First, we assume that policy maximizes *unconditional* present discounted values

$$\max_{i_t,\Phi_t} E_{\eta,\omega}\left[E_t\int_t^\infty e^{-\rho(s-t)}\left[\ln\frac{1}{\omega_s}+\alpha\ln l_s-\chi\frac{l_s^{1+\psi}}{1+\psi}+(1-\alpha)\ln u_s\right]ds\right]$$

The conditional expectation operator $E_t$ is interpreted here as the expectation operator conditional on the value of $\eta$ and of $\omega$ at time $t$. The expectation operator

$E_{\eta,\omega}$ is defined relative to the joint invariant probability distribution of $\eta$ and $\omega$. As in the uncoordinated policy case, *unconditional* present discounted values intuitively represent "commitment".

**Assumption 2**. Second, we assume the same restrictions on macro-prudential policy as in the analysis of the uncoordinated policy.

**Assumption 3**. Third, we assume that monetary policy can implement up to two employment-gap regimes $g_L, g_H$ accordingly

$$g_t = \begin{cases} g_L & \text{if } \eta_t \leq \tilde{\eta}_L \\ \frac{g_H - g_L}{\tilde{\eta}_H - \tilde{\eta}_L}(\eta_t - \tilde{\eta}_L) + g_L & \text{if } \eta_t \in (\tilde{\eta}_L, \tilde{\eta}_H) \\ g_H & \text{if } \eta_t \geq \tilde{\eta}_H \end{cases}$$

The third assumption restricts the set of admissible monetary policies, and, as a consequence, the welfare gains from coordinating monetary and macro-prudential policies (given assumptions 1 and 2).

Intuitively, monetary policy can only implement "simple" employment-gap policies contingent upon the wealth share of financial intermediaries. Simple here means that only two employment-gap regimes are admissible, and that the transitions between employment-gap regimes have to be continuous and linear on the state $\eta$.[34]

Treating the employment gap $g_t$, instead of the policy rate $i_t$, as the control variable of monetary policy imposes no restriction per se. The reason is that the asset pricing condition on deposits establishes a correspondence between $g_t$ and $i_t$. For instance, employment-gap stabilization (i.e. $g_t = 0$ always) is consistent with mimicking the natural interest rate (see the asset pricing condition on deposits under the uncoordinated policy).

[34]The technical advantage of the third assumption is twofold. Firstly, if the employment gap only depends on $\eta_t$, then $\eta_t$ is the only aggregate state variable in the portfolio problems of households and of financial intermediaries (see Appendix A.3). Secondly, as a consequence, leverage, the Tobin's Q and risk-adjusted excess returns solve an ODE system on $\eta$. Price-setting decisions, the inflation rate and the policy rate solve a tractable PDE system on $\eta$ and $\omega$. See the Appendix A.3 for more details.

Overall, the three simplifying assumptions together reduce the optimization problem of the coordinated policy to a static problem with two types of choice variables: $(i)$ the transition states $\tilde{\eta}_L, \tilde{\eta}_H$ and the employment-gap regimes $g_L, g_H$ corresponding to monetary policy; and $(ii)$ the transition states $\eta_L, \eta_H$ corresponding to macro-prudential policy. We solve for the coordinated policy numerically (see Appendix A.3 ).

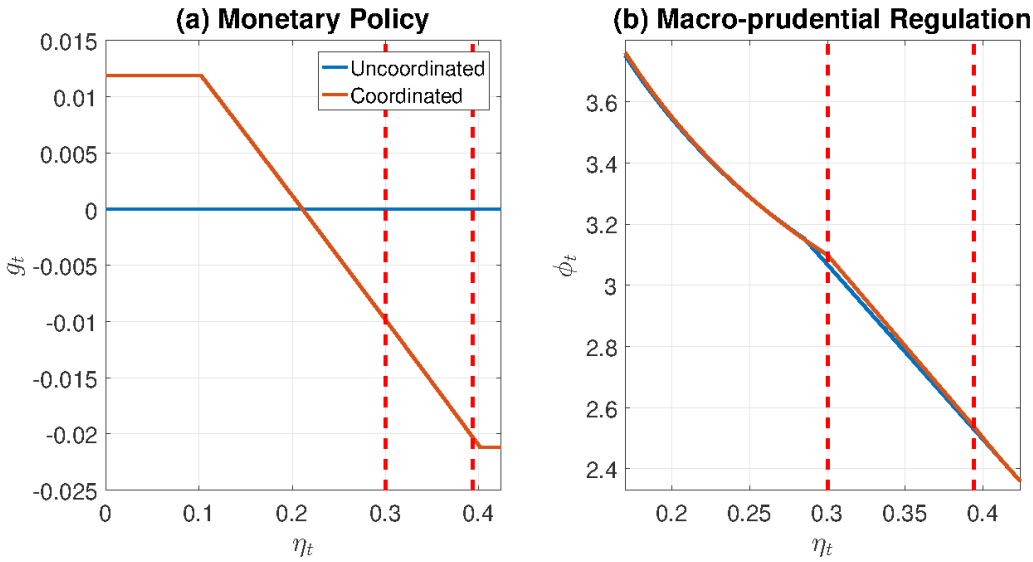



Figure 1.6: A Comparison between the Coordinated and the Uncoordinated Policies

### Monetary Policy

When policies cooperate, monetary policy departs from employment-gap stabilization (equivalently, monetary policy deviates from the natural interest rate) to assist macro-prudential policy in reducing the cost of financial disintermediation (Figure 1.6a). Monetary policy takes into account the side-effects that variability in employment gap brings on price dispersion and on labor misallocation.

**Trade-off** Broadly speaking, monetary policy faces a trade-off between assisting macro-prudential policy and stabilizing employment gap/inflation. Monetary policy can help macro-prudential policy to stabilize the quantity of financial inter-

mediation, since employment-gap policies affect the behavior and the profitability of financial intermediaries. Assisting macro-prudential policy, however, involves costs on price dispersion and on labor misallocation, as it requires implementing counter-cyclical employment-gap policies (i.e. $g_L > g_H$).

**Benefits on Financial Intermediation.** The benefits on financial intermediation follow from the mechanism through which employment-gap policies affect portfolio decisions. Relative to employment-gap stabilization, only state-contingent policies (i.e. $g_L \neq g_H$) affect portfolio decisions. Non-contingent policies (i.e. $g_L = g_H$) do not affect portfolio decisions because they do not affect risk-adjusted excess returns. Intuitively, risk-adjusted excess returns remain the same because the price of capital is a present discounted value of dividend returns. (Non-contingent employment-gap policies do affect dividend returns, but only locally deterministically.[35] As a consequence, the price of capital naturally adjusts to wipe out the effect of dividend returns on risk-adjusted excess returns.) State-contingent employment-gap policies do affect portfolio decisions because they do affect risk-adjusted excess returns. Specifically, temporary high, low, employment gaps positively, negatively, affect risk-adjusted excess returns, because the price of capital responds less than one-to-one to temporary dividend returns.

State-contingent employment-gap policies are beneficial depending on whether they are counter-cyclical (i.e. $g_L > g_H$) or pro-cyclical (i.e. $g_L < g_H$). State-contingent employment-gap policies in general temporarily favor, hinder, financial intermediation depending on whether they temporarily stimulate, slow down, employment. Counter-cyclical policies are beneficial because they temporarily favor,

[35]The dividend return is (see Appendix A.3)

$$r_{k,t} = \frac{1-\alpha}{\alpha}\frac{e^{g_t} l_{*,t}}{u_t \bar{k}} w_t \quad with \quad w_t = y_t \chi e^{\psi g_t} l_{*,t}^{\psi}$$

Employment gaps positively affect dividend returns because they positively affects the wage rate and the ratio of labor to capital services.

In this context, locally deterministically can be safely interpreted as permanently.

hinder, financial intermediation when financial intermediaries need it the most, the least. Namely, when financial intermediaries are poor-capitalized, rich-capitalized. Pro-cyclical policies are detrimental for the opposite reason.[36]

The benefits from counter-cyclical employment-gap policies come from generating more counter-cyclical risk-adjusted excess returns (i.e. negatively steeper as function of $\eta$) (Figure 1.7c, Figure 1.7d). More counter-cyclical risk-adjusted excess returns, in turn, stimulate the profitability of financial intermediation counter-cyclically (Figure 1.8a), and the borrowing capacity counter-cyclically as well (Figure 1.6b). Together these effects help to stabilize the quantity of financial intermediation around relatively high values (Figure 1.8d). The former effect boosts the average wealth share of financial intermediaries (Figures 1.8a, 1.8c); the latter reduces the frequency and the intensity of episodes with deep financial disintermediation (Figures 1.6b, 1.8c, 1.8d). All in all, monetary policy ex-post helps macro-prudential policy to reduce the costs of financial crises.

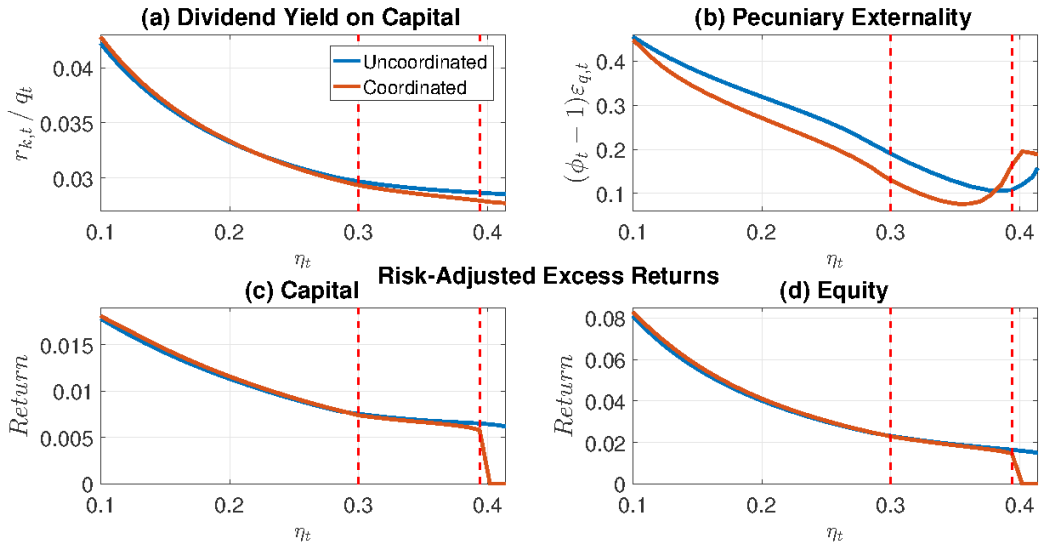



Figure 1.7: A Comparison of the Equilibrium between the Coordinated and the Uncoordinated Economies

[36]Pro-cyclical and counter-cyclical employment-gap policies have similar effects but with opposite directions. We omit the analysis of pro-cyclical policies as they are irrelevant.

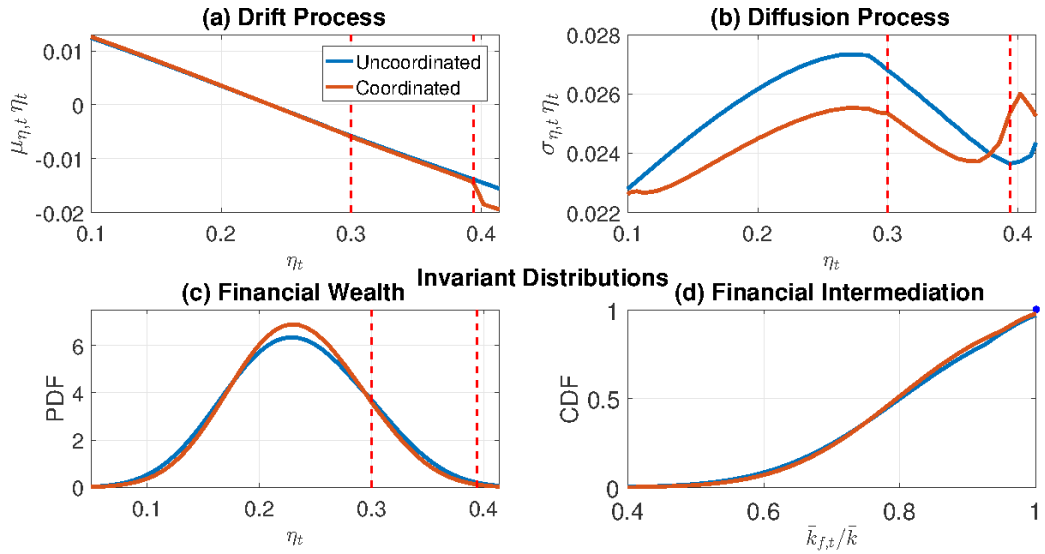



Figure 1.8: A Comparison of the Equilibrium Dynamics between the Coordinated and the Uncoordinated Economies

**Costs on Price Dispersion and on Labor Misallocation.** Assisting macro-prudential policy nonetheless is costly as it requires variability in employment gap (i.e. $g_t \neq 0$). The costs that employment-gap variability brings on price dispersion and on labor misallocation are strictly increasing on $|g_t|$. Additionally, the costs on price dispersion are asymmetric at $g_t = 0$ : specifically, given $|g_t|$, costs are higher for $g_t > 0$ than for $g_t < 0$.

The costs on labor misallocation are strictly increasing on $|g_t|$ because labor deviates more from its efficient level when there is more variability in employment gap.

The costs on price dispersion follow from the interactions between the labor wedge, the marginal production costs and the price-setting behavior of firms.

The employment gap positively affects marginal production costs because it positively affects the labor wedge

$$x_t\left(y_j\right)/y_j = h_t/\omega_t \quad \text{with} \quad h_t = e^{(1+\psi)g_t}$$

Future marginal production costs (similarly, future labor wedges), in turn, positively affect the real prices that firms target. When firms expect the labor wedge to remain stable at one, firms price at marginal cost (i.e. $p_{*,t}/p_t = 1/\omega_t$) because they expect fixed nominal prices to appreciate in real terms at the same rate as future marginal production costs (see subsection 1.3.1). When firms expect the labor wedge to remain above (below) one, firms price above (below) marginal cost because they expect future real prices to eventually fall below (surpass) future marginal production costs.[37]

Future marginal production costs, furthermore, asymmetrically affect the real prices that firms target. The asymmetric behavior of real prices follows from the asymmetric behavior of future profit flows. For any given process $|g_s|$ with $s \geq t$, future profit flows are unbounded from below in high-costs environments (i.e. $g_s > 0$) while they are bounded from below by zero in low-costs environments (i.e. $g_s < 0$). In high-costs environments, future losses are unbounded because fixed nominal prices eventually vanish in real terms. In low-costs environments, future profit flows are bounded from below by zero because fixed nominal prices eventually explode in real terms (future sales quantities, therefore, eventually vanish). Firms target real prices from further away from marginal production costs in high-costs environments than in low-costs environments because $(i)$ the worst possible scenario in terms of future profit flows is worse in the former environment; and $(ii)$ real prices further away from marginal production costs reduce the likelihood and the intensity of the worst possible scenario.

The real prices that firms target, indeed, is the key variable that dictates the evolution of price dispersion (see the law of motion of $\omega_t$ in subsection 1.3.1). From the interaction between the labor wedge, the marginal production costs and the price-setting behavior of firms, follows that $(i)$ there is price dispersion iff the employment

[37]The expectations that firms have concerning the evolution of future real prices follows from the expectations that they have concerning the behavior of future inflation rates (nominal prices remain fixed during the time span in which firms cannot adjust their price). The latter expectations are indeed consistent with the price-setting behavior of firms.

gap systematically deviates from zero (i.e. iff $g_t \neq 0$); and that $(ii)$ price dispersion is more pronounced in high-costs environments than in low-costs environments (price dispersion grows faster when firms target real prices further away from $1/\omega_t$).

**Solution to the Trade-off** The benefits on financial intermediation outweigh the costs on price dispersion and on labor misallocation iff, in the flexible price economy without macro-prudential policy, leverage constraints occasionally bind (equivalently, iff there is a role for macro-prudential policy in the flexible price economy). The reason is that, if policies do not cooperate, monetary policy can eliminate the distortions from nominal rigidities whereas macro-prudential policy cannot eliminate those from financial frictions. Generating a small amount of price dispersion and of labor misallocation therefore entails a second-order loss relative the first-order gain that follows from reducing cost of financial disintermediation.

Provided that monetary policy assists macro-prudential policy, monetary policy departs asymmetrically from employment-gap stabilization (Figure 1.6a). Specifically, monetary policy tilts $g_t$ towards negative values. The asymmetric behavior of monetary policy follows from the asymmetric behavior of price dispersion at $g_t = 0$. Monetary policy still uses $g_t = 0$ as a reference because the costs on price dispersion and on labor misallocation are null at such point, and because firms target real prices closer to one when future labor wedges fluctuate closer to one.

It is worth noticing that if policies cooperate monetary policy behaves in accord with the Greenspan put and with Leaning against the wind.[38] Surprisingly, leaning against the wind (i.e. $g_H < 0$) in itself is not particularly effective at stabilizing the quantity of financial intermediation. The reason is that the levels of $g_L$ and $g_H$ in themselves have only a small effect on portfolio decisions. The spread between $g_L$ and

[38]The Greenspan put cuts the policy rate below the natural interest rate during periods of financial distress. Similarly, it stimulates employment, and economic activity, when financial intermediaries are undercapitalized. Leaning against the wind hikes the policy rate above the natural interest rate during periods of financial booms. Similarly, it slows down employment, and economic activity, when financial intermediaries are well-capitalized.

$g_H$, and their timing, is what really matters instead.[39] Leaning against the wind is therefore primarily useful for mitigating the side-effects on price dispersion.

**Macro-prudential Policy**

Macro-prudential policy behaves similarly independently of whether policies cooperate or not (Figure 1.6b). Specifically, macro-prudential policy always restricts leverage below the natural upper bound when financial intermediaries are locally soft constrained. When policies cooperate, however, macro-prudential policy restricts leverage less severely. The reasons are that macro-prudential policy is locally distortionary, and that monetary policy is already helping to reduce the costs of financial disintermediation.

## 1.4 Quantitative Analysis

We calibrate the model economy to quantitatively assess the costs and benefits from coordinating monetary policy and macro-prudential policy. The figures showed in the previous sections use this same calibration.

### 1.4.1 Calibration

The benchmark economy used for the calibration is the flexible price economy without macro-prudential policy (i.e. $1/\theta,\ 1/\Phi_t \to 0$).

Table 1 describes the parameter values and their corresponding targets in data. The time frequency is annual.

**Panel A: Financial Intermediation Parameters**

[39] Employment-gap regimes $g_L$ and $g_H$ are timed so that the employment-gap policy is countercyclical at the invariant distribution.

Table 1.1: Parameter Values and Targets

| | Parameter | Value | Target |
|---|---|---|---|
| Panel A: Financial Intermediation Parameters | | | |
| $u_h$ | Households' capital utilization rate | 62% | Average Sharpe ratio |
| $\lambda$ | Fraction of divertable assets | 2 | Average leverage multiple |
| $\tilde{\rho}$ | Arrival rate of retirement shock | 10% | Average survival frequency |
| $\kappa$ | Initial capital endowment | 1% | Average wealth/capital ratio |
| Panel B: Technology Parameters | | | |
| $\mu_A$ | Rate of technological progress | 1.5% | Av. utilization-adj. TFP |
| $\sigma_A$ | Fundamental risk | 3.5% | Vol. of utilization-adj. TFP |
| $\alpha$ | Labor share of output | 65% | Literature |
| $\bar{k}$ | Stock of physical capital | 1 | Normalization |
| Panel C: Price-setting Parameters | | | |
| $\theta$ | Arrival rate of price adj. shock | $\frac{3}{2}\ln 2$ | Median frequency of price adj. |
| $\varepsilon$ | Elasticity of substitution | 5 | Literature |
| Panel D: Preference Parameters | | | |
| $\rho$ | Time discount rate | 2% | Literature |
| $\psi$ | Inverse Frisch elasticity of labor | 3 | Literature |
| $\chi$ | Relative utility weight of labor | 2.8 | Average labor hours |
| $\gamma$ | Risk aversion coefficient | 1 | Literature |

The average capital utilization rate of households is 62%.[40] This value targets an average Sharpe ratio of 30% which is standard. The fraction of divertable assets is 2 and targets an average leverage multiple of 3.5.

We interpret the life cycle of financiers as the life cycle of financial intermediary companies. The arrival rate of the retirement shock of financiers targets an average survival frequency of 10 years. This is consistent with Gertler and Kiyotaki (2010).

[40]We assume a linear relationship between $u_{h,t}$ and $\bar{k}_{h,t}/\bar{k}$ to improve the stability of the numerical methods. Specifically,

$$u_{h,t} = m * \bar{k}_{h,t}/\bar{k} + b$$

with $m < 0$ and $-m < b < 1$. This parameterization is consistent with a quadratic-adjustment-cost specification for the utilization rate of physical capital on $\bar{k}_{h,t}/\bar{k}$. We choose $m$ and $b$ to target an average Sharpe ratio of 30%. The (conditional) Sharpe ratio is defined as the ratio of the excess return on capital for financial intermediaries to the volatility of capital

$$Sharpe_t \equiv \frac{r_{k,t}/q_t + \mu_{q,t} - (i_t - \pi_t)}{\sigma_{q,t}}$$

The value of $m$ is $-.55$ and that of $b$ is .75.

The initial capital endowment $\kappa$ targets the average wealth-to-capital ratio in the financial intermediary sector. Using Fed fund data, we estimate an average wealth-to-capital ratio of 20%. This estimate is consistent with Hirakata, Sudo and Ueda (2013). The value of $\kappa$ is 1%.

**Panel B: Technology Parameters**

We parameterize the process of productivity with a Geometric Brownian Motion (GBM) process

$$dA_t/A_t = \mu_A dt + \sigma_A dZ_t$$

The constants $\mu_A$ and $\sigma_A$ match the unconditional mean and the unconditional standard deviation, respectively, of the Utilization-Adjusted Series on Total Factor Productivity (see Basu, Fernald and Kimball 2006; Basu, Fernald, Fisher and Kimball 2006; and Fernald 2014). The value of $\mu_A$ is 1.5% and that of $\sigma_A$ is 3.5%.

The labor share of output is 65%. This value is consistent with the empirical findings in Karabarbounis and Nieman (2014). The aggregate stock of physical capital is 1 which is a normalization.

The elasticity of substitution between intermediate goods is 5. This is consistent with estimates of product demand elasticities in the industrial organization and international trade literature (Berry, Levinsohn and Pakes 1995; Nevo 2001; and Broda and Weinstein 2006).

**Panel C: Price-setting Parameters**

The arrival rate of the price adjustment shock (i.e. the shock that allows firms to adjust their price) is $\frac{3}{2}\ln 2 \simeq 1.04$. This value yields a median frequency of price adjustment of 8 months. Nakamura and Steinsson (2008) finds that the median duration of regular prices ranges from 8 to 11 months (and from 7 to 9 months after including product substitution).

**Panel D: Preference Parameters**

The time discount rate is 2% which yields an annual discount factor of $(1 - e^{-\rho})/\rho \simeq .99$. The Frisch elasticity of labor supply is .5 consistently with the empirical findings in Chetty, Guren, Manoli and Weber (2011). The relative utility weight of labor matches an average share of labor hours of 1/3 per unit of time.

### 1.4.2 Welfare Gains

In the baseline calibration, the annual consumption equivalent gain from coordinating monetary policy and macro-prudential policy is 0.21%. The annual consumption equivalent gain from reducing financial disintermediation is 0.29% while the annual consumption equivalent losses from generating price dispersion and labor misallocation are 0.07% and 0.01%, respectively.

The frequency of financial disintermediation increases but the intensity falls. Financial disintermediation occurs $99.7\% > 99.2\%$ of the time. Episodes with aggregate TFP losses from financial disintermediation beyond 10% and 5% occur $1.54\% < 2.65\%$ and $17.04\% < 20.5\%$ of the time, respectively. The stochastic steady state is always inefficient. Financial disintermediation becomes positively correlated with inflation and price dispersion.

## 1.5 Conclusion

The Global Financial Crisis of 2008 has prompted the development of a relatively new policy instrument: Macro-prudential policy. In addition, the crisis has fostered the debate concerning the costs and benefits from coordinating macro-prudential policy with existing policy instruments such as monetary policy. The first main result in this paper is that, if monetary policy is only concerned with stabilizing inflation, macro-prudential policy should restrict intermediary leverage below its natural

level when financial intermediaries are soft constrained. The second main result is that monetary policy, instead of being exclusively concerned with stabilizing inflation, should coordinate with macro-prudential policy to further stabilize the financial system. Specifically, monetary policy should deviate from the natural interest rate (i.e. the interest rate required for stabilizing inflation), and macro-prudential policy should soften its restrictions on leverage relative to the case in which policies do not cooperate (i.e. the case considered in the first main result). Monetary should deviate both ex-ante and ex-post: during financial crises, monetary policy should target a policy rate below the natural interest rate; once the financial crisis is over, monetary policy should target a policy above the natural interest rate.

The main results in the paper are robust to a series of specifications adopted in the model economy. Firstly, the main results are robust to the source of fundamental shocks (i.e. supply shocks, demand shocks, risk shocks, and so on). Aggregate shocks nonetheless are necessary because a liquidity mismatch problem is required for leverage constraints to generate financial disintermediation. Secondly, the main results are robust to the micro-foundations regarding the price-setting behavior of firms. The latter nonetheless matters for quantitatively assessing the welfare gains from coordination. For instance, state-dependent price-setting behaviors generate larger welfare gains than their time-dependent counterparts, because in the former the distortions from nominal rigidities are less costly (in the former, firms that have a nominal price significantly far away from the optimal nominal price definitely choose to reset their price). Thirdly, and lastly, the main results are robust to the micro-foundations regarding the leverage constraint. The type of leverage constraint nonetheless affects the severity of the costs from financial disintermediation.

# Chapter 2

# Cross-effects of Macro-prudential Policy

## 2.1 Introduction

The Global Financial Crisis of 2008 has underscored the importance of the interaction between monetary policy and macro-prudential policy. A body of literature resulting from the crisis has focused mainly on only one side of the interaction. The body of literature has examined the cross-effects of monetary policy on the primary objective of macro-prudential policy of financial stability. The body of literature nonetheless has mainly disregarded the cross-effects of macro-prudential policy on the primary objective of monetary policy of macroeconomic stability. The unbalanced focus of the body of literature has primarily been the consequence of recent priorities in policy circles: During and in the close aftermath of the Global Financial Crisis of 2008, the primary concern in policy circles has been to gain further understanding on how to use policy tools in general to reduce and/or to prevent severe disruptions in the financial intermediary system.

This paper focuses on the other side of the interaction and investigates the cross-effects of macro-prudential policy on macroeconomic stability. In this paper, macro-prudential policy refers to a capital requirement on financial intermediaries that limits leverage by a multiple of net worth. The resulting limit on leverage is common to all financial intermediaries and is contingent upon the aggregate state of the economy. Macroeconomic stability refers to an ideal situation in which there is full employment and the aggregate price level as well as the inflation rate are stable.

To conduct the analysis, we embed in the framework of Van der Ghote (2017),[1] a zero-lower-bound (ZLB) constraint on the nominal rate that may occasionally bind in equilibrium. In the Van der Ghote (2017) economy, as in a standard New Keynesian economy, the sluggish price-setting behavior of firms implies that real prices in general are not consistent with current marginal production costs nor with current marginal productivity. The resulting distortions in the price system create a labor wedge as well as inefficient price dispersion which in turn generate a departure of aggregate employment from its efficient quantity and a departure of inflation from its structural rate (i.e. macroeconomic instability). Monetary policy can restore and maintain macroeconomic stability by mimicking the natural rate with the nominal rate. The natural rate is the real interest rate in the flexible price economy in which firms reset their nominal price on a continuous basis. The nominal rate is the rate of return on short-term and risk-free nominal debt such as deposits and short-term government bonds. Mimicking the natural rate with the nominal rate induces the same aggregate demand as in the flexible price economy; it also induces firms to set prices in accord with current marginal production costs and with current marginal productivity; and therefore it guarantees macroeconomic stability (Blanchard and Gali 2007). Monetary policy can achieve macroeconomic stability only when the natural rate stays above the ZLB constraint on the nominal rate and the economy stays away from a liquidity

[1]Van der Ghote (2017) is the first chapter in this dissertation.

trap. The nominal rate is subject to a ZLB constraint because risk-free nominal debt is arbitraged against cash assets which by default pay no nominal rate of return.

We show that in the Van der Ghote (2017) economy the natural rate is endogenous. We show furthermore that in such economy the natural rate depends on macro-prudential policy.

In the Van der Ghote (2017) economy, we show that the natural rate can be partitioned into a neo-classical rate and an endogenous natural rate. The neo-classical rate is the real interest rate in the neo-classical economy in which prices are flexible and there are no frictions at all. The neo-classical rate is exogenous; is determined by the fundamentals in the economy; and is independent of macro-prudential policy.

The endogenous natural rate results from frictions in financial markets; and it is determined by the relative capitalization of financial intermediaries to the total wealth in the economy. Frictions in financial markets comprise incomplete financial contracts (which are limited to deposit contracts) and a leverage constraint (that follows from a moral hazard problem and that limits leverage by a multiple of net worth). The leverage constraint puts a limit on the amount of deposits that financial intermediaries can raise from households, and therefore it also puts a limit on the amount of funds that financial intermediaries can channel to firms. In equilibrium, when financial intermediaries are poor-capitalized relative to the total wealth in the economy, the leverage constraint binds, financial intermediaries provide funds to only a fraction of all the firms in the economy, and endogenous aggregate productivity losses materialize.[2] Deposit contracts create a liquidity mismatch problem in the balance sheets of financial intermediaries because deposits are risk-free whereas funds to firms are risky. The liquidity mismatch problem, together with the leverage constraint, implies that in equilibrium the economy recurrently transitions from episodes

[2]Aggregate productivity losses materialize because households (who are less productive at providing funds to firms) are the the agents who provide funds to the remanent firms that cannot get funding from financial intermediaries.

in which financial intermediaries are poor-capitalized, leverage constraints bind, and endogenous aggregate productivity losses materialize, to episodes in which financial intermediaries are well-capitalized, leverage constraints are slack, and endogenous aggregate productivity losses are null, and vice versa. The resulting endogenous fluctuations on aggregate productivity generate endogenous fluctuations on aggregate output, on aggregate consumption, and whence on the natural rate. The endogenous natural rate indeed depends on macro-prudential policy because restrictions on intermediary leverage curb the aggregate quantity of intermediary funds to firms, the endogenous aggregate productivity losses, and aggregate output as well as aggregate consumption.

To examine the effect of macro-prudential policy on the (endogenous) natural rate, we assume that the ZLB constraint on the nominal rate is slack. We consider three simple macro-prudential policy rules. The first two rules are polar opposite. In the first rule, macro-prudential policy does not restrict leverage; in the second rule, macro-prudential policy forbids leverage. The third rule is an intermediate rule between the aforementioned two. In the third rule, macro-prudential policy restricts intermediary leverage to implement the constrained efficient allocation of the flexible price economy.[3] The three rules that we consider capture the notion that macro-prudential policy restricts leverage to reduce the endogenous fluctuations resulting from frictions in financial markets. Furthermore, they capture the notion that tighter macro-prudential policies restrict leverage more severely.

The comparison of the equilibrium under the aforementioned policy rules reveals that tighter macro-prudential policies lift the lower bound of the endogenous nat-

[3]In the Van der Ghote (2017) economy, as in a standard economy with incomplete financial markets and leverage constraints, there is a pecuniary externality that generates excessive leverage, and excessive fluctuations on the endogenous aggregate productivity losses. The reason is that individual financial intermediaries do not internalize how their leverage decisions affect the price of risky assets, the leverage constraint, nor the leverage decisions of the other financial intermediaries in the economy (Lorenzoni 2008; Jeanne and Korinek 2010; Bianchi 2011; Bianchi and Mendoza 2013).

ural rate. Specifically, we obtain that tighter restrictions on intermediary leverage mitigate the liquidity mismatch problem; reduce the fluctuations on the intermediary wealth share and on the endogenous aggregate productivity losses; and whence reduce aggregate output risk as well as aggregate consumption risk. Reductions in aggregate consumption risk depress the value of risk-free securities and, in equilibrium, push the rate of return on risk-free debt, the real interest rate, and the endogenous natural rate, upward. In the specific financially autarkic economy in which macro-prudential policy forbids leverage, we obtain that all endogenous fluctuations vanish, and that the lower bound on endogenous natural rate attains its maximum value of zero. All endogenous fluctuations vanish because financial intermediaries are not subject to a liquidity mismatch problem nor do they concentrate aggregate risk in their balance sheets when they do not take on leverage.

Lastly, we explore the cross-effects of macro-prudential policy on macroeconomic stability. Macro-prudential policy has cross-effects when the natural rate occasionally attains negative values and the ZLB constraint on the nominal rate occasionally prevents monetary policy from mimicking the natural rate. In economies in which macro-prudential policy has cross-effects, we obtain that tighter macro-prudential policies reduce the frequency of liquidity trap episodes potentially preventing the economy from entering into a liquidity trap at all. Furthermore, we obtain that during a liquidity trap, if any, tighter macro-prudential policies reduce the gap between the ZLB and the natural rate, and whence help monetary policy to stimulate aggregate demand, economic activity and aggregate employment, and to reduce deflation and price dispersion.

We find two type of economies in which macro-prudential has cross-effects. We find that macro-prudential policy has cross-effects in secular stagnant economies in which poor economic growth prospectus place the neo-classical rate close to zero. We find that macro-prudential policy also has cross-effects in financially unstable

economies in which severe moral hazard problems and/or strong valuation differences generate large fluctuations on the endogenous aggregate productivity losses and on the endogenous natural rate.

**Related Literature** This paper relates to a body of literature that studies the interactions between monetary policy and macro-prudential policy. The majority of the papers in the literature focuses on the cross-effects of monetary policy on the primary objective of macro-prudential policy of financial stability. We focus instead on the cross-effects of macro-prudential policy on the primary objective of monetary policy of macroeconomic stability. Farhi and Werning (2016) also investigate the cross-effects of macro-prudential policy on macroeconomic stability, but focus on cross-effects based on aggregate demand externalities.

On methodological grounds, our model economy builds upon the works of Calvo (1983); Gertler and Karadi (2011) and Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); and of Van der Ghote (2017).

## 2.2 The Model

The model extends the framework in Van der Ghote (2017) to incorporate a ZLB constraint on the nominal rate that may occasionally bind in equilibrium. We refer the reader to the subsections 1.2.1 to 1.2.5 in Van der Ghote (2017) for a detailed description of the model. We also refer the reader to those subsections for an analysis concerning the optimization problems of the private agents and for the definition of the competitive equilibrium. In this section, we analyze only the behaviors of aggregate output, the real interest rate, social welfare, and of policy, in equilibrium.

### 2.2.1 Aggregate Output

The model economy admits in equilibrium an aggregate production function that is Cobb-Douglas. The inputs in the aggregate production function are aggregate labor $L_t$ and the aggregate stock of physical capital $\bar{K}$. The aggregate stock of physical capital is fixed because neither does physical capital in production depreciate nor is there an investment technology that allows for the accumulation of physical capital across time. Let $Y_t$ denote aggregate output and let $\alpha \in (0,1)$ denote the labor share of output. Aggregate output $Y_t$ satisfies

$$Y_t \equiv A_t \frac{u_t^{1-\alpha}}{\omega_t} L_t^{\alpha} \bar{K}^{1-\alpha}$$

The total factor productivity (TFP) has two components: an exogenous productivity level $A_t$ and an endogenous productivity level $u_t^{1-\alpha}/\omega_t$. The exogenous productivity level $A_t$ evolves stochastically according to the Ito process

$$dA_t/A_t = \mu_{A,t}dt + \sigma_{A,t}dZ_t$$

with drift process $\mu_{A,t}$ and diffusion process $\sigma_{A,t}$. The process $\{Z_t \in \mathbb{R} : t \geq 0\}$ is a standard Wiener process defined on a filtered probability space $(\Omega, \mathcal{F}, P)$. Intuitively, $dZ_t$ represents a shock to the growth rate of aggregate productivity that is normally distributed. The endogenous productivity level $u_t^{1-\alpha}/\omega_t$ is the consequence of price dispersion $\omega_t$ and of the effective capital services rate $u_t$.

#### Price Dispersion

Price dispersion is endogenous and results from the sluggish price-setting behavior of firms (see subsection 1.2.6 in Chapter 1).

Firms produce a continuum of intermediate goods $\{y_{j,t}\}$, with $j \in [0,1]$, using labor and capital services as inputs. Each firm produces a single intermediate good variety $j \in [0,1]$. Firms operate a Cobb-Douglas production technology that has a common labor share of output of $\alpha$ and a common productivity level of $A_t$.[4] Firms compete monopolistically in goods markets and reset their nominal price sluggishly according to Calvo (1983) pricing (see subsection 1.2.1 in Chapter 1). Calvo (1983) pricing allows firms to reset their nominal price only when they are hit by an idiosyncratic Poisson shock that is i.i.d. across firms. A CES aggregator with a constant elasticity of substitution of $\varepsilon > 1$ transforms the intermediate goods $\{y_{j,t}\}$ that firms produce into the final consumption good $y_t$.

In equilibrium, price dispersion $\omega_t$ is the consumption-based price index out of the real prices that firms charge. Specifically,

$$\omega_t \equiv \int_{-\infty}^{t} \theta e^{-\theta(t-s)} \left( \frac{p_{*,s}}{p_t} \right)^{-\varepsilon} ds$$

The parameter $\theta$ is the arrival rate of the Poisson process that allows firms to reset their nominal price. The survival density function $\theta e^{-\theta(t-s)}$ measures the share of firms that have been allowed to reset their nominal price for the last time at time $s \leq t$. The price $p_{*,s}$ is the nominal price (at time $t$) of those firms that reset their nominal price for the last time at time $s \leq t$. Firms that have the possibility to reset their nominal price at any given time $t$ set the same optimal price $p_{*,t}$ because firms in general are identical. The price $p_t$ is the aggregate price level in nominal terms. The function $y_{d,t}(p_j) \equiv (p_j/p_t)^{-\varepsilon} y_t$ is the demand function of intermediate goods that results from the CES aggregator.

[4]The labor share of output $\alpha$ and the productivity level $A_t$ are the same between the production function of firms and the aggregate production function.

The aggregate price level $p_t$ is the consumer price index. Specifically,

$$p_t \equiv \left[ \int_{-\infty}^{t} \theta e^{-\theta(t-s)} p_{*,s}^{1-\varepsilon} ds \right]^{\frac{1}{1-\varepsilon}}$$

The cross-section distribution of intermediate goods prices is $\left\{ \theta e^{-\theta(t-s)}, p_{*,s} \right\}_{s \leq t}$.

Price dispersion is inefficient. The reason is that price dispersion generates a misallocation problem of labor and of capital services across firms. Price dispersion generates quantity dispersion on intermediate goods which is inefficient because the CES aggregator has decreasing marginal returns. Notice that in general $\omega_t \geq 1$ and that $\omega_t = 1$ iff $p_{*,s} = p_t \ \forall s \leq t$. Quantity dispersion on intermediate goods implies an inefficient allocation of labor and of capital services across firms that ultimately depresses endogenous TFP according to $1/\omega_t$.

**Effective Capital Services Rate**

The effective capital services rate $u_t$ is endogenous and results from a leverage constraint that occasionally binds in equilibrium. The leverage constraint occasionally limits the levered positions on capital that financial intermediaries can take as well as the aggregate supply of capital services to firms (see subsection 1.2.6 in Chapter 1).

The potential suppliers of capital services to firms comprise financial intermediaries and households. Potential suppliers transform physical capital into capital services according to a linear production technology. Financial intermediaries have a comparative advantage at providing capital services to firms. Specifically, financial intermediaries can provide capital services at a one-to-one rate whereas households can provide capital services at a rate $u_{h,t} < 1$.

Let $\bar{K}_{f,t}/\bar{K} \in [0,1]$ denote the capital share of financial intermediaries. Let $\bar{K}_{h,t}/\bar{K} \in [0,1]$ denote the equivalent share but for households. In equilibrium, the

capital shares $\bar{K}_{f,t}/\bar{K}$ and $\bar{K}_{h,t}/\bar{K}$ add up to 1 because the market for physical capital clears.[5] The effective capital services rate $u_t$ is the weighted average of $u_{h,t}$ and 1

$$u_t \equiv \bar{K}_{h,t}/\bar{K} * u_{h,t} + \bar{K}_{f,t}/\bar{K}$$

with $\bar{K}_{h,t}/\bar{K} + \bar{K}_{f,t}/\bar{K} = 1$.

In equilibrium, financial intermediaries always hold the largest possible capital share because they have a higher valuation for capital than households. The capital share $\bar{K}_{f,t}/\bar{K}$ and the rate $u_t$ therefore attain their maximum level of 1 when the leverage constraint is slack. The capital share $\bar{K}_{f,t}/\bar{K}$ and the rate $u_t$ are below 1 when the leverage constraint binds, and whence limits the levered positions that financial intermediaries can take on capital.

The leverage constraint results from a moral hazard problem in financial markets and from macro-prudential policy. The moral hazard problem is similar to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). Let $v_t \geq 1$ denote the Tobin's Q of financial intermediaries. Equivalently, $v_t$ measures the marginal value of wealth in the financial intermediary sector. The moral hazard problem limits leverage by the incentive-compatible multiple $\lambda v_t$ of net worth. The inverse of the parameter $\lambda$ is the share of assets that financial intermediaries can walk away with in the Gertler-Karadi-Kiyotaki moral hazard problem (see subsection 1.2.3 in Chapter 1). Macro-prudential policy limits leverage by the multiple $\Phi_t$ of net worth. The leverage multiple $\Phi_t$ is common to all financial intermediaries and depends on the aggregate state. Financial intermediaries take $\Phi_t$ as given.

Let $N_{f,t}$ denote the aggregate net worth of financial intermediaries. Let $q_t$ denote the price of capital. The leverage constraint limits the capital share $\bar{K}_{f,t}/\bar{K}$

[5] We assume that firms cannot hold physical capital (see subsection 1.2.1 in Chapter 1).

accordingly

$$\bar{K}_{f,t}/\bar{K} = \min\left\{\lambda v_t \frac{N_{f,t}}{q_t \bar{K}}, \Phi_t \frac{N_{f,t}}{q_t \bar{K}}, 1\right\}$$

The leverage constraint binds only when financial intermediaries are undercapitalized and lack enough borrowing capacity to absorb all of the aggregate capital stock. Let $\eta_t \equiv N_{f,t}/q_t\bar{K} \in [0,1]$ denote the wealth share of financial intermediaries. The total wealth in the economy is $q_t\bar{K}$ because physical capital is the only real asset. Equivalently, the leverage constraint binds only when $\eta_t < \bar{\eta}_t$ with $\min\{\lambda v_t, \Phi_t\}\bar{\eta}_t \equiv 1$.

### 2.2.2 State Variables and Law of Motions

The state variables together with their law of motions (LoMs) determine the dynamic behavior of the economy. The state variables determine the position of the economy at any given time $t$. The LoMs of the state variables determine the evolution of the economy during the time interval $dt$.

**State Variables** The states variables are $\{A_t, \omega_t, \eta_t\}$. The exogenous productivity level $A_t$ is not relevant for the analysis because the equilibrium outcome is scale invariant with respect to $A_t$.[6] Intuitively, price dispersion $\omega_t$ is a state variable because it summarizes the relevant information in the cross-section distribution of intermediate goods prices. The wealth share $\eta_t$ is a state variable because it summarizes the relevant information concerning the capitalization of financial intermediaries and the aggregate supply of capital services to firms.

**LoM of Price Dispersion** Price dispersion evolves in accord with the evolution of the cross-section distribution of intermediate good prices. The price-setting behavior of firms and the real price $p_{*,t}/p_t$ dictate the evolution of price dispersion.

[6] In our parameterization, we assume that the diffusion processes $\sigma_{A,t}$ is constant (see section 1.4 in Chapter 1).

During the time interval $dt$, a share $\theta dt$ of firms have the opportunity to reset their nominal price. All of the firms that have the opportunity to reset their nominal price during $dt$ set the same nominal price of $p_{*,t}$ because firms reset their price sluggishly (see below). By the end of the time interval $dt$, therefore, a measure $\theta dt$ of intermediate goods prices have the common real price of $p_{*,t}/p_t$.

If the aggregate price level $p_t$ were to remain constant during $dt$, the change on price dispersion would amount to $d\omega_t = \left[(p_{*,t}/p_t)^{-\varepsilon} - \omega_t\right] * \theta dt$. The term $\omega_t$ measures the contribution to price dispersion (at time $t$) of the intermediate goods prices that end up with the common real price of $p_{*,t}/p_t$ (at time $t + dt$). A law of large number applies in the expression for $d\omega_t$ because the idiosyncratic Poisson shock (that allows firms to reset their nominal price) is i.i.d.

The aggregate price level, however, in general does not remain constant during $dt$. The reason is that the aggregate price level is the consumer price index. The aggregate price level responds to changes on the cross-section distribution of intermediate good prices accordingly

$$\frac{dp_t}{p_t} = \frac{\theta}{\varepsilon - 1}\left[1 - \left(\frac{p_{*,t}}{p_t}\right)^{-(\varepsilon-1)}\right] dt + 0 * dZ_t$$

The inflation rate $dp_t/p_t$ indeed affects the rate $d\omega_t/\omega_t$ at which price dispersion evolves. Specifically, because the demand function of intermediate goods is $y_{d,t}(p_j)/y_t = (p_j/p_t)^{-\varepsilon}$, inflation rate $dp_t/p_t$ affects $d\omega_t/\omega_t$ according to $\varepsilon * dp_t/p_t$.

The law of motion of price dispersion therefore is

$$\frac{d\omega_t}{\omega_t} = \left\{\left[\left(\frac{p_{*,t}}{p_t}\right)^{-\varepsilon}\frac{1}{\omega_t} - 1\right]\theta + \varepsilon\frac{\theta}{\varepsilon - 1}\left[1 - \left(\frac{p_{*,t}}{p_t}\right)^{-(\varepsilon-1)}\right]\right\} dt + 0 * dZ_t$$

Mathematically, the law of motion of price dispersion follows from taking the first-order derivative of $\omega_t$ with respect to time $t$. The law of motion of the aggregate price level follows from taking the first-order derivative of $p_t$ with respect to time $t$. Price

dispersion and the aggregate price level are locally deterministic because both $\omega_t$ and $p_t$ are Riemann Integrals. Intuitively, neither does the evolution of price dispersion nor that of the aggregate price level depend on the shock $dZ_t$, because only the firms that can reset their nominal price at the very end of the time interval $dt$ can incorporate in their price the productivity information in the shock $dZ_t$.

For any given state $\{A_t, \omega_t, \eta_t\}$, the real price $p_{*,t}/p_t$ that minimizes the rate $d\omega_t/\omega_t$ at which price dispersion evolves is unique. The optimal real price that minimizes $d\omega_t/\omega_t$ is

$$\frac{p_{*,t}}{p_t} = \frac{1}{\omega_t} \Longleftrightarrow \frac{d\omega_t}{\omega_t} = \frac{dp_t}{p_t} = \frac{\theta}{\varepsilon - 1}\left(1 - \omega_t^{\varepsilon-1}\right) dt \leq 0$$

Intuitively, the real price $p_{*,t}/p_t = 1/\omega_t$ reduces and eventually eliminates price dispersion as fast as possible because at $p_{*,t}/p_t = 1/\omega_t$ the inflation rate $dp_t/p_t$ mirrors the negative of the rate $d\omega_t/\omega_t$ at which the endogenous TPF gains $1/\omega_t$ evolve. A positive amount of time is always required to eliminate price dispersion because firms need time to coordinate on a common nominal price.

In equilibrium, firms nonetheless do not necessarily set prices consistently with $p_{*,t}/p_t = 1/\omega_t$. When they have the opportunity to reset their nominal price, firms set the price that maximizes the present discounted value of their profits flows (see subsection 1.2.1 in Chapter 1). Equivalently, because in equilibrium they set competitive prices, firms set the nominal price at which they break-even in present discounted value. The nominal price that firms set yields the real price

$$\frac{p_{*,t}}{p_t} \equiv \frac{E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} x_s \left[y_{d,s}\left(p_t\right)\right] ds}{E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} \frac{p_t y_{d,s}(p_t)}{p_s} ds}$$

The numerator in the RHS is the present discounted value of the total production costs, $x_s\left[y_{d,s}\left(p_t\right)\right]$. The denominator is the present discounted value of the sales revenues, $p_t/p_s * y_{d,s}\left(p_t\right)$. Both production costs and sales revenues are expressed in real terms and are computed under the assumption that $p_{*,t}/p_t = 1$. Future payoffs are

discounted with the Stochastic Discount Factor (SDF) of a representative household, i.e. $\Lambda_t$, weighted by the survival density function $\theta e^{-\theta(s-t)}$. Intuitively, if the denominator in the RHS exceeds the numerator, firms are willing to target a real price below 1, in order to increase their indirect quantity demanded relative to $p_{*,t}/p_t = 1$ as well as their sales revenues and profits flows. The opposite happens if the numerator exceeds the denominator.

Let $X_t$ denote the numerator in the RHS and let $M_t$ denote the denominator. In the Appendix A.3.1, we show that the processes $X_t$ and $M_t$ are proportional to aggregate output $Y_t$. Additionally, in the Appendix A.3.1, we show that the average processes $x_t \equiv X_t/Y_t$ and $m_t \equiv M_t/Y_t$ satisfy the asset pricing conditions

$$\left(\frac{L_t}{L_{*,t}}\right)^{1+\psi} \frac{1}{\omega_t} * \frac{1}{x_t} dt + E_t\left[dx_t/x_t\right] + \varepsilon \frac{dp_t}{p_t} - (\rho + \theta)\, dt = 0$$

$$1 * \frac{1}{m_t} dt + E_t\left[dm_t/m_t\right] + (\varepsilon - 1)\frac{dp_t}{p_t} - (\rho + \theta)\, dt = 0$$

In this notation, $\rho$ is the discount rate of the representative household and $\psi$ is the inverse of the Frish elasticity of the aggregate labor supply. The process $L_{*,t}$ is the aggregate quantity of labor in the economy with flexible prices (i.e. $1/\theta \to 0$).

The asset pricing conditions reveal that firms set prices consistently with $p_{*,t}/p_t = 1/\omega_t$ only when aggregate labor $L_t$ equals its flexible price quantity $L_{*,t}$. The process $(L_t/L_{*,t})^{1+\psi}\, 1/\omega_t$ is the average production costs at time $t$. The corresponding process at any given time $s > t$ is $(L_s/L_{*,s})^{1+\psi}\, 1/\omega_s$. The average sales revenues at time $t$ (given $p_{*,t}/p_t = 1$) are 1. The corresponding average sales revenues at any given time $s > t$ are $p_t/p_s$ (because nominal prices remain fixed). When aggregate labor $L_t$ equals $L_{*,t}$, firms break-even in present discounted value at $p_{*,t}/p_t = 1/\omega_t$, because

average sales revenues $p_t/p_s * 1/\omega_t$ appreciate in real terms at the same rate as average production costs $1/\omega_s$.[7]

The asset pricing conditions reveal also that firms do not set prices consistently with $p_{*,t}/p_t = 1/\omega_t$ when aggregate labor $L_t$ deviates from $L_{*,t}$. Intuitively, when $L_t > L_{*,t}$, and the economy is over-stimulated relative to the flexible price economy, firms target a real price $p_{*,t}/p_t$ above $1/\omega_t$ because they expect high average production costs. Furthermore, because they expect the other firms to also target real prices above $1/\omega_t$, firms expect inflation when $L_t > L_{*,t}$, which induces them to target real prices even further above $1/\omega_t$, and so on. Similarly, when $L_t < L_{*,t}$, and the economy is over-slowed down relative to the flexible price economy, firms target a real price $p_{*,t}/p_t$ below $1/\omega_t$ because they expect low average production costs. Furthermore, because they expect deflation when $L_t < L_{*,t}$, firms target real prices even further below $1/\omega_t$, which leads to more deflation, and so on. In both cases, the deviation of $p_{*,t}/p_t$ from $1/\omega_t$ yields higher levels of price dispersion relative to $p_{*,t}/p_t = 1/\omega_t$.

**LoM of Financial Wealth Share** The wealth share $\eta_t \equiv N_{f,t}/q_t\bar{K}$ evolves in accord with the profitability of financial intermediaries. The excess returns that financial intermediaries earn over the total wealth in the economy, along with the net transfers that financial intermediaries payout to households, dictate the evolution of $\eta_t$.

Financial intermediaries earn profits from renting out capital services to firms. Financial intermediaries finance their positions on physical capital (i.e. their loans to firms) with their own net worth (i.e. internal financing) and with leverage (i.e. external financing). Let $r_{k,t}$ denote the rental rate of capital services. The total return on internal financing $dR_{f,t}$ is the sum of the dividend yields $r_{k,t}/q_t dt$ and the capital gain/loss rate $dq_t/q_t$. The total return on external financing is the product between the levered capital positions $q_t\bar{K}_{f,t}/N_{f,t} - 1$ and the credit spread $dR_{f,t} - (i_t dt - dp_t/p_t)$.

[7]Mathematically, notice that when $L_t = L_{*,t}$, the process $(x/m)_t \equiv x_t/m_t = 1/\omega_t$ satisfies both asset pricing conditions, as $E_t[dx_t/x_t] = E_t[dm_t/m_t] - d\omega_t/\omega_t$ and $d\omega_t/\omega_t = dp_t/p_t$.

The rate $i_t \geq 0$ is the nominal rate at which financial intermediaries issue deposits. The rate $i_t$ is locally risk-free because deposits promise a fixed return at time $t + dt$.

The net worth of financial intermediaries gross of net transfers to households evolves accordingly

$$dN_{f,t}/N_{f,t} = dR_{f,t} + [dR_{f,t} - (i_t dt - dp_t/p_t)] * \left(q_t \bar{K}_{f,t}/N_{f,t} - 1\right)$$

The wealth share $\eta_t \equiv N_{f,t}/q_t \bar{K}$ evolves according to

$$d\eta_t/\eta_t = \mu_{\eta,t} dt + \sigma_{\eta,t} dZ_t$$

with

$$\mu_{\eta,t} dt = \frac{r_{k,t}}{q_t} dt + (\phi_t - 1)\left[E_t\left[dR_{f,t}\right] - \left(i_t dt - \frac{dp_t}{p_t}\right) - Var_t\left[dR_{f,t}\right]\right] + \left(\frac{\kappa}{\eta_t} - \tilde{\rho}\right) dt$$

$$\sigma^2_{\eta,t} dt = (\phi_t - 1)^2 Var_t\left[dq_t/q_t\right]$$

The process $\phi_t \equiv q_t \bar{K}_{f,t}/N_{f,t}$ is the leverage multiple of financial intermediaries. The first term in $\mu_{\eta,t} dt$ is the excess return on internal financing over the total wealth in the economy. The second term in $\mu_{\eta,t} dt$ plus the term $\sigma_{\eta,t} dZ_t$ amount to the realized excess return on external financing (also over the total wealth in the economy). The excess return on external financial is locally risky because financial intermediaries are subject to a liquidity mismatch problem: The return on physical $dR_{f,t}$ is locally risky whereas the real return on deposits $i_t dt - dp_t/p_t$ is locally risk-free. The liquidity mismatch problem implies that financial intermediaries concentrate aggregate risk in their balance sheets when they take on leverage. The last term in $\mu_{\eta,t} dt$ is the net transfers that financial intermediaries payout to households. In equilibrium, financial intermediaries pay out dividends only when they retire. Financial intermediaries

retire only when they are hit by an idiosyncratic Poisson shock that has an arrival rate of $\tilde{\rho}$. When they retire, financial intermediaries transfer all of their net worth back to the household. Financial intermediaries that retire are replaced with new financial intermediaries, that receive from households a share $\kappa/\tilde{\rho}$ of the aggregate capital stock as initial endowment (see subsection 1.2.3 in Chapter 1).

Figure 2.1 plots the drift process $\mu_{\eta,t}$ and the diffusion process $\sigma_{\eta,t}$ in isolation. Figure 2.1 focuses on a benchmark economy in which prices are flexible and there is no macro-prudential policy, i.e. $1/\theta, 1/\Phi_t \to 0$.[8]

Figure 2.1a shows that the wealth share $\eta_t$ mean-reverts around the stochastic steady state $\eta_{ss}$. The stochastic steady state $\eta_{ss}$ is the state at which the economy would uniformly converge if the $dZ_t$ shocks were shut down. The stochastic steady state satisfies that $\mu_{\eta,t} = 0$.

The wealth share $\eta_t$ mean-reverts around $\eta_{ss}$ because the profitability of financial intermediaries is inversely related to $\eta_t$. When they are undercapitalized, and $\eta_t < \eta_{ss}$ is low, financial intermediaries earn high dividend yields $r_{k,t}/q_t dt$ and high excess returns on internal financing and on external financing (see subsection 1.3.1 in Chapter 1).[9] High dividend yields and excess returns deliver high expected profits which imply a fast recapitalization of financial intermediaries in expectation. When they are well-capitalized, and $\eta_t > \eta_{ss}$ is high, financial intermediaries earn low dividend yields $r_{k,t}/q_t dt$ as well as low excess returns (see subsection 1.3.1 in Chapter 1).[10] Moreover, the low excess returns that financial intermediaries earn are not enough

[8]The behavior of $d\eta_t/\eta_t$ is similar in the more interesting economies in which prices are sticky and there is macro-prudential policy.

[9]The reason is that the rental rate of capital services $r_{k,t}$ is high and the price of capital $q_t$ is low. The rental rate $r_{k,t}$ is high because households hold a large share of the aggregate capital stock and the aggregate supply of capital services is low. The price of capital $q_t$ is low though because households are the marginal investors on capital.

[10]The reason is that the rental rate of capital services $r_{k,t}$ is low and the price of capital $q_t$ is high. The rental rate $r_{k,t}$ is low because financial intermediaries hold all of the aggregate capital stock and the aggregate supply of capital services to firms therefore attains its potential level. The price of capital $q_t$ is high though because financial intermediaries are the marginal investors on capital.

to compensate for their dividend payouts. The conditional expectation $E_t\left[d\eta_t/\eta_t\right]$ is therefore negative and low.

Figure 2.1b complements Figure 2.1a to show that the wealth share $\eta_t$ mean-reverts around $\eta_{ss}$ stochastically. Figure 1b shows also that the fluctuations on $d\eta_t/\eta_t$ (blue line) are large relative to those on $(\phi_t - 1) * dA_t/A_t$ (red dotted line). The term $(\phi_t - 1) * dA_t/A_t$ accounts for the fluctuations on $d\eta_t/\eta_t$ that correspond in size to the fluctuations on the exogenous productivity growth rate $dA_t/A_t$. We refer to the residual fluctuations $d\eta_t/\eta_t - (\phi_t - 1) * dA_t/A_t$ as the endogenous fluctuations on $d\eta_t/\eta_t$.

The endogenous fluctuations on $d\eta_t/\eta_t$ result from the combination of incomplete financial contracts and of leverage constraints that occasionally bind (Lorenzoni 2008; Jeanne and Korinek 2010; Bianchi 2011; Bianchi and Mendoza 2013). Incomplete financial contracts force financial intermediaries to concentrate aggregate risk in their balance sheets when they take on leverage. Leverage constraints, when they bind, force financial intermediaries to reduce their levered capital positions when the latter realizes net worth losses. The combination of the incomplete financial contracts and of leverage constraints (that occasionally bind) creates a positive feedback loop between $dN_{f,t}/N_{f,t},\ d\bar{K}_{f,t}/\bar{K}_{f,t}$ and $dq_t/q_t$ that fuels the endogenous fluctuations on $d\eta_t/\eta_t$ (see subsection 1.2.3 in Chapter 1).[11] The feedback loop indeed fuels large endogenous fluctuations on $d\eta_t/\eta_t$ relative to the constrained efficient allocation. The reason is that the feedback loop entails a pecuniary externality in financial markets that operates through the price of capital. Specifically, because they take the

[11]Specifically, following a negative shock $dA_t/A_t = \sigma_{A\,t} * dZ_t < 0$ that generates negative credit spreads and net worth losses, binding leverage constraints force financial intermediaries to reduce their levered capital positions. The reduction in $\bar{K}_{f,t}/\bar{K}$ generates a drop on $dq_t/q_t$ that is proportionally larger than the drop on $dA_t/A_t$. The reason is that households (who have a lower valuation for capital than financial intermediaries) are the agents who purchase in equilibrium the share of the aggregate capital stock that financial intermediaries sell. The residual drop $dq_t/q_t - dA_t/A_t$ generates further net worth losses $dN_{f,t}/N_{f,t} < dA_t/A_t$ which in turn initiate a positive feedback loop between $dN_{f,t}/N_{f,t},\ d\bar{K}_{f,t}/\bar{K}_{f,t}$ and $dq_t/q_t$. The feedback loop amplifies the effect of the initial negative shock $dA_t/A_t < 0$ on the rates $dq_t/q_t,\ dN_{f,t}/N_{f,t}$ and $d\bar{K}_{f,t}/\bar{K}_{f,t}$ and on $d\eta_t/\eta_t$.

price of capital as given, individual financial intermediaries do not internalize how their sales/purchases of capital affect the strength of the feedback loop. At the time of taking their leverage decisions, therefore, individual financial intermediaries take excessive leverage multiples which worsen the liquidity mismatch problem and strengthen the feedback loop relative to the constrained efficient allocation.

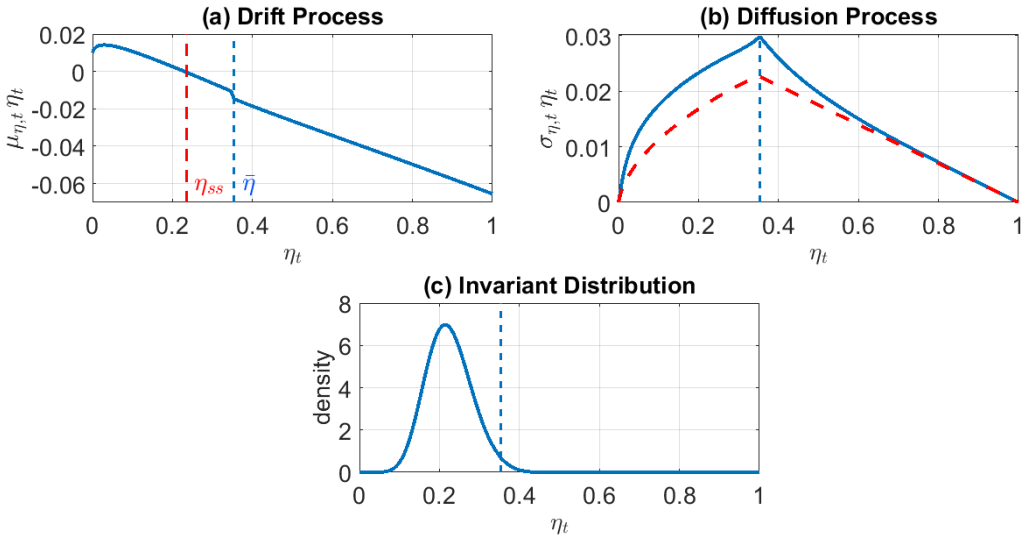



Figure 2.1: Dynamics of the Financial Wealth Share

The drift process $\mu_{\eta,t}$ and the diffusion process $\sigma_{\eta,t}$ together shape the invariant distribution of the wealth share $\eta_t$ (see the Appendix). The invariant distribution of $\eta_t$ together with $\mu_{\eta,t}$ and $\sigma_{\eta,t}$ imply that the economy recurrently transitions from episodes in which financial intermediaries are undercapitalized, and financially constrained, to episodes in which they are well-capitalized, and financially unconstrained, and vice versa (Figure 2.1).

### 2.2.3 Social Welfare

The model economy admits a representative household in equilibrium.[12] The representative household obtains utility from consumption and disutility from labor. The

[12]See subsections 1.2.1 to 1.2.5 in Chapter 1 for a derivation of the representative household.

preferences for consumption are logarithmic. The preferences for labor are CRRA with coefficient $1+\psi$. A parameter $\chi > 0$ weights the disutility from labor in terms of the utility from consumption.

In equilibrium, aggregate consumption equals aggregate output because neither is there investment nor fiscal policy. Log preferences for consumption allows the following representation of utility flows

$$\ln \frac{1}{\omega_t} + \alpha \ln L_t - \chi \frac{L_t^{1+\psi}}{1+\psi} + (1-\alpha) \ln u_t + \ln A_t + (1-\alpha) \ln \bar{K}$$

The first term is the utility losses from price dispersion. The utility flows $\ln 1/\omega_t$ attain their maximum value of 0 when there is no price dispersion and $\omega_t = 1$. The utility flows $\ln 1/\omega_t$ attain negative values when there is price dispersion and $\omega_t > 1$.

The difference between the second term and the third term amounts to the utility gains from labor. The quantity of labor in the flexible price economy $L_{*,t}$ maximizes the utility gains from labor. Specifically,

$$L_{*,t} = \arg \max_{L>0} \left\{ \alpha \ln L - \chi \frac{L^{1+\psi}}{1+\psi} \right\}$$

Intuitively, $L_{*,t}$ maximizes the utility gains from labor because in the flexible price economy intermediate goods prices equal marginal production costs, and input prices equal marginal productivity (see subsections 1.2.1 in Chapter 1). We interpret log-deviations of aggregate labor $L_t$ from its efficient level of $L_{*,t}$ as employment gaps. We interpret deviations of utility gains from labor from its efficient level of $\alpha \ln L_{*,t} - \chi \frac{1}{1+\psi} L_{*,t}^{1+\psi}$ as the utility losses from employment gap.

The fourth term is the utility losses from financial disintermediation. The utility flows $\ln u_t$ attain their maximum level of 0 when financial intermediaries alone provide capital services to firms, and $\bar{K}_{f,t}/\bar{K} = u_t = 1$. The utility flows $\ln u_t$ attain negative

values when both financial intermediaries and households provide capital services to firms, and $\bar{K}_{f,t}/\bar{K},\ u_t < 1$.

Lastly, the terms $\ln A_t + (1-\alpha)\ln \bar{K}$ are exogenous and therefore uninteresting.

### 2.2.4 Policy Rules

Policy instruments comprise monetary policy $i_t$ and macro-prudential policy $\Phi_t$. Monetary policy specifies the process of the nominal interest rate $i_t$.[13] The nominal rate $i_t \geq 0$ is subject to a ZLB constraint that may occasionally bind.

#### Monetary Policy

For simplicity, we consider ad-hoc rules for monetary policy. Specifically, we assume that monetary policy targets the natural interest rate $r_t$. The natural rate $r_t$ is the real interest rate in the analog economy in which prices are flexible. If the natural rate is non-negative, monetary policy sets the nominal rate $i_t$ equal to $r_t$. Otherwise, monetary policy sets $i_t$ to the closest possible value of $r_t$ which is zero. In the second case, we say that the ZLB constraint on the nominal rate binds.

The rule $i_t = \max\{r_t, 0\}$ has well-demarcated effects in terms of social welfare. Specifically, $i_t = \max\{r_t, 0\}$ minimizes the utility losses from price dispersion and from employment gap locally. The effectiveness of the rule $i_t = \max\{r_t, 0\}$ at minimizing the aforementioned utility losses depends on whether the ZLB constraint binds or is slack. If $r_t \geq 0$, and the ZLB constraint on the nominal rate is slack, the rule $i_t = r_t$ stabilizes employment gap on impact and price dispersion as fast as possible (see subsection 1.3.1 in Chapter 1). The rule $i_t = r_t$ stabilizes employment gap on impact because the natural rate implements the same aggregate demand as in the flexible price economy. The rule $i_t = r_t$ stabilizes prices dispersion as fast as possible because firms target the optimal real price of $1/\omega_t$ when $L_t = L_{*,t}$ and employment

[13] The implementation mechanism of monetary policy is the same as in the baseline New Keynesian framework. See Van der Ghote (2017).

gap is null. If $r_t < 0$, and the ZLB constraint on the nominal rate binds, the rule $i_t = \max\{r_t, 0\}$ does not stabilize employment gap on impact nor price dispersion as fast as possible anymore. The rule $i_t = \max\{r_t, 0\}$ does not stabilizes employment gap on impact because monetary policy falls into a liquidity trap. Specifically, monetary policy cannot stimulate aggregate demand enough to implement the aggregate demand of the flexible price economy, and employment gap is condemn to be negative. The rule $i_t = \max\{r_t, 0\}$ does not stabilize price dispersion as fast as possible because the average production costs $(L_t/L_{*,t})^{1+\psi} 1/\omega_t$ attain values below their optimal level of $1/\omega_t$, which implies that firms target real prices $p_{*,t}/p_t$ below $1/\omega_t$, and that the rate $d\omega_t/\omega_t$ deviates from its optimal rate of $\frac{\theta}{\varepsilon-1}\left(1-\omega_t^{\varepsilon-1}\right)dt$. The rule $i_t = \max\{r_t, 0\}$, by pegging $i_t$ to zero when $r_t < 0$, nonetheless, places the aggregate demand and the aggregate labor as close as possible to their counterparts in the flexible price economy. Employment gap and price dispersion therefore fall as much as possible given $i_t \geq 0$.[14]

**Macro-prudential Policy**

Also for simplicity, we consider ad-hoc rules for macro-prudential policy. We consider three rules in total. Two rules are polar opposites. In a first rule, there is no macro-prudential limit on leverage and $\Phi_t = +\infty$. In a second rule, macro-prudential policy forbids leverage and $\Phi_t = 1$. The third rule is an intermediate case. In the third rule, macro-prudential policy maximizes the present discounted value of the utility losses from financial disintermediation taking monetary policy as given (see Chapter 1, sections 1.3.1).

[14]The rule $i_t = \max\{r_t, 0\}$ does not minimize the utility losses from price dispersion and those from employment gap globally, because $i_t = \max\{r_t, 0\}$ does not take into account dynamic effects beyond the time interval $dt$. For instance, the rule $i_t = \max\{r_t, 0\}$ does not internalize its effect on the likelihood of switching between liquidity trap episodes and full-macroeconomic stabilization eventually in the future.

We use ad-hoc policy rules to evaluate the aggregate effects of macro-prudential policy. We pay particular attention to the effect of macro-prudential policy on the natural rate. The effect on the natural rate sheds light on the cross-effect of macro-prudential policy on macroeconomic stability.

## 2.3 Effect of Macro-prudential Policy on the Natural Rate

To evaluate the effect of macro-prudential policy on the natural rate, we consider the case in which the ZLB constraint is slack. We analyze the model economy using similar parameter values to in Van der Ghote (2017).[15]

In equilibrium, because the ZLB constraint is slack, monetary policy mimics the natural rate and $i_t = r_t$. Neither is there employment gap, price dispersion nor inflation. The only relevant state variable is the wealth share of financial intermediaries $\eta_t$.

Macro-prudential policy has aggregate effects because the aggregate quantity of intermediary leverage determines the aggregate supply of capital services to firms. Specifically, the leverage multiple $\phi_t$ determines the effective capital services rate according to $u_t = (1 - \phi_t \eta_t)\, u_{h,t} + \phi_t \eta_t$ and aggregate output according to $Y_t = A_t u_t^{1-\alpha} L_{*,t}^{\alpha} \bar{K}^{1-\alpha} \propto A_t u_t^{1-\alpha}$.[16]

Figure 2.2 illustrates the aggregate effects of the three macro-prudential policy rules under consideration (see subsection 2.2.4). Figure 2.2a focuses on the leverage multiple $\phi_t$ and Figure 2.2b focuses on the natural rate $r_t$.

[15]The only difference with respect to Van der Ghote (2017) is that we target an average Sharpe ratio of 50% (instead of 30%). Higher Sharpe ratios generate larger aggregate effects of macro-prudential policy.

[16]In the flexible price economy, aggregate labor $L_{*,t} = (\alpha/\chi)^{\frac{1}{1+\psi}}$ is constant.

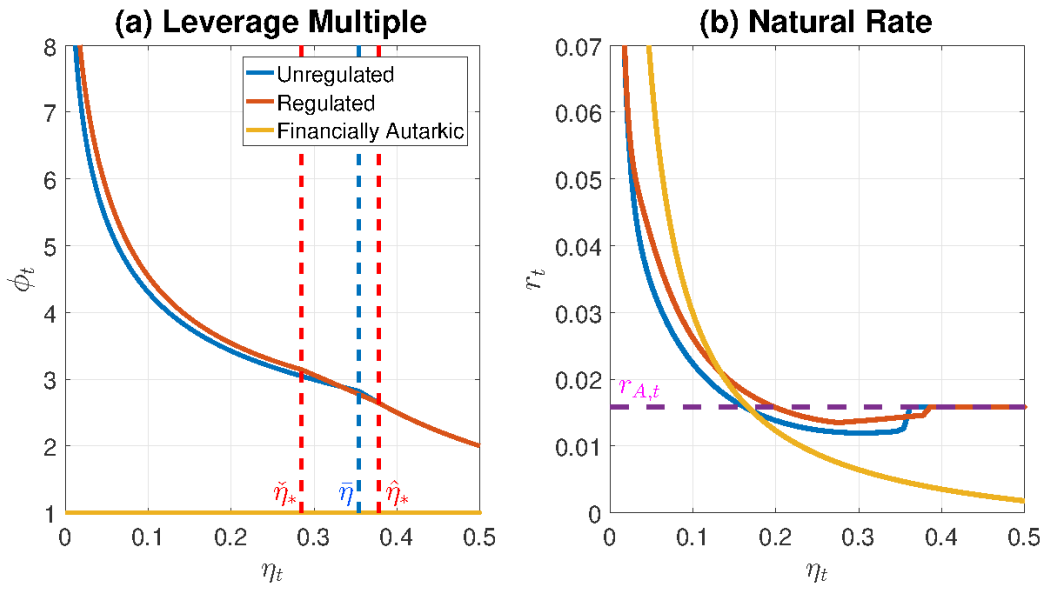



Figure 2.2: Aggregate Effects of Macro-prudential Policy

**Leverage Multiple** In equilibrium, the leverage multiple $\phi_t$ is the minimum between the incentive-compatible limit $\lambda v_t$, the macro-prudential limit $\Phi_t$ and the resource limit $1/\eta_t$. Specifically,

$$\phi_t = \min\left\{\lambda v_t, \Phi_t, 1/\eta_t\right\}$$

In the unregulated economy, i.e. $\Phi_t = +\infty$, macro-prudential policy does not restrict leverage, and the leverage multiple equals its natural upper limit of $\min\left\{\lambda v_t, 1/\eta_t\right\}$. The incentive-compatible leverage constraint $\phi_t \leq \lambda v_t$ binds only when financial intermediaries are undercapitalized, and $\eta_t < \bar{\eta}$ with $\lambda v\left(\bar{\eta}\right)\bar{\eta} \equiv 1$.

In the regulated economy, i.e. $\Phi_t = \Phi_{*,t} < +\infty$, macro-prudential policy restricts leverage occasionally depending on the capitalization of financial intermediaries. Specifically, macro-prudential policy restricts leverage only when financial intermediaries are average-capitalized and $\eta_t \in (\check{\eta}_*, \hat{\eta}_*)$ with $0 < \check{\eta}_* < \hat{\eta}_* < 1$ such that $\lambda v\left(\check{\eta}_*\right) \equiv \Phi_*\left(\check{\eta}_*\right)$ and $\Phi_*\left(\hat{\eta}_*\right)\hat{\eta}_* \equiv 1$. Inside the intermediate region of $\eta_t \in (\check{\eta}_*, \hat{\eta}_*)$, macro-prudential policy restricts leverage according to the optimal

limit $\Phi_{*,t} < \min\{\lambda v_t, 1/\eta_t\}$. Outside the intermediate region of $\eta_t \in (\check{\eta}_*, \hat{\eta}_*)$, macro-prudential policy does not restrict leverage and financial intermediaries behave as in the unregulated economy.

Because the ZLB constraint on the nominal rate is slack, the macro-prudential policy rule $\{\check{\eta}_*, \hat{\eta}_*; \Phi_{*,t}\}$ implements the constrained efficient allocation of the flexible price economy (see Chapter 1, subsection 1.3.1). Specifically, and always in terms of the flexible price economy, the rule $\{\check{\eta}_*, \hat{\eta}_*; \Phi_{*,t}\}$ implements the constrained efficient quantity of the pecuniary externality, the constrained efficient quantity of the feedback loop, and the constrained efficient levels of the endogenous fluctuations. Intuitively, the rule $\{\check{\eta}_*, \hat{\eta}_*; \Phi_{*,t}\}$ restricts leverage only when financial intermediaries are average-capitalized because only in that region financial intermediaries have significant price effects (see Chapter 1, subsection 1.3.1).

In the financially autarkic economy, i.e. $\Phi_t = 1 < \Phi_{*,t}$, macro-prudential policy forbids leverage and $\phi_t = 1$. A forbid on leverage restricts financial intermediaries to take positions on capital that are financed only with their own net worth.

**Natural Rate** In equilibrium, the expected marginal utility return from consumption determines the real interest rate (see subsections 1.2.1 to 1.2.5 in Chapter 1). The real interest rate and the natural rate agree because the ZLB constraint is slack.

The expected marginal utility return from consumption is $-E_t[d\Lambda_t/\Lambda_t]$ and the SDF of the representative household is $\Lambda_t = e^{-\rho t}/Y_t \propto e^{-\rho t}/A_t u_t^{1-\alpha}$. Let $\zeta_t$ denote the endogenous TFP component $u_t^{1-\alpha}$. The natural rate $r_t$ satisfies

$$r_t dt = \rho dt + E_t[dA_t/A_t] + E_t[d\zeta_t/\zeta_t] + Cov_t[dA_t/A_t, d\zeta_t/\zeta_t] - Var_t[dA_t/A_t + d\zeta_t/\zeta_t]$$

The natural rate $r_t$ can be decomposed into the neo-classical rate $r_{A,t}$ and the endogenous natural rate $r_{\zeta,t}$. The neo-classical rate is the real interest rate in the

neo-classical economy in which prices are flexible and there are no financial frictions (i.e. $1/\theta, 1/\lambda \to 0$). The neo-classical rate is exogenous and is determined by the fundamentals in the economy accordingly

$$r_{A,t}dt = \rho dt + E_t\left[dA_t/A_t\right] - Var_t\left[dA_t/A_t\right]$$

The endogenous natural rate is the residual rate after deducting the neo-classical rate from the natural rate. Specifically,

$$r_{\zeta,t}dt = E_t\left[d\zeta_t/\zeta_t\right] - \sqrt{Var_t\left[dA_t/A_t\right]Var_t\left[d\zeta_t/\zeta_t\right]} - Var_t\left[d\zeta_t/\zeta_t\right]$$

The second term in the RHS the covariance $Cov_t\left[dA_t/A_t, d\zeta_t/\zeta_t\right]$.[17] The endogenous natural rate results from the frictions in financial markets: If either financial contracts were to be complete, or moral hazard problems were not to exist, $\zeta_t = 1$ and $d\zeta_t/\zeta_t = 0$ (di Tella 2015 and Van der Ghote 2017). Intuitively, the leverage constraint together with the liquidity mismatch problem generate endogenous fluctuations on the aggregate supply of capital services to firms $u_t\bar{K}$; on the endogenous TFP component $\zeta_t = u_t^{1-\alpha}$; and on aggregate consumption $C_t = Y_t = A_t\zeta_t$. The latter endogenous fluctuations in turn shape the endogenous natural rate $r_{\zeta,t}$. Macro-prudential policy can affect the endogenous natural rate $r_{\zeta,t}$ because restrictions on intermediary leverage curb the leverage multiple $\phi_t$.

In the unregulated economy, and in the regulated economy, the endogenous natural rate $r_{\zeta,t}$ fluctuates stochastically (Figure 2.2b and Figure 2.3b). In the financially autarkic economy, the endogenous natural rate fluctuates deterministically (Figure 2.3b) and in equilibrium remains stable at zero (Figure 2.2b and Figure 2.3a). In the former two economies, financial intermediaries take on leverage, concentrate aggregate

[17]The covariance $Cov_t\left[dA_t/A_t, d\zeta_t/\zeta_t\right]$ is non-negative because the rates $\{dA_t/A_t, d\eta_t/\eta_t, du_t/u_t, d\zeta_t/\zeta_t\}$ co-move non-negatively.

risk in their balance sheets, and endogenous fluctuations are therefore positive and stochastic. In the latter economy, financial intermediaries do not take on leverage nor concentrate aggregate risk in their balance sheets, and all endogenous fluctuations therefore vanish.

Relative to the unregulated economy, in the regulated economy, the endogenous natural rate is higher (Figure 2.2b). The endogenous fluctuations in general are smaller (Figure 2.3b) because the pecuniary externality is constrained efficient. Smaller fluctuations on the growth rate of endogenous TFP in particular reduce aggregate output risk, reduce also aggregate consumption risk, and whence lift the rate of return on risk-free debt, and the endogenous natural rate.[18] Furthermore, in the regulated economy, when $\eta_t$ is low, and they are undercapitalized, financial intermediaries earn relatively large expected profits (Figure 2.3a). The reason is that their borrowing capacity and their leverage multiple are relatively larger (Figure 2.2a). The relative higher growth prospectus in financial intermediation and in endogenous TFP also push the endogenous natural rate upward.

In both the unregulated and the regulated economies, the endogenous natural rate is approximately-u-shaped related to the wealth share of financial intermediaries (Figure 2.2b). The rate $r_{\zeta,t}$ is negative and attains its minimum when $\eta_t$ is intermediate, because the feedback loop, the pecuniary externality, and the endogenous fluctuations on $d\zeta_t/\zeta_t$, attain their maximum values when financial intermediaries are average-capitalized and have significant price effects (see subsection 1.3.1 in Chapter 1). Large stochastic fluctuations on $d\zeta_t/\zeta_t$, in turn, imply high aggregate output risk, as well as high aggregate consumption risk, which depress the rate of return on risk-free debt and the endogenous natural rate. The rate $r_{\zeta,t}$ is positive and attains its highest possible values when $\eta_t$ is low because, whey they are poor-capitalized,

[18] In general, less aggregate consumption risk reduces the value of risk-free assets. In equilibrium, the risk-free rate of return then has to increase, to induce agents to take enough positions to clear the market of risk-free assets.

financial intermediaries earn large profits in expectation (see subsection 2.2.2). Large expected profits, in turn, speed up the recoveries of the wealth share $\eta_t$ and of the endogenous TFP $\zeta_t$ in expectation. The rate $r_{\zeta,t}$ is null when $\eta_t$ is high because financial intermediaries have idle borrowing capacity and absorb all of the aggregate capital stock. (Figure 2.2a). The fluctuations on $u_t = 1$ and on $d\zeta_t/\zeta_t = 0$ then vanish locally.

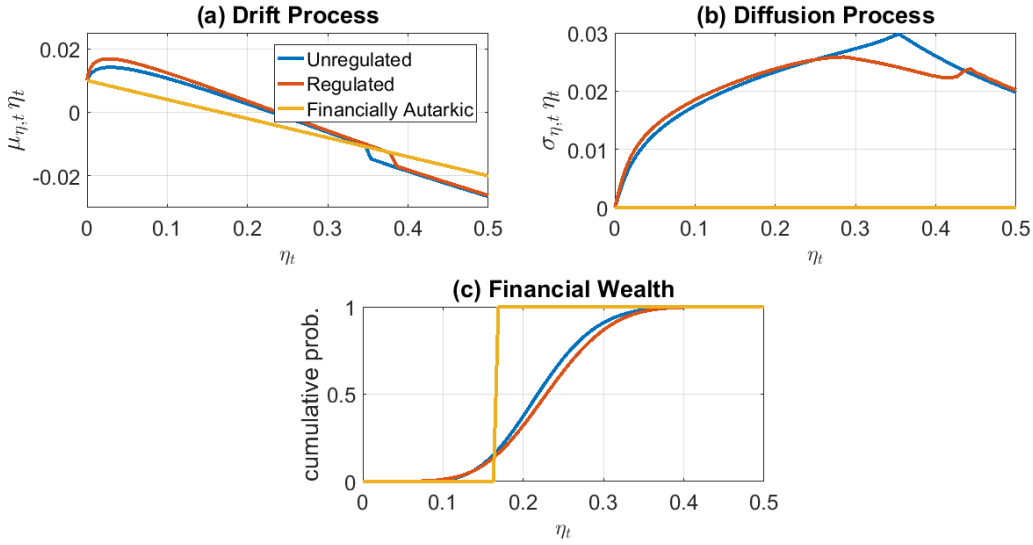



Figure 2.3: Dynamic Effects of Macro-prudential Policy

Let $\underline{r}_\zeta(\Phi_t)$ denote the lower bound of the endogenous natural rate for a given macro-prudential policy $\{\Phi_t\}$. Figure 2.2 together with Figure 2.3 evidence that tighter macro-prudential policies reduce the feedback loop, the pecuniary externality and the endogenous fluctuations, and increase the endogenous natural rate and its lower bound $\underline{r}_\zeta(\Phi_t)$ in equilibrium.

## 2.4 Cross-effects of Macro-prudential Policy on Macroeconomic Stability

In the model economy, macroeconomic stability refers to the situation in which there is no employment gap, nor price dispersion nor inflation. Macroeconomic stability is achieved when monetary policy mimics the natural rate with the nominal rate, and aggregate demand is the same as in the flexible price economy (see subsection 2.2.4).

Macro-prudential policy has cross-effects on macroeconomic stability when the natural rate occasionally attains negative values. In those situations, the ZLB constraint on the nominal rate occasionally prevents monetary policy from mimicking the natural rate, and the economy occasionally enters into a liquidity trap episode in which negative employment gaps, deflation and price dispersion materialize.

The natural rate $r_t$ can be decomposed into the neo-classical rate $r_{A,t}$ and the endogenous natural rate $r_{\zeta,t}$ (see section 2.3). In our parameterization, the neo-classical rate is constant and whence determines the level $r_A$ around which the natural rate fluctuates. The endogenous natural rate $r_{\zeta,t}$ in general fluctuates and its fluctuations (or lack thereof) depend on macro-prudential policy.

For any given macro-prudential policy $\Phi_t$, because the neo-classical rate is constant, the lower bound on the natural rate is $r_A + \underline{r}_{\zeta}(\Phi_t)$. In terms of lower bounds, macro-prudential policy has cross-effects on macroeconomic stability if $r_A + \underline{r}_{\zeta}(+\infty) < 0$. If $r_A + \underline{r}_{\zeta}(+\infty) \geq 0$, macro-prudential policy is irrelevant for macroeconomic stability, because the natural rate is always non-negative, and ZLB constraint on the nominal rate is always slack. None macro-prudential policy can push the lower bound $r_A + \underline{r}_{\zeta}(\Phi_t)$ below $r_A + \underline{r}_{\zeta}(+\infty)$, nor below the ZLB, because restrictions on intermediary leverage below $\min\{\lambda v_t, 1/\eta_t\}$ push the lower bound $\underline{r}_{\zeta}(\Phi_t)$ on the endogenous natural rate upward (see section 2.3).

The parameter values in the model economy ultimately determine whether macroprudential policy has cross-effects. The parameters in the model can be classified into four categories depending on the main function that they serve (see section 1.4 in Chapter 1).

Parameters concerning the price-setting behavior of firms (i.e. the arrival rate $\theta$ of the Poisson shock that allows firms to reset their price; and the elasticity of substitution $\varepsilon$ in the CES aggregator) do not affect the natural rate $r_t$ nor the lower bound $r_A + \underline{r}_\zeta(+\infty)$. Neither do the Frish elasticity of the aggregate labor supply $\psi$ nor the relative utility weight of labor $\chi$. The parameter $\varepsilon$ do not affect $r_t$ nor $r_A + \underline{r}_\zeta(+\infty)$ because in the flexible price economy firms reset their nominal price continuously (i.e. $1/\theta \to 0$) and neither is there price dispersion nor quantity dispersion on intermediate goods. The parameters $\psi$ and $\chi$ do not affect $r_t$ nor $r_A + \underline{r}_\zeta(+\infty)$ either because in the flexible price economy those parameters have no dynamic effects: $\psi$ and $\chi$ affect only the aggregate quantity of labor which is constant.

The remaining parameters in the model economy do affect the natural rate and the lower bound $r_A + \underline{r}_\zeta(+\infty)$. Table 2.1 shows the positive $(+)$, negative $(-)$, or null $(NA)$, effect on the lower bound $r_A + \underline{r}_\zeta(+\infty)$ of an increase in the remaining parameters.

Table 2.1: Effect of Parameters on the Lower Bound of the Natural Rate

| | Parameters | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\mu_A$ | $\sigma_A$ | $1/\lambda$ | $1-u_h$ | $\tilde{\rho}$ | $\kappa$ | $1-\alpha$ |
| Neo-classical rate $r_A$ | $+$ | $+$ | $-$ | $NA$ | $NA$ | $NA$ | $NA$ | $NA$ |
| Lower bound $\underline{r}_\zeta(+\infty)$ | $-$ | $NA$ | $-$ | $-$ | $-$ | $-$ | $+$ | $-$ |

The discount rate $\rho$ has a positive effect on the neo-classical rate $r_A$ and a negative effect on the lower bound $\underline{r}_\zeta(+\infty)$. The effect on $r_A$ is positive because in equilibrium the real interest rate compensates households for their degree of impatience: A higher $\rho$ implies that households are more impatient and that they discount future

consumption more. The effect on $\underline{r}_\zeta(+\infty)$ is negative because physical capital is long-term whereas deposits are short-term. The difference on maturities implies that discount rates have a proportionally larger effect on physical capital, which in turn implies a negative relationship between discount rates and credit spreads. A higher value of $\rho$ then reduces the profitability of financial intermediaries; generates more endogenous fluctuations on $d\eta_t/\eta_t$ and on $d\zeta_t/\zeta_t$; and shrinks the endogenous natural rate and its lower bound $\underline{r}_\zeta(+\infty)$. The discount rate $\rho$ overall has a positive effect on $r_A + \underline{r}_\zeta(+\infty)$ because the effect on the neo-classical rate is direct and whence dominates.

The rate of technological progress $\mu_A$ has a positive effect on the neo-classical rate $r_A$ but has no effect on the lower bound $\underline{r}_\zeta(+\infty)$. The effect on $r_A$ is positive because the rate of technological progress determines the rate of economic growth. The lower bound $\underline{r}_\zeta(+\infty)$ does not depend on $\mu_A$ because the equilibrium outcome is scale invariant with respect to $A_t$.

The fundamental risk coefficient $\sigma_A$ has a negative effect on both the neo-classical rate $r_A$ and the lower bound $\underline{r}_\zeta(+\infty)$. The effect on $r_A$ is negative because in equilibrium the real interest rate indicates the rate of return on risk-free assets: If there is more fundamental risk, and whence more aggregate risk, risk-free assets become more valuable and their rate of return falls in equilibrium. If $\sigma_A$ is higher, financial intermediaries concentrate more aggregate risk per unit of leverage, and the feedback loop, the pecuniary externality, and the endogenous fluctuations, are larger. Higher aggregate output risk as well as higher aggregate consumption risk result, which in turn depress the lower bound $\underline{r}_\zeta(+\infty)$.

In opposite to Brunnermeier and Sannikov (2014), in our model economy, a lower $\sigma_A$ does not generate larger endogenous fluctuations (i.e. the "volatility paradox" result in Brunnermeier and Sannikov 2014, in our model economy, in general does not hold). The reason is that financial intermediaries are subject to a leverage constraint.

The leverage constraint prevents financial intermediaries from taking substantially larger levered positions on capital when $\sigma_A$ is low. The leverage constraint therefore limits the effect of $\sigma_A$ on the feedback loop, on the pecuniary externality and on the endogenous fluctuations.

The remaining parameters in Table 1 affect only the lower bound $\underline{r}_\zeta(+\infty)$. The fraction of divertable assets $1/\lambda$ has a negative effect on $\underline{r}_\zeta(+\infty)$. When financial intermediaries can divert a larger fraction of their assets, moral hazard problems in financial markets become more severe, and the incentive-compatible limit $\lambda v_t$ falls. A lower borrowing capacity intensifies the feedback loop, the pecuniary externality, and the endogenous fluctuations, and therefore reduces the endogenous natural rate, and the lower bound $\underline{r}_\zeta(+\infty)$. The productivity gap $1-u_h$ also has a negative effect on $\underline{r}_\zeta(+\infty)$. When the productivity differences between financial intermediaries and households are larger, differences on their valuations for capital are larger as well, and the feedback loop, the pecuniary externality, and the endogenous fluctuations are larger too. The endogenous natural rate and the lower bound $\underline{r}_\zeta(+\infty)$ are therefore lower.

The effect of the parameters $\tilde{\rho}$ and $\kappa$ on $\underline{r}_\zeta(+\infty)$ depend on their corresponding effect on the net transfers that financial intermediaries payout to households. More and/or more often net transfers deplete the net worth of financial intermediaries faster, and therefore increase the share of time that the wealth share $\eta_t$ spends in regions in which the feedback loop, the pecuniary externality and the endogenous fluctuations are larger. More and/or more often net transfers then reduce the endogenous natural rate and its lower bound $\underline{r}_\zeta(+\infty)$. The average frequency $\tilde{\rho}dt$ at which financial intermediaries payout dividends to households have a positive effect on net transfers. The share $\kappa$ of the aggregate capital stock that starting financial intermediaries receive from households has a negative effect on net transfers.

Lastly, the capital share of output $1-\alpha$ has a negative effect on $\underline{r}_{\zeta}(+\infty)$. A higher share $1-\alpha$ increases the share of capital services on aggregate output as well as the endogenous TFP component $\zeta_t$. The fluctuations on $\zeta_t$ therefore become larger and the lower bound $\underline{r}_{\zeta}(+\infty)$ falls.

Overall, Table 1 together with expression for $r_A$ reveal that secular stagnant economies and/or economies prone to instability in financial markets are more likely to have a negative $r_A+\underline{r}_{\zeta}(+\infty)$. A secular stagnant economy is an economy in which poor fundamentals place the neo-classical rate close to zero. An economy prone to instability in financial markets is an economy in which severe moral hazard problems and/or strong valuation differences generate large endogenous fluctuations and highly negative endogenous natural rates.

When the lower bound $r_A+\underline{r}_{\zeta}(+\infty)$ is negative, and macro-prudential policy has cross-effects on macroeconomic stability, tighter macro-prudential policies in general reduce macroeconomic instability. Tighter macro-prudential policies lift the natural rate and whence reduce frequency and intensity of liquidity traps. Liquidity traps occur less often because higher natural rates fall below the ZLB bound on the nominal rate less often. Even if liquidity traps occur, when they occur, their intensity is lower, because the gap between zero and the natural rate falls.

In the case in which macro-prudential policy minimizes (the present discounted value of) the utility losses from financial disintermediation, either one of two situations may arise. In a first situation, the optimal macro-prudential policy rule $\{\Phi_{*,t};\check{\eta}_*,\hat{\eta}_*\}$ places the natural rate above the ZLB constraint on the nominal rate. In this first situation, a divine coincidence result between macroeconomic stability and financial stability holds, in the sense that both monetary policy and macro-prudential attain their corresponding objectives. Indeed, because the natural rate becomes non-negative as a by-product of using macro-prudential policy to achieve financial stability, monetary policy can effectively achieve macroeconomic stability.

In the second situation, the optimal macro-prudential policy rule $\{\Phi_{*,t}; \check{\eta}_*, \hat{\eta}_*\}$ is not tight enough to place the natural rate above the ZLB constraint. In this second situation, no divine coincidence result between macroeconomic stability and financial stability holds. Nonetheless, macro-prudential policy is still beneficial to monetary policy and for macroeconomic stability, because it reduces the gap between zero and the natural rate.

## 2.5 Conclusion

The interaction between monetary policy and macro-prudential policy is two-sided. In Van der Ghote (2017), we focus mainly on the side concerning the cross-effects of monetary policy on the primary objective of macro-prudential policy of financial stability. In this paper, instead, we focus on the side concerning the cross-effects of macro-prudential policy on the primary objective of monetary policy of macroeconomic stability.

In this paper, we show that tighter macro-prudential policies reduce the aggregate fluctuations resulting from frictions in financial markets; reduce also aggregate output risk and aggregate consumption risk; and whence lift the natural rate. In addition, we show that the resulting effect on the natural rate reduces macroeconomic instability when the natural rate occasionally attains negative values and the ZLB constraint on the nominal rate occasionally prevents monetary policy from mimicking the natural rate. We obtain that macro-prudential policy has cross-effects on macroeconomic stability in secular stagnant economies in which poor fundamentals place the natural rate close to zero. We obtain also that macro-prudential policy has cross-effects in financially unstable economies in which severe moral hazard problems and/or strong valuation differences generate large endogenous fluctuations in financial markets and in the natural rate.

In this paper, we abstract from the social welfare considerations concerning the coordination between monetary policy and macro-prudential policy. We leave for future research an examination of the costs and benefits of coordinating monetary and macro-prudential policies in secular stagnant and/or financially unstable economies.

# Chapter 3

# Liquidity Management, Leverage, and Monetary Policy

## 3.1 Introduction

The provisions of settlement services and of financial intermediary services have traditionally been interrelated. At least since the Renaissance roots of banking in the 15th century, the financial institutions at the center of the settlement system have traditionally taken advantage of their key position to also intermediate funds and credit between borrowers and savers.[1] The modern macroeconomic literature nonetheless has overlooked the interplay between the provisions of settlement services and of financial intermediary services. The main reason has been that the settlement and the financial intermediary systems have functioned smoothly in developed economies during the postwar era.

Motivated by the recent Global Financial Crisis of 2008, this paper examines how the joint provision of settlement services and financial intermediary services af-

[1]See Cipolla (1956) for a description of the banking system in the Mediterranean World of the 5th to the 17th century. See Roberds (2008) and Kahn and Roberds (2009) for a survey of the payment and the settlement systems in the modern U.S. economy.

fects liquidity management and leverage decisions at the financial intermediary level. Additionally, this paper explores the real effects of monetary policy that follow from affecting the real return of liquid assets and the aforementioned liquidity management and leverage decisions.

To conduct the analysis, we develop a standard Gertler and Kiyotaki (2010) economy in which financial intermediaries face also a liquidity management problem. In the model economy, financial intermediaries are good relative to households at allocating funds across non-financial firms. Non-financial firms are mere production units that use credit to produce a final consumption good. In equilibrium, financial intermediaries raise deposits from households to channel funds to non-financial firms beyond the limits given by their own net worth. Financial intermediaries maintain a sufficiently low leverage multiple[2] because they are subject to a moral hazard problem (Gertler and Kiyotaki 2010). The moral hazard problem stipulates that financial intermediaries can divert funds at the penalty of loosing their franchise.

The liquidity management problem originates from the dual role of store-of-value and mean-of-payment that deposits usually serve. In the model economy, in particular, besides being useful for borrowing and saving, deposits are also useful for settling transactions. We take the institutional features of deposits as given.[3] We also take as given the procedure through which households decide to settle transactions with deposits. Following Bianchi and Bigio (2014) and Piazzesi and Schneider (2016), we assume that households randomly reshuffle deposits across financial intermediaries, as a consequence of settling with deposits the transactions that they periodically conduct among each other. The resulting withdrawal risks on deposits affect the cost of leverage and the risk-return profile of financial intermediation because net deposit outflows are subject to a settlement requirement. The settlement requirement

[2]In the model economy, the leverage multiple is the ratio of deposits plus net worth to deposits.
[3]See Stein (2012) for a discussion about the different roles that deposits serve.

obliges financial intermediaries to settle all of their net deposit outflows with reserves. Reserves are nominal assets that serve the role of the legal tender.

At the core of the liquidity management problem are the decisions of how many reserves to hold and of much withdrawal risks to bear. Ex-post, i.e. following the realization of withdrawal shocks, financial intermediaries face liquidity needs if their net deposit outflows exceed their real reserves holdings; they enjoy liquidity excesses if the opposite happens. Those who face liquidity needs have to borrow reserves ex-post to complete the settlement of their net deposit outflows; those who enjoy liquidity excesses can lend their excess reserves over settlement requirements if they want to. Borrowing reserves in general is more costly than what lending reserves is profitable because there is a Monetary Authority that charges a high discount window rate but pays a low interest rate on excess reserves. Besides trading with the Monetary Authority, financial intermediaries can also trade reserves among each other, but successfully only in probability, and at a market rate that in equilibrium is a weighted average of the aforementioned two policy rates. The optimal liquidity ratio[4] that solves the liquidity management problem in general matches the expected marginal benefits from borrowing/lending ex-post less/more reserves to the opportunity cost of holding reserves. Reserves carry the opportunity cost of the foregone real interest on deposits plus the inflation rate, by the Fisher equation, of the nominal interest rate.

We first show that in equilibrium there is a positive interaction between liquidity management and leverage decisions. To this end, we contrast the equilibrium outcome of the economies with and without liquidity. The contrast between the aforementioned two economies reveals that the optimal liquidity ratio reduces the cost of leverage and whence improves the risk-return profile of financial intermediation. The profitability and the Tobin's Q of financial intermediaries therefore is higher in the economy with

[4]In the model economy, the liquidity ratio is the ratio of reserves to deposits.

liquidity, and moral hazard problems are softer while borrowing capacities are larger. Financial intermediaries in general respond to a larger borrowing capacity by increasing their leverage multiple. More leverage implies more deposits, a higher exposure to withdrawal risks, and a therefore higher demand for real reserves holdings.

Monetary policy comprises three independent interest-rate instruments: The nominal interest rate, the discount window rate, and the interest rate on excess reserves. We consider the case in which the Monetary Authority adjusts the nominal interest rate to sustain an annual target of inflation. To explore the real effects of monetary policy, we consider two simple policy experiments. In the first experiment, the target for annual inflation falls by 1%. In the second experiment, the width of the corridor narrows by 0.5%. The corridor here refers to the spread between the discount window rate and the interest rate on excess reserves. In our model economy, there is a corridor system because the market rate is the simple average of the aforementioned two policy rates. As in the policy jargon, in our model economy, narrowing the corridor shrinks the width of the corridor without moving its center, which is the market rate.

In the first experiment, we obtain that liquidity ratios increase. Furthermore, we obtain that liquidity ratios increase proportionally more in a region in which financial intermediaries are undercapitalized and financially constrained. The rationale is that in such region leverage multiples and whence the exposure to withdrawal risks are higher. We obtain also that leverage multiples increase as well: A lower opportunity cost of holding reserves reduces the cost of hedging against withdrawal risks and whence the cost of leverage. A lower cost of leverage, in turn, boosts the profitability of financial intermediation, relaxes agency problems in financial markets, and boosts borrowing capacities. Lastly, we obtain that the economy spends more time in a region in which financial intermediaries are better capitalized, channel further funds to non-financial firms, and the ratio of aggregate output to the aggregate stock of capital is higher.

In the second experiment, we obtain that liquidity ratios increase, or decrease, depending on whether the financial intermediary system is liquid, or illiquid. The financial intermediary system is liquid if financial intermediaries on aggregate hold enough reserves to meet all of their liquidity needs; the financial intermediary system is illiquid if the opposite happens. If the financial intermediary system is liquid, none financial intermediary borrows reserves in the discount window, and narrowing the corridor therefore improves the terms of trade of lending reserves (the interest rate on excess reserves increases) while keeps that of borrowing reserves constant (the market rate remains the same). Financial intermediaries respond to higher lending rates by increasing their liquidity ratio, which in turn reinforces the initial high liquidity status of the financial intermediary system. If the financial intermediary system is illiquid, the opposite happens: None financial intermediary parks reserves in the balance sheet of the Monetary Authority, and narrowing the corridor therefore improves the terms of trade of borrowing reserves (the discount window rate falls) while keeps that of lending reserves constant. Financial intermediaries respond to lower borrowing rates by reducing their liquidity ratio which reinforces the initial high illiquidity status of the financial intermediary system.

The financial intermediary system is liquid or not in equilibrium depending on monetary policy. In particular, for a given width of corridor, we obtain that the financial intermediary system is liquid if the market rate is high relative to the target of inflation.

Lastly, we obtain that the response of leverage multiples to a narrower corridor is independent of the liquidity status of the financial intermediary system. Leverage multiples always increase because a narrower corridor always improves the terms of trade of trading reserves: The market rate remains the same while the discount window rate falls and the interest rate on excess reserves increases. Better terms of

trade reduces the cost of leverage, relaxes agency problems in financial markets, and expands borrowing capacities.

**Related Literature** This paper relates to a body of literature that studies the interactions between the liquidity and leverage at the financial intermediary level and their implications for monetary policy. Following Bianchi and Bigio (2014) and Piazzesi and Schneider (2016), we introduce liquidity concerns based on the dual role of store-of-value and mean-of-payment that deposits usually serve and on the reserve requirements that deposits usually have to satisfy. A key difference with respect to Bianchi and Bigio (2014) is that in our model economy there is no fractionary reserve requirement on deposits. A key difference with respect to Piazzesi and Schneider (2016) is that we work with a continuous-time dynamic stochastic general equilibrium (DSGE) model, that is suitable for capturing the non-linear dynamics associated with occasionally constrained agents. Drechsler, Savov and Schnabl (2015a) also work with a standard continuous-time DSGE model but they introduce liquidity concerns based on the high re-saleability property of reserves in short notice.[5] Another difference with respect to Drechsler, Savov and Schnabl (2015a) is that they focus only on the case in which financial intermediaries fully self-insurance against funding/liquidity risks. The main difference with respect to Brunnermeier and Sannikov (2016) is that they introduce liquidity concerns based on the store-of-value property of assets and on the different dividend payment structures that assets in general provide. Classical articles on this body of literature comprise Holmstrom and Tirole (1998); Bernanke and Blinder (1988); Gurley and Shaw (1964); and Bagehot (1873).

On methodological grounds, our model economy builds upon the works of Gertler and Karadi (2011) and Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); Bianchi and Bigio (2014); and Van der Ghote (2017). We adapt the Gertler and Kiyotaki (2010) economy to continuous time, to account for the non-linear

[5]Intuitively, the idea is that reserves can be sold at a fair price in short notice whereas risky assets have to be sold at a discount.

dynamics associated with occasionally constrained agents. To this end, we use a continuous-time framework that is similar to Brunnermeier and Sannikov (2014) and Van der Ghote (2017). We borrow from Bianchi and Bigio (2014) the market structure of the market for reserves.

## 3.2 The Model

Time is continuous and runs forever. The model is a standard Gertler and Kiyotaki (2010) economy in which financial intermediaries face also a liquidity management problem. The model adapts the Gertler and Kiyotaki (2010) economy to continuous time using a framework that is similar to Brunnermeier and Sannikov (2014) and Van der Ghote (2017).

### 3.2.1 Agents

A continuum of identical households and of financial intermediaries populate the model economy. Households are composed of a continuum of family members of unit measure. Family members share the same preferences for consumption and the same discount rate, $\rho > 0$. There is perfect consumption insurance within the household meaning that all of its family members share their income to always consume the same amount of goods, $c_t \geq 0$.

Family members serve different occupations. A fraction $f \in (0, 1)$ of them are financiers and the remaining fraction are savers. Family members switch their occupation stochastically according to Poisson processes with arrival rate $\tilde{\rho} > 0$ for financiers, and arrival rate $\tilde{\rho} f / (1 - f)$ for savers.

Financiers run the financial intermediaries and savers run the household. Specifically, financiers take the investment portfolio decisions of the financial intermediary company that they manage. Savers choose consumption $c_t$ on behalf of all the family

members that constitute the household. Savers also take the investment portfolio decisions of the household that they manage.

### 3.2.2 Assets and Production Technologies

Asset classes comprise physical capital, deposit contracts, reserves and government bonds.

**Physical capital** Physical capital is a real asset. Physical capital is the key input for producing a final consumption good. There is a linear production technology that transforms physical capital $k_t$ into final output flows $y_t$ according to

$$y_t = ak_t$$

with productivity coefficient $a \in \{a_h, a_f\}$. The value of the productivity coefficient depends on who manages the physical capital. If financial intermediaries manage the $k_t$ units of physical capital, $a = a_f$. Otherwise, if households manage the $k_t$ units, $a = a_h < a_f$. The gap between $a_f$ and $a_h$ measures the comparative advantage that financial intermediaries have at managing physical capital relative to households. Intuitively, the gap $a_f - a_h$ captures the idea that financial intermediaries are good relative to households at allocating funds across non-financial firms; the latter being the agents ultimately involved in the production process of the final consumption good.[6]

Physical capital evolves stochastically according to

$$dk_t/k_t = [I(\iota_t) - \delta]\, dt + \sigma dZ_t \tag{3.1}$$

[6]See Van der Ghote (2017) for a model economy that explicitly incorporates lending/borrowing relationships between financial intermediaries/households and non-financial firms.

The process $Z_t$ is a standard Wiener process defined on a filtered probability space $(\Omega, \mathcal{F}, P)$. The term $dZ_t$ is an aggregate shock that is common to all units of physical capital independently of who manages them. The term $dZ_t$ can also be interpreted as an shock aggregate to the effective units of physical capital.[7] The diffusion coefficient $\sigma$ measures the strength at which the $dZ_t$ shock affects the growth rate of physical capital.

Physical capital depreciates deterministically at the constant rate of $\delta$. The depreciation rate $\delta$ is common to all units of physical capital as well. There is an internal investment technology that transforms $\iota_t k_t$ units of final output into physical capital at the rate of $I(\iota_t)$. The function $I(\iota_t)$ represents a standard investment technology with adjustment costs: $I(0) = 0, \quad I'(\iota) > 0, \; I''(\iota) < 0$.

The total return on capital $dR_{a,t}$ depends on who manages the physical capital. Let $q_t$ denote the price of capital in terms of the final consumption good.[8] The total return $dR_{a,t}$ amounts to the sum of the net dividend yields and the capital gain/loss rate. Specifically,

$$dR_{a,t} \equiv \frac{a - \iota_t}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t}$$

with $a \in \{a_h, a_f\}$. Net dividend yields are the ratio of net dividend returns $(a - \iota_t) k_t$ to the market value of capital holdings $q_t k_t$. Dividend returns are net of expenditures on internal investment. The capital gain/loss rate is the percentage change on the market value of capital holdings during the time interval $dt$.

The total return on capital that financial intermediaries earn is $dR_{a,t}$ evaluated at $a = a_f$ while that that households earn is also $dR_{a,t}$ but evaluated at $a = a_h$. Because the productivity gap $a_f - a_h$ is positive, the total return $dR_{f,t}$ that financial intermediaries obtain is higher than the total return $dR_{h,t}$ that households obtain.

[7] See Brunnermeier and Sannikov (2014) and di Tella (2015) for a detailed discussion.

[8] Physical capital is tradable. Physical capital is traded on the spot at the real price of $q_t$.

We conjecture that the price of capital $q_t$ follows an Ito process with drift process $u_{q,t}$ and diffusion process $\sigma_{q,t}$.[9] Total returns on capital are locally risky and satisfy

$$dR_{a,t} = \left[\frac{a - \iota_t}{q_t} + I\left(\iota_t\right) - \delta + u_{q,t} + \sigma_{q,t}\sigma\right] dt + \left(\sigma_{q,t} + \sigma\right) dZ_t$$

**Deposits** Deposits are financial securities. Specifically, deposits are in zero-net supply; agents can issue as well as save on deposits. Deposits have a short-term maturity and a fixed payment structure: Deposits issued at time $t$ mature at time $t + dt$ and yield a locally risk-free nominal interest rate of $i_t \geq 0$. Let $p_t$ denote the nominal price of the final consumption good. The real return on deposit is then $i_t dt - dp_t/p_t$, being $dp_t/p_t$ the inflation rate between time $t$ and time $t + dt$.

Deposits serve the dual role of store-of-value and mean-of-payment. Besides being useful for borrowing and saving, deposits are also useful for settling transactions.[10] We do not explicitly model the procedure through which depositors decide to use deposits to settle transactions. Instead, we introduce settlement decisions in reduced-form. Our approach is based on Bianchi and Bigio (2014) and Piazzesi and Schneider (2016).

Specifically, depositors settle transactions with deposits stochastically. The settlement of transactions with deposits generates in probability a situation in which net deposit positions across issuers become unbalanced (i.e. gross deposit positions differ before and after settlement for at least two issuers of deposits). For simplicity, we assume that depositors only reshuffle deposits across issuers. The aggregate stock of deposits therefore remains the same before and after settlement.

An aggregate Poisson process $J_t$ dictates whether net deposit positions become unbalanced after settlement. The process $J_t$ is exogenous and has an arrival rate of $\theta$.

[9] The processes $\mu_{q,t}$ and $\sigma_{q,t}$ are endogenous objects to be determined in equilibrium.

[10] Only the agents who save on deposits, i.e. depositors, can use deposits to settle transactions.

When $J_t$ does not jump, i.e. $J_t^+ - J_t = 0$,[11] nothing relevant happens as net deposit positions remain perfectly balanced across issuers. When $J_t$ jumps, i.e. $J_t^+ - J_t = 1$, net deposit positions become unbalanced and are whence reshuffled stochastically across issuers according to a continuous cumulative probability distribution $F$. The domain of the function $F(\omega)$ is the interval $(-\infty, 1]$. A negative realization $\omega < 0$ means that the corresponding issuer receives a share $-\omega$ of net deposit inflows. The share $-\omega$ is computed relative to the stock of deposits before settlement. A positive realization $\omega > 0$ has the same interpretation but for net deposit outflows. The function $F(\omega)$ satisfies that $\int \omega dF(\omega) = 0$, confirming that deposits are just re-allocated across issuers.[12] We interpret a positive shock $\omega$ as a withdrawal shock on deposits.

Deposits are subject to an immediate settlement requirement. The settlement requirement obliges issuers to settle all of their net deposit outflows with reserves, immediately after they occur (i.e. immediately after $J_t$ jumps) and immediately before time $t + dt$ arrives. The settlement requirement implies that withdrawal risks on deposits are potentially costly for issuers. Whether withdrawal risks are costly or not depends on the trading protocol of reserves and on the opportunity cost of holding reserves.[13]

**Reserves** Reserves are nominal assets in non-negative supply. A Monetary Authority issues and distributes reserves. The Monetary Authority pays no nominal interest rate on reserves and, therefore, the real interest rate on reserves equals the

[11] The Poisson processes $J_t$ and $J_t^+$ count number of jumps. The process $J_t$ counts the number of jumps until time $t$. The process $J_t^+$ counts the number of jumps until *immediately* after time $t$. Through the lens of discrete-time models, intuitively, $J_t$ counts the number of jumps up to the morning of date $t$ whereas $J_t^+$ counts the number of jumps up to the night of date $t$. Jumps, if any, occur at noon.

[12] The shock $\omega$ is independent of the size of the issuer.

[13] In our model economy, there is no fractionary reserve requirement on deposits. That is, issuers are not obliged to park a fraction of their deposits on reserves. Incorporating a fractionary reserve requirement is feasible (see Bianchi and Saki 2014 and Piazzesi and Schneider 2016). The model economy can also be accommodated to impose a settlement requirement on *gross* deposit outflows (rather than on *net* deposit outflows).

negative of the inflation rate. The Monetary Authority may nonetheless pay a positive rate of return on *excess reserves* (see below).[14]

Reserves are tradable and are traded both in primary markets and in secondary markets at a real price of $1/p_t$. Reserves are liquid meaning that they are also traded between the moment at which $J_t$ jumps, if any, and time $t + dt$. Reserves are the only liquid asset in the economy. Conditional on a jump $J_t^+ - J_t = 1$, the trading protocol of reserves, and the settlement protocol of net deposit outflows with reserves, are as follows.

Trading and settlement protocols involve two stages. There is no discounting between stages. At the first stage, there is no trading but only settlement. Specifically, at the first stage, issuers have to settle the largest possible share of their net deposit outflows with their own reserves holdings. Let $\bar{\omega}_t \geq 0$ denote the liquidity ratio of issuers just before the jump $J_t^+ - J_t = 1$ occurs.[15] Following this first round of settlement, the reserves holdings that issuers end up with, as well as their liquidity needs, depend on the relative size of the withdrawal shock $\omega$ to that of the liquidity ratio $\bar{\omega}_t$. If $\omega > \bar{\omega}_t$, issuers lose all of their reserves holdings and end up with liquidity needs of $\omega - \bar{\omega}_t$ per unit of initial deposits. Issuers therefore will have to borrow later on $\omega - \bar{\omega}_t$ units of real reserves (per unit of initial deposits) to finish settling all of their net deposit outflows. If $\omega \in [0, \bar{\omega}_t]$, issuers lose a share $\omega/\bar{\omega}_t$ of their reserve holdings but end up without liquidity needs. Issuers indeed end up with *excess reserves* over settlement needs of $\bar{\omega}_t - \omega$ per unit of initial deposits, and can lend those excess reserves later on if they want to. If $\omega < 0$, issuers keep all of their reserves holdings and end up with liquidity excesses as well. Their excess reserves not only comprise their initial reserves holdings but also the reserves inflows that they receive jointly with the net deposit inflows $-\omega$. Reserves inflows per unit of deposits amount to

[14] Introducing a positive nominal interest rate on reserves is feasible.

[15] The liquidity ratio is the ratio of the real reserves holdings to the quantity of deposits issued.

$\Delta_t(\omega;\tilde{\omega}_t)$, with

$$\Delta_t(\omega;\tilde{\omega}_t) \equiv \frac{-\omega}{E_F\left[-\hat{\omega}\mathbf{1}_{\hat{\omega}<0}\right]} E_F\left[\hat{\omega}\mathbf{1}_{\hat{\omega}\in[0,\tilde{\omega}_t]} + \tilde{\omega}_t\mathbf{1}_{\hat{\omega}>\tilde{\omega}_t}\right]$$

In this notation, $\hat{\omega}$ denotes the withdrawal shock of the other issuers in the economy and $\tilde{\omega}_t$ denotes their corresponding liquidity ratio.[16] The expectation operator $E_F[.]$ is defined relative to the cumulative probability distribution function of withdrawal shocks $F$. The first factor in the RHS is the share of the aggregate net deposit outflows that the issuers with withdrawal shock $\omega < 0$ receive. The second factor is the aggregate reserves holdings that the others issuers in the economy use to settle net deposit outflows.

At the second stage, after the first round of settlement concludes, a market for reserves opens. In the market for reserves, only issuers can trade; in particular, the Monetary Authority cannot trade. Issuers who have excess reserves can place lending orders to try to lend their reserves holdings in the market. The other issuers can place borrowing orders to try to borrow reserves. The market for reserves is a directed over-the-counter market (OTC) that has a similar structure to Bianchi and Bigio (2014).[17] Besides trading in the market, issuers can also trade reserves with the Monetary Authority. Issuers who have excess reserves can park their reserves in the balance sheet of the Monetary Authority at a rate of return of $r_{e,t}$. Issuers who need reserves can borrow in the discount window at a rate of return of $r_{w,t}$. The Monetary Authority sets both the interest rate on excess reserves $r_{e,t}$ and the discount window rate $r_{w,t}$ unilaterally. The Monetary Authority commits to trade any quantity of

[16] We conjecture that all issuers set the same liquidity ratio of $\tilde{\omega}_t$. Of course, under our conjecture, $\bar{\omega}_t = \tilde{\omega}_t$.

[17] Specifically, the OTC market for reserves is such that: $(i)$ issuers can place a continuum of orders (but cannot place orders beyond their reserve needs or holdings); $(ii)$ borrowing and lending orders are randomly matched among each other; $(iii)$ bargain takes place between orders (not between the issuers who place the orders); $(iv)$ orders bargain about real transfers according to Nash bargaining; and $(v)$ orders do not bargain collectively (i.e. orders from a same issuer take as given the outcome of the other orders).

reserves at those rates. For simplicity, we express $r_{e,t}$ and $r_{w,t}$ in real terms. We interpret the rates $r_{e,t}$ and $r_{e,t}$ as fee payments within time $t$ (rather than as interest payment between different time periods). We restrict attention to the case in which $r_{w,t} > r_{e,t} \geq 0$, which implies that borrowing reserves at the discount window is costly and that parking reserves in the balance sheet of the Monetary Authority is weakly profitable.

The trading behavior of the Monetary Authority and the OTC market structure of the market for reserves determine the trading outcome of reserves. By trading outcome we mean the trading strategies of issuers; the terms of trade and payoffs that issuers obtain; and the market rate $r_{m,t}$.

The policy rates $r_{w,t}$ and $r_{e,t}$ represent the outside options to trading reserves in the market. The rate $r_{w,t}$ represents the outside option to borrowing reserves. The rate $r_{e,t}$ represents the outside option to lending reserves. The market for reserves admits a single rate of return $r_{m,t}$ (i.e. all trades at the market are executed at the same rate $r_{m,t}$) because bargaining takes place between reserve orders.[18] Furthermore, because reserve orders Nash bargain about real transfers, the market rate $r_{m,t}$ solves

$$r_{m,t} = \arg\max_{\tilde{r}\in\mathbb{R}} \left(r_{w,t} - \tilde{r}\right)^{\xi} \left(\tilde{r} - r_{e,t}\right)^{1-\xi}$$

being $\xi \in [0,1]$ the bargaining power of borrowing orders.

Issuers get better terms of trade in the market than with Monetary Authority because the market rate $r_{m,t}$ is a weighted average of the high discount window rate $r_{w,t}$ and the low interest rate on excess reserves $r_{e,t}$. Therefore, issuers try to place as many successful orders of reserves in the market as possible. The terms of trade that issuers ultimately obtain are given by the effective rate of return for borrowing reserves $r_{b,t}$ and by the effective rate of return for lending reserves $r_{l,t}$. The effective

[18] See Afonso and Lagos (2012) and Bianchi and Bigio (2014) for a detailed discussion. The rationale is that value functions do not affect the outcome of the bargain if orders (rather than issuers) are the ones who bargain.

rates $r_{b,t}$ and $r_{l,t}$ satisfy

$$r_{b,t} = r_{m,t} \min\left\{ \frac{\tilde{\omega}_t}{E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\omega > \tilde{\omega}_t}\right]}, 1 \right\} + r_{w,t} \max\left\{ \frac{E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}\right] - \tilde{\omega}_t}{E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}\right]}, 0 \right\}$$

$$r_{l,t} = r_{m,t} \min\left\{ \frac{E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}\right]}{\tilde{\omega}_t}, 1 \right\} + r_{e,t} \max\left\{ \frac{\tilde{\omega}_t - E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\hat{\omega} > \tilde{\omega}_t}\right]}{\tilde{\omega}_t}, 0 \right\}$$

The effective rates $r_{b,t}$ and $r_{l,t}$ are a weighted average of the market rate $r_{m,t}$ and of the policy rates $\{r_{e,t}, r_{w,t}\}$. The weights in $r_{b,t}$ and $r_{l,t}$ reflect the likelihood of trading orders successfully in the market. Those likelihoods depend on the trading behavior of all the issuers in the economy, and therefore are taken as given by individual issuers. If issuers on aggregate have enough reserves holdings to meet all of their liquidity needs in the first round of settlement, i.e. $\tilde{\omega}_t > E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\omega > \tilde{\omega}_t}\right]$, all borrowing orders trade successfully in the market and $r_{b,t} = r_{m,t}$. The effective lending rate $r_{l,t}$ is a weighted average of $r_{m,t}$ and $r_{e,t}$ because only the share $E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\omega > \tilde{\omega}_t}\right]/\tilde{\omega}_t$ of lending orders trade successfully. In this case, issuers who borrow reserves obtain the best possible terms of trade given the policy rates $\{r_{e,t}, r_{w,t}\}$. Those who lend reserves obtain relatively poor terms of trade. If, alternatively, issuers on aggregate lack enough reserves holdings to meet all of their liquidity needs in the first round of settlement, i.e. $\tilde{\omega}_t < E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\omega > \tilde{\omega}_t}\right]$, the opposite happens. Specifically, all lending orders trade successfully in the market and whence $r_{l,t} = r_{m,t}$. The effective borrowing rate $r_{b,t}$ is a weighted average of $r_{m,t}$ and $r_{w,t}$ because only the share $\tilde{\omega}_t / E_F\left[(\hat{\omega} - \tilde{\omega}_t)\,\mathbf{1}_{\omega > \tilde{\omega}_t}\right]$ of borrowing orders trade successfully. In this second case, issuers who lend reserves obtain the best possible terms of trade while those who borrow reserves obtain relatively poor terms of trade.

A second and final round of settlement begins after the trade of reserves concludes. Payments related to lending/borrowing reserves in the market, and to lending/borrowing with/against the Monetary Authority, are executed simultaneously

with this second round of settlement. We assume that after the second round of settlement concludes deposits are reshuffled back to the position prior to the jump $J_t^+ - J_t = 1$. We assume also that re-allocating deposits and reserves after at the second reshuffle is costless. These two assumptions simplify the analysis: Together, they imply that, at the beginning of time $t + dt$, net deposit positions remain perfectly balanced across issuers, independently of whether $J_t$ has jumped during the time interval $dt$ or not.[19]

**Government Bonds** Government bonds are financial assets in perfectly elastic supply. Government bonds are short-term and pay a locally risk-free nominal interest rate. Government bonds are valid for settling transactions among depositors but not for settling net deposit outflows. The nominal interest rate on government bonds is therefore the same as the nominal deposit rate (government bonds and deposits are perfectly arbritraged). From now on, then, we refer to $i_t$ as the nominal interest rate in the economy.

Government bonds matter only for the implementation of the nominal interest rate $i_t$. The Monetary Authority implements a nominal interest rate through open market operations: That is, the purchase of government bonds with reserves, and vice versa, at the prevailing market prices. Following Drechsler, Savov and Schnabl (2015a), we restrict attention to implementation mechanisms that generate a locally risk-free inflation rate

$$dp_t/p_t = \pi_t dt + 0 * dZ_t \tag{3.2}$$

with $\pi_t dt \equiv E_t \left[ dp_t/p_t \right]$. A locally risk-free inflation rate is consistent with non-financial firms that adjust their nominal price sluggishly according to Calvo (1983) pricing.[20] In subsection 3.2.6, we derive the law of motions for government bonds and

[19]The two simplifying assumptions are consistent with idea that withdrawal shocks on deposits are mere funding/liquidity shocks to financial intermediaries.

[20]See Van der Ghote (2017) for a formal proof and a comprehensive discussion.

for the aggregate stock of reserves that are consistent with a locally risk-free inflation rate.

We assume for simplicity that neither financial intermediaries nor households can take positions government bonds. We assume also that the real interest rate payments on government bonds are financed with lump-sum taxes on households.

### 3.2.3 Financial Intermediaries' Portfolio Problem

Based on the spread $dR_{f,t} - dR_{h,t} > 0$, we conjecture that financial intermediaries borrow to take levered positions on physical capital and that households lend. Let $n_{f,t} \geq 0$ denote the net worth of financial intermediaries. The capital positions that financial intermediaries take $k_{f,t}$ therefore satisfies $q_t k_{f,t} > n_{f,t}$ generically.[21]

The investment portfolio decisions of financial intermediaries comprise $\{k_{f,t}, m_{f,t}/p_t; \iota_t\}$. The process $m_{f,t}/p_t \geq 0$ is the real position that financial intermediaries take on reserves. The investment portfolio decisions of financial intermediaries dictate the evolution of net worth $n_{f,t}$ accordingly

$$\begin{aligned} dn_{f,t} &= dR_{f,t} q_t k_{f,t} - \pi_t m_{f,t}/p_t dt - (i_t - \pi_t)\left(q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}\right) dt \\ &\quad -\tau_{1,t} n_{f,t} dt + \left[R_t\left(\omega; \bar{\omega}_t\right) * \left(q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}\right) + \tau_{2,t}\left(\omega\right) n_{f,t}\right] * \left(J_t^+ - J_t\right) \end{aligned} \tag{3.3}$$

The first line in the RHS describes the real return of the investment portfolio. The first line assumes that withdrawal shocks on deposits do not materialize, namely, that $J_t^+ - J_t = 0$. The quantity $q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}$ is the amount of deposits that financial intermediaries issue. The first term in the second line of the RHS is a tax rate on financial intermediaries. The tax rate $\tau_{1,t}$ is proportional on net worth.

The second term in the second line of the RHS describes the gains/losses in net worth $n_{f,t}$ associated with withdrawal risks on deposits. The function $R_t\left(\omega; \bar{\omega}_t\right)$ mea-

[21] The relationship $q_t k_{f,t} = n_{f,t}$ holds only when financial intermediaries own all of the wealth in the economy.

sures the returns per unit of initial deposits from trading reserves ex-post. Ex-post means following the realizations of the jump $J_t^+ - J_t = 1$ and of the withdrawal shocks on deposits $\omega$. The return $R_t(\omega; \bar{\omega}_t)$ is (see subsection 3.2.2)

$$R_t(\omega; \bar{\omega}_t) \equiv r_{l,t}\left[\bar{\omega}_t + \Delta_t(\omega; \tilde{\omega}_t)\right] \mathbf{1}_{\omega<0} + r_{l,t}(\bar{\omega}_t - \omega) \mathbf{1}_{\omega \in [0,\bar{\omega}_t]} - r_{b,t}(\omega - \bar{\omega}_t) \mathbf{1}_{\omega > \bar{\omega}_t} \quad (3.4)$$

The liquidity ratio of financial intermediaries $\bar{\omega}_t$ is

$$\bar{\omega}_t \equiv \frac{m_{f,t}/p_t}{q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}} \quad (3.5)$$

The total return from trading reserves ex-post is the product between $R_t(\omega; \bar{\omega}_t)$ and the quantity of deposits $q_t k_{f,t} + m_{f,t}/p_t - n_{f,t}$. The process $\tau_{2,t}(\omega) n_{f,t}$ denotes a transfer that financial intermediaries receive ex-post. The transfers $\tau_{2,t}(\omega)$ are proportional to net worth and contingent upon the realization of the idiosyncratic withdrawal shock $\omega$.

Financial intermediaries take the tax rate $\tau_{1,t}$ and the transfers $\tau_{2,t}(\omega)$ as given. The policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ serves only a technical purpose. The purpose of the transfers $\tau_{2,t}(\omega)$ is to guarantee that endogenous variables do not jump along with $J_t$.[22] The purpose of the tax rate $\tau_{1,t}$ is to guarantee that the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ is self-financing. In subsection 3.2.6, we derive the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ that satisfies the aforementioned two properties. Intuitively, endogenous variables do not jump if the transfers $\tau_{2,t}(\omega)$ perfectly insure financial intermediaries against withdrawal risks on deposits. In such case, the net worth $n_{f,t}$ does not jump along with $J_t$. Because the jump $J_t^+ - J_t$ affects directly the evolution of $n_{f,t}$ only, none any other endogenous variable jumps along with $J_t$ either.

[22] From a technical point of view, the analysis is simpler when endogenous variables do not jump. In the conclusion, we discuss the main economic implications that may follow if endogenous variables were allowed to jump along with $J_t$.

Financial intermediaries are subject to a moral hazard problem. The moral hazard problem is similar to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). At every point in time, financial intermediaries have to choose between diverting funds and behaving properly. If they divert funds, financial intermediaries can walk away with a fraction $1/\lambda$ of their capital holdings $k_{f,t}$ at the penalty of losing their franchise.[23] The value of diverting funds is therefore $q_t k_{f,t}/\lambda$. If they behave properly, financial intermediaries stay on business, and obtain the franchise value of their financial intermediary company $V_t$. The value of behaving properly is therefore $V_t$.

The moral hazard problem gives rise to the incentive-compatible constraint

$$q_t k_{f,t} \leq \lambda V_t \tag{3.6}$$

The incentive-compatible constraint guarantees that financial intermediaries never divert funds: Intuitively, if financial intermediaries were to divert funds, creditors would not lend in the first place. The incentive-compatible constraint guarantees also that deposits are de facto non-defaultable. Non-defaultable deposits are consistent with a notion of using deposits for settling transactions.

The objective of financial intermediaries is to maximize the present discounted value of their dividend payouts. Financial intermediaries pay out dividends either when they divert funds or when they retire. In either case, financial intermediaries transfer back to the household their entire net worth. Financial intermediaries retire when the financier in office switches her occupation. When they retire, financial intermediaries close their business after the settlement of net deposit outflows concludes. Financial intermediaries that retire are replaced with new financial intermediaries, that start with an initial endowment of $\kappa/\tilde{\rho}$ shares of the aggregate capital stock.

[23] We assume for simplicity that financial intermediaries cannot walk away with reserves. Relaxing such assumption is feasible. See Gertler and Kiyotaki (2010) for a model economy in which safe assets in general also have a pledgeability ratio below 1.

Financial intermediaries discount future dividend payouts with the stochastic discount factor (SDF) of the household, i.e. $\Lambda_t$,[24] weighted by the survival density function of the financier in office (financial intermediaries never divert funds). We conjecture that the SDF of the household $\Lambda_t$ follows an Ito process (see subsection 3.2.4).

Financial intermediaries solve the portfolio problem

$$\begin{aligned} V_t \equiv & \max_{k_{f,t}, m_{f,t}; \iota_t \geq 0} E_t \int_t^\infty \tilde{\rho} e^{-\tilde{\rho}(s-t)} \frac{\Lambda_s}{\Lambda_t} n_{f,s} ds \\ & s.t. : n_{f,t} \geq 0, \ (3.3), \ (\mathbf{??}), \ (\mathbf{??}), \ (3.4) \end{aligned} \tag{3.7}$$

**Solving the Portfolio Problem** We conjecture that the franchise value of financial intermediaries is linear on net worth

$$V_t = v_t n_{f,t}$$

with $v_t \geq 1$.[25] A linear guess works here because the portfolio problem (3.5) is linear. Intuitively, problem (3.5) is linear because the objective function of financial intermediaries is linear, and because financial intermediaries take assets returns and prices as given (constraint functions are therefore linear as well). The process $v_t$ is the Tobin's Q of financial intermediaries: Equivalently, $v_t$ measures the marginal/average value of wealth in the financial intermediary sector. The process $v_t$ is never below 1 because the Tobin's Q of an hypothetical financial intermediary that can invest only in deposits is always equal to 1.[26] We conjecture that $v_t$ follows an Ito process.

Let $\phi_{.,t}$ denote the portfolio shares of financial intermediaries. Let $\phi_{k,t} \equiv q_t k_{f,t}/n_{f,t}$ denote the portfolio share in physical capital and let $\phi_{m,t} \equiv (m_{f,t}/p_t)/n_{f,t}$ denote

[24] The process $\Lambda_t$ is an endogenous process to be determined in equilibrium.

[25] The process $v_t$ is an endogenous object to be determined in equilibrium.

[26] See subsections 3.2.4 and 3.2.6. The reason is that households are indifferent on the margin between consumption and deposits.

the portfolio share in reserves. The incentive-compatible constraint reduces to the standard financing constraint

$$\phi_{k,t} \leq \lambda v_t$$

The leverage multiple amounts to $\phi_{k,t} + \phi_{m,t}$. The portfolio shares satisfy that $\phi_{k,t} > 1$ generically and that $\phi_{m,t} \geq 0$. The liquidity ratio is the same for all financial intermediaries. Specifically,

$$\bar{\omega}_t = \frac{\phi_{m,t}}{\phi_{k,t} + \phi_{m,t} - 1}$$

**Proposition 6** *The optimality conditions in the portfolio problem of financial intermediaries are four:*

1. *The internal investment condition*

$$I'(\iota_t) = 1/q_t$$

2. *The liquidity ratio condition*

$$-i_t dt + E_F\left[R_{m,t}\left(\omega; \bar{\omega}_t\right)\right] \theta dt \leq 0$$

*with equality if* $\bar{\omega}_t > 0$.

3. *The asset pricing condition on capital*

$$E_t\left[dR_{f,t}\right] + E_F\left[R_{k,t}\left(\omega; \bar{\omega}_t\right)\right] \theta dt - \left(i_t - \pi_t\right) dt + Cov_t\left[d\Lambda_t/\Lambda_t + dv_t/v_t, dR_{f,t}\right] \geq 0$$

*with equality if* $\phi_{k,t} < \lambda v_t$.

4. *The asset pricing condition on the Tobin's Q*

$$\tilde{E}_t\left[dR_{n,t}\right] + \frac{\tilde{\rho}}{v_t} dt + E_t\left[dv_t/v_t\right] - \tilde{\rho} dt + Cov_t\left[d\Lambda_t/\Lambda_t, dv_t/v_t\right] = 0$$

*with*

$$\tilde{E}_t\left[dR_{n,t}\right] \equiv \underbrace{E_t\left[dn_{f,t}/n_{f,t}\right] - \left(i_t - \pi_t\right)dt + Cov_t\left[d\Lambda_t/\Lambda_t + dv_t/v_t, dn_{f,t}/n_{f,t}\right]}_{risk-adjusted\ excess\ return\ on\ equity}$$

**Proof.**[27] ■

The internal investment condition is the same as in Brunnermeier and Sannikov (2014). The internal investment condition implies that $\iota_t$ is positively related with the price of capital $q_t$. Intuitively, when the price of capital is higher, financial intermediaries invest more, because physical capital is more valuable. The internal investment rate solves a completely static problem: the internal investment rate $\iota_t$ only depends on the spot price $q_t$.

The liquidity ratio condition weights the opportunity cost against the marginal benefits of holding reserves. The opportunity cost of holding reserves is the sum of the foregone real interest rate on deposits and the inflation rate: By the Fisher equation, it equals the nominal interest rate $i_t dt$. The marginal benefits of holding reserves are $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$, with

$$R_{m,t}\left(\omega;\bar{\omega}_t\right) \equiv R_t\left(\omega;\bar{\omega}_t\right) + \left[r_{l,t}\mathbf{1}_{\omega<\bar{\omega}_t} + r_{b,t}\mathbf{1}_{\omega>\bar{\omega}_t}\right]\left(1-\bar{\omega}_t\right)$$

The factor $\theta dt$ weights the marginal benefits by the likelihood of a jump during the time interval $dt$. The expectation operator $E_F\left[.\right]$ weights the marginal benefits by the probability distribution $F$ of the withdrawal shocks on deposits. The function $R_{m,t}\left(\omega;\bar{\omega}_t\right)$ is the partial derivative of the total return $R_t\left(\omega;\bar{\omega}_t\right) * \left(\phi_{k,t} + \phi_{m,t} - 1\right)$ with respect to $\phi_{m,t}$. The function $R_{m,t}\left(\omega;\bar{\omega}_t\right)$ reads as follows.

Marginally increasing the portfolio share in reserves $\phi_{m,t}$ brings a direct and an indirect effect. The direct effect is the consequence of raising more deposits: Increasing $\phi_{m,t}$ requires raising more deposits because financial intermediaries have already

[27] See the Appendix B.1.

exhausted all of their net worth in capital positions (i.e. $\phi_{k,t} > 1$). The first term in the RHS accounts for the direct effect. The indirect effect is the consequence of increasing the liquidity ratio $\bar{\omega}_t$. Increasing the liquidity ratio boosts the return $R_t(\omega; \bar{\omega}_t)$ because it alleviates liquidity needs as well as improves liquidity excesses ex-post. For instance, if $\omega > \bar{\omega}_t$, and financial intermediaries happen to be borrowers of reserves, increasing the liquidity ratio saves $r_{b,t}$ per unit of deposits. Similarly, if $\omega < \bar{\omega}_t$, and financial intermediaries happen to be lenders of reserves, increasing the liquidity ratio yields an additional rate of return of $r_{l,t}$ per unit of deposits. The second term in the RHS accounts for indirect effect. The factor $1 - \bar{\omega}_t$ weights by the partial effect that $\phi_{m,t}$ has on $\bar{\omega}_t$ and by the deposits position $\phi_{k,t} + \phi_{m,t} - 1$.

The marginal benefits from holding and whence trading reserves ex-post $R_{m,t}(\omega; \bar{\omega}_t)$ is always non-negative. Furthermore, the marginal benefits $R_{m,t}(\omega; \bar{\omega}_t)$ are fully determined by the liquidity ratio. The liquidity ratio condition therefore implies that financial intermediaries set a positive liquidity ratio $\bar{\omega}_t$, if any, to match the expected marginal benefits from trading reserves ex-post to the opportunity cost of holding reserves. If $i_t dt > E_F[R_{m,t}(\omega; \bar{\omega}_t)]\theta dt$ for all $\bar{\omega}_t \in [0, 1]$, financial intermediaries prefer to set a null liquidity ratio $\bar{\omega}_t = 0$ and to hold no reserves.

The asset pricing conditions on capital and on the Tobin's Q are similar to Van der Ghote (2017). The only difference with respect to them is the term $E_F[R_{k,t}(\omega; \bar{\omega}_t)]\theta dt$.

The asset pricing condition on capital is similar as well to the optimality conditions in the consumption-based asset capital model (C-CAPM). The LHS is the risk-adjusted excess return on capital over deposits that financial intermediaries earn. The first three terms add up to the excess return on capital. The second term is unusual and results from the withdrawal risks on deposits. The function $R_{k,t}(\omega; \bar{\omega}_t)$ is the partial derivative of the total return $R_t(\omega; \bar{\omega}_t) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to

$\phi_{k,t}$. Specifically,

$$R_{k,t}\left(\omega;\bar{\omega}_t\right) \equiv R_t\left(\omega;\bar{\omega}_t\right) - \left[r_{l,t}\mathbf{1}_{\omega<\bar{\omega}_t} + r_{b,t}\mathbf{1}_{\omega>\bar{\omega}_t}\right]\bar{\omega}_t$$

The function $R_{k,t}\left(\omega;\bar{\omega}_t\right)$ has a similar interpretation to $R_{m,t}\left(\omega;\bar{\omega}_t\right)$ but for marginal increases on $\phi_{k,t}$. The term $E_F\left[R_{k,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$ measures the extent up to which marginally increasing $\phi_{k,t}$ affects the expected returns from trading reserves ex-post. The function $R_{k,t}\left(\omega;\bar{\omega}_t\right)$ depends only on the liquidity ratio and is strictly increasing on $\bar{\omega}_t$. The second property in turn implies that higher liquidity ratios boost the excess return that financial intermediaries earn on capital. Intuitively, excess returns are higher when $\bar{\omega}_t$ is higher because financial intermediaries are less exposed to withdrawal risks.

The last term in the LHS, i.e. the covariance $Cov_t\left[d\Lambda_t/\Lambda_t + dv_t/v_t, dR_{f,t}\right]$, measures the compensation for holding capital risk that financial intermediaries demand. The co-movement between $dv_t/v_t$ and $dR_{f,t}$ matters for valuing capital risk because financial intermediaries are subject to financing constraints: Financial intermediaries are then particularly concerned with the co-movement between their marginal value of wealth and the return on their capital investments.

The asset pricing condition on capital describes the preference relationship of financial intermediaries between physical capital and deposits. If the risk-adjusted excess return that they obtain on capital is positive, financial intermediaries strictly prefer capital to deposits, and take levered positions on capital until hitting their financing constraint. Otherwise, financial intermediaries are indifferent between capital and deposits, and are willing to take any position on physical capital and on deposits.

The asset pricing condition on the Tobin's Q describes the behavior of the marginal value of wealth of financial intermediaries. The first term in the LHS is the risk-

adjusted excess return on equity over deposits that financial intermediaries earn. The term $\tilde{E}_t\left[dR_{n,t}\right]$ measures the expected profits that financial intermediaries earn per unit of net worth: Notice that $\tilde{E}_t\left[dR_{n,t}\right]$ is the product of the risk-adjusted excess return on capital and the portfolio share in physical capital $\phi_{k,t}$. The asset pricing condition on the Tobin's Q says that financial intermediaries value wealth more, when they obtain positive and higher risk-adjusted excess returns on capital.

### 3.2.4 Households' Portfolio Problem

Let $n_{h,t} \geq 0$ denote the net worth of households. The portfolio investment decisions of households comprise $\left\{k_{h,t}, m_{h,t}/p_t = 0; \iota_t\right\}$. Households take no positions on reserves because deposits weakly dominate reserves in terms of return: The opportunity cost of holding reserves is always non-negative, $i_t \geq 0$.

The consumption and portfolio investment decisions of households, together with the transfers that they receive, dictate the evolution of the worth $n_{h,t}$ accordingly

$$dn_{h,t} = dR_{h,t} q_t k_{h,t} + \left(i_t - \pi_t\right)\left(n_{h,t} - q_t k_{h,t}\right) dt - c_t dt + Transfers_t \tag{3.8}$$

The quantity $n_{h,t} - q_t k_{h,t}$ is the amount of deposits that households hold. The process $Transfers_t$ denotes the transfers that households receive from financial intermediaries and from the Monetary Authority. The net transfers that households receive from financial intermediaries amount to $\left(\tilde{\rho} N_{f,t} - \kappa q_t K_t\right) dt$. In this notation, upper case variables denote aggregate variables. We specify the transfers that households receive from the Monetary Authority in subsection 3.2.5. For convenience, we automatically deduct from the latter the lump-sum taxes that households pay to finance the interest rate payments on government bonds.

The objective of households is to maximize the present discounted value of their utility flows

$$E_t \int_t^\infty e^{-\rho(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma} ds \tag{3.9}$$

The parameter $\gamma$ is the risk aversion coefficient. The SDF of households is $\Lambda_t \equiv e^{-\rho t} c_t^{-\gamma}$.

**Solving the Portfolio Problem** The portfolio problem of households is to maximize (3.8) subject to $n_{h,t} \geq 0$ and (3.7) .

**Proposition 7** *The optimality conditions in the portfolio problem of households are three:*

*1. The internal investment condition*

$$I'(\iota_t) = 1/q_t$$

*2. The asset pricing condition on deposits*

$$(i_t - \pi_t)\, dt = -E_t [d\Lambda_t/\Lambda_t] \equiv \rho dt + \gamma E_t [dc_t/c_t] - \frac{1}{2}\gamma(\gamma+1) Var_t [dc_t/c_t]$$

*3. The asset pricing condition on capital*

$$E_t [dR_{h,t}] - (i_t - \pi_t)\, dt + Cov_t [d\Lambda_t/\Lambda_t, dR_{h,t}] \leq 0$$

*with equality if* $\bar{k}_{h,t} > 0$.

**Proof.**[28]■

The internal investment condition is the same for both households and financial intermediaries. The reason is that both agents face the same static investment

[28]See the Appendix A.

optimization problem. The internal investment rate $\iota_t$ is then independent of who manages the capital stock.

The asset pricing conditions are similar to the optimality conditions in the C-CAPM. The asset pricing condition on deposits implies that households are indifferent on the margin between consumption and deposits. The asset pricing condition on capital describes the preference relationship of households between physical capital and deposits. The term in the LHS is the risk-adjusted excess return on capital over deposits that households earn. If the risk-adjusted excess return is null, households are indifferent on the margin between capital and deposits. Otherwise, households strictly prefer on the margin deposits to capital, and $\bar{k}_{h,t} = 0$.[29]

### 3.2.5 Monetary Policy and Seigniorage Revenues

#### Monetary Policy

Monetary policy has three independent policy instruments. We characterize policy instruments in terms of rates of return. Monetary policy comprises a process for the nominal interest rate $i_t$; a process for the discount window rate $r_{w,t}$; and a process for the interest rate on excess reserves $r_{e,t}$. Policy instruments generate seigniorage revenues and/or expenditures.

#### Seigniorage Revenues

**Open Market Operations** The Monetary Authority conducts open market operations to implement a process for the nominal interest rate. Open market operations generate seigniorage revenues because government bonds dominate reserves in

[29] We disregard the possibility of the opposite inequality, i.e. " $>$ ", because households are not subject to portfolio constraints. If the opposite inequality were to hold, households would take unbounded positions of physical capital, funded with unbounded negative position on deposits, and, in equilibrium, markets would not clear (see subsection 3.2.6).

terms of return. We assume that the Monetary Authority transfers to the household all seigniorage revenues from open market operations on the spot.

To conduct open market operations, the Monetary Authority issues reserves, and whence borrows at the real rate of return of $-\pi_t dt$, to invest in government bonds at a real rate of return of $(i_t - \pi_t)\, dt$. Gross seigniorage revenues from open market operations therefore amount to $i_t M_t / p_t dt$.[30] The corresponding net seigniorage revenues amount $\pi_t M_t / p_t dt$. The reason is that the Monetary Authority deducts from the gross seigniorage revenues the lump-sum taxes that households pay to finance the interest rate payments $(i_t - \pi_t)\, G_t dt$ on government bonds.

**Discount Window and Excess Reserves** The Monetary Authority trades reserves with financial intermediaries to sustain the rates $r_{w,t}$ and $r_{e,t}$. Lending reserves in the discount window generates seigniorage revenues. Paying interest rates on excess reserves generates seigniorage expenditures. We assume that the Monetary Authority transfers to the household on the spot all net seigniorage revenues from trading reserves with financial intermediaries. Net seigniorage revenues from trading reserves amount to

$$\left[\left(r_{w,t} - r_{e,t}\right) \mathbf{1}\left\{E_F\left[\left(\omega - \bar{\omega}_t\right) \mathbf{1}_{\omega > \bar{\omega}_t}\right] > \bar{\omega}_t\right\} + r_{e,t}\right] * \left[E_F\left[\left(\omega - \bar{\omega}_t\right) \mathbf{1}_{\omega > \bar{\omega}_t}\right] - \bar{\omega}_t\right]$$

per unit of aggregate deposits. If the financial intermediary system is illiquid, i.e. $E_F\left[\left(\omega - \bar{\omega}_t\right) \mathbf{1}_{\omega > \bar{\omega}_t}\right] > \bar{\omega}_t$, financial intermediaries on aggregate lack enough reserves holdings to meet all of their liquidity needs. Financial intermediaries then have to borrow the remanent reserves $E_F\left[\left(\omega - \bar{\omega}_t\right) \mathbf{1}_{\omega > \bar{\omega}_t}\right] - \bar{\omega}_t$ in discount window, and net seigniorage revenues therefore amount to $r_{w,t} * \left[E_F\left[\left(\omega - \bar{\omega}_t\right) \mathbf{1}_{\omega > \bar{\omega}_t}\right] - \bar{\omega}_t\right]$ per unit of aggregate deposits. If the financial intermediary system is liquid, i.e. $E_F\left[\left(\omega - \bar{\omega}_t\right) \mathbf{1}_{\omega > \bar{\omega}_t}\right] < \bar{\omega}_t$, the opposite happens: Financial intermediaries on aggre-

[30] The aggregate stock of real reserves $M_t / p_t$ equals the position that the Monetary Authority takes on government bonds $G_t$. The reason is that all seigniorage revenues are paid on the spot.

gate have enough reserves holdings to meet all of their liquidity needs, and whence park the remanent reserves $\bar{\omega}_t - E_F\left[(\omega - \bar{\omega}_t)\,\mathbf{1}_{\omega>\bar{\omega}_t}\right]$ in the balance sheet of the Monetary Authority. In this second case, net seigniorage expenditures amount to $r_{e,t} * \left[\bar{\omega}_t - E_F\left[(\omega - \bar{\omega}_t)\,\mathbf{1}_{\omega>\bar{\omega}_t}\right]\right]$ per unit of aggregate deposits.

### 3.2.6 Competitive Equilibrium

We restriction attention to a competitive equilibrium in which inflation is locally risk-free and endogenous variables do not jump.

Let $\eta_t \equiv N_{f,t}/q_t K_t$ denote the wealth share of financial intermediaries. Let $\varphi_t \equiv (M_t/p_t)/q_t K_t$ denote the wealth share of reserves. The total wealth in the economy is $q_t K_t$ because physical capital is the only real asset.

**Definition** A competitive equilibrium is a set of stochastic processes adapted to the filtration generated by $Z$ : the price of capital $\{q_t\}$ ; the nominal price $\{p_t\}$ ; the inflation rate $\{\pi_t\}$ ; consumption $\{C_t\}$ ; the capital position of households $\{K_{h,t}\}$ ; the portfolio share in physical capital $\{\phi_{k,t}\}$ ; the portfolio share in reserves $\{\phi_{m,t}\}$ ; the liquidity ratio $\{\bar{\omega}_t\}$ ; the wealth share of financial intermediaries $\{\eta_t\}$ ; the Tobin's Q of financial intermediaries $\{v_t\}$ ; the internal investment rate $\{\iota_t\}$ ; the aggregate capital stock $\{K_t\}$ ; the aggregate stock of nominal reserves $\{M_t\}$ ; the wealth share of reserves $\{\varphi_t\}$ ; the discount window rate $\{r_{w,t}\}$ ; the interest rate on excess reserves $\{r_{e,t}\}$ ; the market rate $\{r_{m,t}\}$ and the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ ; such that:

1. Optimality conditions

   (a) $\{\phi_{k,t}, \phi_{m,t}, \bar{\omega}_t; \iota_t\}$ solve the problem of financial intermediaries

   (b) $\{C_t, K_{h,t}; \iota_t\}$ solve the problem of households

   (c) $\{r_{m,t}\}$ solves the Nash Bargaining problem

2. Market clearing conditions

(a) The market for the consumption good clears

$$C_t + \iota_t K_t = \left[a_f * \phi_{k,t}\eta_t + a_h * (1 - \phi_{k,t}\eta_t)\right] K_t$$

(b) The market for capital holdings clears

$$K_{h,t}/K_t = 1 - \phi_{k,t}\eta_t$$

(c) The market for real reserve holdings clears

$$\phi_{m,t}\eta_t = \varphi_t$$

(d) The OTC market for reserves clears

3. Conditions for law of motions

The nominal price $p_t$ evolves according to (3.2) and the aggregate stock of physical capital $K_t$ evolves according to (3.1) .

4. Condition for locally risk-free inflation rate

The aggregate stock of nominal reserves $M_t$ evolves accordingly

$$\frac{dM_t}{M_t} = \pi_t dt + \frac{d\left(\phi_{m,t}\eta_t\right)}{\phi_{m,t}\eta_t} + \frac{d\left(q_t K_t\right)}{q_t K_t} + \frac{d\left(\phi_{m,t}\eta_t\right)}{\phi_{m,t}\eta_t}\frac{d\left(q_t K_t\right)}{q_t K_t}$$

5. No-jump condition

The policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ is

$$\tau_{1,t} \equiv E_F\left[-R_t\left(\omega; \bar{\omega}_t\right) * \left(\phi_{k,t} + \phi_{m,t} - 1\right)\right]\theta$$

$$\tau_{2,t}(\omega) \equiv -R_t\left(\omega; \bar{\omega}_t\right) * \left(\phi_{k,t} + \phi_{m,t} - 1\right)$$

The market clearing condition for the consumption good guarantees that consumption and investment equal output. The process $\phi_{k,t}\eta_t$ in the RHS is the share of the aggregate capital stock that financial intermediaries manage. The market clearing conditions for asset holdings guarantee that all asset markets clear: The market for deposits automatically clears due to Walras Law. The OTC market clearing condition for reserves determines the effective borrowing rate $r_{b,t}$ and the effective lending rate $r_{l,t}$ in equilibrium. Notice that in equilibrium $\tilde{\omega}_t = \bar{\omega}_t$.

The law of motion for $M_t$ describes the evolution of nominal reserves that implements the locally risk-free inflation rate $dp_t/p_t = \pi_t dt$. The law of motion for $M_t$ follows from applying Ito's Lemma to both sides of the market clearing condition for real reserves holdings. The RHS describes the evolution of the aggregate demand of nominal reserves. The LHS has the same interpretation but for the aggregate supply of nominal reserves.

The transfers $\tau_{2,t}(\omega)$ in condition (5) perfectly insure financial intermediaries against withdrawal risks on deposits. The tax rate $\tau_{1,t}$ in condition (5) guarantees that the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ is self-financing. We assume that $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ can be financed by borrowing and lending at an exogenous real interest rate of $\check{\rho}$, with $\check{\rho} \to 0$. In the limit with $\check{\rho} \to 0$, the tax rate $\tau_{1,t}$ amounts to the expected value of the transfers $\tau_{2,t}(\omega)$.

**Markov Competitive Equilibrium** We conjecture that a Markov equilibrium exists. We conjecture furthermore that the state variables of the Markov equilibrium are $\{\eta, K\}$. The level of the aggregate capital stock $K_t$ is irrelevant for the analysis because the equilibrium outcome is scale invariant with respect to $K_t$.[31] The dynamics of $K_t$ is nonetheless relevant because the evolution of the aggregate capital stock influences the growth rate of the economy (see section 3.4). Both the

[31] Namely, the endogenous variables in the equilibrium are either linear on, or independent of, $K_t$.

level and the dynamics of the wealth share of financial intermediaries $\eta_t$ are relevant (see section 3.4).

The Markov equilibrium adds a consistency condition to the conditions of the competitive equilibrium. The Markov equilibrium (additionally) requires $\eta_t$ to evolve in accord with the conditions of the competitive equilibrium.

## 3.3 Parameter Values and Functional Forms

Table 3.1 below describes the parameter values and the functional forms that we use in our numerical analysis. The time frequency is annual.

Table 3.1: Parameter Values and Functional Forms

| | Parameter Values / Func. Forms | Value | Target |
|---|---|---|---|
| Panel A: Technology | | | |
| $a_f$ | Prod. of fin. intermediaries | 2% | Ratio output/capital |
| $a_h$ | Prod. of households | 1.3% | Average Sharpe ratio |
| $\sigma$ | Fundamental risk | 3.5% | Vol. of adj. TFP |
| $I(\iota)$ | Internal investment technology | $Sqrt(\iota)$ | Literature |
| $\delta$ | Depreciation rate | 1% | Literature |
| Panel B: Financial Intermediation | | | |
| $\lambda$ | Fraction of divertable assets | 2 | Av. leverage multiple |
| $\tilde{\rho}$ | Arrival rate retirement shock | 10% | Av. survival frequency |
| $\kappa$ | Initial capital endowment | 1.5% | Av. wealth/capital ratio |
| Panel C: Settlement | | | |
| $\theta$ | Arrival rate withdrawal shocks | 5 | Average liquidity ratio |
| $F(\omega)$ | Distribution withdrawal shocks | $Logistic$ | Literature |
| $\xi$ | Nash bargaining power | 0.5 | Corridor system |
| Panel D: Preferences | | | |
| $\rho$ | Time discount rate | 2% | Literature |
| $\gamma$ | Risk aversion coefficient | 1 | Literature |

**Panel A: Technology**

The functional form for the internal investment technology $I(\iota)$ is similar to Brunnermeier and Sannikov (2014). Specifically, $I(\iota)$ is

$$I(\iota) \equiv \frac{1}{B}\left(\sqrt{A^2 + 2B\iota} - A\right)$$

Notice that $I(\iota)$ has quadratic adjustment costs.

We choose $A$, $B$ and $a_f$ to match aggregate quantities and prices in the frictionless economy. The frictionless economy is such that there are no financing constraints nor withdrawal risks on deposits (i.e. $1/\lambda, 1/\theta \to 0$).[32] In the frictionless economy, the productivity coefficient $a_f$ equals the ratio of aggregate output to aggregate capital $Y_t/K_t$ (financial intermediaries always manage all of the aggregate capital stock). We set a value of $a_f$ of 2% which is standard. Additionally, in the frictionless economy, the internal investment rate attains its efficient value $\iota_E$, with

$$\iota_E \equiv \frac{q_E^2 - A^2}{2B}$$

The price of capital, and the real interest rate, attain their efficient value $q_E$, and $r_E$, with[33]

$$\begin{aligned} q_E &\equiv B * \left[\sqrt{\rho^2 + 2a_f/B + (A/B)^2} - \rho\right] \\ r_E &\equiv \rho + \left(\frac{q_E - A}{B} - \delta\right)\gamma - \frac{1}{2}\gamma(\gamma+1)\sigma^2 \end{aligned}$$

We set $A$ and $B$ to jointly target a ratio of investment to output $\iota_E/a_f$ of 20% and a real interest rate $r_E$ of 4.05%. A rate $r_E = 4.05\%$ yields an annual real interest rate of 2%.

[32] We assume that in the frictionless economy there is no inflation either. Namely, $\pi_t = 0$.

[33] The efficient price of capital $q_E$ solves the asset pricing condition on capital for financial intermediaries. See the Appendix B.2 for further details.

The depreciation rate of physical capital $\delta$ is set equal to 1% which is standard. The coefficient that measures fundamental risk, i.e. $\sigma$, is set equal to 3.5%. A value $\sigma = 3.5\%$ matches the unconditional standard deviation of the Utilization-Adjusted Series on Total Factor Productivity (see Basu, Fernald and Kimball 2006; Basu, Fernald, Fisher and Kimball 2006; and Fernald 2014). Using such TFP series is consistent with interpreting $dZ_t$ as shocks to the effective units of physical capital.

We look at the original economy with financing constraints and withdrawal risks to assign numerical values to the remaining parameters in the model. In the original economy, the spread between $a_f$ and $a_h$ affects that between $dR_{f,t}$ and $dR_{h,t}$, and, whence, the fluctuations on the return on capital in equilibrium. We set $a_h$ equal to 1.3% to target an average Sharpe ratio of 30%. Van der Ghote (2017) follows a similar methodology.

**Panel B: Financial Intermediation**

We set the fraction of divertable assets $\lambda$ equal to 2 to target an average leverage multiple of 3.5. The arrival rate of the retirement shock of financiers, i.e. $\tilde{\rho}$, targets an average survival frequency of financial intermediaries of 10 years. The initial capital endowment $\kappa$ targets the average wealth-to-capital ratio in the financial intermediary sector. Using Fed fund data, we estimate an average wealth-to-capital ratio of 20%. This estimate is consistent with Hirakata, Sudo and Ueda (2013). The value of $\kappa$ is 1.5%.

**Panel C: Settlement**

The cumulative probability distribution function of withdrawal shocks $F(\omega)$ is similar to Bianchi and Bigio (2014). Using individual US commercial banks Call Reports, Bianchi and Bigio (2014) construct an empirical distribution of withdrawal shocks, and obtain that the corresponding empirical distribution fits a logistic distribution $F(\omega; \mu, s)$ with location $\mu = -0.0029$ and scale $s = 0.022$. We set the location

$\mu$ equal to 0 to guarantee that $\int \omega dF(\omega; \mu, s) = 0$. Additionally, we accommodate the support of the logistic distribution to the open interval $(-\infty, 1)$.

We set the arrival rate $\theta$ of the Poisson process $J_t$ to target an average liquidity ratio of 40%. The value of $\theta$ is 5.

We set a Nash Bargaining coefficient $\xi$ of 0.5. A value $\xi = 0.5$ is consistent with a corridor system of reserves: An even bargaining power between lending and borrowing orders of reserves implies a market rate of $r_{m,t} = (r_{w,t} + r_{e,t})/2$.

**Panel D: Preferences**

We set a time discount rate $\rho$ of 2% which is standard. The risk aversion coefficient $\gamma$ is equal to 1 to obtain log-preferences for consumption.

## 3.4 Liquidity Management and Leverage Decisions

To examine the interactions between liquidity management and leverage decisions we consider monetary policy as given. We consider a monetary policy that is similar to the pre-Global Financial Crisis of 2008 era. Specifically, monetary policy follows a strict-inflation-targeting rule; charges a positive rate in the discount window; and pays no interest rate on excess reserves. We set a target of 2% for annual inflation and a discount window rate of 5%. We study first the economy in which financial intermediaries arbitrary choose to hold no reserves, i.e. $\bar{\omega}_t = \phi_{m,t} = 0$. We interpret this first economy in which there is no liquidity as a benchmark of comparison for the more interesting economy in which liquidity management decisions are endogenous.

### 3.4.1 Benchmark Economy without Liquidity

If $\bar{\omega}_t = \phi_{m,t} = 0$, the equilibrium has two well-demarcated regions (Figure 3.1). The underlying difference between these two regions concerns the size of the borrowing capacity of financial intermediaries. In a first region, financial intermediaries have

a low borrowing capacity relative to the total wealth in the economy. Specifically, financial intermediaries, on aggregate, lack enough borrowing capacity to absorb all of the aggregate capital stock, i.e. $\lambda v_t \eta_t < 1$. In the second region, the opposite happens: Financial intermediaries have a high borrowing capacity relative to the total wealth in the economy, and, on aggregate, they have a enough borrowing capacity to absorb all of the aggregate capital stock, i.e. $\lambda v_t \eta_t \geq 1$.

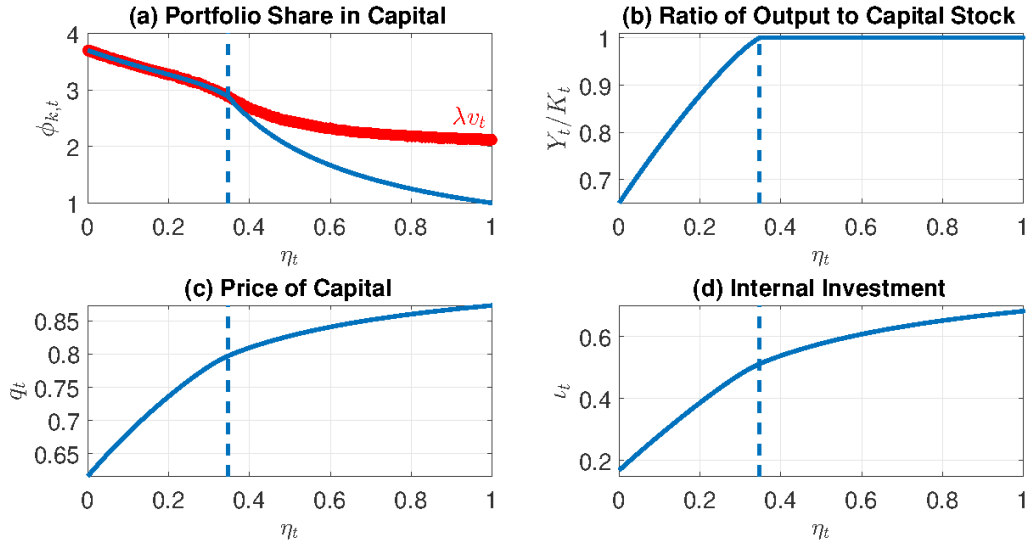



Figure 3.1: The Equilibrium of the Economy without Liquidity

When $\lambda v_t \eta_t < 1$, in equilibrium, both financial intermediaries and households hold physical capital (otherwise, the market for capital would not clear). Households are the marginal investors on capital because they are not subject to financing constraints.

The price of capital reflects the low valuation that households have for physical capital relative to financial intermediaries (Figure 3.1c, LHS of dotted line). The price of capital attains a value considerably below its efficient value of $q_E$, because otherwise households would earn low net dividend yields $(a_h - \iota_t)/q_t$ as well as negative risk-adjusted excess returns. Financial intermediaries borrow until hitting their financing constraint (Figure 3.1a, LHS of dotted line). Financial intermediaries take the largest possible levered position on capital, $\phi_{k,t} = \lambda v_t$, because they earn net high dividend

yields $(a_f - \iota_t)/q_t$ as well as high and positive risk-adjusted excess returns. The ratio $Y_t/K_t \equiv \phi_{k,t}\eta_t a_f + (1 - \phi_{k,t}\eta_t)\, a_h$ attains a value below its efficient level of $a_f$ (Figure 3.1b, LHS of dotted line) because the allocation of physical capital is inefficient. The internal investment rate $\iota_t$ also attains a value below its efficient level of $\iota_E$ but because $q_t << q_E$.

When $\lambda v_t \eta_t \geq 1$, in equilibrium, financial intermediaries manage all of the aggregate capital stock, and households take no positions on capital. Financial intermediaries borrow until clearing the market for capital and whence $\phi_{k,t} = 1/\eta_t \leq \lambda v_t$ (Figure 3.1a, RHS of dotted line). Financial intermediaries become the marginal investors on capital because financing constraints are slack (financial intermediaries cannot borrow until hitting their financing constraint because otherwise the market for capital would not clear).

The price of capital reflects the high valuation that financial intermediaries have for physical capital relative to households (Figure 3.1c, RHS of dotted line). Households refrain themselves from holding capital because they earn low net dividend yields as well as negative risk-adjusted excess returns. The ratio $Y_t/K_t$ attains its efficient level of $a_f$ (Figure 3.1b, RHS of dotted line). The internal investment rate takes a value closer to its efficient value of $\iota_E$ (Figure 3.1d, RHS of dotted line).

The dynamics of the equilibrium differ between the two regions as well. To study dynamic behaviors we decouple time series into a trend variable and a cyclical variable. The cyclical variable measures the fluctuations of the time series around their corresponding trend variable.

Dynamics in general are governed by the law of motions of the state variables $K_t$ and $\eta_t$. The law of motion of the aggregate capital stock $K_t$ dictates the evolution of the trend variables. The reason is that the equilibrium outcome is scale invariant with respect to $K_t$. Because all endogenous variables that exhibit a positive trend have indeed a common trend (i.e. all of them are linear on $K_t$), the law of motion

of $K_t$ also dictates the evolution of the trend rate of economic growth. The law of motion of the wealth share of financial intermediaries $\eta_t$ dictates the evolution of the cyclical variables. Endogenous variables in general exhibit different cyclical behaviors because their corresponding cyclical variables have a different mapping with respect to $\eta_t$.

The trend rate of economic growth evolves accordingly

$$dK_t/K_t = \left[I\left(\iota_t\right) - \delta\right]dt + \sigma dZ_t$$

The trend rate evolves stochastically. The expected trend rate is endogenous and depends on the position of $\eta_t$. Specifically, when the wealth share of financial intermediaries $\eta_t$ is low, the expected trend rate is low as well, because the price of capital is depressed, and the internal investment rate $\iota_t$ is low. The opposite happens when the wealth share of financial intermediaries $\eta_t$ is high.

The wealth share of financial intermediaries $\eta_t$ evolves according to

$$d\eta_t/\eta_t = \mu_{\eta,t}dt + \sigma_{\eta,t}dZ_t$$

with

$$\begin{aligned}
\mu_{\eta,t}dt &= \frac{a_f - \iota_t}{q_t}dt + \\
&\left(\phi_{k,t} - 1\right)\left[E_t\left[dR_{f,t}\right] + E_F\left[R_t\left(\omega;\bar{\omega}_t\right)\right]\theta dt - \left(i_t - \pi\right)dt - Var_t\left[dR_{f,t}\right]\right] \\
&-\phi_{m,t}\left[i_t - E_F\left[R_t\left(\omega;\bar{\omega}_t\right)\right]\theta\right]dt + \left(\frac{\kappa}{\eta_t} - \tilde{\rho}\right)dt
\end{aligned}$$

$$\sigma_{\eta,t}dt = \left(\phi_{k,t} - 1\right)\left(\sigma_{q,t} + \sigma\right)dt$$

The wealth share of financial intermediaries also evolves stochastically. The drift process $\mu_{\eta,t}$ measures the expected growth rate of $\eta_t$. The first three terms in $\mu_{\eta,t}$

add up to the risk-adjusted excess return on the investment portfolio of financial intermediaries over the total wealth in the economy. The first term accounts for the excess return on internal financing, namely, on capital positions funded with net worth. The second term accounts for the risk-adjusted excess return on external financing, i.e. on capital positions funded with deposits. The third term is the effective return from holding reserves.[34] The second and third terms include the policy schedule $\{\tau_{1,t}, \tau_{2,t}(\omega)\}$ that financial intermediaries face in equilibrium (see subsection 3.2.6). The expectation $E_F[R_t(\omega; \bar{\omega}_t)]\theta dt$ in the second term deducts the costs of withdrawal risks that follow from the deposits positions that finance the levered capital positions $\phi_{k,t} - 1$. The same expectation $E_F[R_t(\omega; \bar{\omega}_t)]\theta dt$, but now in the third term, incorporates the benefits of holding reserves into the effective return of reserves. The last term in $\mu_{\eta,t}$ is the net transfers that financial intermediaries payout to households (see subsection 3.2.3).

The diffusion process $\sigma_{\eta,t}$ measures the volatility of the wealth share of financial intermediaries. The process $\sigma_{\eta,t}$ is the product between the levered capital positions $\phi_{k,t} - 1$ and the aggregate risk $\sigma_{q,t} + \sigma$. Aggregate risk amounts to the volatility of the growth rate of the total wealth in the economy, i.e. $Var_t[d(q_t K_t)/q_t K_t] = (\sigma_{q,t} + \sigma)^2 dt$.

Figures 3.2a and 3.2b illustrate the dynamic behavior of $\eta_t$ by isolating the behaviors of $\mu_{\eta,t}\eta_t$ and of $\sigma_{\eta,t}\eta_t$.

The bottom line from Figure 3.2a is that the wealth share of financial intermediaries $\eta_t$ is mean-reverting. The mean at which $\eta_t$ reverts is the stochastic steady state, i.e. the state at which $\mu_{\eta,t} = 0$, (the red dotted line in Figure 3.2a).[35] The bottom line from Figure 3.2b is that fluctuations on $\eta_t$ gets amplified through endogenous fluctuations on the price of capital $q_t$. The red line in Figure 3.2b plots the process

[34] In this particular economy without liquidity, the third term is null, as $\phi_{m,t} = 0$.

[35] The process $\eta_t$ mean-reverts because the first three terms in $\mu_{\eta,t}$ are inversely related to the marginal valuation for capital of the marginal investor (Van der Ghote 2017).

$\sigma_{\eta,t}\eta_t$ assuming that the price of capital is constant, i.e. $\sigma_{q,t} = 0$. The spread between the blue and the red lines is $(\phi_{k,t} - 1)\,\sigma_{q,t}$.[36]

The processes $\mu_{\eta,t}\eta_t$ and $\sigma_{\eta,t}\eta_t$ together shape the invariant distribution of the wealth share of financial intermediaries (see the Appendix A). The invariant distribution of $\eta_t$ shows that the regions $\{\lambda v_t \eta_t < 1\}$ and $\{\lambda v_t \eta_t \geq 1\}$ occur frequently often in equilibrium (Figure 3.2c). Additionally, the invariant distribution of $\eta_t$, together with the processes $\mu_{\eta,t}\eta_t$ and $\sigma_{\eta,t}\eta_t$, show that the economy recurrently transitions from episodes in which financial intermediaries are undercapitalized, and $\{\lambda v_t \eta_t < 1\}$, to episodes in which financial intermediaries are well-capitalized, and $\{\lambda v_t \eta_t \geq 1\}$, and vice versa.

The invariant distribution of $Y_t/K_t$ results from dynamic behavior of the wealth share of financial intermediaries (Figure 3.2d). The ratio $Y_t/K_t$ is pro-cyclical as well as mean-reverting. The ratio $Y_t/K_t$ is pro-cyclical because $Y_t/K_t$ is positively related to $\eta_t$ (Figure 3.1d).

### 3.4.2 Economy with Liquidity

In equilibrium, there are always two well-demarcated regions independently of liquidity management decisions. Specifically, in equilibrium, independently of the processes for $\phi_{m,t}$ and $\bar{\omega}_t$, the regions $\{\lambda v_t \eta_t < 1\}$ and $\{\lambda v_t \eta_t \geq 1\}$ always share the same features that were described for the benchmark economy without liquidity (see subsection 3.4.1). The behavior of the equilibrium nonetheless in general depends on $\bar{\omega}_t$ and $\phi_{m,t}$.

Figure 3.3b shows how to determine the liquidity ratio $\bar{\omega}_t$ in equilibrium. Figure 3.3b plots the responses of $i_t dt$ and $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$ to a one-shot deviation of

[36]The endogenous fluctuations on the price of capital result from the combination of financing constraints and deposit contracts (Van der Ghote 2017). These fluctuations are larger when financing constraints bind, and financial intermediaries have large aggregate effects, because the typical pecuniary externality of economies with incomplete financial markets and occasionally binding financing constraints is present in our model economy as well (Lorenzoni 2008, Jeanne and Korinek 2010, Bianchi 2011, Bianchi and Mendoza 2012, Van der Ghote 2017).

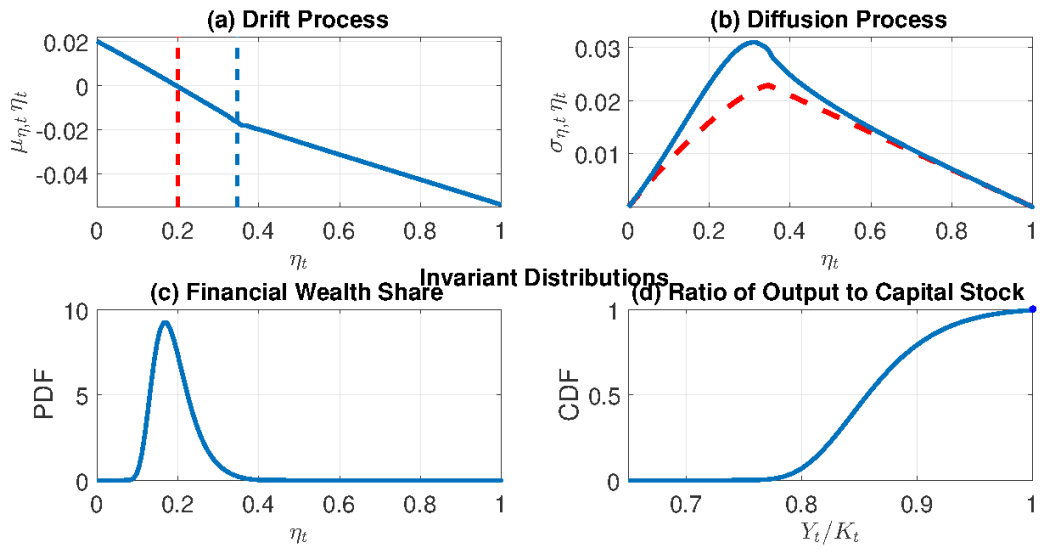



Figure 3.2: The Equilibrium Dynamics of the Economy without Liquidity

the liquidity ratio $\bar{\omega}_t$ from the process that it follows in equilibrium. Figure 3.3b takes the state $\eta_t$ as given. Figure 3.3b shows that a one-shot deviation generates a small effect on the opportunity cost of holding reserves, $i_t dt$, but a large effect on the marginal benefits of holding reserves, $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$. The effect on $i_t dt$ is small because the liquidity ratio does not affect aggregate consumption directly but only indirectly through its effect on the evolution of net worth $n_{f,t}$. The effect on $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$ is large because $R_{m,t}\left(\omega;\bar{\omega}_t\right)$ depends directly on the liquidity ratio (see subsection 3.2.3).

The response functions in Figure 3.3b pin down the value of the liquidity ratio in equilibrium. In equilibrium, if the liquidity ratio is positive, the intersection between $i_t dt$ and $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$ determines the value of $\bar{\omega}_t$. Otherwise, if the liquidity ratio is null, $i_t dt$ has to be weakly greater than $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]\theta dt$ for any given $\bar{\omega}_t \in [0,1]$. The slope of $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]$ in Figure 3.3b determines the sign of the co-movement between $\bar{\omega}_t$ and $i_t$. The slope of $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]$ is negative because the effective lending rate $r_{l,t}$ and the effective borrowing rate $r_{b,t}$ are inversely re-

lated to the liquidity ratio $\bar{\omega}_t$ (see subsection 3.2.2 and Figure 3.3a).[37] The slope of $E_F\left[R_{m,t}\left(\omega;\bar{\omega}_t\right)\right]$ implies that the liquidity ratio and the nominal interest rate negatively co-move in equilibrium. Intuitively, in equilibrium, financial intermediaries reduce their leverage ratios, and whence holds less reserves, when the opportunity costs of holding reserves is high.

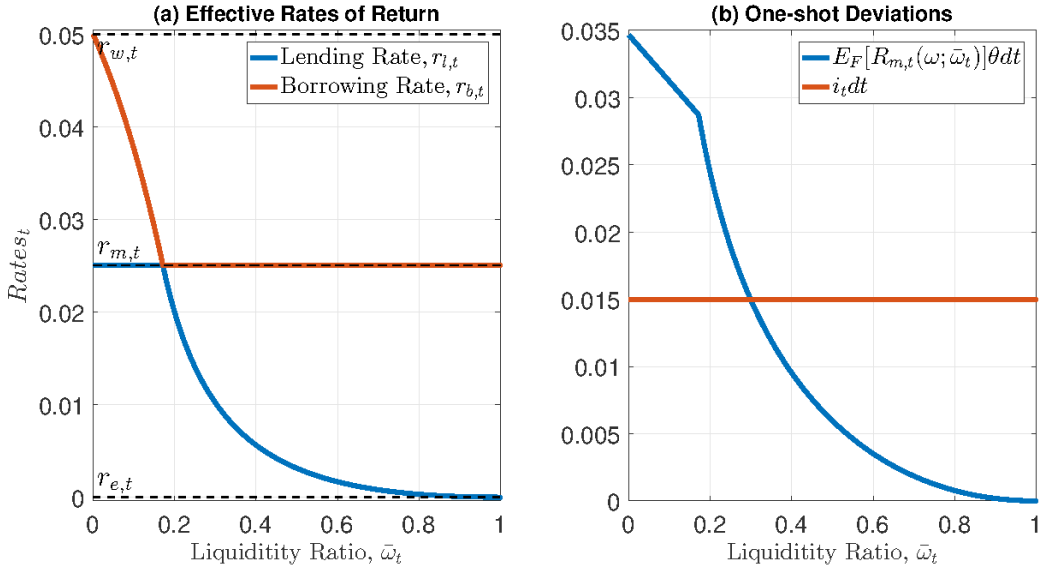



Figure 3.3: Determination of the Liquidity Ratio

Figure 3.4 plots the equilibrium outcome in the economy in which liquidity management decisions are endogenous.

Figure 3.4b plots the liquidity ratio $\bar{\omega}_t$ as a function of the state $\eta_t$. Figure 3.4b shows that liquidity ratio is counter-cyclical. The liquidity ratio in Figure 3.4b is counter-cyclical because the real interest rate, and whence the opportunity costs of holding reserves, are pro-cyclical.[38]

[37] Figure 3a plots the expressions for $r_{l,t}$ and $r_{b,t}$ in subsection 2.2 setting $\tilde{\omega}_t = \bar{\omega}_t$.

[38] The cyclical behavior of the real interest rate depends on the cyclical behaviors of $Y_t/K_t$ and of $\iota_t$. Two counteracting forces shape the cyclical behavior of the real interest rate. On the one hand, a pro-cyclical ratio $Y_t/K_t$ pushes for a counter-cyclical real interest rate. On the other hand, a pro-cyclical investment rate $\iota_t$, together with a pro-cyclical expected trend rate $I\left(\iota_t\right) - \delta$, pushes for a pro-cyclical real interest rate. The second effect in general dominates.

Figure 3.4a plots the portfolio positions in capital $\phi_{k,t}$ as a function of the state $\eta_t$. The purpose of Figure 3.4a is to contrast the behavior of $\phi_{k,t}$ between the economies with, and without, liquidity management decisions. Figure 3.4a shows that $\phi_{k,t}$ is higher in the economy in which liquidity management decisions are endogenous. Figure 3.4a shows also that $\phi_{k,t}$ is higher when the state $\eta_t$ is low, i.e. $\eta_t < \bar{\eta}$, and financing constraints bind, and that the threshold state $\bar{\eta}$ shifts rightward.

The results in Figure 3.4a follow from the positive effects that optimal liquidity ratios have on the borrowing capacity $\lambda v_t$. Optimal liquidity ratios provide a better hedge against withdrawal risk on deposits. Optimal liquidity ratios therefore reduce the costs of leverage $(\phi_{k,t} - 1) E_F \left[R_t \left(\omega; \bar{\omega}_t\right)\right] \theta dt$, that result from the withdrawal risks on the deposits positions that finance the levered capital positions $\phi_{k,t} - 1$. Lower costs of leverage boosts the profitability in financial intermediation as well as the Tobin's Q of financial intermediaries $v_t$. Agency problems in financial markets then relax and the borrowing capacity of financial intermediaries expands.

Figures 3.4c and 3.4d plot invariant distributions. Figure 3.4c shows that the economy with endogenous liquidity management decisions spends more time around states $\eta_t$ in which financial intermediaries are well-capitalized. Figure 4d show that it also spends more time around states in which the ratio $Y_t/K_t$ is higher.

The results in Figures 3.4c and 3.4d also follow from the positive effects that optimal liquidity ratios have on the dynamics of net worth $n_{f,t}$. Optimal liquidity ratios boost the profitability in financial intermediation and whence improve the risk-adjusted excess return on the investment portfolio of financial intermediaries over the total wealth in the economy.

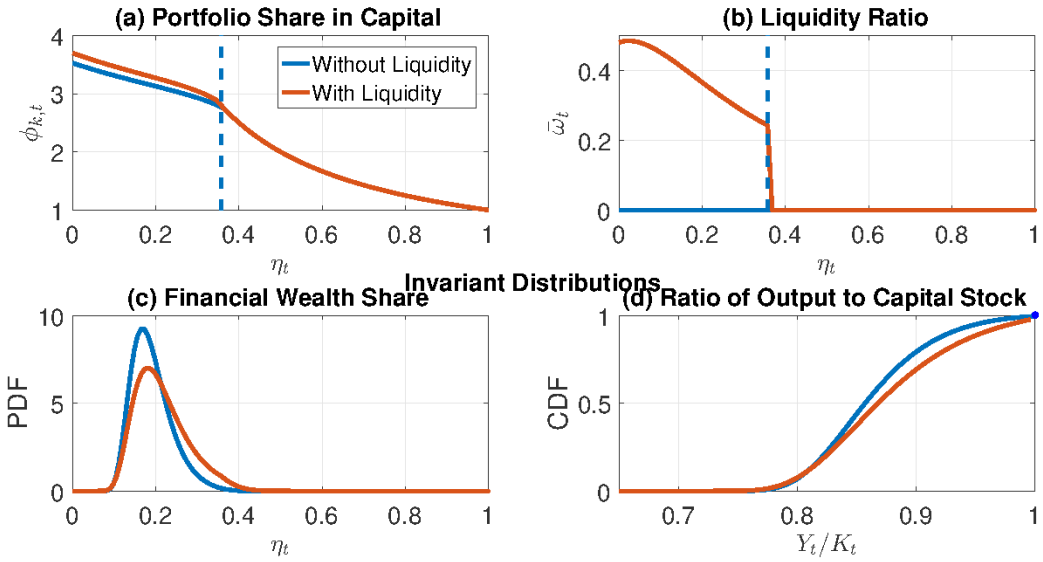



Figure 3.4: A Comparison of the Equilibrium between the Economies with and without Liquidity

## 3.5 Policy Experiments

We conduct two simple policy experiments to examine the real effects of monetary policy. In the first experiment, the target of inflation falls from 2% to 1%. In the second experiment, the width of the corridor (i.e. the spread between the discount window rate and the interest rate on excess reserves) falls by 0.5% leaving the market rate constant. The second experiment explores the real effects of the policy that is traditionally known as narrowing the corridor.

### 3.5.1 Real Effects of Lower Inflation Targets

A reduction in the target of inflation reduces the opportunity cost of holding reserves. The reason is that a lower inflation rate boosts the real rate of return of reserves.

Financial intermediaries respond to a reduction in the target of inflation by increasing their liquidity ratio (Figure 3.5b). Financial intermediaries increase their liquidity ratio relatively more when they are undercapitalized, i.e. when $\lambda v_t \eta_t < 1$, and financing constraints bind, because in that region financial intermediaries take

relatively larger levered positions on physical capital, and whence are relatively more exposed to withdrawal risks on deposits (Figure 3.5a).

The reduction in the target of inflation, combined with the subsequent liquidity responses of financial intermediaries, expand the borrowing capacity $\lambda v_t$. The reason is that a lower opportunity cost of holding reserves, along with a lower exposure to withdrawal risks, boost the profitability of financial intermediation. The Tobin's Q of financial intermediaries $v_t$ therefore increases and agency problems in financial markets relax. The main consequence of expanding the borrowing capacity $\lambda v_t$ is that financial intermediaries take larger levered positions on physical capital when financing constraints bind (Figure 3.5a).

The reduction in the target of inflation affects also the dynamic behavior of the equilibrium outcome. Specifically, when the target of inflation falls, the economy spends more time in states in which financial intermediaries are well-capitalized and the ratio $Y_t/K_t$ is high (Figures 3.5c and 3.5d).

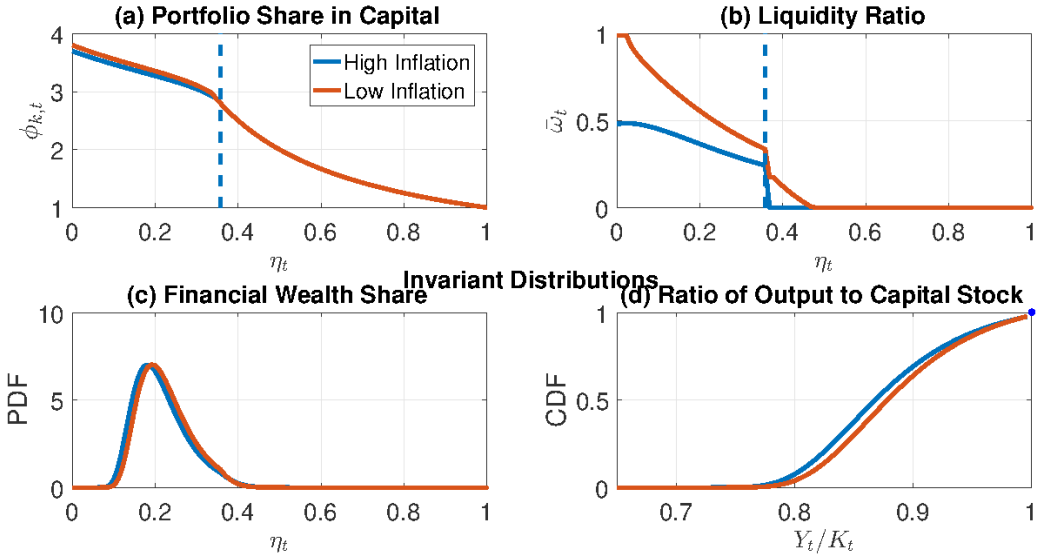



Figure 3.5: Real Effects of Lower Inflation Targets

### 3.5.2 Real Effects of Narrower Corridors

A narrower corridor improves the terms of trade of trading reserves with the Monetary Authority. The reason is that a narrower corridor reduces the discount window rate $r_{w,t}$ as well as increases the interest rate on excess reserves $r_{e,t}$, to keep the market rate $r_{m,t}$ constant.

The liquidity response of financial intermediaries to a narrower corridor depends on how liquid the financial intermediary system initially is (Figure 3.6). If the financial intermediary system is liquid, meaning the economy spends most of its time in the region in which $\bar{\omega}_t > E_F\left[(\omega - \bar{\omega}_t)\,\mathbf{1}_{\omega>\bar{\omega}_t}\right]$, financial intermediaries increase their liquidity ratio $\bar{\omega}_t$ as well as their portfolio share in reserves $\phi_{m,t}$ (Figure 6a). Otherwise, financial intermediaries reduce their liquidity ratio $\bar{\omega}_t$ and $\phi_{m,t}$ (Figure 3.6b).

If the financial intermediary system is liquid, financial intermediaries almost never borrow reserves in the discount window. The reason is that the financial intermediary system almost always has enough reserves to meet all of its aggregate liquidity needs (see subsection 3.2.6). Narrowing the corridor therefore improves the terms of trade of lenders of reserves while keeping constant those of borrowers of reserves. The natural response of financial intermediaries is to increase their liquidity ratio and their liquidity positions.

If the financial intermediary system is illiquid, (i.e. the economy spends most of its time in the region in which $\bar{\omega}_t < E_F\left[(\omega - \bar{\omega}_t)\,\mathbf{1}_{\omega>\bar{\omega}_t}\right]$) the opposite happens. In this second case, financial intermediaries almost never park excess reserves in the balance sheet of the Monetary Authority because the financial intermediary system almost never has enough reserves to meet all of its aggregate liquidity needs. Narrowing the corridor then improves the terms of trade of borrowers of reserves while keeping constant those of lenders of reserves. The natural response of financial intermediaries is to reduce their liquidity ratio and their liquidity positions.

How liquid the financial intermediary system initially is in equilibrium is endogenous and depends on monetary policy. For a given width of corridor, the ratio of the target of inflation $\pi$ to the market rate $r_{m,t}$ determines the liquidity status of the financial intermediary system in equilibrium. If $\pi$ is low relative to $r_{m,t}$, the opportunity cost of holding reserves is low relative to its benefits. As a consequence, liquidity ratios are high, and the financial intermediary system is highly liquid. If $\pi$ is high relative to $r_{m,t}$, the opposite results happens.

For any given initial liquidity status of the financial intermediary, narrowing the corridor reinforces such status (Figure 3.6 again). Specifically, if initially the financial intermediary system is liquid, a narrower the corridor system boosts liquidity ratios $\bar{\omega}_t$ and $\phi_{m,t}$ as well as the share of time that the economy spends in the liquid region of $\{\bar{\omega}_t > E_F\left[(\omega - \bar{\omega}_t)\,\mathbf{1}_{\omega>\bar{\omega}_t}\right]\}$. If initially the financial intermediary system is illiquid, the opposite happens: A narrower the corridor system reduces liquidity ratios $\bar{\omega}_t$ and $\phi_{m,t}$ as well as the share of time that the economy spends in the illiquid region of $\{\bar{\omega}_t < E_F\left[(\omega - \bar{\omega}_t)\,\mathbf{1}_{\omega>\bar{\omega}_t}\right]\}$.

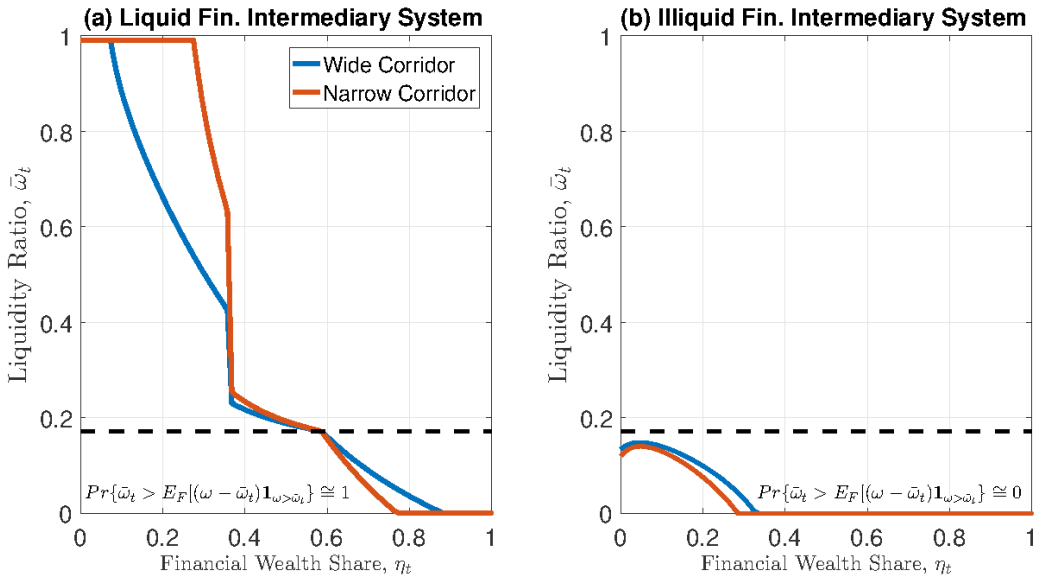



Figure 3.6: Real Effects of Narrower Corridors

The responses of the other endogenous variables to a narrower corridor are independent of the initial liquidity status of the financial intermediary system. The other variables respond mainly to the fact that a narrower corridor improves the terms of trade of trading with the Monetary Authority.

The portfolio share of physical capital $\phi_{k,t}$ always increase, because better terms of trade of borrowing reserves against, and of lending excess reserves to, the Monetary Authority (along with the subsequent liquidity response of financial intermediaries) reduce the costs of leverage associated with withdrawal risks on deposits. The portfolio share $\phi_{k,t}$ increase relatively more when financial intermediaries are undercapitalized, and financing constraints bind, because in that region financial intermediaries are relatively more exposed to withdrawal risks. The economy spends more time in regions in which financial intermediaries are well-capitalized and the ratio $Y_t/K_t$ is high, because financial intermediation because more profitable when leverage costs fall.

## 3.6 Conclusion

The Global Financial Crisis of 2008 has underscored the importance of the financial system in the transmission mechanism of monetary policy. This paper proposes a framework in which the joint provision of settlement services and financial intermediary services matter for assessing the real effects of monetary policy. A key insight that follows from the paper is that interest-rate policies such as policies on the nominal interest rate, on the discount window rate, and on interest rate on excess reserves, affect the interplay between liquidity management and leverage decisions at the financial intermediary level, and whence the course of the real economy.

The analysis conducted in this paper can be extended in two notable directions. A first direction consists in marrying this paper with Van der Ghote (2017). The

resulting marriage would yield a new framework in which the interplay between liquidity management and leverage decisions interacts with sluggish price adjustments. The second direction consists in allowing endogenous variables to jump along with the Poisson process $J_t$. This second extension would relate to the bank runs framework of Gertler and Kiyotaki (2015).

# Appendix A

# Appendix to Chapters 1 and 2

Let $B_{j,t}$ follow an Ito process with drift process $\mu_{B_j,t}$ and diffusion process $\sigma_{B_j,t}$. In the Appendixes, we denote $E_t\left[dB_{j,t}/B_{j,t}\right]$ by $\mu_{B_j,t}dt$ and $Cov_t\left[dB_{1,t}/B_{1,t}, B_{2,t}/B_{2,t}\right]$ by $\sigma_{B_1,t}\sigma_{B_2,t}dt$.

## A.1 Optimality Conditions

The Appendix A.1 derives the optimality conditions in the portfolio problems of households and of financial intermediaries.

### A.1.1 Households

Let $W_t$ denote the value of households. We conjecture that $W_t$ satisfies

$$W_t = W\left(n_{h,t}, J_t\right)$$

where $W : \mathbb{R}^2 \to \mathbb{R}$ is a twice continuously differentiable function, and $J_t$ is a sufficient statistic of the aggregate state variables in the households problem. The process $J_t$ is a scalar. We conjecture that $J_t$ follows an Ito process with drift process $\mu_{J,t}$ and diffusion process $\sigma_{J,t}$.

The value $W_t$ is the solution to the Hamilton-Jacobi-Bellman (HJB) equation

$$\rho W_t = \max_{c_t, l_t, \bar{k}_{h,t} \geq 0} \left\{ \begin{array}{l} \ln c_t - \chi \frac{l_t^{1+\psi}}{1+\psi} + \frac{\partial W_t}{\partial n_{h,t}} \mu_{n,t} n_{h,t} + \frac{\partial W_t}{\partial J_t} \mu_{J,t} J_t + \\ \\ \frac{1}{2} \frac{\partial^2 W_t}{\left(\partial n_{h,t}\right)^2} \left(\sigma_{n,t} n_{h,t}\right)^2 + \frac{\partial^2 W_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \sigma_{n,t} n_{h,t} + \frac{1}{2} \frac{\partial^2 W_t}{(\partial J_t)^2} \left(\sigma_{J,t} J_t\right)^2 \end{array} \right\}$$

where the drift and the diffusion processes for net worth are

$$\mu_{n,t} n_{h,t} = \left( u_{h,t} \frac{r_{k,t}}{q_t} + \mu_{q,t} \right) q_t \bar{k}_{h,t} + \left(i_t - \pi_t\right) \left(n_{h,t} - q_t \bar{k}_{h,t}\right) + w_t l_t + \Pi_t + Div_t - c_t$$

$$\sigma_{n,t} n_{h,t} = q_t \bar{k}_{h,t} \sigma_{q,t}$$

respectively.

The first-order condition with respect to consumption is

$$\frac{1}{c_t} = \frac{\partial W_t}{\partial n_{h,t}}$$

The first-order condition with respect to labor is

$$\chi l_t^{\psi} = w_t \frac{\partial W_t}{\partial n_{h,t}}$$

The first-order condition with respect to physical capital is

$$\left[ u_{h,t} \frac{r_{k,t}}{q_t} + \mu_{q,t} - \left(i_t - \pi_t\right) \right] \frac{\partial W_t}{\partial n_{h,t}} + \sigma_{q,t} \frac{\partial^2 W_t}{\left(\partial n_{h,t}\right)^2} \sigma_{n,t} n_{h,t} + \sigma_{q,t} \frac{\partial^2 W_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \leq 0$$

with equality if $\bar{k}_{h,t} > 0$.

The intra-temporal condition between consumption and labor follows from combining the first two first-order conditions. The asset pricing conditions on deposits and on the price of capital follow from applying the same methodology as in Cox,

Ingersoll and Ross (1985). Specifically, first, replace the first-order conditions in the HJB equation; second, take the first-order condition with respect to $n_{h,t}$ in the expression obtained in the first step; and third, re-arrange the expression obtained in the second step accordingly.

### A.1.2 Financial Intermediaries

The Tobin's Q $v_t$ is the solution to the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}\tilde{\rho} v_t &= \max_{\phi_t} \left\{ \tilde{\rho} + \left[ \tilde{\mu}_{n,t} + \mu_{v,t} - (i_t - \pi_t) + \sigma_{v,t} \tilde{\sigma}_{n,t} - \sigma_{c,t} \tilde{\sigma}_{n,t} - \sigma_{c,t} \sigma_{v,t} \right] v_t \right\} \\ &\quad s.t. : \phi_t \in [1, \min \{ \lambda v_t, \Phi_t \}]\end{aligned}$$

where the drift and diffusion processes of net worth are

$$\tilde{\mu}_{n,t} = \left( \frac{r_{k,t}}{q_t} + \mu_{q,t} \right) \phi_t + (i_t - \pi_t)(1 - \phi_t)$$

$$\tilde{\sigma}_{n,t} = \phi_t \sigma_{q,t}$$

respectively.[1]

The first-order condition with respect to the leverage multiple is

$$\frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) - \sigma_{c,t} \sigma_{q,t} + \sigma_{v,t} \sigma_{q,t} \geq 0$$

[1] To derive the HJB equation, we work with the gain process $G_t$

$$G_t \equiv E_t \int_{-\infty}^{\infty} \tilde{\rho} e^{-\tilde{\rho} s} \Lambda_s n_{f,s} ds = \int_{-\infty}^{t} \tilde{\rho} e^{-\tilde{\rho} s} \Lambda_s n_{f,s} ds + e^{-\tilde{\rho} t} \Lambda_t v_t n_{f,t}$$

The equality at the RHS follows from the definition of $V_t$ and from the conjecture that $V_t = v_t n_{f,t}$. The HJB equation follows from applying Ito's Lemma to the RHS, and from equalizing the resulting drift process to zero. (The drift process of $G_t$ is null because $G_t$ is a martingale: Specifically, $G_t$ is the conditional expectation of a random variable. The drift process of $\Lambda_t$ is $-(i_t - \pi_t)$, and its diffusion process is $-\sigma_{c,t}$).

with equality if $\phi_t < \min\{\lambda v_t, \Phi_t\}$.

The HJB equation yields the condition

$$\tilde{\mu}_{n,t} - (i_t - \pi_t) - \sigma_{c,t}\tilde{\sigma}_{n,t} + \sigma_{v,t}\tilde{\sigma}_{n,t} + \frac{\tilde{\rho}}{v_t} + \mu_{v,t} - \tilde{\rho} - \sigma_{c,t}\sigma_{v,t} = 0$$

## A.2 Distortions and Wedges

The Appendix A.2 derives the aggregate production function and the relationship between the marginal production costs, the productivity wedge and the labor wedge.

### A.2.1 Aggregate Production Function

The market clearing conditions on inputs are

$$\begin{aligned} l_{d,t}(\omega_t y_t) &\equiv \frac{1}{A_t}\left[\frac{\alpha}{1-\alpha}\frac{r_{k,t}}{w_t}\right]^{1-\alpha}\omega_t y_t = l_t \\ k_{d,t}(\omega_t y_t) &\equiv \frac{1}{A_t}\left[\frac{1-\alpha}{\alpha}\frac{w_t}{r_{k,t}}\right]^{\alpha}\omega_t y_t = u_t\bar{k} \end{aligned}$$

Here, $l_{d,t}(\omega_t y_t)$ denotes the aggregate demand of intermediate firms for labor and $k_{d,t}(\omega_t y_t)$ denotes their aggregate demand for capital services.

To derive the aggregate production function, divide the first condition by the second condition and obtain the following expression for $r_{k,t}/w_t$

$$\frac{r_{k,t}}{w_t} = \frac{1-\alpha}{\alpha}\frac{l_t}{u_t\bar{k}}$$

Replace then the expression for $r_{k,t}/w_t$ in the first condition.

### A.2.2 Marginal Production Costs

The marginal production costs are

$$\frac{x_t\left(y_{j,t}\right)}{y_{j,t}} = \frac{1}{A_t}\frac{w_t^{\alpha} r_{k,t}^{1-\alpha}}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}$$

The expression for $r_{k,t}/w_t$ coupled with the intra-temporal condition between consumption and labor imply that

$$\frac{x_t\left(y_j\right)}{y_j} = \frac{h_t}{\omega_t}$$

## A.3 Competitive Equilibrium

The Appendix A.3 characterizes the competitive equilibrium for the different policies under consideration and describes the methodology used to solve for the Markov equilibrium numerically.

### A.3.1 Characterization

#### Flexible Price Economy

In the flexible price economy, $\omega_t = 1$ and $l_t = l_{*,t} = \left(\alpha/\chi\right)^{\frac{1}{1+\psi}}$. The dividend return on capital is given by the marginal productivity of capital services

$$r_{k,t} = \left(1-\alpha\right)\left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1+\psi}}\frac{A_t}{\left(u_t\bar{k}\right)^{\alpha}}$$

**Competitive Equilibrium** The competitive equilibrium is characterized by the following set of conditions

- The asset pricing conditions on capital and on the Tobin's Q

$$u_{h,t}\frac{r_{k,t}}{q_t}+\mu_{q,t}-r_t-\sigma_{y,t}\sigma_{q,t}+\left[(1-u_{h,t})\frac{r_{k,t}}{q_t}+\sigma_{v,t}\sigma_{q,t}\right]\mathbf{1}_{f,t}=0$$

$$\left[(1-u_{h,t})\frac{r_{k,t}}{q_t}+\sigma_{v,t}\sigma_{q,t}\right]\phi_t\mathbf{1}_{f,t}+\frac{\tilde{\rho}}{v_t}+\mu_{v,t}-\tilde{\rho}-\sigma_{y,t}\sigma_{v,t}=0$$

- The law of motion of $\eta_t$

$$\mu_{\eta,t}=\frac{r_{k,t}}{q_t}+\left[\frac{r_{k,t}}{q_t}+\mu_{q,t}-r_t-\sigma_{q,t}^2\right](\phi_t-1)+\frac{\kappa}{\eta_t}-\tilde{\rho}$$

$$\sigma_{\eta,t}=(\phi_t-1)\,\sigma_{q,t}$$

The leverage multiple is $\phi_t=\min\{\lambda v_t,\Phi_t,1/\eta_t\}$. The real interest rate is $r_t=\rho+\mu_{y,t}-\sigma_{y,t}^2$ (this follows from the asset pricing condition on deposits). The process $\mathbf{1}_{f,t}\in\{0,1\}$ indicates whether financial intermediaries are the marginal investors on capital. Namely, $\mathbf{1}_{f,t}=1$ iff $\phi_t=1/\eta_t$.

**Markov Equilibrium** We conjecture that a Markov equilibrium exists. The state variables are $A$ and $\eta$. We conjecture furthermore that the Markov equilibrium is scale invariant with respect to $A$

$$q_t=q(\eta_t)A_t;\qquad v_t=v(\eta_t)$$

where $q,v:[0,1]\to\mathbb{R}_{++}$ are twice continuously differentiable functions.

The Markov equilibrium has two regions. The equilibrium regions differ on the identity of the marginal investor.

For the region in which households are the marginal investors (i.e. $\mathbf{1}_f = 0$ locally), the Markov equilibrium is characterized by the second-order ODE system

$$\left[1-(\phi-1)\left(\varepsilon_q-\varepsilon_y\right)\right]u_h\frac{r_k}{q}+\left(\varepsilon_q-\varepsilon_y\right)\left(\frac{r_k}{q}\phi+\frac{\kappa}{\eta}-\tilde{\rho}-\varepsilon_q\sigma_\eta^2\right)-\rho+$$
$$+\frac{1}{2}\left(\varepsilon_{q_\eta}\varepsilon_q-\varepsilon_{y_\eta}\varepsilon_y\right)\sigma_\eta^2=0$$

$$\left[\left(1-u_h\right)\frac{r_k}{q}+\varepsilon_v\sigma_\eta\sigma_q\right]\phi+\frac{\tilde{\rho}}{v}+\left[\left[u_h+\left(1-u_h\right)\phi\right]\frac{r_k}{q}+\frac{\kappa}{\eta}-\tilde{\rho}-\sigma_q\sigma_\eta\right]\varepsilon_v-\tilde{\rho}+$$
$$+\varepsilon_v\varepsilon_{v_\eta}\frac{\sigma_\eta^2}{2}=0$$

with $\phi=\min\left\{\lambda v,\Phi\right\}$.

For the region in which financial intermediaries are the marginal investors (i.e. $\mathbf{1}_f = 1$ locally), the Markov equilibrium is characterized by the second-order ODE system

$$\left[1-(\phi-1)\left(\varepsilon_q-\varepsilon_y\right)\right]\left(\frac{r_k}{q}+\varepsilon_v\sigma_\eta\sigma_q\right)+\left(\varepsilon_q-\varepsilon_y\right)\left(\frac{r_k}{q}\phi+\frac{\kappa}{\eta}-\tilde{\rho}-\varepsilon_q\sigma_\eta^2\right)-\rho+$$
$$+\frac{1}{2}\left(\varepsilon_{q_\eta}\varepsilon_q-\varepsilon_{y_\eta}\varepsilon_y\right)\sigma_\eta^2=0$$

$$\frac{\tilde{\rho}}{v}+\left[\frac{r_k}{q}-\varepsilon_v\sigma_\eta^2+\frac{\kappa}{\eta}-\tilde{\rho}-\sigma_q\sigma_\eta\right]\varepsilon_v-\tilde{\rho}+\frac{1}{2}\varepsilon_v\varepsilon_{v_\eta}\sigma_\eta^2=0$$

with $\phi=1/\eta$.

The independent variable in the ODE systems is $\eta$. The endogenous functions are $q$ and $v$. The functions $q_\eta$ and $v_\eta$ are the first-order derivative of $q$ and of $v$, respectively, with respect to $\eta$. The function $r_k$ is the dividend return on capital scaled down by the productivity level $A$. The productivity coefficient of households $u_h$ is linear on $\phi\eta$ (see the Calibration section). Aggregate risk satisfies $\sigma_q=\sigma_A/\left[1-(\phi-1)\varepsilon_q\right]$. Notice that we have omitted the time subscript $t$ in the ODE systems.

The ODE systems follow from rearranging the conditions that characterize the competitive equilibrium and from applying Ito's Lemma accordingly.

**Invariant Distribution** Let $\delta_\eta : [0,1] \to \mathbb{R}_{++}$ denote the invariant probability distribution of $\eta$. The function $\delta_\eta$ solves the Kolmogorov forward-equation

$$\frac{\partial}{\partial \eta}\left[\mu_\eta \eta * \delta_\eta\right] + \frac{\partial^2}{\partial \eta^2}\left[\left(\sigma_\eta \eta\right)^2 * \delta_\eta\right] = 0$$

The invariant probability distribution therefore satisfies

$$\delta_\eta \propto \frac{1}{\left(\sigma_\eta \eta\right)^2} \exp\left\{\int_0^\eta 2\frac{\mu_{\tilde{\eta}}\tilde{\eta}}{\left(\sigma_{\tilde{\eta}}\tilde{\eta}\right)^2} d\tilde{\eta}\right\}$$

with $\int_0^1 \delta_\eta\left(\tilde{\eta}\right) d\tilde{\eta} = 1$.

**Economy without Macro-prudential Policy**

In the economy without macro-prudential policy, $\Phi = +\infty$.

We conjecture that there are two equilibrium regions. Specifically, we conjecture that $(i)$ for $\eta < \bar{\eta}$, $\lambda v < 1/\eta$ and $\mathbf{1}_f = 0$; and that $(ii)$ for $\eta \geq \bar{\eta}$, $\lambda v \geq 1/\eta$ and $\mathbf{1}_f = 1$. The threshold state $\bar{\eta}$ is such that $\lambda \bar{v} = 1/\bar{\eta}$ where $\bar{v} \equiv v\left(\bar{\eta}\right)$.

**Economy with Macro-prudential Policy**

In the economy with macro-prudential policy, $\Phi = a_1\eta + a_0 + a_2\mathbf{1}_H$. The constants $a_1, a_0$ are such that $(i)$ $\lambda v_L = a_1\eta_L + a_0$; and $(ii)$ $1/\eta_H = a_1\eta_H + a_0$; with $v_L \equiv v\left(\eta_L\right)$ and $0 < \eta_L < \bar{\eta} < \eta_H < 1$. The constant $a_2$ is such that $\Phi > 1/\eta$ for $\eta > \eta_H$. The function $\mathbf{1}_H$ indicates whether $\eta > \eta_H$.

We conjecture that there are three equilibrium regions. Specifically, we conjecture that $(i)$ for $\eta < \eta_L$, $\lambda v < \min\left\{\Phi, 1/\eta\right\}$ and $\mathbf{1}_f = 0$; $(ii)$ for $\eta \in \left[\eta_L, \eta_H\right]$, $\Phi \leq \min\left\{\lambda v, 1/\eta\right\}$ and $\mathbf{1}_f = 0$; and that $(iii)$ for $\eta > \eta_H$, $1/\eta < \min\left\{\lambda v, \Phi\right\}$ and $\mathbf{1}_f = 1$.

**Optimization Problem** Let $W_F : [0,1] \to \mathbb{R}_-$ denote the *conditional* present discounted value of the aggregate TFP losses from financial disintermediation

$$W_F(\eta_t) \equiv (1-\alpha) E_t \int_t^\infty e^{-\rho(s-t)} \ln u_s ds$$

We conjecture that $W_F$ is twice continuously differentiable. The value $W_F$ solves the HJB equation

$$\rho W_F = (1-\alpha) \ln u + \frac{\partial W_F}{\partial \eta} \mu_\eta \eta + \frac{1}{2} \frac{\partial^2 W_F}{\partial \eta^2} (\sigma_\eta \eta)^2$$

The optimal macro-prudential policy is such that the thresholds $\eta_L, \eta_H$ solve

$$\max_{\eta_L, \eta_H} \int_0^1 W_F(\tilde{\eta}) \delta_\eta(\tilde{\eta}) d\tilde{\eta}$$

**Sticky Price Economy**

In the sticky price economy, in general, $\omega_t \geq 1$ and $l_t \neq l_{*,t}$. The dividend return on capital $r_{k,t}$ is

$$r_{k,t} = e^{(1+\alpha+\psi)g_t} * (1-\alpha) \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1+\psi}} \frac{A_t}{\left(u_t \bar{k}\right)^\alpha} \frac{1}{\omega_t}$$

Let $B_t$ and $M_t$ denote the present discounted value of sales revenues (gross of sales subsidies) and of production costs, respectively. Specifically,

$$B_t \equiv E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} \frac{p_t y_{d,s}(p_t)}{p_s} ds = \theta y_t E_t \int_t^\infty \exp\left\{ - \int_t^s (\theta + \rho - (\varepsilon - 1) \pi_{\tilde{s}}) d\tilde{s} \right\} ds$$

$$M_t \equiv E_t \int_t^\infty \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} x_s [y_{d,s}(p_t)] ds = \theta y_t E_t \int_t^\infty \exp\left\{ - \int_t^s (\theta + \rho - \varepsilon \pi_{\tilde{s}}) d\tilde{s} \right\} \frac{h_s}{\omega_s} ds$$

The equalities at the RHS follow from combining $(i)$ $y_{d,t}(p_j) = (p_j/p_t)^{-\varepsilon} y_t$; $(ii)$ $\Lambda_t = e^{-\rho t}/y_t$; $(iii)$ $p_s/p_t = \exp\left\{\int_t^s \pi_{\tilde{s}} d\tilde{s}\right\}$; together with $(iv)$ $x_t(y_j)/y_j = h_t/\omega_t$.

The unitary values $b_t \equiv B_t/\theta y_t$ and $m_t \equiv M_t/\theta y_t$ satisfy the asset pricing conditions

$$\frac{1}{b_t} + \mu_{b,t} + (\varepsilon - 1)\,\pi_t - \rho - \theta = 0$$

$$\frac{h_t}{\omega_t}\frac{1}{m_t} + \mu_{m,t} + \varepsilon\pi_t - \rho - \theta = 0$$

where $\mu_{b,t}$ and $\mu_{m,t}$ are the drift processes of $b_t$ and of $m_t$, respectively. The asset pricing conditions follow from the Feynman-Kac formula.

**Competitive Equilibrium** The competitive equilibrium is characterized by the following set of conditions

- The asset pricing conditions on capital and on the Tobin's Q (same equations as in the flexible price economy)
- The asset pricing conditions on the unitary sales revenues and on the unitary production costs (i.e. the asset pricing conditions on $b_t$ and on $m_t$)
- The law of motion of $\eta_t$ (same equations as in the flexible price economy)
- The law of motion of $\omega_t$

$$\mu_{\omega,t} = \frac{\theta}{\omega_t}\left[1 - \frac{\varepsilon - 1}{\theta}\pi_t\right]^{\frac{\varepsilon}{\varepsilon-1}} - \theta + \varepsilon\pi_t$$

The leverage multiple is $\phi_t = \min\{\lambda v_t, \Phi_t, 1/\eta_t\}$. The real interest rate is $r_t \equiv i_t - \pi_t = \rho + \mu_{y,t} - \sigma_{y,t}^2$. The inflation rate satisfies

$$\pi_t = \frac{\theta}{\varepsilon - 1}\left[1 - \left(\frac{b_t}{m_t}\right)^{\varepsilon-1}\right]$$

Notice that $p_{*,t}/p_t = m_t/b_t$.

**Markov Equilibrium** We conjecture that a Markov equilibrium exists. The state variables are $A$, $\eta$ and $\omega$. We conjecture furthermore that $(i)$ the Markov equilibrium is scale invariant with respect to $A$; and that $(ii)$ the portfolio problems of households and of financial intermediaries are scale invariant with respect to $A/\omega$;[2]

$$q_t = q(\eta_t) A_t/\omega_t; \qquad v_t = v(\eta_t); \qquad b_t = b(\eta_t, \omega_t); \qquad m_t = m(\eta_t, \omega_t)$$

where $q, v : [0,1] \to \mathbb{R}_{++}$ and $b, m : [0,1] \times [1, +\infty] \to \mathbb{R}_{++}$ are twice continuously differentiable functions.

The Markov equilibrium has the same two regions as in the flexible price economy. The Markov equilibrium is characterized by $(i)$ the same second-order ODE systems as in the flexible price economy but with $r_k = e^{(1+\alpha+\psi)g}\hat{r}_k$; and by $(ii)$ the second-order PDE system

$$\frac{1}{b} + \varepsilon_b \mu_\eta + \tilde{\varepsilon}_b \mu_\omega + \frac{1}{2}\varepsilon_b \varepsilon_{b_\eta} \sigma_\eta^2 + (\varepsilon - 1)\pi - \rho - \theta = 0$$

$$\frac{h}{\omega}\frac{1}{m} + \varepsilon_m \mu_\eta + \tilde{\varepsilon}_m \mu_\omega + \frac{1}{2}\varepsilon_m \varepsilon_{m_\eta} \sigma_\eta^2 + \varepsilon\pi - \rho - \theta = 0$$

The independent variables in the PDE system are $\eta$ and $\omega$. The endogenous functions are $b$ and $m$. The functions $b_\eta$ and $m_\eta$ are the first-order derivative of $b$ and of $m$, respectively, with respect to $\eta$. The functions $\tilde{\varepsilon}_b$ and $\tilde{\varepsilon}_m$ are the elasticity of $b$ and of $m$, respectively, with respect to $\omega$.

It is worth noticing that the ODE systems in $(i)$ are independent of the PDE system in $(ii)$.

**Invariant Distribution** Let $\delta_{\eta,\omega} : [0,1] \times [1, +\infty] \to \mathbb{R}_{++}$ denote the joint invariant probability distribution of $\eta$ and $\omega$. The function $\delta_{\eta,\omega}$ solves the Kolmogorov

[2]Notice that $g_t$ has to be independent of $\omega_t$ for this second conjecture to hold.

forward-equation

$$\frac{\partial}{\partial \eta}\left[\mu_\eta \eta * \delta_{\eta,\omega}\right] + \frac{\partial}{\partial \omega}\left[\mu_\omega \omega * \delta_{\eta,\omega}\right] + \frac{\partial^2}{\partial \eta^2}\left[\left(\sigma_\eta \eta\right)^2 * \delta_{\eta,\omega}\right] = 0$$

**Optimization Problem (Coordinated Policy)** The value of households satisfies

$$W = \frac{1}{\rho}\left[\ln A + \frac{1}{\rho}\left(\mu_A - \frac{1}{2}\sigma_A^2\right) + (1-\alpha)\ln \bar{k}\right] + W_F + W_R$$

The value function $W_F : [0,1] \to \mathbb{R}_-$ is the *conditional* present discounted value of the aggregate TFP losses from financial disintermediation (same definition as in the flexible price economy). The value function $W_R : [0,1] \times [1,+\infty] \to \mathbb{R}$ is the *conditional* present discounted value of the aggregate TFP losses from nominal rigidities.

We conjecture that $W_R$ is twice continuously differentiable. The value $W_R$ solves the HJB equation

$$\rho W_R = \ln \frac{1}{\omega} + \alpha g + \frac{\alpha}{1+\psi}\ln\frac{\alpha}{\chi} - \frac{\alpha}{1+\psi}e^{(1+\psi)g} + \frac{\partial W_R}{\partial \eta}\mu_\eta \eta + \frac{\partial W_R}{\partial \omega}\mu_\omega \omega + \frac{1}{2}\frac{\partial^2 W_R}{\partial \eta^2}\left(\sigma_\eta \eta\right)^2$$

The coordinated policy is such that $\{g_L, g_H, \tilde{\eta}_L, \tilde{\eta}_H; \eta_L, \eta_H\}$ solve

$$\max_{g_L, g_H, \tilde{\eta}_L, \tilde{\eta}_H; \eta_L, \eta_H} \int \left[W_F\left(\tilde{\eta}\right) + W_R\left(\tilde{\eta}, \tilde{\omega}\right)\right] \delta_{\eta,\omega}\left(\tilde{\eta}, \tilde{\omega}\right) d\left(\tilde{\eta}, \tilde{\omega}\right)$$

### A.3.2 Numerical Method

We use spectral methods to solve for the Markov equilibrium numerically. Specifically, we interpolate the endogenous functions with a linear combination of Chebyshev Polynomials of the First Kind. For the ODE systems, we evaluate the interpolation at the Chebyshev nodes using a grid with 190 points. For the PDE systems, we evaluate the interpolation at the Tensor basis (i.e. Tensor product + Cartesian product of Chebyshev nodes) using a grid with $60^2$ points.

We use a non-linear solver to find the coefficients associated with the Chebyshev Polynomials. For the ODE systems, we use as initial guess the equilibrium outcome of the frictionless economy (i.e. $1/\theta, 1/\lambda \to 0$) : Namely, $l_t = l_{*,t};\ \phi_t = 1/\eta_t;\ q_t/r_{k,t} = 1/\rho;\ v_t = 1;\ \omega_t = 1$. For the PDE systems, we use as initial guess the equilibrium outcome of the regulated flexible price economy.

# Appendix B

# Appendix to Chapter 3

Let $B_{j,t}$ with $j = 1, 2$ denote Ito processes with drift process $\mu_{B_j,t}$ and diffusion process $\sigma_{B_j,t}$. In the Appendixes, we use the expressions $E_t\left[dB_{j,t}/B_{j,t}\right]$ and $\mu_{B_j,t}dt$ interchangeably. We also use interchangeably the expressions $Cov_t\left[dB_{1,t}/B_{1,t}, B_{2,t}/B_{2,t}\right]$ and $\sigma_{B_1,t}\sigma_{B_2,t}dt$.

## B.1 Optimality Conditions

In the Appendix B.1, we derive the optimality conditions. Firstly, we derive the optimality conditions of the portfolio problem of financial intermediaries. Secondly, we derive the optimality conditions of the portfolio problem of households.

### B.1.1 Financial Intermediaries

**HJB** Let $G_t$ denote the gain process associated with the franchise value of financial intermediaries $V_t$. The gain process $G_t$ satisfies

$$G_t \equiv E_t \int_0^\infty \tilde{\rho} e^{-\tilde{\rho}s} \Lambda_s n_{f,s} ds = \int_0^t \tilde{\rho} e^{-\tilde{\rho}s} \Lambda_s n_{f,s} ds + e^{-\tilde{\rho}t} \Lambda_t v_t n_{f,t}$$

The equality at the RHS follows from the conjecture that $V_t = v_t n_{f,t}$ and from the definition of $V_t$. The drift process of $G_t$ is null because $G_t$ is the conditional expectation of a random variable. From applying Ito's Lemma to the RHS, and then equalizing the resulting drift process to zero, we obtain the following Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}\tilde{\rho} v_t &= \max_{\phi_{m,t},\phi_{k,t},\iota_t \geq 0} \left\{\tilde{\rho} + \left[\tilde{\mu}_{n,t} + \mu_{v,t} + \mu_{\Lambda,t} + \sigma_{v,t}\tilde{\sigma}_{n,t} + \sigma_{\Lambda,t}\tilde{\sigma}_{n,t} + \sigma_{\Lambda,t}\sigma_{v,t}\right] v_t\right\} \\ & s.t. : \phi_{k,t} \leq \lambda v_t\end{aligned}$$

where the drift and diffusion processes of net worth are

$$\begin{aligned}\tilde{\mu}_{n,t} = &\left[\frac{a_f - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma\right]\phi_{k,t} - \pi_t \phi_{m,t} - (i_t - \pi_t)(\phi_{k,t} + \phi_{m,t} - 1) \\ &- \tau_{1,t} + E_F\left[R_t(\omega, \bar{\omega}) * (\phi_{k,t} + \phi_{m,t} - 1) + \tau_{2,t}(\omega)\right]\theta\end{aligned}$$

$$\tilde{\sigma}_{n,t} = (\sigma_{q,t} + \sigma)\phi_{k,t}$$

respectively. To derive the second line in $\tilde{\mu}_{n,t}$, we have used the conjecture that $v_t$ follows an Ito process (notice that $v_t^+ = v_t$). The Tobin's Q of financial intermediaries $v_t$ is the solution to the HJB above.

**FOC** The first-order condition with respect to the internal investment rate is

$$I'(\iota_t) = 1/q_t$$

The first-order condition with respect to the portfolio share in reserves is

$$-i_t + E_F\left[R_{m,t}(\omega, \bar{\omega})\right]\theta \leq 0$$

with equality if $\phi_{m,t} > 0$. The function $R_{m,t}(\omega, \bar{\omega})$ is the partial derivative of the total return $R_t(\omega, \bar{\omega}) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to $\phi_{m,t}$.

The first-order condition with respect to the portfolio share in physical capital is

$$\frac{a_f - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma - (i_t - \pi_t) + E_F\left[R_{k,t}(\omega, \bar{\omega})\right]\theta + (\sigma_{\Lambda,t} + \sigma_{v,t})(\sigma_{q,t} + \sigma) \geq 0$$

with equality if $\phi_{k,t} < \lambda v_t$.The function $R_{k,t}(\omega, \bar{\omega})$ is the partial derivative of the total return $R_t(\omega, \bar{\omega}) * (\phi_{k,t} + \phi_{m,t} - 1)$ with respect to $\phi_{k,t}$.

**HJB Again** The asset pricing condition for the Tobin's Q satisfies

$$\frac{\tilde{\rho}}{v_t} + [FOC_k]\,\phi_{k,t} + E_F\left[r_{b,t}\mathbf{1}_{\omega > \bar{\omega}_t} + r_{l,t}\mathbf{1}_{\omega < \bar{\omega}_t}\right]\bar{\omega}_t\theta - \tau_{1,t} + E_F\left[\tau_{2,t}(\omega)\right]\theta + \mu_{v,t} - \tilde{\rho} + \sigma_{\Lambda,t}\sigma_{v,t} = 0$$

### B.1.2 Households

**HJB** Let $W_t$ denote the value of households. We conjecture that $W_t$ satisfies

$$W_t = W(n_{h,t}, J_t)$$

where $W : \mathbb{R}^2 \to \mathbb{R}$ is a twice continuously differentiable function, and $J_t$ is a sufficient statistic of the aggregate state variables in the problem of households. The process $J_t$ is a scalar. We conjecture that $J_t$ follows an Ito process with drift process $\mu_{J,t}$ and diffusion process $\sigma_{J,t}$.

The value $W_t$ is the solution to the Hamilton-Jacobi-Bellman (HJB) equation

$$\rho W_t = \max_{c_t, k_{h,t}, \iota_t \geq 0} \left\{ \begin{array}{l} \ln c_t + \frac{\partial W_t}{\partial n_{h,t}}\mu_{n,t}n_{h,t} + \frac{\partial W_t}{\partial J_t}\mu_{J,t}J_t + \\ \\ \frac{1}{2}\frac{\partial^2 W_t}{\left(\partial n_{h,t}\right)^2}\left(\sigma_{n,t}n_{h,t}\right)^2 + \frac{\partial^2 W_t}{\partial J_t \partial n_{h,t}}\sigma_{J,t}J_t\sigma_{n,t}n_{h,t} + \frac{1}{2}\frac{\partial^2 W_t}{(\partial J_t)^2}\left(\sigma_{J,t}J_t\right)^2 \end{array} \right\}$$

where the drift and the diffusion processes for net worth are

$$\mu_{n,t} n_{h,t} = \left[ \frac{a_h - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma \right] q_t k_{h,t} + (i_t - \pi_t)(n_{h,t} - q_t k_{h,t}) + \\ + Transfers_t - c_t$$

$$\sigma_{n,t} n_{h,t} = (\sigma_{q,t} + \sigma) q_t k_{h,t}$$

respectively.

**FOC** The first-order condition with respect to internal investment rate is

$$I'(\iota_t) = 1/q_t$$

The first-order condition with respect to consumption is

$$c_t^{-\gamma} = \frac{\partial W_t}{\partial n_{h,t}}$$

The first-order condition with respect to physical capital is

$$\left[ \frac{a_h - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma - (i_t - \pi_t) \right] \frac{\partial W_t}{\partial n_{h,t}} + (\sigma_{q,t} + \sigma) \frac{\partial^2 W_t}{(\partial n_{h,t})^2} \sigma_{n,t} n_{h,t} + \\ + (\sigma_{q,t} + \sigma) \frac{\partial^2 W_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \leq 0$$

with equality if $k_{h,t} > 0$.

To derive the asset pricing conditions on deposits and on the price of capital, we follow the same methodology as in Cox, Ingersoll and Ross (1985). Specifically, first, we replace the first-order conditions in the HJB equation; second, we take the first-order condition with respect to $n_{h,t}$ in the expression that we derived in the first step; and, third, we re-arrange the expression obtained in the second step accordingly.

## B.2 Competitive Equilibrium

In Appendix B.2, we characterize the competitive equilibrium.

**Competitive Equilibrium** The competitive equilibrium is characterized by the following set of conditions

- The internal investment condition

$$I'(\iota_t) = 1/q_t$$

- The asset pricing conditions on capital and on the Tobin's Q

$$\frac{a_h - \iota_t}{q_t} + I(\iota_t) - \delta + u_{q,t} + \sigma_{q,t}\sigma - (i_t - \pi_t) + \sigma_{\Lambda,t}(\sigma_{q,t} + \sigma) +$$

$$+ \left[\frac{a_f - a_h}{q_t} + E_F\left[R_{k,t}(\omega, \bar{\omega})\right]\theta + \sigma_{v,t}(\sigma_{q,t} + \sigma)\right]\mathbf{1}_{f,t} = 0$$

$$\left[\frac{a_f - a_h}{q_t} + E_F\left[R_{k,t}(\omega, \bar{\omega})\right]\theta + \sigma_{v,t}(\sigma_{q,t} + \sigma)\right]\phi_{k,t}\mathbf{1}_{f,t} +$$

$$\bar{\omega}_t(r_{b,t} - r_{l,t})E_F\left[\mathbf{1}_{\omega>\bar{\omega}_t}\right]\theta + r_{b,t}E_F\left[(\omega - \bar{\omega}_t)\mathbf{1}_{\omega>\bar{\omega}_t}\right]\theta + \frac{\tilde{\rho}}{v_t} + \mu_{v,t}$$

$$-\tilde{\rho} - \sigma_{y,t}\sigma_{v,t} = 0$$

- The reserves holdings condition

$$-i_t dt + E_F\left[R_{m,t}(\omega; \bar{\omega}_t)\right]\theta dt \leq 0$$

with "=" if $\phi_{m,t} > 0$

- The law of motion of $\eta_t$

The leverage multiple is $\phi_{k,t} = \min\{\lambda v_t, 1/\eta_t\}$. The real interest rate is $r_t = \rho + \mu_{c,t} - \sigma_{c,t}^2$ (this follows from the asset pricing condition on deposits). The process $\mathbf{1}_{f,t} \in \{0, 1\}$ indicates whether financial intermediaries are the marginal investors on capital. Namely, $\mathbf{1}_{f,t} = 1$ iff $\phi_{k,t} = 1/\eta_t$.

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