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Essays on the Political Economy of Education Finance

Zhao, Qing

Princeton, NJ : Princeton University

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Essays on the Political Economy of Education Finance

Qing Zhao

A Dissertation
Presented to the Faculty
of Princeton University
in Candidacy for the Degree
of Doctor of Philosophy

Recommended for Acceptance
by the Princeton School of
Public and International Affairs
Adviser: Professor Cecilia Rouse

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Abstract

Local government spending on K-12 public education typically comprises almost half of total spending on K-12 public education in the United States. Local governments typically rely on funds from property tax levies to finance the yearly operating budget for local public schools in their district. These property taxes are often the outcome of a process that is political in nature, since local tax levies may be subject to a voter referendum. This dissertation gives new contributions to the study of these voter referenda and their implications for public finance using both theoretical and empirical approaches.

In an economic downtown, the question of how local funding for schools responds to losses in state aid and disposable income becomes even more pressing. Using an agenda setter model with uncertainty over voter turnout, I develop a systematic approach to analyzing the effects of income and grant aid changes on local property tax levies, overall funding for schools, and the probability of referenda passing. The analysis uses certain properties of indifference curves for normal goods. I also show that budget-maximizing school boards in homogeneous districts choose budget proposals that have high rates of voter support.

The empirical chapter provides an investigation of the outcomes of New Jersey K-12 public school budget referenda. For failed budget proposals, school budget cuts are determined by local governing bodies whose decisions are not specified by law. Using a unique compilation of voting, demographic, and school finance data including school budget proposals and outcomes, this study tests and supports the hypothesis that local governing bodies choose larger reductions for those budget proposals that are larger and more unpopular. Additionally, I analyze the voter support for the referendum and test whether larger proposed tax levies are associated with higher levels of voter opposition. I find that the effect of a larger proposal on the level of voter oppositions depends on the circumstances that surround the larger proposal,

and I propose two theories that explain the different effects. Furthermore, I find that the level of educational attainment of a district is an important predictor for school budget referendum success.

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Contents

Abstract iii

Acknowledgements v

1 Introduction **1**

1.1 The Theoretical Study 3

1.1.1 Heterogeneity and Voter Support 10

1.2 The Empirical Study of Voter Referenda: New Jersey 13

1.3 Further lessons 16

1.3.1 Local Funding in the Pandemic and its Aftermath 18

References **25**

2 The Theory **28**

2.1 Introduction 29

2.2 The Model 34

2.2.1 The District 35

2.2.2 The Voters 37

2.2.3 The School Board under Uncertainty over Turnout 41

2.2.4 The Municipal Body and the Reversion 46

2.3 Changes in Income 48

2.3.1 Support for funding by voters 49

2.3.2 Expected per pupil funding and local contribution 57

2.3.3 Probability of Passing 60
2.3.4 Summary 65
2.4 Changes in Total Aid 66
2.4.1 Case 1 - Fixed local contribution 67
2.4.2 Case 2 - Fixed per pupil funding 74
2.4.3 Summary 79
2.4.4 Comparison of Reversion Schemes 80
2.5 Heterogeneity in Districts 84
2.5.1 Theorems connecting homogeneity to higher probability of passing 86
2.6 Conclusion 93
2.A Appendix 98
2.B Uncertainty over Voter Turnout vs. Preferences 100

References **103**

3 The Empirical Study **106**
3.1 Introduction 108
3.2 The Data 114
3.3 Discussion of the summary statistics 116
3.3.1 Budget proposals and reductions 116
3.3.2 Voter incentives and the school funding formula 119
3.3.3 Voting outcomes for school budget referenda 120
3.3.4 Definitions of Variables 125
3.4 Voter support for school budget referenda 126
3.4.1 Column B: Controlling for Educational Attainment 129
3.4.2 Column C: Controlling for Enrollment 131
3.4.3 Financial Variables 132

3.4.4 Private School Enrollment and the Elderly Population 133
3.4.5 Demographic Variables 134
3.4.6 The Effect of Large Voter Participation 135
3.4.7 Year by Year Analysis and Further Discussion 136
3.5 Municipal reductions to failed budget proposals 137
3.6 Conclusion 145
3.A Figures and Tables 149
3.B Appendix: Further background and statistics 165
3.B.1 Background on Municipalities and Reductions 165
3.B.2 Background on Property Tax Caps in New Jersey 167
3.B.3 Background on State Aid in New Jersey 169
3.B.4 District Budget Variables, Enrollment, and Aid 171
3.B.5 Demographic Variables 173
3.B.6 Diversity indices 176
3.C Theoretical Appendix: The Median Voter Model 178

References **181**

Chapter 1

Introduction

A Study of Local Funding and Voter Support for K-12 Public Education and Further Applications

The United States currently spends several hundred billion dollars per year on K-12 public education, making K-12 public education one of the largest areas of government spending in the country. This spending comes from two primary sources: state level resources, which supply around 47% of funds, and local sources, which supply around 46%[1]. Local governments typically rely on funds from property tax levies to finance the yearly operating budget for local public schools in their district. These property taxes are the outcome of a process that is *political* in nature, since local tax levies are often subject to a voter referendum. This institutional framework requires additional analytical ideas that lie outside the scope of standard economic analysis in order to make predictions regarding economic outcomes and to explain empirical phenomena. The overarching goal of this dissertation is to give new contributions to the study of these voter referenda and their implications for public finance using both theoretical and empirical approaches.

[1] Source: U.S. Census Bureau.

Voter referenda commonly occur in the U.S. when a district's school board requests a large enough increase in local property taxes to help fund local public schools. Specifically, in the vast majority of states, local property tax levies cannot increase beyond a certain state or district-imposed limit on property tax increases, often 1.5-2.5%, without passing a voter referendum in the district to approve the tax increase. The state of New York in particular requires voter referenda to approve any proposed increase in a district's tax levy for schools, regardless of the size. As recently as 2011, the situation in New Jersey was similar. In a typical school funding referendum, participating voters receive information on the ballot about the tax levy proposed by the local school board for funding public schools in the district, and have the option of voting 'Yes' or 'No' to support or oppose the proposed levy. Voter referenda occur either during special school or general elections. Since these referenda play an integral role in determining the funding for K-12 public schools in the U.S., the detailed analysis of the referendum process and its resulting funding levels is a necessary part of understanding the local finance of public schools.

One important aspect of these referenda is that school budget referenda can and do fail in practice. The realistic possibility that school budget proposals fail contributes to the potential instability of local funding for public schools. The failure of a budget proposal can result in the removal or downsizing of educational services. Moreover, whether or not a budget fails, the mere possibility of failure has its own implications. When school boards make their budget proposals, and when voters consider whether to vote 'Yes' or 'No,' they must both consider the possibility that the referendum might fail and what the funding level would be in that case – an important quantity known as the *reversion budget* (Romer & Rosenthal, 1979). Given that referenda are not guaranteed to pass, it is important to study why and when voter referenda for school budgets fail, and what factors influence voter support for school budget referenda overall. In particular, one main contribution that comes from studying this

question is a new theoretical explanation and additional empirical support for the finding that larger districts have lower success rates and hence more volatile local spending.

1.1 The Theoretical Study

The first chapter of this dissertation tackles the theory of voter referenda for school budgets. The goal is to study a model that can be used to answer important policy questions that concern the funding levels for schools and the success rates of budget proposals at the referendum. Some of the central questions that I seek to address are: How do local tax revenues for schools respond to exogenous changes in the district? In particular, if there is a loss of income or state aid in the district, will local expenditures increase to compensate for these losses? To be able to make progress in answering these questions, it is necessary to have a framework that considers the behavior and incentives of the players in the system such as the school board and the voters. With such a model, the aim is then to analyze the effects of policy changes on the proposal chosen by the school board, the resulting expected funding levels for local schools, and the probability that these referenda will pass.

The first task for this theoretical work is therefore to decide upon a model for the interplay between the school board that proposes the budget for schools and the voters who decide whether or not to support the proposal. An important feature that this model should reflect is that voters do not have the option of choosing their most preferred level of spending; rather, the ballot conveys the budget proposed by the school board for the upcoming academic year and voters face only two choices: voting 'Yes' or 'No.' The final outcome depends on whether the proposal earns a large enough share of voter support to pass, which typically requires at least 50% of the votes.

Since voters in school budget referenda do not get to specify their ideally preferred level of expenditure, they rely to an extent on the school board to choose an appropriate budget proposal, or else they receive the reversion budget that is enacted when the proposed budget fails. A key component of the theoretical model is therefore to consider the incentive of the school board in making its proposal; in addition, in cases such as New Jersey where the reversion budget is not dictated by law, the model must also stipulate how the reversion budget is determined.

One classical way of considering the school board's decision is to adopt the *median voter model*. The derivation of this model assumes that there is strong two-sided electoral competition for school board positions as well as perfect voter participation in these elections. In this model, the school board proposes the median of the ideally preferred funding levels of the voters. Assuming that the school board knows with certainty which voters show up to vote and the preferences of these voters, the proposal by the school board always passes and becomes the funding level that is implemented. Thus, in the median voter model without uncertainty, the median voter is the "representative voter" for the district, and his/her desired policy is always the selected policy outcome.

The median voter model, however, may not apply or be the most appropriate model in the case of school budget referenda. In particular, the assumptions for the median voter model are not likely to hold since voter participation in school board elections is typically low, school board competitions are often not two-sided, and the candidates for the school board do not represent the full spectrum of possible funding levels for schools. Furthermore, there is empirical evidence from school budget referenda that is not consistent with the median voter model. For example, one prediction of the median voter model is that budget proposals pass 100% of the time when there is full information and at least half of the time when the school board has some uncertainty over which voters show up or their preferences. However, data,

for example, from from elections in Oregon in the 1970s (Romer & Rosenthal, 1982) and from New Jersey in 2010 (Zhao, 2020a) exhibit years of elections where a sizable majority of local budget proposals failed to pass the referendum. These data would be very unlikely to occur in a median voter setting[2].

In the theoretical components of this thesis, I focus instead on a model in which the school board chooses a proposal that maximizes the expected level of funding for schools while facing uncertainty over which voters turn out to vote. The study of this type of "agenda setter" model of school board behavior started with the pioneering work of Romer and Rosenthal (1978, 1979). In this model, since voters only have a choice of voting 'Yes' or 'No' to the proposal, the school board does not necessarily need to propose the median voter's ideal point in order for the proposal to pass. Instead, if the reversion represents what some voters view as a relatively low level of funding for schools, the school board can use the "threat" of the low reversion to extract additional funding from voters. For school boards with a high taste for education funding, a budget proposal that maximizes expected funding for schools may be the closest they can achieve to their preferred spending level while constrained by the referendum, since proposing their own ideal expenditure level likely may not receive sufficient voter support.

Including school boards that face uncertainty over voter turnout is a key feature in the model, since models that do not incorporate uncertainty lead to the conclusion that all budget proposals would pass in equilibrium. A particular goal of the theoretical component of this thesis is to be relevant to the case of New Jersey referenda, which is the focus of the empirical chapter in this thesis. During the time period of the empirical study, approximately one-third of budget proposals in New Jersey failed to pass the referendum, including more than 58% of proposals in the year 2010. These

[2] For example, 315 of 538 proposed budgets in New Jersey in 2010 failed the referendum. Using a similar calculation as in Romer and Rosenthal (1982), if each proposed budget fails with probability 1/2, the probability of having at least 315 failures is 0.000042.

data show that voter referenda in practice do fail, and demonstrate the importance of having a model that allows for budgets to fail in equilibrium. I therefore consider a model where the school board faces uncertainty over voter turnout, although almost all of the results I obtain also hold if the school board faces uncertainty over voter preferences (Zhao, 2020b, Appendix B). In this model, only one referendum is held in each district and state aid is awarded as a lump sum grant, which is similar to the New Jersey referenda I study in the empirical part of the thesis. The analysis serves as a foundation for considering states with repeated referenda and more complex state aid formulas.

The main contributions of this theory chapter are three-fold. First, I develop an argument based on economics assumptions on preferences that serves as an essential tool for analyzing comparative statics in the model. Second, I use this argument and the model to make predictions about the effects of certain exogenous changes on the expected funding level for schools in a district as well as the probability that referenda will fail. Third, I establish the importance of the role of heterogeneity in induced preferences in determining voter support in the model, and I provide a new explanation for the empirically observed link between voter support and district size.

With a model of a budget-maximizing school board in hand, the first main questions I consider are how local funding for schools will respond to exogenous changes in income or state aid. The answers to these questions have immediate applications to economic policy analysis. For example, what will happen to local funding if a policymaker decides to cut state aid to districts? Also, if a policymaker proposes a tax cut, would the cut indirectly benefit funding for schools by freeing up disposable income for voters? If one could assume the existence of a representative voter for a given district, these questions could generally be solved with standard economic analysis: If there is a tax cut, then assuming educational funding is a normal good, individual demand for education funding would increase when disposable income rises, and

local funding for schools would go up as a result. However, since the median voter model does not necessarily hold, the process of a school board proposal and voter referendum may not result in the will of a representative voter. The challenge then is to develop a method to analyze these policies within the potentially more realistic setting of the agenda setter model.

The first main result of Zhao (2020b) establishes that expected local funding increases in response to an exogenous increase in income under the agenda setter model. This result shows that there is a positive theoretical relationship between school funding and voters' disposable income. The result is the first comparative statics result for the agenda setter model that involves a shifting of the voter's constraint set, and adds to the previous results of Romer and Rosenthal (1978, 1980), who considered exogenous changes in the reversion and, in the context of the flypaper effect, equivalent changes in income and aid in the specific situation where the voter constraint sets overlap. In Romer and Rosenthal (1980), the outcomes of the income and aid changes are compared to each other, which involves comparing two different reversion levels that lie on the same constraint set for the median voter. In Zhao (2020b), in contrast, I study the effect of an income increase compared to the *prior* equilibrium state of the district, which requires new ideas in order to understand the effect of a shifting constraint set for the voters.

To obtain this new result on income changes, I resolve a key issue that is related to the lack of a representative voter. Since voters only have the choice of supporting the proposed budget, which maximizes expected funding, or otherwise supporting the reversion budget if they vote 'No,' the analysis hinges on the way a voter compares a given proposal to the reversion budget. The key argument I develop involves combining the assumption of normal goods with convexity of preferences to establish certain "pinching" and "stretching" properties for pairs of indifference curves, which are necessary for analyzing the voter's decision problem.

In addition to understanding how increases in income affect expected funding, it is also important to analyze what happens to the likelihood that referenda will pass or fail, since establishing the result that expected funding increases does not guarantee that budget proposals will pass. An increase in expected funding could even mean that the proposal becomes much larger and more likely to fail. If referenda become more likely to fail, then local funding could suffer from increased volatility and be less reliable. The second main result of Zhao (2020b) therefore examines how exogenous changes in income impact the probability that the referendum passes, and the expected amount of support the optimal proposal receives. I provide sufficient conditions under which increases in income imply increases in expected voter support and the probability of the referendum passing. These sufficient conditions in part depend on how the income change affects voter heterogeneity in the district. I find that the main effect of an increase in voters' income is increased voter support, while decreases in income tend to decrease voter support and result in a greater likelihood of failed referenda.

The next major task of (Zhao, 2020b) is to analyze the effects of a lump sum decrease in state aid. This analysis has direct applications to policy considerations. When faced with shrinking revenues, especially in times of recession, states often turn to reducing state aid for public schools as one means to reducing government spending. If a policymaker reduces state aid, what is the true impact on education funding? In particular, will local sources of funding increase to compensate at least in part for the loss of aid? On the flipside, a policymaker may increase state aid to certain districts to address a specific need. In this case, to determine the true impact of the policy, it is also important to know whether local funding would change as a result. Would local sources increase to reinforce the gain in state aid revenue, or would the increase in aid indirectly reduce districts' incentives to increase local funding, and therefore help schools by less than the original intent of the policy?

Within the agenda setter framework, I show that expected local funding does indeed increase to compensate for a cut in state aid under relatively general assumptions. This result assumes a condition on the reversion that commonly holds for reversion budgets that are specified by state law, namely that the reversion budget after the cut in state aid corresponds to the same local tax levy whether or not the state aid cut occurs. For example, the reversion may be the previous year's levy (as in Maine), or a multiple of the previous year's levy, which could be the state's cap on property tax increases (as in Massachusetts). This type of reversion functions as a "threat" to voters: If the budget proposal fails, local schools bear the brunt of the loss in aid. Aware of this fact, voters are more willing to support higher funding levels, and the school board's optimal proposal achieves a higher level of expected local funding[3]. I derive this result rigorously using the same "pinching" and "stretching" properties of indifference curves that played a key role in the analysis of an exogenous income change.

The above results show that policies that change state aid to districts may have unintended consequences for school funding that stem from the responses of local funding. A policymaker who increases state aid to districts must anticipate that the real local funding level of the district may fall in response, which might require the policymaker to plan a larger increase in aid than he/she would choose without considering this indirect effect. A policymaker who reduces state aid to a district, in contrast, may be able to expect the blow to school funding to be partially cushioned by increases in local funds.

I also determine how cuts in state aid affect the probability of the referendum passing, and I show that this effect depends in a major way on how a state's reversion budget is structured. For most reversions, where the local tax levy of the reversion does not change in response to changes in aid, the primary effect of a state aid cut is to

[3] Although the local component of expected funding has increased, this new funding amount may not be enough to fully compensate for the loss in aid.

increase voter support for the referendum. However, if the reversion instead ensures that the per pupil funding level corresponding to the reversion budget remains the same despite the change in aid, the main effect of a state aid cut is to decrease voter support for the referendum. The main difference lies in how voters perceive the "threat" of the reversion. In the first case, the reversion is more threatening as the loss in aid falls entirely on schools. However, in the second case, the reversion provides more of a "safety net" that guarantees local schools a certain amount of funding, making the reversion less of a threat. (While this comparison holds for the analysis of a *cut* in state aid, the effects are reversed for an increase in state aid, in which case the target contribution level acts as more of a safety net and the target funding level reversion is more of a threat.) In the empirical study (Zhao, 2020a), I find evidence that, during the time period of the study, the second type of reversion may be a better model for the case of New Jersey, where the reversion is decided by a committee of local governing officials.

1.1.1 Heterogeneity and Voter Support

A key contribution of this thesis is to demonstrate that the level of voter support the agenda setter's proposal receives depends systematically on the degree to which voters in a district are "heterogeneous." Specifically, for the results on changes in income and state aid, heterogeneity in voters' induced preferences over education funding is one of the underlying mechanisms for why referenda become more likely to fail in equilibrium. This heterogeneity can stem from a number of sources, such as differences in induced preferences between parents of public and private school students. The fact that voters in a district disagree on whether or not to support local school budget proposals shows that some degree of voter heterogeneity exists. Having a theoretical framework to explain voter support can help to predict the

circumstances that may lead local funding to be less reliable, and to explain the empirical observations of school budget elections.

The key finding of Zhao (2020b) on heterogeneity is that the degree of voter heterogeneity in a district, as formulated in the model, affects the expected level of support that the school board's optimal proposal receives and the probability of referendum passage. In districts that are more heterogeneous, all else equal, the school board has an incentive to make a proposal with a lower probability of passing and a higher expected level of voter opposition, while in districts that are more homogeneous, the school board has an incentive to choose a less risky proposal (Zhao, 2020b, Theorem 5.3). Furthermore, when the district is close to being completely homogeneous, it is optimal for the school board to choose a proposal whose probability of passing is very close to 1 (Zhao, 2020b, Theorem 5.1)[4]. The intuition for this result is that school boards in homogeneous districts do not gain very much by taking a significant risk with their proposal, since voters in such districts have similar preferences about what funding levels they are willing to support. The low returns of taking a risk make it optimal instead for these school boards to choose proposals that have a high probability of passing[5].

Previously, Romer, Rosenthal, and Munley (1992) studied voter heterogeneity in the context of the median voter model in the absence of uncertainty, where it was shown that greater heterogeneity in voters' induced preferences leads to greater opposition to the median voter proposal. They draw on this result to explain their empirical finding that larger districts have larger proposals and lower rates of referendum success, with the point that larger districts may behave more like budget-maximizers, while school boards in smaller districts may choose proposals that are closer to the

[4] The theorem assumes there is at least one voter whose ideal spending level is greater than the reversion budget.

[5] Although these proposals have a high probability of passing, they may still be relatively large proposals.

will of the median voter. In Zhao (2020a), I similarly find using New Jersey data that larger districts have higher levels of opposition to voter referenda.

In Zhao (2020b), I provide a second explanation for the relationship between district size and referendum success, which is based on the assumption that all school boards choose budget-maximizing proposals. Namely, if all school boards are budget-maximizers facing uncertainty over voter turnout, then in smaller districts, which are expected to be more homogeneous, the school board has an incentive to choose a proposal with a smaller risk of failure. In larger districts, which are expected to be more heterogeneous, the optimal proposal for the school board would be a higher risk proposal, and therefore are usually larger.

The results of Zhao (2020b) on heterogeneity have implications for welfare analysis of the referendum process. One main takeaway is that budget-maximizing school boards in homogeneous districts tend to "emulate" the median voter by choosing proposals with high rates of passage and higher expected levels of voter support. However, their proposal is not actually the median voter proposal, but is instead a different proposal that maximizes the expected budget. If the median voter proposal is considered the politically "ideal" proposal, then the agenda setter's choice of proposal inflates the expected budget for schools[6]. Since this proposal still achieves a large amount of voter support, high levels of voter approval therefore do not imply that the will of the median voter is truly represented, and budget inflation can still occur in these cases.

[6] If there is no uncertainty, the classical agenda setter model predicts that the median voter will receive no surplus relative to the reversion. With uncertainty, the median voter's surplus may be higher or lower, but the expected funding level is still at least as large as the amount the median voter would propose.

1.2 The Empirical Study of Voter Referenda: New Jersey

While important insights can be gained from the theoretical study of the referendum process and voter support in (Zhao, 2020b), certain questions about school budget referenda require an empirical approach. In particular, are the data on budget proposals and voter support for actual voter referenda consistent with the theoretical framework I have studied? Furthermore, the predictions of the theoretical framework for some questions are ambiguous, and an empirical investigation is needed. One such question pertains to the amount of voter support for larger proposals at school budget elections. Is it the case that proposals that are larger receive more opposition, or do larger proposals in fact reflect a higher preference for education funding in the district and receive greater voter support? This question is important because it aims to explain the incentives that are driving certain districts to have higher funding levels for K-12 public schools and more rapid growth in funding.

I investigate these questions and more in the setting of New Jersey school budget referenda during the 2008-09 through 2011-12 budget cycles. During these years, all annual school budget proposals in New Jersey required the passage of a voter referendum, resulting in more than 2000 referenda in the data set. The state of New Jersey is an especially important example in the study of public education finance in the U.S., as it is among the states spending the largest amount on education, both in per pupil and in gross terms[7]. As a result, property taxes in New Jersey are among the highest in the country, and remain a consistent area of concern among New Jersey voters.

While the setting of New Jersey referenda during the years of the study is similar to the model I study in Zhao (2020b), one notable feature that presents a challenge

[7] Source: US Department of Education

to model is the structure of the reversion budget in the event of a failed referendum. In particular, the reversion budgets for school budgets in New Jersey are decided by a group of officials in each municipality known as a municipal body rather than spelled out by state law. Gaining insight into the behavior and decision-making of the municipal body is important because municipal cuts have a significant impact on the growth of funding for K-12 public schools in New Jersey. Their influence is not only due to their ability to reduce failed budget proposals by sizeable amounts, but also due to the indirect threat that the reversion poses for school boards and voters. When school boards choose their budget proposals, and voters cast their vote, they must take into consideration what the reversion budget would be if the budget proposal is defeated.

With these motivations in mind, the empirical study Zhao (2020a) takes on two major tasks. The first task is to analyze the voter support for school budget referenda and to learn about the incentives behind larger budget proposals. In particular, I test whether larger budget proposals are met with larger or smaller amounts of voter support. In addition, I determine whether other variables that theoretically should affect voter support do in fact have a significant effect that is consistent with theory. The second task is to test a theory of the reversion budget in New Jersey, and to gain insight into the behavior of New Jersey municipal bodies. Specifically, I test whether larger and more unpopular proposals are met with larger reductions by the school board. The agenda setter model in Zhao (2020b) leads to the expectation that greater cuts tend to occur for larger budget proposals and for more unpopular proposals; however, it was not known previously if this expectation held empirically for the reversion budgets chosen by the mysterious municipal bodies of New Jersey.

To perform this analysis, I assembled a unique compilation of data from several sources. The data contain both the voting results of the referenda in terms of whether the proposal passed with the number of 'Yes' and 'No' votes and the final budgets, as

well as the original budget proposals and the reductions to the budget proposals that failed. In addition to these quantities, the data also include state and federal aid, and demographic and income characteristics for the districts. It is particularly important to have data on the budget proposal itself, since the size of the proposal is theoretically expected to have a significant effect on the voter support for the referendum. With this combined data, the data set in the empirical chapter improves upon the data in previous studies seeking to study voter support for school budget referenda such as Ehrenberg, Ehrenberg, Smith, and Zhang (2004), which lacks the size of the original budget proposals, and Alexander and Bass (1974); Ehrenberg et al. (2004), which are limited to only having the Pass/Fail result of the elections and the number of 'Yes' and 'No' votes only in a small number of cases. For several results, the observed effect on voter support appears to be more intuitive and consistent with theory than the results of the previous studies of Alexander and Bass (1974); Silverman (2011).

The empirical study Zhao (2020a) thus serves several roles as a complementary paper to Zhao (2020b). One role is that I investigate a question about the incentives behind large budget proposals that cannot be decided by theory alone. Namely, do larger levy proposals incur increased voter opposition, or do larger proposals reflect a district's greater willingness to spend on public education that leads to lower voter opposition? The circumstances that surround the larger proposed levy turn out to be important. A second role is that I test certain refutable predictions of the theory. The theory predicts that factors associated with heterogeneity in voters' induced preferences, which may include the size of the district and the share of K-12 students enrolled in private schools, should be linked to greater voter opposition. In addition, the theory predicts positive relationships between the size of municipal cuts to failed budgets and the size of the proposal or the amount of voter opposition to the proposal. In Zhao (2020a), I find that the data are consistent with these predictions. Finally, the empirical study explores possible theories for describing the reversion in New

Jersey. In particular, I find evidence that the reversion in New Jersey may act as a "safety net" for schools that provides more funding to schools that receive less aid.

1.3 Further lessons

One takeaway point of Zhao (2020a) is that the voter referendum process in New Jersey does have a significant role in constraining the growth of education spending. The year 2010-11, in which the governor declared a state of fiscal emergency, offers compelling evidence. In this year, budget proposals were substantially larger than in other years, and these proposed increases were markedly unpopular, with only 41.1% of proposals passing (Zhao, 2020a, Figure 10). Municipal reductions to the failed budgets amounted to $142 million all together, which corresponds to 24% of the total proposed increase in the tax levy for schools that year.

While the direct impact of municipal reductions on the growth of spending was most profound in 2010-11 in the years of the data set, municipal bodies consistently affected the growth of New Jersey K-12 public school funding during the years 2008-09 through 2011-12 in both observable and unobservable ways. The data show that school boards proposed on average a growth of spending of 3% per year from 2007 to 2011, while the municipal reductions constrained the actual growth in spending to 2.5% per year. What is not observed, however, is the effect that the municipal reductions have on the proposals themselves. If school boards did not have to face the possibility of a failed proposal, the proposed budgets could in principle have been much larger to reflect the school boards' potentially high taste for education spending.

Even so, school boards control the options available to voters, and there is room for school boards to inflate the budget beyond what voters would ideally prefer. Persistent inflation of the budget can become a cause of voter discontentment over the growth of property taxes in their district. A common policy to respond to such voter

discontentment is a property tax cap: a law that prohibits property tax increases above a certain amount that sometimes allows for certain exceptional expense categories to spend beyond the cap. A well-known theory for the emergence of property tax caps is that they are a byproduct of abundant spending in excess of median voter preferences by budget-maximizing school boards[8]. Today, 46 states and the District of Columbia have imposed limits on the growth of property taxes[9].

The findings of Zhao (2020a) indicate that the 2% cap on property tax increases imposed in New Jersey in 2011 had an immediate and substantial impact in curbing spending on education. New Jersey school boards proposed to increase tax levies for schools by $162 million all together in 2011, which is less than half of the total proposed increases in 2008 or 2009, and less than 30% of the total proposed increases of 2010. Proposals were thus overall smaller in 2011, and the 2% cap was binding for many of the school boards' proposals in that year (Zhao, 2020a, Figure 9).

Along with the smaller proposals, the year 2011 experienced a notable increase in voter support that may be partially attributed to the new property tax cap. In 2011, 79.9% of proposals passed, compared to the previous three years where the passage rates were 73.8%, 73.6% and 41.4%. Zhao (2020a, Table 12) shows that proposals in the year 2011 had voter approval that was 2.3 percentage points higher as compared to 2008, controlling for other factors that affect voter support. Furthermore, Table 14 shows that even relatively large proposals in the year 2011 garnered a sizeable amount of support, as the table indicates that the proposed percent increase in the tax levy did not have a statistically significant impact on the percentage of 'No' votes in the referendum outcomes of 2011.

One important policy question is whether the 2% cap leads to a socially optimal level of spending on education. Critics of the cap have argued that many districts in New Jersey are unable to raise sufficient funds to provide a thorough and efficient

[8] See Vigdor (2004) for a survey on theories of property tax cap emergence.
[9] See Paquin (2015).

education for their students. The New Jersey state aid formula specifies a "local fair share" amount that each New Jersey district is expected to raise so that, with the help of state and federal aid, there will be enough funding for schools to meet the standard for an adequate education in New Jersey. In late 2019, the New Jersey Senate proposed legislation to allow some districts to exceed the cap without voter approval in order to levy up to their district's local fair share[10]; however, Gov. Murphy vetoed this proposal. The governor's decision reflects the ongoing concern among New Jersey voters about the state's high property taxes.

Is spending on education in New Jersey too high following the 2% cap? The higher rates of approval of budget proposals in 2011 do not necessarily mean that voters are satisfied with the new spending levels. The approval of a proposal only signifies that a majority of voters prefer the proposal to what they believe would be the outcome of a failed proposal. It may be that a proposal garners a high level of approval but is still significantly larger than the ideal spending levels of voters. Zhao (2020b, Theorem 5.1) underscores this possibility by showing that budget-maximizing school boards in homogeneous districts choose optimal proposals that have a high probability of passing. In particular, high rates of voter approval do not mean that the proposals reflect the preferences of the median voter, and voter discontentment can still be present even if proposals pass by a large margin. If there are indeed budget-maximizing school boards "hiding" in districts with high voter support, then the theory that property tax caps such as the ones in New Jersey arose in the first place as a consequence of agenda setter proposals seems even more plausible.

1.3.1 Local Funding in the Pandemic and its Aftermath

The COVID-19 virus, which first appeared in Wuhan, China before it became widespread across the world, has created a major public health crisis and has led

[10] New Jersey Senate Bill 4289.

to a sharp economic downturn for countries across the globe including the United States. The virus has a mortality rate much higher than the flu, and can spread easily through particles in the air breathed out by infected individuals, as well as through contact with infected surfaces. The virus is all the more dangerous because infected individuals may go several days without experiencing symptoms while still being highly contagious. Consequently, the need to control the transmission of the virus has disrupted virtually every aspect of life and has greatly harmed the economy in the process. The U.S. unemployment rate peaked at 14.7% in April 2020 and remains high at 6.7% as of November 2020[11]. Even among employed individuals, many Americans have experienced lost income, and have had to use money from savings or retirement to pay bills (Minkin, Bennett, & Parker, 2020).

In addition to losses in income in many households, states facing constrained resources[12] due to the pandemic will inevitably make cuts in spending, including spending on K-12 public education, which comprises a large portion of state spending across the U.S. Many states have taken one-time financial actions to avoid cutting education funds[13], but in several states, cuts to education funding are already underway[14]. Meanwhile, the federal government has provided some emergency relief funding through the CARES Act. New York, for example, cut state aid by $1.1 billion, equal to the one-time relief provided by the CARES Act for public schools (McKillip & Sciarra, 2020), and is considering further deep cuts (Beckstead, 2020). The analysis in this thesis can offer some lessons to help inform policymakers who consider such cuts and for school boards who may hope to raise the amount of local funding to maintain their quality of education in the face of state aid cuts.

[11] Source: US Department of Labor Statistics

[12] State revenues have declined by 29% in the second quarter of 2020 compared to the second quarter of 2019. (U.S. Census Bureau, 2020)

[13] See Griffith and Berry (2020).

[14] See `https://edlawcenter.org/research/pandemic-cuts.html`.

One important question for K-12 public school funding in this pandemic is whether local funding will rise to compensate for the cuts to state aid. If not, schools will bear the full impact of the cuts. While the COVID-19 pandemic presents many challenges, there are three particular impacts on districts that are related to the issue of local funding for schools in the context of the theoretical setup in (Zhao, 2020b):

1. For many districts, state aid will decrease in response to the recession.

2. Individual incomes have decreased as a result of the pandemic for many households, especially with drastic increases in unemployment.

3. Furthermore, due to the need for increased sanitation measures including protective equipment, and/or online instruction methods, COVID-19 imposes increases in the costs of education provision (Rice, Miller, Coronado, & Meltzer, 2020).

An important aspect of the COVID-19 pandemic is that, unlike in other recessions, the pandemic has not greatly affected local property values thus far[15].

In view of these multiple effects, the response of local funding may be highly district-dependent. I first analyze the effects of points 1 and 2 together. Holding all else equal, a cut in state aid in a district should lead to some amount of local compensation[16] (Zhao, 2020b, Proposition 4.2). However, if income in the district decreases at the same time, the overall response is ambiguous, since a decrease in income leads to a decrease in expected local tax contributions to schools (Zhao, 2020b, Proposition 3.2). Policymakers should therefore anticipate that local funding levels may respond to cuts in aid (which was the case in the aftermath of the Great Recession of 2008[17]), but increases in local funding levels would be more likely in districts whose

[15] See `https://www.economist.com/finance-and-economics/2020/09/30/why-despite-the-coronavirus-pandemic-house-prices-continue-to-rise`.

[16] This result assumes the reversion tax levy amount does not change depending on aid.

[17] For example, see Chakrabarti, Livingston, and Roy (2014).

residents incur smaller losses in income compared to the size of the state aid cuts. Furthermore, districts that experience larger losses in aid and smaller losses in income may be expected to compensate by larger amounts. In these districts where the effect of the cut in aid dominates, voter referenda should be more likely to occur, and the analysis in Zhao (2020b) can be used to predict how successful the referenda will be, which depends on the structure of the reversion in the state. In most states, the reversion tax levy does not depend on the amount of aid[18], and voter support for local referenda would tend to be relatively high following a loss in aid because voters do not want this loss to fall entirely on local schools (Zhao, 2020b, Theorem 5.2, Corollary 5.2)[19], though losses in income could work against this effect. In contrast, if the reversion acts as a "safety net" that guarantees a certain amount of per pupil funding for schools, the effect of a loss in aid would tend to decrease voter support for the referendum.

If cuts in aid are unavoidable, it may be prudent for states to target the cuts to those districts that are in a better financial position with respect to the pandemic, and to count on these districts to partially compensate through local referenda. If states instead make cuts to districts that have been hit more heavily by the pandemic or those that are taking longer to recover, school funding in these areas could drop substantially since these districts would not be expected to compensate for the losses in aid from the state. While it may be advisable to assign cuts differently across districts, whether or not unequal treatment of districts is politically feasible is a different matter. So far, cuts in aid have not always distinguished between districts; for example, in June 2020, the Georgia legislature enacted a 10% cut in state aid for K-12 public education that was uniform across districts[20]. However, the budget

[18] For example, the reversion budget often corresponds to the state's property tax cap.

[19] The theorem assumes that voter heterogeneity does not substantially increase due to the cut in aid, and also assumes that voters are aware of the impact that the loss in aid has on schools.

[20] See https://gbpi.org/georgia-education-budget-primer-for-state-fiscal-year-2021/

cuts in Ohio, which mostly impacted wealthier areas and preserved funding for high-poverty districts, serve as an example of cuts in state aid due to the pandemic with varying impact on districts (Barnum, 2020).

In addition to the pandemic affecting the aid and local funding for schools, unless current trends can be quickly reversed, the pandemic may also have a political impact in that voter referenda may be less likely to pass in districts that are diverse in terms of income. Studies find that low-income Americans are much more likely to have lost jobs and to have experienced economic hardship than higher income Americans (Minkin et al., 2020). In districts that are diverse in terms of income, the pandemic may therefore have a polarizing effect, in which higher income individuals would remain supportive of school funding referenda, while lower income individuals who have been impacted more by the recession would become less willing to support a given tax levy proposal. With an increased divide between these groups of individuals, a budget-maximizing school board would find it optimal to take more of a risk with its proposal due to the increased district heterogeneity (Zhao, 2020b, Theorem 5.3): In effect, propose a budget on the larger side that carries a high risk of failure, and hope that it will garner enough support from high income individuals in the district and pass. If this effect is widespread, then failed school budget referenda may become more frequent in the future for districts with income-diverse voting populations.

Another way that the pandemic may increase voter heterogeneity is by increasing voter participation. For example, in the 2020 school budget referenda in New York state, the number of votes cast more than tripled compared to the previous year due to the requirement for absentee ballots to be sent to every resident (DiNapoli, 2020). The percent of 'Yes' votes decreased by 4 percentage points compared to the previous year, though the total number of defeated budgets was very similar[21]. Although districts may return to in-person ballots in the near future, New York school boards

[21] Proposed spending increases in 2020 were more conservative than in 2019 (Saunders, 2020).

may face additional uncertainty over voter turnout if the surge in voter participation this year carries over to following years[22].

The effects of increases in the cost of education due to the pandemic (point 3 above) on both local funding responses and voter support may also be significant, and are important to analyze. While increases in educational costs are not one of the exogenous changes considered in Zhao (2020b), the methods can still give some insight into the impact of an increase in the costs of education on the expected level of local funding. In this case, any amount of educational output now requires more funding due to the increases in educational cost. If educational output is measured using units of per pupil funding prior to the pandemic, there are then two effects at work from the increase in cost. First, the given amount of state aid the district receives is less valuable in terms of the educational output that can be generated; namely, there is effectively a decrease in state aid due to the fact that educational output is now more expensive, which shifts a voter's constraint set inward. This effective loss in aid would lead to an increase in expected local funding under the conditions of Zhao (2020b, Proposition 4.2) as voters become more willing to support local measures to compensate. At the same time, during the pandemic, there is an effective "price increase" for funding education, in that any marginal increase in individual expenditure on schools results in a smaller marginal gain in educational output due to educational output being more expensive[23]. This price increase effect is overall ambiguous, but may work against the effective aid decrease if voters have a feeling of hopelessness that their marginal dollar now only has a negligible impact on educational output. For a school board that needs to raise local funds for schools, it is therefore vital to effectively communicate to voters how additional funds for

[22] To compare, New Jersey school districts experienced a higher than normal level of turnout in 2011-12 following the large amount of voter participation the previous year during the fiscal emergency.

[23] Graphically, this effect makes the voter's constraint line steeper and possibly not linear if the increase in cost on specific educational goods and services is not uniform.

schools translate into real educational benefits for students. If school boards can successfully convince voters that the marginal spending on schools matters, then in theory expected local funding may increase in the face of rising costs.

It is also likely that these distinct effects of the pandemic may be present for differing lengths of time, and the long term impact of the pandemic may depend on the outlook for the future. Optimistically, widespread vaccination in the coming year will allow for in-person classes in 2021. Since the aid to districts consists of funding from both state and federal sources, any additional pandemic aid relief on the part of the federal government for K-12 public schools would take some pressure off of local districts to maintain funding levels for schools. One possible outlook for the future is that state aid cuts will occur first, then costs of education will return to normal levels, income levels and business revenues will improve later on, helping state revenues to recover, and finally state aid will eventually be restored. For example, seven years after the Great Recession of 2008, state funding levels remained lower than prior to the recession in most states (Mitchell, Palacios, & Leachman, 2015). In the above scenario, one of the most long-lasting effects of the COVID-19 recession on school funding would be the effects of state aid cuts, which can lead to local compensation from districts and support for referenda under certain conditions as in the preceding analysis. However, this local compensation would not necessarily be enough to fully make up for the losses in aid, particularly for districts that are less equipped to enlarge local tax levies, and it is likely that cuts in aid could negatively impact school funding for several years after they occur.

References

Alexander, A. J., & Bass, G. V. (1974). *Schools, taxes, and voter behavior : an analysis of school district property tax elections.* Rand.

Barnum, M. (2020, May). *A tale of two states: Ohio and New york take different paths to budget cuts for schools.* Chalkbeat. Retrieved from https://www.chalkbeat.org/2020/5/26/21271188/ohio-new-york-schools-budget-cuts

Beckstead, B. (2020, Oct). *State aid cuts could cause four out of 10 school districts to claim insolvency.* NNY 360. Retrieved from https://www.nny360.com/communitynews/education/state-aid-cuts-could-cause-four-out-of-10-school-districts-to-claim-insolvency/article_7f1d6c53-356b-5e4e-887f-f30939f2aa77.html

Chakrabarti, R., Livingston, M., & Roy, J. (2014). Did cuts in state aid during the Great Recession lead to changes in local property taxes. *Education Finance and Policy*, *9*(4), 383-416.

DiNapoli, T. (2020, Oct). *Voting from home: Summary of the 2020-21 school district budget vote.* Office of the New York State Comptroller. Retrieved from https://www.osc.state.ny.us/files/local-government/publications/2020/pdf/summary-of-2020-21-school-district-budget-vote.pdf?utm_medium=email&utm_source=govdelivery

Ehrenberg, R. G., Ehrenberg, R. A., Smith, C. L., & Zhang, L. (2004). Why do school district budget referenda fail? *Educational Evaluation and Policy Analysis*,

26(2), 111–125.

Griffith, M., & Berry, W. (2020, Sept). *COVID-19 and state education budgets: The story behind the numbers.* Learning Policy Institute. Retrieved from https://learningpolicyinstitute.org/blog/covid-state-education-budgets-story-behind-numbers

U.S. Census Bureau. (2020, Sept). *Quarterly summary of state and local government tax revenue for second quarter 2020.* Retrieved from https://www.census.gov/content/dam/Census/library/publications/2020/econ/g20-qtax2.pdf

McKillip, M., & Sciarra, D. (2020, Apr). *New Yorks pandemic adjustment: Depriving resources to students impacted by COVID-19.* Education Law Center. Retrieved from https://edlawcenter.org/news/archives/new-york/new-york%E2%80%99s-pandemic-adjustment-depriving-resources-to-students-impacted-by-covid-19.html

Minkin, R., Bennett, J., & Parker, K. (2020, Oct). Economic fallout from COVID-19 continues to hit lower-income Americans the hardest. *Pew Research Center's Social & Demographic Trends Project*. Retrieved from https://www.pewsocialtrends.org/2020/09/24/economic-fallout-from-covid-19-continues-to-hit-lower-income-americans-the-hardest/

Mitchell, M., Palacios, V., & Leachman, M. (2015). States are still funding higher education below pre-recession levels. *Journal of Collective Bargaining in the Academy*(10), 71.

Paquin, B. P. (2015). *Chronicle of the 161-year history of state-imposed property tax limitations. Working paper.* Lincoln Institute of Land Policy Cambridge.

Rice, K., Miller, G., Coronado, F., & Meltzer, M. (2020, Dec). Estimated resource costs for implementation of CDC's recommended COVID-19 mitigation strategies in pre-kindergarten through grade 12 public schools United States,

202021 school year. *Morbidity and Mortality Weekly Report*. Retrieved from https://www.cdc.gov/mmwr/volumes/69/wr/mm6950e1.htm

Romer, T., & Rosenthal, H. (1978). Political resource allocation, controlled agendas, and the status quo. *Public choice*, *33*(4), 27–43.

Romer, T., & Rosenthal, H. (1979). Bureaucrats versus voters: On the political economy of resource allocation by direct democracy. *The Quarterly Journal of Economics*, *93*(4), 563–587.

Romer, T., & Rosenthal, H. (1980). An institutional theory of the effect of intergovernmental grants. *National Tax Journal*, 451–458.

Romer, T., & Rosenthal, H. (1982). Median voters or budget maximizers: Evidence from school expenditure referenda. *Economic Inquiry*, *20*(4), 556–578.

Romer, T., Rosenthal, H., & Munley, V. G. (1992). Economic incentives and political institutions: Spending and voting in school budget referenda. *Journal of Public Economics*, *49*(1), 1–33.

Saunders, S. (2020, Jun). *School budgets approved by huge margins.* NYSUT Communications. Retrieved from https://www.nysut.org/news/2020/june/school-budget-votes-2

Silverman, R. M. (2011). How unwavering is support for the local property tax?: Voting on school district budgets in New York, 2003-2010. *Journal of Education Finance*, 294–311.

Vigdor, J. L. (2004). Other peoples taxes: Nonresident voters and statewide limitation of local government. *The Journal of Law and Economics*, *47*(2), 453–476.

Zhao, Q. (2020a). An empirical study of voting and school budget cuts in New Jersey. *Thesis, Princeton University*.

Zhao, Q. (2020b). A study of income and grant aid changes on school referenda outcomes under uncertainty over voter turnout. *Thesis, Princeton University*.

Chapter 2

The Theory

Local school funding and voter responses to changes in income and state aid

Abstract

In the current economic downturn, the question of how local funding for schools will respond to losses in state aid and disposable income has become even more pressing. Increasing local property taxes beyond a state-imposed cap typically requires a voter referendum, and voter support for these referenda is not guaranteed. Using an agenda setter model with uncertainty over voter turnout, I develop a systematic approach to analyzing the effects of income and grant aid changes on local property tax levies, overall funding for schools, and the probability of referenda passing. This framework may also be useful to inform both future empirical studies as well as the analysis of historical data. The analysis uses "pinching" and "expanding" properties of indifference curves for normal goods. Additionally, I show that budget-maximizing school boards in homogeneous districts choose budget proposals that have high rates of voter support. This last result provides a new explanation for the heightened level of voter support observed in school budget elections in smaller districts.

2.1 Introduction

The funding of K-12 public education is one of the most significant areas of public expenditure in the United States, and is financed largely through local property taxes as well as state aid. In the face of losses in income or reductions in state aid for education, will local funding increase to compensate for these losses? These questions have become even more pressing during the economic downturn caused by the COVID-19 pandemic. In most states, a voter referendum is required for any school district that tries to increase local tax levies by more than a state-imposed cap, often 1.5-2.5%[1]. At these referenda, voters typically consider a tax levy proposal made by a local school board and face two choices: to vote 'Yes' or 'No'. Will voters be supportive of these referenda when they occur? Gaining insight into these questions requires a thorough analysis of the referendum process and the incentives of the school board that makes the budget proposal.

To decide on a model for the school board, it is important to point out that, in school board elections, the assumptions that are required in the traditional median voter theory are often not likely to hold, and the agenda setter model provides a valuable alternative theory. Namely, the median voter theory assumes that elections for school board positions have strong, two-sided competition with perfect voter participation, while in practice school board elections are often not two-sided, voter participation is often low, and the available candidates do not represent the full spectrum of policy alternatives. When electoral competition is not strong, then the school board might want to propose its own most preferred level of expenditures. However, if the school board's taste for education spending is relatively high compared to that of voters, then such a proposal may be unlikely to pass. For this reason, a school board with high taste for education spending must take into account the preferences

[1] In particular, New York requires voter referenda to approve any proposed increase in spending for schools in a district.

of the voters and the likelihood of their proposal passing. In this case, a proposal that maximizes the expected funding for schools may be the best that the school board can achieve and therefore would come closest to their ideal expenditure level. Thus, school boards may choose budget-maximizing proposals, and would be called "budget-maximizers".

The idea of a proposer with control over the available choices whose goal is to achieve the largest budget was introduced by Niskanen (1971, 1975). Romer and Rosenthal (1978, 1979a) incorporated these features into their pioneering work on voting on school expenditures, and developed a model with a budget-maximizing school board whose budget proposal determines the options available to voters. Specifically, voters have a choice between voting 'for' the school funding proposal or 'against,' and in the event the referendum fails, a 'reversion' or contingency budget is enacted that can be, for example, last year's budget, a minimum budget, or even zero, i.e. closing of the school(s) in the district. Romer and Rosenthal emphasize the importance of the reversion in determining the optimal proposal for the school board, and show that the proposal chosen in the agenda setting model is generally larger than the outcome in the basic median voter model[2]. Since this foundational work, researchers have built on this model in a variety of directions including empirical tests of the model (Romer & Rosenthal, 1979b, 1982; Romer, Rosenthal, & Munley, 1992; Balsdon, Brunner, & Rueben, 2003), analysis of the "flypaper effect" starting with Romer and Rosenthal (1980); Filimon, Romer, and Rosenthal (1982), extensions to repeated referenda (Romer & Rosenthal, 1979a; Romer, Rosenthal, & Ladha, 1984; Morton, 1988), modeling the effects of inflation (Flowers, 1981), and investigation of asymmetric information and moral hazard (Banks, 1990, 1993; Lee, 1995; Chen, 2017).

[2] This result is true assuming there is no uncertainty in the model.

While the above studies lay an important foundation for various applications of the agenda setter model, fundamental questions about this model still have yet to be answered. In particular, How do the size of school budgets and other school finance variables respond to changes in income or grant aid in a district? For example, if aid to the district decreases, do voters compensate by spending more on education? In addition, can the agenda setter model predict what circumstances lead to higher or lower rates of voter approval? In this work, I introduce new ideas with the broad goal of tackling these questions.

The first major task in this paper is to analyze the effect of changes in income and grant aid on school referenda outcomes in the context of a budget-maximizing agenda setter model with uncertainty over voter turnout. The exogenous changes in income or aid can stem from a number of economic reasons, such as a recession, a tax cut, or a decision to cut state aid. Incorporating uncertainty over voter turnout allows the model to be more realistic, as proposals can fail in equilibrium in this setting, which does not occur in the certainty case. Uncertainty over voter turnout was first studied in Romer and Rosenthal (1979a), in which the authors established conditions, under special assumptions about voter participation, that cause a budget-maximizing school board to be worse off or benefit from this uncertainty and from changes in the rate of voter participation. It is reasonable to assume some uncertainty over voter turnout, as voter turnout has been found to be in large part random (Matsusaka & Palda, 1999). In addition to assuming uncertainty over turnout, it is also possible that school boards may have uncertainty over voter preferences, and the majority of the results I obtain also hold under this assumption. Appendix Section 2.B contains more details.

In the analysis in this paper, I show, for example, how to determine the direction of change in the expected funding that results from a change in the voters' income or a lump sum change in grant aid, and how to analyze the way these exogenous

changes affect the probability that referenda proposed by the school board will fail in equilibrium. The probability that referenda fail is important to study, as the failure of a referendum leads to lower funding and to a potential reduction of a district's educational services and staff. If referenda are more likely to fail, then local funding for schools can be volatile and less reliable. Thus, even when I am able to show that expected funding increases, it is still important to study whether or not the probability of passing increases as well in order to understand if funding for education becomes more secure.

In conducting the above analysis, I obtain the first comparative statics results for the agenda setter model that analyze a shift in the voter's constraint set. These results, which are also new for the model without uncertainty, add to the studies of Romer and Rosenthal (1978), which examined the effects of exogenous changes in the reversion, and Romer and Rosenthal (1980), which, in the context of studying the flypaper effect, considered equivalent changes in income and aid where the voter constraint sets overlap. To study the more general income and aid changes in the current work, I develop a new argument involving the relationship between the assumption of normal goods and the property of decreasing marginal returns that leads to certain "pinching" and "expanding" properties of pairs of indifference curves. This argument makes it possible to evaluate how the expected size of the budget responds to changes in voters' incomes or in a district's level of grant aid, and to answer other comparative statics questions in this setting in which there is a lack of a representative voter. The main results of the analysis depend on the structure of the reversion in some cases, and are summarized in Section 2.6.

The final main contribution of this work is to illustrate a new phenomenon in the behavior of budget-maximizing school boards in homogeneous districts, which is that more homogeneous districts should have more popular budget proposals. I do so by analyzing how heterogeneity versus homogeneity affects the solution to the school

board's maximization problem. I find that the school board's budget-maximizing proposal must have a *high* rate of voter approval when the school board is in a more homogeneous district. The intuition is that budget-maximizers in homogeneous districts can increase funding only slightly by choosing a high-risk proposal that has a low probability of passing, since voters in a homogeneous district have similar preferences about what funding levels they are willing to support. Instead, it is optimal for such a school board to choose a low-risk proposal, which typically corresponds to a lower funding level and earns greater support from voters. This result is in contrast to the situation in which the school board knows the voter turnout with certainty, as the budget-maximizing proposal always passes, but with the minimum amount of voter support needed ($\approx 50\%$) in that case.

The result that budget-maximizers in more homogeneous districts choose low risk proposals also supplies an alternative explanation for the empirical result in Romer et al. (1992) that school boards in smaller districts choose smaller proposals with higher levels of voter support. Namely, smaller districts are expected to be more homogeneous, and the result in this work shows that budget-maximizing school boards in homogeneous districts choose proposals with high levels of voter support and approval, thereby "emulating" the median voter. Similarly, Romer et al. (1992) show that, under certain assumptions in the median voter model without uncertainty, more homogeneity in a district leads to a higher level of voter support. One main difference is that it is possible for the budget-maximizing proposal under uncertainty to be much larger than the ideal spending level of the median voter while still garnering a high level of voter support; in general, the budget-maximizing proposal always results in the greatest amount of expected revenue for schools. Thus, the effect of the budget-maximizer in inflating the budget can still occur even in districts where voter approval is high.

While this paper examines the effects of a change in total aid on the expected funding levels for schools, the probability of referenda passing, and the expected proportion of 'No' votes, it also predicts in particular the response of local tax levies to a decrease in state aid. As discussed in Kenyon and Reschovsky (2014), there has been a decline in the funding for public schools in recent years particularly from state sources, resulting in a more important role for local districts. The Great Recession of 2008 led to severe reductions in income and state aid (Leachman & Mai, 2014), and further reductions of this kind may be on the horizon due to the current pandemic. In the time of the Great Recession, local property taxes in the United States increased (Evans, Schwab, & Wagner, 2019), partially compensating for the losses in state aid (Chakrabarti, Livingston, & Roy, 2014). The analysis in this work establishes a theoretical foundation for modeling these responses in local funding levels, and provides conditions under which decreases in state aid may be expected to lead to increases in local funding in the future.

2.2 The Model

First, I introduce the actors in the model and the notation I will be using in the paper. I consider a district $\mathcal{D}$ in which there are three types of players: the school board $\mathcal{S}$, which proposes a level of local contribution, the municipal body $\mathcal{B}$, who determines a level of local contribution in the event of a failed budget proposal, and a set of voters $\mathbb{V}$, each of whom votes to approve or disapprove the proposed budget.

The timing of the model is the following.

1. The school board $\mathcal{S}$ chooses a proposed local contribution level $\overline{E}$ that will appear on the referendum. This local contribution $\overline{E}$ is the total amount that will be collected from property taxes in the district. If the referendum passes,

the total amount of local funding will be an amount $\overline{F}_L$ determined by the district tax price τ_D according to the equation $\tau_D \overline{F}_L = \overline{E}$.

2. Nature ($\mathbb{N}$) chooses a set of voters $\mathbb{V}(\mathbb{N})$ who will participate in voting on the referendum. For example, voters $i \in \mathbb{V}$ may be chosen to participate independently with some probabilities. The probability that no voters participate is assumed to be zero, and the probability distribution of $\mathbb{V}(\mathbb{N})$ is assumed to not depend on the proposal.

3. Each participating voter $i \in \mathbb{V}(\mathbb{N})$ decides to vote 'Yes' or 'No' on the referendum. The outcome of the referendum is decided by majority rule, with the school board proposal passing in the event of a tie.

4. If the referendum passes, then the district will raise an amount $\overline{E}$ of local property tax revenue. The school funding formula of the state dictates how this revenue will be distributed to the districts in the state. If the referendum fails, the amount of property taxes to be collected in the district, E_R, is decided by a municipal body $\mathcal{B}$. This contingency amount is called the "reversion".

In this model, the school board and voters are strategic, whereas the municipal body is not a strategic entity, and I assume that the voters and school board both know the value of the reversion at the time they are making their decisions. In particular, the reversion budget may be a known amount specified by law. Section 2.2.4 contains more details on the reversion.

2.2.1 The District

Following the outcome of the referendum, districts raise a certain amount of local contributions for schools through property taxes. This amount of funding depends on whether the referendum has passed or failed. A referendum typically passes if at

least half of the votes were 'Yes' and fails otherwise. An individual voter's decision problem to vote 'Yes' or 'No' at the poll depends on a comparison between the funding that would result if the referendum passes and the reversion, which is an amount of local funding that is chosen by the municipal body in the event of a failed proposal. The voter's decision problem and the role of the reversion is discussed in detail in Section 2.2.2.

I introduce the notation for local contribution and local per pupil funding and define the role of the district tax price. E will denote the level of local expenditure from local contributions. The local contribution level E in the district will turn out to be one of two values following the referendum:

$$E \in \{E_R, \overline{E}\} \tag{2.1}$$

The local contribution level will be $E = \overline{E}$ if the referendum passes, and will be $E = E_R$ if the referendum fails.

The district faces a tax price of τ_D that is determined by the school finance formula in the state. Thus, if E is the total amount of local revenue raised in the district, then the local component of funding received by local schools, denoted by F_L, is given by

$$\tau_D F_L = E.$$

There are two components of per pupil funding f in the district: a state and federal component and a local component, where $f = f_A + f_L$. The component f_A is the total state aid plus federal aid per pupil, while f_L is the local contribution to per pupil funding that the schools in district receive, which is equal to

$$f_L = \frac{F_L}{\overline{N}} = \frac{E}{\tau_D \overline{N}}, \tag{2.2}$$

where $\overline{N}$ is the number of students in the district.

In this formulation, I assume that the government provides aid to the district as a lump sum amount. In particular, the amount of aid given to the district, f_A does not depend on the amount of local funding that the district raises.

Depending on the outcome of the referendum, the local per pupil funding level can be one of two values

$$f_L \in \{\overline{f}_L, f_R - f_A\} \tag{2.3}$$

where $\overline{f}_L$ and $f_R - f_A$ are the local per pupil funding level if the referendum passes and in the event it fails, respectively. $\overline{f}_L$ and f_R can also be expressed in terms of $\overline{E}$ and E_R as in (2.2). For several results in the paper, it will be simpler to think of the school board as proposing a per pupil funding level $\overline{f}$ with the municipal body choosing a reversion per pupil funding level f_R instead of a proposal with local contribution level $\overline{E}$ and a reversion amount E_R.

2.2.2 The Voters

Next, I derive the constraint and discuss the preferences of voters before introducing the decision problem a voter faces at the polls. Each voter $i \in \mathbb{V}$ possesses some property value P^i and some amount of disposable income Y^i. The total property value in the district is the sum $\sum_j P^j$ and the fraction of the total property value owned by voter i is $\rho^i = \frac{P^i}{\sum_j P^j}$.

The property value of the voter determines the share of local contribution that the voter is accountable for paying. If E is the total revenue collected locally for schools, then the voter $i \in \mathbb{V}$ must contribute an amount

$$E^i = \rho^i E \tag{2.4}$$

Substituting for E using (2.2), equation (2.5) below expresses an individual's local contribution E^i as a function of the local per pupil funding amount f_L.

$$E^i = \rho^i \tau_D \overline{N} f_L \tag{2.5}$$

The individual tax price τ^i is

$$\tau^i \equiv \tau_D \rho^i \overline{N} \tag{2.6}$$

and a voter's local contribution E^i can be expressed in terms of τ^i:

$$E^i = \tau^i f_L. \tag{2.7}$$

Voters have preferences over their consumption C and the amount of per pupil funding f that are represented by utility functions $u_i(C, f)$. I assume that both goods C and f are normal goods and that preferences are convex.

When the district spends E in total, the voter i must spend the portion E^i of his income as in (2.4). The consumption level for voter i is therefore

$$C = Y^i - E^i$$

Substituting equation (2.7) into the above equation and using that $f - f_A = f_L$, the following relationship between consumption and local per pupil funding is obtained:

$$Y^i - C = \tau^i (f - f_A). \tag{2.8}$$

The voter's indirect utility function $V^i(E)$, which describes the voter's preferences over local expenditure levels, can be derived as

$$V^i(E) = u_i(Y^i - E^i, f_A + \frac{E^i}{\tau^i}), \tag{2.9}$$

where the individual local contribution E^i is defined as a function of E by (2.4).

Since u_i is quasiconcave and V^i is defined by restricting u_i to a line segment, the voter's preferences over local contribution represented by $V^i(E)$ are therefore single-peaked.

Similarly, using the relationship between local per pupil funding and expenditures in (2.2), voter preferences over proposed local per pupil funding levels can also be derived. The indirect utility of voter i associated with a proposed per pupil funding level f is given by

$$V^i(f) = u_i(\widehat{C}^i(f), f) \tag{2.10}$$

where $\widehat{C}^i(f)$ is the amount of consumption by voter i when per pupil funding is equal to f

$$\widehat{C}^i(f) = \begin{cases} Y^i & \text{if } f \leq f_A \\ Y^i - \tau^i(f - f_A) & \text{if } f_A \leq f \text{ and } Y^i - \tau^i(f - f_A) \geq 0 \\ 0 & \text{if } f_A \leq f \text{ and } Y^i - \tau^i(f - f_A) < 0 \end{cases} \tag{2.11}$$

The function $\widehat{C}^i(f)$ is the **constraint line** of the voter. This constraint line differs from a budget constraint in that a voter who participates in voting on the referendum is faced with a choice between only two consumption and per pupil funding levels – called the proposal and the reversion – each of which lies on the graph of the constraint line. An example is illustrated in Figure 2.1 below.

The Voter Decision

At the ballot, a participating voter $i \in \mathbb{V}(\mathbb{N})$ compares the indirect utility of a proposed local expenditure level $\overline{E}$ to a reversion expenditure level E_R, which is assumed to be known to the voter. The voter i approves $\overline{E}$ if and only if $V^i(\overline{E}) \geq V^i(E_R)$.

Let $\widetilde{E}^i = \widetilde{E}^i(E_R)$ denote the largest local contribution proposal that the voter will accept for a given reversion local contribution E_R, i.e.

$$\widetilde{E}^i(E_R) = \sup\{E \ : \ V^i(E) \geq V^i(E_R)\}.$$

$\widetilde{E}^i$ can be called the **willingness to contribute** of voter i. It is important to point out that $\widetilde{E}^i$ is a *district* level contribution level and is not the individual contribution of the voter. If voter i's ideal contribution level is less than or equal to the reversion, then $\widetilde{E}^i$ is equal to the reversion E_R.

I use $\widehat{E}^i$ to denote the maximal amount that the individual voter is willing to pay given the reversion. The distinction between $\widetilde{E}^i$ and $\widehat{E}^i$ is that $\widetilde{E}^i$ refers to the district level local contribution, and $\widehat{E}^i$ refers to the local contribution of the individual voter. From (2.4), the relationship between $\widetilde{E}^i$ and $\widehat{E}^i$ is then:

$$\widehat{E}^i = \rho^i \widetilde{E}^i \tag{2.12}$$

The voter's decision problem can also be analogously analyzed in terms of per pupil funding. Let $\tilde{f}^i = \tilde{f}^i(f_R)$ denote the largest per pupil funding level that voter i will accept when the per pupil funding for the reversion is f_R, i.e.

$$\tilde{f}^i(f_R) = \sup\{f \ : \ V^i(f) \geq V^i(f_R)\}$$

An example of the voter decision process is shown in Figure 2.1 below. In this figure, $\overline{E}^i$ is the amount voter i must spend for a given proposal $\overline{E}$ that corresponds to a per pupil funding level of $\bar{f}$.

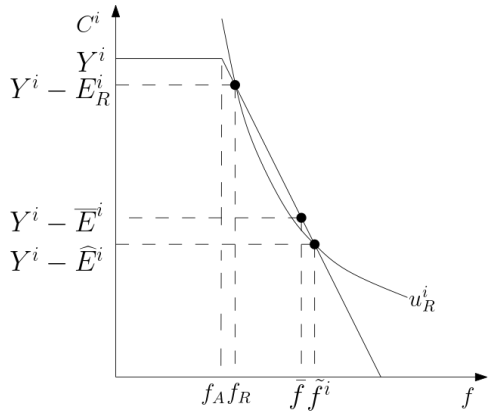


Figure 2.1: Voter i prefers the proposal $(\overline{E}^i, \bar{f})$ to the reversion (E^i_R, f_R). The proposal is smaller than the maximal proposal $(\widehat{E}^i, \tilde{f}^i)$ that the voter is willing to accept and lies on a higher indifference curve.

2.2.3 The School Board under Uncertainty over Turnout

Now, I describe the objective of the school board in this model in a scenario where there is uncertainty over which voters will turn out to vote. I assume that the objective of the school board $\mathcal{S}$ is to make a proposal that will maximize expected revenue. The school board's problem can be stated either in terms of choosing a level of local contribution (i.e. the amount that taxpayers in the district spend on property taxes for schools) or in terms of choosing a level of local per pupil funding.

In terms of local contributions, the school board proposes a local contribution level $\overline{E}$ that will maximize the expected amount of tax revenue collected for schools in the face of uncertainty over voter turnout. Precisely, the school board $\mathcal{S}$ chooses

$$\overline{E}^* = \underset{\overline{E}}{\text{argmax}}\ \pi(\overline{E})\overline{E} + (1 - \pi(\overline{E}))E_R,$$

where $\pi(\overline{E}) = \pi(\overline{E}; E_R)$ denotes the probability that the proposed local contribution $\overline{E}$ will pass the referendum given the reversion amount E_R.

In order to analyze the school board's maximization problem graphically as in Figure 2.2 below, I formulate the school board problem equivalently as maximizing the expected local contribution beyond the reversion local contribution level:

$$\overline{E}^* = \underset{\overline{E}}{\text{argmax}}\ \pi(\overline{E})(\overline{E} - E_R). \tag{2.13}$$

The probability of the proposal passing, $\pi(\overline{E})$, is the probability that the number of participating voters that are willing to support the proposed $\overline{E}$ is at least as large as the number of participating voters who are unwilling to support $\overline{E}$, assuming the proposal passes in the event of a tie.

$$\pi(\overline{E}) = \text{Prob}\left[\#\{i \in \mathbb{V}(\mathbb{N}) \ : \ \widetilde{E}^i \geq \overline{E}\} \geq \#\{i \in \mathbb{V}(\mathbb{N}) \ : \ \widetilde{E}^i < \overline{E}\}\right] \tag{2.14}$$

Graphically, the function $\pi(\overline{E})$ depicts, for each level of $\overline{E}$, the probability that this proposal will pass. The probability that $\overline{E}$ will pass depends on the number of participating voters that vote 'Yes' to the proposed $\overline{E}$. Each individual voter i has a maximal expenditure amount that the voter is willing to accept, $\widetilde{E}^i$, and if $\widetilde{E}^i \geq \overline{E}$, voter i will say 'Yes' to $\overline{E}$. As mentioned previously, the $\widetilde{E}^i$ for each i depends on i's income, property tax share, preferences, and aid to the district as in (2.12) above.

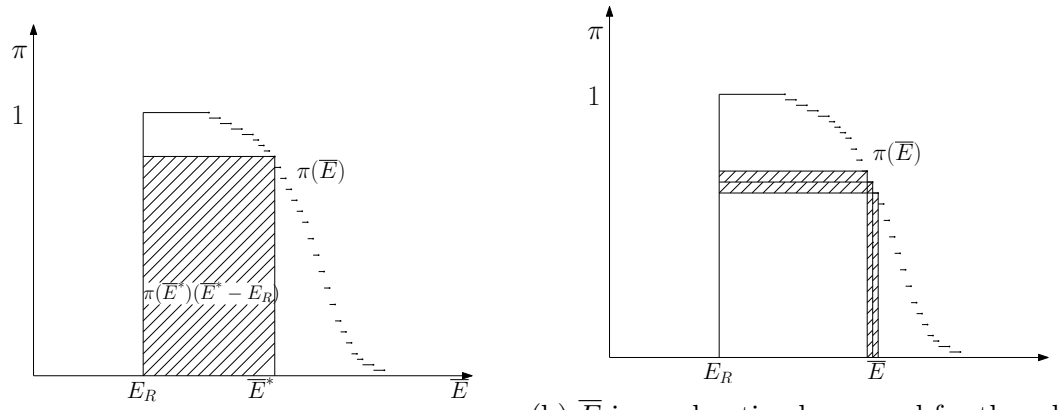
To give an example, I can sort the voters by their $\widetilde{E}^i$ and think about the probability of passing a given $\overline{E}$. For example, suppose there are three voters with $E_R < \widetilde{E}^1 < \widetilde{E}^2 < \widetilde{E}^3$, and suppose that voters 2 and 3 turns out to vote independently with probability $\frac{1}{2}$, while voter 1 turns out to vote with probability 1. If $E_R \leq \overline{E} \leq \widetilde{E}^1$, then all three voters would be willing to accept $\overline{E}$ and no matter which of the voters show up to vote, the proposal will pass, so $\pi(\overline{E}) = 1$. However, if $\widetilde{E}^1 < \overline{E} \leq \widetilde{E}^2 < \widetilde{E}^3$, then the proposal passes if and only if at least one of voters 2

and 3 turns out to vote (since the proposal passes in the event of a tie). In this case, the probability of passing will be $\pi(\overline{E}) = \frac{3}{4} < 1$. Similarly, if $\widetilde{E}^1 < \widetilde{E}^2 < \overline{E} \leq \widetilde{E}^3$, then the proposal will pass if and only if voter 2 does not participate and voter 3 does participate. Thus, $\pi(\overline{E}) = \frac{1}{4} < \frac{3}{4}$.

To describe $\pi(\overline{E})$ graphically, there is a jump discontinuity downwards at each $\widetilde{E}^i$, which signifies the decrease in probability that occurs once $\overline{E}$ goes beyond each $\widetilde{E}^i$ for which $\widetilde{E}^i < \overline{E}$. The function $\pi(\overline{E})$ is therefore nonincreasing and is piecewise constant, taking values between 0 and 1 with a jump discontinuity at each willingness to contribute $\widetilde{E}^i$. The size of the jump discontinuity at $\widetilde{E}^i$ depends on the probability with which voter $i \in \mathbb{V}(\mathbb{N})$ is chosen to participate and the probabilities with which other voters are chosen to participate together with voter i. An example of a possible $\pi(\overline{E})$ is graphed in Figure 2.2.

Figure 2.2 illustrates an example of the school board's maximization problem. The objective of the school board is to choose the proposal that will maximize the area of the shaded rectangle, as in Equation (2.13). In the particular case of Figure 2.2, the reversion local contribution E_R is below the maximal willingness to pay $\widetilde{E}^i$ for all voters. The school board uses marginal analysis illustrated in Figure 2.2b to determine the optimal proposal by comparing losses and gains of choosing other proposals. For example, a small decrease in the proposal from $\widetilde{E}^i$ to $\widetilde{E}^{i-1}$ results in a gain due to a higher probability of passing (since now more voters have willingness to contribute above $\widetilde{E}^{i-1}$), but also a loss due to the smaller local contribution level of the proposal. Graphically, the rectangle being maximized now has a smaller base but a larger height. The marginal gain of decreasing the proposal is illustrated by the horizontal rectangles in Figure 2.2b, while the marginal cost is illustrated in the thin vertical rectangles. When there are a large number of voters, the function $\pi(\overline{E})$ can be approximated by a continuous, decreasing function of $\overline{E}$, and the solution to the

school board's problem can then be approximated by the solution to the continuous problem.



(a) The optimal proposal maximizes the area of the striped rectangle

(b) $\overline{E}$ is a suboptimal proposal for the school board. Marginal gains and losses are shown for small decreases in the proposal.

Figure 2.2: The optimization problem for the school board

In Figure 2.2, I assume that the smallest willingness to contribute $\widetilde{E}^1$ for any voter is greater than the reversion, which implies the probability of passing is equal to 1 between the reversion and the first jump discontinuity at $\widetilde{E}^1$. It is also possible that some voters have willingness to contribute $\widetilde{E}^i$ less than the reversion and would therefore vote 'No' to any proposal greater than the reversion. In this case, proposing any local contribution level above the reversion could have a probability of passing smaller than 1 since a majority of the voters that turn out to vote could have ideal contribution levels less than the reversion. Figure 2.3 illustrates the school board problem for such a scenario. In contrast to the school board problem in the absence of uncertainty, this figure shows that as long as there is a positive probability that some proposal above the reversion will pass, the budget-maximizing school board will always choose a proposal strictly greater than the reversion.

For several results in this paper, I analogously analyze the school board's problem in terms of proposing a per pupil funding level $\bar{f}$ as opposed to a local contribution level $\overline{E}$. The choice of a local contribution level $\overline{E}$ is also equivalently a choice of

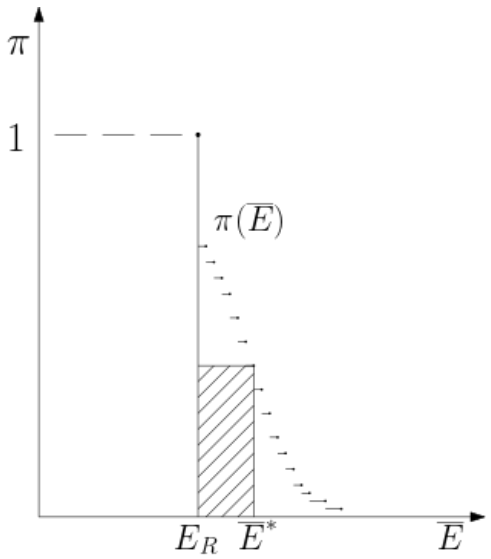


Figure 2.3: School board problem in a case where some voters have ideal points less than the reversion; in this example, $\pi(\overline{E})$ is smaller than 1 for all proposals greater than the reversion.

$\bar{f}$, since $\overline{E}$ and $\bar{f}$ are related by (2.2). Similarly, a reversion local contribution level E_R corresponds to a level of per pupil funding f_R and vice versa. The school board's problem is therefore to choose a proposal that will maximize the expected per pupil funding in the district

$$\bar{f}^* = \underset{\bar{f}}{\text{argmax}}\ \pi(\bar{f})\bar{f} + (1 - \pi(\bar{f}))f_R \tag{2.15}$$

$$\bar{f}^* = \underset{\bar{f}}{\text{argmax}}\ \pi(\bar{f})(\bar{f} - f_R). \tag{2.16}$$

Analogously to $\pi(\overline{E})$, $\pi(\bar{f}) = \pi(\bar{f}; f_R)$ here denotes the probability that the proposed per pupil funding level $\bar{f}$ will pass the referendum given the reversion per pupil funding level f_R. This probability is equal to

$$\pi(\bar{f}) = \text{Prob}\left[\#\{i \in \mathbb{V}(\mathbb{N}) \ : \ \tilde{f}^i \geq \bar{f}\} \geq \#\{i \in \mathbb{V}(\mathbb{N}) \ : \ \tilde{f}^i < \bar{f}\}\right] \tag{2.17}$$

which is the analogous version of Equation (2.14). Additionally, the school board's maximization problem can be depicted by Figure 2.2 with the expenditure axis $\overline{E}$ replaced by an axis for per pupil funding $\bar{f}$.

Assumption 0: I assume that there is at least one voter that may be pivotal with positive probability whose ideal point is greater than the reversion.

If this assumption does not hold, then it is not possible to pass a proposal greater than the reversion, and thus the result of the referendum is always the reversion.

2.2.4 The Municipal Body and the Reversion

In the school board's maximization problem in the previous section, the optimal choice of proposal ($\overline{E}$ and $\bar{f}$) depends on the value of the reversion. The reversion plays a large role in a voter's decision of whether or not to accept a given proposal because the voter compares the utility from the proposal to the utility from the alternative, i.e. the reversion. There are several forms of reversions (or known as contingency budgets) that are possible in the event of a failed referendum. For example, in some states, the reversion that is enacted could be last year's budget, or the contingency budget could be a minimum local contribution level or per pupil funding amount that's mandated by the state, or in some cases it could even mean that schools could be shut down. I assume that, in each district, an entity of the municipality called the municipal body chooses the reversion each time a referendum fails. Unlike the school board and the voters, the municipal body is not a strategic actor in this model.

While the decision-making process of a specific municipal body may be complex, I consider two simple regimes of municipal body behavior for this paper. In the first regime, municipal bodies aim to achieve some target level of local contributions in the event the proposal fails. In this case, the reversion corresponds to a particular property tax rate for the district.

Definition 2.2.1 (Target Local Contribution Level)**.** A municipal body $\mathcal{B}$ has a *target local contribution level* if there is a particular local contribution level E_R that will be enacted by the municipal body in the event of a failed referendum, regardless of other changes in circumstances.

Example: A district may have a Target Local Contribution Level if, in the event of a failed referendum, the resulting funding (i.e. the reversion) consists of setting the amount of property taxes to be raised to fund schools equal to the prior year's level of property taxation (such as in Maine), or equal to the budget that corresponds to the cap on property tax increases imposed by the state (as in Massachusetts, for example). Another example is the case of Arizona, where the failure of a referendum results in schools not receiving any local funding from the district. In these above examples, the reversion is set by law and there is no entity or actor who decides the reversion level; in particular, the "municipal body" is not strategic.

In the second reversion regime, the municipal body instead aims to achieve a target level of per pupil funding for students.

Definition 2.2.2 (Target Funding Level)**.** A municipal body $\mathcal{B}$ has a *target funding level* if there is a particular per pupil funding level f_R that will be enacted by the municipal body in the event of a failed referendum, regardless of other changes in circumstances.

Example: A district may fall under the Target Funding Level reversion regime if, in the event of a failed referendum, the reversion funding level guarantees a minimum level of per pupil funding for students to obtain an adequate education.

One major difference between these two models of the reversion is the way that they respond to a change in total aid. If a reversion has a Target Local Contribution

Level, the reversion tax levy is the same amount whether or not there is a change in aid. Alternatively, if a reversion has a Target Funding Level, then a decrease in aid will cause the reversion tax levy to be larger to achieve the targeted per pupil funding level.

In practice, the Target Local Contribution Level is the most common type of reversion for school funding referenda in the United States when the reversion funding level is specified by law. However, one of the goals of this work is to consider referenda in New Jersey, especially for the time period 2008-2011 that is studied in the empirical work of Zhao (2020). For these elections, the reversion budget is not spelled out by law, but is instead decided by a group of municipal officials (the municipal body). Municipal bodies in New Jersey are members of the municipal government who may be appointed or elected to their positions. The Target Funding Level reversion can be a potential model for a municipal body whose goal is to ensure a sufficient funding level for students while needing to reduce the budget. In particular, in Zhao (2020), I find evidence that municipal bodies during the time period of the data made smaller cuts in districts with lower levels of aid holding other covariates fixed, which is consistent with the assumption of a Target Funding Level reversion.

In the following analyses in this paper, the conclusions pertaining to how income and state and federal aid changes affect school budget referenda outcomes will at times depend on whether the reversion set by the municipal body targets a local contribution level or targets a per pupil funding level.

2.3 Changes in Income

In this paper, I explore the effects of two kinds of exogenous changes: 1) a change in the income of the voters and 2) a lump sum change in total aid, on school referenda outcomes such as the maximal funding proposals that voters would be willing to

accept, the expected funding that results, and the probability of the referendum passing. In this first section, I analyze the effect of an increase in the income of each voter. In the summary following the analysis, I discuss the analogous results for a decrease in income.

2.3.1 Support for funding by voters

I begin by analyzing the effect of an exogenous increase in income on the maximal per pupil funding levels that voters are willing to accept. In order to do so, I consider the new constraint line that a voter i faces following the income increase and the location of the reversion on the new constraint line. I show that the two reversion schemes correspond to the same (f, C) bundle for the voter under the scenario of an income change, and thus, the school referendum outcomes discussed in this section do not depend on which reversion scheme is assumed. Following Proposition 2.3.1 below, I analyze the effect of the income increase on expected per pupil funding and local contribution amounts and on the probability of the referendum passing.

First, I determine whether voters become more willing to support higher funding levels following an increase in their income (i.e. I determine whether each $\tilde{f}^i$, which is the maximal per pupil funding level that voter i is willing to support, increases or decreases). This analysis is important because the solution to the school board's problem is based on the location of the support funding levels $\tilde{f}^i$.

It might seem at first that the supported funding levels $\tilde{f}^i$ can either increase or decrease following an increase in voters' income, in which case it would not be possible to analyze the school board's problem to determine the resulting expected funding levels. Figure 2.4 illustrates an example of an income increase in which the voter becomes less willing to support higher funding levels (i.e. $\tilde{f}^i$ decreases from $\tilde{f}^i_1$ to $\tilde{f}^i_2$). However, in this scenario, these preferences end up violating the assumption that per pupil funding is a normal good. Using the assumption that per pupil funding

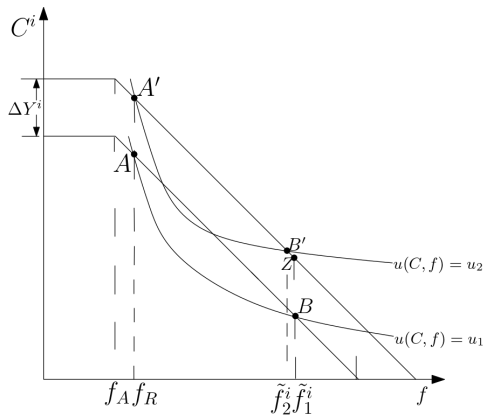


Figure 2.4: Following an increase in income, the voter becomes less willing to support high per pupil funding levels ($\tilde{f}_2^i < \tilde{f}_1^i$). Prop 2.3.1 shows that this situation is not possible when f is a normal good.

is a normal good, I prove the following proposition, which shows that the maximal per pupil funding levels that voters are willing to support must increase following an increase in their income.

Proposition 2.3.1. *Suppose that the disposable income of voter i increases. Assume the voter's ideal point is greater than the reversion prior to the income increase. Then, all else equal, the maximal per pupil funding $\tilde{f}^i$ that voter i is willing to support increases.*

Proof. Suppose that the disposable income for voter i, Y^i, increases to an amount $Y^i + \Delta Y^i$. Let $\tilde{f}_1^i$ be the maximal level of per pupil funding that voter i is willing to support prior to the income increase, and let $\tilde{f}_2^i$ be the maximal level of per pupil funding that voter i is willing to support following the income increase. I want to show that $\tilde{f}_2^i \geq \tilde{f}_1^i$, which means that a situation such as the one in Figure 2.4 cannot occur. Since each $\tilde{f}^i$ is calculated in comparison with the reversion per pupil funding level, I first derive the new constraint line and determine where the reversion per pupil funding level will be located on this new constraint line.

With the increase in income for voter i, the new constraint line can be constructed by considering the school board's problem. For each potential proposal $\overline{E}$, voter i

can obtain the same per pupil funding as prior to the income change, but now enjoys an additional consumption of ΔY^i. Graphically, the constraint line of voter i has expanded upwards by an amount ΔY^i. A given reversion chosen by the municipal body corresponds to a certain per pupil funding level and a certain local contribution level (f_R, E_R^i) for a voter i. The two reversion schemes in this paper allow the municipal body to either set the reversion equal to a target per pupil funding level f_R or alternatively to the local contribution level E_R^i. It is not true in general that targeting the funding level f_R would be equivalent to targeting the local contribution E_R^i. However, in the current analysis of an increase in income, it can be seen that the reversion funding level f_R is the same following the income increase regardless of whether the municipal body targets the local contribution level E_R^i or targets the per pupil funding level f_R. The reason is that $f_R = f_{R,L} + f_A$ is composed of a local funding component $f_{R,L}$ and an aid component f_A. Since the reversion targets the local contribution level E_R^i, the local funding component of the reversion remains fixed following the income increase by (2.2), and the aid component f_A has not changed.

I now show that the maximal level of per pupil funding $\tilde{f}_2^i$ that voter i is willing to support in the event of an income increase is larger than the amount $\tilde{f}_1^i$ that voter i would be willing to support in the absence of the income increase. To do so, I illustrate the voter decision process prior to and following an income increase of an amount ΔY^i in Figure 2.6 below. The points A and A' respectively represent the reversion funding and consumption levels prior to and following the income increase, where the voter i would be required to sacrifice an amount E_R^i of consumption in order to obtain an amount f_R of per pupil funding. The indifference curve through A is given by $u(C, f) = u_1$ and the indifference curve through A' is given by $u(C, f) = u_2$. The constraint lines passing through the points A and A' are represented by the functions $\widehat{C}_1(f)$ and $\widehat{C}_2(f)$ respectively (see Equation 2.11).

One key step in this proof is to use the property of normal goods to show that the slope of the indifference curve through A' is steeper (i.e. larger in magnitude) at A' than is the slope of the indifference curve through A at A. To illustrate this step, it may be helpful to first take a brief detour and examine the implications of the normal goods condition in a utility maximization framework.

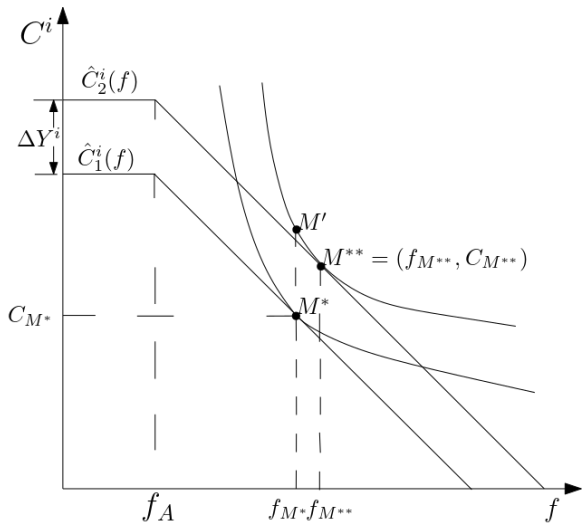


Figure 2.5: Utility maximization under an increase in income

In the typical utility maximization problem in economics, which is illustrated in Figure 2.5, an individual is not given (and therefore not limited to) the choice between the proposal provided by the school board and the reversion, and instead maximizes utility subject to a budget constraint. In this case, suppose the individual's budget constraint is the original constraint line $\widehat{C}_1^i(f)$. I can start with the individual's optimal bundle on $\widehat{C}_1^i(f)$ and use the normal goods condition to compare this optimal bundle with the point corresponding to the new optimal per pupil funding level, which is the point on a higher indifference curve that is tangent to the new constraint line following an increase in income of ΔY^i.

Let M^* be the optimal bundle (f_{M^*}, C_{M^*}) on the original constraint line $\widehat{C}_1^i(f)$, and let M^{**} be the optimal bundle $(f_{M^{**}}, C_{M^{**}})$ under the new constraint line $\widehat{C}_2^i(f)$. By assumption, f is a normal good and thus M^{**} corresponds to a point with more f

compared to M^*. Since the original constraint line $\widehat{C}_1^i(f)$ and the new constraint line $\widehat{C}_2^i(f)$ differ by a constant ΔY^i in the vertical direction, the slopes of the respective indifference curves at tangency points M^* and M^{**} are the same. By convexity, for all $f < f_{M^{**}}$, the slope of the indifference curve through M^{**} is steeper at these points than the slope of this indifference curve at M^*. Let M' be the point on the indifference curve through M^{**} corresponding to a funding level of f_{M^*}. Then, the slope of the indifference curve through M^{**} at M' is larger in magnitude (i.e. more negative) than the slope of the indifference curve through M^* at M^*.

Next, I return to the agenda setter framework considered in this paper, and use the analysis above to show that the assumption that C and f are normal goods implies that $\tilde{f}_2^i \geq \tilde{f}_1^i$. Specifically, I extend the above analysis to the original and new reversion points by comparing the slope of the indifference curve through A' at A' and the slope of the indifference curve through A at A.

Consider the points A and A' and the indifference curves u_1 and u_2 that go through A and A' respectively. I claim that the slope of the indifference curve u_2 at A' is larger in magnitude than the slope of the indifference curve u_1 at A. Let ℓ_A be the line that is tangent to u_1 at A. This line could be thought of as a budget constraint, and A would represent the point on this constraint that is a voter's most preferred point. Since f is a normal good, when income increases, the voter chooses a bundle with more f. Suppose the voter's income increases by an amount such that his new most preferred point is a point on u_2. At this point, the new budget constraint with the same slope as ℓ_A is tangent to u_2. Call this new tangency point $N = (f_N, C_N)$, and define ℓ_N as the line that is tangent to u_2 at N. The slope of the indifference curve u_1 at A is the same as the slope of the indifference curve u_2 at N i.e. the slopes of ℓ_A and ℓ_N are equal. Since preferences are convex, for all $f < f_N$, the slope of u_2 at these points is steeper than the slope of ℓ_N. In particular, at point A', the slope of the indifference curve u_2 is steeper than the slope of ℓ_N. Thus, the slope of u_2

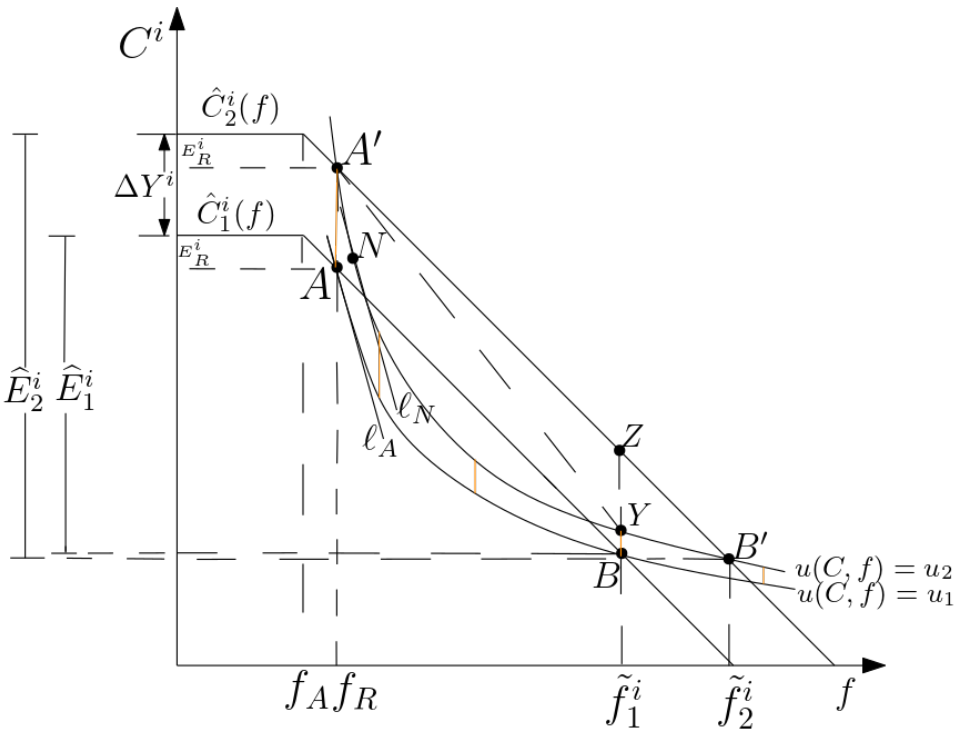


Figure 2.6: Following an income increase, the voter is willing to support a larger level of per-pupil funding. The municipal body may have a target local contribution level or target funding level.

at A' is also steeper than the slope of u_1 at A, i.e. the slope of ℓ_A, since the slopes of ℓ_A and ℓ_N are equal. While A and A' were the points on u_1 and u_2 respectively that correspond to f_R, this comparison of the slopes of u_1 and u_2 can be repeated for any value of f. Thus, I have shown that for each fixed f the slope of u_2 at f is steeper than the slope of u_1 at f. The steeper slope of u_2 compared to the slope at u_1 at any given f implies that for any amount of f, voter i is more willing to trade off consumption for per pupil funding when he has more C (such as at point A') than when he has less C (for example, at point A). This implication of normal goods in f and convexity of preferences can be called 'decreasing marginal returns in consumption'. Since consumption C is also a normal good, the same analysis can be used to show that preferences also exhibit 'decreasing marginal returns in per pupil funding'.

Now, using the above comparison of the slopes of the indifference curves, I want to show that $\tilde{f}_2^i \geq \tilde{f}_1^i$ i.e. that the indifference curve u_2 crosses the new constraint line $\widehat{C}_2^i(f)$ at a level of f to the right of where the original indifference curve u_1 crosses the original constraint line $\widehat{C}_1^i(f)$. I define a point Z on $\widehat{C}_2^i(f)$ that corresponds to $\tilde{f}_1^i$, and I define a point Y on u_2 that corresponds to the same $\tilde{f}_1^i$. I will use the property of decreasing marginal returns in C to show that u_2 must lie below $\widehat{C}_2^i(f)$ at the point Z, namely that point Y is below point Z. *Proof*: First, decreasing marginal returns in C means that at point A', voter i is more willing to trade off consumption compared to point A since he has more consumption at point A' than at A. Furthermore, the same comparison of slopes holds for any given value of f. As a result, the vertical distance between the two indifference curves, which is the length of the orange vertical lines in Figure 2.6, becomes smaller as f increases[3]. Let $\overline{C}_1^i(f)$ be the function whose graph is the indifference curve u_1, and $\overline{C}_2^i(f)$ be the function whose graph is u_2. The vertical distance between u_2 and u_1 is $\overline{C}_2^i(f) - \overline{C}_1^i(f)$. Since it has been shown that $\frac{d\overline{C}_2^i}{df} < \frac{d\overline{C}_1^i}{df}$ for any value of f (since $\overline{C}_2^i$ is steeper than $\overline{C}_1^i$ and their slopes are negative), the vertical distance $\overline{C}_2^i(f) - \overline{C}_1^i(f)$ decreases as f increases:

$$\frac{d}{df}(\overline{C}_2^i - \overline{C}_1^i) = \frac{d\overline{C}_2^i}{df} - \frac{d\overline{C}_1^i}{df} < 0.$$

Therefore, for a given increase in f, such as from f_R to $\tilde{f}_1^i$, the difference in consumption between the two indifference curves becomes smaller:

$$\overline{C}_2^i(\tilde{f}_1^i) - \overline{C}_1^i(\tilde{f}_1^i) < \overline{C}_2^i(f_R) - \overline{C}_1^i(f_R) = \Delta Y^i.$$

[3] Using a similar argument with C and f interchanged, the horizontal distance between the indifference curves becomes larger as f increases and C decreases.

Since the difference in consumption at $\tilde{f}_1^i$ is smaller than the change in income ΔY^i, which is the difference in consumption between the constraint lines, the point Y on the indifference curve u_2 must lie below the point Z on the constraint line $\hat{C}_2^i$.

To finally obtain the conclusion that $\tilde{f}_2^i$ is greater than $\tilde{f}_1^i$, I use the convexity of preferences. Convexity of preferences implies that u_2 lies below the dashed line connecting A' and Y for all values of f between the reversion f_R and $\tilde{f}_1^i$. Therefore, u_2 lies strictly below $\widehat{C}_2^i(f)$ whenever $f_R < f \leq \tilde{f}_1^i$. It follows that the funding level $\tilde{f}_2^i$ at which the higher indifference curve u_2 intersects the new constraint line $\widehat{C}_2^i(f)$ is greater than or equal to $\tilde{f}_1^i$. Graphically, the point B', which corresponds to the point on u_2 at $\tilde{f}_2^i$ is to the right of point B. □

Corollary 2.3.1. *Suppose that the disposable income of voter i increases. Assume the voter's ideal point is greater than the reversion prior to the income increase. Then, all else equal, the maximal local contribution level $\widetilde{E}^i$ that voter i is willing to support increases.*

Proof. Proposition 2.3.1 above showed that the maximal per pupil funding $\tilde{f}^i$ that voter i is willing to support increases following an increase in income, and since $f = f_L + f_A$ for any f, the maximal local per pupil funding voter i is willing to support, $\tilde{f}_L^i$, also increases because f_A has not changed. Using the relationship between $\tilde{f}_L^i$ and $\widetilde{E}^i$ as defined in (2.7), $\widetilde{E}^i$ must also increase when $\tilde{f}^i$ increases in this case of an income increase. □

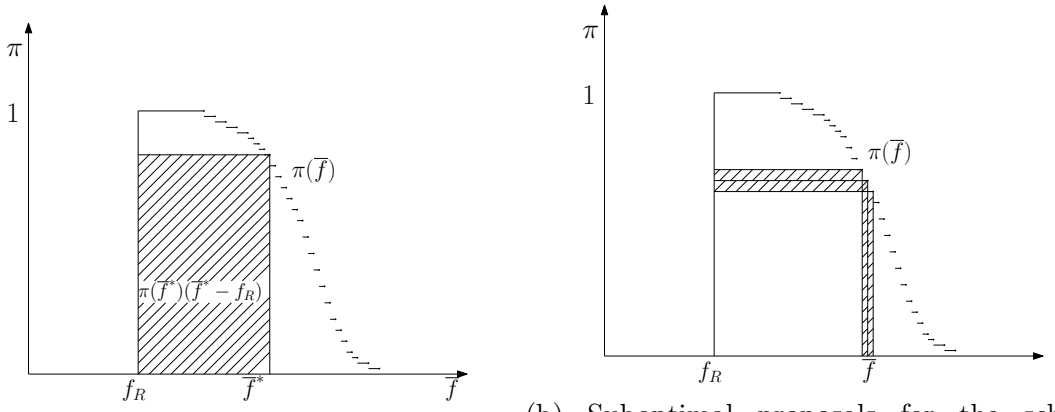
Graphically, the increase in $\widetilde{E}^i$ means that point B' in Figure 2.6, which corresponds to $\tilde{f}_2^i$ following the income increase, **cannot** lie a distance greater than ΔY^i above point B, which corresponds to $\tilde{f}_1^i$.

2.3.2 Expected per pupil funding and local contribution

Now, I show that the expected total per pupil funding and the expected local contribution both increase following an increase in voters' income.

Proposition 2.3.2. *Suppose that the income of every voter in the district increases. Then, all else equal, the expected total per pupil funding in the district increases*[4].

Proof. In this proof, I use the formulation of the school board problem where the school board $\mathcal{S}$ chooses a proposal $\bar{f}$ to maximize the expected per pupil funding as in (2.16). The idea in the proof is to show that the probability of passing for any possible proposal $\bar{f}$ increases, so that the school board cannot be worse off following an increase in the voters' income.



(a) The optimal proposal maximizes the area of the striped rectangle

(b) Suboptimal proposals for the school board. A smaller proposal achieves larger expected per pupil funding.

Figure 2.7: The optimization problem for the school board

Suppose that the income of every voter in the district increases. Let $\pi_1(\bar{f})$ denote the probability that a given proposal $\bar{f}$ passes prior to the increase in income, and let $\pi_2(\bar{f})$ denote the probability that a proposal $\bar{f}$ passes after the increase in income. Let $\bar{f}_1^* = \underset{\bar{f}}{\text{argmax}}\ \pi_1(\bar{f})(\bar{f} - f_R)$ be the optimal proposal in the absence of the income

[4] In the case that every voter who may be pivotal has ideal point below the reversion (which violates Assumption 0), the outcome of the referendum is the reversion, which stays the same rather than increasing.

increase, and let $\bar{f}_2^* = \underset{\bar{f}}{\text{argmax}}\ \pi_2(\bar{f})(\bar{f} - f_R)$ be the optimal proposal following the increase in voters' income. Graphically $\bar{f}_1^*$ and $\bar{f}_2^*$ maximize the area of the striped rectangle in Figure 2.7a prior to and following the income increase respectively. The goal is to show that the school board obtains a greater expected per pupil funding level following the income increase, i.e.

$$f_R + \pi_2(\bar{f}_2^*)(\bar{f}_2^* - f_R) \geq f_R + \pi_1(\bar{f}_1^*)(\bar{f}_1^* - f_R) \tag{2.18}$$

In order to compare the expected per pupil funding levels from prior to and following the income increase, I first consider how the probability of passing $\pi_2(\bar{f})$ following the increase in income compares to the probability of passing $\pi_1(\bar{f})$ before the shift. In (2.17), the probability a proposal passes is the probability that the number of 'Yes' votes is greater than or equal to the number of 'No' votes.

$$\pi(\bar{f}) = \text{Prob}\left[\#\{i \in \mathbb{V}(\mathbb{N}) \ : \ \tilde{f}^i \geq \bar{f}\} \geq \#\{i \in \mathbb{V}(\mathbb{N}) \ : \ \tilde{f}^i < \bar{f}\}\right]$$

Proposition 2.3.1 shows that, after an income increase, the maximal per pupil funding level $\tilde{f}^i + \delta\tilde{f}^i$ that voter i is willing to support will be greater than the corresponding $\tilde{f}^i$ prior to the income increase. As a result, the number of 'Yes' votes for any proposal $\bar{f}$ following the income increase is at least as large as the number of 'Yes' votes prior to the increase for the same proposed $\bar{f}$ regardless of which set of voters $\mathbb{V}(\mathbb{N})$ participates in the voting. Similarly, the number of 'No' votes for any proposal $\bar{f}$ following the income increase is at most as large as the number of 'No' votes prior to the shift regardless of what set of voters $\mathbb{V}(\mathbb{N})$ is chosen. Thus, the probability of passing $\pi_2(\bar{f})$ for any given proposal $\bar{f}$ following the income increase is at least as large as the probability of passing $\pi_1(\bar{f})$ prior to the income increase for the same proposal.

Since $\pi_2(\bar{f}) \geq \pi_1(\bar{f})$ holds for any value of $\bar{f}$, $\pi_2(\bar{f}_1^*) \geq \pi_1(\bar{f}_1^*)$ holds for $\bar{f} = \bar{f}_1^*$. Therefore, $\pi_2(\bar{f}_1^*)(\bar{f}_1^* - f_R) \geq \pi_1(\bar{f}_1^*)(\bar{f}_1^* - f_R)$. Since $\bar{f}_2^*$ was defined to be the per pupil funding level that maximizes expected per pupil funding following the income increase, $\pi_2(\bar{f}_2^*)(\bar{f}_2^* - f_R) \geq \pi_2(\bar{f})(\bar{f} - f_R)$ holds for any other value of $\bar{f}$, including $\bar{f}_1^*$. Thus, $\pi_2(\bar{f}_2^*)(\bar{f}_2^* - f_R) \geq \pi_2(\bar{f}_1^*)(\bar{f}_1^* - f_R) \geq \pi_1(\bar{f}_1^*)(\bar{f}_1^* - f_R)$, and the expected per pupil funding increases: $f_R + \pi_2(\bar{f}_2^*)(\bar{f}_2^* - f_R) \geq f_R + \pi_1(\bar{f}_1^*)(\bar{f}_1^* - f_R)$. □

Corollary 2.3.2. *Suppose that the income of every voter in the district increases. Then, all else equal, the expected local contribution in the district increases.*

Proof. Corollary 2.3.1 showed that the maximal local contribution level $\widetilde{E}^i$ that voter i is willing to support increases. By the same argument as in Proposition 2.3.2 with $\widetilde{E}^i$ in place of $\tilde{f}^i$, it can be seen that the expected local contribution in the district also increases following an increase in voters' income. □

While Proposition 2.3.2 and Corollary 2.3.2 state that expected levels of local contribution and per pupil funding will both increase following an upwards shift in income, they do not make conclusions regarding the size of the optimal proposal by the school board. It may be optimal to either increase or decrease the proposal in order to obtain the higher level of expected funding. However, in the model without uncertainty, it is possible to conclude what happens to the size of the proposal.

Corollary 2.3.3 (Income shift without uncertainty)**.** *Suppose that the income of every voter in the district increases, and assume there is no uncertainty. Then, all else equal, the enacted per pupil funding level $\bar{f}^*$ and the enacted local contribution level $\overline{E}^*$ both increase.*

Proof. In the model without uncertainty, the optimal proposed per pupil funding level $\bar{f}^*$ is equal to the median supported per pupil funding level $\tilde{f}^i$, and this proposal passes with probability 1. Since $\tilde{f}^i$ increases for every voter i following an income increase, the median $\tilde{f}^i$ increases, and therefore the optimal proposal does as well.

Since the amount of aid to the district has not changed, the higher per pupil funding level also implies an increase in the local contribution level by (2.2). □

If the median supported per pupil funding level $\tilde{f}^i$ is equal to the reversion in the above corollary, then the optimal proposal by the school board is the reversion amount, and thus the per pupil funding and local contribution levels also do not decrease in this case.

2.3.3 Probability of Passing

Now, I consider the effect of an income increase on the probability of the referendum passing. Since I established that each voter $i \in \mathbb{V}$ is willing to support a greater level of per pupil funding after an income increase in Proposition 2.3.1, I can show that the probability of passing increases assuming that the increase in the supported per-pupil funding level is uniform across voters. Theorem 2.5.2 shows that the same result also holds if the increases in the supported per pupil funding levels are not too far from uniform.

Thus far in the analysis of an income change, the problem of the school board has been formulated in terms of of choosing a per pupil funding level (or local contribution level) that maximizes expected per pupil funding following the increase in voters' income. For each potential per pupil funding level, the school board considers the probability of that proposal passing, which depends on the maximal proposal, $\tilde{f}_2^i$, that each voter i is now willing to support. In order to think about the probability of the referendum passing, I reformulate the school board's maximization problem to one in which the school board chooses an optimal probability of passing $\overline{\pi}^*$ that maximizes expected per pupil funding.

In this formulation, for each $\overline{\pi}$, $f(\overline{\pi})$ denotes the largest per pupil funding level f that can be obtained with probability at least $\overline{\pi}$. The graph of $f(\overline{\pi})$ can be constructed

by connecting the jump discontinuities in the graph of $\pi(\bar{f})$ and reflecting about the 45 degree line. An example with $N = 7$ voters is shown in Figure 2.8.

Precisely, in terms of $f(\overline{\pi})$, the maximization problem for the school board $\mathcal{S}$ is to choose a probability of passing $\overline{\pi}^*$ that maximizes the expected per pupil funding level, or, as in (2.13), one that maximizes expected per pupil funding above the reversion amount

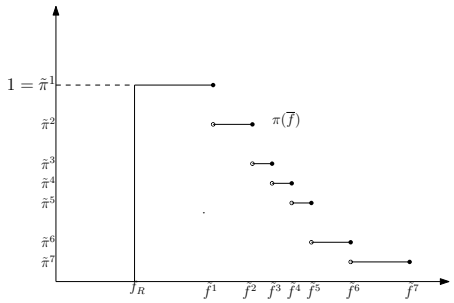
$$\overline{\pi}^* = \underset{\overline{\pi}}{\operatorname{argmax}}\ \overline{\pi}(f(\overline{\pi}) - f_R). \tag{2.19}$$

It is generally not possible to determine how the probability of passing the referendum responds to an increase in the voters' income if the increases in support, $\delta\tilde{f}^i$, are far from uniform across voters. However, with the assumption that the increase $\tilde{f}^i \mapsto \tilde{f}^i + \delta\tilde{f}$ is uniform across voters, I show that the probability of passing must increase after the increase in voters' income. Theorem 2.5.2 in Section 2.5.1 on heterogeneity extends this result to the case of a shift that is "not too far" from uniform.

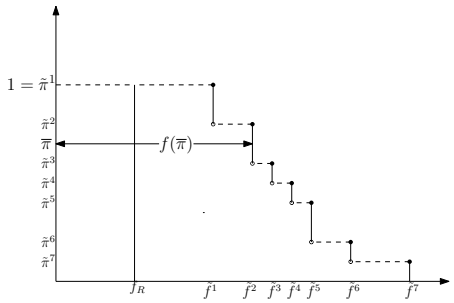
Proposition 2.3.3. *Suppose that there is an increase in the largest per pupil funding proposals that each voter is willing to accept $\tilde{f}^i \mapsto \tilde{f}^i + \delta\tilde{f}$ that is uniform across voters*[5] *(due to an increase in voters' income). Then, all else equal, the probability of passing $\overline{\pi}^*$ increases.*

Proof. The idea behind this proof is to compare the marginal gains and losses from choosing a proposal that is more likely to pass (i.e. a less risky proposal) before and after the increase in voters' income. Suppose that prior to the income increase, $f_1(\overline{\pi})$ is the largest per pupil funding level that will pass the referendum with probability at least $\overline{\pi}$, and $\overline{\pi}_1^*$ is the corresponding probability of passing for the optimal proposal. Following the income increase of ΔY^i for each voter, it was shown in Proposition 2.3.1

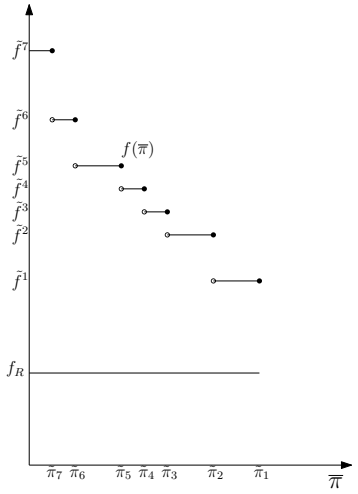
[5] This assumption is only required for those voters with $\tilde{f}^i$ greater than the reversion.



(a) Probability of passing a proposed per-pupil funding level $\overline{f}$



(b) Finding the maximal per pupil funding level that passes with probability at least $\overline{\pi}$



(c) Reflecting figure (b) about the 45° line results in the graph of $f(\overline{\pi})$.

Figure 2.8: Graphing the inverse of $\pi(\overline{f})$ shows the maximum amount of per pupil funding that will pass with probability at least $\overline{\pi}$.

that the maximal level of per pupil funding that voters are willing to accept, $\tilde{f}_2^i$, is greater than or equal to $\tilde{f}_1^i$, by an amount $\delta\tilde{f}^i$. Since this proposition assumes that $\delta\tilde{f}^i$ is uniform across voters, then there is an increase in the function $\tilde{f}^i \mapsto \tilde{f}^i + \delta\tilde{f}$ where $\delta\tilde{f}$ is the size of the uniform increase.

Following this increase, there is a new probability of passing $\overline{\pi}_2^*$ that solves the school board's problem of maximizing expected per pupil funding in (2.19). To show that $\overline{\pi}_2^* \geq \overline{\pi}_1^*$, I compare the marginal returns of a choosing a less risky proposal prior to and following the increase in income and show that the marginal returns of a less risky proposal are higher after the income increase. The marginal returns of choosing a less risky proposal are equal to the marginal gain that comes from a higher probability of passing minus the marginal loss that comes from the proposed funding level being smaller. In Figure 2.9, it can be seen that the loss from choosing a less risky proposal (illustrated by the vertical bars) is the same prior to and following the income increase since $\delta\tilde{f}$ is uniform across voters. However, the marginal gain from choosing a slightly less risky proposal is strictly larger following the income shift, since it is the same gain as prior to the shift plus an additional amount that comes from the increase in $\tilde{f}^i \mapsto \tilde{f}^i + \delta\tilde{f}$. Thus, the marginal returns from choosing a less risky proposal are higher after the income increase compared to beforehand, which means that there is a greater incentive to choose a less risky proposal. Lemma 2.A.2 can now be applied to show that the optimal probability of passing increases. □

Alternatively, it may also be helpful to think about the problem by approximating the school board's objective function in (2.19) with a continuous function that is differentiable in $\overline{\pi}$. In particular, suppose the probability of passing $\overline{\pi}_1^*$ is the solution to the school board's maximization problem. Then $\overline{\pi}_1^*$ satisfies the first order condition $\frac{dF_1}{d\overline{\pi}}(\overline{\pi}_1^*) = 0$, where

$$F_1(\overline{\pi}) = \overline{\pi}(f_1(\overline{\pi}) - f_R)$$

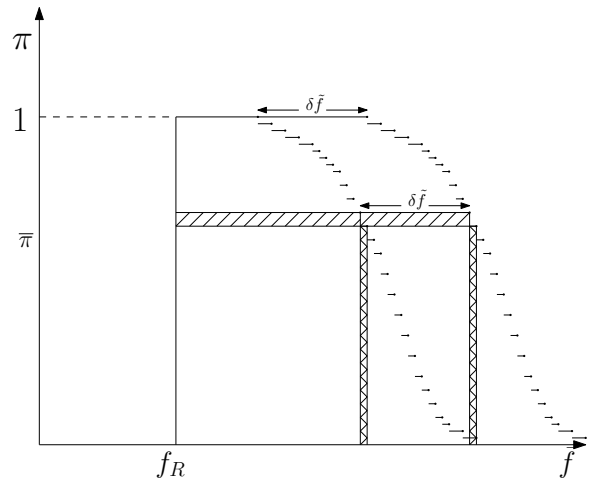


Figure 2.9: Following an increase in voters' income, the school board obtains greater marginal returns from choosing a proposal with increased likelihood of passing if the voter support for per pupil funding increases uniformly. The vertical rectangles in the Figure have the same dimensions when the increase in voter support is uniform.

is the expected per pupil funding beyond the reversion from (2.19). Following the increase in voters' income, there is a new probability of passing $\overline{\pi}_2^*$ that maximizes expected per pupil funding, which satisfies the first order condition $\frac{dF_2}{d\overline{\pi}}(\overline{\pi}_2^*) = 0$, where

$$F_2(\overline{\pi}) = \overline{\pi}(f_1(\overline{\pi}) + \delta\tilde{f} - f_R)$$
$$F_2(\overline{\pi}) = F_1(\overline{\pi}) + \overline{\pi}\delta\tilde{f}.$$

I can compare the objective functions F_2 and F_1 by differentiating with respect to $\overline{\pi}$

$$\frac{dF_2}{d\overline{\pi}}(\overline{\pi}) = \frac{dF_1}{d\overline{\pi}}(\overline{\pi}) + \delta\tilde{f}$$
$$\frac{dF_2}{d\overline{\pi}}(\overline{\pi}) > \frac{dF_1}{d\overline{\pi}}(\overline{\pi}).$$

This calculation shows that if voter support for per pupil funding increases uniformly, then the school board receives higher marginal returns from choosing a proposal with

increased likelihood of passing (as in Figure 2.9). By Lemma 2.A.1, it follows that the optimal probability of passing increases.

2.3.4 Summary

In Section 2.3, I considered an exogenous increase in the income for each voter and analyzed the effects of this income increase on the maximal per pupil funding and local contribution proposals voters would be willing to accept, on the expected per pupil funding and local contribution amounts, and on the probability that the proposal will pass. The results shown in this section have been derived in the case of an income increase for the voters, however, similar results follow for an income reduction of ΔY^i. In particular, the results in Proposition 2.3.1 and Corollary 2.3.1 would hold in the opposite direction for an income reduction. Similarly, the expected per pupil funding and the expected local contribution in the district would decrease in the case of an income reduction for voters. If $\tilde{f}^i \mapsto \tilde{f}^i - \delta\tilde{f}$ in a uniform way across voters, then the probability that the referendum passes $\overline{\pi}^*$ would decline, which can be shown with the same argument as in the proof of Proposition 2.3.3.

Next, I consider a scenario in which the total aid in the district decreases. In a similar fashion to Section 2.3, I will analyze the effect of this decrease in total aid on outcomes such as the maximal local contribution proposals voters would be willing to accept, the expected local contribution amount, and the probability that the proposal will pass. In Section 2.3, it was shown that the reversion scheme of targeting a per pupil funding level corresponded to the same levels of per pupil funding and consumption as the alternative reversion scheme of targeting a local contribution level. However, I will show in the following analysis of a change in the total aid that the results depend on whether the municipal body targets a per pupil funding level or targets a local contribution level.

2.4 Changes in Total Aid

I now analyze the effects of a lump sum change in total aid on school referenda outcomes within the framework of an agenda setter with uncertainty over voter turnout. The decrease in total aid could, for example, be due to a decrease in the state aid or a decrease in the federal aid. For the analysis below, I will focus on the scenario of a decrease in the total aid, and the same analysis can be performed for an increase in total aid. In particular, I will show how a decrease in the total aid affects the maximal local contribution and per pupil funding proposals that voters are willing to accept, the expected local contribution and per pupil funding amounts, and the probability that the referendum will pass.

To begin the analysis, I consider the effect of a lump sum decrease in the total aid on the constraint line faced by the voters. As seen in the income increase scenario, the constraint line is determined such that for each local contribution amount proposed by the school board, the line depicts the largest per pupil funding level that can be afforded by the voter. With a decrease in the total aid, the voter can no longer achieve the same per pupil funding level for each local contribution amount proposed by the school board. In effect, the grant portion of the constraint line has been reduced by the amount of the aid decrease, and the constraint line has shifted inwards. Next, I determine the new maximal local contribution and per pupil funding amounts that each voter would be willing to accept following the decrease in aid. However, this maximal willingness to accept amount depends on the location of the reversion. In contrast to the income increase scenario in which the two reversion schemes – a targeted local contribution amount and a targeted per pupil funding level – were equivalent, in the case of a change in total aid, I will show that the two reversion schemes do not correspond to the same (f, C) bundle. I will consider each scheme, and analyze the effect of a decrease in total aid on school referenda outcomes for each case.

2.4.1 Case 1 - Fixed local contribution

In this first case, the reversion budget in the event of a failed referendum corresponds to a fixed (or targeted) local contribution amount for the voters. Suppose the total aid has decreased by an amount Δf_A such that the new total aid can be denoted by f'_A for each voter, which is less than the original total aid amount f_A. Under the reversion scheme considered in this section, the reversion corresponds to a targeted local contribution amount of E^i_R for each voter i. Prior to the decrease in total aid, the reversion point A corresponded to a per pupil funding level of f_R. Now, after the aid decrease, for an amount of local contribution E^i_R, the voter can no longer afford to achieve f_R but instead can at most achieve $f'_R < f_R$. The reversion now corresponds to a new per pupil funding level that is in contrast to the income increase scenario in Section 2.3 where it was seen that the per pupil funding level f_R under the reversion did not change following the increase in income. Figure 2.10 below depicts voter i's original constraint line $\widehat{C}^i_1(f)$ before the change in total aid and the new constraint line $\widehat{C}^i_2(f)$ following the decrease in total aid.

Using a decreasing marginal returns argument similar to the one in the proof of Propostion 2.3.1, I will show that the maximal local contribution that voter i is now willing to support following a decrease in total aid is at least as large as $\widetilde{E}^i_1$, the maximal local contribution i was willing to support prior to any changes in total aid.

Support for local contribution by voters

Proposition 2.4.1. *Suppose that the total aid for voter i decreases and that the reversion targets a local contribution level. Assume that prior to the aid decrease the voter's ideal point is greater than the reversion. Then, all else equal, the maximal local contribution amount $\widetilde{E}^i$ that voter i is willing to support increases.*

Proof. Denote by $\widetilde{E}^i_1$ the maximal local contribution voter i was willing to support prior to a decrease in total aid, and denote by $\widetilde{E}^i_2$ the maximal local contribution voter

i is willing to support following the aid decrease. The goal is to prove that $\widetilde{E}_2^i \geq \widetilde{E}_1^i$ by showing that there is an increase $\widehat{E}_2^i \geq \widehat{E}_1^i$ in the amount the voter is willing to contribute of his own income. In Figure 2.10 below, which illustrates the voter decision process prior to and following a decrease in total aid by an amount Δf_A, the points A and A' respectively correspond to voter i's reversion funding level before and after the decrease in total aid. I want to show that the point B', the maximal local contribution amount that voter i is willing to support following the decrease in total aid, lies below B, which corresponds to i's maximal local contribution amount prior to the decrease in total aid.

In the income increase analysis in Proposition 2.3.1, I used the property of normal goods to show that preferences exhibited decreasing marginal returns in C and f. In particular, decreasing marginal returns in C implied that voter i was willing to sacrifice more consumption C for any given change in per pupil funding f at a point with more consumption compared to a point with the same per pupil funding but less consumption. This argument was then used to prove that the slope of the indifference curve through the point with more consumption (and equal per pupil funding) was steeper than the slope of the indifference curve through the point with less consumption, and thus the indifference curve through the point with more consumption must take 'longer' before it crosses the new constraint line. A similar argument will be used in the total aid decrease case, except that decreasing marginal returns in f will be the key in the argument instead.

Consider points A and A'. Decreasing marginal returns in f implies that voter i is willing to give up more f to obtain the same amount of consumption at a point where i has more per pupil funding f, such as at point A, than at a point where i has less per pupil funding but equal consumption, such as point A'. This property means that the slope of the indifference curve u_1 at A must be less steep than the slope of the indifference curve u_2 at A', and also that the slope of u_1 must be less steep

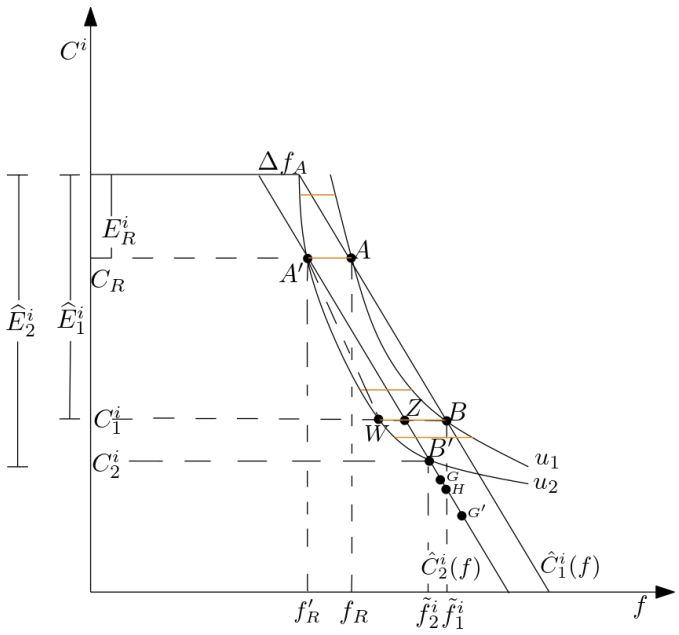


Figure 2.10: Following a decrease in total aid, the voter is willing to support a larger level of local contribution under a target local contribution reversion scheme. The horizontal distance between the indifference curves must increase as C decreases.

than the slope of u_2 at any given level of consumption C. Graphically, decreasing marginal returns in f implies that the horizontal distance between the indifference curves, as depicted by the orange horizontal lines in Figure 2.10, becomes smaller as C increases, and thus becomes larger as C decreases.

Define the point Z to be the point on $\widehat{C}^i_2(f)$ that corresponds to the consumption level C^i_1 of point B, and I define a point W on u_2 that corresponds to the same C^i_1. Since going from point A to point B represents a decrease in consumption, the horizontal distance between A and A' is smaller than the horizontal distance between B and W. Hence, the point W must lie to the left of point Z at the level of consumption C^i_1. In particular, decreasing marginal returns in f means here that the amount of per pupil funding that i is willing to sacrifice to go from B to A is larger than the amount of per pupil funding i is willing to sacrifice to go from the lower per pupil funding level at point W to the point A'.

To obtain that $\widetilde{E}_2^i \geq \widetilde{E}_1^i$, convexity of preferences implies that u_2 lies left of the dashed line connecting A' and W for all values of C between C_1^i and C_R. Therefore, u_2 lies strictly below $\widehat{C}_2^i(f)$ whenever $C_1^i < C < C_R$. It follows that the individual local contribution level $\widehat{E}_2^i$, which corresponds to the consumption level C_2^i and to the point at which the indifference curve u_2 intersects the new constraint line $\widehat{C}_2^i(f)$, is greater than or equal to $\widehat{E}_1^i$. Graphically, point B', which corresponds to the point on u_2 at C_2^i, is below point B. □

While the income increase scenario of Proposition 2.3.1 concluded that there was a corresponding increase in the supported per pupil funding levels $\tilde{f}^i$, in the current scenario of a decrease in total aid under a targeted local contribution reversion scheme, no conclusion is made about whether $\tilde{f}_2^i \lesseqgtr \tilde{f}_1^i$. However, the following corollary shows that the *local* portion of the maximal per pupil funding level a voter is willing to support after a decrease in total aid must increase.

Corollary 2.4.1. *Suppose that the total aid for voter i decreases. Then, all else equal, the maximal local per pupil funding amount $\tilde{f}_L^i$ that voter i is willing to support increases.*

Proof. Proposition 2.4.1 above showed that the maximal local contribution $\widetilde{E}^i$ that voter i is willing to support increases following a decrease in total aid. Using the relationship between E^i and f_L in Equation (2.7) and since τ^i is a constant, $\tilde{f}_L^i$ must also increase when $\widetilde{E}^i$ increases: $\tilde{f}_{L,2}^i \geq \tilde{f}_{L,1}^i$. □

In the following corollaries, even though the quantity $\tilde{f}_L^i$ depends on the voter i, I simplify $\tilde{f}_L^i$ to $\tilde{f}_L$ and no longer include the dependence on i for ease of notation.

Corollary 2.4.2. *Suppose that the total aid for voter i decreases. Then, all else equal, if $\tilde{f}_2^i < \tilde{f}_1^i$, then the decrease in total aid dominated the increase in supported local per pupil funding for voter i, i.e. $|f_A - f'_A| > |\tilde{f}_L^2 - \tilde{f}_L^1|$.*

Proof. Define point H to be the point on $\widehat{C}_2^i(f)$ corresponding to $\tilde{f}_1^i$. If u_2 intersects $\widehat{C}_2^i(f)$ at point H, then $\tilde{f}_2^i = \tilde{f}_1^i$. In order for the maximal per pupil funding level $\tilde{f}_2^i$ that voter i is willing to support following the total aid decrease to be equal to the original maximal per pupil funding level $\tilde{f}_1^i$ that voter i is willing to accept prior to the change in total aid, the change in total aid $|f_A - f'_A|$ must be equal to the change in supported local per pupil funding $|\tilde{f}_L^2 - \tilde{f}_L^1|$. However, if u_2 intersects $\widehat{C}_2^i(f)$ at a point such that $\tilde{f}_2^i < \tilde{f}_1^i$, for example at point G, then it must be the case that $|f_A - f'_A| \neq |\tilde{f}_L^2 - \tilde{f}_L^1|$. Moreover, if $\tilde{f}_2^i < \tilde{f}_1^i$, it must be that $|f_A - f'_A| > |\tilde{f}_L^2 - \tilde{f}_L^1|$. For example, if $|\tilde{f}_L^2 - \tilde{f}_L^1| = 0$, i.e. there is no compensation for the loss in aid, then u_2 would intersect $\widehat{C}_2^i(f)$ at the point Z. In general, if u_2 intersects at the point G to the right of Z but left of H, then the increase in supported local per pupil funding f_L did not fully compensate for the decrease in total aid and $|f_A - f'_A| > |\tilde{f}_L^2 - \tilde{f}_L^1|$. □

Corollary 2.4.3. *Suppose that the total aid for voter i decreases. Then, all else equal, if $\tilde{f}_2^i > \tilde{f}_1^i$, then the increase in local per pupil funding dominated the decrease in total aid, i.e. $|\tilde{f}_L^2 - \tilde{f}_L^1| > |f_A - f'_A|$.*

Proof. Following the same argument as in Corollary 2.4.2, if $\tilde{f}_2^i > \tilde{f}_1^i$, such as at point G', it must be that the increase in supported local per pupil funding more than fully compensated for the decrease in total aid, namely that $|\tilde{f}_L^2 - \tilde{f}_L^1| > |f_A - f'_A|$. □

Expected local contribution

In Proposition 2.4.1, it was shown that the maximal local contribution $\widetilde{E}^i$ that each voter is willing to support increases following a decrease in total aid. Using a similar analysis as the one in Section 2.3.2, I will consider the function $\pi(\overline{E})$ that denotes the probability that a given proposal $\overline{E}$ passes and show that the probability that the proposal passes after the decrease in total aid, $\pi_2(\overline{E})$, is greater than the probability that the proposal passes prior to the aid change, $\pi_1(\overline{E})$, for any given proposal $\overline{E}$. This result can then be used to show that the expected total local

contribution increases after a decrease in total aid. Hence, voters do at least partially compensate for losses of aid, assuming the reversion targets a local contribution level.

Proposition 2.4.2. *Suppose that the total aid of every voter in the district decreases and that the reversion targets a local contribution level. Then, all else equal, the expected total local contribution in the district increases*[6]*.*

Proof. The proof uses the formulation of the school board problem in which the school board $\mathcal{S}$ seeks to choose a proposal $\overline{E}$ that will maximize expected local contributions as in (2.13). Since the proof is analagous to the proof of Proposition 2.3.2, I discuss only the basic steps. Let $\pi_1(\overline{E})$ denote the probability that a given proposal $\overline{E}$ passes prior to the decrease in total aid, and let $\pi_2(\overline{E})$ denote the probability that a proposal $\overline{E}$ passes after the decrease in total aid. Since the $\widetilde{E}^i$ has increased for every voter i, it can be concluded that $\pi_1(\overline{E}) \leq \pi_2(\overline{E})$ for any choice of proposal $\overline{E}$. Using this inequality, it possible to compare the optimal expected local contributions before and after the aid decrease, and thus to conclude that the school board obtains a greater expected total local contribution amount after the decrease in total aid. □

Similarly to Proposition 2.3.2, Proposition 2.4.2 does not conclude what happens to the size of the proposal, and it may be optimal for the school board to either decrease or increase the proposed tax levy in order to obtain the higher expected local contribution. However, in the model without uncertainty, it is possible to determine the change in the size of the proposal.

Corollary 2.4.4 (Aid decrease without uncertainty)**.** *Suppose that total aid decreases and that the reversion targets a local contribution level. Assume there is no uncertainty. Then, all else equal, the proposed and enacted local contribution level increases.*

[6] In the case that every voter who may be pivotal has ideal point below the reversion (which violates Assumption 0), the outcome of the referendum is the reversion, which stays the same rather than increasing.

Proof. In the certainty case, the proposed and enacted contribution level is equal to the median willingness to contribute $\widetilde{E}^i$. Since every voter's willingness to contribute has increased due to the aid decrease, the median $\widetilde{E}^i$ has increased as well. Thus, the local contribution level increases. □

If the median $\widetilde{E}^i$ is equal to the reversion in the above corollary, then the optimal proposal by the school board is the reversion amount, and thus the local contribution level in this case also does not decrease.

Probability of Passing

Similarly to Proposition 2.3.3, it is possible to show that the probability of passing the referendum must increase after a decrease in total aid assuming the increase in the largest proposals that each voter is willing to support following the aid decrease $\tilde{E}^i \mapsto \tilde{E}^i + \delta\tilde{E}$ is uniform across voters. Theorem 2.5.2 then analyzes the case where the shifts in the $\tilde{E}^i$ are not uniform, and Corollary 2.5.2 gives a result on the expected number of 'Yes' votes.

Proposition 2.4.3. *Suppose that there is an increase in the largest local contribution amount that each voter is willing to accept $\tilde{E}^i \mapsto \tilde{E}^i + \delta\tilde{E}$ that is uniform across voters*[7] *(due to a decrease in total aid). Then, all else equal, the probability of passing $\overline{\pi}^*$ increases.*

The specific reversion scheme assumed in this section, namely one that targets a local contribution level, plays a crucial role in obtaining this result. It will be shown in contrast in Section 2.4.2 that in the alternative reversion scheme that targets a per pupil funding level, the probability of passing *decreases* under similar assumptions.

[7] This assumption is only required for voters whose ideal points are initially greater than the reversion.

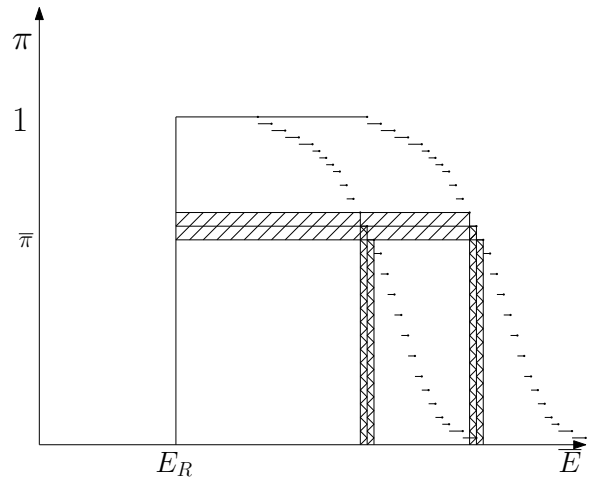


Figure 2.11: Following a decrease in total aid with a fixed local contribution reversion, the school board obtains greater marginal returns from choosing a proposal with increased likelihood of passing if voter support for the level of local contribution increases uniformly. Since the increase in voter support is uniform, the vertical rectangles in the Figure have the same dimensions.

Proof. This proof is analogous to the proof of Proposition 2.3.3, but using the formulation of the school board problem in which the school board chooses a proposal to maximize expected local contribution. □

2.4.2 Case 2 - Fixed per pupil funding

Next, I analyze the effect of a decrease in total aid on school referenda outcomes within a reversion scheme of a fixed (targeted) per pupil funding level. As discussed at the beginning of this section, a fixed per pupil funding reversion does not correspond to the same bundle of goods for the voters as a fixed local contribution reversion. The reason is that with a shift inward of the constraint line by the amount of the total aid decrease Δf_A, the per pupil funding reversion level f_R now corresponds to a different level of consumption and local contribution compared to before the change in total aid. Graphically, while the reversion point A prior to a decrease in total aid corresponds to an amount C_R of consumption, the same level of per pupil funding f_R now corresponds to $C'_R < C_R$ amount of consumption. As seen in the previous

analyses, the location of the reversion matters greatly in determining the new maximal amount of per pupil funding that voter i is willing to support. In these next sections, I derive the effect of the decrease in total aid on school referenda outcomes using the new reversion point A' corresponding to a fixed per pupil funding level f_R.

Support for funding by voters

Proposition 2.4.4. *Suppose that the total aid for voter i decreases and that the reversion is a fixed per pupil funding level. Assume that prior to the aid decrease the voter's ideal point is greater than the reversion. Then, all else equal, the maximal per pupil funding $\tilde{f}^i$ that voter i is willing to support decreases.*

Proof. I denote the original amount of consumption corresponding to the reversion per pupil funding level f_R as C_R represented by point A in Figure 2.12 below. Following the decrease in total aid, f_R now corresponds to a different, lower level of consumption C'_R represented by point A'. By the decreasing marginal returns in consumption argument as in the proof of Proposition 2.3.1, for a given increase in f such as from f_R to $\tilde{f}_1^i$, voter i is willing to sacrifice more consumption at a point where he has more consumption, such as at point A, than at a point with the same per pupil funding level but less consumption, such as at point A'. The slope of the indifference curve u_2 at point A' is therefore less steep than the slope of the indifference curve u_1 at point A. This comparison of the slopes of the indifference curves is true at any given per pupil funding level f, which means that u_1 decreases more quickly than u_2. Let point Z denote the point on $\widehat{C}_2^i(f)$ corresponding to $\tilde{f}_1^i$, and let point X denote the point on u_2 corresponding to $\tilde{f}_1^i$. Then by decreasing marginal returns in C, the point X lies above point Z, which implies that the indifference curve u_2 through point A' lies above the constraint line $\widehat{C}_2^i(f)$ at the per pupil funding level $\tilde{f}_1^i$. Since the slope of u_2 at point A' is steeper than the slope of $\widehat{C}_2^i(f)$, and it was just shown that point X lies above $\widehat{C}_2^i(f)$, by convexity of preferences, there is a unique point between A'

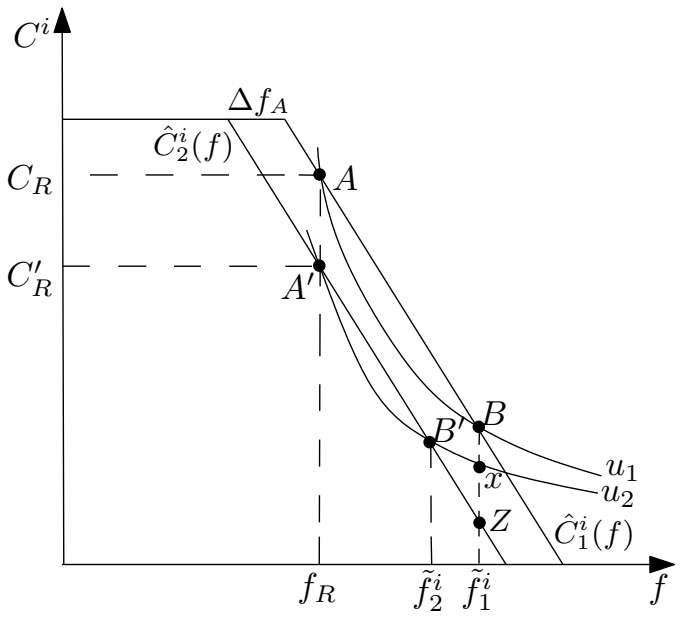


Figure 2.12: Following a decrease in total aid, the maximal per pupil funding that the voter is willing to support decreases under a fixed per pupil funding reversion scheme

and X, call it B', where u_2 intersects $\widehat{C}_2^i(f)$. Since point B' is to the left of point X (which corresponds to $\tilde{f}_1^i$), the point B' corresponding to $\tilde{f}_2^i$ must be to the left of $\tilde{f}_1^i$. Hence, $\tilde{f}_2^i \leq \tilde{f}_1^i$. □

Although this proposition shows that $\tilde{f}_2^i \leq \tilde{f}_1^i$, no conclusive result is established for the maximial local contribution level $\widetilde{E}^i$ that the voter is willing to support following a decrease in aid. Thus, in contrast to the reversion scheme that targets a local contribution level in Section 2.4.1, where Proposition 2.4.1 showed an increase in voters' willingness to contribute, it is not clear whether voters are willing to even partially compensate for losses in aid when the reversion targets a per pupil funding level.

Expected per pupil funding

The expected total per pupil funding is defined in Equation (2.16) as the highest expected total per pupil funding that can be achieved by the school board $\mathcal{S}$.

Proposition 2.4.4 showed that the maximal per pupil funding $\tilde{f}^i$ that voter i is willing to support decreases following a decrease in total aid, and here I show that the expected total per pupil funding decreases after a total aid decrease by using the same argument as in the proof of Proposition 2.3.2.

Proposition 2.4.5. *Suppose that the total aid of every voter in the district decreases. Then, all else equal, the expected total per pupil funding in the district decreases*[8].

Proof. The steps in this proof are analogous to the proof of Proposition 2.3.2. Let $\pi_1(\bar{f})$ denote the probability that a given proposal $\bar{f}$ passes prior to the decrease in total aid, and let $\pi_2(\bar{f})$ denote the probability that a proposal $\bar{f}$ passes after the decrease in total aid. Since $\tilde{f}^i$ has decreased for every voter i, $\pi_1(\bar{f}) \geq \pi_2(\bar{f})$ for any choice of proposal $\bar{f}$. This inequality is the reverse of the inequality in Proposition 2.3.2 for an increase in income, and the optimal expected per pupil funding can be compared from before and after the aid decrease. The result is that the school board obtains a lower expected per pupil funding amount after the decrease in total aid, which is the opposite of the conclusion in Proposition 2.3.2. □

In the case without uncertainty, it is possible to determine what happens to the size of the proposed per pupil funding level.

Corollary 2.4.5. *Suppose that the total aid decreases for every voter in the district and that the reversion targets a per pupil funding level. Assume there is no uncertainty. Then, all else equal, the proposed and enacted per pupil funding level decreases.*

Proof. The proposed per pupil funding level in the certainty case is the median maximal supported per pupil funding level $\tilde{f}^i$, which passes with probability 1. Since every

[8] In the case that every voter who may be pivotal has ideal point below the reversion (which violates Assumption 0), the outcome of the referendum is the reversion, which stays the same rather than decreasing.

$\tilde{f}^i$ has decreased, the median $\tilde{f}^i$ has decreased, and the proposed per pupil funding level therefore decreases. □

Probability of Passing

Proposition 2.4.4 showed that there is a decrease in the maximal per pupil funding proposals that voters are willing to accept following a decrease in total aid. I now analyze the effect of this change on the probability that the referendum will pass. The result I obtain is in contrast to the previous Case 1 of a reversion that targets a local contribution level, where I showed that a decrease in total aid could be associated (under certain assumptions) to an increase in the probability of passing (see Proposition 2.4.3 above and Theorem 2.5.2 below). In the current scenario of a reversion that targets a per pupil funding level, a decrease in total aid leads to a *decrease* in the probability of passing (under similar assumptions) according to Proposition 2.4.6 below.

Proposition 2.4.6. *Suppose that there is a decrease in the largest per pupil funding proposals that each voter is willing to accept* $\tilde{f}^i \mapsto \tilde{f}^i - \delta\tilde{f}$ *that is uniform across voters (due to a decrease in total aid). Then, all else equal, the probability of passing* $\overline{\pi}^*$ *decreases.*

Proof. The proof is analogous to Proposition 2.3.3. Since a *decrease* in the supported per pupil funding levels $\tilde{f}^i \mapsto \tilde{f}^i - \delta\tilde{f}$ occurs in this case, it is found that the probability of passing decreases. □

It is also possible to study nonuniform shifts in the supported per pupil funding levels $\tilde{f}^i$ and to derive a result that is analogous to Theorem 2.5.2 below, which analyzes nonuniform shifts in supported local contribution levels. Furthermore, the pivotal voter argument of Corollary 2.5.2 shows that the expected number of 'Yes' votes decreases under the assumptions of Proposition 2.4.6.

2.4.3 Summary

In the scenarios above where the effect of a lump sum decrease in total aid on school referenda outcomes were analyzed for two reversion schemes, it was shown that a fixed local contribution reversion (Case 1) results in a larger maximal local contribution that voters are willing to support, a larger maximal *local* per pupil funding that voters are willing to support, a higher expected total local contribution amount, and assuming a uniform increase in voters' maximal local contribution levels, an increased probability of passing the referendum. The reversion budget in this scheme now corresponds to a smaller per pupil funding level after the decrease in total aid compared to beforehand. At this lower level of per pupil funding, the voter i is less willing to sacrifice per pupil funding to gain consumption, and as a result the maximal local contribution amount that i is willing to support after the aid decrease $\widetilde{E}_2^i$ is larger than the maximal local contribution amount $\widetilde{E}_1^i$ for voter i before the decrease in total aid. It was also shown that following a decrease in total aid, the expected local contribution amount increases and the probability of the referendum passing also increases assuming a uniform shift in the maximal local contribution amounts that voters are willing to support.

A reversion scheme that targets a fixed per pupil funding (Case 2), in contrast, corresponds to a lower level of consumption after the aid decrease compared to the consumption level of the reversion before the decrease in total aid. The voter in this case is willing to give up less consumption for per pupil funding at the reversion point after the aid decrease compared to the reversion before the change, and the maximal per pupil funding that voter i is willing to support decreases after the decrease in total aid. Proposition 2.4.5 showed that the expected per pupil funding in the district decreases following a decrease in total aid. Assuming a uniform negative shift of $\delta\tilde{f}$ in the maximal per pupil funding proposals supported by the voters, the probability

of passing the referendum also decreases after a decrease in total aid as seen in Proposition 2.4.6.

Similar results regarding the school referenda outcomes can be derived for the scenario of a lump sum increase in the total aid. In particular, given an increase in state aid or an increase in federal aid, the results shown in the Case 1 and Case 2 reversion schemes above hold with the opposite inequalities. Namely, in Case 1 (a targeted local contribution level), for an increase in total aid, the maximal local contribution amount $\widetilde{E}^i$ that voter i is willing to support decreases. In this case, the constraint line $\widehat{C}_2^i(f)$ shifts outward compared to the original constraint line $\widehat{C}_1^i(f)$ before the increase in total aid, and the result follows from the same reasoning as in the proof of Proposition 2.4.1. Corollary 2.4.1 therefore also holds true but in the opposite direction, i.e. the maximal local per pupil funding amount $\tilde{f}_L$ that voter i is willing to support decreases, ceteris paribus, after an increase in total aid. I show that the expected local contribution decreases after an increase in total aid by using the same reasoning as in Proposition 2.4.2. After an increase in total aid, the probability of the referendum passing decreases under a fixed local contribution reversion scheme, which can be proved as in Proposition 2.4.3. For the fixed per pupil funding reversion scheme in Case 2, the results in Propositions 2.4.4, 2.4.5, and 2.4.6 hold in the opposite direction after an increase in total aid. In particular, following an increase in total aid, the maximal per pupil funding that voters are willing to support increases, the expected per pupil funding increases, and the probability of the referendum passing increases under the uniformity assumption.

2.4.4 Comparison of Reversion Schemes

I now compare the two reversion schemes to see which reversion scheme results in higher levels of expected per pupil funding following a decrease in aid. This question

may be useful to a policymaker who is deciding how to structure the reversion rule in a state.

I first show that the maximal per pupil funding level $\tilde{f}^i$ voters are willing to support under a target local contribution reversion after a decrease in total aid is larger than the same voters' maximal per pupil funding levels under the alternative reversion scheme of a fixed per pupil funding amount.

Proposition 2.4.7. *Suppose that the total aid of every voter in the district decreases. Then, all else equal, the maximal per pupil funding $\tilde{f}^i$ that voter i is willing to support is higher under a target local contribution reversion scheme than under a target per pupil funding reversion.*

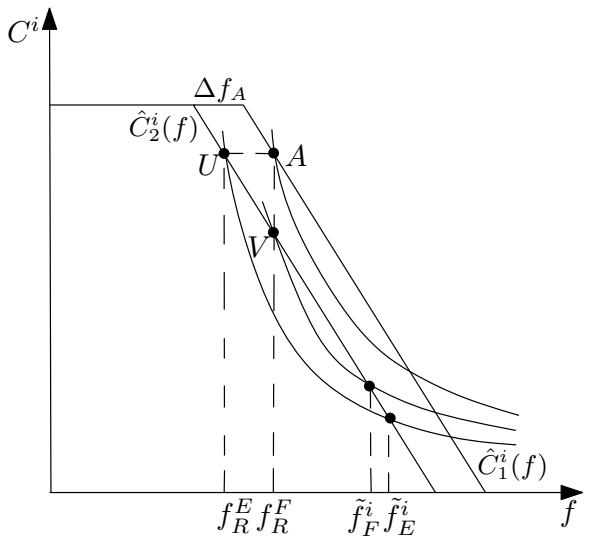


Figure 2.13: Following a decrease in total aid, the maximal per pupil funding that the voter is willing to support under a fixed local contribution reversion is larger than the maximal per pupil funding under a fixed per pupil funding reversion scheme.

Proof. In Figure 2.13 above, let f_R^E (point U) denote the per pupil funding level associated with the reversion point following a decrease in total aid under the targeted local contribution reversion, and let f_R^F (point V) denote the per pupil funding level associated with the reversion point after the aid decrease under the targeted per

pupil funding reversion scheme. I first show that $f_R^E < f_R^F$. Under a targeted per pupil funding reversion, by definition the per pupil funding level associated with the reversion point V, namely f_R^F, is the same as the original per pupil funding level f_R before the change in total aid. Since the total aid has decreased, the constraint line $\widehat{C}_2^i(f)$ has shifted inward compared to $\widehat{C}_1^i(f)$ for values of $f \geq f_A$ by the amount of the aid change Δf_A. Given that the original f_R before the change in total aid is greater than f_A, the point on $\widehat{C}_2^i(f)$ corresponding to the same local contribution level as point A, which is point U, must correspond to a level of per pupil funding less than f_R. Thus $f_R^E \leq f_R^F$.

Next, I show that the maximal per pupil funding level voters are willing to support after a decrease in total aid under a targeted local contribution reversion represented by point U is larger than the maximal per pupil funding level voters are willing to support under a targeted per pupil funding reversion represented by point V. Since $f_R^E \leq f_R^F$, the smaller reversion amount results in an increase in the maximal proposal voters are willing to support, ceteris paribus (Romer & Rosenthal, 1979a). Thus, following a decrease in total aid, the maximal per pupil funding level voters are willing to support under the targeted local contribution reversion, denoted by $\tilde{f}_E^i$, is larger than the maximal per pupil funding level voters are willing to support under the targeted per pupil funding reversion, denoted as $\tilde{f}_F^i$. Graphically, $\tilde{f}_E^i \geq \tilde{f}_F^i$. □

Proposition 2.4.7 shows that, following a cut in aid, the maximal per pupil funding $\tilde{f}^i$ that voter i is willing to support is larger under a reversion that targets local contribution compared to a reversion that targets per pupil funding. While in previous results it was possible to show that a higher supported per pupil funding level $\tilde{f}^i$ results in a higher expected per pupil funding level, I cannot make the same conclusion in this case since the two reversions are not the same. In the following analysis, I explain why no conclusive relationship can be determined.

Let $\pi_E(\bar{f})$ denote the probability that a proposed per pupil funding level $\bar{f}$ passes, following the cut in aid, under a reversion that targets local contribution, and let $\pi_F(\bar{f})$ denote the probability of passing the proposal $\bar{f}$ following the cut in aid under a reversion that targets per pupil funding. Since $\tilde{f}_E^i \geq \tilde{f}_F^i$ for any voter i, $\pi_E(\bar{f}) \geq \pi_F(\bar{f})$ for any possible proposed per pupil funding level $\bar{f}$. As a result, it is possible to say that the expected per pupil funding *above the reversion* is larger under a reversion that targets local contribution. Namely, if $\bar{f}_E^*$ and $\bar{f}_F^*$ are the optimal proposals under reversions that target local contribution and per pupil funding respectively, then the expected per pupil funding level above the respective reversions have the following relationship

$$\pi_F(\bar{f}_F^*)(\bar{f}_F^* - f_R^F) \leq \pi_E(\bar{f}_F^*)(\bar{f}_F^* - f_R^E) \leq \pi_E(\bar{f}_E^*)(\bar{f}_E^* - f_R^E),$$

where the first inequality can be seen from $f_R^E \leq f_R^F$ and the second inequality is true because $\bar{f}_E^*$ maximizes expected funding beyond the reversion amount under a reversion that targets local contribution.

Thus, the reversion that targets local contribution leads to greater expected funding beyond the reversion. However, the total expected per pupil funding amount, $\pi(\bar{f}^*)\bar{f}^* + (1 - \pi(\bar{f}^*))f_R$, which simplifies to

$$f_R + \pi(\bar{f}^*)(\bar{f}^* - f_R)$$

has two terms, one that is the reversion and another that is the expected per pupil funding amount beyond the reversion. Therefore, the expected per pupil funding levels for the two reversion schemes have the following inconclusive relationship

$$f_R^F + \pi_F(\bar{f}_F^*)(\bar{f}_F^* - f_R^F) \quad \lesseqgtr \quad f_R^E + \pi_E(\bar{f}_E^*)(\bar{f}_E^* - f_R^E).$$

Namely, the first terms have the relationship $f_R^F > f_R^E$, while the second terms, which represent expected funding beyond the reversion, have just been shown above to have the opposite relationship.

2.5 Heterogeneity in Districts

I now explore the impact of heterogeneity and homogeneity on the agenda setter's optimization problem, and illustrate that homogeneity in a district causes a budget-maximizing school board to choose proposals with a high probability of passing. I first describe two measures of heterogeneity in a district.

Intuitively, a district can be thought of as homogeneous if all voters in the district are willing to support similar levels of local tax contributions for schools, which means that there is only a small gap between the smallest and the largest values of the voters' willingness to contribute in the district (i.e. the difference $\Delta = \max_i \widetilde{E}^i - \widetilde{E}^1$ is small). I call this definition *global* homogeneity in a district. There is also the idea of *local* homogeneity in a district, which is when the difference in the willingness to contribute between any two nearby voters is small (i.e. the difference $\delta\widetilde{E}^i = \widetilde{E}^{i+1} - \widetilde{E}^i$ is small for all i). These two different notions lead to two different measures of heterogeneity in a district that are defined below, which I will use to prove several theorems that connect homogeneity in a district to a high probability of passing for voter referenda.

Definition 2.5.1. The global degree of heterogeneity of a district is $\Delta \equiv \max_i \widetilde{E}^i - \widetilde{E}^1$. The local degree of heterogeneity at voter i is defined as $\delta\widetilde{E}^i \equiv \widetilde{E}^{i+1} - \widetilde{E}^i$.

The local degree of heterogeneity is similar to the (inverse) density of the supported contribution levels $\widetilde{E}^i$ in the district. When the supported contribution levels of nearby voters are close to one another, the local degree of heterogeneity $\delta\widetilde{E}^i$ is small.

The budget-maximization problem for a school board in a highly homogeneous district is illustrated in Figure 2.14. In this figure, the school board considers the

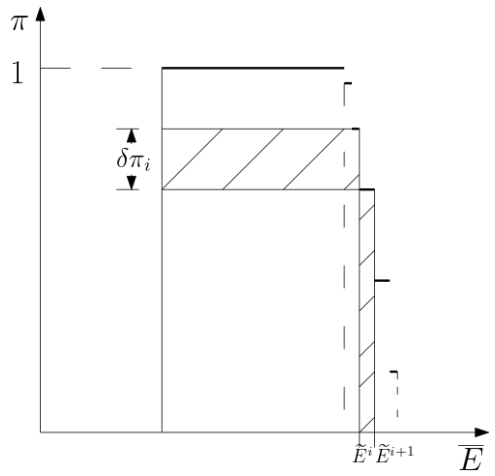


Figure 2.14: The school board problem in a highly homogeneous district

marginal returns of choosing a riskier proposal (i.e. one with smaller probability of passing π) by comparing the marginal cost of choosing a riskier proposal, which equals the area of the horizontal rectangle of height $\delta\pi_i$, to the marginal gain, which equals the area of the thin vertical rectangle of width $\delta\widetilde{E}^i = \widetilde{E}^{i+1} - \widetilde{E}^i$. In this case, the marginal gain of choosing a riskier proposal is small compared to the marginal cost because the local degree of heterogeneity at voter i is very small ($\delta\widetilde{E}^i$ is small compared to $\delta\pi_i$).

This example illustrates how being in a district that is more homogeneous incentivizes a school board to choose a proposal with a higher probability of passing. The reason shown is that increasing the size of the proposal by a small amount rapidly reduces the probability of passing and therefore the expected budget. In this case, it is safer for the school board to choose a proposal with a high probability of passing that is close to the minimal willingness to contribute $\widetilde{E}^1$. In contrast, an example of a heterogeneous district is given in Figure 2.15. In this example, the district is composed of two groups of voters where one group has much lower willingness to contribute than the other. It is optimal for the school board in this district to choose a high risk proposal that aims at the group with higher willingness to contribute.

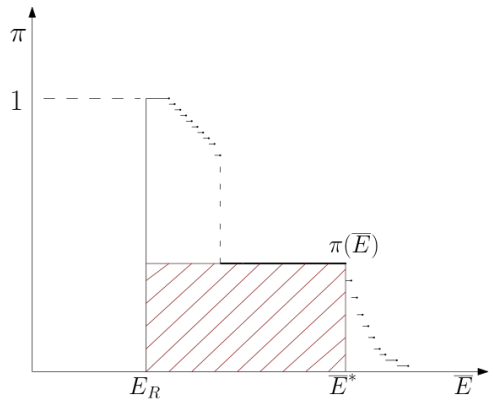


Figure 2.15: The school board problem in a heterogeneous district with two groups of voters

Although the following analyses show that homogeneous districts have higher passing rates, the agenda setter's proposal may still be significantly larger than the ideal spending level of the median voter. The higher passing rates in these districts do not imply that the school boards are aiming to represent the median voter. Rather, it may be the case that the school boards in homogeneous districts behave instead like budget-maximizers to the same extent as they do in heterogeneous districts, but that the solution to the budget-maximization problem in a homogeneous district consists of proposing a budget that is more popular.

2.5.1 Theorems connecting homogeneity to higher probability of passing

I now derive three theorems that show the way in which the degree of heterogeneity in a district corresponds to the probability of the referendum passing. The first theorem focuses on the global degree of heterogeneity, and the second and third theorems are based on the local degree of heterogeneity. All three theorems conclude that more district homogeneity is associated with a higher probability of passing.

In the first theorem, I show that in a district that is globally very homogeneous, the budget-maximizing school board finds it optimal to choose a proposal with a high probability of passing. In this theorem, I assume that all voters who may be pivotal have willingness to contribute $\widetilde{E}^i$ above the reversion.

Theorem 2.5.1. *Suppose that the minimal willingness to contribute $\widetilde{E}^1$ is greater than the reversion i.e. $\widetilde{E}^1 - E_R > 0$. Let $\Delta \equiv \max_i \widetilde{E}^i - \widetilde{E}^1$ be the global degree of heterogeneity in the district. Then the optimal probability of passing π^* for the school board's proposal is greater than or equal to:*

$$\pi^* \geq \frac{\widetilde{E}^1 - E_R}{\widetilde{E}^1 - E_R + \Delta}. \tag{2.20}$$

In particular, if the district is very homogeneous (i.e. if Δ is close to 0), then the optimal probability of passing must be close to 1.

Proof. First, I compare the optimal proposal $\overline{E}^*$ to the proposal $\widetilde{E}^1$. Since $\overline{E}^*$ maximizes expected funding (and thus maximizes expected funding beyond the reversion), and since a proposal of $\widetilde{E}^1$ would pass with probability $\pi(\widetilde{E}^1) = 1$, it follows that

$$\pi(\widetilde{E}^1)(\widetilde{E}^1 - E_R) = \widetilde{E}^1 - E_R \leq \pi^*(\overline{E}^* - E_R).$$

Since $\overline{E}^* \leq \max_i \widetilde{E}^i = \widetilde{E}^1 + \Delta$, then

$$\widetilde{E}^1 - E_R \leq \pi^*(\widetilde{E}^1 + \Delta - E_R).$$

Dividing both sides by $\widetilde{E}^1 + \Delta - E_R$ results in (2.20).

In particular, a very homogeneous district has Δ close to 0, and thus the quantity

$$\frac{\widetilde{E}^1 - E_R}{\widetilde{E}^1 - E_R + \Delta}$$

is very close to 1, which implies that the optimal probability π^* must also be close to 1 using (2.20). □

While Theorem 2.5.1 concludes that the optimal proposal will have a high probability of passing for a very homogeneous district, this low-risk proposal may still be significantly larger than the ideal spending level of the median voter. Thus, the effect of the budget-maximizing school board in enlarging the budget proposal can still occur.

The next theorem examines an increasing shift in voters' willingness to contribute $\widetilde{E}_1^i \mapsto \widetilde{E}_2^i$. This shift in willingness to contribute may be in response to, for example, a policy or event such as a change in aid as studied in the previous sections. An analogous theorem holds for shifts in the maximal per pupil funding levels $\tilde{f}_1^i \mapsto \tilde{f}_2^i$ that the voters are willing to support.

Theorem 2.5.2 (Increases in willingness to contribute)**.** *Suppose there is a shift in the voters' willingness to contribute $\widetilde{E}_1^i \mapsto \widetilde{E}_2^i$ such that the $\widetilde{E}^i$'s remain in order, the reversion E_R remains unchanged, and such that $\widetilde{E}_2^i \geq \widetilde{E}_1^i$ for every voter i. Then as long as the local degree of heterogeneity has not increased by too much, the probability of passing increases.*

The condition that is needed for heterogeneity to "not increase by too much" will be given in the proof.

Proof. The proof uses the marginal analysis that is illustrated in Figure 2.16. In the figure, the thick black step function denotes the probability of passing as a function of the proposal prior to the shift, $\pi_1(\overline{E})$, and the thick orange step function denotes the probability of passing as a function of the proposal following the shift, $\pi_2(\overline{E})$. Since I assume the voters stay in the same order following the shift, the size of the jumps in probability $\delta\pi_i$ remain of the same size, but the locations where the jumps occur shift to the right from $\widetilde{E}_1^i$ to $\widetilde{E}_2^i$.

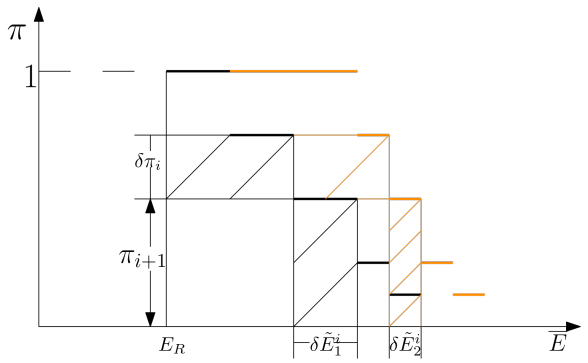


Figure 2.16: An increasing shift in willingness to contribute that does not increase the local degree of heterogeneity

Since the school board uses marginal analysis to choose its budget-maximizing proposal, it is sufficient to show that the marginal returns of choosing a less risky proposal are now greater than what they were prior to the shift in the voters' willingness to contribute. Let MR_1^i be the marginal returns of decreasing the proposal from $\widetilde{E}_1^{i+1}$ to $\widetilde{E}_1^i$ prior to the shift (for $\pi_1(\overline{E})$), and let MR_2^i be the marginal returns of decreasing the proposal from $\widetilde{E}_2^{i+1}$ to $\widetilde{E}_2^i$ following the shift (for $\pi_2(\overline{E})$). The goal is to show that $\text{MR}_2^i \geq \text{MR}_1^i$ for all i. I can then apply Lemma 2.A.2 to show that the probability of passing for the optimal proposal does not decrease.

The marginal returns of choosing a less risky proposal are the marginal gains minus the marginal costs, which are illustrated in Figure 2.16 as

$$\begin{aligned}
\text{MR}_1^i &= \text{MG}_1^i - \text{MC}_1^i, & \text{MR}_2^i &= \text{MG}_2^i - \text{MC}_2^i, \\
\text{MG}_1^i &= \delta\pi_i \cdot \tilde{E}_1^i, & \text{MG}_2^i &= \delta\pi_i \cdot \tilde{E}_2^i, \\
\text{MC}_1^i &= \delta\widetilde{E}_1^i \cdot \pi_{i+1}, & \text{MC}_2^i &= \delta\widetilde{E}_2^i \cdot \pi_{i+1}.
\end{aligned}$$

The assumption that the $\widetilde{E}^i$'s remain in order ensures that the jumps $\delta\pi_i$ and the probabilities π_{i+1} do not change in size following the shift.

As seen in Figure 2.16, the marginal gain has increased by an amount

$$\mathrm{MG}_2^i - \mathrm{MG}_1^i = \delta\pi_i(\tilde{E}_2^i - \tilde{E}_1^i) > 0.$$

Therefore, the marginal returns will increase ($\mathrm{MR}_2^i > \mathrm{MR}_1^i$) as long as the marginal cost has not increased by an amount larger than the increase in the marginal gain. Namely, the marginal returns will increase unless:

$$\mathrm{MC}_2^i - \mathrm{MC}_1^i > \mathrm{MG}_2^i - \mathrm{MG}_1^i,$$

$$\text{which holds iff} \qquad (\delta\widetilde{E}_2^i - \delta\widetilde{E}_1^i)\pi_{i+1} > \delta\pi_i(\tilde{E}_2^i - \tilde{E}_1^i) \qquad \text{for some } i. \tag{2.21}$$

Thus, unless the local degree of heterogeneity increases by "too much" as in (2.21), the marginal returns of choosing a less risky proposal will increase. Using Lemma 2.A.2, the school board is incentivized to choose a less risky proposal following the shift (i.e. a proposal that has a probability of passing $\pi_2(\overline{E}_2^*)$ that is at least as large as the probability of passing $\pi_1(\overline{E}_1^*)$ before the shift). □

Corollary 2.5.1. *Under the assumptions of Theorem 2.5.2, if the local degree of heterogeneity does not increase due to the shift in voters' willingness to contribute, then the probability of passing increases.*

An example of Corollary 2.5.1 is illustrated in Figure 2.16, where it appears that the local degree of heterogeneity has not increased following the shift in the voters' willingness to contribute. In this case, the change in marginal cost (the left hand side of (2.21)) is not positive for any voter i, as illustrated by the size of the orange vertical rectangles compared to the black vertical rectangles.

Corollary 2.5.2. *Under the assumptions of Theorem 2.5.2 or Corollary 2.5.1, the expected number of* Yes *votes increases as well.*

Proof. Let the pivotal voter i^* be the voter with the smallest willingness to contribute $\widetilde{E}^i$ among all voters who vote 'Yes' to the proposal. If $\overline{E}^*$ is the optimal proposal, then $\overline{E}^*$ must be equal to the willingness to contribute of the pivotal voter $\widetilde{E}^{i^*}$, i.e. $\overline{E}^* = \widetilde{E}^{i^*}$. Ordering the voters by their willingness to contribute (which is assumed to be the same ordering before and after the shift), the number of 'Yes' voters is the number of participating voters i having $i \geq i^*$.

Let i_1^* be the pivotal voter prior to the shift, and i_2^* be the pivotal voter following the shift. Then the increase in the probability of passing (Theorem 2.5.2) implies that the pivotal voter has not increased, i.e. $i_2^* \leq i_1^*$. Therefore, regardless of which voters participate, the number of 'Yes' votes following the shift (the number of participating voters with $i \geq i_2^*$) is at least as large as the number of 'Yes' votes prior to the shift (the number of participating voters with $i \geq i_1^*$). Thus, the expected number of 'Yes' votes goes up. □

Theorem 2.5.2 and Corollary 2.5.2 imply that under certain assumptions, policies or events that lead to an increase in the voters' willingness to contribute to education spending will likely result in more popular proposals. However, if the heterogeneity in the district changes by "too much" in response to the policy or event, then this result does not hold. One example of a policy that could lead to an increase in heterogeneity would be if the policy affected some groups of voters much more so than other ones, and a situation such as the one in Figure 2.15 could occur. For example, an economic upturn that disproportionately benefits high income voters could lead to a greater amount of heterogeneity in the district, and it may be optimal for the school board to choose a higher risk proposal aimed at the higher income voters.

Similar to Theorem 2.5.2, the following theorem shows that policies or events that decrease the local heterogeneity in a district will also increase the probability of the referendum passing, assuming that the policy or event does not decrease the voters' willingness to contribute by "too much" as defined below.

Theorem 2.5.3. *Suppose there is a shift in the voters' willingness to contribute* $\widetilde{E}_1^i \mapsto \widetilde{E}_2^i$ *such that the* $\widetilde{E}^i$*'s remain in order, the reversion* E_R *remains unchanged, and such that the local degree of heterogeneity decreases i.e.* $\delta\widetilde{E}_2^i < \delta\widetilde{E}_1^i$ *for all* i*. Then as long as the willingness to contribute does not decrease by too much, the probability of passing increases, as does the expected number of* Yes *votes.*

Proof. The proof follows the same marginal analysis of comparing marginal gains and marginal losses that was used to prove Theorem 2.5.2. In this case, the decrease in the local degree of heterogeneity means that the marginal cost of choosing a less risky proposal has decreased. Therefore, the marginal returns of choosing a less risky proposal will increase unless the marginal gains have decreased by even more. Namely, the marginal returns will increase unless:

$$\mathrm{MC}_1^i - \mathrm{MC}_2^i < \mathrm{MG}_1^i - \mathrm{MG}_2^i,$$

$$\text{which holds iff} \qquad (\delta\widetilde{E}_1^i - \delta\widetilde{E}_2^i)\pi_{i+1} < \delta\pi_i(\tilde{E}_1^i - \tilde{E}_2^i) \qquad \text{for some } i. \tag{2.22}$$

This condition is the same as the condition (2.21) in the proof of Theorem 2.5.2. Thus, unless the voters' willingness to contribute have decreased by "too much" as in (2.22), the marginal returns of choosing a less risky proposal will increase. The result that the expected number of 'Yes' votes increases can be shown using the same steps as in the proof of Corollary 2.5.2. □

While the scenarios in Theorems 2.5.1-2.5.3 and Corollary 2.5.1 vary from one another, there is one unifying takeaway message that can be gleaned from the analysis: In a very homogeneous district, the school board only gains a small amount while risking a lot by choosing a riskier, larger proposal.

2.6 Conclusion

In this paper, I study predictions of school referenda outcomes in a budget-maximizing agenda setter model with uncertainty over voter turnout. In using fundamental economics concepts such as the relationship between normal goods, convexity of preferences, and decreasing marginal returns, I have built a framework for analyzing the effects of changes in voters' income or in a district's level of state aid that may be due to a range of economic events or policies. In contrast to the simpler median voter model, in which the comparative statics consist of studying the optimization problem for a single representative voter, the agenda setter model requires a new argument to perform the analysis. The key idea in the agenda setter setting is to use the normal goods assumption and convexity of preferences to show that indifference curves have specific "pinching" and "expanding" properties, which are illustrated in Figures 2.6 and 2.10. These properties can then be used to analyze the effect on the maximal tax contributions that voters are willing to support for schools, and conclusions can be made regarding specific school referendum outcomes such as the expected amount of school funding and the probability that the referendum passes. Table 2.1 contains a summary of the comparative statics results that I have obtained in this paper. Table 2.2 specifically focuses on the model without uncertainty, and compares the agenda setter predictions I find to the predictions of the median voter model without uncertainty.

In the first scenario of an exogenous increase in voters' income, I show that an increase in income leads to higher expected levels of local tax contributions and per pupil funding. In particular, in the absence of uncertainty, local tax contributions and per pupil expenditures will be higher, and this prediction coincides with the median voter model. The first key point is to show that the maximum levels of local contribution and per pupil funding that voters are willing to support increase following the increase in voters' income (Proposition 2.3.1). If this increase in supported

	Income Increase		Aid Decrease	
Reversion targets	Local Contribution	Per Pupil Funding	Local Contribution	Per Pupil Funding
Expected Local Contribution	Increases		Increases	Unknown
Expected Per Pupil Funding	Increases		Unknown	Decreases
Voter support*	Increases*		Increases*	Decreases*

Table 2.1: Results for agenda setter model with uncertainty over voter turnout. Voter support is either the probability of passing or the expected 'Yes' vote.
*Increases/Decreases if voter heterogeneity does not increase/decrease by "too much."

Scenario	Agenda setter, Target LC Reversion	Agenda setter, Target PPF Reversion	Median Voter
Income Increase	Local tax contributions increase Per pupil funding increases		
Aid Decrease	Local contributions increase Unknown effect on PPF	Unknown effect on LC Per pupil funding decreases	Local contributions increase Per pupil funding decreases

Table 2.2: Median voter model vs. Agenda setter model* without uncertainty
LC = Local tax Contribution; PPF = Per Pupil Funding
*If the median willingness to contribute is below the reversion, the local tax contribution and per pupil funding amounts may remain unchanged in either scenario.

funding levels is not too far from uniform, then the optimal proposal will be more popular; specifically, there will be an increase in the probability of the referendum passing (Proposition 2.3.3, Theorem 2.5.2) and a higher expected number of 'Yes' votes (Corollary 2.5.2). The opposite results hold in the event of a decrease in voters' income, namely lower local tax contributions, lower per pupil funding, and lower voter approval (under the uniformity assumption).

In analyzing the effect of a lump sum change in total aid on school funding outcomes, it is important to distinguish between different types of rules for the reversion, which is the level of funding and local tax contribution that is enacted in the event that the proposal fails. Two different reversion rules are considered: one in which the municipal body targets a specific local tax contribution level, and another in which the municipal body instead targets a per pupil funding level. The target local contri-

bution assumption occurs more frequently in practice (Section 2.2.4). If the reversion targets a local contribution level, then it is shown that an aid decrease will lead to an increase in the expected *local* contribution (Proposition 2.4.2), which means that voters at least partially compensate for losses in aid. This result is consistent with the typical median voter theory. However, it is not clear whether the threat of a low reversion might cause voters to overcompensate for the loss in aid, since it is not known whether the expected per pupil funding decreases or increases. If the reversion instead targets a per pupil funding level, then it is shown that the expected per pupil funding level does not increase, which implies that a decrease in aid causes a loss in expected funding for schools, consistent with median voter theory. However, in this case, it is not known whether there will be any increase in the expected amount of local contribution to partially compensate for the loss in aid.

One main component of this study is to analyze how exogenous changes affect the rate of voter approval for school funding referenda. Studying the rate of voter approval is important because the failure of budget proposals affects the level of educational services a district provides, and lower passage rates can result in local funding becoming more volatile. In a scenario in which voters' incomes have decreased, Theorem 2.5.2 and Corollary 2.5.2 give conditions under which voter approval may decrease as well, making local funding less secure. Using a similar analysis, I find that the effect of a change in aid depends on the structure of the reversion scheme. For reversions that have a target local tax contribution, decreases in aid would lead to higher voter approval under certain assumptions (Proposition 2.4.3, Theorem 2.5.2 and Corollary 2.5.2), making local funding more secure, whereas for reversions that target per pupil funding, decreases in aid would result in *lower* voter approval under similar assumptions (Proposition 2.4.6). The reason for this difference lies in how voters perceive the "threat" of the reversion. When a reversion targets a local contribution level, a reduction in aid makes the reversion more threatening, since the

loss in aid falls entirely on schools if the referendum fails. In contrast, if a reversion targets a per pupil funding level, then the reversion provides more of a "safety net" that guarantees a certain amount of funding for schools despite the loss of aid.

In analyzing the factors that influence voter support, I establish an important relationship between voter approval and homogeneity in a district within the agenda setter model. In particular, compared to heterogeneous districts, homogeneous districts are expected to have higher passage rates and higher levels of voter support, as both theoretical results in the median voter setting and empirical findings illustrate (Romer et al., 1992; Zhao, 2020). I show that this link between homogeneity and high voter approval can be derived as a result of the agenda setter's budget-maximization problem under uncertainty over voter turnout. Specifically, Theorem 2.5.1 illustrates that the passage rate will be high in districts that are globally very homogeneous, and Theorem 2.5.3 shows that policies or events that increase homogeneity in a district tend to increase voter approval, all else equal. The key reason is that a budget-maximizing agenda setter in a homogeneous district has little incentive to choose a high risk proposal with a lower probability of passing, because voters in these districts have similar preferences about what funding levels they support, and a high risk proposal will only accrue small gains in expected funding when compared to lower risk proposals (Figure 2.14). In contrast, more heterogeneity in a district can make it optimal for school boards to choose more high-risk proposals (Figure 2.15).

These results also provide an alternative explanation for one of the main findings of Romer et al. (1992), namely that agenda setting school boards in smaller districts tend to behave more like median voter representatives, while agenda setters in larger districts tend to behave more like budget-maximizers. This finding is based on the empirical observation that school boards in smaller districts tend to have smaller, more popular budget proposals, while school boards in larger districts tend to have larger, less popular budget proposals. This observation is also consistent with an al-

ternative theory given in this paper that 1) smaller districts are more homogeneous, and 2) agenda setters in both larger and smaller districts are choosing proposals to maximize the expected budget in their districts; however, 3) the budget-maximization problem in the smaller, more homogeneous districts results in a more popular (and thus typically smaller) budget proposal. Thus, it is possible for agenda setters in smaller, more homogeneous districts to appear as if they are representing the median voter's interests with more popular proposals, but their more popular proposals may instead be optimal for maximizing the expected budget. Knowing this fact, it is also important to emphasize that even though the budget-maximizing proposals in homogeneous districts may pass by a large margin, these proposals can still be significantly larger than the ideal spending level of the median voter.

Looking forward, there are several worthwhile directions in which the analysis of this work could be extended in future studies. In addition to the changes in income and aid that have been studied here, an important type of exogenous change to consider is a change in an individual's tax price for education. A change in an individual's tax price can occur due to changes in the state aid formula, for example, or to changes in local property values. Graphically, a change in an individual's tax price corresponds to a change in the slope of the constraint line. The methods developed in this work can be extended to analyze such changes, and similar to the case of a change in aid, the type of reversion rule matters. Other state aid rules such as matching grants would also be important to study, but would require additional development of the analysis performed thus far.

While the model in this paper incorporates uncertainty over voter turnout and assumes that the reversion budget is known with certainty, there are several ways in which the model could be extended to be more realistic. For example, it is possible that the school board members do not know the preferences of the voters with certainty (as in Appendix Section 2.B). Also, in the case of New Jersey, where the

reversion is decided by a committee and not dictated by law, the reversion is likely not known with certainty, and it would be practical to incorporate uncertainty over the reversion into the model. While the proposal is always larger than the reversion in the current model, introducing this uncertainty into the model would allow for equilibria where the proposal can be equal to or less than the reversion[9]. This feature of the extended model could provide one explanation for the many failed budgets in New Jersey that do not receive any reductions from the committee.

2.A Appendix

The goal of the following lemmas is to establish a way to compare the maximizers of two functions based on the relationship between their derivatives. In particular, these lemmas will use the comparison of the marginal returns of two functions to give a relationship between the values that maximize those functions. Specifically, if a function $g(x)$ has greater marginal returns to increasing x compared to a function $f(x)$, namely that $f'(x) \leq g'(x)$, then g faces a greater incentive to increase x compared to f, so a maximizer of g should be greater than or equal to a maximizer of f. This concept is formalized in Lemma 2.A.1 below, which is a variation of Athey, Milgrom, and Roberts (1998, Theorem 2.1). Lemma 2.A.2 is the analogous version for discrete maximization problems.

Lemma 2.A.1 (Comparing optimizers)**.** *Let f, g be continuously differentiable, real-valued functions on an interval $[a, b]$. Suppose the marginal returns of f and g satisfy $f'(x) \leq g'(x)$ for all x.*

- *Then, if the maximizers for f and g are unique, the maximizer of g is greater than or equal to the maximizer of f.*

[9] For example, the reversion may be high with a very low probability, and low with a very high probability, and the optimal proposal may be smaller than the high type of reversion.

- *Otherwise, the lowest maximizer of g is greater than or equal to the lowest maximizer of f, and the highest maximizer of g is greater than or equal to the highest maximizer of f.*

Proof. (Lowest maximizers) Let x_f^* be the lowest (least) maximizer of f and x_g^* be the lowest maximizer of g. If the maximizers are unique, then x_f^* and x_g^* are the only maximizers of f and g respectively. If $x_f^* = a$ then $x_g^* \geq x_f^* = a$ since a is the left endpoint. If $x_f^* > a$, then for all $a \leq x < x_f^*$, x is not a maximizer of f since x_f^* is the lowest maximizer, and it follows by the Fundamental Theorem of Calculus that (since it is assumed $f'(x) \leq g'(x)$ for all x),

$$g(x_f^*) - g(x) = \int_x^{x_f^*} g'(t)dt \geq \int_x^{x_f^*} f'(t)dt = f(x_f^*) - f(x) > 0.$$

Since $g(x_f^*) > g(x)$ for all $x < x_f^*$, g cannot have a maximizer less than x_f^*, and it follows that the lowest maximizer of g satisfies $x_g^* \geq x_f^*$.

(Highest maximizers) Let x_f^{**} be the highest (greatest) maximizer of f and x_g^{**} be the highest maximizer of g. If $x_g^{**} = b$ then $x_f^{**} \leq x_g^{**} = b$ since b is the right endpoint. If $x_g^{**} < b$, then for all $x_g^{**} < x \leq b$, x is not a maximizer of g since x_g^{**} is the highest maximizer, and it follows by the Fundamental Theorem of Calculus that

$$0 > g(x) - g(x_g^{**}) = \int_{x_g^{**}}^{x} g'(t)dt \geq \int_{x_g^{**}}^{x} f'(t)dt = f(x) - f(x_g^{**}).$$

Since $f(x_g^{**}) > f(x)$ for all $x > x_g^{**}$, f cannot have a maximizer greater than x_g^{**}, and it follows that the highest maximizer of f satisfies $x_f^{**} \leq x_g^{**}$. □

Lemma 2.A.2. *Let f and g be real-valued functions defined on $\{1, 2, \ldots, N\}$. Suppose the marginal returns defined as $\delta f_i \equiv f(i+1) - f(i)$ and $\delta g_i \equiv g(i+1) - g(i)$ satisfy $\delta f_i \leq \delta g_i$ for all $1 \leq i \leq N-1$.*

- *Then, if the maximizers for f and g are unique, the maximizer of g is greater than or equal to the maximizer of f.*
- *Otherwise, the lowest maximizer of g is greater than or equal to the lowest maximizer of f, and the highest maximizer of g is greater than or equal to the highest maximizer of f.*

The proof involves a few modifications compared to the continuous case. First, instead of using the Fundamental Theorem of Calculus to relate the change in a function with that function's derivative, the following formula is used, which states that the total change in a discrete function can be written as the sum of the marginal changes in that function:

$$g(b) - g(a) = \big(g(b) - g(b-1)\big) + \big(g(b-1) - g(b-2)\big) + \cdots + \big(g(a+1) - g(a)\big)$$
$$g(b) - g(a) = \sum_{i=a}^{b-1} \delta g_i$$

for any $a < b$. Then the proof involves the same steps as in the continuous case by using the comparison of marginal returns of f and g to establish:

$$g(x_f^*) - g(x) = \sum_{i=x}^{x_f^*-1} \delta g_i \geq \sum_{i=x}^{x_f^*-1} \delta f_i = f(x_f^*) - f(x) \qquad \text{for } x < x_f^*$$

$$\text{and} \qquad g(x) - g(x_g^{**}) = \sum_{i=x_g^{**}}^{x-1} \delta g_i \geq \sum_{i=x_g^{**}}^{x-1} \delta f_i = f(x) - f(x_g^{**}) \qquad \text{for } x_g^{**} < x.$$

2.B Uncertainty over Voter Turnout vs. Preferences

Thus far, I have assumed that the school board faces uncertainty over voter turnout, but has complete information over voter preferences. Incorporating uncer-

tainty is an important component of the model, since models without uncertainty lead to an unrealistic conclusion that all budget proposals pass in equilibrium. Since voter turnout has been found to be in large part random (for example in Matsusaka and Palda (1999)), it is reasonable to include uncertainty over voter turnout in the model. At the same time, school boards would likely face some uncertainty over voter preferences, since it would be difficult to gather a voter's full preferences from past voting behavior or conversations, and even harder to do so for an entire voting population.

When a similar model incorporates uncertainty over voter preferences, almost all of the results in this paper continue to hold using the same ideas, although studying uncertainty over voter turnout is particularly tractable. In considering uncertainty over voter preferences, the location of the supported funding levels $\tilde{f}^1 < \tilde{f}^2 < \cdots < \tilde{f}^N$ are not known with certainty, but the school board believes they are distributed according to some probability distribution. Previously, knowing the value of $\tilde{f}^i$ with certainty corresponds to a jump discontinuity in the probability of passing $\pi(\bar{f})$; however, when the value of $\tilde{f}^i$ is uncertain, but is assumed to lie in a given interval, then the probability of passing will be decreasing in this interval based on the probability that voter i is pivotal and $\tilde{f}^i$ is larger or smaller than the proposal $\bar{f}$. Thus, the probability of passing $\pi(\bar{f})$ may have both regions where it is continuous and decreasing as well as jump discontinuities. One example is illustrated in Figure 2.17, where the school board knows the value of $\tilde{f}^i$ for some voters, but is uncertain about the preferences of some of the other voters.

With uncertainty over voter preferences, the objective of the school board is still the same, namely it seeks to maximize expected funding as in formula (2.16), which is equivalent to maximizing the area of the striped rectangle in Figure 2.17. Since $\pi(\bar{f})$ may be continuous and decreasing in some regions, but also have jump discontinuities at certain points, the maximization problem for the school board is more complicated

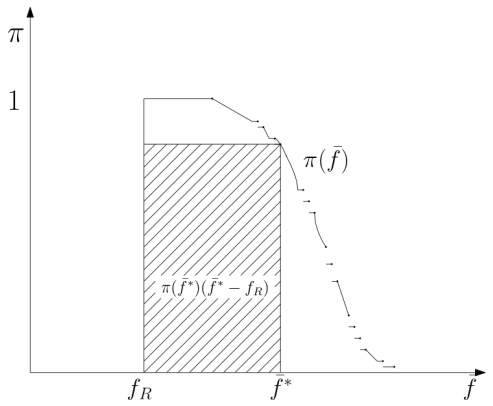


Figure 2.17: School board problem; the school board faces uncertainty both over voter turnout and also over the preferences of some voters

to analyze because it is both continuous and discrete. In this case, the results that are affected are Theorems 2.5.2 and 2.5.3, since they are formulated assuming that the school board's maximization problem is discrete.

References

Athey, S., Milgrom, P., & Roberts, J. (1998). Robust comparative statics.

Balsdon, E., Brunner, E. J., & Rueben, K. (2003). Private demands for public capital: Evidence from school bond referenda. *Journal of Urban Economics*, *54*(3), 610–638.

Banks, J. S. (1990). Monopoly agenda control and asymmetric information. *The Quarterly Journal of Economics*, *105*(2), 445–464.

Banks, J. S. (1993). Two-sided uncertainty in the monopoly agenda setter model. *Journal of Public Economics*, *50*(3), 429–444.

Chakrabarti, R., Livingston, M., & Roy, J. (2014). Did cuts in state aid during the Great Recession lead to changes in local property taxes. *Education Finance and Policy*, *9*(4), 383-416.

Chen, J. (2017). Communication in collective bargaining. *Available at SSRN 2411761*.

Evans, W. N., Schwab, R. M., & Wagner, K. L. (2019). The Great Recession and public education. *Education Finance and Policy*, *14*(2), 298–326.

Filimon, R., Romer, T., & Rosenthal, H. (1982). Asymmetric information and agenda control: The bases of monopoly power in public spending. *Journal of Public Economics*, *17*(1), 51–70.

Flowers, M. R. (1981). Agenda control and budget size: An extension of the Romer-Rosenthal model. *Public Choice*, *37*(3), 579–584.

Kenyon, D. A., & Reschovsky, A. (2014). Introduction to special issue on the property tax and the financing of K-12 education. *Education Finance and Policy*, *9*(4), 373-382.

Leachman, M., & Mai, C. (2014). *Most states funding schools less than before the recession.* Washington, DC: Center on Budget and Policy Priorities.

Lee, K. (1995). Moral hazard in the agenda-setter model. *Southern Economic Journal*, 223–235.

Matsusaka, J. G., & Palda, F. (1999). Voter turnout: How much can we explain? *Public choice*, *98*(3-4), 431–446.

Morton, S. (1988). Strategic voting in repeated referenda. *Social Choice and Welfare*, *5*(1), 45–68.

Niskanen, W. A. (1971). *Bureaucracy and representative government.* Transaction Publishers.

Niskanen, W. A. (1975). Bureaucrats and politicians. *The Journal of Law and Economics*, *18*(3), 617–643.

Romer, T., & Rosenthal, H. (1978). Political resource allocation, controlled agendas, and the status quo. *Public choice*, *33*(4), 27–43.

Romer, T., & Rosenthal, H. (1979a). Bureaucrats versus voters: On the political economy of resource allocation by direct democracy. *The Quarterly Journal of Economics*, *93*(4), 563–587.

Romer, T., & Rosenthal, H. (1979b). The elusive median voter. *Journal of Public Economics*, *12*(2), 143–170.

Romer, T., & Rosenthal, H. (1980). An institutional theory of the effect of intergovernmental grants. *National Tax Journal*, 451–458.

Romer, T., & Rosenthal, H. (1982). Median voters or budget maximizers: Evidence from school expenditure referenda. *Economic Inquiry*, *20*(4), 556–578.

Romer, T., Rosenthal, H., & Ladha, K. (1984). If at first you dont succeed: budgeting

by a sequence of referenda. *Public finance and the quest for efficiency*, 87–108.

Romer, T., Rosenthal, H., & Munley, V. G. (1992). Economic incentives and political institutions: Spending and voting in school budget referenda. *Journal of Public Economics*, *49*(1), 1–33.

Zhao, Q. (2020). An empirical study of voting and school budget cuts in New Jersey. *Thesis, Princeton University*.

Chapter 3

The Empirical Study

An Empirical Study of Voting and School Budget Cuts in New Jersey

Abstract

This paper provides an empirical investigation of the outcomes of New Jersey K-12 public school budget referenda. For failed budget proposals, school budget cuts are determined by local governing bodies whose decisions are not specified by law. The contribution of this work is two-fold. Using a unique compilation of voting data, demographic data, and school finance data including school budget proposals and outcomes, this study tests and supports the hypothesis that local governing bodies choose larger reductions for those budget proposals that are larger and more unpopular. I give an explanation based on a model of school board behavior. Additionally, I analyze the voter support for the referendum and test whether larger proposed tax levies are associated with higher levels of voter opposition. I find that the effect of a larger proposal on the level of voter opposition hinges on the circumstances that surround the larger proposal, and I propose two theories that explain the different

effects. Furthermore, I find that the level of educational attainment of a district is an important predictor for school budget referendum success.

3.1 Introduction

For many years, New Jersey has ranked among the largest in the country in its spending on public education, both in per pupil and absolute terms[1], making the state among the most important in the general study of school finance in the United States. A clear picture of the funding for public schools in New Jersey cannot be obtained without a deep understanding of the local tax levies that comprise a primary source of the funding[2]. Until 2012, these local tax levies had been determined mainly through a democratic voting process that took place during annual school elections in April. In these elections, voters had the choice to vote 'Yes' or 'No' to the question of the general fund levy: an amount presented by the local school board that would dictate the local tax levies raised for schools in the district and also determine the amount of funding schools would receive in the coming fiscal year. Whenever the budget proposed by the school board would fail, which occurred many times every year the elections were held, the decision of the school district's final budget was then placed in the hands of a group of local government officials known as the *municipal body*. Today, this referendum process mainly occurs when a district school board proposes an increase above the statewide cap of 2%. Section 3.B.1 describes further details of this referendum process.

The New Jersey municipal bodies exerted a great influence on the spending for schools for both direct and indirect reasons. Municipal cuts in a single district ranged from nothing at all in many cases to millions of dollars. In the year 2010-11 alone, during a statewide financial crisis in which New Jersey made significant cuts in state aid to districts and over 60% of budget proposals in New Jersey failed the referen-

[1] Source: US Department of Education.

[2] Like New Jersey, most other US states also rely on local tax levies to supply the majority of funding for K-12 public schools (Moser & Rubenstein, 2002). Figures by various levels of government can be found in the National Public Education Finance Survey (NPEFS), which is a part of the Common Core of Data (CCD) from the National Center for Education Statistics (NCES) `https://nces.ed.gov/ccd/stfis.asp`.

dum, municipal cuts totaled all together over $142 million. The true impact of these municipal bodies is, however, likely far greater than these figures would suggest, as the literature on school budget referenda shows that both voter and school board decisions – even for proposals that pass the referendum – depend crucially on the *reversion budget*, which is the budget that is enforced in the event of a referendum failure (Romer & Rosenthal, 1979; Filimon, Romer, & Rosenthal, 1982; Romer & Rosenthal, 1982; Rothstein, 1994; Stevens & Mason, 1996). The reason is that voters and school boards must both consider the probable outcome in the event of a failed budget proposal when they make their own decisions on how to vote or what budget to propose. This reversion budget is exactly what is decided by the municipal body.

The investigation of school finance outcomes in New Jersey in general thus hinges on having a clear understanding of the reductions chosen by the municipal bodies and why some cuts to failed budgets are much larger than others. However, compared to other states in which the reversion budget is specified by state law, understanding the reductions to failed budgets in New Jersey is very difficult as the municipal body decisions are not dictated by law and there are multiple reasonable theories that one may propose to model these entities. What is needed is an empirical investigation of the municipal cuts that is also well-informed by theoretical approaches.

I develop an empirical approach towards predicting which variables influence the municipal reductions based on district level voting, budget proposal, and demographic data in the years 2008-09 to 2011-12. Variables I consider include the size of the budget proposal, the size of the proposed growth in the tax levy, the results of the voting referendum for the proposal, and the amount of per pupil aid given to each district. I examine the hypothesis that proposed increases in the tax levy that are larger and more unpopular are associated with larger municipal cuts.

There are several reasons that, at least in principle, larger and more unpopular proposed increases may be associated with larger cuts. First, the municipal body may

actively respond to a large proportion of 'No' votes and take action to better represent the district by making larger cuts[3]. However, there are reasons to be skeptical of this explanation, especially since other prevailing theories offer alternative explanations and since municipal body members, as opposed to school board members, often are not elected officials. One alternative explanation comes from modeling school board officials as seeking to maximize expected revenue for district schools. The main point is that a budget-maximizing school board can choose a proposal with a high risk of failure, which is then associated with a large cut if it fails. I explain this theory more fully in Section 3.5 below. Using a Tobit analysis to take into account the left-censored nature of municipal reduction figures, I find that the data support the hypothesis that larger and more unpopular proposed increases are indeed associated with larger municipal reductions.

Voter support in response to tax levy proposals

Prior to investigating the municipal cuts, an important question is why budgets fail in the first place and what factors are associated with lower levels of voter support for the school budget referenda. Studying the determinants of voter support is important because the referenda, which provide the threat of failure of budget proposals, were the central means by which voters exerted partial control over growth in spending in their district during the years in which elections were prevalent, and remain important today for proposed increases above 2%, which require voter approval. Having a better understanding of what proposals earn a greater amount of opposition is of interest both to school boards aiming to avoid the financial repercussions of failed budget proposals and to policymakers seeking to predict the financial implications of policies that can affect school spending. Furthermore, it is natural

[3] Barr and Dee (2016) find using New Jersey data that tax levies per pupil decrease following a failed proposal. This finding shows that municipal reductions are significant and positive when they occur, but does not show that municipal bodies respond to the results of the election when choosing the reduction.

to study the variables affecting the percent 'No' vote as a preliminary step towards investigating the municipal cuts, since the data on municipal reductions consist of exactly those proposals that received at least 50 percent 'No' votes. Finally, in view of the economic consequences of the recent pandemic, proposed increases above 2% that require voter approval may be expected to become more common in the near future due to the stress of the current recession.

A central question I address in this study is whether larger proposed tax levies are associated with greater amounts of voter opposition. The theoretical analysis of this question is not completely straightforward. One might think at first that an exogenous increase in the proposed tax levy for a fixed population of voters should result in an increase in the fraction of voters who oppose the proposal. However, tax levy proposals are not expected to be exogenous, but rather should be the outcome of a deliberate decision process, and the voting population is not the same when one examines multiple districts. A more detailed analysis should consider a model for how the school board determines the tax levy proposal. I consider the two main approaches to modeling the proposal in the school finance literature, which are the median voter model, which assumes the school board proposes the ideal funding level of the median voter (see Appendix 3.C), and the budget-maximizer model under uncertainty (Romer & Rosenthal, 1979; Zhao, 2020), which assumes the school board seeks to maximize expected funding for schools[4].

For both of these models, there are two competing predictions as to whether larger budget proposals should be empirically associated with larger or smaller levels of voter opposition. The answer depends on what causes the school board's proposal to be larger in the first place. There are two particular mechanisms that can lead

[4] The median voter model is expected to apply in regimes where there is strong electoral competition for school board positions, while the budget-maximizer model is expected to apply in cases where electoral competition is relatively weak and the desired funding levels of school board members are relatively high. In either model, assuming the presence of uncertainty over voter turnout or preferences is important since models without uncertainty predict that all budget proposals will pass.

to a school board's optimal proposal being larger, which could be called the "support hypothesis" and the "disagreement hypothesis," and these hypotheses lead to contrasting implications.

The support hypothesis is that a budget proposal being larger reflects that voters in the district are uniformly more willing to support higher levels of education funding. In this case, Corollary 5.2 of Zhao (2020) shows that the optimal proposal for a budget-maximizer will have a lower expected percentage of 'No' votes as long as voters' induced preferences are not too heterogeneous[5]. A similar result holds in the median voter model with uncertainty under certain symmetry assumptions (see Appendix 3.C). In summary, the support hypothesis implies that higher proposed levies would be associated with *lower* levels of voter opposition, regardless of whether the median voter model or the budget-maximizer model is assumed. The alternative disagreement hypothesis is that higher tax levies are subject to more disagreement among voters, where some voters are willing to support larger levies while others are not. In this case, the district is more heterogeneous in terms of what funding levels voters are willing to support, and Theorem 5.3 of Zhao (2020) shows[6] that the budget-maximizing proposal in such a district will have a *higher* expected percentage of 'No' votes as long as the levels of spending that voters are willing to support are not too high. Appendix 3.C explains how such heterogeneity of induced preferences can also lead to greater voter opposition to higher proposals in the median voter model as well. Finally, it is also possible that a district is both highly heterogeneous and also willing to support high levels of education spending, in which case it is unclear whether the expected percentage of 'No' votes would be relatively high or low.

[5] The conclusion of (Zhao, 2020, Corollary 5.2) regarding lower expected percentage of 'No' votes also applies if the voters are willing to support higher funding levels due to the reversion budget being lower, in which case the support hypothesis still holds.

[6] Theorem 5.3 of Zhao (2020) examines an decrease in district heterogeneity, but the analysis applies in reverse for increases in heterogeneity.

To distinguish which of these hypotheses appears more prevalent in the case of New Jersey, I test whether larger proposed percent increases in the local tax levies and larger tax levies per person are associated with higher levels of voter opposition. The resulting two coefficients reflect the effects of two different ways that the proposal can increase: In one case, I isolate the effect of a larger proposed percent increase when the levy per person is held fixed, while in the other case, I consider the effect of having a larger levy per person in districts with similar proposed percent increases in the levy. In each case, the coefficient reflects a relationship between the size of the proposal and the percentage of 'No' votes that gives insight into the underlying reason that the proposal is increasing under each of the two different circumstances. Finding a positive relationship provides evidence for an increase in the heterogeneity of induced preferences in the district according to the disagreement hypothesis. Finding a negative relationship provides evidence that there is some uniform increase in the funding levels voters are willing to support according to the support hypothesis.

In addition to evaluating the effects of proposed tax levy increases and higher tax levies per person on the level of voter opposition, I also determine the effects of several other variables that are expected to be linked to voter support, including notably the level of educational attainment in a district. The question of which variables affect voter support in school budget referenda has been of interest to a number of researchers including Alexander and Bass (1974); Ehrenberg, Ehrenberg, Smith, and Zhang (2004); NYSED (2005); Silverman (2011). A strength of the current study is the assembly of a unique data set that includes both the size of the original budget proposals and the exact voting results for a large number of school budget elections, in addition to a number of other demographic and financial district characteristics. Among other results, I find that higher levels of educational attainment of a district are clearly associated with lower levels of opposition for voter referenda. In the process of analyzing the effects of the explanatory variables, I compare the results

with predictions of the median voter and budget-maximizer models; however, the main goal of this section is to determine certain effects that the theories do not resolve. Section 3.4 discusses the full results of the study.

The findings obtained in this work on the main determinants of school budget voting outcomes in New Jersey therefore contribute to the literature on voter opposition to K-12 public school referenda. However, voter opposition to referenda only captures part of the picture of school finance in New Jersey, as a key piece of the local funding of school districts ultimately rests in the hands of the municipal bodies. Despite the importance of the municipal bodies in determining the New Jersey districts' reversion budgets, there has been no systematic study of the behavior of these municipal bodies thus far in the existing school finance literature. In the second main contribution of this paper, I provide the first investigation of the main factors that affect the budget cuts chosen by municipal officials in New Jersey. Since one of the main findings is that higher opposition to the referendum corresponds to larger municipal reductions, the additional study of voter opposition in this paper serves as a complement to the analysis of the municipal cuts. Together the two projects combine to shed light on the unique picture of K-12 public school finance in the important state of New Jersey.

3.2 The Data

All together, I collected 2,262 general fund tax levy proposals from annual New Jersey district school elections during the four years 2008-09 through 2011-12. Each tax levy proposal represents a proposal for a specific district in a specific year. The proposals come from 572 school districts in New Jersey, which consolidated into 559 districts beginning in 2010.

The full data consist of: voting outcomes for the school budget referendum including the number of voters who voted 'Yes' and 'No', district budgetary information such

as the tax levy and the total operating budget for each district, municipal reduction quantities for defeated budget proposals, and district demographic characteristics including information on student enrollment, racial composition, poverty, age, income, private school, educational attainment, and population[7]. For the outcome of the election including the number of 'Yes' votes and the number of 'No' votes, I use data from the NJ School Board Association.

I use school district budget data from the NJ Department of Education, and for each district and year in the sample, the data contain both the individual district's contribution to schools from local tax sources and the total operating budget for each district. The district's contribution to schools includes for each year the district's tax levy that is anticipated for the upcoming school year following the school election and any municipal reductions, the tax levy that was revised for the current school year, and the actual tax levy for the previous year. For example, for the year 2008-09, the data include the "anticipated" tax levy for the year 2008-09, and updated figures for the years 2007-08 and 2006-07. While the tax levy is the district's contribution to the school budget, the total operating budget includes additional support including state and federal aid.

Since a primary goal of the study is to understand the municipal reductions that are made to the defeated school budgets, it was necessary to obtain data on the school budget proposals that failed and the resulting cuts that were made by the municipality. For the years of the study from 2008-09 to 2011-12, information on the school district budget proposals that failed came from the NJ Department of Education. For each year in the sample, the data include the districts in which the school budget question failed during the school election along with the failed proposed budget, municipal and commissioner reductions, and the resulting certified tax levy for each of these districts.

[7] Data for the full range of variables were not available for all instances of tax levy proposals.

I use data from the NJ Department of Education to obtain information on total K-12 public school enrollment in the district. Data on other district demographic characteristics such as age, racial composition, education level, income, population, and private school enrollment came from the U.S. Census American Community Survey. Section 3.B.5 contains a detailed discussion of the demographic variables.

3.3 Discussion of the summary statistics

3.3.1 Budget proposals and reductions

Of the 2,262 observations of budget proposals in the data, there are 2,173 for which voting data were available. Many of these latter proposals (711 = approximately 32.7%) did not earn a majority of votes in the popular referendum and were reviewed for a reduction by the district municipal body. In most cases (604 of the 711 reviewed budgets) the municipal body chose to reduce the proposed budget by a positive amount, but a significant number of proposals that were considered for reduction (the remaining 107) incurred a zero reduction instead. In general, the municipal bodies cannot increase the proposed tax levy for a failed referendum, and thus the municipal reductions are always non-negative amounts. Figure 3.A.1 shows the municipal reductions as a fraction of the proposed tax levy (with 3 outliers removed for clarity of presentation). This figure also illustrates the left-censored nature of the municipal reduction quantities.

In terms of magnitude, the proposed tax levies are comparable in size to the tax levy of the previous year for the same district, as described in Section 3.B.4 below, but are typically larger. The median percent proposed increase from the previous year in the data was 3%, with percentile ranges described in Table 3.1, and proposed increases

of 2% or 4% occurred frequently in the data[8]. Most proposals in the data set (around 81.4%) requested a positive increase in the levy, although some (8.3%) requested a decrease in the local levy and other proposals (10.3%) were for maintaining the previous year's tax levy. The sum of all levy increases proposed in New Jersey during the four year period of the data amounted to a total proposed increase of $1.46 Billion in the tax levy. This total corresponds to a 3% proposed increase[9] per year compared to the initial total of $11.6 Billion in local levies in 2007.

Table 3.1: Proposal and reduction sizes

Variable	Median	25-75%ile	10-90%ile	5-95%ile	Min-Max	Obs.
Proposed Percent Increase from Previous Year's Levy	3.0%	1.3%-4.0%	0%-5.5%	-0.9%-7.9%	-37%-82%	2262
Total Reduction	$117k	$3k-$354k	$0-$946k	$0-$1.24M	$0-$7.63M	711
Reduction as Percent of Positive Proposed Increase	32%	14%-57%	0%-100%	0%-104%	0%-1940%	645

Final row uses only observations for which the proposed increase was positive

The municipal reductions to failed proposals, though typically smaller than the corresponding proposed increases in the tax levy, amount to a sizable fraction of the proposed increases overall. Among the 645 reductions that were associated with a positive proposed increase, the median reduction as a portion of proposed increase was 32% of the proposed increase (see Table 3.1). Figure 3.A.2 provides a histogram for the logarithm of the reductions. The largest reductions in the data for a single district and year were on the order of $6-$7.6 million. Taking all reductions across districts into account, the total of all local levies in New Jersey grew from $11.61 billion in 2007 to $12.84 billion in 2011, representing a growth of 2.5% per year, which is markedly smaller than the 3% increase per year called for by the proposed levies.

[8] There were 210 proposals requesting an increase of 2% to within .01 percentage points, and 419 proposals requesting an increase of 4% to within .01 percentage points.

[9] In terms of inflation, the value of the dollar decreased by 2.1% per year on average during this period.

These figures illustrate that the municipal reductions had a substantial direct effect on the growth of spending for education in New Jersey, but do not capture the indirect effects of municipal reductions, which may also be sizable. Namely, since school boards must take into account the realistic possibility of a reduction following a failed referendum when making their budget proposals, the threat of a municipal reduction imposes an indirect effect on the size of proposed budgets, regardless of whether the proposals ultimately pass or fail. Additionally, voters must also take into account the same possible reduction by the municipal bodies when deciding whether or not to support the referendum. Therefore, the above figures should be viewed as only a part of the true impact of the municipal reductions on the growth of spending on K-12 public education in New Jersey.

It is important to highlight that both proposed tax levy increases as well as reduction figures are substantially larger in the year of the financial emergency in 2010 than in the other three years of the data. In this year, the governor made large cuts in state aid to many districts, and districts made efforts to compensate for the losses of aid[10]. In gross terms, the total of all proposed levies in 2010 amounted to a proposed increase of $593 million as compared to the average of $289 million per year for the other three years of the data set. Figure 3.A.3 highlights how the tax levy proposals in 2010 were all together larger compared to the other years. The municipal reductions followed a similar pattern, amounting to $142 million total in the year 2010, which corresponds to 24% of the total proposed increase in the tax levy that year, as compared to $30.5 million per year on average in the other years. The fact that the municipal reductions were larger in 2010 can be seen both in Figure 3.A.4, which compares the reduction as a percentage of the proposed tax levy for 2010 against the

[10] Losses in state aid were one of the exceptional expense categories that could allow district school boards to request budgets above the 4% cap on property tax increases in place at that time. See Section 3.B.2.

other years, and in Figure 3.A.5, which shows the distribution of the logarithm of the municipal reduction in 2010 as compared to the other years.

It is also important to point out that proposed tax levy increases (and also reduction figures) are substantially smaller in 2011 than in the prior three years of the data. For example, the total of all proposed increases amounted to $162 million in 2011, whereas the total of all proposed increases averaged $352 million per year in 2008-09 through 2009-10, and was even higher ($593 million) in 2010. Figure 3.A.6 compares the proposed percent increase in the tax levy in 2011 to the previous three years of the data. The figure shows that proposals of 4% were common prior to 2011, and proposals prior to 2011 were typically larger, whereas in 2011, proposals of 2% were common and the proposals were generally smaller.

The reason for this phenomenon is likely due to the fact that a 4% property tax cap was in place in the budget years 2008-09 through 2010-11, whereas a 2% property tax cap was implemented starting in 2011-12. Both caps included exceptions for certain expense categories, but the 4% cap had far more exceptional categories than the 2% cap, which made it easier for school boards to make proposals that exceeded the 4% cap. This larger number of exceptions is likely part of the reason why the share of proposals that exceeded the 4% cap (34.5% of proposals in 2008-09 through 2010-11) was larger than the share that exceeded the 2% cap (29.3% in 2011-12). In particular, the 4% cap included losses of state aid as an exceptional expense category, which was important in 2010, while the 2% cap does not include losses of state aid as an exceptional category. Section 3.B.2 explains these property tax caps in more detail.

3.3.2 Voter incentives and the school funding formula

During the time period spanned by the data, the budget proposals of school boards give voters an accurate reflection of how much revenue will be raised and

allocated to schools. Namely, the entirety of the general fund levy remains in the district to fund local schools, and the amount of state aid received by the district does not depend on how much is raised locally. Thus, the amount of tax revenue required to raise $1 for local schools is exactly $1 in local tax revenue; namely, every district faces an *inverse tax price* equal to one. This feature of the New Jersey school funding formula contrasts aid formulas in other states, and also contrasts the previous funding formula in New Jersey, which used income tax revenue in a matching scheme that rewarded districts for raising higher local tax revenues. See Section 3.B.3 for a full description of the school funding formula in New Jersey.

For the current study, the above features of the state aid formula mean that it is not necessary to include a district's tax price as one of the variables in any regression, since all districts face a tax price equal to one. If, instead, different districts faced different tax prices, as it was prior to the School Funding Reform Act of 2008, then it would be important to include the tax price as one of the variables in the main regressions of the current study[11].

3.3.3 Voting outcomes for school budget referenda

In all four years of the data, it was common for voter referenda to meet a considerable amount of opposition, especially in the year 2010 of the fiscal emergency. The percentage of 'No' votes overall had a mean of 45.6 percent 'No' votes and a standard deviation of 10.4 percentage points. Figure 3.A.7 and Table 3.2 both illustrate this distribution. Among those proposals that failed, half had at least 54.8% 'No' votes, while the most one-sided elections reached up to 93% 'No' votes. Of proposals that passed the referendum, the median election had 58.4% 'Yes' votes, while the most one-sided elections passed unanimously.

[11] Studies on both the theory of voter referenda (Romer & Rosenthal, 1979) and empirical studies of voter referenda (Rubinfeld, 1977; Brokaw, Gale, & Merz, 1990a; Stevens & Mason, 1996) indicate that voters actively respond to the tax price they face in considering a given referendum.

Table 3.2: Percent 'No' votes

Variable	Median	25-75%ile	10-90%ile	5-95%ile	Min-Max	Obs.
Percent 'No' votes	45.5%	39% - 52%	33% - 58%	30% - 62%	0% - 93%	2173
Percent 'No' votes 2010	51.3%	48% - 57%	41% - 62%	38% - 64%	0% - 91%	538
Percent 'No' votes 2008, 2009, 2011	43.8%	38% - 50%	32% - 55%	28% - 60%	0% - 93%	1635

Among the 572 districts of New Jersey in the data set, 68% of districts (389) had at least one failed proposal during the four year period of the data set, and many districts (218) experienced more than one failed proposal in this period. Figure 3.1 demonstrates these facts and summarizes the percent of districts that experienced a given number of failures during the sample period.

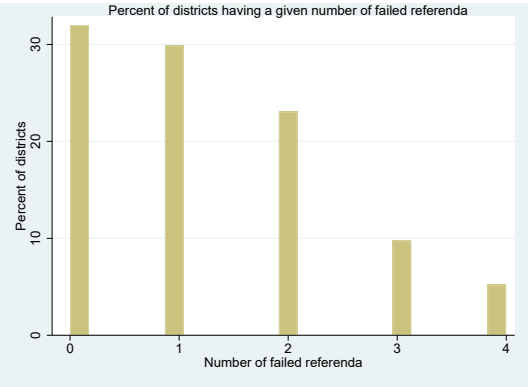


Figure 3.1: NJ districts having a given number of failures from 2008-09 to 2011-12

Table 3.3: Number of passed vs. failed proposals by year

Year	Pass	Fail	Total
2008	405	143	548
2009	405	145	550
2010	223	315	538
2011	429	108	537

The 'Total' varies due to limited data and district consolidation.

The year 2010 had a statistically significant influence on the voting outcomes, although it was common for a considerable portion of budget proposals to fail in each year of the data set. The proportion of failed referenda in 2010 was much higher than the other years, as 58.6% of budget proposals failed in 2010 as opposed to 24.3% of budgets in the other three years (see Table 3.3 for a year-to-year summary). More specifically, the distribution of the percentage of 'No' votes shifted in 2010 from a mean of 43.7 percent 'No' votes, $\sigma = 10.1$ percentage points, in the other three years to a mean of 51.3 percent 'No' votes, $\sigma = 9.2$ percentage points, in 2010. Figure 3.A.8 demonstrates this effect by comparing the percentage of 'No' votes for the elections in 2010 versus the percentage of 'No' votes for the other three years. The CDF plot in Figure 3.A.9 shows furthermore that the percent 'No' vote in 2010 first order stochastically dominates the other years away from the extreme tails. Table 3.4 shows the results of a basic regression of the percent 'No' variable on the year, and indicates a statistically significant increase in the percentage of 'No' votes in 2010 in comparison to 2008-09. The year 2011, in contrast, had a statistically significantly increase in voter support in comparison to 2008-09, and the CDF plot of Figure 3.A.10 compares the voter support in 2011 to the other years.

Another prominent feature of the voting in 2010 is that the number of voters participating in the school elections was much larger for the elections in that year. The median number of voters in 2010 was 1430 voters, substantially larger than the median of 823 voters in the other three years. The regression results of Table 3.5 illustrate the magnitude of this large increase, showing that voter participation in the year 2010 went up by 79 percentage points[12] compared to 2008 (controlling for population). The same table shows that the year 2011 experienced a statistically significant increase in turnout compared to 2008-09, though not as sizable as 2010. In general, the number of voters participating in a given school budget referendum reflects the overall size of

[12] The voter participation is larger by a factor of $e^{0.585} - 1 \approx 0.79$ in the year 2010.

Table 3.4: Percent 'No' vs. Year, 2008-2011

	Percent 'No'
2009.YEAR	-1.090
	(0.596)
2010.YEAR	6.323***
	(0.600)
2011.YEAR	-2.662***
	(0.600)
Constant	44.98***
	(0.422)
No. of observations	2173

OLS, standard errors in parentheses, 2008.YEAR omitted
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

the district, and, as seen in Table 3.5, grows slightly slower than proportional to the population of the district. Figure 3.A.11 illustrates the percentage of the total district population participating in the election, while Figure 3.A.12 provides a histogram of the logarithm of the total number of participating voters.

Table 3.5: ln of Participating Voters vs. ln of Population and year

	logVoters
ln(Population)	0.888***
	(0.0100)
2009.YEAR	0.0434
	(0.0316)
2010.YEAR	0.585***
	(0.0316)
2011.YEAR	0.142***
	(0.0317)
Constant	-1.552***
	(0.0942)
No. of observations	2157

OLS, standard errors in parentheses, 2008.YEAR omitted

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

No. of obs. smaller due to limited population data

3.3.4 Definitions of Variables

Table 3.6: Table of variables

Proposed Percent Increase in Levy	Proposed percent increase in general fund levy
ln(Levy$_{t-1}$/Population)	log of (previous year's local tax levy / total district population)
ln(Population)	Logarithm of total district population
i.YEAR	Indicator variable for year [i]
% with a Bachelor's Degree	Percent of district having attained at least a bachelor's degree
ln(Enrollment/Population)	log of (total K-12 public school enrollment/district population)
ln(Median Income)	Logarithm of median income
% Low Income	Percent of population with income less than $25k
Total Aid	Measure of total aid = Total operating budget - Local tax levy
ln(Per Pupil Aid)	Logarithm of (Total Aid / Total Enrollment)
isSDA	1 if SDA (formerly Abbott) district, 0 else; see Appendix 3.B.5
% K-12 Students in Private School	Percent of K-12 students enrolled in private schools
% Over 65	Percent of population over 65 years old
% White	Percent of population identifying as White
% Black	Percent of population identifying as Black
% American Indian	Percent of population identifying as American Indian
% Asian	Percent of population identifying as Asian
% Hawaiian	Percent of population identifying as Hawaiian
% Other	Percent of population identifying as Other
% Two or More Races	Percent of population identifying as Two or more races
Diversity Index	Measure of racial diversity in a district; see Appendix 3.B.6
ln(#Voters/Population)	Logarithm of (total number of voters / total population)
Percent 'No'	Percent of voters in election voting 'No'
Reduction Pct	Municipal reduction as a percent of the proposed tax levy

3.4 Voter support for school budget referenda

I now test the effects of several variables that are expected to be linked to voter opposition to school budget proposals using data from New Jersey for the years 2008-09 through 2011-12. A central hypothesis I test is that larger proposals and proposed increases are associated with higher levels of voter opposition. The contribution of this study is important in that it gives the first investigation of New Jersey school budget referenda using a robust data set that includes the important year of the fiscal emergency in 2010.

Table 3.7 below presents the main regression, which fits the percentage of 'No' votes, which represents the voter opposition to the proposed tax levy, against financial and demographic characteristics of the district and of the proposal. The model specification is:

$$\begin{aligned}\text{PercentNo}_{it} = \alpha &+ \beta_P(\text{Proposed Percent Increase in Levy}) \\ &+ \beta_L \ln(\text{Levy}_{t-1}/\text{Population}) + X_{it}\beta + \text{YEAR}_t\gamma + \nu_{it},\end{aligned}$$

where X_{it} is a vector of characteristics of the district i in year t; YEAR_t is a vector of year indicator variables (including 2009, 2010, and 2011 with 2008 omitted); and the ν_{it} are i.i.d. normal random errors[13].

Since the central question under investigation is whether larger proposals are associated to larger levels of voter opposition, the main coefficients of interest are those measuring the effects of the proposed percent increase in the levy (β_P), and the logarithm of the previous year's levy per person (β_L) on the percentage of 'No' votes. These coefficients measure how voter opposition responds to larger proposals in two

[13] PercentNo, as illustrated in Figure 3.A.7, is essentially normally distributed. Using the voting logit log(PercentNo/(100-PercentNo)) instead leads to results that are qualitatively the same with very small differences. However, the nearly unanimous elections, which become outliers, greatly affect the coefficients and the 5 unanimous elections would need to be treated.

Table 3.7: Percent of 'No' votes vs. district and proposal variables

	% No (A)	% No (B)	% No (C)	% No (D)	% No (E)
β_P: Proposed Percent	0.657***	0.704***	0.712***	0.703***	0.649***
Increase in Levy	(0.0699)	(0.0681)	(0.0678)	(0.0686)	(0.0682)
β_L: ln(Levy$_{t-1}$/Population)	-1.595***	1.599***	2.834***	2.176***	1.772**
	(0.359)	(0.457)	(0.516)	(0.626)	(0.638)
ln(Population)	1.165***	1.441***	1.656***	1.701***	2.105***
	(0.185)	(0.182)	(0.186)	(0.189)	(0.201)
2009.YEAR	-1.069	-1.099*	-1.116*	-1.052*	-1.196*
	(0.547)	(0.532)	(0.529)	(0.527)	(0.519)
2010.YEAR	5.054***	4.937***	4.823***	4.854***	2.427***
	(0.562)	(0.547)	(0.544)	(0.544)	(0.613)
2011.YEAR	-1.637**	-1.629**	-1.772**	-1.723**	-2.323***
	(0.560)	(0.545)	(0.543)	(0.546)	(0.548)
% with a Bachelor's		-0.161***	-0.182***	-0.246***	-0.253***
Degree		(0.0149)	(0.0154)	(0.0249)	(0.0257)
ln(Enrollment/Population)			-2.597***	-2.445***	-2.436***
			(0.514)	(0.614)	(0.611)
ln(Median Income)				3.733*	2.191
				(1.738)	(1.759)
% Low Income				-0.0218	0.0448
				(0.0587)	(0.0586)
ln(Per Pupil Aid)				-0.132	-0.0670
				(0.332)	(0.333)
isSDA				-1.648	-0.176
				(1.210)	(1.226)
% K-12 Students in				0.0252	0.0577*
Private School				(0.0261)	(0.0264)
% Over 65				-0.0528	-0.137**
				(0.0406)	(0.0432)
ln(#Voters/Population)					4.442***
					(0.560)
Constant	43.70***	24.45***	8.879	-24.37	2.818
	(3.161)	(3.556)	(4.689)	(21.16)	(21.39)
R^2	0.1794	0.2226	0.2320	0.2425	0.2716
N	2109	2109	2109	2109	2109

OLS, standard errors in parentheses, N = 2109 observations due to limited available data

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Column E controls for (%Amer.Ind., %Asian, %Black, %Hawaiian, %Other, %TwoOrMoreRaces)

These variables are not statistically significant and have not been displayed

different circumstances. The coefficient β_P refers to districts with similar tax burdens per person where one district proposes a larger increase, while β_L refers to districts where one district has an overall larger tax levy per person (and both districts propose a comparable percent increase). Omitting one of these variables results in only minor changes in the results.

To properly test the support and disagreement hypotheses, I must first start with a model that has a minimal number of covariates so that induced preferences are allowed to vary freely when the proposal increases. Column A of Table 3.7 provides such a test by fitting the percentage of 'No' votes against the proposed percent increase in the levy, the logarithm of the previous year's levy per person, the logarithm of the population, and the year. The main findings are that β_P is statistically significant and positive, meaning that larger proposed percent increases in the levy meet greater opposition, while β_L is statistically significant and negative. The finding that $\beta_L < 0$ means that districts that imposed larger tax levies per person the year before have *higher* levels of voter support for their referendum in the current year[14].

The finding that the coefficients β_L and β_P have opposite signs suggests that there are different underlying reasons behind the two types of proposal increases that are related to these coefficients. According to the support hypothesis, the finding that $\beta_L < 0$ is consistent with the idea that larger levies per person in the previous year may be associated with uniform increases in the funding levels that voters are willing to support. For districts that implement similar tax levies per person in the previous year, the finding that $\beta_P > 0$ suggests that larger proposed percent increases in the levy may be associated with greater heterogeneity in induced preferences in the district, as in the disagreement hypothesis. Overall the finding that higher proposed levies are associated with greater voter opposition is consistent with most empirical

[14] In interpreting this coefficient, I assume that the population of the district does not fluctuate drastically in a single year, since the population in the current year is used rather than the one from the previous year.

studies of school budget referenda (Alexander & Bass, 1974; Romer, Rosenthal, & Ladha, 1984; Romer, Rosenthal, & Munley, 1992; Brunner & Balsdon, 2004; NYSED, 2005).

Column A also controls for the population of the district, and I find that the effect of having a larger population on the level of voter opposition is statistically significant and positive. This finding is consistent with theoretical models of the referendum process. Namely, districts with larger populations are likely to have greater heterogeneity in terms of induced preferences over education spending. Greater heterogeneity of induced preferences has been shown in various models to imply greater opposition to the referendum (see Romer et al. (1992) for the median voter case and Zhao (2020) for the case of the budget-maximizer model under uncertainty).

3.4.1 Column B: Controlling for Educational Attainment

The support hypothesis states that the coefficient β_L is negative in Column A for the reason that districts with higher tax levies per person are those that are uniformly willing to support higher funding levels, which can lead in theory to higher voter support in either the budget-maximizer model (Zhao, 2020, Corollary 5.2) or in the median voter model (Appendix 3.C). If this theory is correct, then controlling for variables that strongly correlate with uniform increases in the unobservable funding levels voters are willing to support should diminish the mechanism that causes β_L to be negative, and may possibly change the sign of the coefficient. One variable that may have this property is the level of educational attainment in a district.

Educational attainment may be an omitted variable for the regression of Column A, since this variable may be expected to correlate with both the expected percentage of 'No' votes as well as the tax levy per person, which is a main variable of interest. First, to determine the theoretical relationship between educational attainment and the percentage of 'No' votes in a district, it is important to know that

voter support will increase when there are uniform increases in the funding levels voters are willing to support (see (Zhao, 2020, Corollary 5.2) for the budget-maximizer case and Appendix 3.C for the median voter case under symmetry). Intuitively, voters in districts with higher levels of educational attainment may be uniformly willing to support larger amounts of education funding. In this case, educational attainment should have a negative relationship with the percentage of 'No' votes in theory. Empirically, Ehrenberg et al. (2004) have found that the level of educational attainment of a district has a positive effect on the probability of a district referendum passing in New York school budget elections (although other studies do not obtain the same result (Alexander & Bass, 1974; Lankford, 1985)). Second, it is likely that districts with a more educated population would have higher levels of local taxation per person to fund schools, not only because more educated people tend to have higher incomes, but also because they may place a higher value on education and be more willing to trade consumption for education spending even with income held fixed. Table 3.A.1 confirms that, in this data set, educational attainment has a positive effect on the local levy per person for schools in New Jersey districts even when controlling for income. Thus, if educational attainment is an omitted variable, then the coefficient β_L of Column A may have a downward bias, and β_L can be expected to increase upon controlling for educational attainment.

In the regression of Column B in Table 3.7, I control for the educational attainment of the district using the percent of the district population with at least a bachelor's degree. In this regression, β_L changes sign from negative to positive and is statistically significant. The change in the sign of β_L indicates that districts with higher levels of taxation per person have lower levels of support for the referendum holding the other covariates fixed, which is consistent with the "disagreement hypothesis" discussed in the introduction.

Similarly to Ehrenberg et al. (2004), I find that districts with higher educational attainment have greater voter support for their budget proposals. The greater level of support for the budget proposals likely reflects that districts with a more educated population have a generally higher willingness to support education funding.

3.4.2 Column C: Controlling for Enrollment

Another variable that may impact the level of voter support is the fraction of the total population that consists of students enrolled in K-12 public schools in the district. Districts that have a greater fraction of the population enrolled in K-12 public schools would also likely have a greater share of the voting population being parents with children enrolled in K-12 public schools[15]. As a result, districts with a large fraction of the population enrolled in K-12 public schools may be expected to have higher tastes for education funding overall, similar to districts with a large percentage of people having attained a bachelor's degree. If these higher tastes for education translate into uniform increases in the funding levels voters are willing to support, then as in the case of educational attainment, one may expect districts with a greater fraction of the population enrolled in K-12 public schools to have higher levels of voter support for the referendum.

The share of the population enrolled in K-12 public schools may also be positively related to the tax burden per person to fund schools in the district because having a greater number of students in public schools that need to be educated requires a greater amount of funding to be raised in part from local taxpayers. Since the share of the population enrolled in K-12 public schools is likely to be correlated with both the local levy per person for schools as well as the percentage of 'No' votes, it is included to help address potential omitted variable bias. Therefore, controlling for

[15] Data on this share of parents in the voting population are not available.

K-12 public school enrollment as a fraction of the district population may lead to an increase in the positive value of the coefficient β_L.

Column C in Table 3.7, which controls for the natural log of the share of the population enrolled in K-12 public schools (ln(Enrollment/Population)), is consistent with the above expectations. The coefficient of this variable is negative, which means that districts with a higher share of the population enrolled in K-12 public schools have a higher level of support for the voter referendum. This finding accords with the Tiebout sorting hypothesis (Tiebout, 1956), which states that voters decide to locate in a district that matches their preferences for public good expenditures when relocation costs are minimal. In addition, in Column C, the value of the coefficient of the logarithm of the previous year's tax levy per person, β_L, increases compared to Column B upon controlling for ln(Enrollment/Population), similar to the effect on β_L of controlling for educational attainment between Columns A and B.

3.4.3 Financial Variables

The financial situation of a district might impact voters' tastes for education funding and the overall level of support for public school funding referenda. For example, Ehrenberg et al. (2004) find a negative relationship between the probability of a referendum failing and the proportion of the population below the poverty line. In Columns D-E of Table 3.7, I consider the effects of four district financial variables: the logarithm of the median income, the percentage of the population with income below $25k per year, the logarithm of the per pupil aid in the district, and whether or not the district is classified as an SDA district[16].

Among these variables, the only one that obtains statistical significance is the logarithm of median income, which is significant at the 95% confidence level in the regression of Column D. However, this variable is no longer statistically significant in

[16] SDA districts in New Jersey, formerly called Abbott districts, are a group of school districts with an especially high level of need for assistance. See Section 3.B.5 for more information.

the regression of Column E, which controls for demographic variables and the fraction of the district population that participates in the voting. Furthermore, upon performing an F-test I do not find the four financial variables to be jointly significant. Previous studies have also considered median income as a predictor for the passage rate of budgets, and have similarly found a lack of a statistically significant effect (Alexander & Bass, 1974; Ehrenberg et al., 2004). A key point is that the current study as well as these previous studies all control for the level of educational attainment in the district. Without controlling for educational attainment, the logarithm of median income has a statistically significant, negative relationship with the percent of 'No' votes.

3.4.4 Private School Enrollment and the Elderly Population

Parents of children in private schools and elderly people in a district may both have different incentives for supporting or rejecting public school funding referenda compared to other people in the district. Parents of children in private schools in particular face a lower incentive to support public school spending compared to their public school counterparts since their children are enrolled outside the public school system. This lower incentive for parents of children in private schools to support public school funding creates heterogeneity in induced preferences for public school funding in a district, which leads in theory to a reduced level of support for voter referenda (see (Zhao, 2020, Theorem 5.3) for the budget-maximizer case and Appendix 3.C for the median voter case). This theory is consistent with the empirical findings of Ehrenberg et al. (2004), which finds that a higher rate of private school enrollment leads to lower levels of support for voter referenda.

It is not clear a priori what the effect of the elderly population on voter support should be, since the literature on elderly support for general school finance initiatives contains conflicting conclusions about whether elderly voters have greater or lesser

support for such initiatives (see Lambert, Clark, Wilcox, and Park (2009) for a review). Previous studies focused specifically on school finance referenda have found that a greater percentage of elderly people is associated with lower opposition to referendum passage (Ehrenberg et al., 2004; Brokaw, Gale, & Merz, 1990b). An explanation for this finding given in Ehrenberg et al. (2004) is that elderly people may have grandchildren whose schools they wish to support. Another possibility is that elderly people support higher quality education in the hopes of maintaining the resale value of their homes, as the quality of education in the district becomes capitalized into their property values. The role of capitalization in elderly support of local school spending has been explored in several studies in the education finance literature, such as Bergstrom, Rubinfeld, and Shapiro (1982); Harris, Evans, and Schwab (2001); Ladd and Murray (2001); Brunner and Balsdon (2004); Lambert et al. (2009); and Hilber and Mayer (2009).

In Columns D-E, I control for the percentage of K-12 students enrolled in private school, which acts as a proxy for the fraction of the voting population consisting of parents with children in private school, as well as the percentage of the population that is over the age of 65. In the final regression of Column E, both these variables are statistically significant: the effect of private school enrollment on voter opposition is positive, while the effect of the proportion of elderly people is negative. Both these results are consistent with the findings of Ehrenberg et al. (2004).

3.4.5 Demographic Variables

It is reasonable to expect that demographic characteristics of a district may affect that district's level of voter support. For instance, Priest and Fox (2005) find using data in Charlotte-Mecklenberg, NC that minority groups appear to be more supportive of school bond measures. On the other hand, Silverman (2011) finds using New York referendum data that the percentage of a district's population consisting

of minorities has a statistically significant, positive effect on the percentage of 'No' votes. An explanation for this finding offered in Silverman (2011) is that the presence of more minorities may cause a decrease in support among white voters in the district.

In Column E, I test for the influence of minority groups by controlling for the proportion of the district population identifying as various racial groups including: American Indian, Asian, Black, Hawaiian, Other, and as two or more races. I do not find that any particular group has a statistically significant effect on the level of voter support for the referendum. Furthermore, upon performing an F-test I do not find the group of variables describing race to be jointly significant.

3.4.6 The Effect of Large Voter Participation

Lastly, one important political variable that has been found to be related to voter support is the rate of voter participation in the election. Studies have generally found that larger rates of participation in elections are associated with higher levels of opposition to the referendum (Carter & Savard, 1961; Piele & Hall, 1973; Alexander & Bass, 1974; Dunne, Reed, & Wilbanks, 1997; Pecquet, Morris Coats, & Yen, 1996; Peirce, 2017), although not all studies obtain this conclusion (Rubinfeld, 1977; Rubinfeld & Thomas, 1980). The classical explanation for greater participation resulting in greater opposition is that: 1) a larger and more unpopular proposal gives incentive for a larger voting population to endure the implicit cost of participating in the election because the issue becomes more important, and 2) these new voters tend to be more likely to vote 'No' than the typical electorate for a school finance referendum. Nonetheless, in their study of New Jersey school budget referenda from 2002-03 to 2008-09, Barr and Dee (2016) find that in years following a failed budget proposal both voter support and voter participation simultaneously increase.

In Column E, I control for the number of participating voters as a fraction of the total district population, which serves as a proxy for the fraction of voters that

choose to participate among the total number of eligible voters. I find that elections in which a greater share of the population participates have greater opposition to the referendum, which provides additional support to the above theory that greater turnout may be motivated by increased opposition to the budget proposal.

3.4.7 Year by Year Analysis and Further Discussion

In Table 3.A.2, I test the percentage of 'No' votes against proposal and districts characteristics for each year individually. In the years 2008-09 through 2010-11, I find that the effect of larger proposed percent increases on voter opposition is positive and significant at the 99.9% level. However, in the year 2011-12, I do not find a statistically significant positive relationship. This lack of significance in 2011-12 may be surprising at first given the clear evidence for a positive effect in the other years and in other empirical studies.

A possible explanation for the absence of a statistically significant positive relationship in 2011-12 may be related to: 1) the fact that 2011-12 was the first year that the 2% cap on property tax increases was in place, and 2) the way in which this cap may result in an increase in voter approval for large (2%+) increase proposals among districts with high taste for education spending. Namely, a district with high taste for education spending may prefer or at least be willing to support a levy increase larger than 2%. Prior to 2011-12, such a district might face a budget proposal substantially larger than 2% that would not garner as much voter support. However, with the proposal capped at 2% in 2011-12, even districts with high taste for education spending would face a proposal for an increase of only 2%, which would then receive a high approval rating from voters.

I find strong evidence for a positive relationship between the proposed percent increase in the local tax levy and the level of voter opposition, in contrast to Silverman (2011) who instead finds that it is the proposed percent increase in *total* spending

including aid as opposed to the local spending that matters in his study of New York referenda. Silverman's finding is counterintuitive because voters should care more about the increase in the local levy, since that is the amount they are being asked to pay. Although I do find that that the proposed percent increase in total spending has a small, positive effect on the percent of 'No' votes when it is added to the regression of Table 3.7, Column E ($p = 0.047$, standardized $\beta = 0.049$), it is much less clearly economically significant than the proposed percent increase in the local tax levy ($p = 0.000$, standardized $\beta = 0.189$). One possible reason for the difference in these results is that New York voters only see the total spending on the ballot. Thus, if they are not fully informed about the aid component (which in New York is determined after the referendum process), they may not be able to discern the local portion.

Finally, in this current study, it was not possible to obtain information on every variable that could predict the voter support for school budget referenda, and there are a number of variables whose inclusion could improve upon the results for further studies such as whether a district is an urban or rural district (Lentz, 1999; Bowers, Metzger, & Militello, 2010) and including the percentage of the voting population consisting of blue-collar vs. white-collar workers (Alexander & Bass, 1974), although this demographic characteristic correlates strongly with the educational attainment and income level of a district, which I do include.

3.5 Municipal reductions to failed budget proposals

Next, I analyze the behavior and actions of municipal bodies when they make cuts to failed budget proposals. I test the hypothesis that larger and more unpopular proposals are subject to larger reductions by the municipal body.

The main regression of this section fits the municipal reduction as a percent of the proposed levy (Reduction Pct) against characteristics of the proposal and the district, including the outcome of the voting on the proposal embodied in the variable (Percent 'No') from Section 3.4. First, I point out that there are many instances where the municipal body made a reduction of zero to the failed budget proposal, and municipal bodies in general do not add additional funds to the failed levy proposal. Thus, municipal reductions must be non-negative amounts, and the variable [Reduction Pct] has a distribution shaped like a left-censored bell curve (as seen in Figure 3.A.1). Therefore, it is most appropriate to apply a Tobit analysis to take into account the left-censored nature of the data[17]. The full results of the Tobit regression for the municipal reduction as a percent of the levy are given in[18] Table 3.8. To incorporate the zero reductions, which make up 15% of all the reductions, it is clear from Figures 3.A.1 and 3.A.2 that the reduction as a percent of the proposal, as opposed to the total amount of the reduction or its logarithm, is the most appropriate variable to use to measure the municipal cuts[19].

The model specification for the main regression for the study of municipal reductions is

$$\text{ReductionPct}_{it} = \max\{\alpha + X_{it}\beta + \beta_{\text{No}}\,\text{PercentNo}_{it} + \text{YEAR}_t\gamma + \nu_{it}, 0\},$$

where ReductionPct_{it} is the municipal reduction as a percent of the proposal for district i in year t, X_{it} is a vector of characteristics of district i and the proposal in district i in year t used in the regression on the percent of 'No' votes from Section 3.4, PercentNo_{it} is the percent of 'No' votes for the referendum in district i in year t,

[17] Although a Tobit model is more appropriate for these data, the qualitative results do not change when I use an OLS model instead.

[18] Several demographic variables in the regression that were not significant at the 95% level, including PctAmerInd, PctAsian, PctHawaiian, and PctTwoRaces, were omitted from the table.

[19] Figure 3.A.2 also suggests that there are a number of municipal reductions that would have been very small compared to the proposal that are chosen instead to be a reduction of exactly zero.

YEAR_t is a vector of year indicator variables, and the ν_{it} are i.i.d. normal random variables.

The main regression results are consistent with the hypothesis that larger and more unpopular proposals (proposals with higher proposed percent increases in the levy or higher percent 'No' votes) are associated with larger reductions. In particular, the proposed percent increase in the levy and the percentage of 'No' votes are the two most economically significant variables in the main regression of Column B in terms of standardized coefficients. It is also notable that the only variable that turns out to be a statistically significant predictor of the municipal reductions in Column B that is also statistically significant in predicting the percentage of 'No' votes in Section 3.4 is the proposed percent increase in the local levy.

At first, the finding that a larger percentage of 'No' votes is associated with a larger reduction may seem to indicate that municipal bodies actively respond to the result of a referendum to satiate the voters. For example, it is conceivable that in light of a failed proposal, municipal bodies may pay attention to the degree of voter disapproval (as measured by the percent of 'No' votes), and choose the magnitude of their cuts accordingly to reflect the level of dissatisfaction. However, there are alternative models that would predict the same relationship between voter support and the size of the municipal reduction. For example, this finding is also consistent with a model of the proposal and reduction process in which: 1) the reversion budget chosen by the municipal body does not change in response to the voting, and 2) the school board seeks to maximize expected school funding while facing uncertainty over voter turnout or preferences. In this model, proposals that result in a larger percentage of 'No' votes generally correspond to proposals by the school board with a greater risk of failure. Proposals with a greater risk of failure are then precisely those proposals that are large compared to the reversion, since voters compare the

Table 3.8: Municipal reduction as percent of the proposed levy

	All years (A)	All years (B)	Only 2010 (C)
Proposed Percent	0.236***	0.231***	0.189***
Increase in Levy	(0.0197)	(0.0203)	(0.0295)
Percent 'No' Votes	0.0535***	0.0615***	0.0626***
	(0.0105)	(0.0111)	(0.0175)
ln(Per Pupil Aid)	0.347***	0.397***	0.480**
	(0.0928)	(0.116)	(0.166)
isSDA	-1.388***	-1.176**	-1.216
	(0.362)	(0.400)	(0.648)
$\ln(\text{Levy}_{t-1}/\text{Population})$	0.386**	0.352	0.160
	(0.145)	(0.225)	(0.361)
ln(Population)	0.120	0.104	0.119
	(0.0621)	(0.0696)	(0.111)
2009.YEAR	-0.00692	0.00877	
	(0.196)	(0.194)	
2010.YEAR	0.416*	0.311	
	(0.172)	(0.193)	
2011.YEAR	0.0802	0.0577	
	(0.216)	(0.218)	
% with a Bachelor's		0.0124	0.0194
Degree		(0.00967)	(0.0153)
ln(Enrollment/Population)		-0.170	0.00571
		(0.213)	(0.344)
ln(Median Income)		-0.748	-1.858*
		(0.610)	(0.929)
% Low Income		-0.0313	-0.0584
		(0.0209)	(0.0346)
% Over 65		-0.0113	-0.00985
		(0.0142)	(0.0232)
% K-12 Students in		-0.00524	0.0274
Private School		(0.00817)	(0.0143)
% Black		0.0154*	0.0185
		(0.00643)	(0.0140)
ln(#Voters/Population)		0.150	0.698
		(0.216)	(0.426)
Constant	-9.474***	-1.510	13.48
	(1.834)	(7.506)	(11.29)
No. of observations	685	685	306
Pseudo-R^2	0.0880	0.0967	0.0722

Tobit; standard errors in parentheses, 2008.YEAR omitted.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. No. of observations varies due to limited data.

Not displayed: %American Indian, %Asian, %Hawaiian, %Other, %Two or More Races.

size of the proposal to the reversion when they decide how to vote. Therefore, a larger percent 'No' vote would generally occur together with a larger municipal reduction.

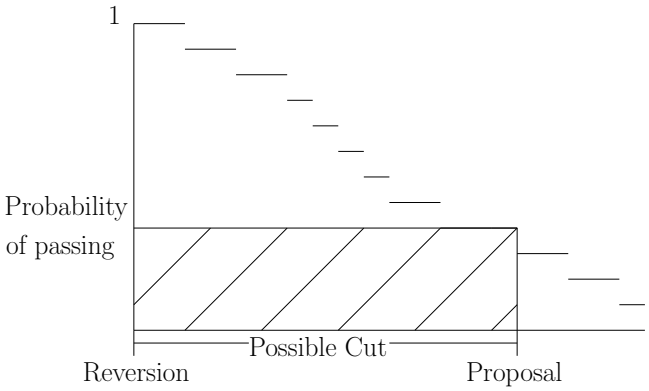


Figure 3.2: A school board selecting a high-risk proposal to maximize expected revenue

Figure 3.2 illustrates an example of a school board choosing a proposal with a high risk of failure and therefore a large reduction in the event of failure. In this figure, the reversion corresponds to a certain level of per pupil funding that does not respond to the outcome of the voting. The step function represents, for any proposed per pupil funding level[20], the probability of passing the referendum. As seen in the figure, the larger the proposed per pupil funding level, the lower the probability that the proposal passes. School boards in this model seek to maximize expected per pupil funding, which is equivalent to maximizing the expected per pupil funding beyond the reversion amount, and this latter quantity is the area of the striped rectangle. In this particular example, the school board chooses a proposal with a relatively low probability of passing, which results in a large gap between the proposal and the reversion. These proposals that have a low probability of passing are the ones that have a comparatively large expected proportion of 'No' votes. In the event that such a proposal fails, the municipal cut consequently must be large since there is a large gap between the proposal and reversion amounts. Thus, there is a relationship between a

[20] In New Jersey, a budget proposal consists of a local tax levy figure. This amount translates to a particular level of per pupil funding since the aid to the district is already projected.

high expected proportion of 'No' votes and large municipal cuts that is a consequence of the model rather than being due to the municipal body actively responding to the voting. A detailed analysis of this model can be found in Zhao (2020).

This analysis shows that the model studied in Zhao (2020), which assumes budget-maximizing behavior of school boards along with nonresponsive behavior of the municipal body, is consistent with the main relationships found in the data. However, the data do shed light into what improvements can be made to make the model more realistic. Specifically, the many instances of zero reductions that occur in the data cannot be directly explained by the model of Zhao (2020) since a school board that maximizes expected revenue would not choose a proposal that results in a zero reduction. One way to improve the model to allow for zero reductions would be to incorporate uncertainty over the reversion budget[21].

The empirical finding in the current study that districts with a greater amount of per pupil aid tend to have larger reductions provides an interesting insight into the behavior of the municipal bodies. Municipal bodies face at least two conflicting priorities in choosing their reduction amount, and it is not clear a priori which priority would be most important. One possible priority would be to ensure that taxes are not too high for residents in their municipality by aiming for a certain level of local tax contribution: a reversion budget that imposes a local tax levy of $\overline{T}$ dollars. The reduction is then the difference between the proposed tax levy and $\overline{T}$. If municipal bodies behave in this manner, municipal reductions would not be sensitive to changes in aid, all else equal, since the reversion tax levy $\overline{T}$ that the municipal body aims for would be unaffected by changes in aid, and since the proposal is being held constant. The data, however, indicate instead that increases in aid are associated with increases in the size of the municipal reduction.

[21] For example, suppose the reversion budget takes on a low value f_L with high probability or a high value f_H with low probability. The school board may find it optimal to make a proposal that is larger than f_L but still smaller than f_H, since f_L is the more likely reversion, and the reduction will be zero in the less likely scenario that the reversion is f_H.

A different, possibly conflicting priority for municipal bodies to have would be to ensure that funding for students is not too low by choosing a budget reduction that, for example, targets a certain level of (per pupil) funding for schools: $\bar{f}$ dollars per pupil. In this latter scenario, there should be a *positive* relationship between aid and the reduction since the proposal is held fixed and since an increase in aid means that the local tax levy required to achieve the municipal body's desired per pupil funding level $\bar{f}$ would become smaller[22]. The finding that districts with greater amounts of aid per pupil receive larger cuts therefore suggests that a sizable portion of municipal bodies are not targeting a level of local contribution, but instead may be aiming to achieve a certain level of per pupil funding. At the very least, this observed relationship between per pupil aid and the size of the reduction does strongly suggest that the amount of per pupil aid offered to the district is an important factor for municipal bodies in choosing the amount of the reduction.

While the finding on the effect of per pupil aid supports the theory that municipal bodies aim to achieve a certain level of per pupil funding as a main priority, the takeaway from the finding on enrollment is less clear. If municipal reductions do target a certain level of per pupil funding, at first glance, it would be expected that districts with greater enrollment figures would have smaller reductions. The reason is that when enrollment is higher, achieving any given level of per pupil funding requires a larger amount of total funding and thus a greater tax levy when per pupil aid is fixed, which would require reversion budgets to be larger and hence municipal reductions to be smaller (since the proposed increase is also fixed). Although it is found in this work that enrollment has a negative effect on reductions, the effect is not statistically significant.

[22] The relationship between $\bar{f}$ and aid is $\bar{f}$ = (Local Levy + Aid)/Enrollment. For a fixed proposal, a decrease in the reversion tax levy implies the reduction would be larger.

The finding that SDA districts[23] tend to have smaller reductions than non-SDA districts may have multiple explanations. One possible explanation is that an SDA district naturally requires a greater amount of funding to provide an adequate education to its students. Since municipal reductions need to meet the requirements of an adequate education in order to be approved by a commissioner, municipal bodies in SDA districts may be likely to agree with school boards and support a higher level of local levies to provide an adequate level of funding. This explanation would be consistent with the previously described theory that the per pupil funding level of a district is an important part of the municipal body's decision making process. It is also conceivable that the label of being an SDA district alone rather than a specifically greater level of need motivates the municipal bodies to make smaller reductions in order to support the district's effort to improve its quality of education.

In this study, I also find (with $p = 0.014$) that districts with a higher percentage of the population identifying as Black tended to receive larger reductions. One possible reason is that the percent of the population identifying as Black may be a proxy for a higher level of ethnic diversity in the district. Including an index that measures the diversity of the district (discussed in more detail in Appendix Section 3.B.6), the variable [PctBlack] is no longer statistically significant ($p = 0.309$), and the statistical significance is mostly absorbed into the variable [DiversityIndex] ($p = 0.107$)[24]. Moreover, when [PctBlack] is removed and [DiversityIndex] is used instead, [DiversityIndex] becomes highly statistically significant with $\beta = 0.676$ and $p = 0.006$.

[23] Section 3.B.5 contains a description of SDA districts, which are a group of especially high need districts in New Jersey (formerly known as "Abbott districts").

[24] This finding also holds when alternative measures of diversity are considered such as the Shannon entropy, but does not hold when the Hirschman-Herfindahl index is used instead.

3.6 Conclusion

From the above data and analysis, several important conclusions can be drawn regarding the outcome of school budget referenda in New Jersey during the years 2008-09 through 2011-12. First, it is evident that many budget proposals during this period failed to pass the popular referendum. In addition, these failures had a large impact on the funding for K-12 public schools, as these failed proposals became subject to reductions by the municipal bodies in the districts. Even when the budgets did pass, in many cases the margin of passing was slim (see Figure 3.A.7), and many budget proposals in each year faced sizable opposition.

Certain patterns emerge regarding which proposals receive a higher level of opposition, and a main finding of the study is that the proposed percent increase in the local tax levy is positively related to the level of opposition to the referendum. The observed positive relationship between the proposed percent increase in the levy and the opposition to the referendum holds clearly in every year 2008-09 through 2010-11, but not in 2011-12. The lack of this finding in 2011-12 may be explained by the fact that 2011-12 was the first year in which the 2% cap on property tax increases came into effect. Namely, in this year, the cap may have been binding for districts that typically have a higher taste for education spending, and these capped proposals may have received more support in 2011 compared to prior years in which the proposals would have been larger.

Based on the findings and their implications for the support and disagreement hypotheses, I find that different types of increases in the proposed levy appear to occur for distinct underlying reasons that depend on the surrounding circumstances. Before controlling for key predictor variables, I find that districts with higher tax levies per person have lower levels of voter opposition. This finding is consistent with the support hypothesis, which asserts that larger levies per person may be associated with uniform increases in the funding levels voters are willing to support. However,

upon controlling for the educational attainment in the district, I find that the opposite relationship between tax levy per person and voter opposition emerges. This finding continues to hold when controlling for further covariates such as the income level of the district. Furthermore, I find that larger proposed percent increases in the levy are associated with higher voter opposition. These latter findings are consistent with the disagreement hypothesis, which states that larger levies may reflect heterogeneity in the funding levels voters are willing to support.

The current study identifies several further factors that influence the opposition to school budget referenda. Districts with larger populations tend to have lower support for the budget, which may be due to the greater amount of heterogeneity in larger districts. Districts with a larger share of the population enrolled in K-12 public schools have a higher level of support for the budget. Similar to Ehrenberg et al. (2004), I also find that districts with a more educated population had greater support for the referendum. Likewise, districts with a higher proportion of elderly people or a lower proportion of students in private school were more supportive of the referendum. With data on the percent of 'Yes' and 'No' votes to school budget referenda as well as information on the size of the proposals for New Jersey school districts, one of the additional contributions of the current study beyond Ehrenberg et al. (2004) is to show that the findings still hold when 1) the percent of 'No' votes is used to measure voter support rather than simply the probability of the proposal passing, and also when 2) one controls for the very important variable that is the size of the proposals themselves.

Another key contribution of the current study is to provide the first empirical investigation of the size of municipal reductions to failed budget proposals in New Jersey, which had a sizable impact in partially curbing the growth of local tax levies in New Jersey during the time period of the study. Performing this empirical investigation is essential to forming the picture of school finance in New Jersey, since the

decisions of New Jersey municipal bodies are not dictated by state law. The main findings of the investigation are that municipal reductions are higher for larger proposed tax levy increases, for proposals that receive a greater amount of opposition in the referendum, and for districts that receive a larger amount of aid.

These findings also provide insight into the behavior of general municipal bodies that would otherwise be a mystery. For example, I find that there is some evidence that municipal bodies choose their reductions in order to ensure a certain level of (per pupil) funding for schools. Evidence for this idea comes from the finding that districts that receive a greater amount of aid experience larger reductions, and from the finding that larger proposed levy increases correspond to higher reductions. In addition, the finding that more unpopular proposals tend to receive larger reductions may at first seem to imply that municipal bodies actively respond to the will of voters in the district. However, the same conclusion can be obtained from other models of school board and municipal body behavior. In particular, these findings are consistent with the theory that school board budget proposals aim to maximize the expected budget for schools, and that municipal bodies seek to achieve a certain level of (per pupil) funding for the district that does not depend on the proposal or the amount of voter support the referendum receives.

The year 2010-11 in which the governor declared a state of fiscal emergency in New Jersey requires special attention. In this year, the governor instituted heavy cuts to state aid, though federal aid increased at the same time. School boards in 2010-11 requested larger local tax levy increases compared to other years, with nearly half of the tax levy proposals requesting increases over the 4% property tax cap in place at that time. These proposals met heavy opposition from voters at the April elections, and a majority of the proposals failed to pass the referendum. These larger, failed proposals then received substantial reductions by the New Jersey municipal bodies. Even controlling for many factors that influence voter support, there was

greater opposition to the referendum in 2010, which may have been in part due to the governor encouraging voters to vote down proposals that did not freeze teacher salaries for the year of the fiscal emergency. Nonetheless, despite these distinctive features of the year 2010-11, the reductions chosen by municipal bodies followed the same pattern as in the other years, as they were primarily predicted by the proposed percent increase in the levy, the percentage of 'No' votes in the referendum, and the level of per pupil aid available to the district.

The current study and its findings point towards directions of future research that should be useful for informing policy, both in New Jersey and beyond. The increase in the number of relatively large proposals that passed in 2011-12 indicates that the 2% cap, which remains in place today, strongly limits the spending of New Jersey districts beyond the spending constraints imposed by the referendum process. Furthermore, while school finance elections have been uncommon in New Jersey since 2012-13, if another state of emergency takes place such as in 2010, a surge of districts proposing tax levy increases above the 2% cap may be expected. This study provides a step forward towards estimating how voters will support or oppose such increases, how municipal bodies will reduce those proposals that fail, and the overall extent to which local districts may be expected to compensate for losses in state and federal aid. Estimating these quantities is important for evaluating the true financial impact of a state aid cut. A natural direction for future research would be to incorporate a structural model of the referendum process such as in Romer et al. (1992) in an effort to provide further estimates of the effects of such education finance policies.

3.A Figures and Tables

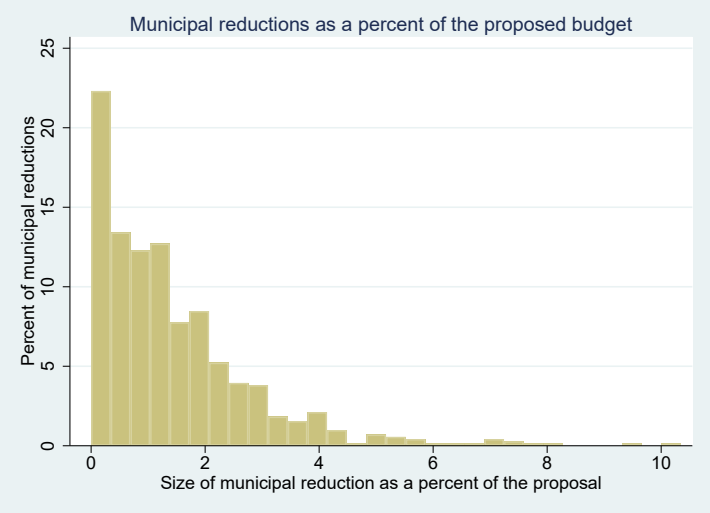


Figure 3.A.1: Municipal reductions as a percent of the proposal

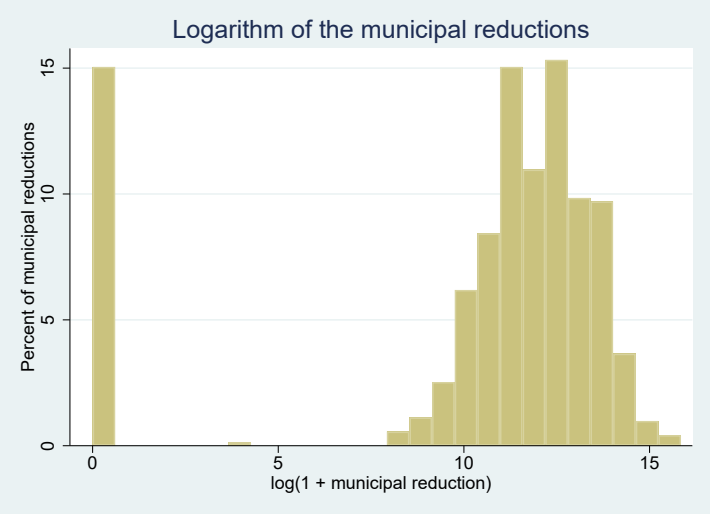


Figure 3.A.2: The natural logarithm of municipal reductions to failed proposals

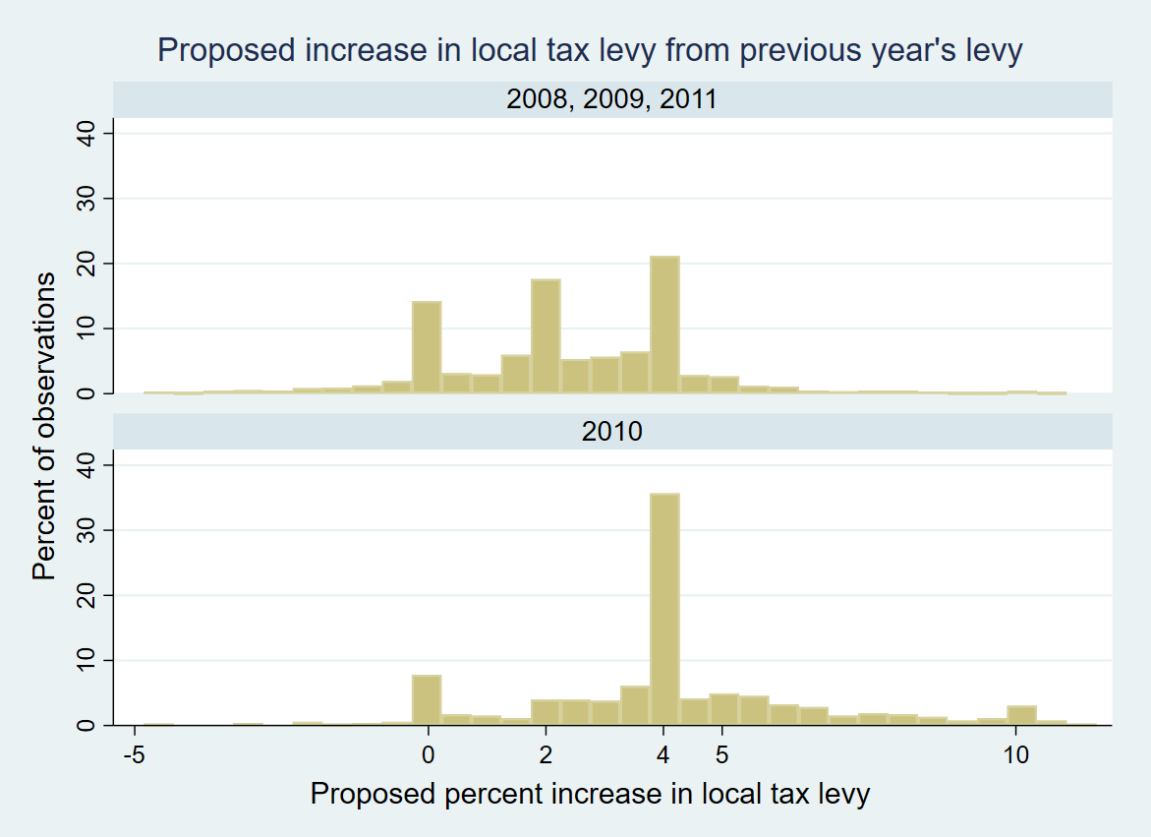


Figure 3.A.3: Proposed percent increases in general fund levy, 2010 vs. other years

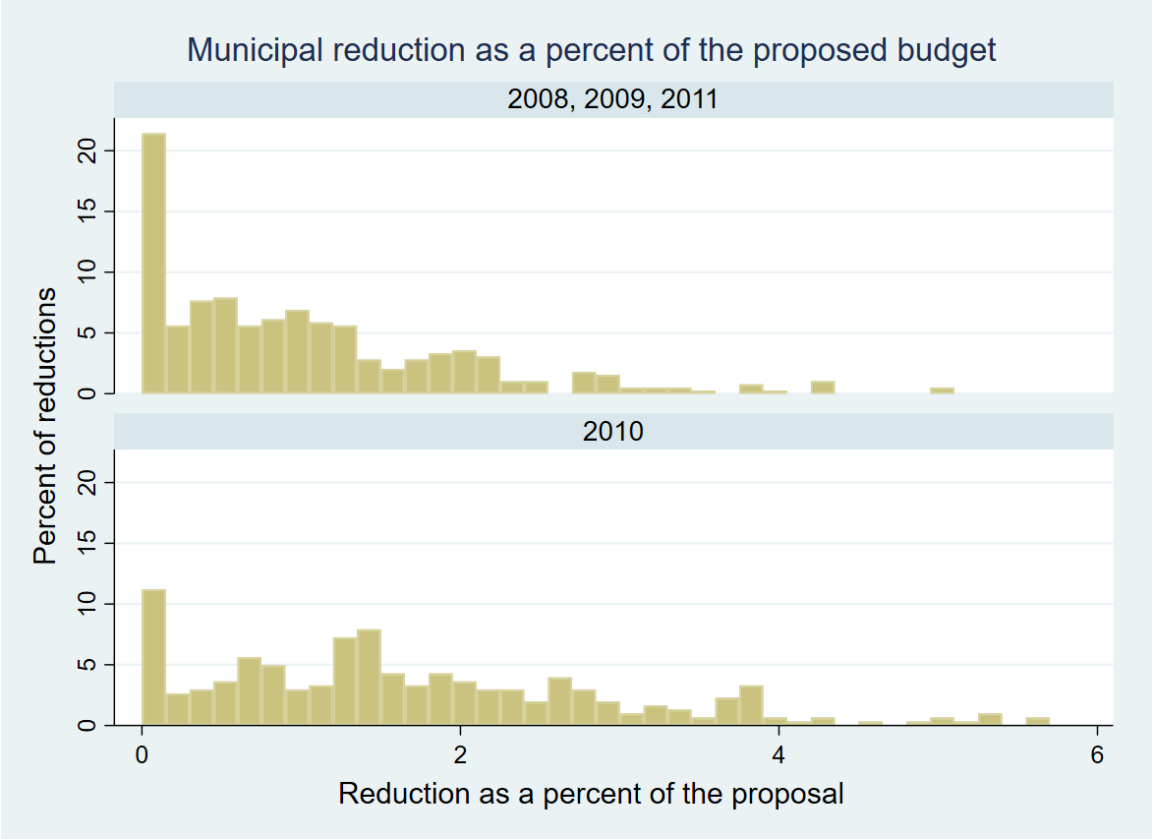


Figure 3.A.4: Municipal reductions as a percent of the proposed budget, 2010 vs. other years

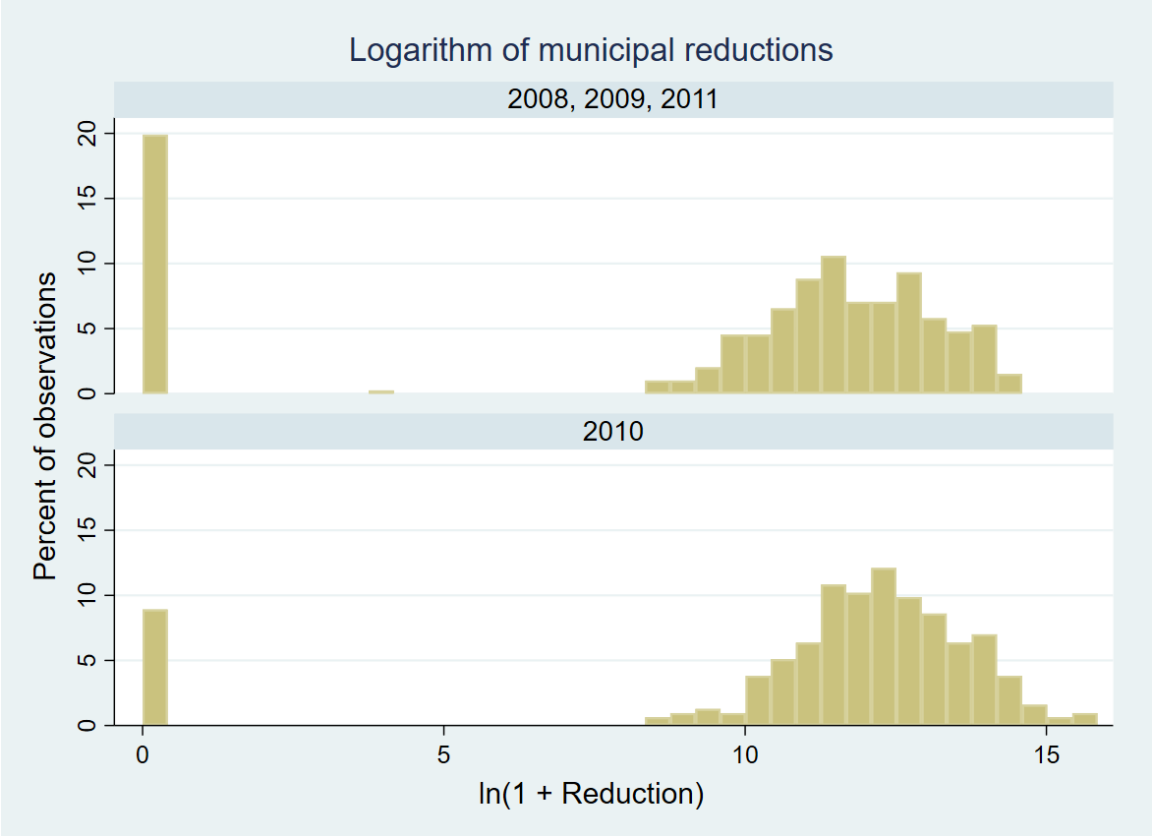


Figure 3.A.5: Comparing the sizes of municipal reductions, 2010 vs. other years

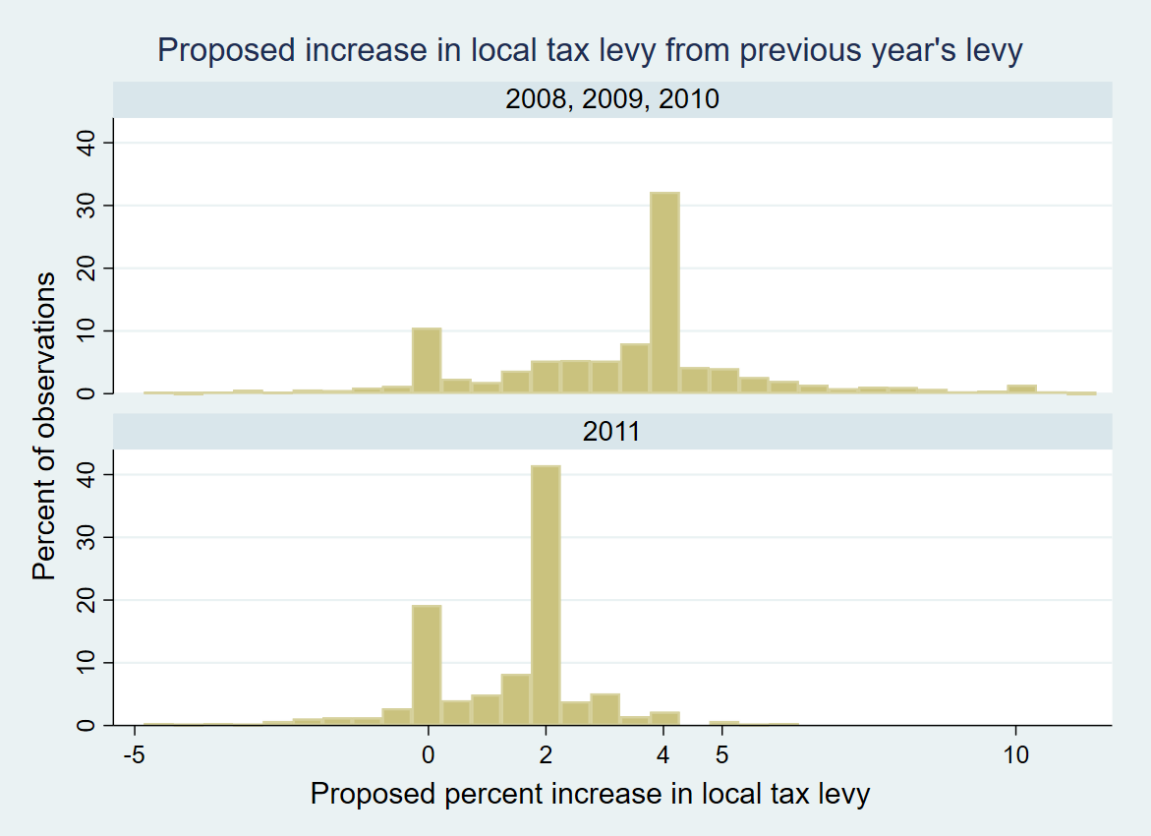


Figure 3.A.6: Proposed percent increases in general fund levy, 2011 vs. other years

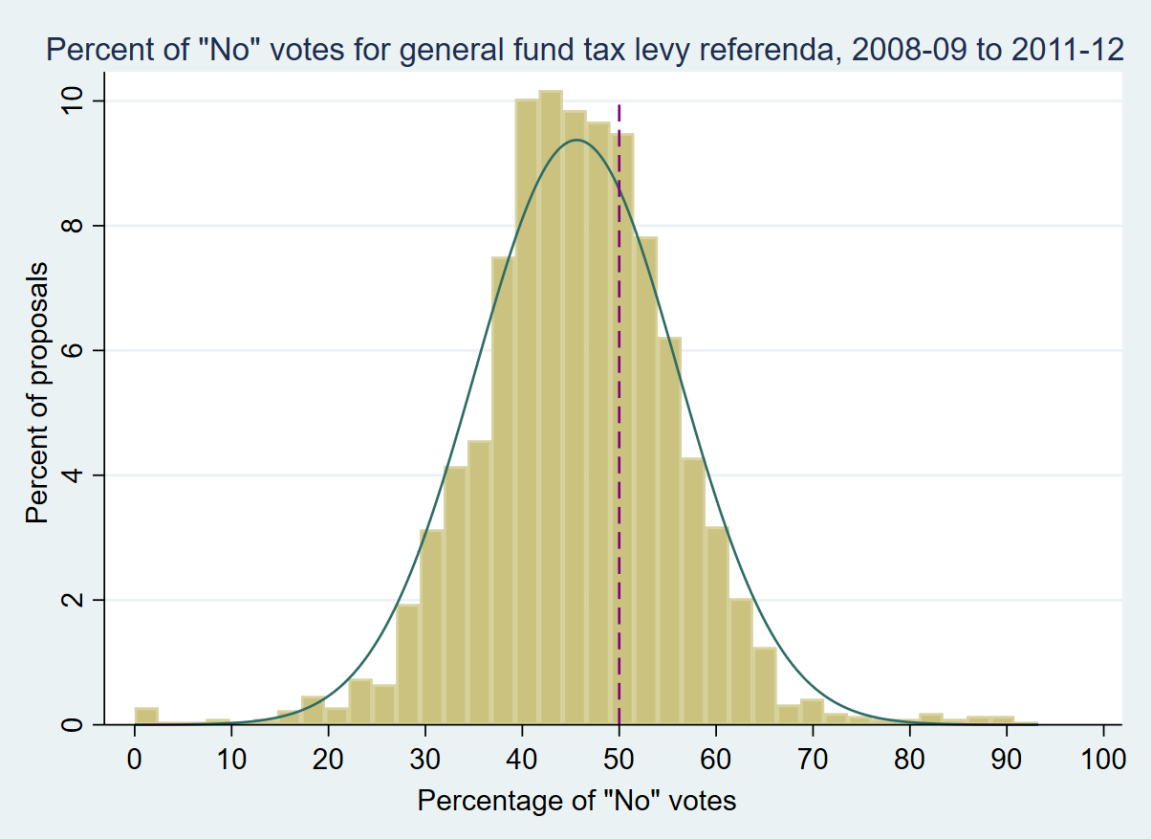


Figure 3.A.7: Percent of 'No' votes for general fund tax levy proposals, 2008-09 to 2011-12

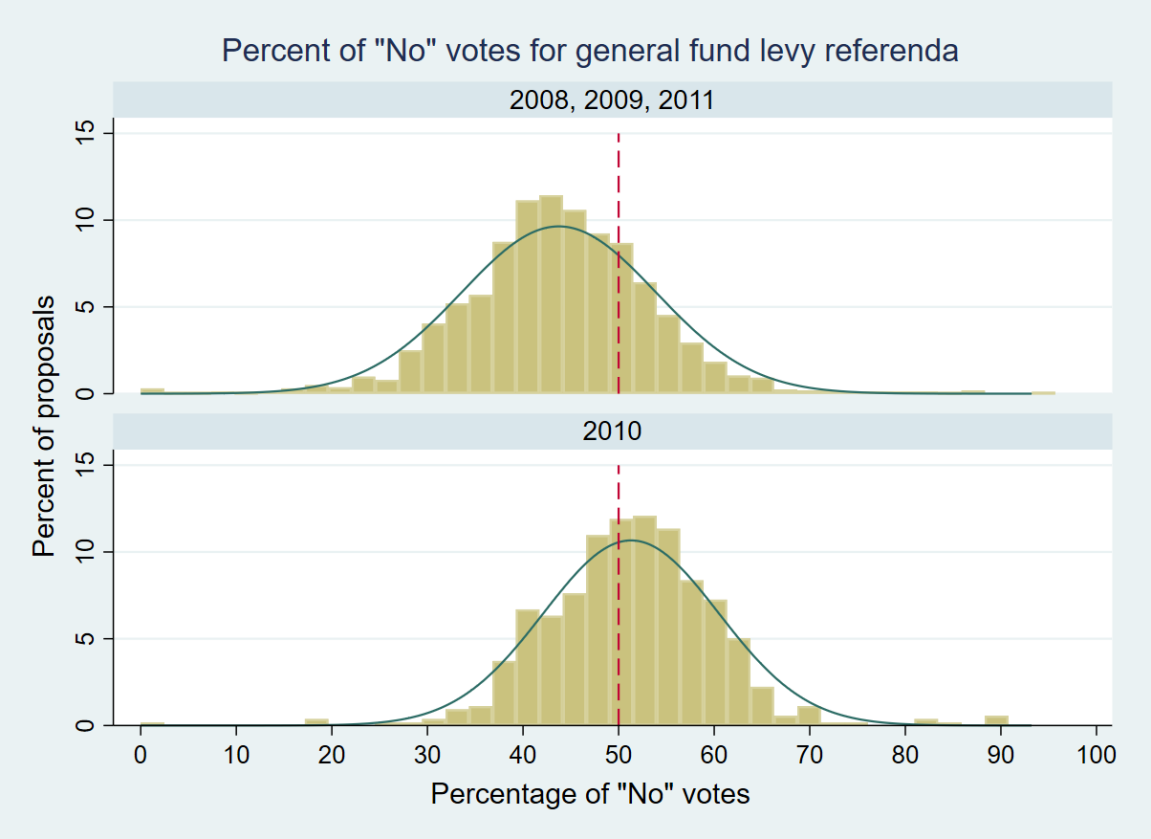


Figure 3.A.8: Percent of 'No' votes for general fund tax levy proposals, 2010 vs. other years

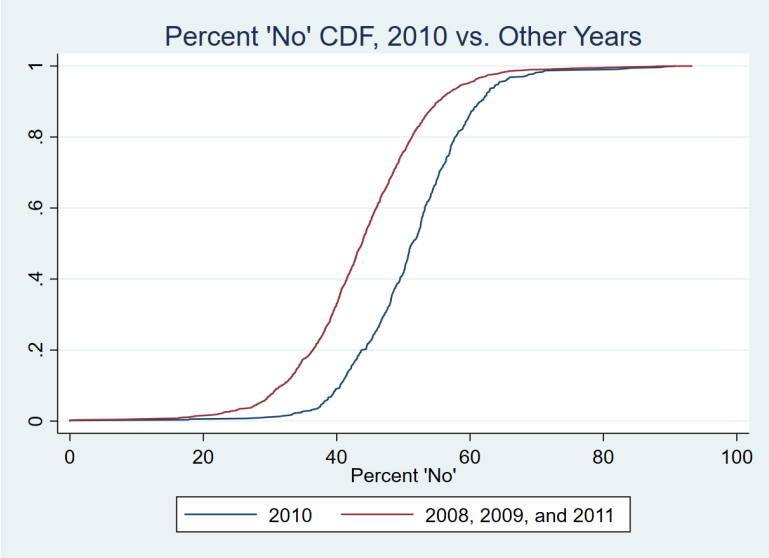


Figure 3.A.9: Percent No CDF, 2010 vs. other years

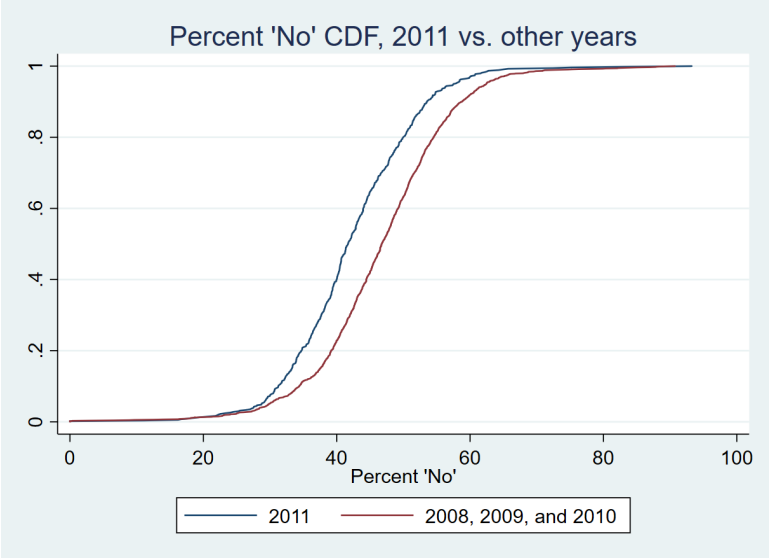


Figure 3.A.10: Percent No CDF, 2011 vs. other years

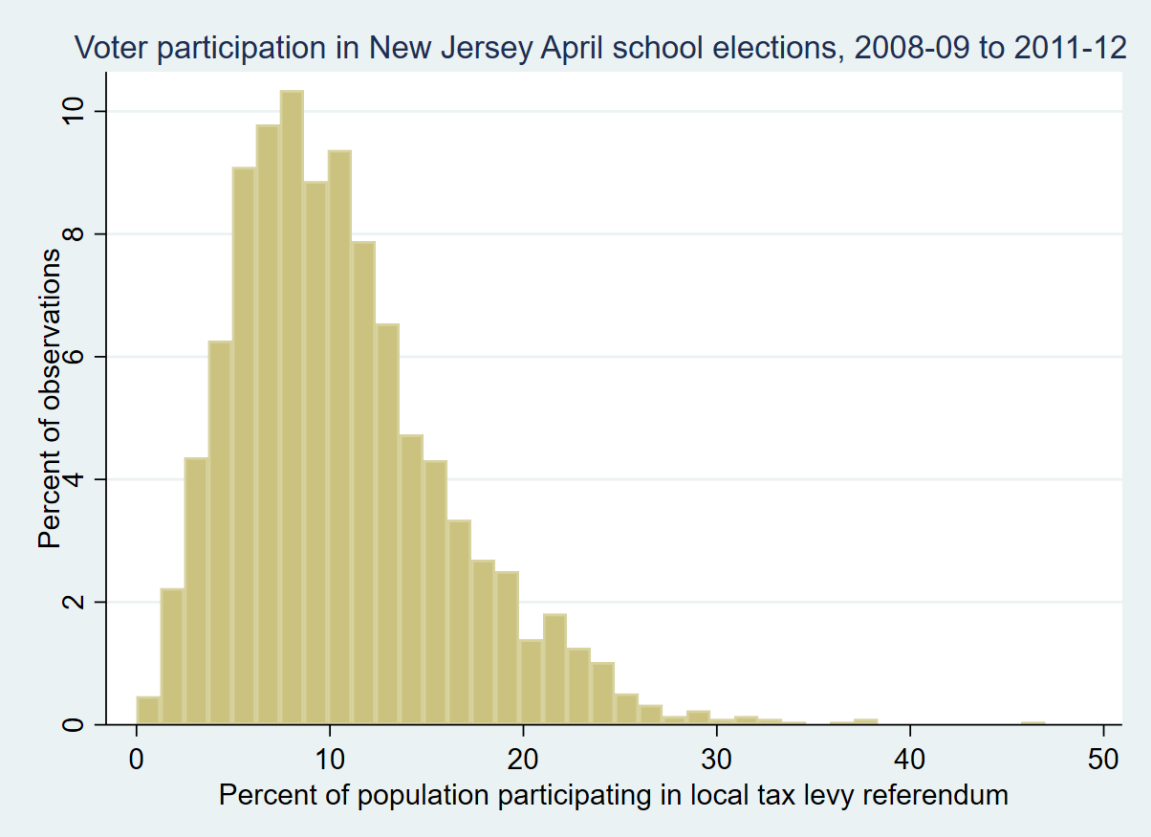


Figure 3.A.11: Percent of population voting in local tax levy referendum

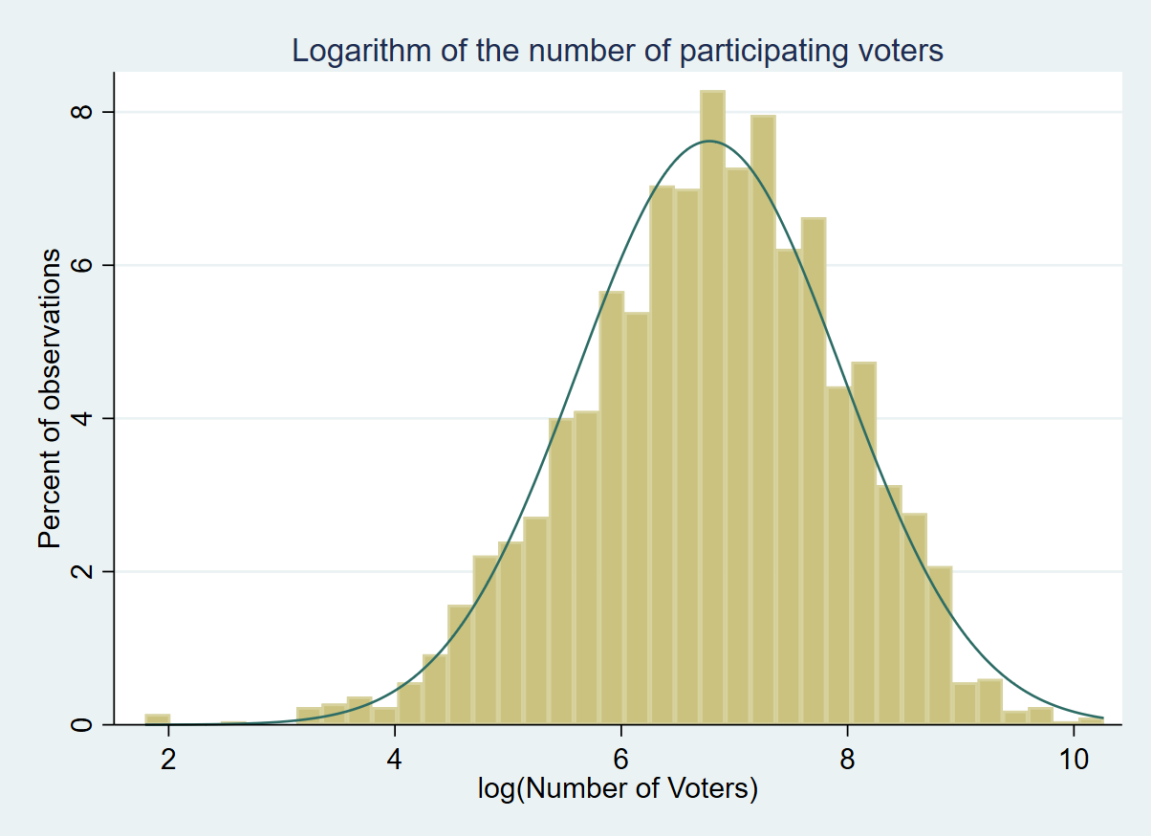


Figure 3.A.12: Logarithm of the number of voters

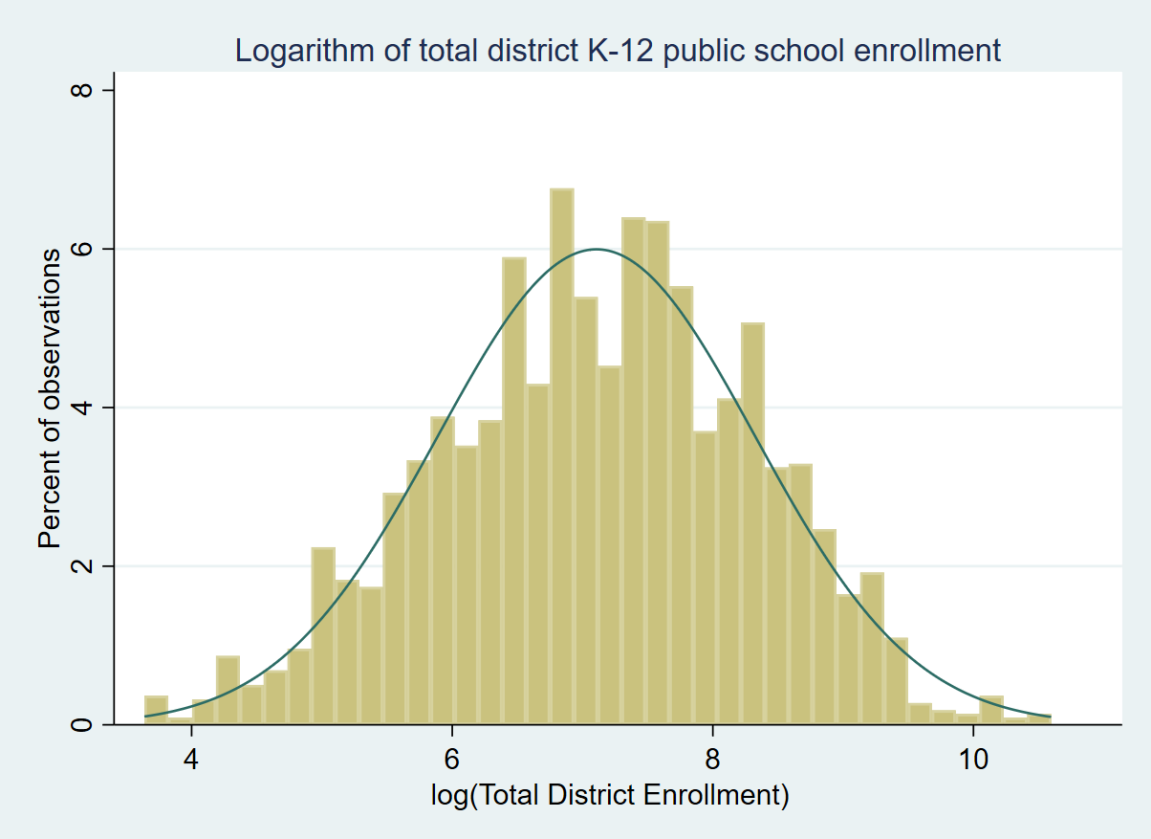


Figure 3.A.13: Logarithm of total district K-12 public school enrollment in New Jersey

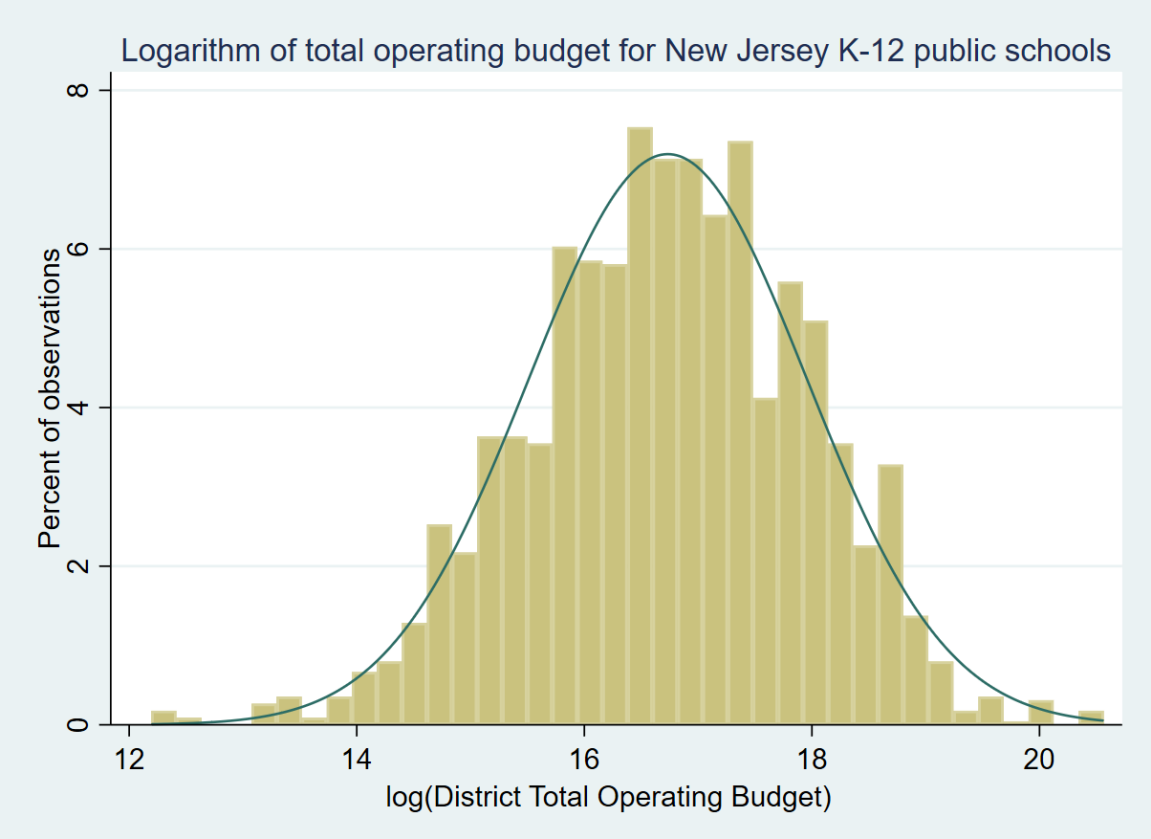


Figure 3.A.14: Logarithm of total district operating budget for K-12 public schools in New Jersey

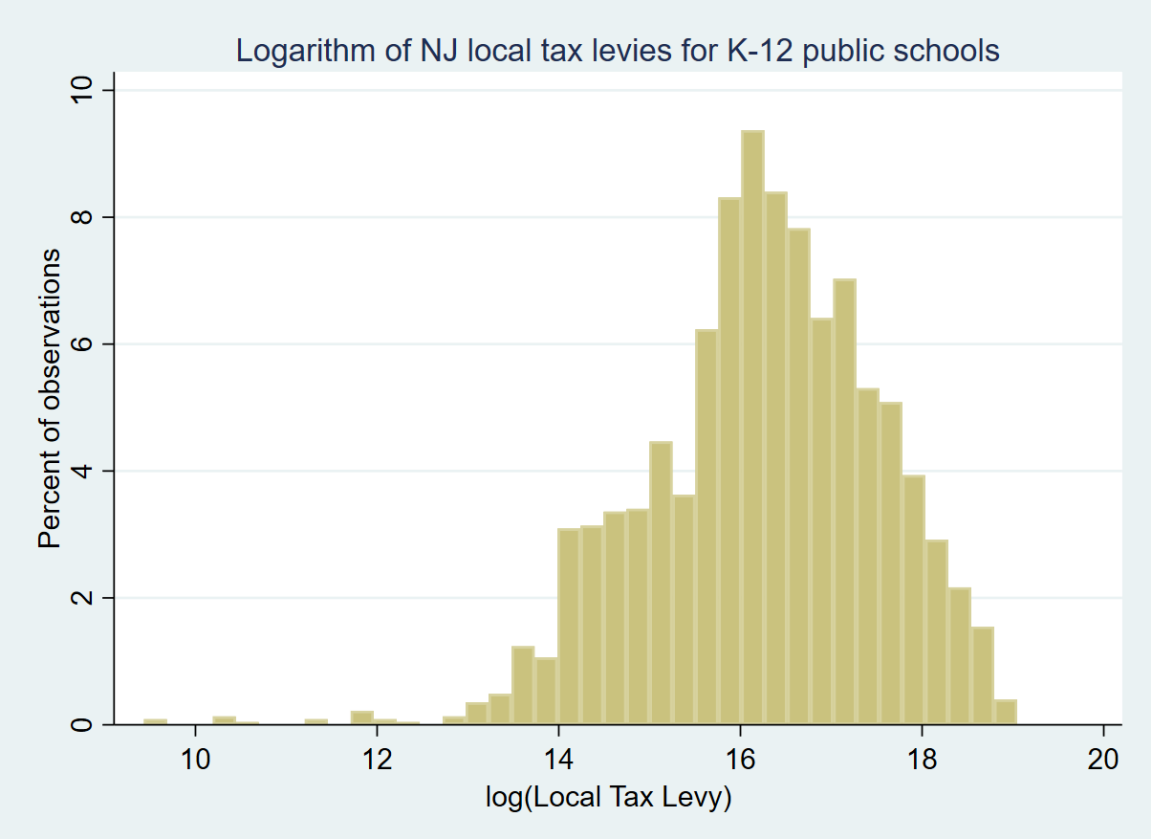


Figure 3.A.15: Logarithm of district general fund levy for K-12 public schools in New Jersey

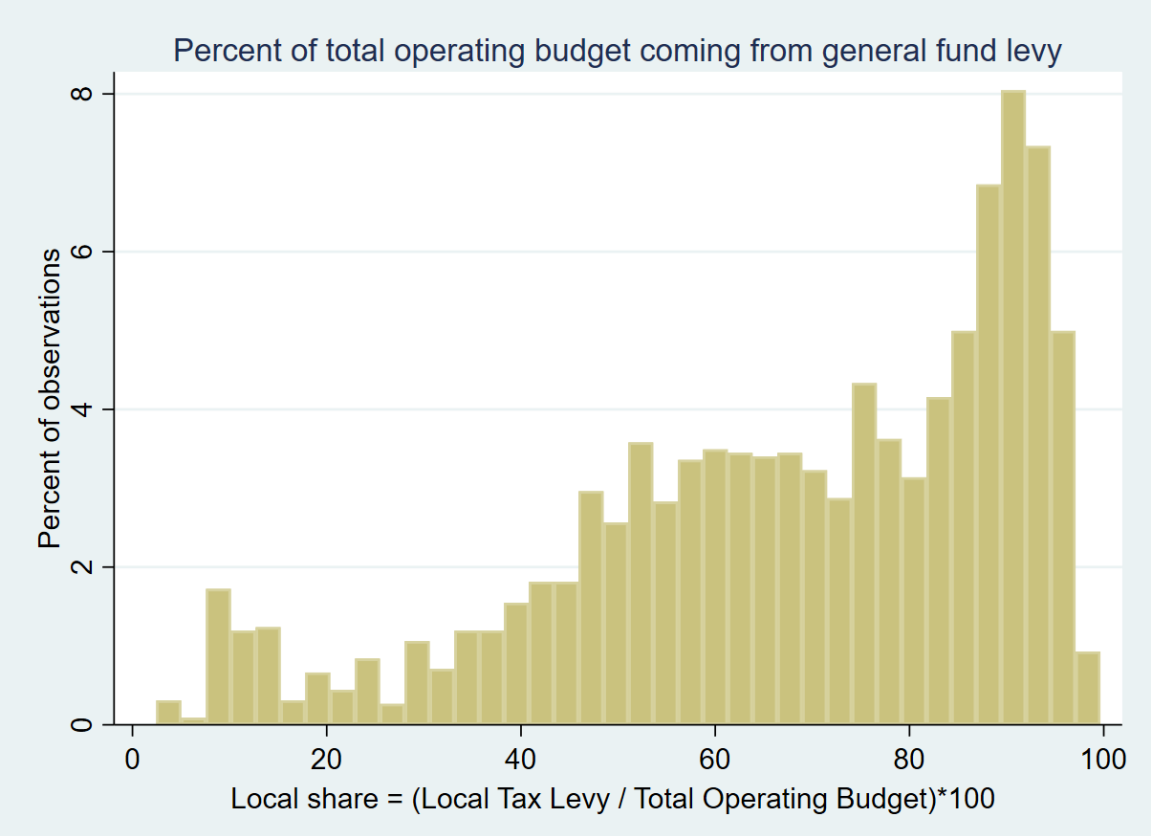


Figure 3.A.16: Local share of school funding for New Jersey districts

Table 3.A.1: log Levy per Person vs. Educational Attainment and Income

	$\ln(\text{Levy}_{t-1}/\text{Population})$
% with a Bachelor's Degree	0.0136^{***}
	(0.000955)
ln(Median Income)	0.439^{***}
	(0.0479)
Constant	1.595^{**}
	(0.512)
No. of observations	2109
R^2	0.4311

OLS. Standard errors in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Data are restricted to those used in Table 3.7.

Table 3.A.2: Effects of district and proposal variables on Percent 'No' by year

	2008 % No	2009 % No	2010 % No	2011 % No
Proposed Percent	0.649***	0.910***	0.720***	0.290
Increase in Levy	(0.139)	(0.142)	(0.108)	(0.206)
ln(Levy$_{t-1}$/Population)	1.369	3.837**	1.491	0.0268
	(1.263)	(1.330)	(1.176)	(1.303)
ln(Population)	2.168***	3.189***	1.189**	1.871***
	(0.404)	(0.423)	(0.375)	(0.401)
% with a Bachelor's	-0.291***	-0.280***	-0.201***	-0.220***
Degree	(0.0493)	(0.0534)	(0.0494)	(0.0524)
ln(Enrollment/Population)	-0.465	-4.601***	-2.664*	-1.892
	(1.206)	(1.290)	(1.103)	(1.254)
ln(Median Income)	3.261	3.987	4.670	-3.052
	(3.356)	(3.704)	(3.279)	(3.634)
% Low Income	0.0710	0.265*	-0.0496	-0.0837
	(0.114)	(0.122)	(0.111)	(0.122)
ln(Per Pupil Aid)	-0.968	-0.0348	0.539	-0.387
	(0.840)	(0.850)	(0.513)	(0.612)
isSDA	-5.049	2.759	-0.512	0.586
	(2.579)	(2.542)	(2.240)	(2.407)
% Over 65	-0.155	-0.243**	-0.0506	-0.0918
	(0.0862)	(0.0902)	(0.0811)	(0.0856)
% K-12 Students in	0.0583	0.0761	0.0427	0.0185
Private School	(0.0512)	(0.0533)	(0.0497)	(0.0552)
% Black	0.0625	-0.00329	-0.0483	0.0922*
	(0.0411)	(0.0439)	(0.0398)	(0.0452)
% American Indian	0.811	-0.494	-0.361	-0.666
	(0.817)	(1.012)	(0.848)	(0.946)
% Asian	0.0676	-0.00775	-0.148*	0.0305
	(0.0663)	(0.0657)	(0.0595)	(0.0618)
% Hawaiian	-2.826	-3.206	-1.610	7.263*
	(4.298)	(3.281)	(2.965)	(2.834)
% Two or More Races	0.215	0.386	0.651*	-0.0398
	(0.347)	(0.350)	(0.269)	(0.306)
ln(#Voters/Population)	2.479*	6.091***	1.463	5.672***
	(0.996)	(1.130)	(1.343)	(1.115)
Constant	0.741	-45.64	-24.51	81.70
	(41.41)	(45.82)	(39.58)	(43.68)
No. of observations	528	529	526	526
R^2	0.1801	0.2689	0.2147	0.1845

OLS, standard errors in parentheses. No. of obs. varies due to limited data.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. % Other omitted

3.B Appendix: Further background and statistics

3.B.1 Background on Municipalities and Reductions

In the time period of the data set, New Jersey contains 572 school districts, which were consolidated into 559 districts beginning in 2010. Each school district services one or more local jurisdictions known as municipalities, and municipalities are the local units that comprise the 21 counties in New Jersey. Municipalities in New Jersey can be one of five types, including borough, city, town, township, and village, and are governed by one of several different forms of municipal government. Each municipality typically has a mayor and a number of other officials who preside over the legal functions of the municipality, which include the provision of public goods and services (for example, maintenance of a police force, a fire department, garbage collection, recycling, etc.)[25]. A school district may contain one or more schools within the district. In some cases, school districts have a sending-receiving relationship with other districts, meaning that the district uses local property tax revenues to send a number of students within the district to schools located in another nearby district.

In New Jersey, the municipal bodies consist of official representatives from each municipality in a school district who form the committee that decides on possible cuts to failed budget proposals. These municipal officials generally have other responsibilities in the municipal government besides their role in the municipal body. The municipal body members may either be elected or appointed to their positions in the municipal government. The number of officials serving on a given municipal body depends in general on the municipality and its government. The school boards in New Jersey districts, meanwhile, consist of a number of elected school board members that are elected during April school elections during the time period of the data set. These April school elections are also the same elections in which, during the time

[25] Municipal services are also paid for mainly by local property taxes.

period of the data set, voters consider the question of the general fund levy for their school district's local school funding.

In the event that the budget proposed by the school board fails to earn the support of a majority of voters, the school board has the responsibility of presenting the failed budget to the municipal body. The municipal body then has the task of determining, with the consultation of the school board, how much money is necessary to be raised for schools from property taxes. If the municipal body decides to reduce the school board's proposed budget, the municipal body makes specific line-item cuts to the budget. The municipal body then sends the reduced budget to to the county commissioner for approval. The commissioner receiving the reduced budget can decide to impose additional reductions to the budget.

A school board whose proposed budget is reduced by the municipal body does not have to adhere to the specific line-item recommendations of the municipal body. The school board may instead adjust the budget as it sees fit within the total amount allowed following the municipal cuts. In addition, a school board whose budget was reduced by the municipal body may make an appeal to the county commissioner for the restoration of funds. Within the data set used in these studies, the commissioner rarely makes changes to the reductions proposed by the municipal body, as there are only three instances of budgets being increased by the commissioner, and five instances of non-negligible budget reductions by the commissioner.

Prior to the 2011-12 election cycle, the state of New Jersey passed a law that imposed a cap of 2.0% on the proposed increase on the general fund levy (excluding certain exceptional expense categories). Additionally the law imposed further guidelines relating the municipal reduction process to the state aid formula. According to this law, any municipal body that proposes a reduced budget that is below the "adequacy budget" for that district (see Section 3.B.3 below) is required to present an explanation of how the reduced budget still achieves the standards of a "thorough

and efficient education" required by New Jersey state law. The law in general allows school boards to appeal the reductions to the county commissioner on the basis of the need for long term planning and budgeting. In cases where the reduced budget is below the adequacy budget, the school board may also appeal the reductions on the basis that they fail to provide a thorough and efficient education. The law specifies that a commissioner who considers such an appeal should take into account fluctuations in enrollment in the district, the history of voter approvals or rejections of proposed budgets, the effect on the local tax levy, and possible effects on the ability of the district to fulfill its contractual obligations.

Legislation that passed in 2012 allows school districts to move their annual April school elections to coincide with the general election in November, and furthermore, for districts that move their elections, eliminates the requirement of the annual school budget referendum if the proposed general fund levy does not exceed the 2% cap. The overwhelming majority of New Jersey districts did decide to move their elections in 2012 (or soon after), and have chosen budgets to remain under the 2% cap. For these reasons, voting over school budgets has become much less frequent following the 2011-12 school elections. Whenever voting on school budget referenda does occur, research suggests that the timing of the general election in November is less favorable for the passing of school finance initiatives compared to the April elections in which they were held previously due to the characteristics of the electorate that will likely turn out to vote Filer and Kenny (1980); Peirce (2017); Pecquet et al. (1996); Dunne et al. (1997).

3.B.2 Background on Property Tax Caps in New Jersey

Within the time frame of the current study (2008-09 to 2011-12), two different laws were in place that imposed restrictions on the property tax increases that school boards and municipal governments were allowed to propose.

The first limit on property tax increases, which was signed into law in 2007 and remained in place for the 2008-09 through 2010-11 budget cycles, was a cap that limited property tax increases proposed by school boards or municipal governments to at most a nominal 4%. This 4% cap, however, included 14 exceptional expense categories that allowed school boards or municipal governments to propose property tax increases in excess of 4% if such a tax increase could be justified by expenditures in one or more of the exceptional categories. A few examples of exceptional expense categories included debt service, pension contributions, health care costs, energy costs, insurance, the replacement of reduced state education funds, and any purpose for which the district's Local Finance Board grants a waiver. A proposed increase above the 4% cap that was not justified by the exceptional categories could still be proposed; however, the referendum had to earn a 60% majority of voters to pass in this case.

Due to the multitude of exceptional expense categories, school boards were commonly able to justify general fund levy increases beyond 4%. In fact, more than 30% of proposals between the 2008-09 and 2010-11 budget years exceeded the 4% cap. In particular, it is worth pointing out that the replacement of reduced state education funds was one of the exceptional expense categories. In 2010-11, when a state fiscal emergency resulted in large reductions in state aid, school boards were thus allowed to submit budget proposals larger than 4% that could make up for some of the losses in state aid by drastically increasing local property taxes.

In 2010, New Jersey passed a more stringent law that remains in place today and restricts nominal proposed increases in the property tax to 2% beginning with the 2011-12 elections[26]. The legislature modeled the 2% cap after a 2.5% property tax cap implemented in Massachusetts starting in 1980 that allowed Massachusetts to lead the nation in educational outcomes despite having lower spending levels than New Jersey Barro (2010). Similar to the previous 4% cap, the 2% cap can be exceeded if

[26] See `https://nj.gov/dca/divisions/dlgs/hottopics/lfns/11/2011-3r.pdf` for more information on the 2% cap.

additional expenses are justified by certain exceptional expense categories. However, the 2% cap allows for far fewer exceptional expense categories. The four exceptional categories include: capital expenditures and debt service, pension contributions and accrued liability, health care costs, and extraordinary costs associated with a declared emergency. It is also still possible to exceed the 2% cap even if the expenses do not fall under the exceptional expense categories. However, to do so, the school board must specify on the ballot the amount by which the maximum allowable levy is exceeded both in percentage and in absolute terms along with justification for such a proposed increase. This obligation may discourage school boards from proposing levies beyond the cap. The 2% cap may then be exceeded if the proposal passes by a simple majority of voters.

3.B.3 Background on State Aid in New Jersey

In order to discuss the incentives underlying the voting on school finance issues, it is necessary to understand the context of the state aid formula that is in effect when school boards choose their proposals and voters decide on how to vote. Beginning with the 2008-09 school year and continuing through the time period of the data set, New Jersey allocated aid to school districts according to the rules described in the School Finance Reform Act (SFRA) of 2008[27]. This act provided for three broad categories of aid: Equalization Aid, Categorical Aid, and Adjustment Aid. (In today's state aid formula, adjustment aid is no longer present.) The state finances the various types of aid primarily through income taxes and allocates aid to each district as a supplement to the amount raised locally through the general fund levy. I emphasize two aspects of the state aid formula, which are that 1) every district keeps 100% of the property taxes it raises, and 2) the amount of aid given is independent of the amount raised locally.

[27] See `https://www.state.nj.us/education/sff/` for a detailed explanation of the SFRA.

The NJ state government calculates the amount of Equalization Aid given to a district by determining what is known as the Adequacy Budget for the district, which estimates the minimal cost to provide a "thorough and efficient" education to students in the district, and subtracting from the Adequacy Budget the Local Fair Share, which estimates how much tax revenue a given district can be expected to raise for schools:

Equalization Aid = Adequacy Budget - Local Fair Share.

The calculation of the Adequacy Budget takes into account the number of students in a district and sums up the estimated cost of the resources needed to educate them for that academic year. This calculation assigns an "average" elementary school student a certain "Base Per-Pupil Amount," representing the amount required to provide a "thorough and efficent" education at the elementary school level, and assigns an "average" middle or high school student a larger weight corresponding to a higher cost of education. Students that face even higher costs of education, such as those having Limited English Proficiency, those requiring special education, or those enrolled in free or reduced lunch programs, receive even higher weights corresponding to their higher costs of education. The Adequacy Budget is then the sum of the total of the estimated costs of educating all the students in the district.

Once the Adequacy Budget for a district is determined, the second step in calculating Equalization Aid is to determine the Local Fair Share for the district, which is an estimate of how much the district may be expected to raise for local schools from property taxes. The following formula determines the Local Fair Share for a district

Local Fair Share = [(Total Property Value)*(Property Rate)+(Total Income)*(Income Rate)]/2.

The Property Rate and Income Rate are parameters that the state government determines annually that ensure, for example, balancing of the budget. Once the Local

Fair Share is determined, Equalization Aid for each district is the Adequacy Budget minus the district's Local Fair Share.

In addition to Equalization Aid, the state provides aid in the form of Categorical Aid and Adjustment Aid. The state gives Categorical Aid to districts to support several kinds of educational expenses, such as: security aid, transportation aid, one-third of special education costs (the other two-thirds are accounted for in the Adequacy Budget), preschool aid, and extraordinary aid for special education. Except for the last category of extraordinary aid, which the state provides as a reimbursement for a portion of the district's actual costs of educating students with extraordinary special education needs, for the other categories the state computes the amount of aid to be given by multiplying the per-pupil cost for the aid category by the number of students eligible for that type of aid.

The purpose of Adjustment Aid is to ensure that no district receives less state aid than it did in the school year prior to passing the SFRA. More precisely, districts receive Adjustment Aid if the sum of their Equalization Aid and Categorical Aid is less than 102% of the district's state aid during the 2007-08 school year. In this case, the amount of Adjustment Aid will make up the difference so that the total state aid equals 102% of the state aid received in the 2007-08 school year.

3.B.4 District Budget Variables, Enrollment, and Aid

A prominent feature of the data is that there is a very large variation in size among the districts across New Jersey in terms of both finances and population. All of the statistics in this work that reflect the size of the district such as total enrollment, school operating budget, and local tax levy figures reflect this variety in district size. All of these variables tend to have highly skewed distributions; however, the logarithms of most of the district size variables essentially follow a normal distribution. It is notable, however, that the local tax levy, despite reflecting district size, does

not have a lognormal distribution. This difference is ultimately due to an uneven allocation of state and federal aid across New Jersey districts.

The district level total enrollment variable is closely tied to district size and reflects the great variety in New Jersey district sizes by exhibiting an approximately lognormal distribution. Table 3.3 highlights the variation in size across New Jersey districts by enrollment, showing that a typical district may have enrollment near the median of 1280 students, with almost 25% of districts having over 3000 students and another 25% of districts having at most 530 students. Figure 3.A.13 demonstrates the lognormal distribution of enrollment. The largest district in the data set had total enrollment of nearly 40,000 students, while the smallest district enrollment was around 40 students, again illustrating the great variety in district size.

Similarly, the total operating budget in a district also reflects the size of the district and approximately follows a lognormal distribution as seen in Figure 3.A.14. Table 3.3 also illustrates the degree to which district size varies in terms of the total operating budget, showing that the median district operating budget in the sample is near $19 million, but that the 25%-75%ile range, which contains half of the district operating budgets, varies from approximately $8.15 to $44.1 million. The most extreme cases include budgets of under $13,000 at the minimum and budgets of over $850 million at the maximum. Overall, the total of all school operating budgets across New Jersey averaged $21 billion per year during the sample period.

Table 3.3: Total Operating Budget, Local Levies, and Enrollment

Variable	Median	25-75%ile	10-90%ile	5-95%ile	Min-Max	Obs.
Total Operating Budget	$19M	$8.2M-$44M	$3.8M-$85M	$2.4M-$129M	$13k-$853M	2262
Local Tax Levy	$12M	$5.4M-$27M	$1.9M-$56M	$1.3M-$78M	$12.5k-$187M	2262
Enrollment	1300	529-2950	245-5830	152-8680	38-40,000	2188

No. of observations varies due to limited data

In contrast to the total operating budget, the local tax levies in New Jersey districts do not follow a lognormal distribution even though the local tax levy in a district does reflect the district's size. This difference is illustrated by comparing Figures 3.A.14 and 3.A.15, and implies that local tax levies are not proportional to enrollment or to the total operating budget. The reason that the total operating budget and the local tax levy differ in distribution is that New Jersey districts receive state and federal aid in a nonuniform manner based on district needs according to New Jersey state aid rules and federal aid criteria. As a result, the size of the local levy as a proportion of the total operating budget varies substantially, especially when considering low income and SDA districts, which receive a large amount of aid.

Figure 3.A.16 provides a histogram of the local tax levy as a percent of the total operating budget. All together, the total of all local tax levies across New Jersey districts amounted to $12.4 billion per year averaged over the four year data set. This amount accounts for more than half of the $21 billion per year total operating budget for schools in New Jersey during this period.

The difference between the total operating budget and the local tax levy is encapsulated in the total aid variable [TotAid]. This variable consists of the state plus federal aid given to a district, and also may include a part of the operating budget that comes from other sources outside of the local tax levy. As a measure of district size, the variable [TotAid] takes on a large range of values from hundreds of dollars at minimum to over $750 million at maximum, and has a median value of $4 million. Similar to other district size variables discussed above, the logarithm of [TotAid] has an essentially normal distribution.

3.B.5 Demographic Variables

I provide basic information on several demographic characteristics of New Jersey that may play an important role in understanding the voter support for school

districts' annual tax levy proposals. These variables include measurements of the composition of the district in terms of race, age, gender, income levels, private school attendance, educational attainment, and population. Although I do not expect such demographic variables to be important predictors of municipal reductions to failed tax levy proposals, it is reasonable to expect that at least some of these variables may be important in determining the passing or failing of the proposed budgets.

I use several variables to describe the racial composition in the district, including the percent of the population identifying as White (%White), Black (%Black), American Indian (%AmerInd), Asian (%Asian), Hawaiian (%Hawaiian), Other (%Other), or as Two or more races (%TwoRaces). Table 3.4 summarizes the means and standard deviations of these variables.

Table 3.4: Race variables, NJ, 2008 - 2011

	%White	%Black	%Asian	%AmerInd	%Hawaiian	%Other	%TwoRaces
Mean	81.46	7.4	5.69	0.21	0.02	3.46	1.73
Stdev	16.8	12.14	7.15	0.44	0.11	5.53	1.37
Max	100	93.15	55.6	7.24	2.58	49.69	12.47

Based on 2,245 available district-year observations

I use the variable [% Over 65], which indicates the percent of a district's population that is over the age of 65, to measure the fraction of the population in a district that is elderly. The mean of [% Over 65] is 14.7% with a standard deviation of 6.2 percentage points. The values of [% Over 65] range from 2.5% to more than 66% of the district population being over 65.

The variable [% K-12 Students in Private School] indicates the percentage of enrolled K-12 students in a district that enroll in a private school. This variable has a mean of 11.7% with a standard deviation of 8.1 percentage points.

I use the variable [ln(Population)] to represent the total population of a district. In contrast to the total population in a district, which has a highly skewed distribution, the variable [ln(Population)], which is the logarithm of the total district population,

has an essentially normal distribution away from the tails of the distribution. The total population of school districts in New Jersey varies widely, from merely dozens of people to over 275,000 within a district.

I use two variables to measure district income composition. To measure the average income in a district, I use the logarithm of district median income, which has a less skewed histogram than the median income itself. The mean of the median income variable during the time period of the data set is approximately $82,000 per year with a standard deviation of $28,000 per year, and the full range of median incomes goes from approximately $25,000 per year up to about $200,000 per year. I also use the variable [% Low Income] to indicate the percentage of the district population earning less than $25,000 per year. The variable [% Low Income] has a mean of 14% with a standard deviation of 7.6 percentage points. In the most extreme districts, the percentage of the population earning less than $25,000 may range from 0% at lowest to 52% at largest.

To measure the level of educational attainment of the population in a district, I consider, similar to Ehrenberg et al. (2004), the percent of people in the district that have attained at least a bachelor's degree. This variable has a mean of 36.4% with a standard deviation of 16.7 percentage points, and ranges in value from a minimum of 3.8% to a maximum of 86.3% in a district.

There is a specific group of districts in New Jersey known as "SDA districts" (formerly known as Abbott districts) that are especially in need of aid from the government to provide an adequate education for their students. I use the indicator variable [isSDA] to represent these districts in the analysis. The name "SDA district" derives from the requirement that the New Jersey School Development Authority (SDA) oversees all school building and renovation projects in these districts, which are entirely financed by the state. Thirty-one of the districts in NJ are classified as SDA districts. These districts have especially low levels of financial resources to

use to finance education in their district, and consistently have failed to perform at an adequate level on statewide standardized tests. SDA districts also in general have a large population of at-risk students or students that require an especially high level of resources to obtain a thorough and efficient education as required by state law. Among the SDA districts, three districts[28] are state-controlled during the time period of the data set. These districts had exhibited a consistent failure to improve their status as an underachieving district. Instead of having a school board that is elected at the district level, the state-controlled districts have a state-appointed board of school estimate that plans the budgets and manages educational efforts in the district. The goal of the state-appointed board is to eventually return control of the district to local school boards when the educational circumstances of the district have improved and stabilized.

3.B.6 Diversity indices

Borrowing from the ecology literature, researchers often use the "true diversity of order 1" to measure the diversity of species in a population Tuomisto (2010), and the analogous diversity measure in the context of this work is given by

$$\text{DiversityIndex} = \frac{1}{\rho_1^{\rho_1}\rho_2^{\rho_2}\cdots\rho_R^{\rho_R}} = \prod_{i=1}^{R}\rho_i^{-\rho_i},$$

where $\rho_i \in [0,1]$ represents the proportion of the population (of the district) that identifies as racial group i, and R is the number of distinct racial groups in the district. With this formulation, a district predominantly composed of a single racial group will have diversity close to 1, while a district with n racial groups of equal size will have diversity n. In the data of the current study, there are 7 different racial groups: White, Black, Asian, American Indian, Hawaiian, Other, and Two or more

[28] The three state-controlled districts during this time period are Newark City, Jersey City, and Paterson City.

races, so the diversity index takes values between 1 and 7. Larger values represent districts that are more "diverse." Including this index of diversity in the study of municipal reductions results in [%Black] no longer being a statistically significant variable in the main regression of Section 3.5.

The Shannon entropy is another index of diversity that researchers in other fields of study, such as information theory and computer science, use to measure the diversity of data. The Shannon entropy is defined as the logarithm of the above [DiversityIndex]. Controlling for diversity in the present work using the Shannon index also results in [%Black] no longer being a statistically significant variable in the study of municipal reductions of Section 3.5.

Another measure of diversity, which researchers in economics commonly use (see for example Alesina and Ferrara (2005)), is the Hirschman-Herfindahl index (or the Fractionalization index), which is given by

$$\text{Fractionalization} = 1 - \sum_{i=1}^{n} \rho_i^2,$$

where n is the number of different racial groups in the district and ρ_i is the proportion of the district population identifying as racial group i. If the entire population of the district is the same racial group, this index has a value of 0, representing the least amount of diversity. The largest value of the index is $(1 - \frac{1}{n})$, which occurs when a district has n equally sized different racial groups. Controlling for the diversity of a district using the [Fractionalization] index does not affect the significance of [%Black] in the analysis of municipal reductions in Section 3.5.

3.C Theoretical Appendix: The Median Voter Model

I show below that in the median voter model, larger proposed increases may be associated to either increases or decreases in the level of voter support depending on the mechanism that causes the increase in the proposal. The two mechanisms I consider are called the "support hypothesis" and the "disagreement hypothesis."

The Model Suppose that there are voters $i = 1, \ldots, N$ with induced preferences over per pupil funding, f, that are single-peaked. The voters are ordered by their ideal points f_*^i, which are the per pupil funding levels at which the voters obtain their maximal utility. Assume that preferences are symmetric about the ideal points, i.e. the voters are indifferent between $f_i^* + h$ and $f_i^* - h$ for any h.

Suppose that the reversion funding level f_R, which is the funding level in the event of a referendum failure, is known to the voters. I assume in this model that there is uncertainty over voter turnout, namely that Nature chooses the set of participating voters according to some probability distribution, rather than assuming that all voters participate with certainty.

The budget proposal on the ballot represents a certain per pupil funding level $\bar{f}$. To decide how to vote, each voter compares the utility of $\bar{f}$ to the reversion level f_R, and there is a per pupil funding level $\tilde{f}^i$ that is the largest per pupil funding level that the voter will support in the referendum (given the particular reversion). If the voter's ideal point f_*^i is less than or equal to the reversion f_R, then $\tilde{f}^i = f_R$. Otherwise, the largest funding level $\tilde{f}^i$ that the voter is willing to support is greater than her ideal point f_*^i (and also greater than the reversion). Due to the symmetry, $\tilde{f}^i = f_*^i + (f_*^i - f_R)$ in the latter case.

Let voter M be the median voter and assume the median ideal point $f_*^M > f_R$ is greater than the reversion. The proposal in the median voter model is defined to be the median ideal point $\bar{f} = f_*^M$. The voters that favor the budget proposal over the reversion are those with supported funding levels $\tilde{f}^i \geq f_*^M$, which includes 1) all voters $i \geq M$ with ideal points greater than or equal to the median ideal point, and 2) all voters that have ideal points below the median but still for whom $\tilde{f}^i \geq f_*^M$ holds. The other voters, for whom $\tilde{f}^i < f_*^M$, will vote against the proposal if they participate.

Although a majority of the voters prefer the proposal $\bar{f} = f_*^M$ to the reversion, it is possible that the proposal might still fail to pass the referendum due to uncertainty over voter turnout, since a majority of the voters that are chosen to participate may be ones with $\tilde{f}^i < f_M$.

The Support Hypothesis Suppose that the proposal $\bar{f}$ now becomes larger due to an exogenous uniform increase in the ideal funding levels of voters: $f_*^i \mapsto f_*^i + \Delta$. I assume that this shift in ideal points does not change the reversion and also does not affect the way in which Nature chooses voters at random. This uniform shift also implies that the proposal $\bar{f}$, which equals the median ideal point f_*^M, increases by Δ: $f_*^M \mapsto f_*^M + \Delta$.

Next, I show that the expected percentage of 'No' votes decreases following the shift (or remains the same). The main point is that when ideal points increase by Δ, the supported funding levels $\tilde{f}^i = f_*^i + (f_*^i - f_R)$ increase by 2Δ for any voter i whose ideal point is greater than or equal to the reversion prior to the shift. Thus, every voter who would approve the proposal prior to the shift (those with $\tilde{f}^i \geq f_*^M$ before the shift) would still approve the proposal following the shift, and there may be additional voters that were initially unsupportive ($\tilde{f}^i < f_*^M$ prior to the shift) that become supportive after the shift takes place. Regardless of which set of voters

participates, the number of supportive voters increases (or remains the same), and thus the expected percentage of 'No' votes does not increase following the shift.

Applying the Support Hypothesis to the Data When the support hypothesis is tested empirically, the shift of ideal points in the district corresponds to comparing two different districts that have similar population sizes where one district supports higher funding levels compared to the other.

The Disagreement Hypothesis In the following scenario, I show that it is possible for the expected proportion of 'No' votes to increase rather than decrease when an increase in the proposal is accompanied by increases in district heterogeneity. Suppose that the ideal points f_*^i increase only for some of the voters rather than for all of the voters. In particular, suppose that initially there is a group of voters i with $P \leq i \leq Q$ who support the referendum, where $P < Q < M$ are less than the median. Now, assume that the ideal points of all voters i in the group $Q < i \leq N$ increase by a large amount Δ, while the ideal points of the other voters remain fixed. Then, the proposal f_*^M increases by Δ, and the voters in the group $Q < i \leq N$ still support the referendum. However, voters in the group $P \leq i \leq Q$ may no longer support the referendum since the maximal supported funding levels $\tilde{f}^i$ for this group have been assumed not to increase, whereas the proposal f_*^M is now larger. Then, in this case, the set of voters who support the referendum has become smaller if Δ is large enough, and thus the expected proportion of 'No' votes has increased. This example therefore illustrates how increases in the proposal that are accompanied by increases in district heterogeneity can lead to decreases in voter support.

References

Alesina, A., & Ferrara, E. L. (2005). Ethnic diversity and economic performance. *Journal of economic literature*, *43*(3), 762–800.

Alexander, A. J., & Bass, G. V. (1974). *Schools, taxes, and voter behavior : an analysis of school district property tax elections*. Rand.

Barr, A. C., & Dee, T. S. (2016). Property taxes and politicians: evidence from school budget elections. *National Tax Journal*, *69*(3), 517–544.

Barro, J. (2010). Do property-tax caps work? Lessons for New Jersey from Massachusetts. Civic report no. 62. *Manhattan Institute for Policy Research*.

Bergstrom, T. C., Rubinfeld, D. L., & Shapiro, P. (1982). Micro-based estimates of demand functions for local school expenditures. *Econometrica: Journal of the Econometric Society*, 1183–1205.

Bowers, A. J., Metzger, S. A., & Militello, M. (2010). Knowing the odds: Parameters that predict passing or failing school district bonds. *Educational Policy*, *24*(2), 398–420.

Brokaw, A. J., Gale, J. R., & Merz, T. E. (1990a). The effect of tax price on voter choice in local school referenda: Some new evidence from Michigan. *National Tax Journal*, 53–60.

Brokaw, A. J., Gale, J. R., & Merz, T. E. (1990b). Explaining voter behavior toward local school expenditures: The impact of public attitudes. *Economics of Education Review*, *9*(1), 67–72.

Brunner, E., & Balsdon, E. (2004). Intergenerational conflict and the political economy of school spending. *Journal of Urban Economics*, *56*(2), 369–388.

Carter, R. F., & Savard, W. G. (1961). *Influence of voter turnout on school bond and tax elections*. US Department of Health, Education, and Welfare, Office of Education.

Dunne, S., Reed, W. R., & Wilbanks, J. (1997). Endogenizing the median voter: Public choice goes to school. *Public Choice*, *93*(1-2), 99–118.

Ehrenberg, R. G., Ehrenberg, R. A., Smith, C. L., & Zhang, L. (2004). Why do school district budget referenda fail? *Educational Evaluation and Policy Analysis*, *26*(2), 111–125.

Filer, J. E., & Kenny, L. W. (1980). Voter turnout and the benefits of voting. *Public Choice*, *35*(5), 575–585.

Filimon, R., Romer, T., & Rosenthal, H. (1982). Asymmetric information and agenda control: The bases of monopoly power in public spending. *Journal of Public Economics*, *17*(1), 51–70.

Harris, A. R., Evans, W. N., & Schwab, R. M. (2001). Education spending in an aging America. *Journal of public economics*, *81*(3), 449–472.

Hilber, C. A., & Mayer, C. (2009). Why do households without children support local public schools? Linking house price capitalization to school spending. *Journal of Urban Economics*, *65*(1), 74–90.

Ladd, H. F., & Murray, S. E. (2001). Intergenerational conflict reconsidered: County demographic structure and the demand for public education. *Economics of education review*, *20*(4), 343–357.

Lambert, D. M., Clark, C. D., Wilcox, M. D., & Park, W. M. (2009). Public education financing trends and the gray peril hypothesis. *Growth and Change*, *40*(4), 619–648.

Lankford, R. H. (1985). Preferences of citizens for public expenditures on elementary

and secondary education. *Journal of Econometrics*, *27*(1), 1–20.

Lentz, C. (1999). Predicting school referenda outcomes: Answers from Illinois. *Journal of Education Finance*, *24*(4), 459–482.

Moser, M., & Rubenstein, R. (2002). The equality of public school district funding in the united states: A national status report. *Public Administration Review*, *62*(1), 63–72.

NYSED, N. Y. S. E. D. (2005). *School budget vote failures: Risk factors*. Albany, NY: The State of New York, Fiscal Analysis and Research Unit.

Pecquet, G. M., Morris Coats, R., & Yen, S. T. (1996). Special versus general elections and composition of the voters: Evidence from louisiana school tax elections. *Public Finance Quarterly*, *24*(2), 131–147.

Peirce, W. S. (2017). Why would bond referenda ever fail? Do they? In *Explorations in public sector economics* (pp. 93–107). Springer.

Piele, P. K., & Hall, J. S. (1973). *Budgets, bonds, and ballots: Voting behavior in school financial elections*. Lexington Books.

Priest, T. B., & Fox, L. P. (2005). Minority support for school bonds in charlotte-mecklenburg: A cautionary note. *Education and Urban Society*, *37*(2), 193–203.

Romer, T., & Rosenthal, H. (1979). Bureaucrats versus voters: On the political economy of resource allocation by direct democracy. *The Quarterly Journal of Economics*, *93*(4), 563–587.

Romer, T., & Rosenthal, H. (1982). Median voters or budget maximizers: Evidence from school expenditure referenda. *Economic Inquiry*, *20*(4), 556–578.

Romer, T., Rosenthal, H., & Ladha, K. (1984). If at first you dont succeed: budgeting by a sequence of referenda. *Public finance and the quest for efficiency*, 87–108.

Romer, T., Rosenthal, H., & Munley, V. G. (1992). Economic incentives and political institutions: Spending and voting in school budget referenda. *Journal of Public Economics*, *49*(1), 1–33.

Rothstein, P. (1994). Learning the preferences of governments and voters from proposed spending and aggregated votes. *Journal of Public Economics*, *54*(3), 361–389.

Rubinfeld, D. L. (1977). Voting in a local school election: A micro analysis. *The Review of Economics and Statistics*, 30–42.

Rubinfeld, D. L., & Thomas, R. (1980). On the economics of voter turnout in local school elections. *Public Choice*, 315–331.

Silverman, R. M. (2011). How unwavering is support for the local property tax?: Voting on school district budgets in New York, 2003-2010. *Journal of Education Finance*, 294–311.

Stevens, J. B., & Mason, R. (1996). Political markets, property tax referenda, and local school spending. *Public Choice*, *86*(3-4), 257–277.

Tiebout, C. M. (1956). A pure theory of local expenditures. *Journal of political economy*, *64*(5), 416–424.

Tuomisto, H. (2010). A consistent terminology for quantifying species diversity? Yes, it does exist. *Oecologia*, *164*(4), 853–860.

Zhao, Q. (2020). A study of income and grant aid changes on school referenda outcomes under uncertainty over voter turnout. *Thesis, Princeton University*.