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Three Papers on Logic and Contingency

Cohen, Eliya

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THREE PAPERS ON LOGIC AND CONTINGENCY

ELIYA COHEN

A DISSERTATION
PRESENTED TO THE FACULTY
OF PRINCETON UNIVERSITY
IN CANDIDACY FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

RECOMMENDED FOR ACCEPTANCE
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Abstract

This dissertation consists of three papers related to logic and contingency. Each paper interacts with and is critical of methodologies that are cautious to the potential implications of metaphysical intuition for metaphysical theorizing.

The first paper defends an account of future contingents for those attempting to remain neutral about the metaphysics of time. According to the influential account of MacFarlane, only the truth of sentences is relativized to contexts of assessment, possible situations in which the use of an expression might be assessed. I argue instead for a new account, on which both the truth of future contingent sentences—e.g. 'there will be a sea-battle tomorrow'—and content of certain subsentential constituent expressions—e.g. 'Newman-1' introduced as the first born child of the twenty-second century—are relativized to contexts of assessment.

The second paper defends first-order contingentism, roughly the view that it is contingent what individuals there are, against a new argument for its opponent, necessitism, called the Adjunction Argument. The methods employed in this argument make central use of formalism and abduction, relying in particular on the strength certain principles can lend to theories in which they figure. I argue that many contingentists endorse a conception of existence, which makes the relevant formal principles governing quantification inappropriate by their own lights.

The third paper develops a variable domain model theory with accessibility to interpret the language of second-order propositional modal logic and establishes a number of incompleteness results. The results are extended to a related class of logics validating a higher-order comprehension principle. Although the results are primarily formal, they are also of interest to the debate over modal ontology and the recent attention to arguments against the efficacy of comprehension principles compatible with higher-order contingentism, the view that it is contingent what properties and propositions there are.

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My debts go at least as far back as my first mathematics and philosophy classes taught at Reed College. Mark Hinchliff and Paul Hovda first introduced me to the core questions of this dissertation, to the interactions between logic, contingency, and methodology, and the late Tom Wieting, who taught my first course in mathematical logic, introduced me to formal questions related to the final chapter.

This dissertation wouldn't have developed the way it did without these early influences, and without the influence of my advisors Harvey Lederman, Boris Kment, and Gideon Rosen. Harvey taught me a patience for rigor and an intolerance for the unintelligible, Boris a cautious attitude toward formal approaches and to my own intuitions, and Gideon to trust problems over solutions. A special thanks is due to Peter Fritz for inspiration and guidance, especially in the final chapter.

In many ways, this dissertation represents a fraction of the interests I developed at Princeton and the influence I had from the faculty and students. Of those not already mentioned above, I'm particularly indebted to John Burgess, Hans Halvorson, and Ben Morison from the faculty, and Alisabeth Ayars, Claudia Dumitru, Robbie Hirsch, Ian McKeachie, Patrick Miller, Nat Tabris, Alice Van't Hoff, and Daniel Vitek from the student body, when they were students. Finally, I'd like to thank the entire faculty, administration, and student body at Princeton, my cohort, the cohort above, and Daniel Berntson, Sam Berstler, Haley Brennan, Gabe Broughton, David Builes, Rachel Christy, Troy Cross, Katie Cruz, Simon Cullen, Tom Davies, Branden Fitelson, Sam Fullhart, Dan Garber, Liz Harman, Dan Hoek, Austin Hounsel, Jason Iuliano, Maddy Kilbride, Brendan Kolb, Daniel Kranzelbinder, Kyle Landrum, Adam Lerner, Michal Masny, Sarah McGrath, Alex Meehan, Joe Moore, Alejandro Naranjo Sandoval, Samuel Preston, Chris Register, David Schroeren, Gabe Shapiro, Simon Shogry, Mack Sullivan, Aarthy Vaidyanathan, Ian Walling, Megan Wicks, Claudia Yau, Erik Zhang, Snow Zhang, and Christen Zimecki. I'd like to thank my family: Isaac Cohen, Marielle Cohen, Rachel Erwin, Barbara Hart, Conor Meara, and Noah Meara. Finally, I owe thanks and apology to those I've inevitably left out.

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Chapter 1: Future contingents and content relativism

Abstract. The semantic problem of future contingents has largely been discussed as a problem of how to handle future contingency at the sentential or propositional level, as with the classic Aristotelian sentence, 'There will be a sea-battle tomorrow'. In this paper, I discuss an extension of the problem: how should we give a general semantics for future contingents that applies not only to future contingency at the sentential and propositional level, but also at the level of subsentential constituents? Metaphysics-first views, that incorporate or reject a privileged future, can easily give a unified semantics. I focus on views which attempt to remain neutral about the metaphysics of time. I argue that the position best suited to give a general semantics for future contingents while adopting this background assumption is a hybrid truth and content relativist view: relativize the truth of sentences and the content of certain subsentential constituents to contexts of assessment, possible situations in which the use of an expression might be assessed.[1]

1.1 Introductory remarks

We think and talk about the future. We hope, bet, promise, and predict. Looking back, we reflect on and assess our past thoughts and claims about what will be, and seem to rightly judge them to be accurate or not. There's no problem squaring our future talk and attitudes with views of time in which there is one privileged future. The difficulty is in squaring them with views for which

[1]I would like to thank Mark Hinchliff, Paul Hovda, Boris Kment, Harvey Lederman, Gideon Rosen, and many other students and professors at Reed and Princeton for helpful comments and discussions.

there is no privileged future, where we could find ourselves living in one of many different possible continuations of today.

Much of the literature until John MacFarlane's truth relativism was driven by metaphysics: start with the metaphysical view and develop a semantics around it, usually but not always at the expense of one or another intuition.[2] MacFarlane's innovation was to show that we could give a metaphysically neutral semantics that respected the prevailing judgments on both sides of the debate over whether or not there is a privileged future. He presents a semantics that respects the sense in which we can look back on an utterance made in the past and say that it was then accurate without assuming a view on which there is only one privileged way things could go.

Yet, future contingency isn't only a feature of sentences, and it's unclear how a view that adopts the neutral background assumption should handle cases where future contingency is a part of constituent expressions or their introduction, as with the definite description 'the first born child of the twenty-second century' and the name 'Newman-1' introduced by it.

The problem addressed in this paper is just this: what view should we adopt to give a more general semantics of future contingents on the background assumptions set out by MacFarlane? I start by presenting the problem of future contingents and the extension of it. I discuss three classes of responses in the literature: privileged future views, no-privileged future views, and neutral views. The former two can give unsurprising extensions of their original positions. I argue that the position best suited to give a general semantics for future contingents while adopting the neutral background assumptions is a hybrid truth and content relativist view, relativizing the truth of sentences and the content of certain subsentential constituents to contexts of assessment, possible situations in which the use of an expression might be assessed.

1.2 The problem of future contingents

The basic trouble with future contingency and truth is in squaring the ubiquity of future thought and talk with the possibility that the future is really unsettled, that tomorrow isn't just an epistemic contingency—something that might be this way or that for all we know—but a real contingency, as if time branched and each branch represented one of many possible continuations of the present, no one of them privileged so as to have the resources to make future judgments true (or false) now, unless made true by facts obtaining now.[3] Since if tomorrow is really unsettled, isn't there no such

[2] MacFarlane 2003, 2014.

[3] Roughly, branching requires that which worlds or timelines are possible varies with time. Here we are interested only in future branching.

thing as *the* future, but many possible ones? And wouldn't we be in error to think that we can rightly judge, from our perspective now, yesterday's thoughts and claims as then (in)accurate, when yesterday it was possible to find ourselves in a different today?[4]

It's tempting to think that there is no way to square the two, that no-privileged future views are incompatible with the truth of our future-directed and reflective thought and talk.[5] If the two really are incompatible, it would mean that our semantic theories of future-directed talk have serious metaphysical underpinnings: to vindicate our talk about the future would be to vindicate privileged future views. Conversely, it would mean that our physical and metaphysical theories of time have semantic consequences, that to take no-privileged future views seriously would be to accept that our future-directed talk and attitudes are fundamentally confused. Depending on how seriously we take these attitudes and speech, that consequence could be a knock against no-privileged future views in favor of privileged future ones, and even against some prominent physical theories.[6]

We might wonder whether we can dissolve the tension. To succeed in this, one would have to give a semantics that could vindicate ordinary thought and talk about the future in a way that is compatible with no privileged future. We could do this by giving a semantics that assumes there is no privileged future. But it isn't just interesting to be able to adopt physical or metaphysical theories of time without having to accept the widespread error of our future-directed talk and thought, that is, to separate the semantics from the metaphysics. It would also be interesting to separate the metaphysics from the semantics, to be able to give a semantics that doesn't assume any one or other theory of time.

To do so would change the debate over the problem of truth for future contingents, which is often framed and responded to largely in terms of the metaphysics. When we consider the future contingent sentence, 'there will be a sea-battle', it seems we are torn between two intuitions, the indeterminacy intuition and the determinacy intuition.[7]

On the one hand, there appear to be many possible future histories concerning sea-battles, and factors now which are not yet determined that could affect which one becomes the actual future. It seems improper then to assign such a statement a truth-value such as true or false when nothing now settles the case. This is the *indeterminacy intuition.* From our perspective in the present, future contingents seem neither true nor false.

[4]The distinction between privileged futures and no-privileged futures may not be perfect. There are interpretations of quantum mechanics that have branching possibilities, no privileged future, and yet no contingency or indeterminacy. The naive distinction will do for our purposes.

[5]Lewis 1986, 199-209.

[6]See, e.g., Barrett 2001.

[7]MacFarlane 2003.

On the other hand, we wonder if we can coherently claim that all possibilities consistent with the past and present (including the laws of nature) are not-true now without denying that one of them will be true tomorrow. Our future beliefs and claims seem assertible and credible now. And, when amidst a roaring sea-battle, it seems that someone can truly say, "Your assertion yesterday turned out to be true," or "was accurate," as if we were just uncovering what was already there.[8] This is the *determinacy intuition.* (i) From a position now looking forward, it makes sense to say we can have true beliefs and can make true assertions about the future and, (ii), from a position in the future looking back, future contingent sentences appear to express propositions with definite truth-values. This paper focuses on (ii), on retrospection and not assertion.[9]

The problem is said to arise from the apparent incompatibility of these intuitions. On the orthodox view, the proposition expressed by a sentence is relativized only to a *context of utterance*, a possible situation in which an expression can be used.[10] The indexical sentence 'I'm a philosopher' does not express the same proposition across contexts of utterance. Such sentences are *context-sensitive* because the proposition expressed varies across different contexts: at a context where Biden says it, it expresses the proposition *Biden is a philosopher*, at a context where Trump says it, it expresses the proposition *Trump is a philosopher*.[11] The orthodox view stops relativization here. The proposition expressed by 'there will be a sea-battle tomorrow', relative to a context of utterance centered on today, August 12th, just is the proposition that there be a sea-battle on August 13th; and if that proposition lacks a truth-value now, it cannot gain one later.[12] Conversely, if the proposition turns out to be true, then it cannot have been neither true nor false when first said.[13] Hence, assuming orthodoxy, the determinacy and indeterminacy intuitions are incompatible.

1.2.1 Three classes of responses

The semantic literature on future contingents usually proceeds by starting with a metaphysical position and generating a semantics in light of it.

There are the *no-privileged future types*: those who adopt a metaphysics that embraces the open future, none of which are privileged, and either (i) a three-valued semantics, according to which future contingents receive the truth-value INDETERMINATE, (ii) a view on which all future contingents

[8] MacFarlane 2014, 325.

[9] MacFarlane bites the bullet and claims we shouldn't assert future contingents. He also considers weakening a requirement on assertion. Ultimately, I think assertion is a problem for his view and the view presented in this paper, but we won't address it further. See MacFarlane 2014, 230-232.

[10] Kaplan 1989, 591.

[11] '1+1=2' is *context-insensitive* since it expresses the same proposition across contexts of utterance.

[12] I will use 'tomorrow' for readability, but nothing important hinges on the indexical. We could change the phrase to, 'there is a sea-battle on August 13th, 2022' said today, August 12, 2022.

[13] MacFarlane 2014, 322.

are false because 'WILL(P)' means roughly 'P is true in all possible futures', or (iii) a supervaluationist semantics, according to which future contingents have some kind of semantic indeterminacy, where they may be true on some possible future histories and false on others but are neither true nor false now, or it is indeterminate what the truth-value of a future contingents is, but it is nevertheless determinately true that it is either true or false.[14]

Then there are the *privileged future types*: those who, for instance, adopt (i) determinism, roughly, the view for which the laws of nature and a complete account of the state of the universe at any time t together entail the state of the universe at every subsequent time, (ii) standard four-dimensionalism, according to which the unique actual future is the only future, even if it is not determined by the state of the world in the present together with the laws, or (iii) a thin red line view which posits an actual future among the many possible ones.[15] Typically for these views, any utterance about the future is true just in case it is true in the future or the real or actual future.

Instead of giving a semantics that assumes or rejects a privileged future, John MacFarlane proposes rejecting the orthodox view that stops relativization at the context of utterance. In doing so, he presents a semantics for future contingents that vindicates our future-directed talk without assuming any metaphysical view, in particular without assuming a privileged future view, which can clearly vindicating our future talk and thought.

He succeeds in this by relativizing future contingents to both contexts of utterance and *contexts of assessment*, or possible situations in which the use of an expression might be assessed.[16] A sentence is *assessment-sensitive* just in case it expresses a proposition at a context of utterance that yields different *truth-values* from different contexts of assessment. Indexical expressions such as 'I'm a philosopher' are clear examples of context-sensitive sentences. The clearest candidates for assessment-sensitive sentences are taste statements, although even those are contentious. My sister says, "Licorice is tasty." Based on our different taste preferences, from my context of assessment, what she said is false. From my sister's context of assessment, the very same proposition is true.

The claim MacFarlane is making is that, unlike context sensitive sentences, assessment sensitive sentences and the propositions they express don't have truth-values on their own, but only relative to what the context of assessment may supply. This can be made more dramatic if we put it as a point about properties: there is a monadic property F and object x, such that x instantiates F relative to some contexts but not others. This is quite different than context sensitivity, which isn't a view on which propositions themselves vary in truth value across different contexts; rather, the context of

[14] Łukasiewicz 1967; Todd 2021; Thomason 1970; Fine 1975.
[15] Belnap and Green 1994; Lewis 1986; Sider 2001.
[16] MacFarlane 2014, 60.

utterance determines what proposition is expressed by a sentence and that proposition, whatever it is, has a single truth value.

Later, we will go into more detail about content relativism, but it's worth mentioning it up front to distinguish it from truth relativism and context sensitivity. As with truth relativism, content relativism involves assessment sensitivity, but instead of the truth value of propositions varying across contexts of assessment, it is the propositional content itself that varies. According to content relativism, when my sister says, "Licorice is tasty" in context of utterance c, she expresses the proposition *according to Marielle's taste preferences, licorice is tasty* from her context of assessment a_1 and the proposition *according to Eliya's taste preferences, licorice is tasty* from my context of assessment a_2. One utterance, two propositions. On MacFarlane's truth relativist view, what proposition a sentence expresses is fixed by the context of utterance and does not vary with the context of assessment, unlike content relativism, where it's the content and not the truth that varies with the context of assessment.

It's important to see how content relativism and context-sensitivity are different. A context-sensitive expression such as 'I'm a philosopher' expresses a single content at a context of utterance c; that content is simply determined by c. When Biden say it, it expresses the content *Biden's a philosopher*; when Trump says it, it expresses the content *Trump's a philosopher*. But these are distinct utterances. According to content relativism, a single utterance may express multiple propositions from different contexts of assessment.

Now that we understand truth relativism and nearby views, we turn back to future contingents. MacFarlane argues that future contingents are truth relative. Take the sentence said yesterday by General Sota, "There will be a sea-battle tomorrow." Given that we are amidst a raging and blood-soaked sea, MacFarlane claims that the proposition expressed is true as uttered yesterday and assessed from today. If in fact there is no privileged future and it was contingent yesterday whether there would be a sea-battle, the proposition is neither true nor false as uttered and assessed from yesterday. The very same proposition expressed by 'there will be a sea-battle tomorrow' at a single context of utterance can be true, false, and neither from different contexts of assessment.

Note that MacFarlane is a truth relativist about future contingents in the sense that the context of assessment is used to determine truth, but he isn't actually committed to there being truth relative future contingents; so long as there is only one future, no future contingent sentence actually varies in truth across contexts of assessment. Call this a *semantic truth relativist*. Because he is committed to remaining neutral about the metaphysics, his view entails the possibility of truth relative sentences given the possibility of no-privileged future. (A more precise discussion of truth relativism can be

found in the appendix below.)

For the purposes of this paper, we discuss these three classes of views under the terminology of no-privileged future views, privileged future views, and neutral views, the lattermost being a view that gives a semantics for future contingents neutral to the debate of whether or not there is a privileged future.

1.3 Extended problem of future contingents

Future contingency isn't just a feature of sentences or propositions. It can occur at the constituent level as well, as with terms or expressions introduced in a way that depends on what happens in the future, e.g. future contingent names, names apparently referring to individuals most naturally described as not yet existing. The tension between no-privileged future views and our future-directed talk and retrospective accuracy judgments naturally extends to an analogous problem with these expressions. Consider the following:

> BOXCAR: Horse breeders Alex and Jaime are speaking to each other after having watched American Pharaoh win the 2015 Kentucky Derby. Alex says to Jaime, "I've arranged for American Pharaoh to breed with my best mare. I bet you $10,000 that the foal resulting from that breeding will win the 2022 Kentucky Derby." Jaime accepts and asks Alex if he has come up with a name for his future horse. Alex says, "Boxcar," "Boxcar will win the 2022 Kentucky Derby."

Naively, it seems that Alex and Jaime have made a bet in 2015 about a future individual, Boxcar, and its winning the 2022 Kentucky Derby.[17] Now that it is 2022, they look back and assess the claim that Boxcar won, judge it to be inaccurate, and Alex pays Jaime. As before: the problem is not to find some combination of views that vindicates the retrospective assessment of future tense claims as true or false. A privileged future view can easily do that, taking 'Boxcar will win' as uttered in 2015 as true then and now if and only if the foal resulting from the breeding wins in 2022. Or adopting a view about the name 'Boxcar', such as a descriptivist theory of content wherein, e.g., the meaning of 'Boxcar' just is *the foal resulting from the breeding*.[18]

The challenge is to square our talk and attitudes with a range of relevant metaphysical and semantic possibilities, including possibilities on which the future is really unsettled. Since if it's

[17] If bets pose problems, other sentences will do, e.g. 'Newman-1, the first born child of the twenty-second century, will be male'.

[18] I assume that there is no reference requirement for a name to be a name. 'Santa Claus' is a name whether or not we take it to be vacuous.

really unsettled which horse results from that breeding, there is no talking about *the* individual Boxcar as there is no unique referent (or any referent at all), but the possibility of many. On this view, Alex's 2015 statement, when made, did not express a (unique) full proposition, since there was then no fact of the matter as to the referent of 'Boxcar'. But if no truth-evaluable proposition was expressed, how can we now assess, from our perspective in 2022, the statement in retrospect as (in)accurate or about this particular horse, when it was then possible that it was this horse and not Rich Strike that won or that 'Boxcar' could have referred to something other than what it in fact does? Shouldn't we just say that thanks to reference failure, nothing that can be true or false was said?

It seems, as before, that we are torn between two intuitions. On the one hand, it doesn't seem settled in 2015 what the name refers to and there appear to be many possibilities of individuals that could satisfy the description or be the referent of the name, depending on different breeding dates or ovum and sperm cells. All this, plus the original concern that no matter which horse is picked out, some futures witness Alex's foal's winning and some witness its losing.[19] So not only do we feel the force of the original indeterminacy intuition, but it's exacerbated by the fact that we suspect the phrase to fail to refer or be indeterminate in reference, (and if we aren't simple descriptivists) to have vacuous or indeterminate content.

On the other hand, when 2022 rolls around, it seems that we can say that it was this horse we were talking about and that the assertion turned out to be inaccurate. Here too, we feel the force of the original determinacy intuition, exacerbated by the fact that from a position in 2022 looking back, futuristic names appear to refer to a unique individual, have a content like any other name, and sentences in which they occur express propositions about that individual.

Two new problems arise from the apparent incompatibility of these intuitions. The PROBLEM OF REFERENCE: on the orthodox view of reference, the referent of a name is determined at a context of utterance, but doesn't vary across contexts.[20] The name 'Boxcar' as uttered at a context c, if vacuous or indeterminate in reference, cannot be said to (as still uttered at c) gain a referent later.[21]

[19]The example consists of two contingent bits: which horse will be the resulting foal and whether the resulting foal wins. These can come apart, e.g., if it's settled now that the resulting foal will lose, but not settled which horse the resulting foal will be.

[20]Names aren't typically considered context sensitive at all, though the referent of a name is generated by the context of utterance in that the name picks out the thing that is, e.g., conventionally or causally related in the right way to an object that underwent an original reference fixing. Whatever looks like variation in reference is usually considered to be a matter of two names that sound alike.

[21]The change in referent is not the same as the sense in which 'the president of the United States' changes referent over time. In cases like, 'The president of the United States is always at least 35 years old', the description so used isn't really a referring expression, although some may think that it is and in that case it's similar to, 'The president of the United States is busy', referring exactly to one person at a single context of utterance. The case of 'the foal resulting from the breeding' as uttered in 2015 seems to have no referent from the perspective of 2015 but has one (still uttered in 2015) from the perspective of 2022. See below for why this is also not like Madagascar cases.

Conversely, if the name 'Boxcar' as uttered in 2015 refers to this particular horse in 2022 it cannot have been indeterminate or vacuous to start. It seems we're forced to pick between its always referring or never referring at a specific occasion of use.

The first problem arises for names and descriptions like 'Boxcar' and 'the foal resulting from the breeding'. In the context of our example, the second problem is one for 'Boxcar' but not the definite description and can only be generated for non-simple descriptivist views. For clarity, we will assume a direct reference theory of content, but the argument extends to a broader class of positions.

The PROBLEM OF CONTENT: on the orthodox direct reference theory of content, the meaning of a name is its referent, what it denotes in the context of utterance, but typically doesn't vary across contexts. If the word 'Boxcar' as used in 2015 is somehow defective and ought to be treated like 'Santa Claus' or 'Homer'—e.g. vacuous or referring to a concept or something like the null element—it cannot at the same context of utterance gain a different meaning later on. Conversely, if when assessing the 2015 utterance in 2022 we want to say that the content of 'Boxcar' refers to a particular horse, then it always refered at that context.[22]

For the rest of the paper, I will present five mutually inconsistent premises, and argue that the best way for neutral views to account for the general class of future contingents is to adopt a hybrid truth and content relativist view. The argument below will be a more regimented version of the problem of content in the case of future contingents, with a focus on expressions like 'Boxcar'. We could adopt a number of views about what these terms are grammatically, what they refer to if anything, and what they mean if anything. But, the project isn't just to give a consistent set of theses, but a consistent and *neutral* set of theses. Just as we set aside any particular view about the metaphysics of time when discussing MacFarlane's solution to the original problem, we will attempt to do the same here, and respect the prevailing intuitions as we address this incarnation of the problem of future contingents.

1.4 Mutual inconsistency

We start with two theses meant to express the indeterminacy and determinacy intuitions, and the apparent tension between no-privileged future views and retrospective judgments:

[22] It's worth noting that the problem doesn't essentially rely on names. Names like 'Boxcar' are vivid but not exclusive to the class of constituent expressions we're interested in. We could introduce the predicate 'is spark' to mean, *is happy* if it rains tomorrow and *is sad* if it doesn't. Furthermore, the problem may extend to other expressions relativists are interested in, such as taste statements or epistemic modals. However, in these cases, the original reference fixing may establish a single stable referent at the context of utterance. This option is not available for future contingents when there's a possibility of no privileged referent.

NO PRIVILEGED FUTURE: There are at least two distinct possible futures metaphysically resulting from antecedent events, conditions, and laws, *and* there is nothing now that privileges one future over another

RETROSPECTION: There exist some true future contingent statements with the occurrence of a future contingent expression like 'Boxcar', i.e., introduced by a futuristic description. By this, I mean two things: (i) some Boxcar-statements express full propositions, absolutely or relative to a context of utterance c and from a context of assessment a, where c is in the past of a and (ii) the propositions expressed by Boxcar-statements are sometimes true, either absolutely or relative to c from a, where c is in the past of a.[23]

The first thesis, NO PRIVILEGED FUTURE, is designed for *genuine* contingency of the future. It rules out determinist, thin red line, and four-dimensionalist views. The second thesis, RETROSPECTION, is meant to apply to sentences such as 'Boxcar will be male', such that they express some proposition at and from relevant contexts that is either absolutely true or true relative to a context of utterance and assessment.

The next three theses have more to do with semantics.

NAME: 'Boxcar' is a name.

DIRECT REFERENCE: the semantic value of a name is its referent.

The latter thesis rules out many descriptivist theories of content which claim that the content of a name is a uniquely identifying description. It is a simplification, and the argument will work for any theory of names that gives Boxcar-style names different semantic values depending on how the contingent future plays out.[24] These may seem like unduly contentious theses, theses that could easily be denied, but they aren't given a commitment to neutrality between privileged and no-privileged future views. We will discuss exactly why below.

The final thesis is the negation of content relativism and content pluralism for future contingents. A discussion of each is needed. Recall that for the truth relativist, some proposition yields different truth-values at different contexts of utterance and assessment. A truth relativist about taste statements might say that the sentence 'Licorice is tasty' said by my sister expresses a single proposition—*Licorice is tasty*—true from my sister's context of assessment and false from mine. A *content relativist* about taste statements says that my sister's utterance expresses a true proposition *according to Marielle's taste preferences, licorice is tasty* from my sister's context of assessment and a

[23]This thesis assumes that only complete propositions are truth-evaluable.
[24]The semantic value needn't be a referent; it could be a property.

false proposition *according to Eliya's taste preferences, licorice is tasty* from my context of assessment.[25] For a single expression at a context of utterance, the content relativist says that there may be a *different proposition* expressed from different contexts of assessment. That is:

> CONTENT RELATIVISM: there is an expression s that expresses a proposition p at c from some context of assessment a and a proposition p^* distinct from p at c from some context of assessment a^* distinct from a.

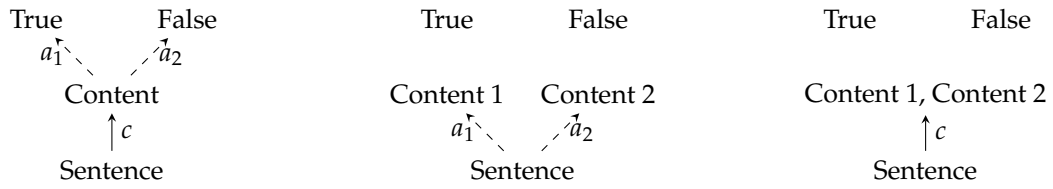

Figure 1.1: Truth Relativism, Content Relativism, and Content Pluralism

It is important to notice how content relativism differs from *content pluralism*:

> CONTENT PLURALISM: there is an expression s that expresses distinct propositions p and p^* at a single context of utterance c.

Pluralism does not require contexts of assessment as content relativism and truth relativism do. To see the difference, consider a drill sergeant, who says to a group of a hundred recruits, "If your mother knew the pain I am going to put you through, she would never have let you enlist." A content pluralist might say that at the specified context of utterance, the drill sergeant expresses multiple propositions corresponding to each recruit.[26] Conversely, according to the content *relativist* the content of the expression depends on the context of assessment; there is *only one* proposition expressed from each audience member's context, because the proposition expressed only arises when we fix a context of assessment. For the *truth relativist*, there is *only one* proposition expressed from the context of utterance, not plurally from the context of utterance and not only one from each context of assessment (but many broadly construed). We can see a simplified visual of the difference between truth relativism, content relativism, and content pluralism in Figure 1 above. Since we will be making use of a thesis that rejects both content pluralism and content relativism, we will give a single name to the combined negated thesis for typical future contingents:

> CONTENT STABILITY: for any typical future contingent with the occurrence of a future contingent expression like 'Boxcar' and context of utterance c, (a) no more than one

[25] Content relativism is compatible with truth relativism.
[26] MacFarlane 2014, 74.

content is expressed and (b) if that sentence has content p from context of assessment a, then it has content p from any other context of assessment a_i.[27]

1.4.1 The argument

Assume NO PRIVILEGED FUTURE, RETROSPECTION, NAME, DIRECT REFERENCE, and CONTENT STABILITY for reductio.[28] Consider the sentence said by Alex at c, where 'Boxcar' is introduced as the foal resulting from the breeding,

(1) Boxcar will be male.

Without loss of generality, we say that (1) uttered at c is an example of a true future contingent with the occurrence of a term introduced by means of a futuristic description. By RETROSPECTION, it is true absolutely or as assessed from some context a later than c. By NO PRIVILEGED FUTURE, there are two distinct possible futures with respect to the foal resulting from the breeding: future 1 at which Bob, a male, is born and future 2 at which Alice, a female, is born. Alice is not identical with Bob. (1) is a genuinely contingent sentence and there are no privileged links between the present and either of the futures. But, since the sentence is true absolutely or relatively and expresses a complete proposition by RETROSPECTION, what is the content of 'Boxcar'? By NAME and DIRECT REFERENCE, 'Boxcar' is not a hidden description and its semantic value is its referent. Then, what does it refer to, if anything? Either it fails to refer, refers to one thing from c, multiple things from c, or different things relative to different contexts of assessment. If 'Boxcar' at c fails to refer to anything, then 'Boxcar is male' cannot be true absolutely or true relatively, since only complete propositions are truth evaluable. That is, the case where 'Boxcar' fails to refer contradicts RETROSPECTION because the sentence cannot be true (absolutely or relatively). If 'Boxcar' refers to Bob (or Alice and not both) at c, then one future is privileged, which contradicts NO PRIVILEGED FUTURE. If 'Boxcar' both refers to Bob and refers to Alice at c, then content pluralism is true, which contradicts CONTENT STABILITY. If 'Boxcar' at c refers to Bob from context of assessment a_1, Alice from context of assessment a_2, and neither from context of assessment c, then content relativism is true, which contradicts CONTENT STABILITY. We have exhausted the possibilities. Either NO PRIVILEGED FUTURE, RETROSPECTION, NAME, DIRECT REFERENCE, or CONTENT STABILITY is false.

[27] We limit our focus to *typical future contingents*. These are sentences which, when you remove their futurity and contingency, are typically regarded as expressing one proposition across contexts of utterance or assessment after reference is fixed and ambiguities resolved. For instance, 'there is a sea-battle' is typically regarded as expressing one proposition and 'Boxcar is male' is regarded as expressing one proposition when the referent of 'Boxcar' is fixed.

[28] Given some simple compositional semantics, wherein the meaning of a complex expression is determined by the meaning of its parts plus rules used to combine them.

1.5 Privileged and no-privileged future views

Determinists, four-dimensionalists, and thin red line theorists will simply deny NO PRIVILEGED FUTURE, directly corresponding to their acceptance of the determinacy intuition and of a privileged future. There needn't be a difference between future contingents with occurrences of names like 'Boxcar' and ones without such occurrences.[29] All future contingents are treated as true if and only if they are true in the (actual) future.

Three-valued and gappy logicians will deny RETROSPECTION. All future contingents can also be given a uniform treatment, expressing INDETERMINATE, FALSE, or gappy propositions.

1.6 Neutral views

In order to examine how neutral views should respond to the argument, it's worth stating their commitments. First, they tout a metaphysical neutrality; they take no stance on whether or not there is a privileged future. Second, neutral views show that the indeterminacy and determinacy intuitions are compatible. 'There will be a sea-battle tomorrow', is true uttered yesterday and assessed from today amidst the blood-soaked sea. This handles the determinacy intuition, as we can truly say, "Sota's assertion turned out to be true. What he said was accurate." If there is no privileged future and it is possible to find ourselves in a peaceful or fighting tomorrow, 'there will be a sea-battle tomorrow' and the proposition it expresses are neither true nor false relative to a context of assessment before today. This handles the indeterminacy intuition, as the sentence is neither true nor false assessed from yesterday. In short:

> NEUTRALITY Neutral views (i) take no stance on whether or not there is a privileged future and (ii) respect the prevailing intuitions on both sides of that debate.

Neutral views are of interest to those who believe semantic theories should try to give a theory of linguistic meaning that avoids baking in controversial metaphysical views. They're also of interest to no-privileged future views, for if a neutral view like MacFarlane's is successful, privileged future views can no longer use future-directed and reflective thought and talk in their favor.

Yet, if we take expressions with Boxcar-names seriously, truth relativism as it's currently defended does not provide a neutral semantics for future talk and retrospective judgments, as we now discuss.

[29] Of course, the views are consistent with thinking the reference is unsuccessful in some way.

1.6.1 Truth relativism

MacFarlane's view does take a stance on reference and content. He tells us that the extension of a term and its content is the object it refers to, to be determined by the context of utterance, not assessment.[30]

If we take this to apply to Boxcar-sentences, MacFarlane's view has the consequence that—given no privileged future—the content of 'Boxcar' is vacuous, indeterminate, or otherwise treated like a fictional name at the context of utterance. This is to give up on the retrospective accuracy judgments and thereby give up on RETROSPECTION and NEUTRALITY.

The main reason for accepting RETROSPECTION is that no one would balk at the retrospective assessment of the utterance, 'Boxcar will be male'. Everyone—regardless of theory—knows how those assessments ought to go, even the no-privileged future types. Any semantics that can't make sense of those assessments is dubious; that's the point of the retrospective objection to no-privileged future views. Such an objection is meant to be dissolved by neutral views and separating Boxcar-cases out as a special class of cases goes against the very project of neutrality. It fails to establish that the indeterminacy and determinacy intuitions are compatible. It undermines the determinacy intuition and sides with no-privileged future views.

For the remainder of the paper, I evaluate each premise in terms of how its negation fairs against NEUTRALITY. I conclude that the best way to give a consistent and metaphysically neutral semantics for future talk and thought that respects the prevailing intuitions on both sides of the debate, to give a semantics that vindicates retrospective judgments that doesn't assume a privileged future view, is to adopt a hybrid truth and content relativist position, where both the truth of future contingent sentences and the content of futuristic names such as 'Boxcar' are relativized to contexts of assessment.

1.6.2 No privileged future and retrospection

It's clear enough that neutral views shouldn't take a stance on the truth of NO PRIVILEGED FUTURE: rejecting it undermines NEUTRALITY by assuming a privileged future.[31]

It's also clear enough that neutral views should accept RETROSPECTION. To reject it would be to reject retroactive judgments but for the special cases of future contingents with occurrences of futuristic names. We discussed the primary problem with this in the last section: the main objective

[30] See MacFarlane 2014, 152. His main concern is to single out his position as an opponent of content relativism.

[31] Entertaining it for the sake of argument just is the project of trying to give a semantics for future talk that doesn't assume a privileged future, to show the intuitions are compatible.

of a neutral view is to vindicate RETROSPECTION without taking a stance on the metaphysics.

It also fails to respect a prevailing intuition that many no privileged future theorists have: there's no important difference between 'Boxcar will be male' and 'there will be a sea-battle tomorrow'. Both are true iff they are true in the (actual) future. 'Boxcar' is quite like 'Neptune' when the name was first introduced by a definite description before the planet had been observed.

Everyone in the debate is interested in understanding the semantics of future directed talk, all future directed talk. Since future contingents with names like 'Boxcar' are a kind of future contingent sentence, a kind of future talk, the proponent of NEUTRALITY who rejects RETROSPECTION for these cases needs to explain why. But the sort of reason they would give—assuming certain theories of content—would only be compelling to those who accept a privileged future.[32]

1.6.3 Direct reference and name

As with RETROSPECTION, the denial of either DIRECT REFERENCE or NAME is only really motivated by intuitions held by those in the no-privileged future camp. If there is a privileged future, the sentence is true (or false, and not both) uttered and assessed from now. It's natural for someone who believes in a privileged future to think that 'Boxcar' is a name just like any other, that the referent is temporally distant much like the referent of 'Biden' is spatially distant from me when I use it. One would expect a neutral semantics to respect the retrospective judgment that it is a referring name with content.[33]

This brings up a broader question about what exactly we want to be neutral about. Originally, we just wanted to make sense of our intuitions about our retrospective accuracy judgments without assuming a metaphysics of time. This seems to be about assigning the right truth-values to whole sentences and propositions, rather than the content or reference of parts of sentences.

But the content and reference of names are important to the goal of assigning truth-values to whole sentences and propositions. To claim that a sentence like 'Boxcar will be male' as uttered now and assessed after the twenty-second century is true depends on what the meaning and referent of 'Boxcar' is.

[32] Many proponents of direct reference believe that there are true future contingents with occurrences of names most naturally described as referring to things that don't exist yet. Salmon 1998 and Soames 2002, 92-3.

[33] It bears repeating: the problem isn't essentially about names.

1.6.4 Madagascar

One might wonder if 'Boxcar' is like Gareth Evans's Madagascar case, a case in which a single name-type shifts reference over time.[34] According to Evans, the term 'Madagascar', originally a native term for an inland part of Africa, seemed to shift reference when Marco Polo mistakingly took it to apply to an island off the coast of Africa, after which it became common for people to use it to refer to that island. We might wonder if 'Boxcar' is like 'Madagascar', with a shift in reference: before the foal's birth it doesn't refer, after which it does.

The difference is 'Boxcar' doesn't shift reference across contexts of utterance. 'Madagascar', as used before Marco Polo, refers to the inland part of Africa, no matter our perspective when we evaluate it in the future. It would be a mistake to judge the native as speaking about the island. They're different instances of use. The Boxcar case is about different contexts of assessment. 'Boxcar' seems to fail to refer as uttered and assessed before the foal's birth. And yet, I seem to judge it to have a referent when the *very same* utterance is assessed after the birth. The problem of this paper is a problem about token utterances of sentences involving Boxcar-type names, whereas the Madagascar phenomenon is about name-types.

1.6.5 Content pluralism

Consider the denial of CONTENT STABILITY by accepting *content pluralism.* Content pluralists about future contingents say 'Boxcar will be male' as uttered before the foal's birth expresses two propositions, *Alice is male* and *Bob is male*, on the assumption that there is no privileged future.

The view doesn't vindicate RETROSPECTION. Looking back, content pluralism has it that the sentence expresses (at least) two propositions when uttered, one true and one false; therefore, it seems natural to say even in the future that the sentence was neither true nor false. This is in contrast with the intuition that the sentence retrospectively turned out to be true (or false) and not neither. Furthermore, the utterer seems to have no intention of saying multiple things as they do in paradigm cases like, 'Jesus loves you'. Although knowingly ignorant of the referent, they intend to refer to one thing.

1.6.6 Content relativism

The final option is to deny CONTENT STABILITY by accepting *content relativism*. This view upholds NEUTRALITY. Assume direct reference and no-privileged future. Uttered and assessed from c, 'Boxcar'

[34] Evans 1973.

is vacuous and fails to refer, handling the new indeterminacy intuition. Uttered at c and assessed from a_2 after the birth when Alice is first-born 'Boxcar' refers to the unique individual Alice. We can say the content of the name is Alice uttered in c and assessed from a_2 and the sentence 'Boxcar will be male' is about that single individual and that Alex said something inaccurate.

Just as with truth relativism, content relativism about future contingents is a semantic view that isn't committed to the existence of content relative sentences, just their possibility: were there no privileged future, there would be content relative future contingents.

Note that none of this entails a commitment to content relativism across all cases of future contingency, in the sense that 'there will be a sea-battle tomorrow' varies content across contexts of assessment. We want a way of making sense of how 'there will be a sea-battle tomorrow' said yesterday can come out true today since there is a sea-battle, without assuming a privileged future. MacFarlane shows us how to do that with truth relativism. So too, we want a way of making sense of how 'Boxcar will win the 2022 Kentucky Derby' said in 2015 comes out false after the 2022 Kentucky Derby once Rich Strike wins, in a way that doesn't assume a privileged future and that is compatible with the view that the content of 'Boxcar' is its referent. The best way to do that, to have truth variance across branches (if there are any) and to respect retrospection, is to relativize the content of 'Boxcar'. This doesn't mean that we have to relativize the content of all future contingents like 'there will be a sea-battle tomorrow'.[35]

It is objectionably disunified to adopt a hybrid truth and content relative view? I think not. Truth relativism alone cannot account for a more general class of retrospective judgments. This would be to prefer unification at the expense of plausibility and the project of neutrality. One could, alternatively, adopt a unified content relativist view, but this is an over-zealous attitude to content relativity and unification that would fail to accommodate the intuition that in the case of 'there will be a sea-battle', the *same content* could be neither true nor false today, true tomorrow from one context, and false tomorrow from another. 'There will be a sea-battle' has some sort of stability in its content that 'Boxcar will be male' doesn't seem to have.

Furthermore, in the case of 'Boxcar', we didn't relativize the content of any *future contingency*; we relativized the content of the name because the name picks out different people in different future contexts. The kind of truth relativism adopted here is different than the kind of content relativism adopted here. The former relativizes the truth of sentences based on *occurrences* of future contingency in the sentence, the latter relativizes the content of expressions that are themselves

[35]That is, unless we had a view on which words like 'tomorrow' are like 'Boxcar', introduced by way of a futuristic description and whose content should be e.g. extensional. I would expect the content to be treated like most indexicals, but that doesn't entail a view on which that content is tied to a specific world or branch.

introduced by future contingency but have no future contingency in their syntax. This suggests we should adopt the hybrid view, truth relativism as it applies to future contingency that arises in the syntax (and analogously for the proposition) and content relativism as it applies to expressions introduced by future contingency.

Objections to content relativism

The main objections to content relativism are usually discussed as the view applies to other kinds of expressions, such as taste statements. Here, I discuss a one prominent problem: content relativists can't make sense of disagreement.[36] Whether or not this is a problem for content relativism about taste statements, it doesn't arise for the case of future contingents.[37]

Consider the problem of disagreement for content relativists about taste statements: we have disagreements about what is tasty, but if content relativism is correct we shouldn't dissent to anyone's taste claims. This is because, according to the content relativist, there is no shared propositional content for which I assent and my sister dissents. It seems that if the content relativist is correct, we should instead simply agree that licorice is both *tasty-according-to-Eliya's-tastes* and *not tasty-according-to-Marielle's-tastes*, rather than what it seems, which is that I assent and Marielle dissents to *Licorice is tasty*.

Whether or not this is a problem for content relativists about taste, the problem doesn't arise in the case of future contingents. Contexts of assessment in the case of future contingents aren't agent bound nor can they "interact" in a single conversation as they do in the case of taste disagreement. We can't get the problem going unless we can say there are disagreements with people across branches, which there aren't. In fact, we don't take any stand on whether or not there are distinct branches, nor do the contexts naturally attach to different agents like they do in taste cases.

1.7 Concluding remarks

In this paper, I extended the semantic problem of future contingents, which traditionally focuses on future contingency as a feature of sentences or propositions, to constituents. Neutral views, like John MacFarlane's, provide a semantics for future-directed talk and retrospective accuracy judgments without assuming any metaphysical theory about whether there is a privileged future. MacFarlane's

[36] To start, see MacFarlane 2014, 72-6.

[37] There are many other interesting problems in the literature, e.g., content relativism either collapses to truth relativism or isn't capable of making sense of explicit relativizations, that it can't make sense of eavesdropping cases, an so on. For the sake of space, I don't include these discussions, but the same response applies: the problems for content relativism about non-future contingent expressions arise because the contexts of assessment are bound to agents. They don't arise for future contingents because the contexts attach to whole situations at a time and world.

theory works well when applied to cases where future contingency is a feature of the sentence or proposition expressed, but it was shown to be less clear how his theory should handle cases where future contingency is a feature of constituents of sentences or propositions, like in the case of 'Boxcar'. While opponent positions have an easy time giving a consistent story across a broad range of future contingents, neutral view are best able to maintain their position if they adopt a hybrid truth and content relativist position.

1.8 Appendix: Truth relativism about future contingents

To understand the truth relativism more precisely, we will need more clarity about concepts like times, worlds, and branching. Following MacFarlane, the approach will be broadly acceptable to people who hold a number of different metaphysical views. For a more thorough discussion of below, consult MacFarlane (2014). We start with a totally ordered set of *times*, a set of *worlds*, a domain function D mapping a world and time to a set of objects, and a valuation function V mapping an atomic predicate, world, and time to an extension (e.g. a set of objects, or tuples of objects) that satisfy the predicate, given its actual meaning, in that world and time. Worlds are restricted to those nomologically possible ones. Possibility is diachronic, what is possible at one time may not be possible at another, where—intuitively—a world w_2 is accessible from w_2 at time t just in case w_2 and w_1 have a common past but differ in their futures. We need a notion of *overlapping branches*, or possible histories, which are forward-looking; that is, if w_2 is accessible from w_1 at t, then w_2 is accessible from w_1 at all times $t' \leq t$.

In such a framework, a possible occasion of utterance will be common to multiple overlapping branches or histories. At a context c, a sentence will be true if the proposition it expresses is true at all circumstances of evaluation compatible with c, where a *circumstance of evaluation* of a proposition is a situation in which it is appropriate to evaluate it for truth or falsity.[38] So, a future-tensed proposition, as uttered and assessed from now, is true if and only if it is true in all possible branches overlapping now. When General Sota says, "One plus one will be two tomorrow," what he has said is true assessed from the present because there is no way the future could be such that one and one is not equal to two. This applies to contexts other than the present as well. A future-tensed proposition expressed at a context of utterance c, as assessed from a context a later than c, is true if and only if it is true in all possible branches overlapping the later context a.

Let c be the context of utterance, a the context of assessment, b a branch, $B(e)$ the set of all

[38] MacFarlane 2014, 76-77 (See p. 76, n. 7).

branches overlapping at a context e, and t_c the time of utterance. We use $[S]^c_{(b,t_c)}$ to denote the extension, the truth value, of the sentence S at c, and index (b, t_c).[39] a and c are elements of some b. That is, they can be understood as points or nodes on the branches. A sentence S is true as used at c and assessed from a if and only if for all $b \in B(c, a)$, $[S]^c_{(b,t_c)}$= TRUE, where

$$B(c,a) = \begin{cases} B(a) \text{ if } B(a) \subset B(c) \\ B(c) \text{ otherwise.} \end{cases}$$

When c is in the past of a, $B(c, a) = B(a)$. Otherwise, $B(c, a) = B(c)$.[40,41] The definition says that (i) when c is in the past of a, someone at a should take an assertion made at c to be accurate only if its content is true at all the branches overlapping at a, and (ii) when c is not in the past of a, someone at c should take the assertion to be accurate only if its content is true at all the branches overlapping at c. The context of utterance fixes the time and the context of assessment fixes the branches.

[39] A context determines a unique *index* whose coordinates are set to the corresponding features of the context. In the case of future contingents, the context determines an index composed of a branch and a time of utterance. We define truth at a context recursively from truth at a context and index. For further discussion, see MacFarlane 2014, 57-8.

[40] MacFarlane 2014, 226. I have changed some of the letters from MacFarlane's original.

[41] This is what MacFarlane calls a postsemantics, which in the case of future contingents is meant to explain retrospective accuracy judgments. Allowing for assessment sensitivity, MacFarlane makes the following distinction: where the semantics proper is the truth of a sentence at a context and index, the postsemantics is the truth of a sentence at a context of use and context of assessment.

Chapter 2: Contingentism, Existence, and Quantification

Abstract. This paper presents a response to Goodman's Adjunction Argument for necessitism, the view that necessarily everything necessarily is something, on behalf of contingentists. The methods employed in Goodman's argument make central use of formalism and abduction, relying in particular on the strength certain principles can lend to theories in which they figure. I argue against this as an advantage for necessitists, in broadly three ways. I defend a contingentist view that incorporates these principles, and so has the same theoretical advantage on this front. I present an amendment to the principles for a different set of contingentist views and defend it with a *tu quoque*. Finally, I argue for the position that many contingentists who endorse a certain conception of existence make the relevant formal principles governing quantification inappropriate by their own lights.[1]

2.1 Introduction

Necessitism is the view that necessarily everything necessarily is something, or more formally, $\Box \forall x \Box \exists y (y = x)$, where $\Box$ is read as metaphysical necessity familiar from Kripke (1972) and the quantifiers are read as unrestricted. *Contingentism* is the negation of necessitism.

It can be tempting to look at these views for the first time and think there couldn't be a serious debate: contingentism is just obviously true. After all, isn't it obvious that you could have failed to

[1]I'd especially like to thank Daniel Berntson, Boris Kment, Harvey Lederman, Gideon Rosen, and Alice Van't Hoff for discussion and comments.

exist and that there could have been something there isn't? Your parents might never have met and Wittgenstein could have had children but didn't.[2]

But necessitists have pointed out that it really isn't so obvious that you could have failed to exist or that there could have been something there isn't. All that's obvious is that you could have failed to be born and that Wittgenstein doesn't have a child but could have. Contingentists can account for the truth of these modal judgments with contingent *existence*; but necessitists can account for them by using the notion of *concreteness*: you're contingently concrete and Wittgenstein's possible child is contingently non-concrete—even though both of you exist necessarily.[3]

If necessitists are right that our modal judgments don't point decisively in favor of contingentism, some such as Williamson (2013) have suggested that we should take abductive considerations—considerations of theory choice—and formal approaches more seriously, as a way of settling the question whether everything exists necessarily.

Against this background, Goodman (2016a) presents what he calls the Adjunction Argument, employing a formal approach and drawing heavily on abductive methods. But instead of focusing on modality and higher-order modal comprehension principles as Williamson (2013) and others have, Goodman focuses on theories of propositional granularity. This paper provides a systematic assessment of Goodman's argument, focusing on contingentist responses to it.

The most notable assumption in Goodman's argument is ADJUNCTION. ADJUNCTION's main virtue is the strength it lends to theories in which it figures; along with some assumptions about propositional entailment, ADJUNCTION allows us to uniquely specify the first-order quantifier. One objective of this paper is to clarify what exactly it means to uniquely specify the quantifier and why this result might be desirable.

My primary concern, however, for Goodman's argument is the principle of ADJUNCTION. Even if Goodman is right that ADJUNCTION lends strength to theories in which it figures, I argue that a necessitist theory that adopts ADJUNCTION is no better than a contingentist theory that cannot uniquely specify the quantifier. The necessitist theory either involves a new bit of ideology, *chunky*—a downstream relative to *contingent concreteness* that's either primitive or analyzed as *grounded in the concrete*—to account for entrenched modal judgments, or else cannot account for such judgments. But as we will see, the former theory cannot specify the extension of this term uniquely. So both necessitist and contingentist views have a bit of ideology whose extension they cannot specify;

[2]See Williamson 2013, King 2016, and a response by Williamson 2016 for a discussion of when a view should be considered a live option.

[3]For more complicated cases, they introduce a more nuanced notion of *chunkiness*, which is either primitive or typically analyzed as *grounded in the concrete*, to be described below.

and so neither is abductively superior to the other in this respect: *tu quoque*. As for the necessitist theory that doesn't adopt new ideology, it's unclear how we should weigh such a theory against a contingentist theory that does account for entrenched modal judgements.

I start in section 2.2 by discussing the background methodology, both to introduce the ideology of *concreteness* and *chunkiness* used later in the *tu quoque*, but also to lend support to the significance of this added ideology. I then present Goodman's Adjunction Argument in section 2.3, clarify how ADJUNCTION uniquely specifies the first-order quantifiers, present a generalization of the argument, and discuss why this result is desirable. In section 2.4, I defend a variety of contingentism that can accept ADJUNCTION. In section 2.5, I present a free-logical amendment to ADJUNCTION that makes use of primitive vocabulary. This amendment is arguably as strong as the original principle. It does, however, seem push the problem back: what uniquely specifies the new vocabulary? I defend the use of this vocabulary with a *tu quoque*: if there's a problem with this contingentist vocabulary, there's a problem with the necessitist vocabulary *chunky*. This brings out a contingentist conception of existence and quantification that makes the relevant formal principles governing quantification inappropriate by their own lights. In section 2.6, I begin to articulate this contingentist conception of existence brought out by the discussion. Inferential rules for the quantifiers look like sufficient conditions on existence. It's no wonder that Williamsonian necessitists might treat existence and quantification as "logical" or "trivial"—for many of them, it takes nothing to exist.[4] It is also no wonder that contingentists, and certain necessitists, would reject this treatment—the world has to be a certain way for something to exist. Finally, while Dorr and Goodman have identified a interesting feature of a quantifier, it's a further question whether this quantifier characterizes *real* existence, and what resources we have to answer such a question.

2.2 Abductive and other weak considerations

Part of an ongoing necessitist challenge to contingentists is that contingentist logics are inferior to necessitist ones and that abductive considerations favor necessitism. In the late 20th century, the focus was on first-order modal logic and the Barcan Formula and its converse with formal challenges from Linsky and Zalta (1994, 1996) and Williamson (1998, 2002). Necessitism is a theorem of the simple quantified modal logic that includes the standard rules of a classical first-order theory of quantification and the system S5 of propositional modal logic. Many contingentists respond by

[4]The concept *logical* suggests a distinction between logical and non-logical vocabulary. This distinction might be confused. I merely mean to use it suggestively, as a name for a list of things like entailment, disjunction, predication, etc, as opposed to vocabulary like essence, grounds, etc.

adopting Kripke models with variable domains and logics with restricted quantifier or identity rules, roughly corresponding to positive and negative free logics. An important development in the overall objection that contingentist logics are inferior to necessitist ones comes from Williamson (2013). This time, the focus was on higher-order modal logic and modalized comprehension principles. Williamson argues that the only defensible forms of higher-order modal logic entail a version of necessitism for higher-order entities: every proposition, property, etc., is necessarily identical to something. He further argues that hybrid views that would view higher-order entities as necessary existents while allowing first-order entities to exist contingently are untenable, and hence that we should embrace necessitism at every type. Some have more recently defended hybrid views (Goodman 2016b; Skiba 2022; Fairchild (unpublished draft)) or responded by developing competing contingentist model theories (Fritz 2016, 2017) and, analogous to the earlier debates, restricted quantifier or abstraction and reduction rules, roughly corresponding again to positive and negative free logics.[5] The argument under dispute in this paper also presents an important development in the overall objection, this time focusing primarily on theories of propositional grain.[6]

One concern that arises about these abstract approaches to the debate is just how forceful these consideration are in terms of recommending that we believe in necessitism. Arguments are going to be diagnostically valuable, putting increasingly more pressure on their opponents to clarify their views, but how convincing are they?

The concern is centered on the belief that there is a commonsensical appeal of contingentism over necessitism. Contingentist formal systems may be complicated, and they may have surprising implications. But so long as it remains indisputably obvious that some things exist contingently, pointing out the complications will not amount to an argument for necessitism. For that purpose, the necessitist needs to soften the blow by showing that her view can offer some surrogate for commonsensical contingentism, to give some account for our entrenched modal judgments.

Necessitists, then, are at least incentivized to account for our entrenched modal judgments to weaken any contingentist strategy that claims an advantage on this front. This section describes how they do this with the added ideology of *chunkiness* and *concreteness*, which will later be used in part of my response to Goodman's argument.

[5] See also the last chapter of this dissertation.

[6] This isn't to say that there aren't interesting connections between propositional granularity, modality, and in particular the modal ontological debate that have been discussed for some time. The difference here is the use of abductive methods in this connection.

2.2.1 Weakening contingentist claim to advantage

Necessitists have pointed out that many of the so-called obviously true modal judgments that favor contingentism are really one-step removed from what we really find commonsensical: e.g. what's obvious or commonsensical is that you could have failed to be born, not that you could have failed to be something.

Where contingentists explain these judgments with existence, necessitist introduce the ideology of *is contingently non-concrete* or *is contingently grounded in the concrete*, which can account for our entrenched judgments of contingency in a way compatible with necessary existence. That is: where contingentists say, you might not have been born and so might not have existed, necessitists say, you might not have been born and so might not have been concrete.

There is a *prima facie* implausibility to the view that a concrete thing could have been non-concrete. It doesn't seem that I am even possibly like a number in the significant way that being possibly non-concrete attributes to me: that is, possibly existing without any physical qualities. Intuitively, the number two and I have something actually in common that distinguishes us from Wittgenstein's possible children. The contingentist can tell us what this is: the number two and I exist; Wittgenstein's possible children do not. The necessitist says that that all of us exist, and that Wittgenstein's possible children resemble the number two in being non-concrete. So what is the necessitist's way of describing what the number two and I have in common that distinguishes us from Wittgenstein's possible children, while still accounting for the intuitive difference between me and the number two—I'm intuitively contingent in some way that numbers are not? It may be true to say that actual people and numbers are *as concrete as they could have been*, whereas Wittgenstein's possible children are not. But this seems to miss out on the idea that the number and I have a kind of *positive reality* that Wittgenstein's merely possible children palpably lack. In other words, is there a necessitist account for the intuitive sense of what *exists* by contingentist lights rather than just what *contingently exists* by their lights?[7] Furthermore, in these situations where I am not concrete, it would seem that I wouldn't have any qualities that would distinguish me from other contingently non-concrete things. Contingently non-concrete things are queer, differing in modal properties without differing in categorical properties.

One thing the necessitist could say is that while the number two necessarily instantiates its essential properties, both Wittgenstein's possible child and I only do so contingently. In actuality,

[7] I don't mean to suggest that necessitists need a surrogate for every contingentist truth. It isn't in general a good test of the tenability of a view that it can find a neutral equivalent of every statement of their opponent. These, however, are arguably well-entrenched judgments. See Goodman 2016b.

two and I instantiate our essential properties and Wittgenstein's possible child does not. But this would be a rather odd use of essential. In standard discussions, your essential properties are the properties you have in every world in which you exist.[8] According to this route, the necessitist would be saying that Wittgenstein's possible child and I can exist without our essential properties. Furthermore, this wouldn't help us distinguish between me and Wittgenstein's possible child in situations where neither of us is concrete.

So if an essential property is a property a thing has in every world in which it exists, then the necessitist has to revise the usual account of the essences of ordinary things. And so necessitists give an account on which our essential properties just are conditional in form. I'm not essentially human; I'm essentially human if concrete. An electron isn't essentially a charged particle; it's essentially charged if concrete, etc. This helps with individuation. This one would have been a charged particle if it had been concrete; that one would have been a human. I am possibly concrete; two is not. So they are distinct. But this then loses what the previous approach had going for it: namely, that it captured what two and I actually have in common that Wittgenstein's possible child does not, since now we all have our essential properties necessarily. Furthermore, we may also want to know what grounds the modal or essential properties of a non-concrete thing. Many want to say that the modal is grounded in the non-modal. For the necessitist, the modal floats free of the non-modal. A better route might be to keep the modal essential properties but say that two and I both actually have a non-trivial battery of categorical (non-modal) properties, whereas Wittgenstein's possible children do not. But this approach seems to fall apart when we consider other potentially contingent things, like impure sets, which we discuss next.

Even if it's mysterious or controversial how the necessitist story goes, they have given some story. So when it comes to weakening the apparent contingentist advantage over necessitists to account for our entrenched modal intuitions, the necessitist has done her job. Still so—if we're in the game of taking abductive considerations seriously, then the necessitist loses plausibility points by insisting that there are indefinitely many non-concrete things in the world that differ from one another only in modal respects and not in categorical respects. That's like saying that I have a bottle full of pills that differ massively in their causal powers—one pill makes you larger, one makes you smaller—but which differ only in their causal powers and not in their chemical makeup. Of course, we can bite that bullet if we have to, but it's still a bullet.

[8] Fineans reject this account as too coarse-grained, but that's not the issue now.

2.2.2 Clarifying the ideology

What has become a bit more challenging for necessitists is how to account for the intuitive contingency of impure sets, e.g. $\{Socrates\}$ and, singular propositions, e.g. *Biden is a politician* and the intuitive dependence these thing have on Socrates and Biden, respectively. What's interesting about this for us is that it clarifies the added necessitist ideology, what it means or is dependent on, which is crucial to my response to Goodman's argument later on.

Many contingentists and necessitists agree that impure sets, e.g. $\{Socrates\}$ and, singular propositions, e.g. *Biden is a politician* depend in some way on Socrates and Biden, respectively.[9] Contingentists are inclined to say that $\{Socrates\}$ exists contingently because Socrates does, but because the necessitist denies contingent existence, she will look for a surrogate for the sense in which $\{Socrates\}$ could have failed to exist by the contingentist lights and the sense in which $\{Socrates\}$ depends on Socrates, in a way that ensures some difference between $\{Socrates\}$ and the number two.

Their normal surrogate won't do: when considering Socrates, the surrogate for $\ulcorner X$ exists contingently$\urcorner$ was $\ulcorner X$ is contingently concrete$\urcorner$. But we can't say that about $\{Socrates\}$; it can't be that $\{Socrates\}$ is contingently concrete, since sets are never concrete entities.[10] The necessitist instead may say, $\{Socrates\}$ is contingent in the sense that it could have been *grounded in the concrete*, and since this is also true for Socrates, necessitists make this their general point: unlike the number two, Socrates and $\{Socrates\}$ are contingently grounded in the concrete.[11]

This necessitist story amounts to a big difference between impure sets and ordinary individuals. When ordinary individuals are not concrete, their only interesting properties are modal. When impure sets are not (grounded in the) concrete, they have most of their usual properties: they are sets, they have certain members, those members may be non-concrete, etc. Nothing modal is going on there. This may not be a point against necessitism, but it does amount to a notable difference between necessitism and contingentism. Contingentists can think that impure sets and individuals are on a par: they have the same metaphysical status in worlds in which the individual is not concrete; they just don't exist. Necessitists must think that impure sets are still replete with

[9] Many necessitists agree that the dependence holds, and Williamson even argues against hybrid views, view for which individuals contingently exist but all propositions and sets exist necessarily.

[10] I will assume the widespread view that these are abstract things, but nothing I say ultimately hangs on the point that they are not concrete. On some conceptions, things like impure sets count as concrete because they, or parts of them, exist in space and time.

[11] It's worth pointing out that some believe that ordinary mathematical objects exist only contingently. There are possible worlds in which there are no numbers, pure sets, etc., since nominalism is a consistent view of what there is, the thought goes. We may believe that contingentism and necessitism should at least be able to articulate such a view, but this view would make trouble for the necessitist we've been considering. The number two is not contingently grounded in the concrete, so what would the surrogate be?

categorical properties even if the underlying individuals are non-concrete; so the set and its member have very different metaphysical statuses when the member is non-concrete.

Consider also the temporal analogue to necessitism. There is a clear, even empirically verifiable, shift in the metaphysical status of things like me—I'm concrete now and won't be in a hundred years. There is no clear shift in status of things like impure sets.[12] It's hard to understand what the metaphysical change really is when a set exists and is grounded in the concrete and a set exists and isn't so grounded. Why is being *contingently grounded in the concrete* significant in a way that being *contingently grounded in the tall* isn't? There's no significant change in $\{Socrates\}$. After all, even in worlds in which Socrates is not concrete, the set $\{Socrates\}$ exists, since all its members exist. $\{Socrates\}$ is still grounded in Socrates, regardless of what his properties are.

The most notable reason to pause is that the necessitist makes unannounced and essential use of *grounding* as a new bit of ideology. As a matter of sociology, necessitists tend to be allergic to this sort of hyperintensional metaphysics. It would be surprising that they would retreat to an appeal to grounding in this way.

Later, we will discuss in detail how these points bear on the Adjunction Argument: the added ideology of *is chunky*, which is primitive or typically analyzed as *is grounded in the concrete*—meant to play a similar theoretical role that existence and quantification play for contingentists—plays a role in weakening the necessitist claim to a theoretical advantage over contingentists in the sense Goodman suggests.

2.3 Goodman's Adjunction Argument

In this section, I present Goodman's Adjunction Argument for necessitism. I will then focus on his defense of ADJUNCTION, which he takes to be primarily abductive: many independently motivated theories of propositional entailment make ADJUNCTION an attractively strong principle, a principle that can uniquely specify the quantifier. The goal here is to clarify what it is to uniquely specify the quantifier, what is assumed to achieve this result, and why this result is attractive. For the former, I present an explicit proof and generalize it. The generalization will inform the discussion in later sections where I investigate contingentist-inspired amendments to ADJUNCTION that also uniquely specify the semantic contribution of the quantifiers. I then move on to defend the motivation for wanting to uniquely specify—or as Goodman says, pin down—the first-order quantifiers.

[12] Of course, we aren't demanding that it be empirically verifiable.

2.3.1 The argument

Goodman's argument makes use of two unfamiliar connectives, both dyadic operators and both denoted by the same symbol $\leq$. The first takes sentences as arguments and may be pronounced as follows: for formulas ϕ and ψ, read $\ulcorner \phi \leq \psi \urcorner$ as $\ulcorner$that ϕ entails that $\psi \urcorner$. The second takes monadic predicates as arguments and may be pronounced as follows: for predicates F and G, read $\ulcorner F \leq G \urcorner$ as $\ulcorner$being F entails being $G \urcorner$. These are not intended as definitions or analyses, but merely pronunciation schemes. An important part of Goodman's argument is concerned with how to interpret these connectives to make the argument compelling. While the interpretation of $\leq$ is up for grabs, it is meant to have obvious properties in common with any notion of entailment, including reflexivity and transitivity.

We furthermore employ abstraction and reduction principles familiar from the $\lambda-$calculus. For any formula ϕ, the expression $\ulcorner(\lambda x.\phi)\urcorner$ is a monadic predicate that takes expressions of the same syntactic type as x as arguments and in which all occurrences of x are bound. Reduction and abstraction allow us to form predicates from formulas and vice versa using expressions we already have. Consider a simple example: the standard treatment of λ expressions allow us to take a closed formula like 'Ted is sitting and eating' and form the complex predicate, pronounced here as, 'is such that it is sitting and eating' such that when we apply it to 'Ted' we get something that (standardly) entails and is entailed by the original formula.[13] Consider the following principles:

REFLEXIVITY

$\phi \leq \phi$

TRANSITIVITY

$(\phi \leq \psi) \wedge (\psi \leq \chi) \rightarrow (\phi \leq \chi)$

APPLICATION

$F \leq G \rightarrow Fa \leq Ga$

REDUCTION

$(\lambda x.\phi)a \leq \phi[a/x]$, where a is free for x in ϕ[14]

[13] This is a simple case. The $\lambda-$calculus is a powerful tool for creating functions from formulas, even in rather complicated cases. Bacon (2017) gives a good example: if we wished to express the fact that the complex predicate 'being necessarily F' instantiates the higher-order concept 'applying to a', it doesn't seem we would be able to express 'being necessarily F' with $\Box Fx$—which is an open sentence—nor would we be able to use $\Box F$—which isn't well formed. Employing lambda expressions, however, makes the task simple: we use $\lambda x \Box Fx$ for 'being necessarily F' and λXXa for 'applying to a'. And we may write, $(\lambda XXa)\lambda x \Box Fx$ to express the fact that 'being necessarily F' instantiates 'applying to a'.

[14] That is, x is everywhere substitutable for a in ϕ if for any variable y that appears in ϕ it is not the case that a appears in the scope of a quantifier of the form $\forall y$ or $\exists y$ within ϕ.

ABSTRACTION

$\phi[a/x] \leq (\lambda x.\phi)a$, where a is free for x in ϕ

ADJUNCTION[15]

$\phi \leq \forall x\psi \leftrightarrow (\lambda x.\phi) \leq (\lambda x.\psi)$, where x is not free in ϕ

NECESSITATION

$\phi \leq \psi \rightarrow \Box(\phi \rightarrow \psi)$

The other principles besides ADJUNCTION are fairly familiar. To help understand how to read ADJUNCTION a bit better, consider the following phrasing: for proposition p and monadic predicate F, $p \leq \forall xFx \leftrightarrow (\lambda x.p) \leq (\lambda x.Fx)$, where x is not free in p. This could be read: p entails everything is F just in case being such that p is the case entails being such that you're F. ADJUNCTION should strike the reader as related to the classical introduction and elimination rules for the quantifier, as follows:

UI $\forall x\phi \rightarrow \phi[a/x]$, where a is free for x in ϕ

Gen: if $\vdash \phi$, then $\vdash \forall x\phi[x/a]$, where x is free for a in ϕ.

Instances of ADJUNCTION can be thought of as reformulations of these rules in terms of relations between properties rather than relations between propositions (i.e. the right hand side of ADJUNCTION is the universal introduction rule when p is replaced for $\forall xFx$, trivially satisfying the left hand side: being such that everything is F entails being such that you are F).[16] It is a principle that tells us when propositions entail generalizations given only information about when one property entails another. Even before discussing the options for a definition of entailment, these principles are intuitively motivated on any interpretation of entailment. And, as Goodman observes, they imply necessitism:

1. $\forall x\exists y(y = x) \leq \forall x\exists y(y = x) \rightarrow (\lambda x.\forall x\exists y(y = x)) \leq (\lambda x.\exists y(y = x))$ (ADJUNCTION)
2. $(\lambda x.\forall x\exists y(y = x)) \leq (\lambda x.\exists y(y = x))$ (1, REFLEXIVITY)
3. $(\lambda x.\forall x\exists y(y = x))a \leq (\lambda x.\exists y(y = x))a$ (2, APPLICATION)
4. $\forall x\exists y(y = x) \leq (\lambda x.\forall x\exists y(y = x))a$ (ABSTRACTION)
5. $(\lambda x.\exists y(y = x))a \leq \exists y(y = a)$ (REDUCTION)

[15]The name of the principle comes from terminology in category theory. Universal generalization, an operator taking properties to propositions, is the *right adjoint* of vacuous predicate-abstraction, an operator taking propositions to properties, while existential generalization is the *left adjoint* of vacuous predicate-abstraction. See Dorr (2014).

[16]Consider also the existential version of ADJUNCTION: for monadic concept F and proposition p, $\exists xFx \leq p \leftrightarrow \lambda x.Fx \leq \lambda x.p$. Similar to universal quantification, the right hand side of this principle is the existential elimination rule when p is replaced for $\exists xFx$, trivially satisfying the left hand side: being such that you are F entails being such that there is an F.

6. $\forall x \exists y(y = x) \leq \exists y(y = a)$ (3,4,5 TRANSITIVITY)

7. $\Box(\forall x \exists y(y = x) \rightarrow \exists y(y = a))$ (6, NECESSITATION)

8. $\Box\, \exists y(y = a)$ (7, K, $\Box(\forall x \exists y(y = x))$)[17]

9. $\Box\, \forall x \Box\, \exists y(y = x)$ (8, RN, Gen)[18]

The argument is clearly valid. But whether or not it's compelling depends on how entailment is understood.

One natural thought would be to define $\phi \leq \psi$ as $\Box(\phi \rightarrow \psi)$, modal entailment. But this won't do. The argument would fail to be effective against contingentists who believe that having a property entails existing, a view we may call *serious actualism*, and that predicates always denote real properties. They would happily reject ABSTRACTION on such an interpretation, as $\phi[a/x] \leq (\lambda x.\phi)a$ would become $\Box(\phi[a/x] \rightarrow (\lambda x.\phi)a)$, which we may read as necessarily, if $\phi(a)$ then being such that ϕ applies to a, the latter of which entails that a has a property if we assume that all predicates denote real properties, and so by serious actualism entails that a exists. Now, contingentists might think that while it's possible that you didn't exist, it is not possible that you had the property of nonexistence, and so would reject $\Box(\phi[a/x] \rightarrow (\lambda x.\phi)a)$ and thereby reject $\phi[a/x] \leq (\lambda x.\phi)a$.

These dialectical issues on modal interpretations of entailment don't just arise for contingentists who believe that having a property entails existing. Contingentists who think having properties and standing in relations aren't existence entailing will be unmoved by the argument above if we define $F \leq G$ modally as $\Box\, \forall x \Box(Fx \rightarrow Gx)$ or as $\Box\, \forall x(Fx \rightarrow Gx)$, for the needed instances of ADJUNCTION and APPLICATION, respectively. After all, the needed instance of ADJUNCTION states that, if everything exists entails everything exists, then being such that everything exists entails being such that you exist. The latter phrase becomes, e.g., necessarily everything x is such that necessarily if x is such that everything exists then x exists, which should fail for any purportedly contingent thing. That is, it is possible on such a view for you to be such that everything exists without yourself existing. Similarly for the needed instance of APPLICATION, which basically applies these properties to an individual a. If we can refer to merely possible individuals like Curly-0, the merely possible bald man in Quine's doorway, such contingentists might think that Curly-0 is such that everything exists, but not such that he exists.

[17] We assume K and $\Box(\forall x \exists y(y = x))$ are uncontroversial here.

[18] RN: If $\vdash \phi$, then $\vdash \Box\phi$. Gen: if $\vdash \phi$, then $\vdash \forall x \phi[x/a]$, where x is free for a in ϕ. Note that both must hold on logics of modality and identity. It isn't uncommon for contingentists to reject full RN, which would block the step from 8 to 9. For contingentists who do so, we may restrict the argument to the penultimate step, concluding the necessary existence of any arbitrary individual a, which isn't full blown necessitism, but is nonetheless standardly rejected by contingentists. I won't evaluate RN in this paper.

Goodman's own approach is to define entailment as disjunctive containment, that is, in terms of disjunction and a pair of related notions of 'identification', written $\equiv$, as follows: Define $\phi \leq \psi$ as $(\phi \vee \psi) \equiv \psi$ and $F \leq G$ as $(\lambda x.Fx \vee Gx) \equiv G$, where $\equiv$ is higher-order indistinguishability. That is, we can define the notions of identification where the arguments are sentences or predicates using higher-order quantification, i.e. (i) for sentences ϕ and ψ and variable O of the same syntactic category as monadic sentential operators like negation, define $\phi \equiv \psi$ as $\forall O(O\phi \leftrightarrow O\psi)$ and (ii) for predicates F and G and variable X that takes monadic predicates of individuals as arguments, define $F \equiv G$ as $\forall X(XF \leftrightarrow XG)$.

On this interpretation of $\leq$, NECESSITATION isn't just true by definition (as it would be on the interpretation of $\leq$ as modal entailment), but follows from Goodman's interpretation of $\leq$ and further uncontroversial modal principles (see below for explicit proof). APPLICATION is a natural generalization of the notion of properties and REDUCTION and ABSTRACTION are baked into a standard account of the lambda calculus.

Of course, contingentists may still press on these principles, and below we will consider one such strategy, but the dialectic becomes notably more interesting when entailment is defined in terms of disjunction and identification rather than in modal terms. This is because—instead of rejecting principles with modal instances that obviously contradict various entrenched contingentist theses—contingentists will have to reject more general claims about property and propositional entailment, which needn't be defined modally.

For instance, consider ABSTRACTION again as an example: it's one thing to reject a modal interpretation of ABSTRACTION—on some contingentist views there are straightforward counterexamples—but it's a somewhat different commitment to reject a non-modal interpretation. It isn't a matter of what might have been, but a matter of propositional granularity. Herein lies the novelty of Goodman's argument. It presents a case for necessitism based primarily on general claims about propositional granularity rather than claims about modality.

2.3.2 Adjunction

The only principle that needs a special discussion at this point is ADJUNCTION. What motivates accepting it? After all, contingentist hackles might be up once they realize that it draws heavily on standard classical quantifier inference rules, which many standardly restrict. And for a certain class of contingentists, the instance of ADJUNCTION in the first premise looks bad: why would it be that being such that everything exists entails that you exist, on any interpretation of entailment? At this

point, it seems we need some sort of argument for ADJUNCTION.

Goodman claims that ADJUNCTION is motivated abductively. His idea is as follows: Given Goodman's interpretation of $\leq$, ADJUNCTION is a principle which provides necessary and sufficient conditions for which propositions entail which generalizations in terms of disjunction and identification among properties. And it turns out that given fairly light assumptions about propositional granularity, ADJUNCTION gives us a result that Goodman claims makes the principle attractively strong: it uniquely specifies, or "pins down", the universal quantifier, when we assume that the definition of entailment we've focused on is reflexive and anti-symmetric. These assumptions, under the present interpretation of entailment, rule out certain fine-grained conceptions of propositions, as does REFLEXIVITY, which becomes the claim that disjunction is idempotent ($\phi \vee \phi \equiv \phi$). I'll also need to assume a principle of FUNCTIONALITY, see below. Any structured theory of propositions or otherwise fairly fine-grained view of propositions will have a way out, but contingentist who prefer semi fine-grained or coarse-grained views, will have to look elsewhere for a response.

At this point, Goodman's argument needs clarification: first, what exactly does it take to uniquely specify the quantifier and, second, why might this result be desirable?

Uniquely specifying the quantifier

Here, I present an explicit proof of the unique specification of the quantifiers when ADJUNCTION is assumed. I then present a generalization of this proof. It turns out that unique specification follows when we replace ADJUNCTION with any principle that says which propositions entail which generalizations but makes no use of the quantifier on the right-hand side of the biconditional.[19]

To prove that ADJUNCTION uniquely specifies the universal quantifier, we will need to assume:

FUNCTIONALITY: if $\forall X(FX \equiv GX)$, then $F \equiv G$

REFLEXIVITY: $\phi \leq \phi$ for all ϕ

ANTI-SYMMETRY: if $\phi \leq \psi$ and $\psi \leq \phi$, then $\phi \equiv \psi$

ADJUNCTION: $\phi \leq \forall x\psi \leftrightarrow (\lambda x.\phi) \leq (\lambda x.\psi)$, where x is not free in ϕ

A first-order quantifier $\forall$ is a thing of type $(e \to t) \to t$ that generalizes over things of type e.[20] This, of course, doesn't distinguish quantifiers from other things of the same type, but that's just our question: what property of properties is the quantifier and is there only one? Whatever could

[19] That is, for instance, while ADJUNCTION, i.e. $\phi \leq \forall x\psi \leftrightarrow (\lambda x.\phi) \leq (\lambda x.\psi)$, has no instance of a quantifier for the right-hand condition, FREE ADJUNCTION, $\phi \leq \forall x\psi \leftrightarrow (\lambda x.\phi) \leq (\lambda y.\forall x((y = x) \to \psi))$ does. The former but not the latter can be used for unique specification. We will discuss this and another principle in a later section.

[20] Types are types of expressions and determine how expressions combine. Here, we start with type e, those of individual type, like you or me, and type t, which are typically interpreted as those of propositional type, those which are true or false. First-order quantifiers take monadic properties (type $e \to t$) as inputs and produce propositions as outputs.

uniquely specify the quantifier, then, must have some relation to what we take to be crucial to quantification, and it's natural to think its inferential profile will play some part. For clarity:

> We will say THE CORE PRINCIPLES to refer to the conjunction of REFLEXIVITY, ANTI-SYMMETRY and FUNCTIONALITY.

Below, I will sometimes say "ADJUNCTION holds for operator A". By this, I mean the following:

> ADJUNCTION(A): $\phi \leq A x \psi \leftrightarrow (\lambda x.\phi) \leq (\lambda x.\psi)$, where x is not free in ϕ.

The following meta-proof establishes equivalence of any operator A such that ADJUNCTION holds for operator A, i.e. ADJUNCTION(A).

Theorem 1. *Given* THE CORE PRINCIPLES, ADJUNCTION($\forall$) *and* ADJUNCTION($\forall^*$), *for arbitrary* $\forall$ *and* $\forall^*$: $\forall \equiv \forall^*$.[21]

Proof. For some arbitrary ψ, replace ϕ with $\forall x \psi$ in ADJUNCTION($\forall$) to get:

$$\forall x \psi \leq \forall x \psi \leftrightarrow (\lambda x.\forall x \psi) \leq (\lambda x.\psi)$$

By REFLEXIVITY, $\forall x \psi \leq \forall x \psi$. Together these yield (1) $\lambda x.\forall x \psi \leq \lambda x.\psi$. For the same ψ, replace ϕ with $\forall x \psi$ in ADJUNCTION($\forall^*$) to get:

$$\forall x \psi \leq \forall^* x \psi \leftrightarrow (\lambda x.\forall x \psi) \leq (\lambda x.\psi)$$

This with (1) yields: (2) $\forall x \psi \leq \forall^* x \psi$. By symmetric reasoning, $\forall^* x \psi \leq \forall x \psi$. From this, (2), and ANTI-SYMMETRY, $\forall x \psi \equiv \forall^* x \psi$. Strictly speaking, this should be read as $\forall(\lambda x.\psi) \equiv \forall^*(\lambda x.\psi)$ (see footnote 18). Since ψ in $\lambda x.\psi$ is arbitrary and by Gen, we can infer an instance of the antecedent of FUNCTIONALITY, from which we deduce $\forall \equiv \forall^*$, as desired. □

What Theorem 1 proves is that if ADJUNCTION holds for any operator of type $(e \to t) \to t$, they are $\equiv$ related. On the background assumption that that establishes identity, we get uniqueness.[22]

The result generalizes to amendments to ADJUNCTION that keep the left-hand side of the biconditional fixed and the right-hand side is unchanged by replacements of any operator in the place of $\forall$. More precisely: let Δ be a schema in which ϕ and ψ may occur, as well as $\forall$. (More will need

[21] It might strike some readers as odd that we've put an identity between two quantifiers, since they might be used to thinking of quantifiers as variable binding sentential operators, taking sentences to other sentences. Our language doesn't handle quantifiers in this way; that is, syncategorimatically. In higher-order logic, variable binding is handled only by $\lambda-$terms. Strictly speaking, quantifiers aren't sentential operators that bind variables; rather, they are predicate constants. While I will for readability write $\ulcorner \forall x \phi \urcorner$, strictly speaking I mean $\ulcorner \forall(\lambda x.\phi) \urcorner$.

[22] It's worth mentioning that $\equiv$ is defined with higher-order quantifiers. It might seem dubious that higher-order quantifiers are used to determine first-order quantifiers. All we need, however, is an intelligible conception of higher-order identity. We could define it in terms of higher-order indistinguishability, but for now I will set this concern aside for the purposes of this paper and just assume this implies identity. For literature on the Identity Identity, see Dorr 2016 and Bacon and Russell 2019.

to be said later about what restrictions to place on Δ.) We write $\Delta[\forall^*/\forall]$ as usual for the result of replacing all occurrences of $\forall$ with $\forall^*$. We then use PRINCIPLE$^\Delta$ to mean $\phi \leq \forall x\psi \leftrightarrow \Delta$. As above, we say that PRINCIPLE$^\Delta$ holds for quantifier $\forall^*$ to mean:

> PRINCIPLE$^\Delta(\forall^*)$: $\phi \leq \forall^* x\psi \leftrightarrow \Delta[\forall^*/\forall]$,where x is not free in ϕ.

We will need the following to prove our result:

> STABILITY: PRINCIPLE$^\Delta$ is *stable* just in case for any things of type $(e \to t) \to t$ (candidates for quantifiers) $\forall^\dagger$ and $\forall^*$ such that PRINCIPLE$^\Delta(\forall^\dagger)$ and PRINCIPLE$^\Delta(\forall^*)$: $\Delta[\forall^\dagger/\forall] = \Delta[\forall^*/\forall]$.

One last principle we'll need is substitution. For propositions p, q:

> SUBSTITUTION: $p = q \to (\phi \to \phi[p/q])$

The following meta-meta-proof establishes equivalence of any operator A such that PRINCIPLE$^\Delta(A)$.

Theorem 2. *Given* THE CORE PRINCIPLES, SUBSTITUTION, STABILITY *of* PRINCIPLE$^\Delta$, PRINCIPLE$^\Delta(\forall^\dagger)$ *and* PRINCIPLE$^\Delta(\forall^*)$ *for arbitrary* $\forall^\dagger$ *and* $\forall^*$*:* $\forall^\dagger \equiv \forall^*$.

Proof. For some ψ, replace ϕ with $\forall^\dagger x\psi$ in PRINCIPLE$^\Delta(\forall^\dagger)$ to get:

$$\forall^\dagger x\psi \leq \forall^\dagger x\psi \leftrightarrow \Delta[\forall^\dagger/\forall]$$

By REFLEXIVITY, $\forall^\dagger x\psi \leq \forall^\dagger x\psi$. Together these yield (1) $\Delta[\forall^\dagger/\forall]$. For the same ψ, replace ϕ with $\forall^\dagger x\psi$ in PRINCIPLE$^\Delta(\forall^*)$ to get:

$$\forall^\dagger x\psi \leq \forall^* x\psi \leftrightarrow \Delta[\forall^*/\forall]$$

This with STABILITY and SUBSTITUTION yields:

$$\forall^\dagger x\psi \leq \forall^* x\psi \leftrightarrow \Delta[\forall^\dagger/\forall]$$

This with (1) yields: (2) $\forall^\dagger x\psi \leq \forall^* x\psi$. By symmetric reasoning, $\forall^* x\psi \leq \forall^\dagger x\psi$. From this, (2), and ANTI-SYMMETRY, $\forall^\dagger x\psi \equiv \forall^* x\psi$, which by FUNCTIONALITY and assuming we can generalize on the arbitrary ψ ensures $\forall^\dagger \equiv \forall^*$, as desired. □

Note that both proofs conclude that the quantifiers are $\equiv$ related, which we're interpreting as higher-order indistinguishability. We don't need to assume the substantial and more controversial claim that higher-order indistinguishability *just is* identity. All that is required is the weaker claim that higher-order indistinguishability *implies* identity, which follows given fairly weak principles about quantification and identity.

The interest of Theorem 2 is that it shows how we can uniquely pin down the quantifier no matter what Δ expresses so long as STABILITY holds with the other requisite premises. Of course, everyone is in the business of imposing some constraints on what can be expressed in Δ. No one would want to endorse a principle in which Δ is tautological or says something irrelevant to quantification.

Furthermore, note that neither Theorem 1 nor Theorem 2 requires all the premises needed in the Adjunction Argument to necessitism. As we will see in later sections, there are (i) contingentist views compatible with the premises necessary for Theorem 1 and (ii) admissible contingentist amendments to ADJUNCTION compatible with the premises in Theorem 2. That is, contingentists can in principle support a stable amendment to ADJUNCTION. First, we need to discuss why it might be desirable to have a principle which, given certain background assumptions, uniquely specifies the quantifier.

The unique specification of the quantifier is desirable

Strictly speaking, ADJUNCTION is a claim about the identity of propositions. And we've seen, in the last subsection, how it can uniquely specify the quantifiers. The main purpose of this section is to explain why this result might be desirable and to motivate the appeal of the methodology used to defend it.

Though not necessary, it would be interesting to have a metasemantic story for our logical vocabulary. After all, it's tempting to think that vocabulary such as the conjunction symbol and the quantifiers are determinately meaningful. These bits of vocabulary aren't like proper names; their reference isn't determined by some causal connection. And so, we would like a story that tells us in virtue of what are things like the conjunction symbol and quantifiers determinately and uniquely meaningful, even if we don't have a story about what that meaning is.

One very tempting starting point is to think that their theoretical role—in this case, their functional profile—plays an important part in uniquely determining their meaning. Goodman, following Dorr, gives us a story for how that might work for the quantifiers, drawing on other attractive stories given for other logical operators, like the conjunction symbol. The semantic contribution of the conjunction symbol can be uniquely specified by analogues of its functional patterns understood as entailments. One can be more or less fine-grained about the identity conditions for logical vocabulary. But one simple and elegant story goes like this: if you assume that $\phi \wedge \psi$ entails ϕ, that ϕ and ψ entail $\phi \wedge \psi$, TRANSITIVITY, ANTI-SYMMETRY, FUNCTIONALITY, and that $\equiv$ implies identity, then the conjunction symbol picks out the unique thing that satisfies the entailment analogues of the functional profile of conjunction. This is not to suggest that we have a story or account of what the conjunction symbol *means* or how it is *defined*. What we can now say is that there's a unique thing

that satisfies the functional profile, so long as we also constrain ourselves to certain assumptions about propositional grain.

We would like to have an analogous story for the quantifiers, and this is what ADJUNCTION can do. It allows us to extend the attractive story, according to which logical operators are uniquely determined by entailment analogues of their functional roles along with some assumptions about propositional grain, to quantifiers. Given the background assumptions specified above, ADJUNCTION uniquely characterizes the semantic contribution of the quantifier in terms of its functional profile, in an analogous way we did above for truth functional connectives like the conjunction symbol.

So far, all we've said in favor of ADJUNCTION is that it gives us a solution to the problem of semantic indeterminacy; it tells us that there is a unique semantic contribution of the quantifier. But that's only a reason to accept ADJUNCTION if there is no alternative principle or alternative metasemantic story that also suffices for that purpose.

So: is there any reason to think that the particular method invoked is the only or best method for fixing the quantifiers? After all, we don't in general expect that the metasemantic story for concepts like *essence* or *ground* should go this way, the way of being pinned down uniquely with the use of logical vocabulary alone—although there have been some attempts to give logical accounts. The most widely accepted method has been to defend the intelligibility of these concepts through examples, to show that no other concepts can account for these examples, characterize their relationship to other concepts, to show that they're theoretically fruitful, etc. But we don't give anything like a definition or analysis of them.

There are a number of things someone might say. For instance, (i) while there isn't (yet) a story uniquely specifying the concepts of *ground* or *essence* in such basic terms, if there were and it didn't fly in the face of something we're bound to accept, we would have reason to adopt it, even if it led to certain controversial metaphysical conclusions. It's always preferable to have an analysis or unique specification when we can manage it. (ii) Quantifiers are logical and the kind of metasemantic story we should demand for the quantifiers is different from one we can expect from metaphysical concepts like *ground*. Quantifiers are bits of logic and they should be characterized logically, but no such demand should be placed on concepts like *ground*, which are worldly.

Goodman and Dorr give us a solution to the problem of specifying determinate meaning for the quantifiers. If this story is the only game in town, that's an excellent reason to accept it. And if contingentists cannot accept ADJUNCTION with the required background, if there are no other stories available, and if abductive considerations like the strength ADJUNCTION lends to theories in which it figures can tip the scales, then we have reason to accept necessitism.

This paper defends contingentism against Goodman's argument. I start defending a contingentist adoption of ADJUNCTION against Goodman's objections to it. I then discuss an alternative to ADJUNCTION that satisfies STABILITY, and defend it with a *tu quoque*. I end by discussing the contingentist treatment of existence that makes the relevant formal principles governing quantification inappropriate by their own lights.

2.4 Contingentism with ADJUNCTION

It should be clear by Theorems 1 and 2 that it isn't necessitism, but coarse-graining, that pins down the quantifier with ADJUNCTION. It's natural to wonder, then, can contingentists in principle accept ADJUNCTION and the coarser-grained principles necessary to pin down the quantifier?

Consider a contingentist who believes in *serious actualism*, the view that having a property or standing in a relation entails existing.[23] Goodman notes that this contingentist would likely worry about ABSTRACTION and NECESSITATION. On the assumption that true predication denotes having a property, contingentists who believe in serious actualism might worry about $\Box(\neg\exists x(x = a) \rightarrow (\lambda x.\neg\exists y(y = x))a)$ for any purportedly contingent being a. After all, if we understand λ-terms as denoting real properties, then it isn't possible for you to not exist while having the property of not existing, since the latter entails that you exist by serious actualism. So: on any interpretation of propositional entailment that supports NECESSITATION, ABSTRACTION has instances directly in conflict with serious actualist commitments, assuming that true predication denotes having a property. Rejecting either ABSTRACTION or NECESSITATION doesn't just block this undesirable instance, it also blocks the derivation of necessitism in the Adjunction Argument.

Now, there's also nothing stopping the serious actualist from accepting a coarse or semi-coarse grained view of propositions, nor is there anything really stopping her from endorsing all the premises necessary for Theorem 1.[24] That is, contingentists could accept THE CORE PRINCIPLES, the interpretation of $\leq$ as disjunctive containment, and the claim that higher-order indistinguishability entails identity. After all, it isn't ADJUNCTION that entails necessitism, nor is it ADJUNCTION with the other premises necessary for Theorem 1 that entails necessitism. In principle then, there are contingentist views that are compatible with all premises necessary for Theorem 1, who can resist some of the other principles used to argue for necessitism.

[23] Serious actualism is related, but not identical, to the being constraint: **BC** $\Box\forall x_1 ... \Box\forall x_n \Box(Rx_1...x_n \rightarrow \exists y(y = x_i))$ for $1 \leq i \leq n$. See Williamson 2013. Serious actualism and the being constraint are compatible with both contingentism and necessitism. See Adams (1981); Plantinga (1983, 1985); Fine (1985); Kment (2014).

[24] Fine-grained views of propositions, most notably structuralism—roughly, the view that propositions have structures analogous to those of sentences—have been under attack recently. See Goodman (2017), originally Russell (1903). For a response, see Kment (forthcoming) and Wilhelm (unpublished draft).

Once the contingentist accepts these premises, however, she can't reject NECESSITATION, since it follows on Goodman's stipulated interpretation of $\leq$. NECESSITATION recall is $\phi \leq \psi \rightarrow \Box(\phi \rightarrow \psi)$, where $\phi \leq \psi$ is interpreted as $(\phi \vee \psi) \equiv \psi$, i.e. $\forall O(O(\phi \vee \psi) \leftrightarrow O\psi)$. On this interpretation, $\phi \leq \psi$ is definitionally equivalent to $(\phi \vee \psi) \equiv \psi$. From an analogue of Leibniz's Law for $\equiv$, it follows that $\Box(\psi \rightarrow \psi) \leftrightarrow \Box((\phi \vee \psi) \rightarrow \psi)$. From uncontroversial propositional modal reasoning, it follows that $\Box(\phi \rightarrow \psi)$, and we get NECESSITATION.[25]

Goodman takes issue with the kind of contingentist who would reject ABSTRACTION on serious actualist grounds. None of Goodman's objections have direct bearing on the Adjunction Argument nor on the prospect of pinning down quantification, but it's worth discussing them here as a way of showing that the view is defensible.

Goodman's first objection is that serious actualism "appears to be inconsistent with the truth of the English sentences contingentists usually use to articulate and motivate their view."[26] The sentences he has in mind are, 'possibly something is possibly nothing' and 'there is actually something that could have been nothing', which appear to include complex predicates like 'is possibly nothing' or 'is nothing' rather than sentential operators like 'it is not the case that' or 'it could have been the case that'.

Contingentists who accept serious actualism shouldn't be moved by this initial objection. First, contingentists usually articulate and motivate their views in terms of existence, 'possibly there is something that possibly fails to exist'. This doesn't sound like it involves the predication of existence except within the scope of the negation operator. It's unclear, then, if there's a *prima facie* objection here at all. Goodman himself wouldn't even say that '*a* exists' ascribes a property to *a*, since according to him the quantifier expresses a second-order property, in this case, that being identical to *a* has at least one instance. If sentences about existence are used to motivate and articulate contingentism, and if we're interpreting quantification in the way Goodman is, it wouldn't be that we've ascribed anything like a property of existence to *a*.

But even setting all that aside, it's not unreasonable to think these ordinary judgments aren't attributing properties to individuals even if—in English—there is predication of a term. When we ordinarily think that I might not have existed, we're not thinking that I could have had the property

[25] We have to assume a restricted rule of NECESSITATION that applies to uncontroversial propositional tautologies. From $\Box(\psi \rightarrow \psi)$ and $\Box(\psi \rightarrow \psi) \leftrightarrow \Box((\phi \vee \psi) \rightarrow \psi)$, we get $\Box((\phi \vee \psi) \rightarrow \psi)$, from which $\Box(\phi \rightarrow \psi)$ follows in addition to $\Box(((\phi \vee \psi) \rightarrow \psi) \rightarrow (\phi \rightarrow \psi))$, axiom K, and MP.

[26] Goodman, 2016a, p172. To be clear, his arguments are really against anyone who accepts the being constraint—see earlier footnote. His points really are more forceful against this person and not the broader class of serious actualists, who don't need to take a specific stand on how to formalize her view. The advantage of the being constraint is that it's clear; it says that true predication entails existence. However, that's strictly more than serious actualism is committed to, which doesn't imply any stance about whether true predication entails having a property.

of not existing; we're thinking that the situation could have been such that nothing was me. It's a judgment about a possibility, not a judgment about a possible property attribution. It's a judgment about a way the world could have been, not clearly a judgment about a way I could have been. Even if 'I might not have existed' sounds like it takes the form $\Diamond \neg Ea$ where E is a property, it still doesn't seem to ascribe a property to me of possibly not existing. Furthermore, 'existence' as a predicate has a longstanding reputation of not acting—grammatically speaking—like most other predicates.[27] It is simply not outlandish to treat these judgments operationally rather than predicatively. And it wouldn't be particularly forceful to object to the use of a symbol or phrase as an abbreviation, as with the operational treatment of existence in sentences such as 'possibly something is possibly nothing'.

Goodman goes on to press the serious actualist in two other ways. First, he wants to know what demarcates real and merely apparent predicates.

Everyone needs to make a principled decision about how to understand the lambda operator and when predicates denote properties, and there are several legitimate ways to use it. It could be that lambda operators are devices for introducing metaphysically committal predicates, predicates that denote real properties, but they don't have to be. They also could fail to denote real properties or be understood as shorthands for more complicated phrases. If every predicate ascribes a property, then of course serious actualists would deny ABSTRACTION. By the lights of the serious actualist, 'Fa' entails a's existence whenever F expresses a property. Maybe you can abstract on $\phi(a)$ only when $\phi(a)$ entails $\exists x \phi x$ or that $\phi(a)$ and $E(a)$, where E is a primitive property of existence. Strictly speaking, however, serious actualism is consistent with ABSTRACTION so long as some predicates fail to denote real properties of things. Goodman's arguments, then, are more forceful against the person who accepts the being constraint (**BC** $\Box \forall x_1 ... \Box \forall x_n \Box (Rx_1 ... x_n \rightarrow \exists y(y = x_i))$ for $1 \leq i \leq n$) rather than the more general view of serious actualism, which doesn't itself say anything about what it takes in the language to denote a property.

The serious actualist Goodman has focused on, however, the one who rejects ABSTRACTION, believes that predication with the use of lambdas is metaphysically committal. Goodman's request for clarification is interesting; that is, it would be interesting and illuminating to know how, in general, we should restrict ABSTRACTION, which predicates denote real properties and which are merely apparent. But the fact that we have to say more here isn't a decisive objection to a view, just as asking the necessitist to clarify her position about *chunkiness*, ungrounded modal properties, and

[27] See Moore (1936) for the grammatical differences between 'exists' and more typical predicates like 'growls'. The end of the paper, however, complicates this with modal talk.

atypical uses of *essences* isn't a decisive objection to their view. So in short, interesting question; the contingentist should try to clarify, but their position is no less mysterious than the necessitist's.

Goodman moves on to a second objection: we have a pre-theoretical practice of using open formulas to introduce new expressions, including meaningful predicates, by explicit definition, with the understanding that when we use such a predicate the resulting formula is intersubstitutable with the original. Predicate abstraction is a convenient notation for *ad hoc* definition; it follows this pre-existing practice. We should thereby be able to infer all non-vacuous instances of ABSTRACTION and REDUCTION from corresponding instances of REFLEXIVITY—but we can't if ABSTRACTION is denied or restricted.

It is correct that this is our practice in extensional contexts like mathematics or beginning logic, but it isn't clear the point extends to hyperintensional or intensional contexts. A simple attitudes case should make this clear: Vixens are female foxes, but someone could be such that they believe x is a vixen but not believe x is a female and a fox. Even in beginning logic, we introduce these cases as intuitive failures of substitutivity. If Goodman is right that our practices surrounding ABSTRACTION in extensional contexts work in the way he describes, why think that this applies also to modal cases? That is, his point is most forceful when we consider ABSTRACTION alone as merely saying that $\phi(t)$ entails $(\lambda x.\phi)t$, is isn't as clearly true for the stronger claim that $A\phi(t)$ entails $A(\lambda x.\phi)t$ for any operator A.

The serious actualist will not accept the combination of views that allows us to λ-abstract on any formula and then generalize, $\exists X$, on any λ-term, if the quantifier is meant to range over properties. If lambda operators merely serve to introduce shorthand notation, then ABSTRACTION is okay—even to the serious actualist—but then $\lambda x.\phi$ cannot also express a property, and we therefore cannot infer $\exists x(x = t)$ from $(\lambda x.\phi)t$. Of course, this would likely result in an adoption of a fine-graining, rather than a rejection of ABSTRACTION.

Goodman makes it clear that he is focusing on coarse-grained views, but it's certainly worth mentioning that most serious actualists who would reject ABSTRACTION, or second-order existential generalization, would do so because they would also reject the coarse-grained assumptions or the Fregean conception of quantifiers as properties of properties.

These contingentists distinguish between propositions like *Biden could have failed to exist* and *Biden could have had the property of non-existence*. That is a modal reason for fine-graining, for rejecting ANTI-SYMMETRY, since it could be that ϕ and $\lambda x.\phi$ are mutually entailing but not identical or higher-order indistinguishable, and not intersubstitutable in all contexts.

Furthermore, contingentists who accept serious actualism would likely reject the Fregean con-

ception of quantifiers as properties of properties that we made use of above. When we regiment the sentence 'Something is sitting and eating' in the familiar first-order way, we get:

$\exists x(Sx\&Ex)$

there is something such that it is sitting and it is eating.

In the Fregean conception, we get

$\exists(\lambda x.Sx\&Ex)$

the property of sitting and eating has at least one instance.

Once you distinguish between the formula ϕ and a predicate $\lambda x.\phi$, you should also distinguish between saying that something meets the first condition and saying that something meets the second condition. And presumably, once you've done this, you should take $\exists x\phi x$ to express that there is something that meets the first condition and not that something meets the second condition. That is counter to the second-order predicate treatment of quantifiers which makes no distinction between the two and defines existence by reference to the second condition: Biden could have failed to exist and so something could have failed to exist is—to the serious actualist—distinct from saying that Biden could have failed to exist and so the property of possibly not existing has at least one instance. To her, the former is true and the latter is false.

The primary point of this section was to show that some contingentists—a defensible, mainstream group of them—can pin down quantification in the way presented in Goodman's paper by accepting ADJUNCTION on Goodman's interpretation of $\leq$ with all requisite assumptions about propositional granularity. The proof of necessitism from ADJUNCTION and other principles essentially relies on ABSTRACTION, so contingentists can reject that argument by rejecting ABSTRACTION, as may be well-motivated by certain contingentist positions. But the proof of the uniqueness theorem for the quantifiers from ADJUNCTION and other principles makes no use of ABSTRACTION. So that proof goes through even if we reject ABSTRACTION for the reasons given above.

2.5 Contingentism with an amendment to ADJUNCTION

In this section, I show that—in principle—contingentists can amend ADJUNCTION in a way that supports STABILITY, which we showed in Theorem 2 is required for pinning down quantification. Any view that accepts ADJUNCTION pins down the quantifier with fewer resources than the amended principle; however, this is only a theoretical advantage in isolation. I argue that this is no advantage for a full necessitist theory that will either include additional ideology like *chunkiness*, which is not

uniquely specified, or will not be able to account for our entrenched modal judgments. Contingentists who accept ADJUNCTION are the only ones who may claim a theoretical advantage on this front.

2.5.1 Amendment to ADJUNCTION

Earlier, we talked about how certain contingentists will want to reject the following instance of ADJUNCTION from premise 1, which reads

$$\forall x \exists y(y = x) \leq \forall x \exists y(y = x) \rightarrow (\lambda x.\forall x \exists y(y = x)) \leq (\lambda x.\exists y(y = x))$$

Paraphrased: everything exists entails everything exists only if being such that everything exists entails being such that you exist.

Contingentists who believe there are terms that refer to merely possible beings and believe that having a property or standing in a relation does not entail existing will reject premise 1. The property of being such that everything exists clearly does not entail the property of being such that you exist. After all, Curly-0 is such that everything exists but not such that he exists. Goodman discusses one amendment to ADJUNCTION in light of this:

FREE ADJUNCTION

$\phi \leq \forall x \psi \leftrightarrow (\lambda x.\phi) \leq (\lambda y.\forall x((y = x) \rightarrow \psi))$, where x is not free in ϕ.

Paraphrased: ϕ entails everything is ψ just in case being such that you are ϕ entails being such that you are ψ if you exist.

He rejects this principle as a substitute for ADJUNCTION because of its logical weakness, for its inability to uniquely specify the quantifiers. Assuming that $(\lambda y.\forall x(y = x \rightarrow (Fx \rightarrow \psi)))$ and $(\lambda y.\forall x(Fx \rightarrow (y = x \rightarrow \psi)))$ are mutually entailing, TRANSITIVITY of entailment in predicate position gives us the following instance of FREE ADJUNCTION:

RESTRICTED FREE ADJUNCTION

$\phi \leq \forall x(Fx \rightarrow \psi) \leftrightarrow (\lambda x.\phi) \leq (\lambda y.\forall x(Fx \rightarrow ((y = x) \rightarrow \psi)))$, where x is not free in ϕ.

If FREE ADJUNCTION holds for the unrestricted quantifier, then it holds for all restricted quantifiers under any non-universal condition F as well:[28]

$\phi \leq \forall^F x \psi \leftrightarrow (\lambda x.\phi) \leq (\lambda y.\forall^F x((y = x) \rightarrow \psi))$, where x is not free in ϕ.

[28] See Dorr 2014

FREE ADJUNCTION does not satisfy STABILITY, since the quantifier is featured on the right-hand side of the biconditional. By contrast, ADJUNCTION fails when the quantifier is read as restricted by any non-universal condition F, when we assume that entailment is interpreted in a way that validates REFLEXIVITY, since line 2 of the Adjunction Argument becomes $(\lambda x.\forall x(Fx \rightarrow \exists y(Fy \wedge (y = x)))) \leq (\lambda x.\exists y(Fy \wedge y = x))$. Clearly, this is false because everything satisfies the former condition while every non-F fails the latter.

Consider the following alternative to FREE ADJUNCTION:

EXISTENCE ADJUNCTION

$\phi \leq \forall x\psi \leftrightarrow (\lambda x.\phi) \leq (\lambda x.(Ex \rightarrow \psi))$, where x is not free in ϕ,

where E is not defined quantificationally, but as a primitive existence predicate.[29] Strictly speaking, on the assumption that E is unique, EXISTENCE ADJUNCTION provides the contingentist with a free-logical amendment to ADJUNCTION that supports STABILITY. STABILITY, recall, is what is crucial in Theorem 2 to generalize Goodman's result of pinning down the quantifier.

The kind of free logic motivated by the view that would adopt EXISTENCE ADJUNCTION is one that would likely adopt a positive semantics, where $a = a$ is a theorem, whether or not a denotes, and classical quantifier rules are restricted, for instance

UIR: $\forall x\phi(Ea \rightarrow \phi(a/t))$, where a is free for x in ϕ

since the kind of view that would reject the instance of ADJUNCTION in premise 1 of the adjunction argument is one who thinks that having a property or standing in a relation doesn't entail existing and some terms denote merely possible beings.[30]

EXISTENCE ADJUNCTION is not weak in the way that FREE ADJUNCTION is weak. For every non-universal condition F, line 2 of the Adjunction Argument becomes $(\lambda x.\forall x(Fx \rightarrow \exists y(Fy \wedge (y = x)))) \leq (\lambda x.(Ex \rightarrow \exists y(Fy \wedge y = x)))$. Clearly, this is false because everything satisfies the former while every non-F fails the latter, just as with ADJUNCTION. Note that—on the intended interpretation—E is a universal property, in the sense that $\forall xEx$ is a theorem. The framework is free in the sense that Ea is nonetheless sometimes false. Relatively speaking, then, EXISTENCE ADJUNCTION is as strong as ADJUNCTION.

[29] So long as we include the vocabulary of identity, $=$, it is well known that E is redundant. Where a is a variable or singular term, Ea can everywhere be replaced by $\exists x(x = a)$. The line I'm taking in this paper, is that E is primitive nonetheless, not to be defined in terms of $\exists$, despite their equivalence.

[30] Roughly, negative semantics treats all atomic sentences involving empty names as false, positive semantics treats at least some atomic sentences involving empty names as true, and neutral semantics treats all atomic sentences involving empty names as neither true nor false.

2.5.2 Tu quoque

The amendment does respond to the stated challenge to give a contingentist friendly alternative to ADJUNCTION that supports STABILITY, but it may strike the reader as insufficient. We don't have a story that tells us how to uniquely specify E. Unless the contingentist earns her right to this vocabulary, she seems stuck with a theory that appeals to expressions which aren't uniquely pinned down in the relevant way.

It's true that, when we consider ADJUNCTION and EXISTENCE ADJUNCTION in isolation, the latter seems to push the problem back. However, ADJUNCTION should not be considered in isolation; it is supported by the strength it lends to theories in which it figures. Here, I argue that the necessitist theory that adopts ADJUNCTION is on no better footing than a contingentist that adopts EXISTENCE ADJUNCTION, so long as the necessitist theory is adopted with ideology like *chunkiness*: the added ideology of *is chunky* also cannot be pinned down in the way ADJUNCTION makes claim to pin down quantification. These properties are meant to play a similar theoretical role that *existence* plays for the contingentist, and against the contingentist who takes quantification to not be pinned down by ADJUNCTION, necessitism would have no real theoretical advantage overall.

Necessitists are incentivized to account for the truth of our entrenched modal judgments with *chunkiness*. In section 2.2, we saw that this ideology ends up being primitive or depending on concepts like *grounding*. Start with the assumption that we want something in common between the contingent status of things like you and me, impure sets like $\{Socrates\}$, and propositions like *Biden is a politician*. We saw that necessitists couldn't simply analyze our judgments in terms of *concreteness*, which is a bit of ideology contingentists also have, because that wouldn't account for the contingency of impure sets. We can't make use of *concreteness* and set *containment* or *membership*, because that wouldn't account for propositions. *Chunkiness* can account for the broader class if it's taken as a primitive or is analyzed, but it's unclear how we would analyze it without *grounding* or other hyperintensional resources. Necessitists are incentivized to a degree to adopt a fine-grained picture as well. This is clearly at odds with the Adjunction Argument, which makes use of a number of coarse-grained assumptions. Because of this, it seems the view the necessitist would presumably take is one on which *chunkiness* is primitive, but then everyone needs a metaphysical primitive that will be no better off than the contingentist primitive E.

The necessitist may hope to define *chunkiness* in terms of grounding. The received view in the literature on grounding is that grounding is an unanalyzable metaphysical primitive.[31] Definitions

[31] See, for instance, Schaffer 2009, Rosen 2010, Fine 2012.

or analyses are sometimes given, but never widely accepted.[32] We can get a grip on the notion from examples, helping us specify how the concept relates to and is different from others, what its application conditions are, and so on. A systematic theory, a complete definition, a proof of unique specification may be desirable, but not required to regard the notion as legitimate. The lack of unique specification isn't particularly damning for the case of *grounding*. Not every notion can be uniquely specified. Everyone in the debate then will make use of some primitive that seems to do similar theoretical work, and is no better off than the contingentist's primitive of existence.

Perhaps the necessitist's *chunky* and the contingentist's *existence* work the way *grounding* does. So long as the necessitist needs to introduce ideology like *is chunky* to account for the truth of statements similar to what contingentists account for with quantifiers, and because *is chunky* cannot be uniquely specified, there is no theoretical advantage for anyone who can uniquely specify the quantifiers. If there's a problem with one, there is a problem with the other. If it's admissible to introduce this non-logical vocabulary for the necessitist, it is admissible for the contingentist. Once this vocabulary is introduced and deemed admissible, it can be used for whatever theoretical purpose one wants, including specifying the quantifier. There is, then, no advantage the necessitist has over the contingentist to uniquely pin down the quantifier because the contingentist can use admissible ideology to specify quantification on their view that's on a par with necessitist ideology like *chunky*.

The necessitist may still insist that they do have an advantage: they can characterize quantification in something like logical terms, not using primitive vocabulary like *existence*, and the contingentist cannot. Quantification is logical! they may say, and a logical characterization is good if we can have one.

It's unclear what this use of logical would mean, but it is true that quantification is specified using fewer resources: is that kind of characterization preferred? As we will discuss in the next section, there's reason to think that certain necessitists will see this sort of consideration about kinds of resources we can use to characterize quantification in a very different way than a contingentist and certain necessitists will. Existence and quantification are—for the contingentist—more "worldly" than "logical", like *grounding*, doing more work and requiring more resources. So the insistence that quantification itself should be specified in a certain way isn't so forceful against the contingentist, who won't think of quantification like certain necessitists do.

It's clear that ADJUNCTION, if it's a point in favor of anyone, is a point in favor of a semi-coarse-

[32]For instance, Bricker 2006, but makes use of ideology like *fundamentality*; Correia 2013 in terms of *essence*; Correia and Skiles 2019 in terms of propositional identification and closely related ideology, roughly, that p grounds q when $p \equiv (q \vee r)$, given moderately fine-grained principles of property individuation.

grained view of properties and propositions, rather than a point for contingentists or necessitists. If it favors either party in the debate over modal ontology, it will have to be considered in a package of views.

In this paper, we've primarily considered four classes of views: contingentists who add a primitive *existence* predicate to specify quantification, necessitists who specify quantification with ADJUNCTION and add a primitive concept of *chunkiness* to weaken the contingentist claim to advantage over accounting for entrenched modal judgments, necessitists who adopt ADJUNCTION but bite the bullet on entrenched judgments, and contingentists who accept ADJUNCTION without primitive vocabulary. The theory that seems to do best in the terms set out by Goodman—all else equal—is the lattermost contingentist who rejects ABSTRACTION, adopts ADJUNCTION and the other principles set out for the derivation of Theorem 1. Nonetheless, in the next section, we will discuss why contingentists might want to reject ADJUNCTION in any case.

2.6 Existence and quantification

In this section, I argue that many contingentists endorse a conception of existence that makes the relevant formal principles governing quantification in the adjunction argument inappropriate by their own lights. The details of this point are difficult to make precise, but are worth gesturing at because they are centrally important to the methodology behind endorsing ADJUNCTION. Even the serious actualist described in section 2.4, who could in principle accept ADJUNCTION, may reject ADJUNCTION for reasons described in this section.

Central to the modal ontological debate are two critical questions: what does it take for something to exist and are these conditions contingent or necessary? I start by describing how many contingentists may justifiably reject ADJUNCTION, since instances of it look like sufficient conditions on existence that they may object to. In fact, certain necessitists may even object to the conditions specified by ADJUNCTION. This is related to the first critical question of what it takes for something to exist. I move to the second critical question in the following subsection. Contingentists think that something's existence is tied to contingent conditions. As a result, it does metaphysically significant, demarcating modal work for the contingentists, as *chunkiness* does for certain necessitists. We wouldn't expect these more worldly notions to be pinned down without more significant resources than ADJUNCTION provides. Finally, logicians can describe the implications of a formal theory—e.g. it can uniquely specify the quantifier—and Dorr and Goodman have identified an interesting quantifier with an interesting implication, but it's a further question to ask whether it

denotes *real*, *metaphysical* existence. Who and what decides that question here is up for debate, since, as I've shown, necessitists and contingentists both have a mix of issues in their favor and it is not clear what combination of strengths and weaknesses we should pick.

2.6.1 Sufficient conditions for existence

The classical inferential rules for quantifiers look like conditions on existence. Existential introduction, e.g., looks like a condition on which (i) being the subject of true atomic predication is sufficient for existence or (ii) being an object of reference is sufficient for existence. Furthermore, instances of ADJUNCTION are reformulations of the classical introduction and elimination rules in terms of relations between properties (e.g. the right hand side of ADJUNCTION is the universal introduction rule when p is replaced for $\forall xFx$, trivially satisfying the left hand side: being such that everything is F entails being such that you are F.).[33]

How might necessitists and contingentists react to these conditions? What does it take for something to exist?

Some contingentists may agree with these conditions, but many will not. After all, contingentism just is the thesis that existence is contingent—so some thing's existence is tied to contingent conditions. This is part of why many contingentists reject classical logic for a free logic.

The kind of contingentist who might endorse these sufficient conditions for existence believes in serious actualism. As discussed in section 2.4, such a contingentist might adopt ADJUNCTION and use it to pin down the semantic contribution of quantification. They needn't, however, adopt ADJUNCTION. Serious actualists could endorse these conditions while thinking they don't exhaust the characterization of existence. Maybe there are other important sufficient or necessary conditions on existence: e.g. instantiating your essential properties. Reality, they might think, has to be some way in order for "something" to exist, and perhaps ADJUNCTION underperforms; it characterizes the quantifiers in a way that fails to fully capture the conditions under which a thing exists.

Other contingentists might reject the conditions. They might think that being an object of reference or the subject of true atomic predication isn't sufficient for existence. They may believe we refer to Socrates, maybe even Vulcan (the planet causing perturbations in Mercury's orbit), Botticelli's Venus, Newman-1 (the first born child of the 22nd century), Curly-0 (the merely possible bald man in Quine's doorway), or the merely possible child resulting from the fusion of this ovum and that sperm, but in no case do we refer to some existing thing. If that is true, reference is not

[33]Consider also the existential version of ADJUNCTION: for monadic concept F and proposition p, $\exists xFx \leq p \leftrightarrow \lambda x.Fx \leq \lambda x.p$. Similar to universal quantification, the right hand side of this principle is the classical existential elimination rule when p is replaced for $\exists xFx$, trivially satisfying the left hand side: being such that you are F entails being such that there is an F.

sufficient for existence. Similarly, some contingentists think there are cases of true atomic predication of terms that do not denote existing things, such as 'Socrates is self-identical'. So again if that's true, true atomic predication is not sufficient for existence. Reality, they might think, has to be some way in order for "something" to exist, and ADJUNCTION misfires; it doesn't just underperform, but characterizes the quantifiers in a way that provides false conditions under which things exists.

For contingentists who think that reality plays a role in characterizing when something exists, who interpret quantificational expressions as expressions about existence, and who think that logical vocabulary alone cannot express what reality has to be like in order for "something" to exists, a treatment of the quantifier as specified by ADJUNCTION is going to seem too demanding.

This motivates a general defense of the view that contingentists are warranted in believing more resources than ADJUNCTION uses are necessary to specify the quantifier, to specify existence.

Even some necessitists will think these conditions are false. We could imagine necessitists who give substantial answers to the question what does it take to exist: some think the only things that could exist are the actually existing things, while others think the only things that do and could exist are those basic, unalterable things whose configuration may change.[34] We could also imagine that these necessitists think there is true atomic predication or reference that would push them to reject the conditions specified by ADJUNCTION, and who might adopt a free logic.

The Williamsonian necessitist described in this paper, however, treats existence in a way that's tempting to describe as trivial, since the existence of some things isn't generated by a clear way the world is. For contingentists and non-Williamsonian necessitists described above, the world has to be a certain way for something to exist. And while we can describe a story to help us wrap our minds around how it might be true that it takes nothing for some special class of thing to exist, e.g., the null set, it's more difficult to understand how this works with the case of you, or mountains, or other such thing in situations where you all exist but not concretely. The actual state of things, for Williamsonian necessitists, doesn't determine whether or not you exist; even if you can be described as having distinct modal properties, you aren't always categorically different than some other non-concrete things like your parents. You just exist, or not. So in a sense—and I apologize for the use of 'trivial' and 'significant'—for these kinds of Williamsonian necessitists, reference and true atomic predication are trivially sufficient for existence, not substantially sufficient for existence in the way they are for contingentists who adopt these conditions.

[34] While we are not attempting historical exegesis, some have interpreted Spinoza and Wittgenstein, respectively, to have adopted these views.

2.6.2 Metaphysical significance of existence

Existence is metaphysically significant for contingentists in a way that it is not for the necessitist. It's the thing that does demarcating work to tell us the difference between me, the number two, and Wittgenstein's possible child in actuality and across modal situations. It explains the relevant dependences $\{Socrates\}$ and *Biden is a politician* have on Socrates and Biden, respectively.

Existence may have some significance for some necessitists, but not in the same way as for the contingentist. For the necessitist: everything has it, couldn't have failed to have it, and it couldn't have been that something had it without actually having it. When it comes to existence, the modal is quite like the actual. It does no distinguishing work, neither actually nor modally. Reality may have to be some way for something to exist, but those conditions will never be contingent conditions. Everything just is, always, necessarily.

Contingentists treat existence very differently. Even though everything exists, some things could have failed to exist, and it could have been the case that there was something that there actually isn't. For many contingentists, existence does demarcating work: some things could have failed to exist while others couldn't. It's not trivial. It's what is significantly different about a situation in which you are alive and a situation in which you were never born. Existence may not be trivial for the necessitist in the sense that the number two and you and I have to exist, and necessarily so. It might be that only actual things could have existed. So reality may have to be some way for us all to exist, but not in the same sense as for the contingentist, where existence is tied to contingent conditions. And contingentists must describe, under what conditions, "something" exists, and then specify which of those conditions are contingently met. Just the same, necessitists must describe, under what conditions, something is *chunky*.

If our view of quantification tracks our view of existence, then it might not be surprising that some necessitists would be happy with the treatment of the quantifier we get with ADJUNCTION, but they would be more like the Williamsonian necessitist, who thinks that there are ungrounded modal facts, and so existence is in a sense trivialized. Contingentists and some necessitists will see ADJUNCTION as too limited or a mischaracterization, as not using enough resources. Existence is metaphysically significant and worldly to the contingentist, perhaps in a way much like many treat *grounding* or causation—and those concepts, we mentioned, are widely accepted to be only characterizable, not fully specifiable, by logical vocabulary alone. That is, for the contingentist, the metaphysics of being is much more like the metaphysics of grounding, causation, mass, charge. Yes, it would be nice if we could specify grounding uniquely—it adds strength to a theory by answering

the question of whether the vocabulary is uniquely specified that would otherwise be hard to answer—but it's unsurprising that it can't. And to some contingentists, the same thing might be said about existence, that we need more resources, worldly, primitive, maybe even hyperintensional ones, than the minimal ones we use with ADJUNCTION.

In the necessitist framework, no one would expect *chunky* to be uniquely specified by minimal resources a principle such as ADJUNCTION offers. It shouldn't be surprising or damaging to the contingentist that quantification isn't specified uniquely. After all, they think existence is substantial in the sense that reality has to be some way for something to exist, that the conditions on existence are connected more deeply to the way the world is. If we interpret quantification and its inferential profile as stating true conditions on existence, then those who think those conditions are contingent won't think they're expressible with minimal vocabulary and will find ADJUNCTION wanting as a principle to uniquely specify the quantifier.

2.6.3 Quantifiers and *real* existence

Pinning down the quantifiers with ADJUNCTION may be desirable, and it gives us some reason to investigate the quantifier characterized by ADJUNCTION, but it's not obvious that it characterizes *real* existence or that it is so attractive that we should give up contingentism to get such a result. It might be that the semantic values of the "real" quantifiers aren't as ADJUNCTION characterizes them. Or it might be that the metasemantic story for the quantifiers shouldn't be like the one we give for the conjunction symbol above. Dorr, who investigates the quantifier pinned down by ADJUNCTION and the CORE PRINCIPLES, points out, "so long as the completeness condition holds there is nothing to stop us from stipulatively introducing new quantifier-words which do have them [the characteristics attributed by ADJUNCTION], and using these words to raise a new array of challenging metaphysical questions."[35] It's potentially fruitful to think that this quantifier is metaphysically interesting, and Goodman has proved that, given certain assumptions, it is a necessitist quantifier. But why should we think it characterizes real existence?

It's not obvious when a bit of ideology should be the primary business of logicians, but just the fact that logicians take in interest in something is not enough to settle that question. Burgess (1999) makes a relevant point in the vicinity:

> The question Which is the right system of tense logic? is not one for the logician: the logician can indicate how this or that or the other system corresponds to this or that or

[35] Dorr 2014.

> the other theory of the nature of time, but which is the right theory of the nature of time is a question for the physicist. Similarly, the question Which is the right system of mood logic [modal logic as logic of inevitability, of metaphysical necessity]? would seem to be one not for the logician, but for the metaphysician. By contrast, the question Which is the right system of validity or demonstrability logic? cannot be passed off by logic to some other discipline.[36]

Picking the right theory of tense is primarily the business of physicists; picking the right theory of metaphysical modality is primarily the business of metaphysicians. While this goes beyond what Burgess says, part of the job of the physicist and metaphysician is telling us if the specification of the tense and modal operators makes sense. Logicians take an interest in tense and modal operators, but does that mean a formal advantage is really an advantage?

When we're in the business of giving a story for the quantifiers, perhaps the same point applies. Even though $\forall$ and $\exists$ are logical in the sense that they feature in logics; insofar as they characterize *real* existence, their characterization may be one not fully at home within logic. It's one thing to tell us an interesting feature of a certain symbol under certain conditions, but it's another thing to say that that symbol denotes real existence. To a contingentist: existence is something that does significant and worldly modal metaphysical work: so it would seem it is primarily the job of the metaphysician to tell us if some characterization we've found in a logic accurately or inaccurately describes real existence. But these sentiments are part of a wider sensibility about the role of logic in metaphysics that go beyond the scope of the paper.

The contingentist who accepts all this will still need a story for the specification of existence. When we're looking for such a story, we will first take all the core truths about something. If those determine a unique referent, we're done and that is the best case scenario. If they determine many referents, the theory underdetermines the identity of the thing. The contingentist and necessitist disagree about what those core truths are about existence, and so it wouldn't be surprising that their stories differ. Certain necessitist and certain contingentists may find themselves in the situation where one referent is already determined by what they take to be the core truths about quantification and existence. For the contingentists who have a different starting point, the referent of quantification may be underdetermined. Maybe that's okay.

[36] Burgess 1999.

2.7 Conclusion

An interesting feature of Goodman's Adjunction Argument is his methodology. If our modal judgments fail to favor either contender in the modal ontological debate, if necessitism and contingentism fare equally well with respect to these judgments, what resources do we have to argue for what there might have been? Goodman uses an interesting abductive method, starting first with a formal theory of quantification, identity, and propositional and property entailment, and from there argues that the scales tip in favor of necessitism. This method is similar to Williamson's (2013) method; it employs a formalism-first approach and abductive considerations of strength. But instead of focusing on abductive methods as they apply to modality and higher-order modal comprehension principles, Goodman focuses on abduction and theories of propositional granularity, although the distinctions are not sharp.

This paper pressed against Goodman's argument and methodology in a number of ways on behalf of the contingentist. I argued that contingentists can accept stable principles to pin down the quantifiers by accepting ADJUNCTION and the other premises necessary for Theorem 1 or by amending ADJUNCTION to a free version and accepting the other premises necessary for Theorem 2. I defended the contingentist who adopts the amendment with a *tu quoque*. The necessitist who makes use of added vocabulary to weaken the contingentist claim to being able to account for entrenched modal judgments is at no advantage. So long as the added ideology is admissible, contingentist uses of *existence* as primitive vocabulary are admissible. In fact, necessitists can either (i) bite the bullet and not account for our entrenched judgments, while also not having added vocabulary or (ii) introduce added vocabulary like *chunkiness* that seems to require primitive or hyperintensional resources to specify. In the first instance, we may ask how to weigh the apparent advantage of specifying the quantifier against the apparent advantage of being able to account for entrenched judgments. In the second instance, it seems the necessitist, in requiring hyperintensional resources, isn't coarse-grained enough to specify quantification with the use of ADJUNCTION.

In the next major section, I discussed how contingentists seem to think of quantification and existence in a way quite different than the Williamsonian necessitist, and may wish to adopt different but equally admissible stories for quantification. Specifying the inventory of things that exists can still be substantive for necessitists; necessitists will have to say that it takes something for something to exist, but not contingently, and that story can vary across different views. Nonetheless, contingentists think of existence in a rather more substantial way: reality has to be some way in

order that "something" can exist and some of those conditions could have failed to obtain. If the inferential profile of the quantifiers determines sufficient conditions on existence, then contingentists may object to any principle that can pin down quantification with this profile if they believe it cannot fully or correctly account for what reality needs to be like for "something" to exist. It might be rather more like grounding or causation: characterizable but not uniquely specifiable with logical vocabulary alone.

I've spoken in a conservative way throughout the paper, accepting the abductive methodology. I wanted to say at the close that I'm cautious about abductive considerations and unsure if they are ever reasons to favor views, especially in the case of metaphysics and logic. I suspect that, ultimately, to really evaluate each view fully is too difficult to be illuminating in many metaphysical cases, especially since—unlike in some scientific or ordinary life cases—we can neither check our hypotheses against future data nor rely on agreed upon starting points.

Chapter 3: Incompleteness of Propositionally Quantified Modal Logics on Variable Domain Structures with Accessibility

Abstract. A variable domain model theory with accessibility is developed to interpret the language of second-order propositional modal logic. It is shown that propositionally quantified modal logics whose modalities are S5 and weaker as characterized by the full class of such models are recursively isomorphic to second-order logic and thus not recursively axiomatizable. The result is then extended to a related class of logics validating a natural higher-order comprehension principle. While the primary objective of this paper is to explore formal systems, many of the model theoretic decisions are informed by contingentist views. Given the attention to higher-order formal arguments against contingentism, the results of this paper might be of some interest to those concerned with that debate.[1]

[1]I'd like to thank Boris Kment, Harvey Lederman, Gideon Rosen, Daniel Vitek and especially Peter Fritz for comments and guidance.

3.1 Introduction

The study of propositional modal logic with the addition of propositional quantifiers goes at least as far back as Lewis, Langford, & Lamprecht's 1932 *Symbolic Logic*. It was developed further by Kripke in a short discussion at the end of his 1959 paper and Fine in his dissertation, results of which were summarized and extended in a publication the next year.[2] These earlier discussions established results on *constant* domain structures with *no accessibility relation*—i.e., structures in which the domain of the propositional quantifiers does not vary from world to world and whose corresponding logic is equivalent to S5. More precisely, Fine (1970) and Kaplan (1970) showed that the set of valid sentences in the language $L_{\exists}$, a propositionally quantified modal language to be defined below, as interpreted over sets of worlds, with $\Box$ ranging over all worlds, and $\exists$ ranging over *all* subsets, is recursively axiomatizable and even decidable.[3,4]

When an *accessibility relation* is added (and when the modality is S4.2 or weaker), the corresponding logic turns out to be recursively isomorphic to full second-order logic and hence undecidable (See Kaminsky and Tiomkin (1996)).[5,6] Fritz (2017) shows, among other results, that when *variable domains* are introduced as constrained by model theory he calls 'equivalence systems', the $L_{\exists}$ logic of the class of all equivalence systems is recursively isomorphic to second-order logic.[7] Fritz's discussion assumes S5, since his model-theory is without an accessibility relation. To date, no one has studied the expressive power of an extension of this system that incorporates both an accessibility relation and a domain of quantification that can vary from world to world. The focus of this paper is to do just this, to investigate further the expressive power of propositionally quantified modal logics on structures with both *variable domains* and an *accessibility relation*.

There's a significant jump in complexity when we move from a propositionally quantified modal logic corresponding to a model theory with no accessibility relation and constant domain to a model theory incorporating *either* an accessibility relation or varying domains. A system that incorporates *both* ought to be no less complex than the systems that incorporate either; but—interestingly—neither of the earlier independent results or methods can easily be applied to the integrated case.[8] The

[2] Kripke 1959; Fine 1969, 1970. Around the same time, more discussion of propositional quantifiers in modal logic surfaced in Bull 1969; Kaplan 1970; and, Gabbay 1971.

[3] Fine 1970.

[4] Kaplan 1970.

[5] Kaminski & Tiomkin 1996.

[6] A recursive isomorphism is a bijective recursive function.

[7] When restricting the quantifier to an existential operator for propositional identity, Fritz (2017) shows that his logic S5E is complete with respect to the class of all equivalence systems. The natural extension of the axiomatization by a comprehension principle turns out not to completely axiomatize the logic of coherent equivalence systems where the quantifiers are restricted to propositional identity. See Fritz 2017 for details.

[8] Note, however, that Kaminsky and Tiomkin are not working with the same model theory as Fritz.

purpose of this paper is to establish this result. I begin by extending Fritz's varying domain model theory with the addition of an accessibility relation, which I call 'constellation families'. I then prove that there is a recursive isomorphism between full second-order logic and the propositionally quantified $L_{\exists}$ logic of constellation families whose modalities are S5 and weaker on the full class of frames and on two subclasses of frames called coherent and quasicoherent systems. These systems are of interest because they validate a natural higher-order comprehension principle.

I adopt the general approach of much of the recent literature, which focuses on possible worlds semantics and on expressive power, which typically establishes non-axiomatizability.[9] It's important to note, however, that other recent papers focus on algebraic approaches[10] and broadly metaphysical and epistemological applications.[11]

While this paper's primary aim is to explore formal systems, it is also greatly informed by the metaphysical debate over contingentism and necessitism and the higher-order formal arguments presented in Williamson 2013.[12] In light of this, the results may be of some interest to those concerned with that debate.

3.2 Model theory and semantics

In this section, I lay out the model theory and semantics for *constellation families*, a generalization of Fritz's *equivalence systems* by the addition of an accessibility relation.[13] The latter is discussed first.

Core to Fritz (2016)'s possible worlds model theory is a notion of indistinguishability among worlds: metaphorically speaking, worlds are indistinguishable (relative to a given world) when they "look the same" from the "perspective" of the given world, even if they are in fact distinct and distinguishable from the "perspective" of another world. The model theory is of interest to propositional contingentists—proponents of the view that it is contingent what propositions exist—and has the potential for other philosophical applications.

To make the notion of indistinguishability precise, Fritz's systems, called equivalence systems, associate with each world w an equivalence relation $\sim_w$, relating two worlds just in case they cannot be distinguished at w. Formally,

Definition 1. *An* equivalence system *on a set W is a function $\sim$ mapping every $w \in W$ to an equivalence*

[9] See, e.g. Kremer 1993, 1997, 2018; Ghilardi & Zawadowski 1995; Kaminski, M. & Tiomkin, M. 1996; Antonelli & Thomason 2002; Kuhn 2004; Zach 2004; ten Cate 2006; Kuusisto 2008, 2015; Fritz 2017, 2018.

[10] Holliday 2017; Holliday & Litak 2018; Ding 2018, 2020.

[11] French 2003; Belardinelli & van der Hoek 2015, 2016; Fritz & Lederman 2015; Belardinelli, van Ditmarsch, & van der Hoek 2016; Fritz 2016, 2017, 2018; Fan & Liau 2017; Belardinelli, van der Hoek, & Kuijer 2018; Ding 2020; Bednarczyk & Demri 2019.

[12] Williamson 2013.

[13] See Fritz 2016, 2017.

relation $\sim_w$ on W.

Definition 2. *Given a set W and equivalence system $\sim$, we say that $\mathcal{F} = \langle W, \sim \rangle$ is an* equivalence frame.

I adopt the convention of referring to an equivalence frame by its associated equivalence system $\sim$. Following and refining Stalnaker's (2012, Appendix A) development of models for propositional contingentism, Fritz begins with the assumptions that propositions are identified with sets of worlds and that for each world the propositions there are form a complete atomic Boolean algebra. Equivalence systems may then be used for the purposes of modeling contingency in what propositions there are by taking the propositions at a world w of set W to be generated by the relation of indistinguishability: for any proposition p and set $s \subseteq W$ indistinguishable from a world w, either $s \subseteq p$ or $s \cap p = \emptyset$. That is, each equivalence system $\sim$ on W generates a domain function $D(w) : W \to \mathcal{P}(\mathcal{P}(W))$ for each world $w \in W$ satisfying the following: Proposition $p \in D(w)$ iff p is closed under $\sim_w$.

Fritz then defines truth and validity relative to an equivalence frame and assignment function $a : \Phi \to \mathcal{P}(W)$ in the standard way, except for the clause for the propositional quantifier, which is defined relative to the propositional domain function. It is a feature of this set-up that a sentence letter p can be true at w relative to a model even if $a(p)$ is not in $D(w)$ relative to the model. p is true at w iff w is a member of $a(p)$. But w can be a member of a proposition that does not exist at w. i.e., a proposition that is not closed under indistinguishability relative to w. So it is a feature of the set-up that $\phi(p)$ does not entail $\exists p \phi p$. There are failures of existential generalization. His main result is that the logic of the full class and of interesting subclasses of equivalence systems are recursively isomorphic to second-order logic.

Fritz's frames validate axiom 5 ($\Diamond \phi \to \Box \Diamond \phi$) since $\Box$ ranges over all worlds; that is, his semantics takes $\Box p$ to be true at a world in a model when p is true at all worlds in the model. Needless to say, systems that validate 5 aren't the only interesting systems to consider. Of course, it is of independent formal interest to explore a broader class of frames validating modal axioms weaker than 5, to consider the extension to systems with nontrivial accessibility relations. But in addition to this independent interest, logics whose modalities are S5 might not be the right logics for the philosophical views Fritz's system has and could be used to model. Metaphysical necessity might correspond to a logic with a modality weaker than S5.[14] Moreover, there are independent

[14]This might be for essentialist reasons or for reasons having to do with the broadest necessity. See Chandler 1976 and Salmon 1989 for familiar essentialist based arguments that metaphysical modality is weaker than S5. More recently, see Teitel 2019 for an argument that certain contingentists and essentialist assumptions are in tension with axioms B and 4 and Ditter 2020 that there is some reason to believe the logic of metaphysical necessity is exactly S4. The lattermost argument, however, is framed in the context of a logic that validates necessitism. Bacon 2018 argues that the logic of the broadest necessity, under

propositional contingentist-inspired reasons to adopt a logic with a modality that doesn't validate one or some of axioms 5, 4, or B.[15] The recent formal work on these logics, then, is limited for the purposes of contingentists who reject axioms 5, 4, or B for the logic of contingentism. By no means exhaustively, these considerations motivate the study of the following extension of Fritz's system.

In Fritz's equivalence systems, necessity is understood simply as truth in all worlds. In our extension—constellation families—necessity is understood relative to a world w as truth in all worlds accessible from w. We begin in the usual way:

Definition 3. *A* frame *is a tuple* $\langle W, R\rangle$ *where* W *is a set and* $R \subseteq W \times W$*, (analogously,* $R : W \to \mathcal{P}(W)$*).*

W is understood as the set of worlds and R as a relation of accessibility between worlds. For a set W and worlds $u, v \in W$, I will use the notation Ruv to denote that v is accessible from u, and the notation $R(w)$ to denote the set of worlds accessible from w.[16]

We follow Fritz in exploring models that include, in addition to a set of worlds, W, a function $\approx$ from W to $W \times W$ designed to capture the notion of indistinguishability among worlds. Like Fritz, we suppose that $\approx_w$ is always an equivalence relation. Unlike Fritz, because our models include accessibility relations, we impose a further constraint on $\approx$, viz., that the worlds inaccessible from a given world are indistinguishable relative to w.[17] So in general, if a world can see two worlds, then it "makes a claim" to the distinguishability of two worlds (whether they're indistinguishable or not). I assume the converse holds for distinguishability but not for indistinguishability. Thus:

Definition 4. *A* constellation family *on a frame* $\langle W, R\rangle$ *is a function* $\approx$ *mapping every* $w \in W$ *to an*

a specific interpretation of *broadest*, is no weaker than S4, and offers a series of philosophical arguments that it doesn't validate the S5 principle. He also presents some compelling arguments that metaphysical necessity is not the broadest necessity, and that it could still correspond to a logic corresponding to S5. The latter arguments can be resisted without resisting the former arguments; that is, one could conclude that metaphysical necessity just is the broadest necessity and corresponds to a logic weaker than S5.

[15]The arguments are presented for contingentism about individuals, but can be transposed to contingentism about propositions. For instance, one might think that Socrates' existence is only possible at a world when he has the property of possibly existing. If serious actualism is true and individuals can't have properties without existing, then Socrates can't have the property of possibly existing at worlds where he doesn't exist. So even if it's true that Socrates exists (and possibly exists), it isn't necessarily possible that Socrates exists. This thought motivated a rejection of B and 5. For more thorough treatments of these issues, see Adams 1981 and Kment 2014.

[16]As is usual, I use the metaphorical language of 'worlds' for elements of W and 'a world w can see' and 'worlds accessible from w' for the members of $R(w)$.

[17]This choice is a simplification for the formalism. It does, however, bring up interesting questions about its motivation. For if we consider Chandler/Salmon-type reasons for thinking the 4 principle may fail, then it seems that two worlds may be inaccessible and yet still distinguishable. E.g. It might be impossible that Woody is made out of material b and impossible that Woody is made out of material c, and yet we still may be able to distinguish materials b and c and hence distinguish these impossible situations. Furthermore, if we consider applications to awareness and knowledge, I may know that a given proposition fails to obtain, but still be aware of distinctions between ways for the propositions to hold. E.g. I may know that a ball is not red, while still being aware of the distinction between it being maroon and being crimson. So long as we maintain all other choice points in the paper, I believe there will be no problem changing the requirement of the indistinguishability of inaccessibles. However, it's hard to see how that is motivated without also changing the restriction, below, that propositional domains are generated only by accessible worlds. In the case where we change both, we run into some preliminary problems defining the sentence we need in the object language to establish our complexity result, and so will need to be part of future work for the time being. Thanks to Peter Fritz and Boris Kment for pushing me on these points.

equivalence relation $\approx_w$ *on* W *subject to the following constraint: for* $u, v, w \in W$*, if* $u, v \notin R(w)$*, then* $u \approx_w v$*.*

It will also be useful to have notation available for the restriction of $\approx_w$ for a given $w \in W$ only relating worlds accessible from w:

Definition 5. *A* minimal constellation family *on a frame* $\langle W, R \rangle$ *is a function* $\approx^*$ *mapping every* $w \in W$ *to an equivalence relation* $\approx^*_w = \approx_w |_{R(w) \times R(w)}$.

Definition 6. *Given a frame* $\langle W, R \rangle$ *and constellation family* $\approx$*, we say that* $\mathcal{F}^c = \langle W, R, \approx \rangle$ *is a* constellation frame.

I will adopt the convention of referring to a constellation frame by its associated constellation family $\approx$. We can represent them, generalizing from Fritz (2016), as in the following diagram:[18]

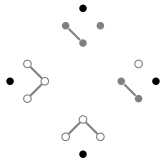

Figure 1

A constellation family $\approx$ based on n worlds is drawn using n *constellations* of n dots arranged in a circle, with lines connecting dots of the ith constellation according to $\approx_i$, representing indistinguishability.[19] We adopt the convention of arranging the dots clockwise with 1 on top. The dot representing i in the ith constellation is distinguished by being black instead of gray. So the ith constellation represents a world i that is represented in the same position at each constellation in the constellation family. Open circles represent the worlds inaccessible from the "home" world of a constellation. A line is always drawn for vacuous indistinguishability between them. Consider the example in Figure 1 above. World 2 cannot see world 1 and so the second constellation has an open circle where world 1 is represented. World 3 cannot see any other worlds other than itself, and so all worlds other than 3 in the third constellation are open circles and there is a line between them.

We assume, following Fritz, that propositions are sets of worlds and that for each world the propositional domain at that world forms a complete atomic Boolean algebra. Because of this, the propositional domain of a given world can be defined in terms of its associated equivalence relation.

[18] The original tikz code used to produce the following diagram was developed by Peter Fritz. I'd like to thank him for sharing this with me. I've modified the code for the diagrams with open circles to represent inaccessibility.

[19] The benefit of using the terminology of 'constellation families,' 'constellations,' and 'worlds' as opposed to just 'worlds' is that we have a clear way of referring to the group of worlds corresponding to one world's "perspective" by its constellation.

For Fritz, the propositions that exist at w just are the members of the algebra of propositions generated by the equivalence classes of $\sim_w$, where those equivalence classes are the atoms. That is, for every equivalence relation $\sim$ on a set W, we let $\mathcal{A}(\sim) = \{\bigcup X : X \subseteq W/\sim\}$, where A/B is the quotient set, the set of equivalence classes of the equivalence relation B on set A. So for every equivalence system $\sim$ on W, the domain function of $\sim$ is the function $D^{\sim}_w : W \to \mathcal{P}(\mathcal{P}(W))$ such that for all $w \in W$, $D^{\sim}_w = \mathcal{A}(\sim_w)$.[20] That is, the domain $D(w)$ of a given world $w \in W$ of an equivalence system $\langle W, \approx\rangle$ is the set of subsets of W that are closed under $\approx_w$.

In our system, we make a simple change: the domain $D(w)$ of a given world $w \in W$ of a constellation frame $\langle W, R, \approx\rangle$ is the set of subsets of W that are closed under $\approx^*_w$, limiting the elements of w's propositional domain to be composed only of worlds that are accessible from w.[21] And we note that for all $p \subseteq R(w)$, $p \in \mathcal{A}(\approx^*_w)$ just in case for all $u, v \in R(w)$, $u \approx^*_w v$ only if $u \in p$ iff $v \in p$. More precisely, the domain of constellation families is defined as follows:

Definition 7. *For every constellation family $\approx$ of a frame $\langle W, R\rangle$, the* domain function *of $\approx$ is the function $D^{\approx}_w : W \to \mathcal{P}(\mathcal{P}(W))$ such that for all $w \in W$, $D^{\approx}_w = \mathcal{A}(\approx^*_w)$.*

For ease, we use the notation $D(w)$ for the domain of each $w \in W$ on a pre-specified constellation family $\approx$. So for the following,

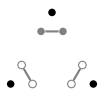

Figure 2

the set of propositions at 1 is, $\{\{\}, \{1\}, \{2,3\}, \{1,2,3\}\}$, while the set of propositions at 2 is, $\{\{\}, \{2\}\}$.

To define a semantics, assume the following formal language:

Definition 8. *Let $L_\exists$ be the set of formulas built from $(,)$, the Boolean operators $\neg$ and $\wedge$, an existential quantifier $\exists$ binding proposition letters, a modal operator $\Box$, and a countably infinite set of proposition letters Φ. Other Boolean operators, $\Diamond$, and $\forall$ will be used as abbreviations as usual. The set of formulas is defined in the usual way.*

[20] Fritz 2017, 204.

[21] This choice to restrict the propositional domains to accessible worlds may also be some cause for alarm. For one, we may think: if the accessibility relation is not transitive, then the set of accessible worlds is a proposition whose negation doesn't exist. Later in the proof, we will define a predicate E, which may be read as existence, in terms of necessary equivalence rather than identity. Necessary equivalence doesn't license substitutivity, and so isn't sufficient for identity. This has downstream results for the comprehension principle defined below, where propositions are not closed under negation, but something weaker. As I was thinking about it in the early stages of writing this paper, propositional identity is contingent. We don't in this system with accessibility have the resources we naturally would for a stronger notion of propositional identity. To speak metaphorically, what a world considers to be Top is the necessary proposition, which may, as a matter of contingency, just be a proposition which another world views as contingent. The choice to redefine propositional domains in terms of the accessible and inaccessible worlds seems doable, although it will have to remain an option for another paper. I'd like to thank Peter Fritz for discussions.

From the definition of domain, we have a straightforward way of interpreting $L_\exists$ on constellation families:

Definition 9. *An $L_\exists$−formula being* true *relative to a constellation family $\approx$ on a set W, a world w, and an assignment function $a : \Phi \to \mathcal{P}(W)$ is defined by the following clauses:*

$$
\begin{aligned}
&\approx, w, a \vDash p && \text{iff } w \in a(p)\\
&\approx, w, a \vDash \neg\phi && \text{iff not } \approx, w, a \vDash \phi\\
&\approx, w, a \vDash \phi \wedge \psi && \text{iff } \approx, w, a \vDash \phi \text{ and } \approx, w, a \vDash \psi\\
&\approx, w, a \vDash \Box\phi && \text{iff } \approx, v, a \vDash \phi \text{ for all } v \in R(w)\\
&\approx, w, a \vDash \exists p\phi && \text{iff there is a } P \in D^{\approx}_{w} \subseteq \mathcal{P}(R(w)) \text{ such that } \approx, w, a[P/p] \vDash \phi
\end{aligned}
$$

where $a[P/p]$ is the assignment function which maps p to P and every other proposition letter q to $a(q)$.

As usual, $\approx, w \vDash \phi$ indicates ϕ is true relative to all assignment functions (i.e. $\approx, w \vDash \phi$ if $\approx, w, a \vDash \phi$ for all assignment functions a) and analogously, $\approx, a \vDash \phi$ when ϕ is true relative to all worlds, and $\approx \vDash \phi$ when ϕ is true relative to all worlds and assignment functions.

Note that existential generalization isn't valid: a may assign p a proposition that contains w but which does not exist at w. That is, p is true at w relative to a, but doesn't exist.

Definition 10 (Validity)**.** *An $L_\exists$-formula ϕ is* valid *on a constellation family $\approx$ (a class C of constellation families) if $\approx \vDash \phi$ (for every constellation family $\approx$ in C). An $L_\exists$−formula ϕ is* satisfiable *on a constellation family (a class of constellation families) if its negation is not valid on it. The $L_\exists$-logic of a class of constellation families is the set of $L_\exists$-formulas valid on it.*

Definition 11. *$L_\exists C$=$\{\phi : \phi$ is valid on all constellation families$\}$*

We pronounce '$L_\exists C$' as 'the $L_\exists$ logic of constellation families'.

3.3 Incompleteness

In this section, I prove that there is a recursive embedding of full second-order logic into the logic of constellation families whose modalities are S5 and weaker—whose frames include, as a subclass, frames with an accessibility relation that is transitive, reflexive, and symmetric. In the next two sections, I show a correspondence between broadly two subclasses of constellation families that validate a comprehension principle, called coherent and quasicoherent systems, and prove

incompleteness on logics of quasicoherent and coherent systems S5 and weaker. The technique is shown to fail to extend to some coherent and quasicoherent systems that don't include 5-frames, specifically the one whose modality is S4.1.

The results rely on the following two definitions and theorem from recursion theory. The definitions are given for sets, A and B, of natural numbers, but they can be taken to apply to sets of formulas of languages whose syntax can be recursively arithmetized.

Definition 12. *A is* 1-reducible *to B ($A \leq_1 B$) if, for some 1-1 recursive function f, $x \in A$ iff $f(x) \in B$. (Post 1944; see Odifreddi 1989, p. 324)*

Note that $\leq_1$ is reflexive and transitive.

Definition 13. *A is* recursively isomorphic *to B if, for some 1-1 onto recursive function, f, $x \in A$ iff $f(x) \in B$. (Post 1944; see Odifreddi 1989, p. 324)*

Theorem 3 (Isomorphism Theorem)**.** *A is recursively isomorphic to B iff $A \leq_1 B$ and $B \leq_1 A$. (Myhill 1955; see Odifreddi 1989, p. 325)*

My technique will be to embed monadic second-order logic with a pairing function into the full S5 logic of constellation families. This is enough to get the incompleteness results, as monadic second-order logic with a pairing function is recursively isomorphic to full second-order logic. I will now state this more precisely:

Definition 14. *Let P be a binary relation on a non-empty set S. Suppose that for every $x, y \in S$ there is a $z \neq x, y \in S$ such that for all $w \in S$, $\langle w, z\rangle \in P$ just in case $w = x$ or $w = y$. Such a P is a* pairing relation*. (For convenience, define a subset of P that only includes the special pairings: $P_p = \{\langle x, z\rangle, \langle y, z\rangle \in P$: for all $w \in S$, $\langle w, z\rangle \in P$ just in case $w = x$ or $w = y\}$.)*

Definition 15. *Let LM be a language based on a countably infinite set of first-order variables $x, y, z, ...$, a countably infinite set of monadic second-order variables $X, Y, Z, ...$, and a binary relation Q, whose formulas are constructed from atomic formulas of the form Xy and Qxy using Boolean operators $\neg$ and $\wedge$ and first- and second-order quantifiers of the form $\exists x$ and $\exists X$, respectively.*

Define truth *of such a formula relative to a set D, pairing relation P on D, and an assignment function a mapping each first-order variable to a member of D and each monadic second-order variable X to a subset of D, using the following clauses:*

$D, P, a \vDash Xy$ *iff* $a(y) \in a(X)$

$D, P, a \vDash Qxy$ *iff* $\langle a(x), a(y)\rangle \in P$

$D, P, a \vDash \exists x A$ *iff there is a* $d \in D$ *such that* $D, P, a[d/x] \vDash A$

$D, P, a \vDash \exists X A$ *iff there is a monadic relation* $F \subseteq D$ *such that* $D, P, a[F/X] \vDash A$

and the usual Boolean clauses, with $a[d/x]$ *and* $a[F/X]$ *defined as in Definition 9.*

Dropping one or both of the parameters P and a in $D, P, a \vDash \phi$ *indicates that* ϕ *is true relative to all pairing functions or all assignment functions; e.g. an LM-formula is* true *in a set D, written* $D \vDash A$*, if* $D, P, a \vDash A$ *for all assignment functions a and pairing relations P. Let SM be the set of LM-sentences (closed formulas) true in all sets.*

Although SM is in several ways more restrictive than full second-order logic as it is usually defined, these differences can also be ignored for present purposes, because of the following fact established by Gurevich 1985:

Fact 1. *There is a recursive embedding of full second-order logic in SM.*[22]

Constructing a recursive embedding of SM in the $L_{\exists}$-logic of constellation families thus suffices to establish that the latter is recursively isomorphic to full second-order logic.

It's worth mentioning that there is a notable difference in the proof strategy adopted in this paper and the strategies employed in Fritz (2017) and Kaminsky and Tiomkin (1996). Core to both Fritz and Kaminsky & Tiomkin's proofs is employment of a technique of referring in the object language to a constant set of atomic propositions on the whole domain of worlds. This is not possible in the current system. Fritz works with a varying domain but no accessibility; Kaminsky & Tiomkin work with accessibility but a constant domain. These features enable both Fritz and Kaminsky & Tiomkin to refer to a single stock of propositions: Kaminsky & Tiomkin through the propositional quantifiers since the domain of propositions is constant (all worlds have the same stock of propositions), Fritz through the modal operators since propositional domains are defined by worlds and all worlds see all worlds (all worlds have the same stock of worlds, so to speak). One suggestive way to think about it: while Kaminsky & Tiomkin work with a constant domain of propositions, Fritz works with constant domain of worlds. So for Fritz: even though each world might differ in what propositions *exist*—the quantifiers range over different sets of propositions relative to each world—the way the modal operator is defined allows Fritz to nonetheless reference the whole system of worlds, which

[22]The definition of pairing used by Gurevich is different than the one used here. Gurevich defines pairing as a ternary predicate P on a non-empty set S such that for every $x, y \in S$ there is $z \in S$, with $\langle x, y, z\rangle \in P$, and for every $z \in S$ there is at most one pair $\langle x, y\rangle$ with $\langle x, y, z\rangle \in P$. Three notable differences: (i) the definition used in this paper, and not Gurevich's, is consistent with there being a $z \in S$ that is P-related to more than two things, (ii) Gurevich's relation is ordered and the one in this paper is not, and (iii) Gurevich's and not the definition used in this paper is consistent with there being cases in which an element pairs itself (and possibly also something else). These do not matter as the proof can be easily altered to go through with the fact that arbitrary binary relations can be coded by symmetric irreflexive binary ones. See Nerode and Shore, 1980, p.184.

in turn define the propositions, and so act in a way that stabilizes the propositional domain within the object language.

In the system of this paper, for which worlds can differ both in what propositions there are and what worlds there are (or are accessible from), neither the quantifiers nor the modal operators can be used to single out a constant stock of propositions across the whole domain of worlds. This is the unique challenge of the system we work with here and requires a somewhat different approach to the ones used in previous papers.

The basic idea, then, for this paper is to work with the propositions which are atomic at a given world. The construction can be spelled out as follows: In a particular world w, say that an atomic proposition P *pairs* atomic propositions Q and T (distinct from P) just in case for every atomic proposition S, P entails S is an atom just in case $S = Q$ or $S = T$. Assume that for any false atomic propositions Q and T, there is a false atomic proposition P which pairs them. With this, we can simulate monadic second-order logic with a pairing relation as follows: singular quantification is translated as quantification over false atomic propositions, monadic second-order quantification is translated as quantification over false propositions, pairing is given by the modal definition of *pairs*, and application is modal entailment. The rest of this section makes this way of simulating monadic second-order logic with a pairing relation precise and shows how to turn it into a recursive embedding of SM in the $L_{\exists}$ logic of constellation families.

For readability, we will introduce the following two abbreviations for the atoms at w and the false atoms at w:

Definition 16. *Let $\approx$ be a constellation family on a set W with accessibility relation R and assignment function a. For a world $w \in W$:*

$$AT_w := W/\approx^*_w$$
$$ATF_w := W/\approx^*_w - [w]_w$$

Where $[x]_y$ is the equivalence class x is a member of for y in AT_y defined by $\approx_y$. The following simulates pairing in the setting of constellation families:

Definition 17. *Let $\approx$ be a constellation family on a set W with accessibility relation R, $w \in W$, and assignment function a:*

$P \in AT_w$ pairs_w $Q, T \neq P \in AT_w$ *just in case for all $S \in AT_w$ and $v \in P$: $S \in AT_v$ iff $S = Q$ or $S = T$.*

For convenience, I will say that $P \in AT_w$ pairs_w $Q \in AT_w$ just in case there is a $T \in AT_w$ such that P $pairs_w$ Q and T.

I will often say $\ulcorner P\ pairs_w\ Q \urcorner$ instead of $\ulcorner P \in AT_w\ pairs_w\ Q \in AT_w \urcorner$, and so on, for readability. Note that pairs_w tells us that the only atoms of w in $v's$ atomic domain are P and Q. Of course, other propositions not included in w's atomic domain could exist at v. Another way of thinking about it is that the only core distinctions that w and v share are those corresponding to P and Q.

The simulation relies on there being a world w such that for any of its false atomic propositions Q and T there is a false atomic proposition P that pairs_w them. A constellation family satisfying this constraint will be called $pairing_w$:

Definition 18. *Let $\approx$ be a constellation family on a set W with accessibility relation R and let a be an assignment function. $\approx$ is* pairing_w *(for $w \in W$) iff the following condition is satisfied:*

For every $Q, T \in ATF_w \neq \emptyset$, there is a $P \neq Q, T \in ATF_w$ that $pairs_w$ them.

We will introduce a number of syntactic abbreviations:

$$\phi = \psi := \Box(\phi \leftrightarrow \psi)$$

$$E\phi := \exists r(r = \phi)$$

$$atom(\phi) := E\phi \wedge \Diamond \phi \wedge \forall r(\Box(\phi \rightarrow r) \vee \Box(\phi \rightarrow \neg r))$$

It's worth noting that this use of $=$ can fail Leibniz's Law. That is, it may be that $\phi = \psi$ but $\Box\Box\phi$ and $\neg\Box\Box\psi$.

Lemma 4. *Let $\approx$ be a constellation family on a set W, R an accessibility relation, and a an assignment function. Then:*

(i) $\approx, w, a \vDash \phi = \psi$ *iff* $\{v \in R(w) : \approx, v, a \vDash \phi\} = \{v \in R(w) : \approx, v, a \vDash \psi\}$

(ii) $\approx, w, a \vDash E\phi$ *iff* $\{v \in R(w) : \approx, v, a \vDash \phi\} \in D(w)$

(iii) $\approx, w, a \vDash atom(\phi)$ *iff* $\{v \in R(w) : \approx, v, a \vDash \phi\} \in AT_w$

Proof.

(i) $\approx, w, a \vDash \phi = \psi$ *iff* $\approx, w, a \vDash \Box(\phi \leftrightarrow \psi)$

iff for all $v \in R(w), \approx, v, a \vDash \phi \leftrightarrow \psi$

iff for all $v \in R(w), \approx, v, a \vDash \phi$ *iff* $\approx, v, a \vDash \psi$

iff for all $v \in R(w), v \in a(\phi)$ *iff* $v \in a(\psi)$

iff $\{v \in R(w) : \approx, v, a \vDash \phi\} = \{v \in R(w) : \approx, v, a \vDash \psi\}$

(ii) $\approx, w, a \vDash E\phi$ *iff* $\approx, w, a \vDash \exists r(r = \phi)$

iff there is a $p \in D(w)$ *s.t* $\approx, w, a[p/r] \vDash r = \phi$

iff there is a $p \in D(w)$ *s.t* $\{v \in R(w) : \approx, v, a[p/r] \vDash r\} =$

$\{v \in R(w) : \approx, v, a \vDash \phi\}$

iff $\{v \in R(w) : \approx, v, a \vDash \phi\} \in D(w)$

$(iii \Rightarrow)$ *Assume* $\approx, w, a \vDash atom(\phi)$

then $\approx, w, a \vDash E\phi \wedge \Diamond \phi \wedge \forall r(\Box(\phi \rightarrow r) \vee \Box(\phi \rightarrow \neg r))$

Since $\approx, w, a \vDash E\phi$, *there is a* $p = \{v \in R(w) : \approx, v, a \vDash \phi\} \in D(w)$.

Note that $p \subseteq a(\phi)$.

Since $p \in D(w)$, *there is some* $X \subseteq AT_w$ *s.t* $p = \cup X$

We know that $\cup X \neq \emptyset$, *since* $\approx, w, a \vDash \Diamond \phi$

Since $\approx, w, a \vDash \forall r(\Box(\phi \rightarrow r) \vee \Box(\phi \rightarrow \neg r))$, *all* $r \in D(w)$

are s.t $a(\phi) \subseteq r$ *or* $a(\phi) \subseteq \neg r$

Since $X \neq \emptyset$, *pick arbitrary* $q \in X$ *(note* $q \in AT_w$*)*

Then, $a(\phi) \subseteq q$ *or* $a(\phi) \subseteq \neg q$

But $q \subseteq a(\phi)$ *since* $q \subseteq \cup X = p \subseteq a(\phi)$

And $a(\phi) \not\subseteq \neg q$ *or else* $q \subseteq \neg q$ *which is impossible since* $q \in AT_w$

and $\emptyset \notin AT_w$

Hence, $a(\phi) \subseteq q$

From this and since $q \subseteq a(\phi), a(\phi) = q$

And so, $a(\phi) = \{v \in R(w) : \approx, v, a \vDash \phi\} \in AT_w$, *as desired.*

$(iii \Leftarrow)$ *Assume* $p = \{v \in R(w) : \approx, v, a \vDash \phi\} \in AT_w$

First, $p \in D(w)$ *since* $p \in AT_w \subseteq D(w)$ *and so* $\approx, w, a \vDash E\phi$

Second, there is a $v \in R(w)$ *s.t* $v \in p$ *since* $\emptyset \notin AT_w$ *and*

$\cup AT_w = R(w)$, *and so* $\approx, w, a \vDash \Diamond \phi$

Finally, consider arbitrary $q \in D(w)$. *So for some* $X \subseteq AT_w$,

$q = \cup X$. *Since* $p \in AT_w$, *either* $p \in X$ *or* $p \notin X$.

If $p \in X$, *then* $p \subseteq q$. *If* $p \notin X$, *then* $p \subseteq R(w) - q$.

Hence, either $p \subseteq q$ *or* $p \subseteq \neg q$.

That is, either for all $v \in R(w), \approx, v, a \vDash p \rightarrow q$

or for all $v \in R(w)$ $\approx, v, a \vDash p \rightarrow \neg q$.

So $\approx, w, a \vDash \Box(p \rightarrow q) \vee \Box p \rightarrow \neg q$

Since q arbitrary, $\approx, w, a \vDash \forall r(\Box(p \rightarrow r) \vee \Box(p \rightarrow \neg r))$

Together, $\approx, w, a \vDash E\phi \wedge \Diamond \phi \wedge \forall r(\Box(\phi \rightarrow r) \vee \Box(\phi \rightarrow \neg r))$

That is $\approx, w, a \vDash atom(\phi)$.

□

The simulation of LM in $L_\exists$ can now be introduced formally as a function mapping each $LM-$formula A to an $L_\exists-$formula A^*. To do so, fix an injective function which maps each variable ζ of LM (first- or second-order) to a proposition letter p_ζ of $L_\exists$. Define such a function using the following recursive clauses:

$$(Xy)^* := \Box(p_y \rightarrow p_X)$$
$$(Qxy)^* := \Box(p_y \rightarrow atom(p_x))$$
$$(\neg A)^* := \neg A^*$$
$$(A \wedge B)^* := A^* \wedge B^*$$
$$(\exists x A)^* := \exists p(atom(p) \wedge \neg p \wedge A^*)$$
$$(\exists X A)^* := \exists p A^*$$

In Qxy, y is the one that pairs x if it pairs.

In order to show that this has the intended effect, it suffices to consider a correspondence between

assignment functions for LM to assignment functions for $L_\exists$ which map p_x, for a first-order variable x, to an atom of a world w, and p_X, to a union of atoms of w (a proposition of w), as follows:

Definition 19. *Assignment function a^* for LM and assignment function $a : \Phi \to \mathcal{P}(W)$ for $L_\exists$ are* coding$_w$ *if the following holds:*

(i) for all variables x of LM, $a^*(x) \in D$ iff $a(p_x) \in AT_w$, and
(ii) for all variables X of LM, $a^*(X) \subseteq D$ iff $a(p_X) \in D(w)$.[23]

In particular, when a^* and a are coding$_w$, note $a^*(x) \in a^*(X)$ iff $a(p_x) \subseteq a(p_X)$. It's also crucial for the validity of the proof to understand that given a and a^* are coding$_w$, for all $v \in a(p_x)$ or $v \in a(p_X)$, $v \in R(w)$. This is clear because the above definition ensures p_x and p_X always get mapped to elements of AT_w and $D(w)$, respectively, and $\cup AT_w = \cup D(w) = R(w)$.

We will also need a correspondence between P and $\approx$ when P is a pairing relation and $\approx$ pairing$_w$:

Definition 20. *Pairing relation P of a model of SM and $\approx$ of a constellation family that is pairing$_w$* correspond$_w$ *just in case $\langle a^*(x), a^*(y)\rangle \in P$ iff for all $v \in a(p_y)$, $a(p_x) \in AT_v$ for assignment functions a for $L_\exists$ and a^* for LM. In particular, $\langle a^*(x), a^*(y)\rangle \in P_p$ iff $a(p_y)$ pairs$_w$ $a(p_x)$.*

The next lemma shows that the mapping $\cdot^*$ functions as intended:

Lemma 5. *Let $\approx$ be a constellation family on a set W, R an accessibility relation on W, and a an assignment function for $L_\exists$ such that $\approx$ is pairing$_w$ for some world w. If $D = ATF_w$, P a pairing relation on D such that P and $\approx$ correspond$_w$, and a^* an assignment function for LM such that a and a^* are coding$_w$, then for every LM-formula A,*

$$D, P, a^* \vDash A \text{ iff } \approx, w, a \vDash A^*$$

[23] The left hand side of the biconditional statements are always satisfied.

Proof. By induction on the complexity of A.

$D, P, a^* \vDash Xy$ *iff* $a^*(y) \in a^*(X)$ *(by semantics)*

iff $a(p_y) \subseteq a(p_X)$ *(by definition of coding$_w$)*

iff for all $v \in R(w), v \in a(p_y)$ *only if* $v \in a(p_X)$ *(by semantics and coding$_w$)*

iff for all $v \in R(w), \approx, v, a \vDash p_y \to p_X$ *(by semantics)*

iff $\approx, w, a \vDash \Box(p_y \to p_X)$ *(by semantics)*

iff $\approx, w, a \vDash (Xy)^*$ *(by definition of $\cdot^*$)*

$D, P, a^* \vDash Qxy$ *iff* $\langle a^*(x), a^*(y)\rangle \in P$ *(by semantics)*

iff for all $v \in R(w), v \in a(p_y)$ *only if* $a(p_x) \in AT_v$ *(by Definition 20 and coding$_w$)*

iff for all $v \in R(w), \approx, v, a \vDash (p_y \to atom(p_x))$ *(by semantics and Lemma 2 and coding$_w$)*

iff $\approx, w, a \vDash \Box(p_y \to atom(p_x))$ *(by semantics)*

iff $\approx, w, a \vDash (Qxy)^*$ *(by definition $\cdot^*$)*

$D, P, a^* \vDash \neg A$ *iff not* $D, P, a^* \vDash A$ *(by semantics)*

iff not $\approx, w, a \vDash A^*$ *(by IH)*

iff $\approx, w, a \vDash \neg A^*$ *(by semantics)*

iff $\approx, w, a \vDash (\neg A)^*$ *(by definition of $\cdot^*$)*

$D, P, a^* \vDash A \wedge B$ *iff* $D, P, a^* \vDash A$ *and* $D, P, a^* \vDash B$ *(by semantics)*

iff $\approx, w, a \vDash A^*$ *and* $\approx, w, a \vDash B^*$ *(by IH)*

iff $\approx, w, a \vDash A^* \wedge B^*$ *(by semantics)*

iff $\approx, w, a \vDash (A \wedge B)^*$ *(by definition of $\cdot^*$)*

$D, P, a^* \vDash \exists x A$ *iff there is a* $d \in D$ *s.t* $D, P, a^*[d/x] \vDash A$ *(by semantics)*

iff there is a $d \in ATF_w$ *s.t* $D, P, (a[d/p_x])^* \vDash A$ *(by definition of $\cdot^*$,*

coding$_w$, and assumption that $D = ATF_w$*)*

iff there is a $d \in AT_w$ *and* $d \notin [w]_w$ *s.t* $\approx, w, a[d/p_x] \vDash A^*$ *(by IH)*

iff there is a $d \in D(w)$ *s.t* $\approx, w, a[d/p_x] \vDash atom(p_x) \wedge \neg p \wedge A^*$ *(by Lemma 2)*

iff $\approx, w, a \vDash \exists p(atom(p) \wedge \neg p \wedge A^*)$ *(by semantics)*

iff $\approx, w, a \vDash (\exists x A)^*$ *(by definition of $\cdot^*$)*

$D, P, a^* \vDash \exists X A$ *iff there is a monadic relation* $F \subseteq D$ *s.t* $D, P, a^*[F/X] \vDash A$ *(by semantics)*

iff there is a $\cup F \in D(w)$ *s.t* $D, P, (a[\cup F/p_X])^* \vDash A$ *(by definition of* $\cdot^*$ *and coding*$_w$*)*

iff there is a $\cup F \in D(w)$ *s.t* $\approx, w, a[\cup F/p_X] \vDash A^*$ *(by IH)*

iff $\approx, w, a \vDash \exists p A^*$ *(by semantics)*

iff $\approx, w, a \vDash (\exists X A)^*$ *(by definition* $\cdot^*$*)*

□

The next task is to formulate the model-theoretic condition of being pairing$_w$ in $L_\exists$.

Lemma 6. *For any constellation family* $\approx$ *on set* W *with* R *accessibility relation, world* $w \in W$*, and assignment function* a*:*

$\approx, w, a \vDash \forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)))$ *iff* $\approx$ *is pairing*$_w$*.*

Note that the quantifier $\exists^{FA}$ restricts to the false atoms of w, and similarly for $\forall^{FA}$ (recall how atom(ϕ) is defined above). Also note that because we are quantifying only over atoms (never mind false atoms), we can conclude from $\approx, n, a[S/s] \vDash atom(s)$ that $S \in AT_n$, for some $n \in W$. For similar reasons, we can conclude $S = Q$ from $\approx, n, a[Q/q, S/s] \vDash s = q$.

Proof. Assume $\approx, w, a \vDash \forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)))$. This is true iff for all $P, Q \in ATF_w$ there is an $T \neq P, Q \in ATF_w$ such that for all $S \in ATF_w$, $\approx, w, a[P/p, Q/q, T/t, S/s] \vDash \Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q))$, by semantics. By semantics again, this is true iff for all $P, Q \in ATF_w$ there is a $T \neq P, Q \in ATF_w$ such that for all $S \in ATF_w$: for all $v \in R(w)$, $\approx, v, a[P/p, Q/q, T/t, S/s] \vDash t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)$. By semantics and Lemma 2, this is true iff for all $P, Q \in ATF_w$ there is a $T \neq P, Q \in ATF_w$ such that for all $S \in ATF_w$: for all $v \in R(w)$, if $v \in T$, then $S \in AT_v$ iff $S = Q$ or $S = P$.[24] By Definition 17, this is true iff for all $P, Q \in ATF_w$ there is a $T \neq P, Q \in ATF_w$ that pairs$_w$ them. By Definition 18, $\approx$ is pairing$_w$. □

We now prove that there is a recursive embedding of SM into the $L_\exists$ logic of constellation families whose modalities are S5 and weaker.

Lemma 7. *There is an embedding of* SM *into* $L_\exists$ *logics of constellation families S5 and weaker. The mapping* $\tau : \phi \mapsto (\forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q))) \rightarrow \phi^*)$ *embeds* SM *into* $L_\exists C$*.*

[24] Note that 'for all $v \in R(w)$, if $v \in T$' is redundant.

Proof. We show that τ embeds SM into $L_\exists$ logics of constellation families whose modalities are S5 and weaker, in the sense that $\phi \in SM$ iff $\tau(\phi) \in L_\exists C$.

We start with the left-to-right direction: If $\tau(\phi) \notin L_\exists C$, then there is a constellation family $\approx$ on a set W with accessibility relation R, a world $w \in W$, and assignment function a that functions well (i.e. for all variables x and X of LM, $a(p_x) \in AT_w$ and $a(p_X) \in D(w)$),[25] such that $\approx, w, a \vDash \forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)))$ and $\approx, w, a \nvDash \phi^*$. By the former and Lemma 4, $\approx$ is pairing$_w$. Let $D = ATF_w$. Define a^* such that (i) for any first-order variable x of LM, a^* maps x to an element of D iff a maps propositional variable p_x to an atom of w and (ii) for any second-order variable X of LM, a^* maps X to a monadic relation on D iff a maps propositional variable p_X to a union of atoms of w.[26] It follows from this that (i) for all variables x of LM, $a^*(x) \in D$ iff $a(p_x) \in AT_w$, and (ii) for all variables X of LM, $a^*(X) \subseteq D$ iff $a(p_X) \in D(w)$. By Definition 19, a and a^* are coding$_w$. Let P be such that $\langle a^*(x), a^*(y)\rangle \in P$ iff for all $v \in a(p_y)$, $a(p_x) \in AT_v$. By Definition 20, P and $\approx$ correspond$_w$.

Claim 1: P is a pairing relation on D. Consider arbitrary $a^*(x), a^*(y) \in D = ATF_w$. Since a and a^* are coding$_w$ and $D = ATF_w$, $a^*(x) = a(p_x) \in ATF_w$ and $a^*(y) = a(p_y) \in ATF_w$. $\approx$ is pairing$_w$, and so there is an $a(p_z)$ such that it pairs$_w$ $a(p_x)$ and $a(p_y)$. By construction of P, $\langle a^*(x), a^*(z)\rangle \in P$ and $\langle a^*(y), a^*(z)\rangle \in P$. For uniqueness of the pair, consider $\langle a^*(v), a^*(z)\rangle \in P$ for some $a^*(v) \in D$. By construction of P, $a(p_z)$ pairs$_w$ $a(p_v)$. Since $\approx$ is pairing$_w$ and $a(p_z)$ pairs$_w$ $a(p_x)$ and $a(p_y)$, $a(p_v) = a(p_x)$ or $a(p_v) = a(p_y)$. And so, $a^*(v) = a^*(x)$ or $a^*(v) = a^*(y)$.

Since $D = ATF_w$, $\approx$ is pairing$_w$, a and a^* are coding$_w$, P and $\approx$ correspond$_w$, and P is a pairing relation, by Lemma 3, $D, P, a^* \nvDash \phi$. That is, $\phi \notin SM$, as desired.

We now establish the right-to-left direction: If $\phi \notin SM$, then there is some set D, pairing relation

[25] Such an assignment function can be chosen given the definition of $\cdot^*$. It is simple to prove that given the mapping $\tau : \phi \mapsto (\forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q))) \rightarrow \phi^*)$: For all $\tau(\phi) \notin L_\exists C$, there is a constellation family $\approx$ on a set W with accessibility relation R, $w \in W$, and assignment function a such that $\approx, w, a \nvDash \tau(\phi)$, and for all variables x and X of LM, $a(p_x) \in AT_w$ and $a(p_X) \in D(w)$. For all ϕ, there is a constellation family $\approx$, world $w \in W$ with accessibility relation R on W, and assignment function a such that $\approx, w, a \vDash \forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)))$ but $\approx, w, a \nvDash \phi^*$. By the former and by Lemma 4, $\approx$ is pairing$_w$. The inductive clauses are immediate. For the first two atomic clauses: Roughly, for the first clause where $\phi = Xy$, there must be a $b \in W$ such that $b \in R(w)$ and $\approx, b, a \nvDash p_y \rightarrow p_X$. Define a such that $b \in a(p_y) \in AT_w$ and $b \notin a(p_X) \in D(w)$. For the second clause where $\phi = Qxy$, there must be a $b \in W$ such that $b \in R(w)$ and $\approx, b, a \nvDash p_y \rightarrow atom(p_x)$. Define a such that $b \in a(p_y) \in AT_w$ but $a(p_x) \in AT_w$ and $a(p_x) \notin AT_b$ (that is, b does not pair a).

[26] Note that it doesn't matter if a maps some propositional variable to something outside of ATF_w. In defining a^*, we only care about the restricted part of a that does map to ATF_w.

P, and assignment function a^* such that $D, P, a^* \nvDash \phi$. First, define a new D^*, P^*, and a^{**} in the following way:

Define D^* as follows: $x \in D$ iff $\{x\} \in D^*$. Define P^* such that $\langle x, y\rangle \in P$ iff $\langle\{x\}, \{y\}\rangle \in P^*$ and a^{**} such that $\{a^*(x)\} = a^{**}(x)$ and $a^*(y) \in a^*(X)$ iff $a^{**}(y) \in a^{**}(X)$. (D and D^* are clearly isomorphic. Consider the function that takes x to its singleton. Similarly, P and P^* are isomorphic.) By definition, P^* is pairing on D^*.

By induction on the complexity of ϕ, it's clear that $D^*, P^*, a^{**} \vDash \phi$ iff $D, P, a^* \vDash \phi$. All cases are straightforward, consider:

$D^*, P^*, a^{**} \vDash Qxy$ *iff* $\langle a^{**}(x), a^{**}(y)\rangle \in P^*$ *(by semantics)*

iff $\langle a^*(x), a^*(y)\rangle \in P$ *(by construction of* P^**)*

iff $D, P, a^* \vDash Qxy$ *(by semantics)*

By the contrapositive of the left-to-right direction and by assumption, $D^*, P^*, a^{**} \nvDash \phi$. Next, we define $\approx, w$, and a as follows:

Let $W = D \cup \{w\}$, for $w \notin D$. Define $AT_w = D^* \cup \{\{w\}\}$, which in turn determines the relation $\approx_w$. (Note: $ATF_w = D^*$.) Next, we ensure that $\approx$ is pairing$_w$. Define AT_b for each $b \neq w \in W$, which determines the $\approx_b$'s, as follows:

For $\langle a, b\rangle \in P$, let $\{a\} \in AT_b$ and $\{x \in W : \langle x, b\rangle \notin P\} \in AT_b$.

Finally, define a such that (i) for all variables x of LM, $a^{**}(x) \in D$ iff $a(p_x) \in AT_w$, and (ii) for all variables X of LM, $a^{**} \subseteq D$ iff $a(p_X) \in D(w)$. By Definition 19, a and a^{**} are coding$_w$.

Note that R is universal by construction. We now must show that $\approx$ is pairing$_w$ and that P^* and $\approx$ correspond$_w$, as we've already constructed the system such that $D^* = ATF_w$, P^* is a pairing relation, and a and a^{**} are coding$_w$.

Claim 2: $\approx$ is pairing$_w$. Take arbitrary $\{a\}, \{b\} \in ATF_w$. Since P^* is a pairing relation on D^* (and since $ATF_w = D^* \neq \emptyset$ and a and a^{**} coding$_w$): there is a $\{v\} \neq \{a\}, \{b\} \in D^*$ such that for all $\{u\} \in D^*$, $\langle\{u\}, \{v\}\rangle \in P^*$ iff $\{u\} = \{a\}$ or $\{u\} = \{b\}$. By construction of the AT_v's (and

since $ATF_w = D^* \neq \emptyset$ and a and a^{**} coding$_w$): there is a $\{v\} \neq \{a\}, \{b\} \in ATF_w$ such that for all $\{u\} \in ATF_w$, $\{u\} \in AT_v$ iff $\{u\} = \{a\}$ or $\{u\} = \{b\}$. From this and since all elements of AT_w are singletons, there is a $\{v\} \neq \{a\}, \{b\} \in ATF_w$ such that for all $\{u\} \in ATF_w$ and $x \in \{v\}$, $\{u\} \in AT_x$ iff $\{u\} = \{a\}$ or $\{u\} = \{b\}$. That is, there is a $\{v\} \in ATF_w$ that pairs$_w$ $\{a\}$ and $\{b\}$. We can generalize since $\{a\}$ and $\{b\}$ were arbitrary.

Claim 3: P^* and $\approx$ correspond$_w$. Suppose $\langle \{a\}, \{b\} \rangle \in P^*$. Then, $\{a\} \in AT_b$ (by construction of the AT_v's, definition of domain, and since a and a^{**} are coding$_w$). Since $\approx$ is pairing$_w$ and since the elements of AT_w are singletons, for all $v \in \{b\}$, $\{a\} \in AT_v$. Suppose, for the other direction, that for all $v \in \{b\}$, $\{a\} \in AT_v$. Since $b \in \{b\}$: $\{a\} \in AT_b$. By this and since $ATF_w = D^*$ and a and a^* are coding$_w$, $\langle \{a\}, \{b\} \rangle \in P^*$.

Hence, by Lemma 3, $\approx, w, a \nVdash \phi^*$. By Claim 2 ($\approx$ is pairing$_w$) and Lemma 4, $\approx, w, a \vDash \forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)))$. And so, $\tau(\phi) \notin L_\exists C$, as desired. □

Theorem 8. *There is a recursive embedding of second-order predicate logic into the $L_\exists$ logic of all constellation families whose modalities are S5 and weaker.*

Proof. $\cdot^*$ is evidently recursive. The result follows directly from this, Fact 1, and Lemma 5. □

The following corollaries follow in turn from this:

Corollary 1. *The $L_\exists$ logic of all constellation families whose modalities are S5 and weaker is recursively isomorphic to second order logic.*

See Kremer 1993 for details.

Corollary 2. *The $L_\exists$ logic of all constellation families whose modalities are S5 and weaker is not recursively axiomatizable.*

3.4 Coherence and incompleteness

In the last section, we established a recursive isomorphism between the logics corresponding to the full class of constellation families whose modalities are S5 and weaker. We now look at a restriction on the full class, called coherent constellation families, which validate a comprehension principle, and show that the logics corresponding to coherent constellation families whose modalities are S5 and weaker are recursively isomorphic to second order logic. It is also shown that the technique

cannot be applied to the coherent logic whose modality is S4.1 to establish incompleteness. We begin with a definition of coherence, which requires the following:

Definition 21. *A permutation f on W in a frame $\langle W, R, \approx \rangle$ is an* automorphism *of $\approx$ just in case*

- *for all $w, u, v \in W$, $v \approx_w u$ iff $f(v) \approx_{f(w)} f(u)$, and*
- *for all $w, u \in W$, Rwu iff $Rf(w)f(u)$.*

For $w \in W$, let $aut(\approx)$ be the set of automorphisms of $\approx$ and $aut(\approx)_w$ the subset of members of $aut(\approx)$ mapping w to itself (the first forms a group, and the second is sometimes called the stabilizer subgroup of w).

Definition 22. *A constellation family $\approx$ on a frame $\langle W, R, \approx \rangle$* coheres *if for all $w, v, u \in W$ such that $v \approx_w^* u$, there is an $f \in aut(\approx)_w$ such that $f(v) = u$ and $f \subseteq \approx_w$.*

One motivation for studying coherence is its relation to a comprehension principle, and it turns out that this principle is valid on coherent constellation families.[27] The existing propositions are closed under Boolean operations, guaranteed in all constellation families by the fact that propositional domains form (complete atomic) Boolean algebras. As Fritz notes, coherence is a structural criterion that not only guarantees that propositions are closed under Boolean operations as above, but also that propositions are closed under definability using arbitrary $L_\exists-$formulas. In $L_\exists$, we can express this as the requirement of validating the following comprehension schema, $C_\exists$ below, where ϕ is any $L_\exists-$formula and $pl(\phi)$ is the set of proposition letters free in ϕ.

Definition 23. *The following is valid on coherent constellation families.*

$C_\exists$: $\vdash (\bigwedge_{p \in pl(\phi)} Ep) \rightarrow E\phi$

The proposition denoted by ϕ exists if the propositions denoted by all the variables free in ϕ exist. As noted, the comprehension principle is not valid on the class of all constellation families, consider the following example from Fritz:

Figure 14

[27] This is of interest, for instance, to higher-order contingentists in search of contingentist comprehension principles. See Williamson 2013.

Notice that $\approx, 1 \vDash \neg E \forall p \Box Ep$ and 1 relates 1 and 2 but 2 doesn't. World 1 has the propositions $\{\}$ and $\{1,2\}$, whereas world 2 has the propositions $\{\}, \{1\}, \{2\}$, and $\{1,2\}$. $\forall p \Box Ep$ is the proposition $\{1\}$ because it's true of 1 and not 2, but because 1 relates itself to 2, both worlds need to appear in all the same sets in 1's domain. Hence, $\{1\} \notin D(1)$. The strategy Fritz employed in this example was to make the system incoherent by not respecting the simplest condition of coherence, that a world sees itself as distinct, and then constructed a case where not having $\{1\}$, the proposition a world has if it sees itself as distinct from all other worlds, created a problem for a more complex sentence. Of course, there are no propositional constants in $\forall p \Box Ep$ and $\{\} \in D(1)$ so we get the failure of comprehension that we were aiming for.

We will explore the relationship between comprehension and various modal systems. To prove comprehension is valid on all coherent constellation families, we start by showing that truth in constellation families is invariant under automorphisms. Above, we extended our permutation f on the set of worlds W to a permutation on propositions $\mathcal{P}(W)$, letting $f.P = \{f(w) : w \in P\}$ for all $P \subseteq W$. Now, we extend the permutations on the set of worlds W to a permutation on assignments a. For any automorphism f of a constellation family $\approx$ and assignment function a, we let $f.a$ be the assignment function mapping each $p \in \Phi$ to $f.(a(p))$.

Theorem 9. *Let $\langle W, R, \approx \rangle$ be a constellation family, $w \in W$, and f an automorphism of $\approx$.*

(i) For any $P \subseteq W$, $P \in D(w)$ iff $f.P \in D(f(w))$

(ii) For any assignment function a and $\phi \in L_{\exists}$, $\approx, w, a \vDash \phi$ iff $\approx, f(w), f.a \vDash \phi$.

Proof. (i) $P \notin D(w)$ just in case $\exists v, u \in R(w)$ such that $v \in P$, $u \notin P$, and $v \approx^*_w u$, by definition of $D(w)$. $v \in P$ iff $f(v) \in f.P$; $u \notin P$ iff $f(u) \notin f.P$; $v \approx^*_w u$ iff $f(v) \approx^*_{f(w)} f(u)$ by f being an automorphism. So then, $P \notin D(w)$ iff there are $v, u \in R(w)$ such that $f(v) \in f.P$, $f(u) \notin f.P$, and $f(v) \approx^*_{f(w)} f(u)$. Since f is a permutation, this is the case iff $\exists v, u \in W$ such that $v \in f.P$, $u \notin f.P$, and $v \approx^*_{f(w)} u$, i.e. $f.P \notin D(f(w))$, as desired.

(ii) By induction on the structure of ϕ. Only the cases of $\exists$ and $\Box$ are of interest.

$\approx, w, a \vDash \exists p\phi$ iff $\approx, w, a[P/p] \vDash \phi$ for some $P \in D(w)$(by semantics)

iff $\approx, f(w), f.(a[P/p]) \vDash \phi$ for some $P \in D(w)$(inductive hypothesis)

iff $\approx, f(w), (f.a)[f.P/p] \vDash \phi$ for some $P \subseteq W$ such that $f.P \in D(f(w))$(by (i))

iff $\approx, f(w), f.a \vDash \exists p\phi$(by semantics)

$\approx, w, a \vDash \Box\phi$ iff $\approx, v, a \vDash \phi$ for all $v \in R(w)$(by semantics)

iff $\approx, f(v), f.a \vDash \phi$ for all $v \in R(w)$(IH)

iff $\approx, f(v), f.a \vDash \phi$ for all $f(v) \in R(f(w))$(by second clause in aut)

iff $\approx, f(w), f.a \vDash \Box\phi$(by semantics)

□

Definition 24. *The following are axiom schemas or rules of the logic of $L_\exists$ of normal K modal constellation families*

PL: Any substitution instance of a propositional theorem.

K: $\vdash \Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

Nec: $\vdash \phi$ *entails* $\vdash \Box\phi$

MP: $\vdash \phi$ *and* $\vdash \phi \to \psi$ *entail* $\vdash \psi$

US: $\vdash \phi$ *entails any uniform substitution instance of* ϕ

Gen: $\vdash \phi \to \psi$ *entails* $\vdash \phi \to \forall p\psi$*, where p does not occur free in* ϕ

UIE: $\vdash (\forall p\phi \wedge EP) \to \phi[P/p]$*, where P is free for* ϕ

$\Box E$*:* $\vdash \Box(\phi \leftrightarrow \psi) \to (E\phi \leftrightarrow E\psi)$

$C\top$*:* $\vdash E\top$

$C\neg$*:* $\vdash E\phi \to E\neg\phi$

$C\wedge$*:* $\vdash (E\phi \wedge E\psi) \to E(\phi \wedge \psi)$

Where $pl(\phi)$ is the set of proposition letters free in ϕ.

Theorem 10 (Soundness)**.** *If a formula is derivable from the logic defined above, then it is valid on all constellation families.*

Proof. As usual by induction on the complexity of the formulas. □

Corollary 3 (Soundness of comprehension)**.** *If a formula is derivable from the logic defined above plus $C_\exists$, then it is valid on all coherent constellation families.*

Proof. The proof follows as a corollary to Theorem 8 above and Theorem 11 below. □

3.4.1 Incompleteness

The following establishes that logics corresponding to coherent constellation families whose modalities are S5 and weaker are recursively isomorphic to full second-order logic.

Lemma 11. *There is an embedding of SM into the $L_\exists$ logic of coherent constellation families with a modality S5 or weaker ($L_\exists Co$). The mapping $\tau : \phi \mapsto (\forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q))) \rightarrow \phi^*)$ embeds SM into the $L_\exists Co$*

Proof. From Lemma 5, since if $\phi \in SM$ then $\tau(\phi) \in L_\exists C$, in particular $\tau(\phi) \in L_\exists Co$. For the other direction: If $\phi \notin SM$, then there is some set D, pairing relation P, and assignment function a^* such that $D, P, a^* \nVdash \phi$.

Define D^* as follows:

$$\text{For all } x \in D\text{: } \{x_1, x_2\} = [x]_w \in D^*\text{, such that } x_1 = \{x\}\text{, and } x_2 = \{\{x\}\}.^{28}$$

Define P^* as follows: $\langle a, x\rangle \in P$ iff $\langle [a]_w, [x]_w\rangle \in P^*$. Define a^{**} such that $[a^*(x)]_w = a^{**}(x)$ and $a^*(y) \in a^*(X)$ iff $a^{**}(y) \in a^{**}(X)$. It's clear that D and D^* are isomorphic, similarly for P and P^*, and that P^* is pairing on D^*. By induction on the complexity of ϕ, it's clear that $D^*, P^*, a^{**} \vDash \psi$ iff $D, P, a^* \vDash \psi$. By this and by assumption $D^*, P^*, a^{**} \nVdash \phi$.
Next, we define $\approx, w, a$ as follows. Let $W = \bigcup D^* \cup \{w\}$, where $w \notin D$. Let $D^* \cup \{\{w\}\} = AT_w$. (Note: $D^* = ATF_w$.)

Define AT_{y_1}, AT_{y_2} for all $y \in D$, which determines $\approx_{y_1}, \approx_{y_2}$, as follows:

For $x, y \in D$ such that $\langle x, y\rangle \in P$: $[x]_w \in AT_{y_1}, AT_{y_2}$.
For $x, y \in D$ such that $\langle x, y\rangle \notin P$: $\{x_1\}, \{x_2\} \in AT_{y_1}, AT_{y_2}$.
And that is all.

Note that R is universal by construction. Finally, define a such that for all first- or second-order variables ζ of LM, $a^{**}(\zeta) = a(p_\zeta)$. It clearly follows that a is such that (i) for all variables x of LM,

[28] All that is required is $x_1 \neq x_2$ and for all $d \neq x \in D$, $x_1 \neq d_1, d_2$ and $x_2 \neq d_1, d_2$.

$a^{**}(x) \in D$ iff $a(p_x) \in AT_w$ and (ii) for all variables X of LM, $a^{**}(X) \subseteq D$ iff $a(p_X) \in D(w)$. For all variables p of $L_\exists$ that don't correspond to a variable x or X of LM, let $a(p) \in \mathcal{P}(W) - AT_w$. By Definition 19, a and a^{**} are coding$_w$.

We need to show that $\approx$ is coherent, pairing$_w$, and that P and $\approx$ correspond$_w$, since we know that $D^* = ATF_w$, P^* is pairing, and a and a^{**} are coding$_w$.

Coherence: Without loss of generality, suppose $v \approx_n^* u$. By construction, either $v = u$ or $v, u \in [v]_w$. For the former, let f be the identity function. For the latter, let $f(v) = u, f(u) = v$ and for all $d \neq u, v \in W$, $f(d) = d$. All we have to do is establish that f is an automorphism. Since R is universal: trivially, for all $x, y \in W$, Rxy iff $Rf(x)f(y)$. To show for all $x, y, z \in W$, $x \approx_z y$ iff $f(x) \approx_{f(z)} f(y)$, consider first (without loss of generality) $z = v$. By construction, for all $n \in W$, $n \in [n]_v$ iff $n \in [n]_u$. It follows that $x \approx_z y$ iff $f(x) \approx_{f(z)} f(y)$. Consider second that $z \neq u, v$. Assume $x \approx_z y$. We only need to check the case of $x = u$ (without loss of generality). Then $y = u$ or $y = v$. Clearly:

$$u \approx_z u \xrightarrow{f, f^{-1}} v \approx_z v \text{ and } u \approx_z v \xrightarrow{f, f^{-1}} v \approx_z u.$$

Assume $x \not\approx_z y$. We only need to check the case of $x = u$ (without loss of generality). Then $y \neq u, v$.

$$u \not\approx_z y \xrightarrow{f, f^{-1}} v \not\approx_z y.$$

It follows that $x \approx_z y$ iff $f(x) \approx_{f(z)} f(y)$. This establishes the f is an automorphism. The other conditions follow immediately from construction of f. Hence, $\approx$ is coherent.

Claim 4: $\approx$ is pairing$_w$. Consider arbitrary $[a]_w, [b]_w \in ATF_w$. Since P^* is a pairing relation on D^* (and since $ATF_w = D^* \neq \emptyset$ and a and a^{**} are coding$_w$), there is an $[v]_w \neq [a]_w, [b]_w \in D^*$ such that for all $[u]_w \in D^*$, $\langle [u]_w, [v]_w \rangle \in P^*$ iff $[u]_w = [a]_w$ or $[v]_w = [b]_w$. By construction of the $\approx_x$'s (and since $ATF_w = D^* \neq \emptyset$ and a and a^{**} are coding$_w$), there is a $[v]_w \neq [a]_w, [b]_w \in ATF_w$ such that for all $[u]_w \in ATF_w$ and $x \in [v]_w$, $[u]_w \in AT_x$ iff $[u]_w = [a]_w$ or $[u]_w = [b]_w$. That is, there is a $[v]_w \in ATF_w$ that pairs$_w$ $[a]_w$ and $[b]_w$. We can generalize since $[a]_w$ and $[b]_w$ were arbitrary.

Claim 5: P^* and $\approx$ correspond$_w$. Suppose $\langle [a]_w, [b]_w \rangle \in P^*$. Then, $[a]_w \in AT_v$ (by construction of the $\approx_x$'s, definition of domain, and since a and a^{**} are coding$_w$). Since $\approx$ is pairing$_w$, then for all $v \in [b]_w$: $[a]_w \in AT_v$. Suppose, for the other direction, that for all $v \in [b]_w$: $[a]_w \in AT_v$. By definition

$[b]_w, [a]_w \in ATF_w$. And since $ATF_w = D^*$ and since a and a^* are coding$_w$, $\langle [a]_w, [b]_w \rangle \in P^*$.

Hence, by Lemma 3, $\approx, w, a \not\Vdash \phi^*$. Since $\approx$ is pairing$_w$, by Lemma 4, $\approx, w, a \vDash \forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \rightarrow (atom(s) \leftrightarrow s = p \vee s = q)))$. And so, $\tau(\phi) \notin L_{\exists} Co$. □

The proof of Lemma 9 alone suffices to prove the original Lemma 5. The lemmas are kept distinct because of the added complexity of coherence to the construction and because it makes clear where the important step in the proof is for proving non-axiomatizability on different classes of frames.

Modality of S4.1

The proof technique doesn't work for coherent frames for some systems that don't include S5 frames, e.g. whose modality is S4.1.[29] Consider the following:

Proof. If the pairing function is surjective, then for every $x \in D$ there is some $y \in D$ such that $\langle x, y \rangle \in P$. Given that we've defined pairing to exclude the possibility of anything pairing itself (recall, x, y can only be paired by $z \neq x, y$)—then it will be impossible to construct a corresponding constellation family where w sees a world that only sees itself (that is, we cannot construct a frame for which R satisfies $\forall w \exists u(Rwu \wedge \forall v(Ruv \rightarrow u = v))$). For example:

Consider $\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, ($\mathbb{N} = \{1, 2, ...\}$) for unordered $\{i, j\}$ such that $i \leq j$,

$$\{i, j\} \mapsto \begin{cases} i + \frac{(i+j)(i+j-2)}{4} & \text{if } i + j \text{ is even} \\ i + \frac{(i+j-1)^2}{4} & \text{if } i + j \text{ is odd} \end{cases}$$

Call this function g_1. It's evidently surjective.[30] It isn't yet pairing as defined in Definition 14. There are three problematic cases: $g_1(1,1) = 1, g_1(1,2) = 2$, and $g_1(1,3) = 3$. To avoid this, simply define a function g_2, such that $g_2(i,j) = g_1(i,j)$, for $i \leq j$, except when $i = 1$ and $j = 1, 2, 3$ or when $i = 10$ and $j = 10, 20, 30$. For these cases: $g_2(i,j) = g_1(i * 10, j * 10)$ and $g_2(i * 10, j * 10) = g_1(i,j)$.[31]

For all $i \leq j$: $g_2(i,j) > j$ for all i, j except $\{10, 10\}$, $\{10, 20\}$, and $\{10, 30\}$. In any case, it is clear that there is no x such that $g_2(y,z) = x$ and $x = y$ or $x = z$.

□

[29] Recall that S4.1 is not intermediate between S4 and S5. While it is stronger than S4, it is incomparable to S5.

[30] Let $g_1(k) := \{g_1(i,j) : i + j = k\}$. For all i, j such that $i + j = k$, $g_1(k)$ is a set of consecutive numbers. (Notice that the second term will be the same for any i, j such that $i + j = k$.) Then notice that the minimum of $g_1(k+1)$ is one more than the maximum of $g_1(k)$.

[31] That is, $g_2(1,1) = g_1(10,10) = 100$, $g_2(1,2) = g_1(10,20) = 220$, $g_2(1,3) = g_1(10,30) = 390$, $g_2(10,10) = g_1(1,1) = 1$, $g_2(10,20) = g_1(1,2) = 2$, $g_2(10,30) = g_1(1,3) = 3$.

3.5 Quasicoherence

As it turns out, one does not need all the clauses in the condition of coherence to validate comprehension: w does not need to map to itself.

Definition 25. *A constellation family $\approx$ on a frame $\langle W, R, \approx\rangle$* quasicoheres *if for all $w, v, u \in W$ such that $v \approx^*_w u$, there is an $f \in aut(\approx)$ such that $f(v) = u$ and $f \subseteq \approx_w$.*

To prove that quasicoherence suffices for the validity of $C_\exists$, the following lemma shows that if a world sees itself as indistinct from another, the worlds accessible from one just are those accessible from the other.

Theorem 12. *If $\approx$ is quasicoherent and $1 \approx_1 2$, then $u \in R(1)$ iff $u \in R(2)$.*

Proof. Assume that $u \in R(1)$ but $u \notin R(2)$ for reductio. Then there is an $f \subseteq \approx_1$ such that $f(1) = 2$. Since $u \notin R(2)$, f is an automorphism, and $f(1) = 2$, there is an $x \in W$ such that $f(x) = u$ and $x \notin R(1)$, But then $f \not\subseteq \approx_1$ since $x \not\approx_1 u$ when $x \notin R(1)$ and $u \in R(1)$, which is absurd.

The other direction follows analogously. □

Theorem 13. *$C_\exists$ is valid on the class of quasicoherent constellation families.*

Proof. Consider any quasicoherent constellation family $\approx$ on a frame $\langle W, R, \approx\rangle$, $w \in W$, and $\phi \in L_\exists$. It suffices to show that for every assignment function $a : \Phi \mapsto D(w)$, $\approx, w, a \vDash E\phi$, since a is the function that guarantees that all propositional constants are mapped to only members of w's propositional domain and hence each p is such that $\approx, w, a \vDash Ep$. $\approx, w, a \vDash E\phi$ is the case iff for all $u, v \in R(w)$, if $u \approx^*_w v$, then $\approx, v, a \vDash \phi$ iff $\approx, u, a \vDash \phi$. If $u, v \in R(w)$ and $v \approx^*_w u$, then by quasicoherence, there is an $f \in aut(\approx)$ such that $f(v) = u$ and $f \subseteq \approx_w$. Claim: $a = f.a$. It suffices to show that $a(p) = f.(a(p))$. Suppose $z \in f.(a(p))$, then there is an $n \in a(p)$ such that $f(n) = z$. Since f is quasicoherent, $f \subseteq \approx_w$, and so $n \approx_w z$. Since $n \in a(p) \in D(w) \subseteq \mathcal{P}(R(w))$, $n \in R(w)$. The only way they can be indistinguishable and $n \in R(w)$ is if $z \in R(w)$. Hence, $n \approx^*_w z$. Since $a(p) \in D(w)$, $n \approx^*_w z$ only if $(n \in a(p)$ iff $z \in a(p))$. Thus $z \in a(p)$.

For the other direction, assume $z \in a(p)$. (We want to show that $z \in f.(a(p))$). Consider $f(z) \in f.(a(p))$.

Claim: $f.(a(p)) \in D(w)$. Either $f(w) = w$ or not. First: If $f(w) = w$, then clearly $f.(a(p)) \in D(w)$ since $a(p) \in D(w)$ and by Theorem 7.(i) $f.(a(p)) \in D(f(w)) = D(w)$, the lattermost of which follows in this case. Second: if $f(w) \neq w$, assume for reductio that $f.(a(p)) \notin D(w)$. Then there is $u, v \in R(w)$ such that $u \approx^*_w v$ and $u \in f.(a(p))$ but $v \notin f.(a(p))$. Since $u, v \in R(w)$, $u, v \in R(f(w))$

by Theorem 10, which means–along with the fact that $f.(a(p)) \in D(f(w))$ by Theorem 7(i)—that $u \not\approx^*_{f(w)} v$. But if $u \approx^*_w v$ and $u \not\approx^*_{f(w)} v$, then $\approx$ wouldn't be quasicoherent because there is no $x \in W$ such that $f(x) = u$.

$$u \not\approx_{f(w)} v \xrightarrow{f^{-1}} x \not\approx_w u$$

since otherwise $f \not\subseteq \approx_w$.

Since $f.(a(p)) \in D(w), z \approx^*_w f(z)$ only if $(z \in f.(a(p))$ iff $f(z) \in f.(a(p)))$. And so, $z \in f.(a(p))$. By these, $a = f.a$. With Theorem 7(ii), $\approx, v, a \vDash \phi$ iff $\approx, u, a \vDash \phi$ as desired. □

The following proof establishes that quasicoherent constellation families with an accessibility relation of S5 or weaker are recursively isomorphic to full second-order logic.

Lemma 14. *There is an embedding of SM into the $L_\exists$ logic of quasicoherent constellation families ($L_\exists Q$) with accessibility relation corresponding to the logic of S5 and weaker. The mapping $\tau : \phi \mapsto (\forall^{FA} p, q \exists^{FA} t \neq p, q \forall^{FA} s(\Box(t \to (atom(s) \leftrightarrow s = p \vee s = q))) \to \phi^*)$ embeds SM into the $L_\exists Q$*

Proof. Follows directly from Lemma 9. □

3.6 Conclusion

Propositionally quantified modal logics whose modalities are S5 and weaker valid on the full class of constellation families and on coherent and quasicoherent systems are recursively isomorphic to full second order logic. There are then a number of interesting open questions about whether completeness or incompleteness can be established for systems valid on other interesting subclasses of the models. It is also open whether we can establish a result when we use constellation families to interpret a language that drops propositional quantification and adds a propositional operator. Finally, it is open how these systems and models might be used in philosophical application, such as propositional contingentism or awareness.

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