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Three Essays on the Economics of Shadow Banking

Huang, Ji

Princeton, NJ : Princeton University

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Three Essays on the Economics of Shadow Banking

Ji Huang

A Dissertation
Presented to the Faculty
of Princeton University
in Candidacy for the Degree
of Doctor of Philosophy

Recommended for Acceptance
by the Department of
Economics
Advisers: Markus K. Brunnermeier and Yuliy V. Sannikov

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Abstract

This dissertation contains three self-contained essays that investigate the economics of shadow banking.

In Chapter 1, I incorporate shadow banking modeled as off-balance-sheet financing in a standard continuous-time macro-finance model. In this model, regular banks pursue regulatory arbitrage by extending businesses outside the regulatory perimeter via shadow banking. In the absence of regulatory authorities, shadow banking creates an enforcement problem, which leads to an endogenous constraint on leverage for shadow banking. Shadow banking adds to financial instability because tightening market discipline in economic downturns forces shadow banks to sell assets at fire-sale prices to regular banks. Overall, financial instability as a function of financial regulation is U-shaped rather than monotonically decreasing. This new framework can comprehensively evaluate the impact of different regulatory regimes on both the regulated and unregulated banking sectors.

In Chapter 2, I modify the framework developed in Chapter 1 and modernize Minsky's "Financial Instability Hypothesis" in an economy where the financial market consists of both regular banking and shadow banking. As in Chapter 1, the friction with off-balance-sheet financing constrains the borrowing capacity of shadow banking, although shadow banking provides cheaper credit than regular banking does. Chapter 2 shows that the decline of asset volatility in economic upturns speeds up the growth of shadow banking. The overheated expansion of shadow banking exposes the economy to the risk of a self-fulfilling systemic run; in turn, this risk then may lead to the sudden collapse of the shadow banking system.

In Chapter 3, I investigate two inseparable explanations for US banks' liquidity problem in the 2007-09 financial crisis: the malfunction of the securitization market and the loan commitment drawdowns. I find that US banks issue more C&I loan com-

mitments when they perceive more loan securitization potential. A critical missing variable problem results when only one of the two is applied as an explanatory variable. I find that the exposure to C&I loan commitment takedown is the dominant influence that explains US banks' liquidity problem from July 2007 to June 2008 and that from July 2008 to June 2009 the freeze of the securitization market is the dominant influence.

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Chapter 1

Banking and Shadow Banking

1.1 Introduction

The 2007-09 global financial crisis brought shadow banking—bank-like activities undertaken by unregulated financial entities and off-balance-sheet vehicles—to light in a worldwide discussion about the connection between financial instability and financial regulation. Over the course of the discussion, a new school of thought has emerged, arguing that since financial activities can always migrate to the unregulated sector, tightening financial regulation could endanger financial stability.[1] This view challenges the conventional wisdom that the government can always secure financial stability by strengthening financial regulation.

Are the two schools of thought entirely incompatible for the purposes of policy-making? Or, is there a more comprehensive guideline for regulatory authorities to

[1]For instance, Brunnermeier et al. (2009) mention the financial migration from the regulated to the unregulated sector and discuss the relevant regulatory perimeter issue. The IMF's October 2014 *Global Financial Stability Report* shows that countries that have a large shadow banking sector typically implement tight financial regulatory rules (IMF, 2014). Richard Berner, Director of the Office of Financial Research, mentioned the risk of financial migration towards the unregulated shadow banking sector in his speech at the Money Marketeers of New York University on October 15, 2014 (Berner, 2014).

follow? Does financial migration always have substantial economic and financial consequences? Is it necessary for the government to arbitrarily extend its regulatory perimeter to preempt any financial migration?

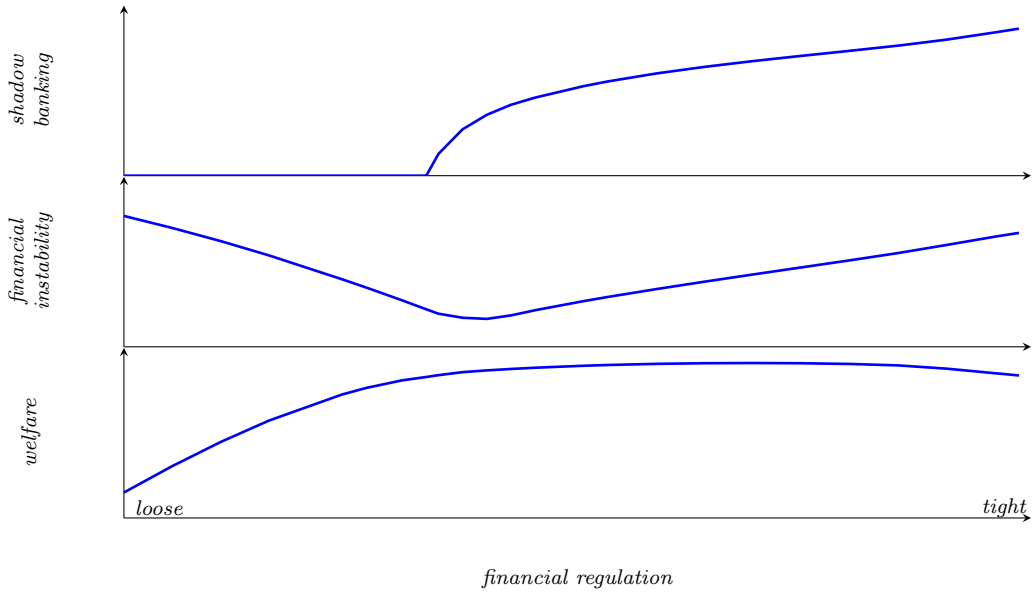


Figure 1.1: This stylized figure illustrates one of the chapter's main results: as regulatory authorities tighten financial regulation the shadow banking system emerges and expands (upper panel); financial instability initially declines and later rises (middle panel); social welfare initially increases and later diminishes (lower panel).

To address all these questions, this chapter theoretically investigates those banking activities which chase regulatory arbitrage by migrating from the regulated traditional banking sector to the unregulated shadow banking sector. We argue that when financial regulation is sufficiently lenient, such financial migration is negligible, and in these circumstances, tightening regulation lowers financial instability and improves social welfare. However, when financial regulation becomes stringent enough, financial migration towards the shadow banking sector is sizable, and further strengthening of regulation heightens financial instability and worsens social welfare. Thus, this chapter reconciles the two seemly contradictory schools of thought about financial instability within a single framework. This framework can also evaluate the welfare

implications of different regulatory regimes in light of their impacts on the shadow banking sector.

This chapter underscores the fact that the borrowing capacity of shadow banking relies on market discipline and, more importantly, on the level of financial regulation that regular banks face. This differs from the standard perception that a financial firm's internal optimal choice determines the amount of its shadow banking businesses (Plantin, 2014). This chapter highlights that if the channel connecting the borrowing capacity of shadow banking to financial regulation is turned off, tightening regulation can always lower financial instability because the contraction of the regular banking sector caused by strict regulation dominates the expansion of the shadow banking sector.

Similar to Thomas and Worrall (1988), Kehoe and Levine (1993), and Kocherlakota (1996), a standard enforcement problem in our model gives rise to the maximum borrowing capacity of shadow banking. The enforcement problem originates from the institutional details of shadow banking. To be consistent with such institutional details, this chapter models regular banking as a regular bank's *on-balance-sheet financing* and shadow banking as the regular bank's *off-balance-sheet financing*. To regulatory authorities, a regular bank paints its shadow bank (i.e., off-balance-sheet vehicle) as a legally separate entity to circumvent any regulation of financial activities operated under the guise of the shadow bank. To creditors, however, the regular bank paints the shadow bank as part of its own business. Due to the absence of an authority, an enforcement problem arises because the creditors of a shadow bank cannot force its parent regular bank to protect them if the shadow bank is in trouble.

We next argue that tighter financial regulation causes the larger borrowing capacity of shadow banking; that is, the higher leverage that a regular bank can obtain via shadow banking. As in Thomas and Worrall (1988), Kehoe and Levine (1993),

and Kocherlakota (1996), if a regular bank defaults on the obligations of its shadow bank, then the creditors of the shadow bank stop lending to the shadow bank, which deprives the regular bank of its opportunities for regulatory arbitrage. The cost to the regular bank of such default is, therefore, the present value of the future regulatory arbitrage benefits that shadow banking offers. With more stringent regulation, comes greater opportunity for regulatory arbitrage. Thus, the cost of default is larger in economies with tighter regulation, and the leverage of shadow banking is higher in such economies.

The fact that the leverage of shadow banking is endogenously determined involves an interesting feedback loop between the cost of default and the leverage of shadow banking. A lower cost of default leads to a greater incentive to default and a narrower shadow banking channel. A narrower shadow banking channel offers less benefits to regular banks, which implies a lower cost of default. When financial regulation is sufficiently loose, the cost for regular banks to default is small. The feedback loop amplifies the effect of the small cost of default so profoundly as to rule out shadow banking completely (upper panel of Figure 1.1).

We embed our modeling of banking and shadow banking in a standard continuous-time macro-finance framework (Brunnermeier and Sannikov, 2014). In this framework, because of a constraint on equity financing, banks can use leverage only to finance their investments. We choose this macro-finance framework because it allows us to model financial instability as an endogenous risk generated by the financial system itself. Financial regulation that limits the use of bank leverage is essential, since the excessive use of bank leverage leads to high endogenous risk and causes a pecuniary externality for the entire economy (Lorenzoni, 2008; Stein, 2012). Shadow banking emerges as the response of regular banks to financial regulation.

The solution of a continuous-time macro-finance model characterizes the full dynamics of an economy. With this advantage, our model captures three salient dynamic features of shadow banking observed during the 2007-09 financial crisis: the pro-cyclicality of shadow banking, the reintermediation by shadow banks conducting fire sales of assets to regular banks, and the sudden collapse of the shadow banking system. By analyzing both leverage dynamics and endogenous risk dynamics, we uncover a general equilibrium channel through which shadow banking adds to financial instability.

To facilitate the illustration of the above amplification channel, we now explain why shadow banking is pro-cyclical in our model. In economic booms, both endogenous risk and the market leverage of regular banking are low.[2] Since endogenous risk is low, a shadow bank is unlikely to be in trouble, and thus creditors are willing to lend to it. When the leverage of a regular bank is low, the regular bank must have enough incentives and resources to protect its shadow bank when a negative macroeconomic shock causes troubles for both its shadow bank and itself. Hence, regular banks have low incentives to default during economic booms. These low incentives to default, in turn, raise the leverage of shadow banking. In addition, the feedback loop between the cost of default and the leverage of shadow banking amplifies the expansion of shadow banking in economic booms.

Shadow banking increases financial instability as a general equilibrium effect in our model. The leverage of shadow banking is high in economic upturns. Since they are immune to financial regulation, shadow banks can accumulate substantial amounts of assets in upturns. When a negative macroeconomic shock hits the economy, the shrinking shadow banking channel forces shadow banks to offer assets to regular banks

[2]The banking sector's countercyclical leverage is a standard result in continuous-time macro-finance models such as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013). This result is consistent with findings based on market leverage, while the book leverage indicates the opposite case.

at fire-sale prices (i.e., reintermediation). Since regular banks are reluctant to acquire assets from shadow banks due to financial regulation, asset prices have to decline so significantly that regular banks are willing to purchase those assets. The degree of financial instability in the system naturally accelerates.

We next elaborate on the intuition behind the U-shaped relationship between financial regulation and financial instability and the hump-shaped relationship between financial regulation and social welfare (middle and lower panels of Figure 1.1). When regulation is loose enough, shadow banking is negligible. Tighter regulation of regular banking leads to lower financial instability and higher economic welfare. When regulation is sufficiently tight, considerable amounts of banking activities shift to the shadow banking sector thanks to the ample funding capacity of shadow banking. More stringent regulation causes a larger shadow banking system and higher financial instability, which diminishes social welfare.

Related Literature. The literature on shadow banking is fast growing and diverse. Different papers model shadow banking in drastically different ways. Adrian and Ashcraft (2012) provide a thorough survey of this growing literature. In this chapter, we try to categorize models of shadow banking along two dimensions: the motive for shadow banking and the type of negative externalities caused by shadow banking.

The existence of shadow banking can be demand/preference driven. In Gennaioli et al. (2013a), infinitely risk-averse households only value securities' worst scenario payoffs, and shadow banking can increase such payoffs by pooling different assets together. In Moreira and Savov (2014), the preference specification of households leads directly to a demand for the liquid securities that shadow banking generates.

The second motive for shadow banking is regulatory arbitrage as discussed in this chapter. Luck and Schempp (2014), Ordonez (2013), and Plantin (2014) are papers that fall into this category.

Models of shadow banking differ in the type of the externalities that shadow banking causes. The first category includes non-pecuniary externalities. In Plantin (2014), shadow banking exposes the real sector to counter-productive uncertainty. In both Luck and Schempp (2014) and Gennaioli et al. (2013a), creditors of shadow banking suffer from unexpected default caused by bank runs or crises. Generally, investments financed by shadow banking in these models have worse or more volatile fundamentals than those financed by regular banking.

Unlike the first group of papers discussed above, in Moreira and Savov (2014) and in this chapter shadow banking does not involve any investments of inferior quality. Instead, we focus on the pecuniary externality that the individual choices of shadow banks for optimal leverage cause excessive endogenous risk because individual agents do not take into account the price impact of their actions in the competitive equilibrium.

Since the main result of Plantin (2014) is in the same spirit as ours, we highlights two other differences between the papers. First, financial regulation in Plantin (2014) constrains the issuance of risky securities, and regulation in this chapter curbs the use of risk-free debt and leverage. Second, the size of shadow banking is a bank's internal optimal choice in Plantin (2014), whereas the size of shadow banking relies on the credit market or is market-based in this chapter.

This chapter is also related to the literature on pecuniary externalities. One closely related paper is Bianchi (2011), whose quantitative examination of the non-pecuniary externality of excessive borrowing in a dynamic general equilibrium model highlights that raising the cost of borrowing can improve welfare.

In terms of methodology, this chapter follows the emerging literature that consider economies with financial frictions in a continuous-time setting. This literature includes Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012b, 2013).

The methodology employed in this literature has the advantage of allowing the exact characterization of full equilibrium dynamics. In our model, the tractability allows us to explicitly relate the leverage constraint to endogenous risk. Several papers, such as Adrian and Boyarchenko (2012), Danielsson et al. (2012), and Phelan (2012), have similarly related the leverage constraint to endogenous risk by assuming various forms of Value-at-Risk constraints. The leverage constraint in this chapter originates from the financial friction that bankers could strategically default on securities generated by shadow banking. The leverage of shadow banking is endogenously determined by the leverage constraint in equilibrium, which is in line with the market-based concept of shadow banking. In addition, it draws the connection between shadow banking activities and financial regulation. Lastly, the microfounded leverage constraint sets the stage for the systemic event in our model, where bankers default if the constraint does not hold.

This chapter is structured as follows. We lay out the baseline model in Section 1.2. In Section 2.3, we characterize the non-sunspot equilibrium of the baseline model, in which the shadow banking system never collapses on the equilibrium path, and illustrate the main results with numerical examples. Section 1.4 explores the welfare and policy implications of the proposed framework, and Section 1.5 highlights the fact that the endogenous leverage constraint for shadow banking is essential for the U-shaped result and demonstrates the robustness of the main results by varying agents' preferences.

1.2 The Baseline Model

To analyze how shadow banking changes conventional ideas about financial regulation and financial instability, this section will model both shadow banking and reg-

ular banking and their interaction within a standard continuous-time macro-finance framework developed by Brunnermeier and Sannikov (2014). We will also specify the dynamic portfolio choice problem for each agent and list the equilibrium conditions that are characterized in Section 2.3.

1.2.1 Model Setup

The general model setup is standard in the literature on financial frictions. As in Kiyotaki and Moore (1997), He and Krishnamurthy (2012b), and others, our model has heterogeneous agents: productive bankers and less-productive households. Bankers can raise funds from households through both regulated regular banking, modeled as on-balance-sheet financing, and through unregulated shadow banking, modeled as off-balance-sheet financing.

Technology and Preferences

We consider a continuous-time infinite-horizon economy with two types of goods: durable physical capital goods and non-durable consumption goods.

At any time $t \in [0, \infty)$, a banker holding capital k_t produces consumption goods y_t according to

$$y_t = ak_t.$$

Households also have a linear production technology

$$y_t^h = a^h k_t^h,$$

although they are less productive; that is, $a^h < a$. Both bankers and households have the investment technology that an agent inverts $g(\iota_t)k_t$ units of consumption goods

into $\iota_t k_t$ units of new capital, where

$$g(\iota_t) = \iota_t + 0.5\phi\left(\iota_t - \delta\right)^2$$

with δ denoting the depreciation rate.[3] Thus, the capital stock held by each agent grows at the rate $\iota_t - \delta$ in the absence of any shock.

The exogenous aggregate shock to the economy is driven by a Poisson process $\{N_t\}_{t=0}^{\infty}$ with intensity λ. Whenever the Poisson shock hits the economy, the capital stock held by each agent drops by a constant proportion, κ.[4] The law of motion of the aggregate capital K_t is

$$dK_t = K_{t-}\left(\iota_{t-} - \delta\right)dt - K_{t-}\kappa dN_t,$$

conditional on all agents choosing the same investment rate ι_{t-}, where K_{t-} denotes $\lim_{s\uparrow t} K_s$; i.e., the left limit of the process $\{K_s, s \geq 0\}$ in period t. For purposes of exposition, we interpret time $t-$ as the period right before time t.

We assume that bankers have logarithmic utility that households are risk neutral, and that both types of agents have a time discount rate ρ. The expected discounted lifetime utility of a banker is

$$E_0\left[\int_0^{\infty} e^{-\rho u}\ln\left(c_u\right)du\right]. \tag{1.1}$$

We also assume that a banker's utility becomes negative infinity if her consumption is negative, and that households can have negative consumption. In Section 1.5.2, we modify the baseline model such that households have Epstein-Zin preferences.

[3] We choose the functional form of capital adjustment cost to be consistent with Christiano et al. (2005), He and Krishnamurthy (2012a), and many other quantitative macroeconomic models.

[4] The capital in the model should be measured in efficiency units, and both the investment and the Poisson shock affect the quality of capital. The setup yields the tractability of the model, since the economy is scale-invariant with respect to the aggregate capital stock. Other recent papers that also use a macroeconomic capital quality shock include Gertler and Karadi (2011) and Gertler et al. (2012).

Physical Assets

There is no friction in the market for capital goods. The market price of capital goods is in units of consumption goods, denoted by q_t. The law of motion of q_t is denoted by

$$dq_t = q_{t-}\mu^q_{t-}dt - q_{t-}\kappa^q_t dN_t,$$

where μ^q_t and κ^q_t are endogenously determined in equilibrium. In addition, κ^q_t is allowed to be stochastic in each period t.

In period t, in the absence of a negative shock, the rate of return for a banker holding physical capital is

$$\frac{a - g(\iota_t)}{q_t} + \iota_t - \delta + \mu^q_t.$$

Other than the dividend yield $(a - g(\iota_t))/q_t$, there are two sources of gain from holding capital: the growth in the banker's capital stock $\iota_t - \delta$ and the rise in the price of capital μ^q_t. Similarly, there are two types of risk for holding capital: exogenous and endogenous. Exogenous risk is the possible κ proportional decline in the banker's capital stock, which is the direct consequence of the exogenous Poisson shock. Endogenous risk is the κ^q_t proportional change in the price of capital, which is the general equilibrium effect of the Poisson shock. Endogenous risk affects the banker's investment return through its impact on the $1 - \kappa$ proportion of physical capital left to the banker given the hit of the Poisson shock. Formally, the rate of return for bankers from holding capital is

$$\underbrace{\left[\frac{a - g(\iota_{t-})}{q_{t-}} + \iota_{t-} - \delta + \mu^q_{t-}\right]}_{\equiv R_{t-}} dt - \kappa^Q_t dN_t,$$

where $\kappa_t^Q \equiv \kappa + (1-\kappa)\,\kappa_t^q$. Similarly, the rate of return for households holding capital is

$$\underbrace{\left[\frac{a^h - g(\iota_{t-}^h)}{q_{t-}} + \iota_{t-}^h - \delta + \mu_{t-}^q\right]}_{\equiv R_{t-}^h} dt - \kappa_t^Q dN_t.$$

The Financial Market and Regulatory Authority

The financial market is incomplete. The following four assumptions detail the incompleteness of the financial market.

Assumption 1 *Households do not hold equity issued by other agents.*

A banker can establish a regular bank. Via regular banking, bankers issue short-term debt and equity to finance their holdings of physical capital. The regulatory authority imposes the regulation in Assumption 2 on regular banks.

Assumption 2 *Regular banks' debt financing is taxed at rate τ_t in period t; total tax revenue is instantly redistributed back to regular banks as lump-sum subsidies, whose amounts are proportional to bankers' net worth.*

Under the regulation, regular banks have to pay tax τ_t for each dollar they raise. Even though there is a tax rebate, the tax rate τ_t affects the optimal leverage of a regular bank in period t because the tax rebate is distributed as a lump-sum subsidy. The lump-sum tax rebate setup exactly cancels the wealth effect of tax τ_t.

To circumvent the above regulation, a banker can sponsor a shadow bank and earn the residual value of the shadow bank each period as a management fee. In the real world, this activity is often called off-balance-sheet financing.

Assumption 3 *The regulatory authority treats a shadow bank as a regular bank, if bankers hold the equity of the shadow bank.*

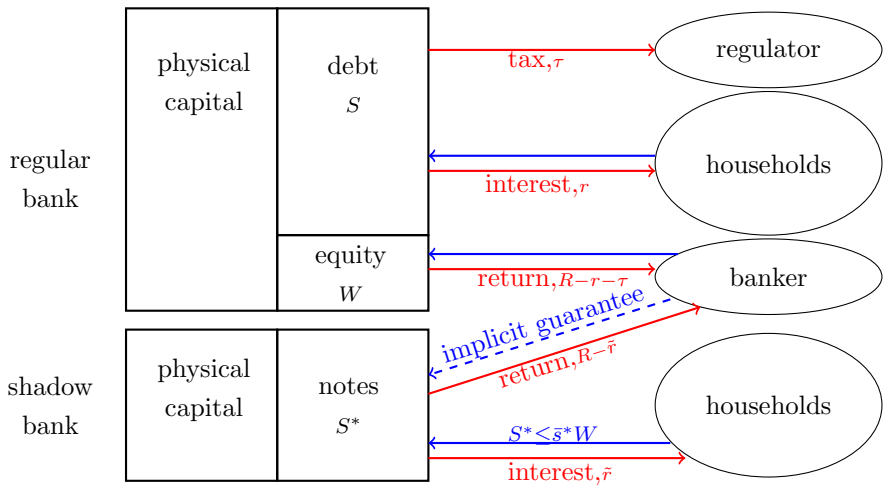


Figure 1.2: This figure details the financial side of the model. A regular bank's debt financing is taxed at rate τ. Households hold debt issued by regular banks and enjoy the rate of return r. Bankers earn regular banks' residual values at rate $R-r-\tau$ as their equity. Bankers also obtain shadow banks' residual value at rate $R-\tilde{r}$ as their guarantors, where $\tilde{r}$ is the rate of return that shadow banks promised to their household creditors. Bankers extend implicit guarantees to shadow banks. The maximum size of a shadow bank is the maximum leverage of shadow banking $\bar{s}$ times its banker's net worth W.

We name debt issued by shadow banks *note* to distinguish it from that issued by regular banks. Notes are also short term. Assumptions 1 and 3 imply that shadow banks are all debt financed. Any drop in the asset value of a shadow bank causes its creditors to suffer unless its sponsor bails it out. Assumption 9 specifies the structure of the note market and how households manage to secure the safety of their investments in shadow banks.

Assumption 4 *In the note market in any period t,*

i. a positive measure of households can form a unit, and the unit offers a one-period note contract $(\bar{s}_t^*, \tilde{r}_t)$;

ii. if a banker accepts the contract, she can borrow (through shadow banking) up to $\bar{s}_t^*$ times her net worth in period t and pay the principal and interest at rate $\tilde{r}_t$ in the following period.

iii. The market excludes bankers who default and allows them to come back at rate ξ.

1.2.2 Problems for Bankers and Households

Suppose a banker's net worth is W_{t-} in period $t-$. S_{t-} denotes the value of debt that she raises via regular banking. The excess return from holding capital goods funded by regular banking is

$$S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)dt-S_{t-}\kappa_t^Q dN_t,$$

where r_t is the risk-free rate.

The banker also manages the shadow bank. The size of the shadow bank is S_{t-}^* in dollar terms. The banker earns the difference between the return from the capital investment $R_{t-}S_{t-}^*$ and the interest $\tilde{r}_{t-}S_{t-}^*$ promised to creditors. The size of the shadow bank is limited by the leverage constraint specified by the note contract $\left(\bar{s}_{t-}^*,\tilde{r}_{t-}\right)$

$$S_t^*\leq\bar{s}_t^*W_t. \tag{1.2}$$

In addition, the banker has a strategic choice $\mathcal{D}_t$ to make if a Poisson shock hits in period t. Given that the Poisson hits the economy, if the banker does not default ($\mathcal{D}_t=0$), she bears the loss $S_{t-}^*\kappa_t^Q$ for creditors of her shadow bank; otherwise, she does not bear the loss. Thus, a banker's dynamic budget constraint is

$$\begin{aligned} dW_t &= \left(W_{t-}R_{t-}+S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+S_{t-}^*\left(R_{t-}-\tilde{r}_{t-}\right)+\pi_{t-}W_{t-}-c_{t-}\right)dt \\ &\quad -\left(W_{t-}\kappa_t^Q+S_{t-}\kappa_t^Q+\left(1-\mathcal{D}_t\right)S_{t-}^*\kappa_t^Q\right)dN_t, \end{aligned} \tag{1.3}$$

where π_t is the ratio of subsidy to net worth and c_t is the banker's consumption in period t. Since the banker may use a mixed strategy, let $\mathbf{d}_t$ denote the probability that $\mathcal{D}_t = 1$.

Taking $\{q_t, r_t, \tau_t, \tilde{r}_t, \pi_t, \bar{s}_t^*\}_{t=0}^{\infty}$ as given, the banker chooses $\{c_t, S_t, S_t^*, \iota_t, \mathbf{d}_t\}_{t=0}^{\infty}$ to maximize her expected lifetime utility (1.1) subject to the leverage constraint (1.2) and the dynamic budget constraint (2.2).

Households can invest in both capital goods and financial instruments issued by regular and shadow banks. S_t^h denotes the value of capital that household h holds, and n_t the value of notes that it holds. The wealth W_t^h of the household evolves according to

$$dW_t^h = \left(W_{t-}^h r_{t-} + S_{t-}^h \left(R_{t-}^h - r_{t-}\right) + n_{t-}\left(\tilde{r}_{t-} - r_{t-}\right) - c_{t-}^h\right) dt - \left(S_{t-}^h + \mathbf{d}_t n_{t-}\right) \kappa_t^Q dN_t. \tag{1.4}$$

Here, we assume that all bankers follow the same rules on default decisions in a given period. Formally, a household maximizes

$$U_0^h = E_0\left[\int_0^{\infty} e^{-\rho t} c_t^h dt\right]$$

by choosing $\left\{c_t^h, n_t, S_t^h\right\}_{t=0}^{\infty}$.

1.2.3 Equilibrium

We make the following assumption to guarantee that the wealth share of bankers would not be large enough to undo all financial frictions.

Assumption 5 *Each banker retires independently at rate χ. If a banker retires, she can only save her wealth and earn risk-free rate r_t*

$\boldsymbol{I} = [0,1]$ and $\boldsymbol{J} = (1,2]$ denote sets of bankers and households, respectively. Individual bankers and households are indexed by $i \in \boldsymbol{I}$ and $j \in \boldsymbol{J}$.

Definition 1 *Given the initial endowments of capital goods* $\left\{k_0^i, k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$ *to bankers and households such that*

$$\int_0^1 k_0^i di + \int_1^2 k_0^j dj = K_0,$$

and a locally deterministic tax rate process $\{\tau_t\}_{t=0}^{\infty}$*, an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by* $\{N_t\}_{t=0}^{\infty}$*: the price of capital* $\{q_t\}_{t=0}^{\infty}$*, risk-free rate* $\{r_t\}_{t=0}^{\infty}$*, the maximum leverage of shadow banking* $\{\bar{s}_t^*\}_{t=0}^{\infty}$*, the interest rate on notes* $\{\tilde{r}_t\}_{t=0}^{\infty}$*, the ratio of subsidy to net worth* $\{\pi_t\}_{t=0}^{\infty}$*, wealth* $\left\{W_t^i, W_t^j\right\}_{t=0}^{\infty}$*, capital holdings* $\left\{k_t^i, k_t^j\right\}_{t=0}^{\infty}$*, investment decisions* $\{\iota_t^i\}_{t=0}^{\infty}$*, default decisions* $\{\mathbf{d}_t^i\}_{t=0}^{\infty}$*, and consumption* $\left\{c_t^i, c_t^j\right\}_{t=0}^{\infty}$ *of banker* $i \in \boldsymbol{I}$ *and household* $j \in \boldsymbol{J}$*; such that*

1. $\left\{W_0^i, W_0^j\right\}$ *satisfy* $W_0^i = q_0 k_0^i$ *and* $W_0^j = q_0 k_0^j$*, for* $i \in \boldsymbol{I}$ *and* $j \in \boldsymbol{J}$**;**
2. *bankers solve their problems given* $\{q_t, r_t, \tau_t, \tilde{r}_t, \pi_t, \bar{s}_t^*\}_{t=0}^{\infty}$*;*
3. *households solve their problems given* $\{q_t, r_t, \tilde{r}_t, \mathbf{d}_t^i, i \in \boldsymbol{I}\}_{t=0}^{\infty}$*;*
4. *the budget of the regulatory authority is balanced;*
5. *markets for both consumption goods and capital goods clear*

$$\int_0^1 c_i^i di + \int_1^2 c_t^j dj = \int_0^1 \left(a - g\left(\iota_t^i\right)\right) k_t^i di + \int_1^2 \left(a^h - g\left(\iota_t^j\right)\right) k_t^j dj, \tag{1.5}$$

$$\begin{aligned} \int_0^1 k_t^i di + \int_1^2 k_t^j dj &= K_t, \\ \text{where } dK &= \left(\int_0^2 \left(\iota_t^i - \delta\right) k_t^i di\right) dt - \kappa K_t dN_t; \end{aligned} \tag{1.6}$$

6. the note market clears

Given this definition, the debt market automatically clears by Walras' Law.

1.2.4 Financial Frictions

It is worthwhile to summarize three types of financial frictions in the baseline model. First, we do not allow for equity issuance in the model (Assumption 1). The restriction on equity financing gives rise to the standard balance sheet amplification mechanism in the financial friction literature (Krishnamurthy, 2010). Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012b) provide microfoundations for this restriction. Our model naturally inherits the amplification mechanism discussed in these two papers.

The second friction is the intervention of the regulatory authority (Assumption 2). Financial regulations are necessary given that the level of leverage chosen by individual bankers may not be the socially optimal because borrowers do not internalize the external negative impact of their private leverage choices, as discussed by Lorenzoni (2008) and Stein (2012). Section 1.4.1 shows that the tax on regular banking in Assumption 2 can improve bankers' welfare by adjusting the leverage choices of regular banks and reducing the pecuniary externality.

The third financial friction is the leverage constraint on shadow banking (Assumption 9). One implication of assumptions 1 and 3 is that shadow banks issue no equity. This is realistic because the equity portion of shadow banks in the real world, such as special purpose vehicles and money market mutual funds, is typically very thin.

1.3 Financial Instability in the Non-Sunspot Equilibrium

In this section, we characterize the non-sunspot equilibrium of the baseline model with numerical examples. With the model characterization, we will present our main result that the relationship between financial instability and financial regulation displays a U shape.

1.3.1 Characterizing the Non-Sunspot Equilibrium

It is convenient to characterize the non-sunspot equilibrium because it does not involve a sudden collapse of the shadow banking sector. In addition, endogenous risk κ_t^q is deterministic in each period t. So is the loss for each dollar of investment in capital goods, κ_t^Q.

Households' Optimal Choices

ι_t denotes a household's investment in period t. The expression of R_t^h implies that the optimal level of ι_t maximizes

$$\frac{-\iota_t - 0.5\phi\left(\iota_t - \delta\right)^2}{q_t} + \iota_t.$$

The first-order condition yields an expression that the optimal investment rate is a function of the price of capital, q_t

$$\iota_t = \delta + \frac{q_t - 1}{\phi}. \tag{1.7}$$

Bankers have the same investment function $\iota(\cdot)$ since they have the same investment technology.

Given that households are risk-neutral, the following conditions must hold in equilibrium to be consistent with the households optimal consumption and portfolio choice.

Proposition 1 *A household's portfolio choices* $\{S_t^h, n_t, t \geq 0\}$ *satisfy*

$$r_t = \rho \tag{1.8}$$

$$R_{t-}^h - r_{t-} \leq \lambda \kappa_t^Q, \quad = \quad \text{if } S_{t-}^h > 0, \tag{1.9}$$

$$\tilde{r}_{t-} - r_{t-} \leq \lambda \mathbf{d}_t \kappa_t^Q, \quad = \quad \text{if } n_{t-} > 0, \tag{1.10}$$

for all $t \geq 0$.

This proposition assumes that all bankers follow the same default strategy $\mathbf{d}_t$ in a given period.

Bankers' Optimal Choices

For bankers' optimal choices, we begin with the result that bankers never have negative net worth. Intuitively, bankers are risk-averse, and negative net worth leads to negative consumption and infinite negative utility.

Lemma 1 *If the initial net worth of a banker* $W_0 > 0$*, it is never optimal for the banker to have negative net worth.*

Proof. See Appendix. ■

A corollary of Lemma 1 is that a banker's overall leverage must be small enough that her net worth will never be wiped out. Therefore, a banker never defaults on her regular bank obligations.

Corollary 1 *A banker's overall leverage has an upper bound in any period; in particular,*

$$(W_{t-} + S_{t-})\,\kappa_t^Q + S_{t-}^*\kappa_t^Q \mathbf{1}_{(d_t>0)} < W_{t-}$$

always holds.

We next apply the stochastic control approach to solve for a banker's optimal consumption and portfolio choices. Consider a banker who can access shadow banking. We first conjecture and later verify that the banker's continuation value function take the form

$$J_t \equiv E_t\left[\int_t^\infty e^{-\rho u}\ln(c_u)du\right] = \frac{ln(W_t)}{\rho} + h_t,$$

where W_t is the banker's net worth in period t, and h_t is a separably additive term that depends on market conditions and evolves endogenously according to

$$dh_t = h_{t-}\mu_{t-}^h dt - h_{t-}\kappa_t^h dN_t.$$

Second, we conjecture that, if the banker defaults, her continuation value is $\hat{J}_t = ln(W_t)/\,\rho + \hat{h}_t$, where $\hat{h}_t$ follows

$$d\hat{h}_t = \hat{h}_{t-}\mu_{t-}^{\hat{h}} dt - \hat{h}_{t-}\kappa_t^{\hat{h}} dN_t.$$

Now, we are ready to spell out the Hamilton-Jacobian-Bellman equation for the banker's optimal control problem

$$\begin{aligned}\rho J_{t-} &= h_{t-}\mu^h_{t-} + \chi\left(J^r_t(W_t) - J_{t-}\right) \\ &+ \max_{c_{t-},S_{t-},S^*_{t-},\mathbf{d}_t} \left\{ \begin{array}{c} \frac{1}{\rho W_{t-}}\left(W_{t-}(R_{t-}+\pi_{t-}) + S_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right) + S^*_{t-}\left(R_{t-}-\tilde{r}_{t-}\right) - c_{t-}\right) \\ +\lambda\left(1-\mathbf{d}_t\right)\left(\frac{1}{\rho}\ln\left(W_{t-}-\left(W_{t-}+S_{t-}+S^*_{t-}\right)\kappa^Q_t\right) + h_{t-}(1-\kappa^h_t)\right) \\ +\lambda\mathbf{d}_t\left(\frac{1}{\rho}\ln\left(W_{t-}-\left(W_{t-}+S_{t-}\right)\kappa^Q_t\right) + \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)\right) - \lambda J_{t-} + \ln(c_{t-}) \end{array} \right\},\end{aligned}$$

where $J^r_t(\cdot)$ is the banker's continuation value function if she retires in period t.

Proposition 2 *A banker's choices of optimal consumption, optimal portfolio weights, and default decision* $\{c_t, s_t, s^*_t, \mathbf{d}_t\}_{t=0}^{\infty}$ *satisfy*

$$c_t = \rho W_t, \tag{1.11}$$

$$\left(1+s_{t-}\right)\kappa^Q_t + s^*_{t-}\kappa^Q_t \mathbf{1}_{(\mathbf{d}_t>0)} < 1, \tag{1.12}$$

$$R_{t-} - r_{t-} - \tau_{t-} \leq \frac{\lambda\left(1-\mathbf{d}_t\right)\kappa^Q_t}{1-\left(1+s_{t-}+s^*_{t-}\right)\kappa^Q_t} + \frac{\lambda\mathbf{d}_t\kappa^Q_t}{1-\left(1+s_{t-}\right)\kappa^Q_t}, \quad = \text{ if } s_{t-}>0, \tag{1.13}$$

$$R_{t-} - \tilde{r}_{t-} \geq \frac{\lambda\left(1-\mathbf{d}_t\right)\kappa^Q_t}{1-\left(1+s_{t-}+s^*_{t-}\right)\kappa^Q_{t-}}, \quad = \text{ if } s^*_{t-} < \bar{s}^*_{t-}, \tag{1.14}$$

and

$$\left\{ \begin{array}{ll} \mathbf{d}_t = 1, & \text{if} \quad \frac{1}{\rho}\ln\left(1-\left(1+s_{t-}\right)\kappa^Q_t\right) + \hat{h}_t > \frac{1}{\rho}\ln(1-\left(1+s_{t-}+s^*_{t-}\right)\kappa^Q_t) + h_t, \\ \mathbf{d}_t \in [0,1], & \text{if} \quad \frac{1}{\rho}\ln\left(1-\left(1+s_{t-}\right)\kappa^Q_t\right) + \hat{h}_t = \frac{1}{\rho}\ln\left(1-\left(1+s_{t-}+s^*_{t-}\right)\kappa^Q_t\right) + h_t, \\ \mathbf{d}_t = 0, & \text{otherwise,} \end{array} \right. \tag{1.15}$$

for all $t \geq 0$, *where* $s_t = S_t/W_t$, $s^*_t = S^*_t/W_t$, $\hat{h}_t = \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)$, *and* $h_t = h_{t-}(1-\kappa^h_t)$.

Proof. See Appendix. ■

We will interpret these expressions in order. Logarithmic agents always consume ρ fraction of their current net worth, as equation (2.6) shows. Inequality (2.7) guaran-

tees that the banker's net worth will be positive if a negative shock hits the economy. Equations (2.8) and (2.9) are first-order conditions with respect to the banker's portfolio choices. Equation (1.15) characterizes the banker's default decision given a negative shock.

To consider a banker's strategic choice, we need to characterize her optimal choice if she cannot access shadow banking because of default. The law of motion of her net worth $\hat{W}_t$ in the case of default is

$$d\hat{W}_t = \left(\hat{W}_{t-}(R_{t-}+\pi_{t-}) + \hat{S}_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right) - \hat{c}_{t-}\right)dt - \left(\hat{W}_{t-}\kappa_t^Q + \hat{S}_{t-}\kappa_t^Q\right)dN_t.$$

The HJB equation for the banker is

$$\begin{aligned}\rho\hat{J}_{t-} &= \hat{h}_{t-}\mu_{t-}^{\hat{h}} + \chi\big(J_t^r\big(\hat{W}_t\big) - \hat{J}_{t-}\big) + \xi(J_t - \hat{J}_{t-}) \\ &+ \max_{\hat{c}_{t-},\hat{S}_{t-}} \left\{ \begin{array}{c} \ln\left(\hat{c}_{t-}\right) + \frac{1}{\rho\hat{W}_{t-}}\big(\hat{W}_{t-}(R_{t-}+\pi_{t-}) + \hat{S}_{t-}\big(R_{t-}-r_{t-}-\tau_{t-}\big) - \hat{c}_{t-}\big) \\ +\lambda\big(\frac{1}{\rho}\ln\big(\hat{W}_{t-} - \big(\hat{W}_{t-}+\hat{S}_{t-}\big)\kappa_t^Q\big) + \hat{h}_{t-}\big(1-\kappa_t^{\hat{h}}\big) - \hat{J}_{t-}\big) \end{array} \right\}.\end{aligned}$$

Similar to Proposition 9, we list the banker's first-order conditions

$$\begin{aligned}\hat{c}_t &= \rho\hat{W}_t \\ \left(1+\hat{s}_{t-}\right)\kappa_t^Q &< 1 \qquad (1.16) \\ R_{t-}-r_{t-}-\tau_{t-} &= \frac{\lambda\kappa_t^Q}{1-\left(1+\hat{s}_{t-}\right)\kappa_t^Q} \qquad (1.17)\end{aligned}$$

The Note Market and the Enforcement Problem

Our analysis of the note market begins with the optimal portfolio choice problem for a banker who intends to default if a negative shock hits in the subsequent period. To maximize the growth of her net worth, the banker borrows up to $\bar{s}_{t-}^*$ times her net

worth via shadow banking and adjusts her portfolio weight on regular banking $\tilde{s}_{t-}$ so that

$$R_{t-} - r_{t-} - \tau_{t-} = \frac{\lambda \kappa_t^Q}{1 - \left(1 + \tilde{s}_{t-}\right)\kappa_t^Q}.$$

In addition, she needs to make sure that her continuation value with default in the event of a negative shock is larger than that without default

$$\frac{1}{\rho} \ln\left(1 - \left(1 + \tilde{s}_{t-}\right)\kappa_t^Q\right) + \hat{h}_t > \frac{1}{\rho} \ln\left(1 - \left(1 + \tilde{s}_{t-} + \bar{s}_{t-}^*\right)\kappa_t^Q\right) + h_t,$$

where $h_t = h_{t-}\left(1 - \kappa_t^h\right)$ and $\hat{h}_t = \hat{h}_{t-}\left(1 - \kappa_t^{\hat{h}}\right)$.

Given this banker's choices, we argue that no banker intends to default, if the maximum leverage of shadow banking $\bar{s}_{t-}^*$ satisfies the enforceability constraint

$$\frac{1}{\rho} \ln\left(1 - \left(1 + \tilde{s}_{t-}\right)\kappa_t^Q\right) + \hat{h}_t = \frac{1}{\rho} \ln\left(1 - \left(1 + \tilde{s}_{t-} + \bar{s}_{t-}^*\right)\kappa_t^Q\right) + h_t. \tag{1.18}$$

We now justify this statement. (s_{t-}, s_{t-}^*) denotes the optimal portfolio choice in period $t-$ for a banker who intends to honor her shadow bank obligations in period t. The above no-default statement is true because

$$\begin{aligned}
& \frac{s_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + s_{t-}^*\left(R_{t-} - r_{t-}\right)}{\rho} + \frac{\lambda}{\rho} \ln\left(1 - \left(1 + s_{t-} + s_{t-}^*\right)\kappa_t^Q\right) + \lambda h_t \\
> & \frac{\tilde{s}_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + \bar{s}_{t-}^*\left(R_{t-} - r_{t-}\right)}{\rho} + \frac{\lambda}{\rho} \ln\left(1 - \left(1 + \tilde{s}_{t-} + \bar{s}_{t-}\right)\kappa_t^Q\right) + \lambda h_t \\
= & \frac{\tilde{s}_{t-}\left(R_{t-} - r_{t-} - \tau_{t-}\right) + \bar{s}_{t-}^*\left(R_{t-} - r_{t-}\right)}{\rho} + \frac{\lambda}{\rho} \ln\left(1 - \left(1 + \tilde{s}_{t-}\right)\kappa_t^Q\right) + \lambda \hat{h}_t.
\end{aligned}$$

The inequality holds since (s_{t-}, s_{t-}^*) is the banker's optimal choice given that she intends to protect her shadow bank; the equality is true because of the enforceability

constraint (1.18). Overall, we observe that it is strictly better for the banker to bail out her shadow bank in trouble.

This enforceability constraint is "too" tight because creditors can increase $\bar{s}^*_{t-}$ by a sufficiently small amount that bankers still strictly prefer to protect their shadow banks in period t. To simplify equilibrium analysis, we exclude the possibility that creditors raise the maximum leverage of shadow banking above $\bar{s}^*_{t-}$ with Assumption 6.

Assumption 6 *The creditors of a shadow bank always want to rule out its default in the worst scenario that the leverage of its sponsoring regular bank equals* $\tilde{s}_{t-}$.

The rationale for Assumption 6 is as follows: since the creditors of a shadow bank cannot influence the leverage choice for its sponsoring regular bank, those creditors are very cautious, and they attempt to exclude the shadow bank's default in the worst case for them; that is, the banker intends to default and, accordingly, maximizes the leverage of her regular bank. In Appendix 1.7, we analyze a relatively loose enforceability constraint, with which we can characterize the market-clearing note contract $(\bar{s}^*_{t-}, \tilde{r}_{t-})$ only under Assumptions 1 - 9. Appendix 1.7 also shows that Assumption 6 is not essential for our main results.

Assumption 6 avoids shadow banks' default, which is consistent with Assumption 1. This is because if a shadow bank defaults whenever a negative shock hits, then notes issued by the shadow bank are essentially combinations of risk-free debt and equity. However, households cannot hold equity (Assumption 1). Since no shadow bank defaults given the enforceability constraint (1.18), the no-arbitrage condition implies that the rate of return from holding notes equals the risk-free rate, i.e., $\tilde{r}_t = r_t$.

We characterize a banker's optimal choices given the note contract $\{\bar{s}^*_t, \tilde{r}_t\}_{t=0}^{\infty}$ in Proposition 3.

Proposition 3 *Given the note contract* $\{\bar{s}_t^*, \tilde{r}_t\}_{t=0}^{\infty}$*, a banker's optimal portfolio weights* $\{s_t, s_t^*\}_{t=0}^{\infty}$*, and default decisions* $\{\mathbf{d}_t\}_{t=0}^{\infty}$ *satisfy* $\mathbf{d}_t = 0$*,*

$$(1 + s_{t-} + s_{t-}^*)\kappa_t^Q < 1,$$

$$R_{t-} - r_{t-} - \tau_{t-} \leq \frac{\lambda \kappa_t^Q}{1 - \left(1 + s_{t-} + s_{t-}^*\right)\kappa_t^Q}, \quad = \quad \textit{if } s_{t-} > 0,$$

$$R_{t-} - r_{t-} \geq \frac{\lambda \kappa_t^Q}{1 - \left(1 + s_{t-} + s_{t-}^*\right)\kappa_t^Q}, \quad = \quad \textit{if } s_{t-}^* < \bar{s}_{t-}^*,$$

for all $t \geq 0$*.*

To fully specify $\bar{s}_{t-}^*$, we need to know the difference between h_t and $\hat{h}_t$, which is denoted by H_t. The following proposition characterizes H_t, which is interpreted as the cost for a banker to default on her shadow bank obligations in period t.

Proposition 4 *Given that* $ln(\cdot)/\,\rho + h_t$ *is the continuation value function of a banker who can access shadow banking and* $ln(\cdot)/\,\rho + \hat{h}_t$ *is the continuation value function of a banker who defaults in period* $t-$*,*

$$H_t \equiv h_t - \hat{h}_t = E_t \left[\int_t^{\infty} \exp\left(-\left(\rho + \xi + \chi\right)(u - t) \right) f_u \; du \right], \tag{1.19}$$

where f_u *equals*

$$\frac{1}{\rho} \left(\begin{array}{c} \overbrace{\left(s_{u-} + s_{u-}^* - \hat{s}_{u-}\right)\left(R_{u-} - r_{u-} - \tau_{t-}\right)}^{\textit{higher leverage benefit due to cheap credit}} + \overbrace{s_{u-}^* \tau_{u-}}^{\textit{tax benefit}} \\ + \lambda \underbrace{\left(\ln\left(1 - \left(1 + s_{u-} + s_{u-}^*\right)\kappa_u^Q\right) - \ln\left(1 - \left(1 + \hat{s}_{u-}\right)\kappa_u^Q\right) \right)}_{\textit{high risk due to high leverage}} \end{array} \right).$$

Proof. See Appendix. ■

f_u is the tax benefit (i.e., regulatory arbitrage) that shadow banking offers in period u, and is essentially the difference between the growth rate of net worth for bankers

who can use shadow banks and the growth rate of net worth for bankers who cannot. H_t is the present value of future tax benefits f_u that a banker will lose if she defaults in period t. The discount factor is the banker's time discount factor plus the "come-back" intensity ξ and the retirement rate χ, because once bankers return to the shadow banking sector or retire, the advantage of accessing shadow banking effectively disappears.

We next highlight the feedback loop between the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^{\infty}$ and the cost of default $\{H_t\}_{t=0}^{\infty}$. First, the enforceability constraint (1.18) implies that the maximum leverage of shadow banking relies on a banker's cost of default to her shadow bank's obligations. Second, the probabilistic representation of the cost of default (4) indicates that the maximum leverage of shadow banking directly affects how costly default is for bankers.

This feedback loop gives rise to an equilibrium where shadow banking does not exist. Conjecture that $\{\bar{s}_t^* = 0\}_{t=0}^{\infty}$. The probabilistic representation (4) implies $\{H_t = 0\}_{t=0}^{\infty}$, and the enforceability constraint (1.18) justifies the conjecture. Therefore, the economy has an equilibrium in which shadow banking does not exist. We summarize the conclusion of these arguments in the following proposition.

Proposition 5 *There exists a non-sunspot equilibrium where shadow banking does not exist, that is,* $\{\bar{s}_t^* = 0, H_t = 0\}_{t=0}^{\infty}$.

This equilibrium is degenerate, and hereafter we label it the "bad" equilibrium since productive bankers are unable to leverage up via shadow banking. There might be a non-degenerate equilibrium, however, where shadow banking exists, which is labeled the "good" equilibrium.

Equilibrium selection is beyond the scope of this chapter. Given the importance of shadow banking in the real world, we assume that the "good" equilibrium prevails when both "good" and "bad" equilibria exist.

Miscellany

We group discussions of equilibrium conditions 4 and 5 in Definition 1 together, since all of them are straightforward. The budget of the regulatory authority is balanced if $\pi_t = s_t \tau_t$ for $t \geq 0$. Since households are risk-neutral, the market for consumption goods clears automatically. The market for physical capital clears if the fractions of physical capital held by bankers and households sum to 1. Let ψ_t denote the fraction of physical capital held by bankers, which equals $(1 + s_t + s_t^*)\omega_t$.

In our model, as in other continuous-time macro-finance papers, the wealth distribution matters for the dynamics of the economy. Later, we will capture the dynamics of an equilibrium with the bankers' wealth share $\omega_t \equiv \int_0^1 W_t^i di \Big/ q_t K_t$. Lemma 5 characterizes how ω_t evolves.

Lemma 2 *The law of motion of* ω_t *is*

$$
\begin{aligned}
d\omega_t &= \omega_{t-}\mu_{t-}^{\omega} dt - \omega_{t-}\kappa_t^{\omega} dN_t, && (1.20)\\
\textit{where } \mu_t^{\omega} &= R_t + s_t\left(R_t - r_t\right) + s_t^*\left(R_t - \tilde{r}_t\right) - \mu_t^q - \mu_t^K - \rho - \chi, && (1.21)\\
\textit{and } \kappa_t^{\omega} &= \frac{\left(s_{t-} + s_{t-}^*\right)\kappa_t^Q}{1 - \kappa_t^Q}. && (1.22)
\end{aligned}
$$

Proof. See Appendix. ■

1.3.2 Markov Equilibrium

Our model has the property of scale-invariance with respect to K. This means that, for a given equilibrium in an economy with initial capital $\left\{k_0^i, k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$, there exists an equivalent equilibrium with the same laws of motion of ω_t, q_t, and H_t in any economy with initial capital $\left\{\varsigma k_0^i, \varsigma k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$, where $\varsigma \in (0, \infty)$.

Equations (1.2) − (4) can characterize an equilibrium specified by Definition 1. The scale-invariance property implies that we can characterize an equilibrium that is Markov in ω with a modification of Assumption 2.[5]

Assumption 7 *In period t, the tax rate τ_t equals $\min\{\tau, \tau s_t\}$, where τ is a positive constant and s_t denotes $\int_0^1 S_t^i \, di \Big/ \int_0^1 W_t^i \, di$; the tax rate is τ for bankers who are unable to use shadow banking due to default.*

Note that the tax rate τ_t at any time t depends only on the aggregate variable. Thus, individual bankers always take the tax rate as given.

In the Markov equilibrium, the dynamics of all endogenous aggregate variables $\{q_t, H_t\}_{t=0}^{\infty}$ can be fully described by the law of motion of the state variable and functions $q(\omega)$ and $H(\omega)$, which are defined over the domain $(0, \bar{\omega}]$. Thanks to Ito's Lemma, we derive the law of motion of $\{q_t, H_t\}_{t=0}^{\infty}$.

$$\mu_t^q = \frac{q'(\omega_t)}{q(\omega_t)} \omega_t \mu_t^{\omega}, \tag{1.23}$$

$$\kappa_t^q = \frac{q(\omega_{t-}) - q(\omega_{t-}(1 - \kappa_t^{\omega}))}{q(\omega_{t-})}, \tag{1.24}$$

$$\mu_t^H = \frac{H'(\omega_t)}{H(\omega_t)} \omega_t \mu_t^H, \tag{1.25}$$

$$\kappa_t^H = \frac{H(\omega_{t-}) - H(\omega_{t-}(1 - \kappa_t^{\omega}))}{H(\omega_{t-})}. \tag{1.26}$$

The following proposition describes a system of delay differential equations and their boundary conditions, which define function $q(\omega)$ and $H(\omega)$.

Proposition 6 *$q(\omega)$ and $H(\omega)$ are defined over $(0, \bar{\omega}]$. Given $(\omega, q(\omega'), H(\omega'), 0 < \omega' < \omega)$, we compute $(q'(\omega), H'(\omega))$ using the following procedure:*

[5] A natural tax policy is that $\tau_t = \tau$ for $t \geq 0$. Given this policy, endogenous risk κ_t^q jumps as shadow banks become the marginal buyer of physical capital, which complicates the computation of an equilibrium. Our setup specified in Assumption 7 makes the process that shadow banks become marginal buyers smooth and simplifies the computation.

1. *Conjecture that* $\psi < 1$, *find* $s + s^*$ *such that*

$$\frac{a - a^h}{q} - \tau = \frac{\lambda \kappa^Q}{1 - (1 + s + s^*)\kappa^Q} - \lambda \kappa^Q,$$

equations (2.25) *and* (2.27) *hold. Derive* $(\kappa^\omega, \kappa^q, \kappa^Q)$ *according to Ito's Lemma and* μ^q *based on equation* (2.8). *Also compute* ψ.

2. *If* $\psi < 1$ *does not hold, then* $\psi = 1$ *and* $s + s^* = 1/\omega - 1$. *Similarly, derive* $(\kappa^\omega, \kappa^q, \kappa^Q, \kappa^H)$ *according to Ito's Lemma and* μ^q *based on equation* (2.8).

3. *Given* $s + s^* (= \tilde{s})$ *and* κ^Q, *compute* $\bar{s}^*$ *such that equation* (1.18) *holds and then we derive* s. *Also, derive* μ^ω *according to equation* (2.24).

4. *Compute* $q'(\omega)$ *according to equation* (2.26).

5. *Finally, compute* f *based on* (4) *and then derive* $H'(\omega)$ *according to*

$$(\rho + \xi + \chi) H(\omega) = f + \omega \mu^\omega H'(\omega) + \lambda \left(H(\omega (1 - \kappa^\omega)) - H(\omega) \right). \tag{1.27}$$

Boundary conditions are

$$\mu^q(\bar{\omega}) = \mu^H(\bar{\omega}) = \mu^\omega(\bar{\omega}) = 0,$$
$$\lim_{\omega \to 0} q(\omega) = \underline{q} \text{ and } \lim_{\omega \to 0} H(\omega) = 0,$$

where $\underline{q}$ *satisfies*

$$a^h - \delta - \frac{\underline{q}^2 - 1}{2\phi} = \rho \underline{q}. \tag{1.28}$$

For the "bad" equilibrium, we need to solve only a single differential with respect to $q(\omega)$, since $H(\omega) = 0$ for all $\omega \in (0, \bar{\omega}]$.

Equilibrium Uniqueness

Within the class of Markov equilibria, we can establish under what conditions the "bad" equilibrium (where shadow banking does not emerge) is unique. To achieve this result, we define mapping Γ which takes the cost of default function $H(\omega)$ as input,

$$\Gamma H(\omega) = E_t \left[\int_t^{\infty} \exp(-\left(\rho + \xi + \chi\right)(u-t)) f(\omega_u) du \middle| \omega_t = \omega \right]$$

where

$$f(\omega) = \frac{1}{\rho} \left(\begin{array}{c} \left(s(\omega) + s^*(\omega) - \hat{s}(\omega)\right)\left(R(\omega) - r(\omega) - \tau(\omega)\right) + s^*(\omega)\tau(\omega) \\ +\lambda \left(\ln\left(1 - \left(1 + s(\omega) + s^*(\omega)\right)\kappa^Q(\omega)\right) - \ln\left(1 - \left(1 + \hat{s}(\omega)\right)\kappa^Q(\omega)\right)\right) \end{array} \right),$$

$$s^*(\omega) \leq \bar{s}^*(\omega),$$

and

$$\bar{s}^*(\omega) = \left(1 - \exp\left(-\rho H(\omega)\right)\right)\left(\frac{1}{\kappa^Q(\omega)} - \left(1 + \tilde{s}(\omega)\right)\right).$$

Equation (4) clearly shows that $H(\omega)$ is a fixed point of the mapping Γ. As we have noted, the mapping Γ might allow for two fixed points: one leads to the "good" non-degenerate equilibrium, and the other yields the "bad" degenerate equilibrium. The following theorem provides a sufficient condition, under which the "bad" equilibrium is unique.

Theorem 1 *If $\tau < \left(\rho + \xi + \chi\right)\kappa$, the mapping Γ is a contraction mapping with the fixed point $H(\omega) = 0$ for all $\omega \in (0, \bar{\omega}]$.*

Proof. See Appendix. ■

To show that Γ is a contraction mapping, we justify that Γ satisfies Blackwell's sufficent conditions if $\tau < (\rho + \xi + \chi)\kappa$.

The feedback loop illustrated earlier explains why Γ could be a contraction mapping. Suppose the magnitude of the exogenous shock κ increases permanently in an economy where shadow banking exists. Bankers' incentives to default increase, and these heightened incentives lower the maximum leverage of shadow banking $\bar{s}^*$ (the enforceability constraint (1.18)). The decline in the leverage of shadow banking $\bar{s}^*$ reduces the cost of default H (the probabilistic representation (4)), which, in turn, lowers the leverage of shadow banking. This cycle could completely eliminate any shadow banking in equilibrium. The following analyses will explore the economic implications of changes in other parameters such as the tax rate τ.

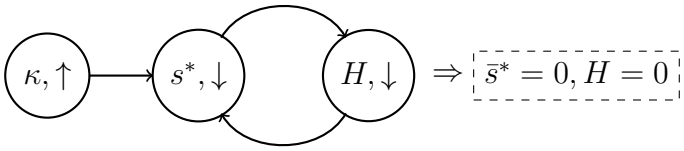


Figure 1.3: Feedback loop

1.3.3 Numerical Example

In this section, we present the main dynamic properties of the baseline model via a numerical example. Thanks to the global solution provided by the continuous-time approach, we are able to show the endogenous variables as functions of the state variable (bankers' wealth share ω) as well as the dynamics of the economy at any state.

Calibration and Moments

We restrict the choice of parameter values by calibrating our model. Table 2.1 lists calibrated parameters, and Table 1.2 in Appendix 2.8 contains moments generated by the model and their targets. We set the time discount factor ρ to 3% to match the real

Table 1.1: Parameters

Parameter		Choice	Target
Preference and Technology			
ρ	time discount rate	3%	Campbell and Cochrane (1999)
χ	retirement rate	16%	average Sharpe ratio
a	bankers' productivity	22.5%	investment rate
a^h	households' productivity	10%	highest Sharpe ratio
δ	depreciation rate	10%	Christiano et al. (2005)
ϕ	capital adjustment cost	3	Christiano et al. (2005)
Shocks			
λ	Poisson shock intensity	1	volatility of growth rate
κ	Poisson shock magnitude	4%	of bankers' wealth
Frictions			
τ	regulation	3%	leverage
ξ	"come-back" rate	6%	ratio of securitization

interest rate estimated by Campbell and Cochrane (1999). Bankers' retirement rate χ is set at 16% to target the average Sharpe ratio. We set bankers' productivity at 22.5% so that the average investment-to-capital ratio is close to 11% (He and Krishnamurthy, 2012a). The productivity of less-productive households is chosen at 10% to match the fact that the Sharpe ratio during the 2007-09 financial crisis was approximately 15 times the average level (He and Krishnamurthy, 2012a). Choices of the depreciation rate and the capital adjustment cost ϕ are standard in the macroeconomic literature (Christiano, Eichenbaum, and Evans, 2005). We set the Poisson shock parameters to target the conditional volatility of the growth rate of bankers' wealth in distressed periods and in non-distressed periods. The distressed periods are defined as periods with the lowest 33% Sharpe ratios. The regulation parameter, tax rate τ, is set at 3% to target the average leverage of the entire banking sector. We set the intensity with which bankers can re-access shadow banking after default at 6% to target the ratio of securitization by non-agency issuers in the third quarter of 2006. Appendix 2.8 contains both the data and procedure used to calculate this ratio.

Capital Misallocation and the Price of Capital

The constraint on equity issuance (Assumption 1) leads to capital misallocation and to the low price of capital in economic downturns, where bankers' wealth share is low. Panel a in Figure 2.3 shows that the fraction of capital goods that bankers hold $\psi(\omega)$ is weakly increasing in bankers' wealth share ω. When bankers are wealthy enough to hold all physical capital, there is no capital misallocation. Since bankers use leverage, they can hold all capital goods when they seize approximately 25% of the total wealth in the economy. If their wealth share declines substantially, bankers have to downsize their asset holdings; otherwise, their risk exposure would be too large because of high leverage. As the share of bankers' wealth diminishes, bankers hold a declining fraction of capital goods, capital misallocation deepens, and aggregate productivity declines. Capital misallocation depresses the price of capital, since capital goods are less valuable in the possession of less-productive households. Panel b in Figure 2.3 shows that the price of capital $q(\omega)$ is increasing in bankers' wealth share ω. Since the high rate of return comes with the low price of capital, the excess return from holding capital is decreasing in the state variable (Panel d in Figure 2.3).

Endogenous Risk, Leverage, and Pecuniary Externality

Besides causing capital misallocation and depressing the price of capital, the no-equity-financing friction also generates endogenous risk through the balance sheet amplification mechanism. When an aggregate negative shock hits the economy, bankers' net worth declines disproportionately because of the leverage effect, and bankers' overall leverage shoots up unfavorably (Panel c in Figure 2.3). Since all bankers are pressured to sell capital goods, the price of capital deteriorates, which further erodes bankers' net worth and leads to another round of asset sales.

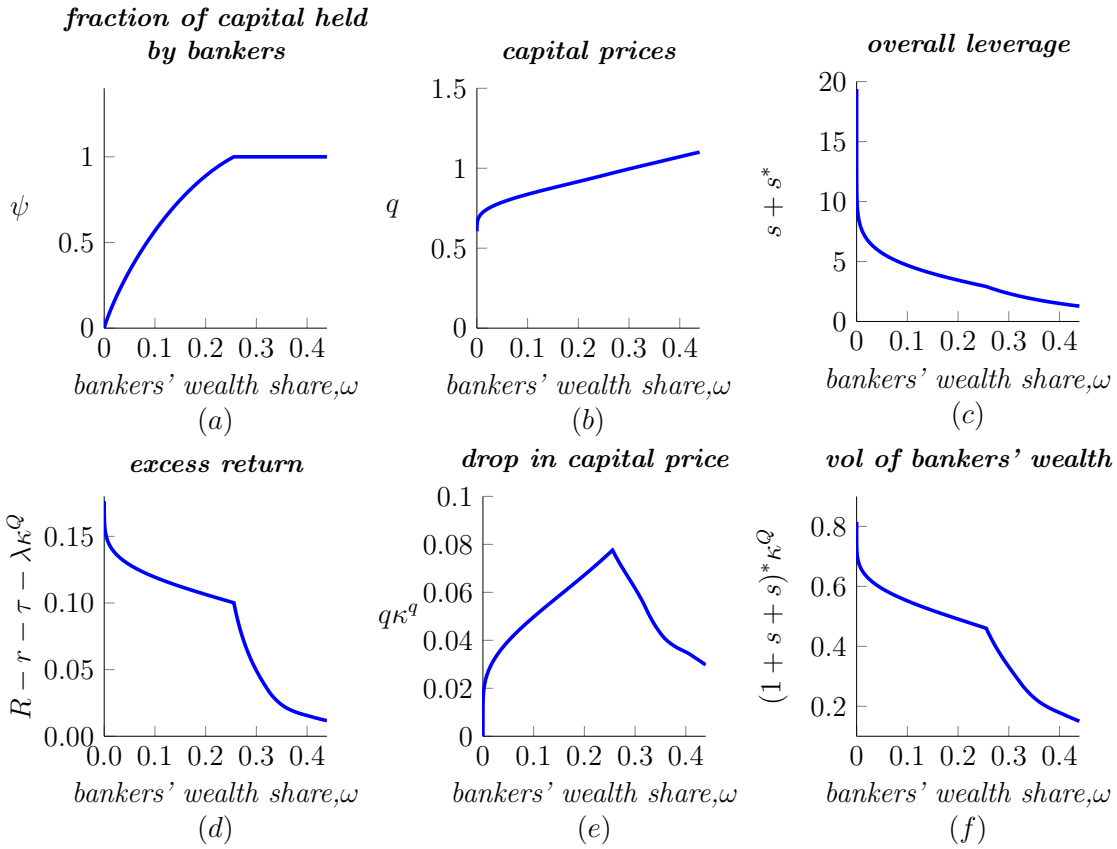


Figure 1.4: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For parameter values, see Section 1.3.3.

Endogenous risk caused by the balance sheet amplification varies with different states of the economy (Panel e in Figure 2.3). When bankers are about to conduct fire sales of physical capital to households, endogenous risk is at its most significant. When bankers' wealth share is either very large or very small, endogenous risk is small for the following reasons.[6] When bankers have a small proportion of wealth, they hold only a small fraction of physical capital in the economy. A negative shock causes a small decline in the price of capital because the magnitude of bankers' total asset sales is tiny. When bankers' wealth share is very large, their overall leverage is mechanically

[6]In He and Krishnamurthy (2012b) and He and Krishnamurthy (2013), endogenous risk does not decline as the wealth share of productive agents goes to zero because they assume that only productive agents can hold assets. Therefore, in their papers, the pressure to sell does not decline even when productive agents become very poor.

low (Panel c in Figure 2.3). A negative shock has a small impact on bankers' net worth and, similarly, has a small impact on the price movement of capital .

A pecuniary externality exists in the competitive equilibrium because individual bankers do not internalize the aggregate impact of their individual leverage choices on endogenous risk. The leverage decision of each banker has zero impact on endogenous risk. However, the aggregation of all bankers' leverage choices matters considerably. Therefore, the socially optimal leverage choice is quite distinct from the private optimal choice in the competitive market. The tax on regular banking adjusts bank leverage and improves social welfare. Welfare issues are discussed in detail in Section 1.4.1.

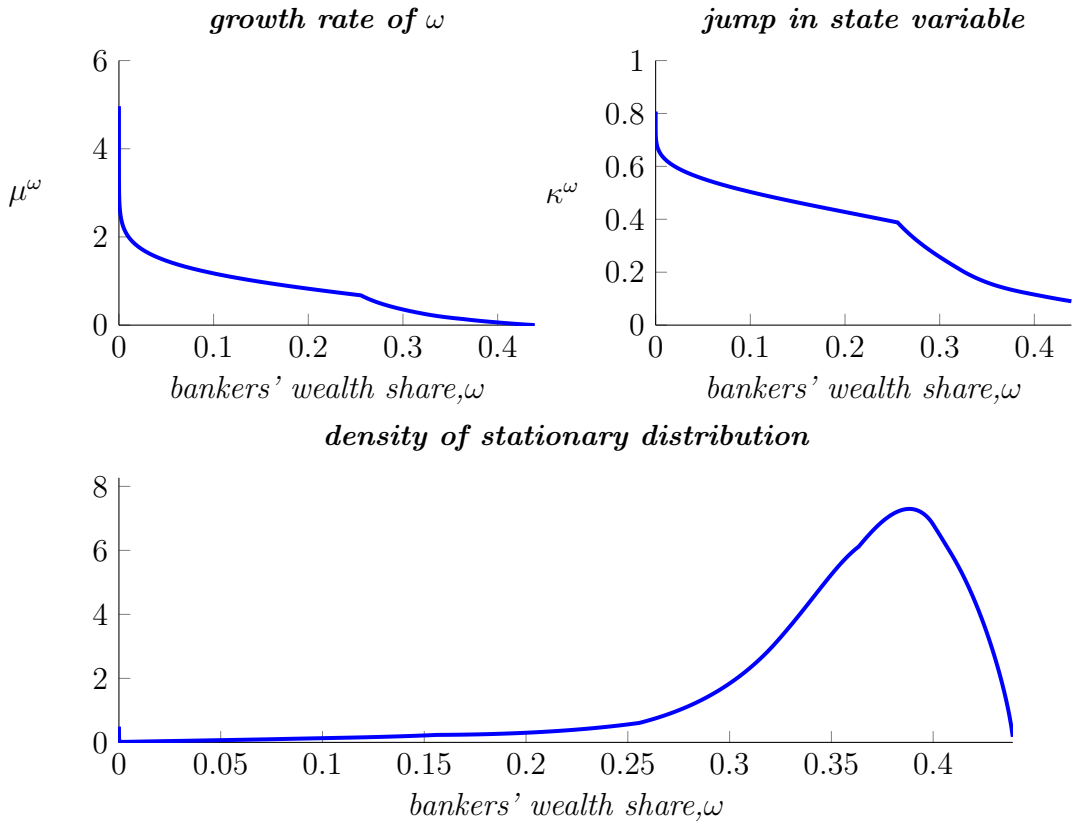


Figure 1.5: μ^ω (upper left panel),κ^ω (upper right panel), and the density of stationary distribution (lower panel) as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. Appendix 1.8 contains the formula and algorithm used to numerically solve for the density function of the stationary distribution. For parameter values, see Section 1.3.3.

The Dynamics of the State Variable.

We next focus on the evolution of the state variable. Equation (2.23) describes its motion. Upper plots in Figure 1.5 present values of μ^ω and κ^ω at different states. In states where bankers' wealth share is small, the growth rate of the state variable μ^ω is high owing to bankers' high leverage $s+s^*$ (Panel c in Figure 2.3) plus the high rate of return from holding physical capital (Panel d in Figure 2.3). The proportional decline in the state variable is also large in recessions because of bankers' high leverage.

The lower panel of Figure 1.5 shows that for most of the time bankers hold about 38% of the wealth in the economy. An economy is rarely in a situation where bankers hold only a little wealth because the low price of capital and high returns from holding capital help bankers to quickly build up their wealth and pull the economy out of recessions. Bankers' wealth share never exceeds the point $\bar{\omega}$, where $\mu^\omega(\bar{\omega}) = 0$, because bankers retire randomly.

1.3.4 The Feedback Loop in Shadow Banking

The feedback loop between the maximum leverage of shadow banking $\{\bar{s}_t^*\}_{t=0}^\infty$ and the cost of default $\{H_t\}_{t=0}^\infty$ is the driving force underpinning our main results: *i*) shadow banking is pro-cyclical; *ii*) shadow banking adds to financial instability through reintermediation; *iii*) financial regulation can raise or reduce financial instability under different circumstances. This section explains each of the three main results using dynamic and comparative statics analyses.

Dynamic Result : Pro-cyclical Leverage of Shadow Banking

We show that the leverage of shadow banking $\{s^*\}_{t=0}^\infty$ is pro-cyclical (Panel *a* in Figure 1.6). To demonstrate this, we rearrange the enforceability constraint (1.18)

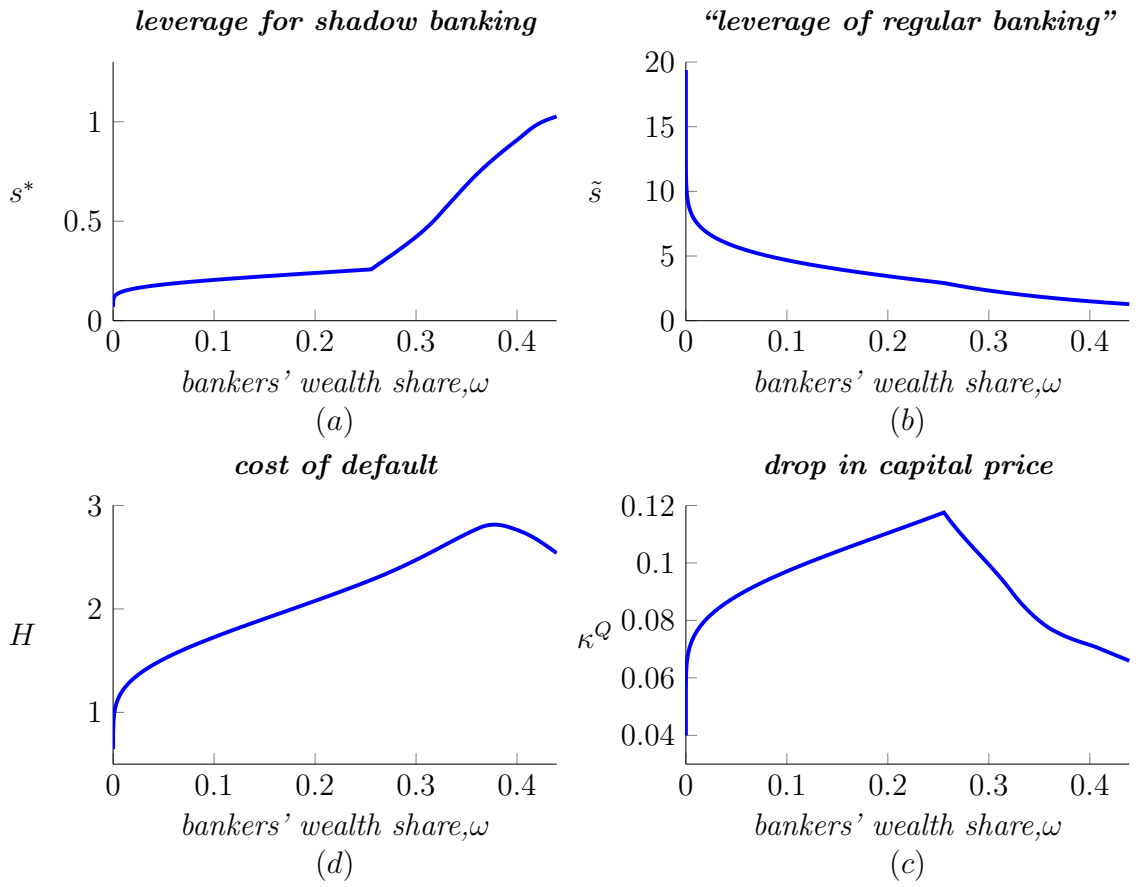


Figure 1.6: Pro-cyclical Shadow Banking
This figure presents the leverage of shadow banking s^*, "the leverage of regular banking" $\tilde{s}$, asset volatility κ^Q, and the cost of default H as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For parameter values, see Section 1.3.3.

and find that three factors affect the maximum leverage of shadow banking in period $t-$: *i*) a banker's exposure to regular banking business, $\tilde{s}_{t-}$, *ii*) the loss to one dollar of investment, κ_t^Q, *iii*) the cost of default, H_t.

$$\bar{s}_{t-}^* = (1 - \exp(-\rho H_t))(\frac{1}{\kappa_t^Q} - (1 + \tilde{s}_{t-})). \tag{1.29}$$

Next, we illustrate how the three factors work in a dynamic setting by considering the example of economic booms. At the same time, we will highlight the importance of the feedback loop between $\{s_t^*\}_{t=0}^{\infty}$ and $\{H_t\}_{t=0}^{\infty}$. Suppose the economy is in a boom where the bankers' share of wealth is high, a high price of capital prevails, and a low

rate of return induces bankers to take low leverage (Panel b in Figure 1.6), thereby lowering their incentive to default. The low asset volatility that often characterizes such a boom reduces the likelihood of bankers' defaulting (Panel c in Figure 1.6). Therefore, the leverage of shadow banking is high in economic booms (Panel a in Figure 1.6). Given that the leverage of shadow banking is high in economic booms, equation (4) implies that the cost of default is also high (Panel d in Figure 1.6), which raises the maximum leverage of shadow banking further as the enforceability constraint (1.18) demonstrates. The shadow banking sector expands substantially during economic booms as a result of this feedback loop.

Comparative Statics

We next compare different economies and see how the presence of shadow banking changes the conventional understanding of financial instability and its connection to financial regulation. First, we examine economies with and without shadow banking and show that shadow banking increases financial instability. Next, we vary parameter τ and explain that there is a U-shaped relationship between financial instability and the regulation of the traditional banking sector.

Reintermediation. The combined effect of pro-cyclical shadow banking and reintermediation increases endogenous risk in the economy. In our model, the pro-cyclicality of shadow banking means that shadow banks purchase a large number of assets in economic upturns. The cost of funding the balance sheets of regular banks is expensive. Therefore, in economic booms, the scale of asset accumulation by shadow banks exceeds that which regular banks would pursue in absence of an accompanying shadow bank system. If an adverse shock hits the economy, shadow banks must divest large amounts of assets as the leverage constraint tightens, and regular banks are reluctant to acquire these assets because it is so expensive to expand their own

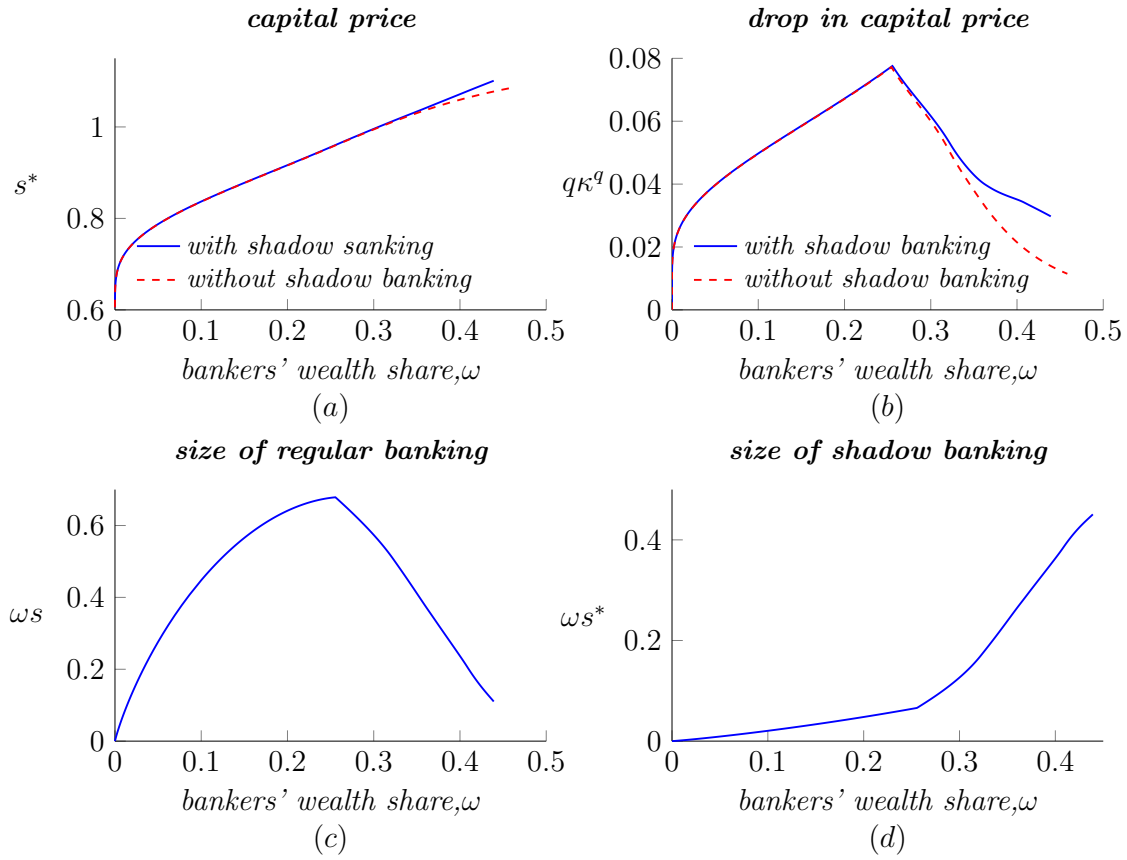


Figure 1.7: Reintermediation
This figure presents the price of capital (blue solid line in upper left panel), the jump in the price of capital (blue solid line in upper right panel), the size of debts (lower left panel), and the size of notes (lower right panel) as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium. For comparison, this figure also shows the price of capital (red dashed line in upper left panel) and the jump in the price of capital (red dashed line in upper right panel) as functions of the state variable ω in the "bad" equilibrium. For parameter values, see Section 1.3.3

balance sheets. As a result, the price of capital declines more than it would if there were no shadow banking in the economy (Panel b in Figure 1.7). Finally, the decline in the price of capital erodes bankers' net worth and increases their leverage, $s + s^*$, which causes increased asset volatility and forces a further decline in the leverage of shadow banking. Overall, our calibrated model shows that shadow banking increases the price volatility of capital $q\kappa^q$ by 31% on average.

Shadow Banking: Innocent or Not? The answer is "Yes and No" in our model. The answer is "Yes" because all shadow banks hold the same type of physical capital as regular banks. The overall investment quality in the economy does not deteriorate because of the appearance of shadow banking. Thus, if we focus on the asset side of the shadow banking system, no one can blame shadow banking for increasing financial instability. Moreover, even if we move to the liability side, a single shadow bank can borrow up to the limit and still cause no harm. The answer is also "No" because when shadow banks expand in economic upturns, bankers fail take into account the negative externality of the asset fire sales that occur in economic downturns.

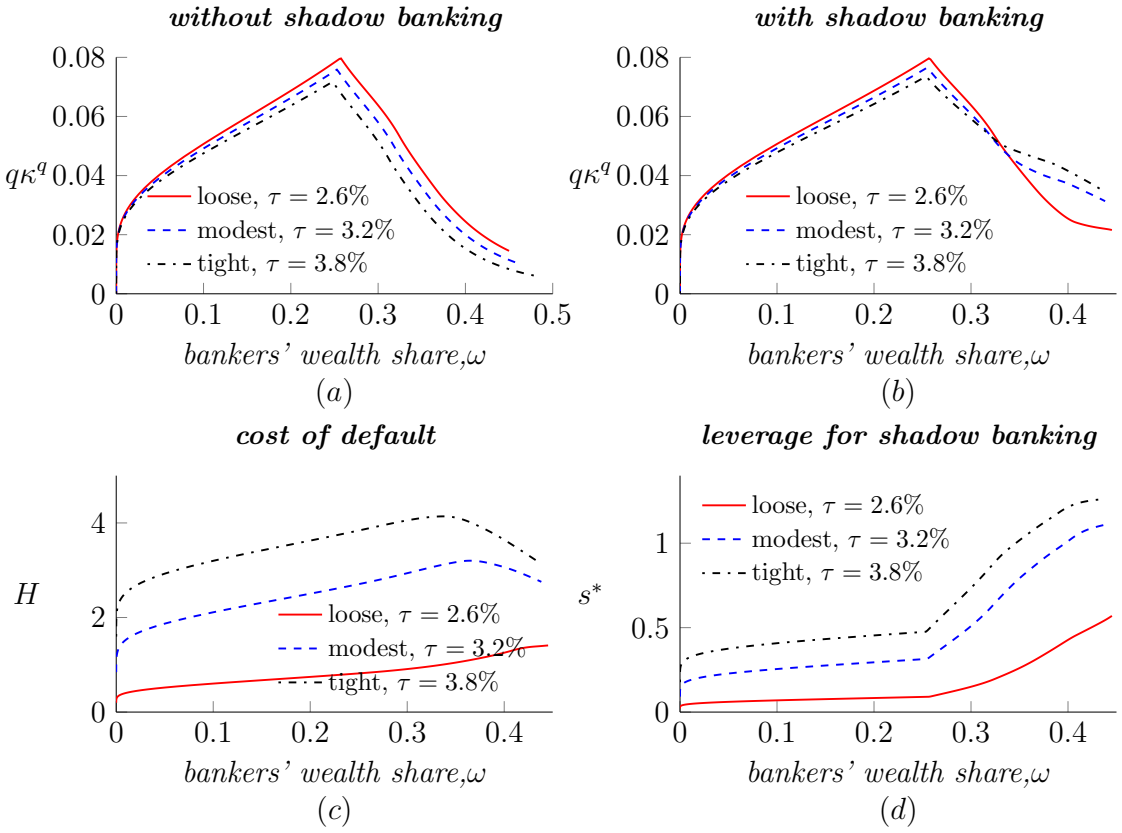


Figure 1.8: Regulatory Paradox
This figure shows the price of capital in the "bad" equilibrium (upper left panel), the price of capital in the "good" equilibrium (upper right panel), the cost of default (lower left panel), and the leverage of shadow banking (lower right panel). The red solid line is for the economy with loose financial regulation ($\tau = 2.6\%$); the blue dashed line for the economy with modest regulation ($\tau = 3.2\%$); the black dash-dot line for the economy with tight regulation ($\tau = 3.8\%$). For other parameter values, see Section 1.3.3

Regulatory Paradox. The conventional wisdom that tough regulation always secures financial stability may not hold when we take shadow banking into account. In the "bad" equilibrium without shadow banking, if the regulatory authority tightens regulation by raising τ, banks' leverage and the price volatility of capital decline accordingly (Panel *a* in Figure 1.8). However, in the "good" equilibrium where shadow banking plays a crucial role, the economy with tighter regulation actually experiences higher market risk (Panel *b* in Figure 1.8). The intuition is as follows: if regulation becomes tighter, bankers will face higher tax burdens if they default. Thus, tighter regulation comes with a larger cost of default, as equation (4) indicates (Panel c in Figure 1.8). Furthermore, a larger cost of default gives rise to a higher leverage for shadow banking. Thus, the shadow banking sector is larger in economies with more stringent regulation. Since shadow banking adds to financial instability, tough regulation imposed on regular banks can jeopardize financial stability.

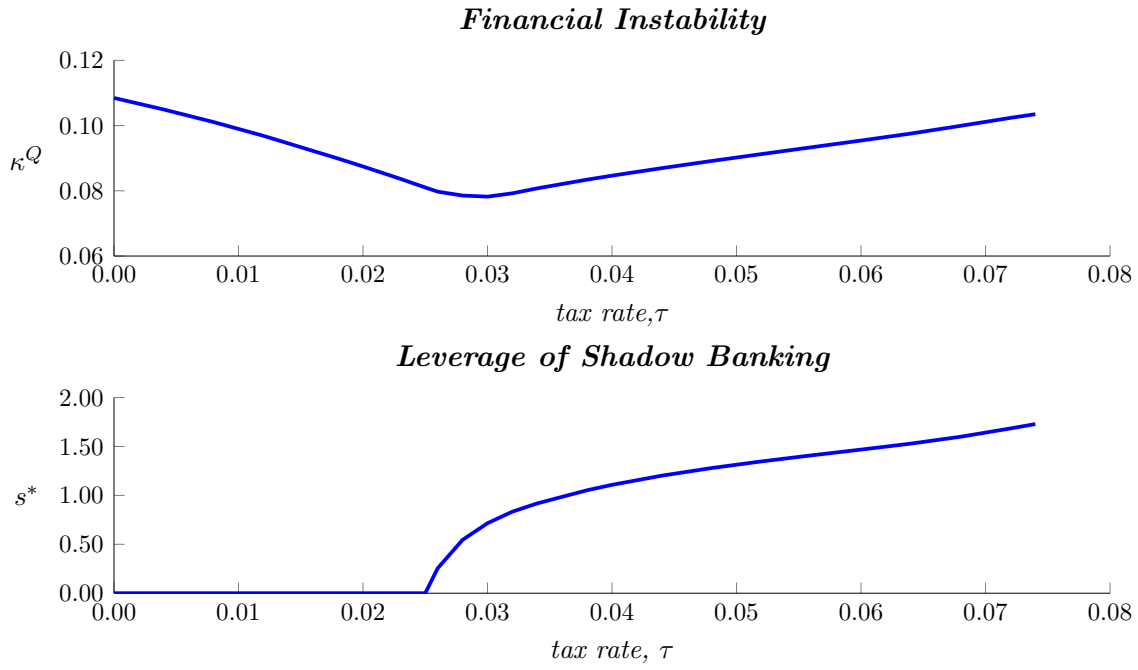


Figure 1.9: Regulatory Smile
This figure shows the investment risk κ^Q (upper panel) and the leverage of shadow banking (lower panel) at the stochastic steady state of economies with different tax rates τ. The stochastic steady state is the state where $\omega\mu^\omega - \lambda\omega\kappa^\omega = 0$. We assume that the "good" equilibrium prevails if it exists. For other parameter values, see Section 1.3.3

Regulatory Smile. The regulatory paradox does not mean that relaxing financial regulation always reduces financial instability. It depends on the relative size of the shadow banking system. Recall the feedback loop discussed earlier. If the regulatory authority lowers the tax rate τ, both the benefit of shadow banking and the cost of default decline, which, in turn, lowers the maximum leverage of shadow banking and further reduces the cost of default. The feedback loop can be so significant that the shadow banking system simply does not emerge. In our numerical example, when τ declines from 3% to 2.5% (lower panel in Figure 1.9), the shadow banking system disappears. In the regime where the level of financial regulation is lenient enough to eliminate the shadow banking system, the conventional wisdom that tightening regulation secures financial stability still holds.

Our model emphasizes the non-monotonic relationship between financial instability and financial regulation in the presence of shadow banking. The instability-regulation relationship is actually U-shaped: when regulations are relaxed enough, the size of shadow banking is small and financial instability diminishes with decreased regulatory stringency; when regulation is so tight that a large shadow banking system emerges, the reverse is true. This is how our framework reconciles and accommodates the two "contradictory" schools of thought on financial regulation.

1.4 Welfare and Policy Implications

In this section, we analyze the welfare and policy implications of our framework. First, we characterize the optimal tax rate that maximizes agents' welfare in the presence of shadow banking. Second, we find that counter-cyclical regulation can generally improve financial stability and welfare because such counter-cyclical regulation alleviates the risk of the asset fire sales between shadow banks and regular banks. Third,

we show that our regulatory smile result still holds when the price control analyzed in the baseline model is replaced by a quantity control (e.g., capital requirement constraint). Lastly, we extend our baseline model to consider the impact of monetary policy on shadow banking.

1.4.1 Welfare Implications of the Baseline Model

To avoid the problem of welfare aggregation, we reinterpret our model as one that consists of a representative banker and a representative household. Let K_0 denote the total capital stock in period 0. Note that the banker's wealth share ω_0 is exactly the fraction of capital goods that she owns. Thus, her net worth in period 0 is $\omega_0 K_0 q_0$, and the net worth of the representative household is $\big(1-\omega_0\big)K_0 q_0$. Without loss of generality, we assume that $K_0 = 1$. Hence, the welfare pair of the representative household and banker is $\big((1-\omega_0)q_0, \ln(\omega_0 q_0)/\,\rho + h_0\big)$. We will focus on the welfare of the representative banker because the household's welfare does not rely directly on two critical variables in policy discussions: economic growth and financial instability.

In the following welfare analysis, we first concentrate on the "bad" equilibrium (red dashed lines in Figure 1.10) without shadow banking and consider the balance between economic growth and financial instability. Second, we proceed to the "good" equilibrium (blue solid lines in Figure 1.10) where shadow banking plays a crucial role, and emphasize that a model can mislead welfare analysis if it does not incorporate shadow banking.

Financial regulation can improve the banker's welfare in the "bad" equilibrium as the red dashed lines in Figure 1.10 show. The underlying mechanism is that as the tax rate τ increases from zero, the welfare benefit brought about by the decline in the volatility of the banker's wealth (Panel b in Figure 1.11) outweighs the welfare cost caused by the drop in the growth of banker's wealth (Panel a in Figure 1.11). The

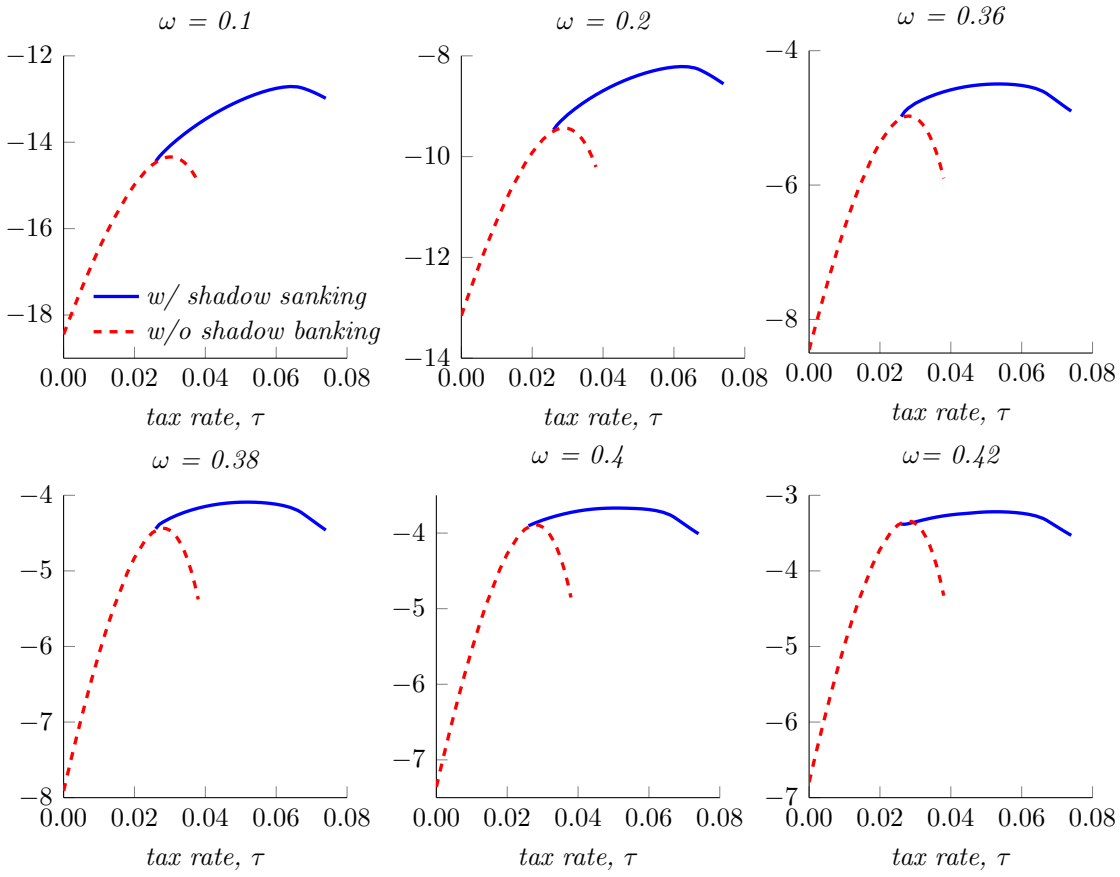


Figure 1.10: Welfare
This figure shows the level of the banker's continuation value $\ln(\omega q)/\rho + h$ in 6 different states ($\omega = 0.1$ in upper left panel, $\omega = 0.2$ in upper middle panel, $\omega = 0.36$ in the upper right panel, $\omega = 0.38$ in the lower left panel, $\omega = 0.4$ in the lower middle panel, and $\omega = 0.42$ in the lower right panel), given different tax rates. For parameter values other than τ, see Section 1.3.3.

unregulated competitive equilibrium ($\tau = 0$) is sub-optimal for the banker because of the pecuniary externality that she does not internalize the negative impact of her leverage choice on endogenous risk κ_t^q. This result essentially justifies the legitimacy of financial regulation in our model.

We now move to the "good" equilibrium and discuss the optimal tax rate in light of shadow banking. In contrast to the "bad" equilibrium, the rise in tax rate τ increases both the growth and the volatility of the banker's wealth because *i*) the shadow banking sector expands as regulation tightens, and *ii*) the growth of shadow banking

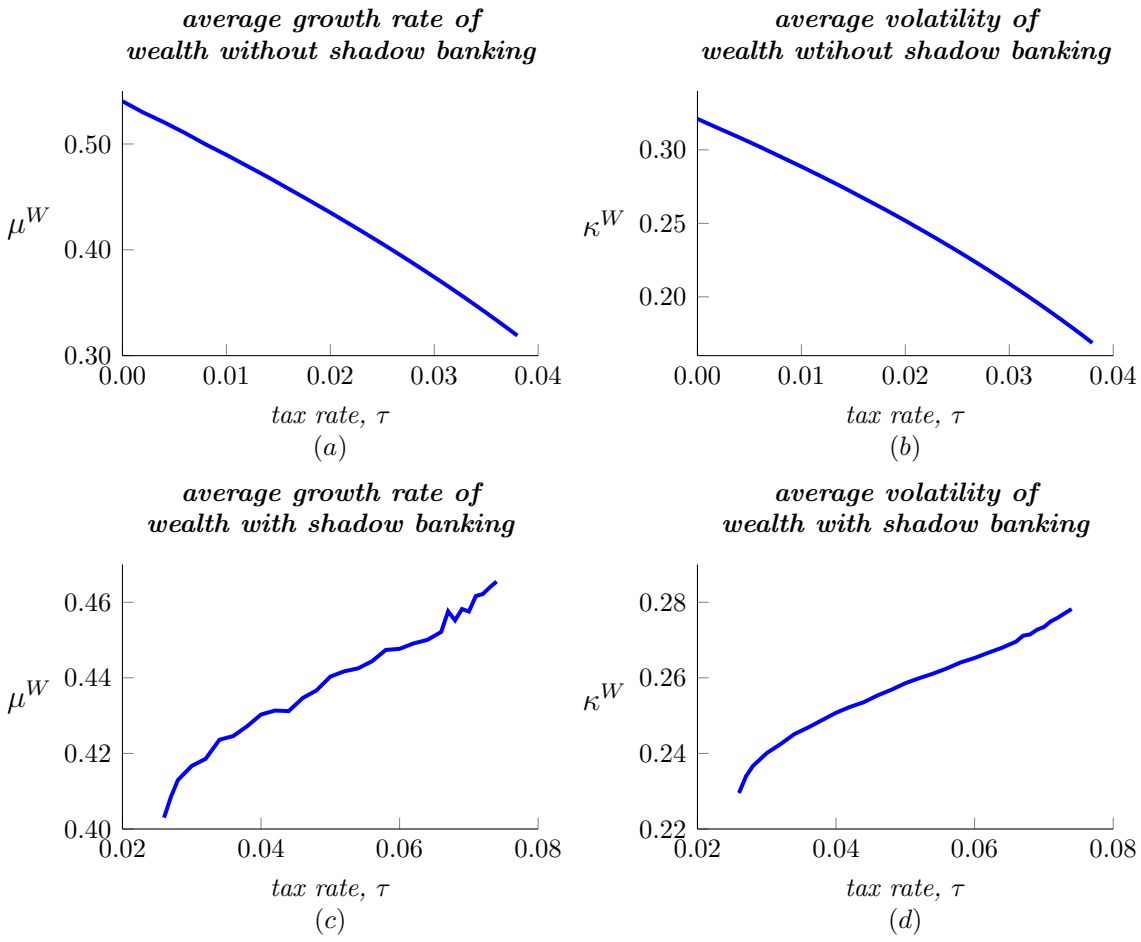


Figure 1.11: Welfare
This figure shows how the average growth rate (left panels) and the average volatility (right panels) of the representative banker's wealth change as the tax rate changes in economies without shadow banking (upper panels) and with shadow banking (lower panels). We use the stationary distribution to calculate moments. For parameter values other than τ, see Section 1.3.3.

more than offsets the negative effect of regulation. In our numerical example, the growth of shadow banking contributes to the banker's welfare because, as the shadow banking sector begins to grow, the growth benefit dominates the cost of increased risk. However, if regulation is too stringent, further tightening will heighten financial instability, the negative effect of which actually dominates the benefit of economic growth. Therefore, our baseline model indicates the optimal level of the tax rate with the impact of shadow banking taken into account.

Figure 1.10 shows that the optimal tax rate in the "good" equilibrium differs from that in the "bad" one. Although extremely tight regulation hurts the banker's welfare in both "good" and "bad" equilibria, the underlying reason is different for each of the "good" and "bad" cases. In the "bad" equilibrium, tightening regulation has the negative effect of slowing economic growth (Panel a in Figure 1.11). However, in the "good" equilibrium, raising tax rate τ increases financial instability (Panel d in Figure 1.11). Therefore, a model that omits shadow banking will completely misguide the policy-making process if the economy has a large or fast-growing shadow banking sector.

1.4.2 Counter-Cyclical Regulation

In this section, we substitute the constant tax rate regulation (Assumption 7) with a counter-cyclical regulation specified by the following assumption.

Assumption 8 *In period t, the tax rate* τ_t *equals* $\min\{\tau(\omega_t), \tau(\omega_t)s_t\}$. $\tau(\omega)$ *is defined by* $\left(\underline{\tau}\left(\tilde{\omega}-\omega\right)+\bar{\tau}\omega\right)/\tilde{\omega}$, *where* $\underline{\tau}$, $\bar{\tau}$, *and* $\tilde{\omega}$ *are constants,* $\underline{\tau}<\bar{\tau}$, *and* $\tilde{\omega}$ *is larger than* $\bar{\omega}$.

The interpretation of Assumption 8 is that the regulatory authority alleviates the tax burden on the regular banking sector in recessions and discourage the bankers' use of leverage in economic booms. We use the same algorithm to solve for the equilibrium of the modified model.

Counter-cyclical tax rate regulation outperforms constant regulation in a number of aspects, as we illustrate in Figure 1.12. The price of capital is higher with the counter-cyclical regulation than it is in the baseline model (Panel a in Figure 1.12) because bankers can assume higher leverage in economic downturns due to the low tax rate on leverage (Panel b in Figure 1.12). For the same reason, it is easier

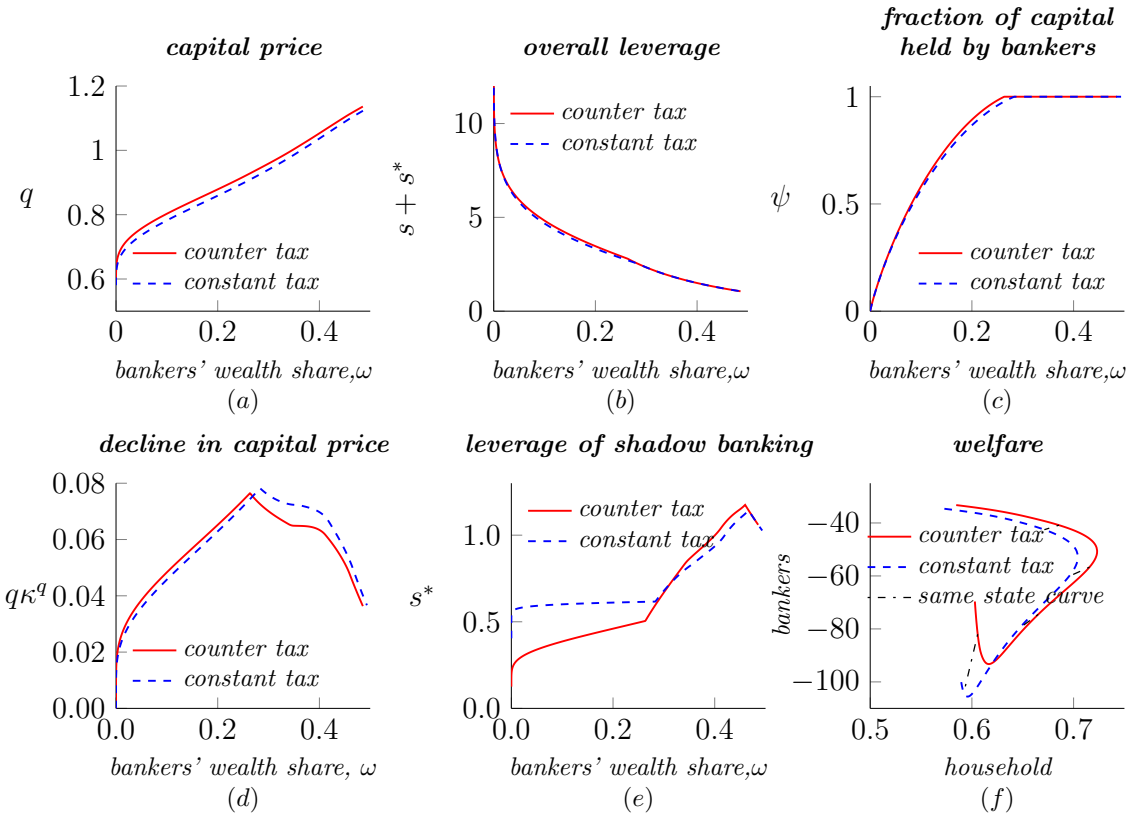


Figure 1.12: Counter-Cyclical versus Constant Regulation.
This figure compares the "good" equilibrium of an economy with a counter-cyclical tax rate (red solid line) and that of an economy with a constant tax rate (blue dashed line) based on seven endogenous variables in the equilibria of two economies: the price of capital (upper left panel), overall leverage (upper middle panel), fraction of capital held by bankers (upper right panel), decline in the price of capital (lower left panel), leverage of shadow banking (lower middle panel), and continuation values of bankers and households (lower right panel). The economy with the counter-cyclical policy has an average tax rate 0.0445, which is calculated based on the stationary distribution, and the economy with the constant tax rate policy has a tax rate 0.0445. The black dash-dot lines link the household-banker conitinuation value pairs $\big((1-\omega)q, \ln(\omega q) + h\big)$ at the same states of the two economies. For other parameter values, see Section 1.3.3.

for bankers to seize all physical capital in the modified model (Panel c in Figure 1.12). In addition, the counter-cyclical regulation enhances the financial stability of an economy (Panel d in Figure 1.12): when shadow banks divest assets in fire sales during economic downturns, the counter-cyclical regulation implies that regular banks are more willing to purchase assets than they are when they face the constant tax rate. Therefore, the counter-cyclical regulation mitigates the impact of asset fire sales on the price of capital. Although the counter-cyclical regulation alleviates financial instability, it makes the leverage of shadow banking more volatile because bankers face a higher cost of default in economic upturns than they face in economic downturns, and thus the leverage of shadow banking is much higher in upturns than it is in downturns (Panel e in Figure 1.12). Given the foregoing discussion, it is readily evident that counter-cyclical tax rate regulation improves the welfare of both bankers and households (Panel f in Figure 1.12).

The "regulatory paradox" result still holds in the model with the counter-cyclical policy, although the financial market is more stable. The left panel of Figure 1.13 shows that counter-cyclical regulation can substantially enhance financial stability when the average tax is high. This is true despite of the fact that the volume of reintermediation is larger in the economy with counter-cyclical regulation (right panel in Figure 1.13). This is because regular banks face declining borrowing costs when they need to raise funds to acquire assets dumped by shadow banks in economic downturns. It is this property of counter-cyclical regulation that substantially lowers the financial instability of an economy.

1.4.3 Quantity Control

In this section, we investigate a modified model, in which the regular banking sector is subject to a quantity control rather than the price control of the baseline model.

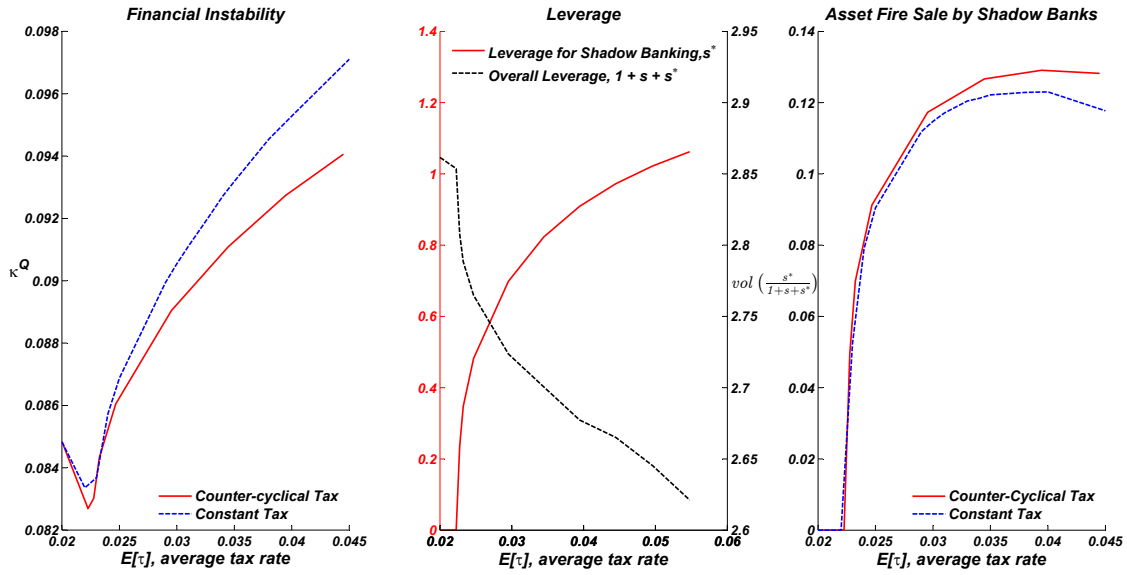


Figure 1.13: Financial Instability and Counter-Cyclical Regulation
This figure shows how the average volatility (red solid line, left panel), the average leverage of shadow banking (red solid line, middle panel), the average overall leverage (black dashed line, middle panel), and the volatility of the securitization ratio (red solid line, right panel) change with the average tax rate in the "good" equilibrium of an economy with a counter-cyclical tax rate policy. All moments are based on the stationary distribution. For comparison, left and right panels show how the average volatility and the volatility of the securitization ratio changes, respectively, as the tax rate moves in an otherwise fixed economy. For parameter values, see both Section 1.3.3 and Figure 1.12.

In particular, we consider the capital-requirement constraint that commercial banks in the real world often face, and this constraint essentially imposes an upper bound $\bar{s}$ for a regular bank's liability-to-equity ratio.

Given a quantity-oriented capital-requirement constraint, the expected lifetime utility maximization problem for a banker who can access shadow banking differs from that in the baseline model in two respects. First, the banker faces a capital-requirement constraint $S_t \leq \bar{s}W_t$. Second, there is neither a tax nor a lump-sum subsidy in the dynamic budget constraint

$$dW_t = \left(W_{t-}R_{t-} + \left(S_{t-} + S^*_{t-}\right)\left(R_{t-} - r_{t-}\right) - c_{t-}\right)dt - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa^Q_t dN_t.$$

There are two similar changes for the maximization problem of bankers who cannot access shadow banking. We use the same numerical procedure to solve for the Markov equilibria of the modified model. The choice of parameter values follows: $\rho = 4\%$, $\chi = 1\%$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, and $\bar{s} = 4$.

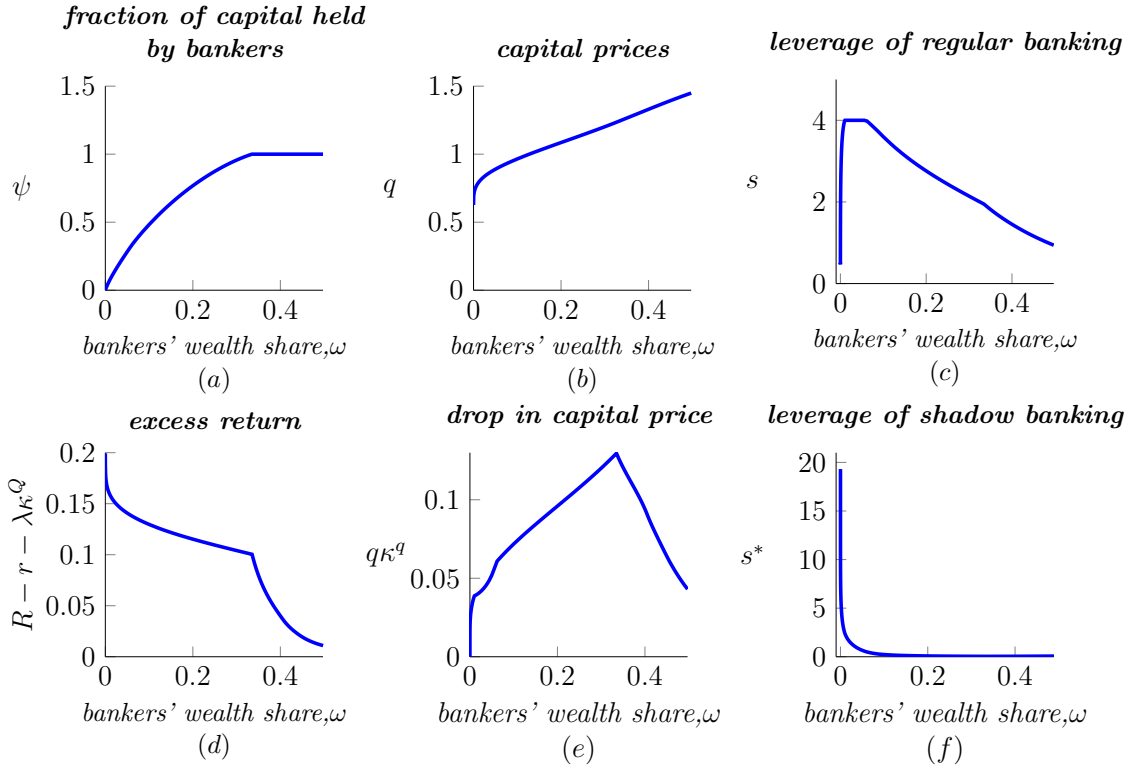


Figure 1.14: $\psi, q, (s + s^*), R - r - \tau, q\kappa^q$, and $\left(1 + s + s^*\right)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in "good" equilibrium of the modified model with households of Epstein-Zin preference. For the choice of parameter values, see Section 1.4.3.

We first focus on the dynamics of endogenous variables and then move to the regulatory smile result of this quantity-control model. A number of endogenous variables, such as the price of capital, the leverage of regular banking, the excess return of running regular banks, and the price volatility of capital, have dynamics similar to the baseline model (Panels a-e in Figure 1.14). The leverage dynamics of shadow banking change drastically as the price control is replaced by the quantity control. The different dynamics result from the fact that when bankers' share of wealth is small, the excess return is high, and the incentives to build up leverage are strong. In

these states of high excess return and leverage buildup, if a banker defaults, then she can raise funds only through regular banking, which allows for only comparatively low leverage. Therefore, when bankers' share of wealth is small, the cost of default is extremely large, and the leverage of shadow banking is very high. This property is absent in the baseline model because bankers do not face a binding leverage constraint for their regular banking. Thus, in the baseline model, both the cost of default and the leverage of shadow banking in economic downturns are low.

Constant quantity control is less appealing than price control because it prevents productive bankers from leveraging when the entire economy needs them to do so. When bankers' wealth share is small, the aggregate productivity could be as low as households' productivity if the capital-requirement constraint is binding for regular banks and shadow banking is not available. We can see that micro-prudential policies such as a capital-requirement constraint may not benefit the overall economy, even though it capably contains the credit risk of individual banks.

Figure 1.15 shows that the regulatory smile result of our baseline model continues to hold when regular banks face quantity control. If financial regulation is very lenient, the leverage of shadow banking is low and financial instability is high. As financial regulation tightens (i.e., the maximum leverage of regular banking $\bar{s}$ declines), financial instability initially diminishes. However, if the regulation is so tight that the shadow banking sector becomes sizable, tighter regulation causes higher financial instability.

1.4.4 Monetary Policy and Shadow Banking

Although this chapter is primarily about financial regulation, in this section we digress and discuss how shadow banking responds to certain changes in monetary policy. We relate our model to recent work by Drechsler et al. (2014), which associates the

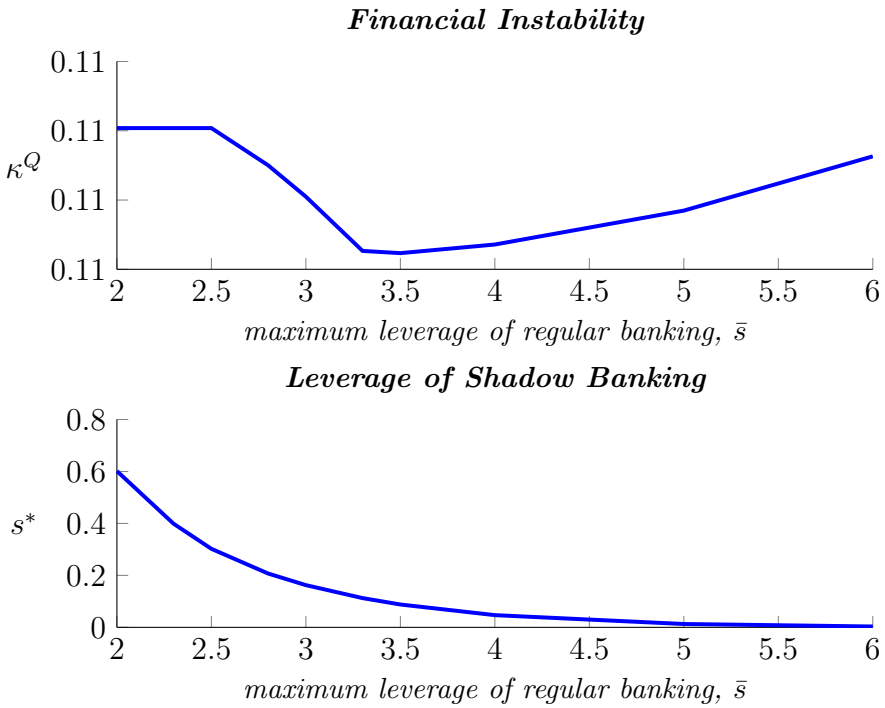


Figure 1.15: This figure shows the investment risk κ^Q (upper panel) and the leverage for shadow banking (lower panel) at the stochastic steady state of different economies with different capital-requirement constraints in the "good" equilibrium of the modified model. The stochastic steady state is the state where $\omega\mu^\omega - \lambda\omega\kappa^\omega = 0$. For the choice of parameter values, see Section 1.4.3.

nominal interest rate and banks' reserve requirement to the use of bank leverage. We will show that their setup has the same effect as our model's tax on regular banking.

With reference to Drechsler et al. (2014), we next interpret the tax on regular banking as the reserve-requirement constraint and associate the tax to the nominal interest rate. Consider a regular bank that issues debt worth S_{t-}. Due to the reserve requirement, the bank must hold reserves worth νS_{t-}. If we interpret reserves as the numéraire, the rate of return for holding reserves in real terms is the negative of the inflation rate because the real value of reserves is the inverse of the price level of

consumption goods.[7] Given the reserve requirement, the dynamic budget constraint of the banker becomes

$$\begin{aligned} dW_t &= \left(W_{t-}R_{t-} + S_{t-}\left(R_{t-} - r_{t-}\right) - \nu S_{t-}\left(i_{t-}dt + r_{t-}dt\right) + S^*_{t-}\left(R_{t-} - \tilde{r}_{t-}\right)\right)dt \\ &- c_{t-}dt - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa^Q_t dN_t, \end{aligned}$$

where i_{t-} is the inflation rate. The tax rate τ_t in the baseline model is equivalent to $\nu(i_t + r_t)$, the product of the reserve requirement parameter ν and the nominal interest rate $i_t + r_t$. By adding the specification that the central bank redistributes its revenue back to bankers in proportion to their net worth, it then becomes evident that tight monetary policy has the same effect as tight financial regulation in the baseline model.

1.5 Robustness

To identify what specification is critical for our main results, we vary the setup of the baseline model along two dimensions: the cost of default and the preference of households. We find that it is crucial to have the cost of default depend on financial regulation in the baseline model and that households' preferences are not essential for our main results.

[7] Given that each unit of consumption goods is equivalent to p_t units of reserves, the inflation rate is dp_t/p_t. The rate of return for holding reserves in real terms is

$$\frac{d\left(\frac{1}{p_t}\right)}{\frac{1}{p_t}} = \frac{-\frac{dp_t}{p_t^2}}{\frac{1}{p_t}} = -\frac{dp_t}{p_t}.$$

1.5.1 Exogenous Leverage Constraint for Shadow Banking

For "regulatory paradox" result to obtain, the maximum leverage of shadow banking must be dependent on financial regulation. To demonstrate this, we characterize a variant of the baseline model where the endogenous cost of default $\{H_t\}_{t=0}^{\infty}$ is replaced by a constant $\bar{H}$.[8] To characterize the equilibrium of the modified model, we need to solve only a single delayed differential equation with respect to $q(\omega)$.

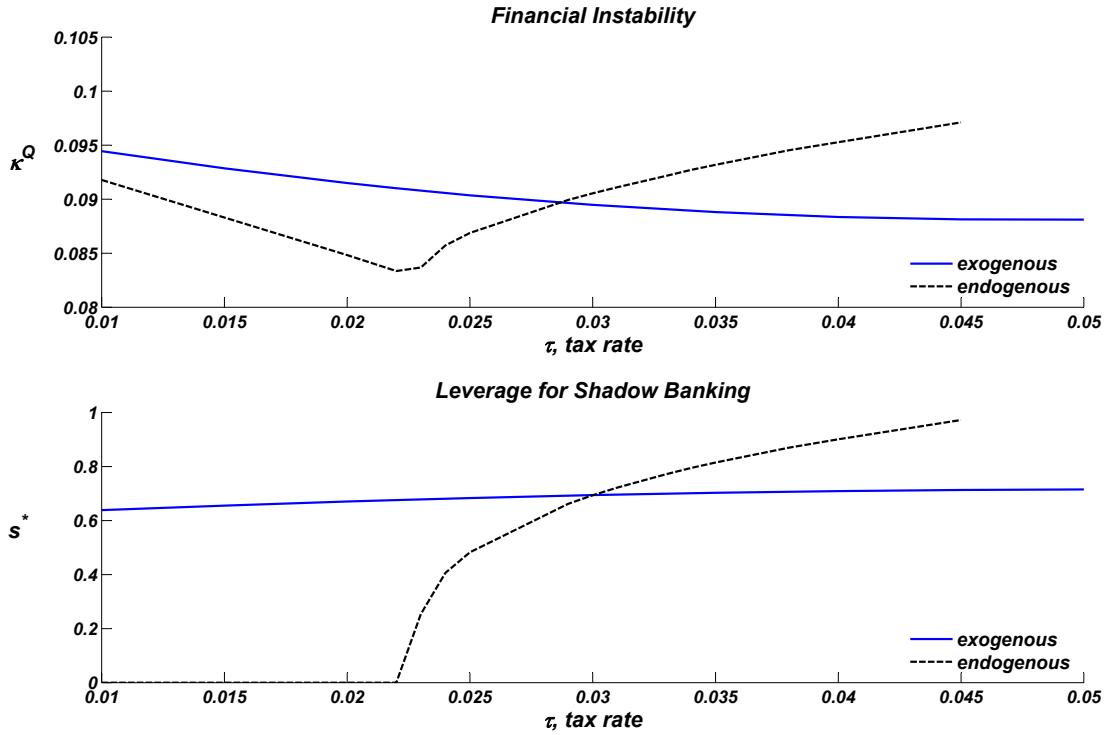


Figure 1.16: Financial Instability and Financial Regulation.
This figure shows how the average volatility (the blue solid line in the upper panel) and the average leverage for shadow banking (the blue solid line in the lower panel) changes with the tax rate in economies where bankers face exogenous leverage constraint for shadow banking. For comparison, two panels also display their counterparts in the baseline model (the black dashed line). The exogenous cost of default H equals 2.7709, which is the average cost of default in the calibrated model in Subsection 1.3.3. For other parameter values, see Section 1.3.3.

Figure 1.19 in Appendix 1.11 shows that some of the basic results of our baseline model are preserved in the modified model. For instance, the leverage of shadow

[8] In Chapter 2, I construct a special scheme to deter bankers from defaulting on their shadow bank obligations such that the cost of default is a constant.

banking is still pro-cyclical. Recall that the maximum leverage of shadow banking depends on the cost of default H, the asset volatility κ^Q, and the leverage of regular banking $\tilde{s}$.

$$\bar{s}^*_{t-} = (1 - \exp(-\rho \bar{H}))(\frac{1}{\kappa^Q_t} - (1 + \tilde{s}_{t-})).$$

Although the cost of default is constant in the modified model, the leverage of regular banking is counter-cyclical and asset volatility is low in upturns. Hence, the pro-cyclicality of shadow banking still holds.

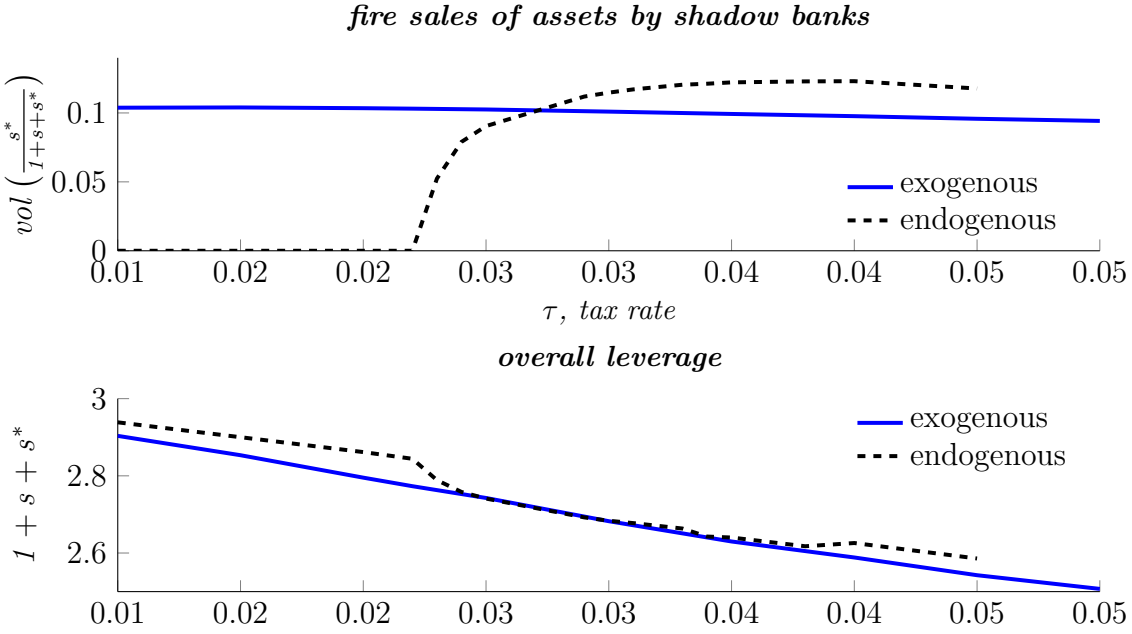


Figure 1.17: Asset Fire Sales and Financial Regulation.
This figure shows how the volatility of the securitization ratio $s^*/(1+s+s^*)$ (blue solid line in the upper panel) and the bankers' average overall leverage (blue solid line in the lower panel) changes with the tax rate in economies where bankers face an exogenous leverage constraint for shadow banking. For comparison, both panels also display their counterparts in the baseline model (black dashed line). The exogenous cost of default H equals 2.7709, which is the average cost of default in the calibrated model in Section 1.3.3. For other parameter values, see Section 1.3.3.

The "regulatory paradox" result does not hold in the modified model, as shown in the upper panel of Figure 1.16. Two facts drive this outcome. First, the size of the shadow banking sector does not change substantially with financial regulation (lower panel of

Figure 1.16). When the regulatory authority raises the tax on regular banking, there are not many banking activities that migrate to the shadow banking sector. Second, when the level of financial regulation varies, the magnitude of the reintermediation does not change as significantly as it does in the baseline model (upper panel of Figure 1.17). Since the amount of shadow banking and the degree of reintermediation do not change significantly with financial regulation in the modified model, neither do we observe any significant change in financial stability caused by shadow banking. The primary reason that tighter regulation leads to reduced financial instability is simply because the overall leverage of the economy declines as financial regulation tightens (lower panel of Figure 1.17).

1.5.2 Households with Epstein-Zin Preference

In this section, we investigate whether our main results are robust to the preference specification of households. In particular, we modify the baseline model so that households have Epstein-Zin preferences with the time discount rate ρ, the relative risk-aversion coefficient γ, and the elasticity of intertemporal substitution b^{-1}. To simplify the characterization of the model, we modify Assumption 5 such that bankers become households at rate χ.

In the modified model, a household chooses $\{c_t^h, S_t^h, n_t, t \geq 0\}$ to maximize

$$U_0^h = E_0 \left[\int_0^\infty f\left(c_s^h, U_s^h\right) ds \right],$$

where

$$f\left(c^h, U^h\right) = \frac{1}{1-b} \left\{ \frac{\rho \left(c^h\right)^{1-b}}{\left(\left(1-\gamma\right) U^h\right)^{\frac{\gamma-b}{1-\gamma}}} - \rho \left(1-\gamma\right) U^h \right\}$$

and

$$U_t^h = E_t \left[\int_t^\infty f\left(c_s^h, U_s^h\right) ds \right], \text{ for } t > 0,$$

subject to the dynamic budget constraint

$$dW_t^h = \left(W_{t-}^h r_{t-} + S_{t-}^h \left(R_{t-}^h - r_{t-}\right) + n_{t-}\left(\tilde{r}_{t-} - r_{t-}\right) - c_{t-}^h\right) dt - S_{t-}^h \kappa_t^Q dN_t.$$

In solving for the equilibrium of the modified model, we need to characterize the optimal consumption and portfolio choices of households. To do so, we conjecture that the continuation value of a household with net worth W_t^h has the following functional form

$$V_t^h = V\left(\zeta_t, W_t^h\right) \equiv \frac{\left(\zeta_t W_t^h\right)^{1-\gamma}}{1-\gamma},$$

where ζ_t follows

$$d\zeta_t = \zeta_{t-}\mu_{t-}^{\zeta} dt - \zeta_{t-}\kappa_t^{\zeta} dN_t.$$

As in Di Tella (2012), ζ_t is interpreted as the continuation value multiplier of a household's net worth since the function $V\left(\zeta_t, W_t^h\right)$ is homogeneous of degree $1-\gamma$ with respect to W_t^h. Given our conjecture, the Hamilton-Jacobi-Bellman equation of the household's dynamic programming problem is

$$0 = \max_{c^h, S^h} \left\{ f\left(c_t^h, V_t^h\right) + \mathcal{D}^{c^h, S^h} V\left(\zeta_t, W_t^h\right) \right\}, \tag{1.30}$$

where

$$\begin{aligned} \mathcal{D}^{c^h, S^h} V\left(\zeta_t, W_t^h\right) &= \left(W_{t-}^h r_{t-} + S_{t-}^h \left(R_{t-}^h - r_{t-}\right) + n_{t-}(\tilde{r}_{t-} - r_{t-}) - c_{t-}^h\right) \zeta_{t-}^{1-\gamma} \left(W_{t-}^h\right)^{-\gamma} \\ +\mu_{t-}^{\zeta}\left(\zeta_{t-} W_{t-}^h\right)^{1-\gamma} &+ \lambda\left(\frac{\left(\left(\zeta_{t-} - \zeta_{t-}\kappa_t^{\zeta}\right)\left(W_{t-}^h - S_{t-}^h \kappa_t^Q\right)\right)^{1-\gamma}}{1-\gamma} - \frac{\left(\zeta_{t-} W_{t-}^h\right)^{1-\gamma}}{1-\gamma} \right). \end{aligned}$$

We summarize the key results of the problem in the following proposition.

Proposition 7 *Each household's optimal consumption weight* $\left\{\hat{c}_t^h, t \geq 0\right\}$*, optimal portfolio weight* $\{s_t^h, n_t\}_{t=0}^{\infty}$*, and the process* $\{\zeta_t\}_{t=0}^{\infty}$ *satisfy*

$$\left(\hat{c}_t^h\right)^b = \frac{\rho}{\zeta_t^{1-b}}, \tag{1.31}$$

$$R_{t-}^h - r_{t-} \leq \frac{\lambda \kappa_t^Q}{\left(1 - s_{t-}^h \kappa_t^Q\right)^\gamma}\left(1 - \kappa_t^\zeta\right)^{1-\gamma}, \;\; = \;\; \textit{if } s_{t-}^h > 0, \tag{1.32}$$

$$\tilde{r}_{t-} \leq r_{t-}, \; = \;\; \textit{if } n_{t-} > 0, \tag{1.33}$$

$$\begin{aligned} 0 \;\; &= \;\; \frac{\rho\left(\hat{c}_{t-}^h\right)^{1-b}}{(1-b)\zeta_{t-}^{1-b}} - \frac{\rho}{1-b} + r_{t-} + s_{t-}^h\left(R_{t-}^h - r_{t-}\right) - \hat{c}_{t-}^h + \mu_{t-}^\zeta \\ &+ \;\; \lambda\left(\frac{\left(\left(1-\kappa_t^\zeta\right)\left(1 - s_{t-}^h\kappa_t^Q\right)\right)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma}\right), \end{aligned} \tag{1.34}$$

where $\hat{c}_t^h = c_t^h / W_t^h$ *and* $s_t^h = S_t^h / W_t^h$*.*

Two market-clearing conditions in the modified model differ from their counterparts in the baseline model. First, the risk-free rate, which is constant in the baseline model, is jointly determined by the optimal portfolio choices of both bankers and households (equations 1.32 and 1.33) and the dynamics of the continuation value multiplier $\{\zeta_t\}_{t=0}^{\infty}$ (equation 2.14). Second, the market for consumption goods does not clear automatically in the modified model since households are risk-averse. Given households' optimal consumption choice condition (2.12), we have the market-clearing condition for consumption goods in period t,

$$a\psi_t + a^h\left(1 - \psi_t\right) - g(\iota_t) = \left(\rho\omega_t + \rho^{\frac{1}{b}}\zeta_t^{\frac{b-1}{b}}\left(1 - \omega_t\right)\right) q_t.$$

As in our previous analyses, we focus on the Markov equilibrium of the modified model where shadow banking exists. To characterize the equilibrium, we need to

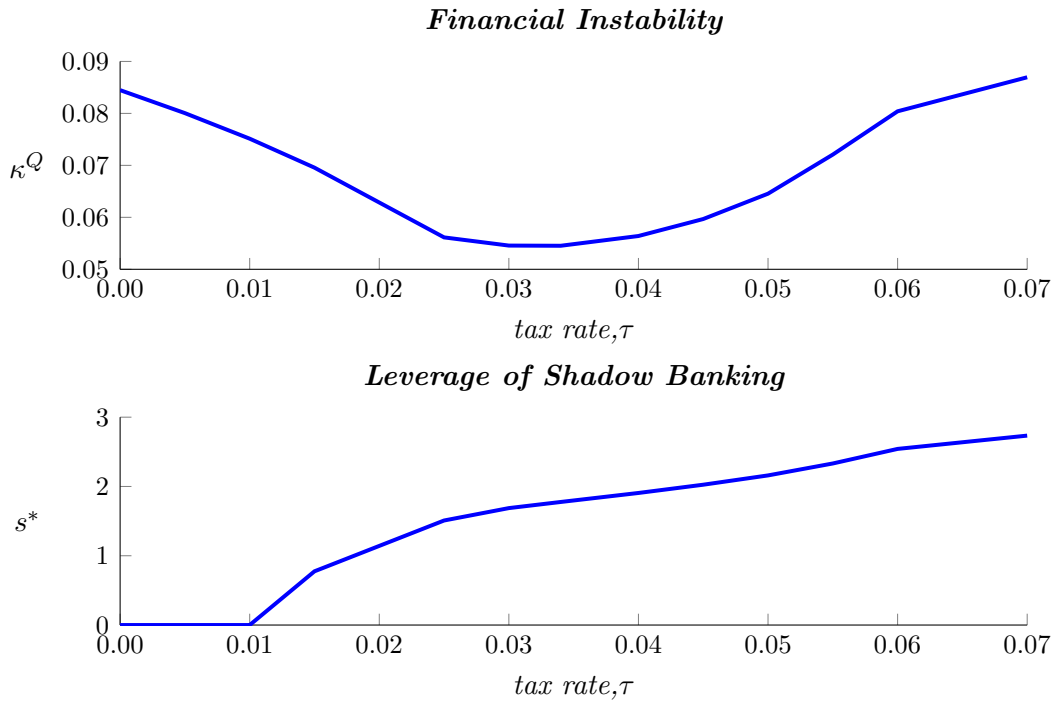


Figure 1.18: This figure shows the investment risk κ^Q (upper panel) and the leverage for shadow banking (lower panel) at the stochastic steady state of different economies with different tax rates τ in the modified model with households of Epstein-Zin preference. The stochastic steady state is the state where $\omega\mu^\omega - \lambda\omega\kappa^\omega = 0$. We assume that the "good" equilibrium prevails if it exists. For the choice of parameter values, see Section 1.5.2.

solve a delay differential equation with respect to $\zeta(\omega)$ in addition to differential equations for $q(\omega)$ and $H(\omega)$. Results found in the baseline model survive in the modified model, as the following numerical example shows. The choice of parameter values is that $\rho = 4\%$, $\gamma = 2$, $b = 0.5$, $\chi = 0.1$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, $\tau = 3\%$, and $\xi = 5\%$. Figure 1.20 in Appendix 1.11 shows that the main endogenous variables have dynamic properties similar to those in the baseline model. The regulatory smile result holds in the modified model, as Figure 1.18 shows.

1.6 Final Remarks

This chapter provides a framework for evaluating financial regulatory rules in the modern financial environment where the unregulated shadow banking sector plays a

critical role. Our framework explicitly takes into account the unintended and indirect influence of bank regulation on the shadow banking sector. Thus, our framework offers a more comprehensive toolkit for policy evaluation.

The framework proposed in this chapter could be extended in the following two directions. The first and most straightforward follow-up work is to characterize the social planner's constrained efficient regulatory rule by, say, a process of tax rate $\{\tau_t, t \geq 0\}$. However, this exercise requires a completely new methodology that can characterize the set of *all* competitive equilibria under *all* sorts of regulations. Recall that this chapter focuses only on Markov equilibria with a single state variable.

Second, how creditors respond to the default of a shadow bank could also be endogenized as an equilibrium outcome. One can calibrate this extension by feeding it with the deep parameters found in credit markets.

1.7 Appendix A: Alternative Enforceability Constraint

Recall that the enforceability constraint (1.18) is so tight that creditors of a shadow bank can raise the maximum leverage for shadow banking up to the level that they are not exposed to the risk of the shadow bank's default. In this section, we derive an enforceability constraint that is more lenient than (1.18). Under such an enforceability constraint, we do not have to resort to Assumption 6 so as to characterize the equilibrium of the market for notes (i.e., financial instruments issued by shadow banks).

For purposes of exposition, (s_{t-}, s^*_{t-}) denotes the optimal leverage choice for a banker who does not intend to default if a negative shock hits the economy; $(\tilde{s}_{t-}, \bar{s}^*_{t-})$ denotes

the optimal leverage choice for a banker who intends to default if a negative shock hits.

We first derive the condition under which a banker prefers $(s_{t-}, s^*_{t-}, \mathbf{d}_t = 0)$ over $(\tilde{s}_{t-}, \bar{s}^*_{t-}, \mathbf{d}_t = 1)$, and second demonstrate the condition under which a banker voluntarily bails out her shadow bank in trouble given her portfolio choice (s_{t-}, s^*_{t-}).

The HJB equation listed in Section 1.3.1 yields the first condition:

$$\begin{aligned} & \frac{s_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+s^*_{t-}\left(R_{t-}-r_{t-}\right)}{\rho}+\frac{\lambda}{\rho}\ln\left(1-\left(1+s_{t-}+s^*_{t-}\right)\kappa^Q_t\right)+\lambda h_t \\ \geq & \frac{\tilde{s}_{t-}\left(R_{t-}-r_{t-}-\tau_{t-}\right)+\bar{s}^*_{t-}\left(R_{t-}-r_{t-}\right)}{\rho}+\frac{\lambda}{\rho}\ln\left(1-\left(1+\tilde{s}_{t-}\right)\kappa^Q_t\right)+\lambda\hat{h}_t, \end{aligned}$$

where $h_t = h_{t-}(1-\kappa^h_t)$ and $\hat{h}_t = \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)$. To have a simpler condition, we focus on a special case that the leverage constraint for shadow banking is binding (i.e., $s^*_{t-} = \bar{s}^*_{t-}$). First-order conditions discussed in Section 1.3.1 imply that $s^*_{t-} = \bar{s}^*_{t-}$ and $s_{t-} + \bar{s}^*_{t-} = \tilde{s}^*_{t-}$. In this case, the first condition reduces to

$$\bar{s}^*_{t-} \leq \frac{\rho\lambda H_t}{R_{t-}-r_{t-}-\tau_{t-}}.$$

The second condition is:

$$\frac{1}{\rho}\ln\left(1-\left(1+s_{t-}+\bar{s}^*_{t-}\right)\kappa^Q_t\right)+h_t \geq \frac{1}{\rho}\ln\left(1-\left(1+s_{t-}\right)\kappa^Q_t\right)+\hat{h}_t,$$

which is similar to the enforceability constraint (1.18). The second condition is relatively loose because the enforcement problem becomes less severe when the leverage of a banker's regular bank is $s_{t-}(< \tilde{s}_{t-})$. In the special case $s^*_{t-} = \bar{s}^*_{t-}$, the second condition becomes

$$\bar{s}^*_{t-} \leq (1-\exp(-\rho H_t))(\frac{1}{\kappa^Q_t}-(1+s_{t-})).$$

Next, we argue that given Assumptions 1 - 9 the note contract that clears the credit market for shadow banking satisfies $\tilde{r}_{t-} = r_{t-}$ and

$$\bar{s}^*_{t-} = \min\{(1 - \exp(-\rho H_t))(\frac{1}{\kappa^Q_t} - (1 + s_{t-})), \frac{\rho\lambda H_t}{R_{t-} - r_{t-} - \tau_{t-}}\}. \tag{1.35}$$

First, if $\bar{s}^*_t$ is higher than the right-hand side of condition (1.35), then $\mathbf{d}_t = 1$ in equilibrium, which means a note in period $t-$ is the combination of risk-free debt and equity. However, households cannot hold equity by Assumption 1. Second, if $\bar{s}^*_{t-}$ is lower than the right-hand side of condition (1.35), then the competition among creditors will push up $\bar{s}^*_{t-}$. The no-arbitrage condition implies that $\tilde{r}_{t-} = r_{t-}$.

The enforceability constraint (1.18) is tighter than the alternative enforceability constraint (1.35) because $\bar{s}^*_{t-}$ that satisfies equation (1.18) is less than the right-hand side of equation (1.35).

Figure 1.21 - 1.23 in Appendix 1.11 show that main results demonstrated in Section 2.3 still hold if the maximum leverage of shadow banking in equilibrium is replaced by equation (1.35). Therefore, Assumption 6 is not essential for our results. The choice of parameter values for numerical examples shown by Figure 1.21 - 1.23 is that $\rho = 3\%$, $\chi = 0.15$, $a = 0.225$, $a^h = 0.1$, $\delta = 10\%$, $\phi = 3$, $\lambda = 1$, $\kappa = 4\%$, $\tau = 3\%$, and $\xi = 5\%$.

1.8 Appendix B: Stationary Distribution

We present the condition under which the stationary distribution of ω is nondegenerate.

Proposition 8 *If $\alpha_\pi > -1$, where α_π is a root of the equation*

$$(1+\alpha_\pi)\underline{\mu}^\omega = \lambda\left((1-\underline{\kappa}^\omega)^{-(1+\alpha_\pi)} - 1\right), \tag{1.36}$$

then the stationary density of ω is nondenegerate.

Proof. $\pi(\omega)$ denotes the stationary density of ω. Suppose $\pi(\omega)$ is nondegenerate, then $\pi(\omega)$ satisfies

$$0 = -\frac{d(\pi(\omega)\mu^\omega\omega)}{d\omega} - \lambda\pi(\omega) + \lambda\pi(\hat{\omega}(\omega))\frac{d\hat{\omega}(\omega)}{d\omega},$$

where $\hat{\omega}$ is such that $\omega = \hat{\omega}\left(1-\kappa^{\hat{\omega}}\right)$. The above equation is based on the Kolmogorov Forward Equation. To prove that $\pi(\omega)$ is nondegenerate, we need to verify that the solution of the above equation has a finite integral over $[0, \underline{\omega}]$. Since $\underline{\omega}$ is the upper bound of ω by construction, what we need to show essentially is that the integral of $\pi(\omega)$ between 0 and ε is finite, where ε is a small positive number. Around $\omega = 0$, the solution of the above differential equation can be approximated by $C_\pi\omega^{\alpha_\pi} + o(\omega^{\alpha_\pi})$, where C_π is a positive constant. After plugging this approximation into the above Kolmogorov Forward Equation, we derive (1.36). Hence, if there is a root of (1.36) that is larger than -1, then the integral of $\pi(\omega)$ over $(0, \varepsilon)$ is finite. ■

Algorithm. In addition, we could numerically compute the density function of the stationary distribution of ω based on the above Kolmogorov Forward Equation. We first solve for the differential equation with respect to $\Pi(\omega) = \pi(\omega)\mu^\omega\omega$

$$0 = -\Pi'(\omega) - \frac{\lambda\Pi(\omega)}{\mu^\omega\omega} + \lambda\frac{\Pi(\hat{\omega}(\omega))}{\mu^{\hat{\omega}}\hat{\omega}}\frac{d\hat{\omega}(\omega)}{d\omega},$$

and recover $\pi(\omega)$ from $\pi(\omega) = \Pi(\omega)/(\mu^\omega\omega)$. The algorithm used to solve for the density fuction starts from $\bar{\omega}$ instead of 0. This is because we only need to solve an

ODE when the state variable is around $\hat{\omega}$. To understand this, notice that $\bar{\omega}$ is the upper bound of the state variable, which means there does not exist a state $\hat{\omega}$ such that $\bar{\omega} = \hat{\omega}\left(1 - \kappa^{\hat{\omega}}\right)$. Therefore, $\pi(\hat{\omega}(\omega)) = 0$ when ω is around $\bar{\omega}$. As we proceed from $\bar{\omega}$ towards 0, we can use our calculated results to interpolate the value of Π at any $\hat{\omega}$ since $\hat{\omega}$ has to be larger than ω.

1.9 Appendix C: Proofs

Proof of Lemma 1. Now suppose in period t, the banker's net worth W_t is negative. The law of motion for the banker's net worth is

$$dW_t = \left(W_{t-}R_{t-} + S_{t-}\left(R_{t-} - r_{t-}\right) + S^*_{t-}\left(R_{t-} - r_{t-}\right) - c_{t-}\right)dt - \left(W_{t-}\kappa^Q_t + S_{t-}\kappa^Q_t + S^*_{t-}\kappa^Q_t\right)dN_t$$

Given a fixed time T, we can construct a new measure under which

$$R_{s-} - r_{s-} = \tilde{\lambda}_{t-}\kappa^Q_s$$

for each time s between t and T. Under this new measure,

$$\tilde{E}_t\left[W_T \exp\left(-\int_t^T r_u du\right) + \int_t^T \left(c_s \exp\left(-\int_t^s r_u du\right)\right) ds\right] \leq W_t < 0$$

Suppose the banker retires at the stopping time S with positive net worth W_S. After the banker retires, her net worth evolves as

$$dW_{S+u} = W_{S+u}\left(r_{S+u} - \rho\right) du.$$

Her net worth in period $S+s$ is $W_S \exp\left(\int_S^{S+s} r_u du - \rho s\right)$. It is easy to see that

$$\lim_{s\to\infty} E_S\left[\exp\left(-\int_S^{S+s} r_u du\right) W_{S+s}\right] = 0.$$

Thus, if T is large enough, $E_t\left[W_T \exp\left(-\int_t^T r_u du\right)\right]$ could be arbitrarily small. Since $W_t < 0$, the consumption of the banker must be negative at some point between t and T with a strictly positive probability. Since the banker has logarithm preference, the banker's expected lifetime discounted utility in period t must be negative infinity. Therefore, we show that it is never optimal for the banker to have negative net worth.

■

Proof of Proposition 9. (2.7) holds because of Lemma 1. We proceed to show (2.8) and (2.9).

Given the conjecture that the banker's continuation value function $\ln(W)/\rho + h$, we can establish the following Hamilton-Jacobi-Bellman (HJB) equation

$$\rho(\tfrac{\ln(W_{t-})}{\rho} + h_{t-}) = h_{t-}\mu^h_{t-} + \xi_r\left(\tfrac{\ln(W_t)}{\rho} + \tfrac{r_{t-}-\rho}{\rho} - (\tfrac{\ln(W_{t-})}{\rho} + h_{t-})\right) + \max_{c_{t-},S_{t-},S^*_{t-},\mathbf{d}_t}$$

$$\left\{\begin{array}{c} \ln(c_{t-}) + \frac{1}{\rho W_{t-}}\left(W_{t-}R_{t-} + S_{t-}(R_{t-} - r_{t-} - \tau_{t-}) + S^*_{t-}(R_{t-} - \tilde{r}_{t-}) + \Pi_{t-} - c_{t-}\right) \\ +\lambda(1-\mathbf{d}_t)\left(\frac{1}{\rho}\ln\left(W_{t-} - \left(W_{t-} + S_{t-} + S^*_{t-}\right)\kappa^Q_t\right) + h_{t-}(1-\kappa^h_t)\right) \\ +\lambda\mathbf{d}_t\left(\frac{1}{\rho}\ln\left(W_{t-} - (W_{t-} + S_{t-})\kappa^Q_t\right) + \hat{h}_{t-}(1-\kappa^{\hat{h}}_t)\right) - \lambda(\frac{\ln(W_{t-})}{\rho} + h_{t-}) \end{array}\right\},$$

where $\Pi_t = S_t\tau_t$. (2.8) and (2.9) are first-order conditions with respect to s_{t-} and s^*_{t-}. Furthermore, $\{h_t, t \geq 0\}$ must satisfy

$$\begin{aligned}
\rho h_{t-} &= h_{t-}\mu^h_{t-} + \chi\left(\frac{r-\rho}{\rho} - h_{t-}\right) \\
&\quad + \log(\rho) + \frac{1}{\rho}\left(R_{t-} + s_{t-}\left(R_{t-} - r_{t-}\right) + s^*_{t-}\left(R_{t-} - \tilde{r}_{t-}\right) - \rho\right) \\
&\quad + \lambda\left(1 - \mathbf{d}_t\right)\left(\frac{1}{\rho}\ln\left(1 - \left(1 + s_{t-} + s^*_{t-}\right)\kappa^Q_t\right) + h_{t-}\left(1 - \kappa^h_t\right)\right) \\
&\quad + \lambda\mathbf{d}_t\left(\frac{1}{\rho}\ln\left(1 - \left(1 + s_{t-}\right)\kappa^Q_t\right) + \hat{h}_{t-}\left(1 - \kappa^{\hat{h}}_t\right)\right) - \lambda h_{t-}
\end{aligned}$$

To complete the argument, the transversality condition

$$\lim_{t\to\infty} E\left[\exp\left(-\rho t\right)\left(\ln(W_t)/\,\rho + h_t\right)\right] = 0.$$

also must hold ■

Proof of Lemma 5. W_t denotes $\int_0^1 W^i_t di$. In a Markov equilibrium, (2.2), the optimal choice of bankers, and the market-clearing for notes, and the balanced budget of the regulatory authority imply that

$$\begin{aligned}
dW_t &= W_{t-}\left(\left(R_{t-} + s_{t-}\left(R_{t-} - r_{t-}\right) + s^*_{t-}\left(R_{t-} - r_{t-}\right) - \rho - \chi\right)dt - \left(s_{t-} + s^*_{t-}\right)\kappa^Q_t dN_t\right) \\
&= W_{t-}\left(\left(R_{t-} + s_{t-}\left(R_{t-} - r_{t-}\right) + s^*_{t-}\left(R_{t-} - r_{t-}\right) - \rho - \chi\right)dt - \left(s_{t-} + s^*_{t-}\right)\kappa^Q_t dN_t\right).
\end{aligned}$$

Note bankers retire at the intensity χ. Next, consider the scaling factor $1/\left(q_t K_t\right)$.

$$d\left(q_t K_t\right) = q_{t-}K_{t-}\left(\left(\mu^q_{t-} + \mu^K_{t-}\right)dt - \kappa^Q_t dN_t\right),$$

and

$$d\left(\frac{1}{q_t K_t}\right) = \left(\frac{1}{q_{t-}K_{t-}}\right)\left(-\left(\mu^q_{t-} + \mu^K_{t-}\right)dt + \frac{\kappa^Q_t}{1-\kappa^Q_t}dN_t\right).$$

Then,

$$\begin{aligned} d\omega_t &= \omega_{t-}\left(\mu^{\omega}_{t-}dt - \kappa^{\omega}_t dN_t\right) \\ \text{where } \mu^{\omega}_t &= R_{t-} + s_{t-}\left(R_{t-} - r_{t-}\right) + s^*_{t-}\left(R_{t-} - r_{t-}\right) - \mu^q_{t-} - \mu^K_{t-} - \rho - \chi \\ \text{and } \kappa^{\omega}_t &= \frac{\left(s_{t-} + s^*_{t-}\right)\kappa^Q_t}{1 - \kappa^Q_t}. \end{aligned}$$

■

Proof of Proposition 4. Without loss of generality, we focus a banker with net worth W_t in period t and explicitly express her continuation value in different cases.

We start with the case that the banker is retired. Since logarithmic agents only consumer ρ fraction of their net worth, the growth rate of her net worth is $r_{t+v}-\rho$ in period $t+v$. Hence, the banker's net worth in period $t+u$ will be $W_t \exp\left(\int_0^u r_{t+v} - \rho\ dv\right)$. The banker's continuation value in period t is

$$\begin{aligned} & \int_0^{\infty} \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right) + \int_0^u r_{t+v} - \rho\ dv\right) du \\ = & \frac{\ln\left(W_t\right)}{\rho} + \frac{\ln\left(\rho\right)}{\rho} + \int_0^{\infty} \exp\left(-\rho u\right) \int_0^u r_{t+v} - \rho\ dv\ du \\ = & \frac{\ln\left(W_t\right)}{\rho} + \frac{\ln\left(\rho\right)}{\rho} + \frac{1}{\rho}\int_0^{\infty} \exp(-\rho v)\left(r_{t+v} - \rho\right)\ dv, \end{aligned}$$

which is denoted by $\ln\left(W_t\right)/\,\rho + h^r_t$.

We use the same idea to express the continuation value of a banker who can access shadow banking. Given the banker's optimal portfolio choices $\left(s_{t+u}, s^*_{t+u}\right)$, if she does not retire in period $t+u$, her net worth is

$$W_t \exp\left(\begin{array}{c} \int_0^u \left(R_{t+v} + s_{t+v}\left(R_{t+v} - r_{t+v}\right) + s^*_{t+v}\left(R_{t+v} - r_{t+v}\right) - \rho\right) dv \\ + \int_0^u \ln\left(1 - \left(1 + s_{t+v} + s^*_{t+v}\right)\kappa^Q_{t+v}\right) dN_{t+v} \end{array}\right)$$

Let $t+T$ denote the stopping time that the banker retires. Her continuation value in period t is

$$
\begin{aligned}
& E_t\left[\begin{array}{c}\int_0^T \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right)+\int_0^u R_{t+v}+s_{t+v}\left(R_{t+v}-r_{t+v}\right)+s_{t+v}^*\left(R_{t+v}-r_{t+v}\right)-\rho\ dv\right)du \\ +\int_0^T \exp\left(-\rho u\right)\int_0^u \ln\left(1-\left(1+s_{t+v}+s_{t+v}^*\right)\kappa_{t+v}^Q\right)dN_{t+v}\ du+\exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho}+h_{t+T}^r\right).\end{array}\right] \\
= & \frac{\ln\left(W_t\right)}{\rho}+\frac{\rho\ln\left(\rho\right)}{\rho+\chi} \\
+ & E_t\left[\int_0^\infty \exp\left(-\left(\rho+\chi\right)v\right)\left(\begin{array}{c}R_{t+v}+s_{t+v}\left(R_{t+v}-r_{t+v}\right)+s_{t+v}^*\left(R_{t+v}-r_{t+v}\right)-\rho \\ +\lambda\ln\left(1-\left(1+s_{t+v}+s_{t+v}^*\right)\kappa_{t+v}^Q\right)\end{array}\right)dv\right] \\
+ & E_t\left[\int_0^\infty \chi\exp\left(-\left(\rho+\chi\right)v\right)h_{t+v}^r\ dv\right],
\end{aligned}
$$

which we denote as $\ln\left(W_t\right)/\ \rho+h_t$.

Finally, we consider the case that the banker who cannot use shadow banking but obtain such opportunity at intensity ξ. Let $\hat{s}_{t+u}$ denote her optimal portfolio choices and T_ξ the stopping when the banker obtain the access to shadow banking. Her continuation value in period t is

$$
\begin{aligned}
& E_t\left[\begin{array}{c}\int_0^{\min\left(T,T_\xi\right)} \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right)+\int_0^u R_{t+v}+\hat{s}_{t+v}\left(R_{t+v}-r_{t+v}-\tau_{t+v}\right)+s_{t+v}\tau_{t+v}-\rho\ dv\right)du \\ +\int_0^{\min\left(T,T_\xi\right)} \exp\left(-\rho u\right)\int_0^u \ln\left(1-\left(1+\hat{s}_{t+v}\right)\kappa_{t+v}^Q\right)dN_{t+v}\ du \\ +\exp\left(-T_\xi\rho\right)\left(\frac{\ln\left(W_{t+T_\xi}\right)}{\rho}+h_t\right)\mathbf{1}_{\left(\min\left(T_\xi,T\right)=T_\xi\right)}+\exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho}+h_{t+T}^r\right)\mathbf{1}_{\left(\min\left(T_\xi,T\right)=T\right)}\end{array}\right] \\
= & E_t\left[\begin{array}{c}\int_0^T \exp\left(-\rho u\right)\left(\ln\left(\rho W_t\right)+\int_0^u R_{t+v}+s_{t+v}\left(R_{t+v}-r_{t+v}\right)+s_{t+v}^*\left(R_{t+v}-r_{t+v}\right)-\rho\ dv\right)du \\ +\int_0^T \exp\left(-\rho u\right)\int_0^u \ln\left(1-\left(1+s_{t+v}+s_{t+v}^*\right)\kappa_{t+v}^Q\right)dN_{t+v}\ du\exp\left(-T\rho\right)\left(\frac{\ln(W_{t+T})}{\rho}+h_{t+T}^r\right)\end{array}\right]- \\
& E_t\left[\int_0^{\min\left(T,T_\xi\right)}\exp(-\rho u)\int_0^u\left(\begin{array}{c}\left(\left(s_{t+v}+s_{t+v}^*-\hat{s}_{t+v}\right)\left(R_{t+v}-r_{t+v}-\tau_{t+v}\right)+s_{t+v}^*\tau_{t+v}\right)dv \\ +\left(\ln\left(1-\left(1+s_{t+v}+s_{t+v}^*\right)\kappa_{t+v}^Q\right)-\ln\left(1-\left(1+\hat{s}_{t+v}\right)\kappa_{t+v}^Q\right)\right)dN_{t+v}\end{array}\right)du\right] \\
= & \ln\left(W_t\right)/\ \rho+h_t-H_t,
\end{aligned}
$$

where H equals

$$E_t\left[\int_0^\infty \exp\left(-\left(\rho+\chi+\xi\right)v\right)\left(\begin{array}{c}\left(s_{t+v}+s_{t+v}^*-\hat{s}_{t+v}\right)\left(R_{t+v}-r_{t+v}-\tau_{t+v}\right)+s_{t+v}^*\tau_{t+v}\\ +\lambda\left(\ln\left(1-\left(1+s_{t+v}+s_{t+v}^*\right)\kappa_{t+v}^Q\right)-\ln\left(1-\left(1+\hat{s}_{t+v}\right)\kappa_{t+v}^Q\right)\right)\end{array}\right)dv\right]$$

■

Lemma 3 *H is bounded.*

Proof of Lemma 3. It is easy to see that H is bounded from below by 0. Suppose there are two bankers with the same net worth in period t, banker 1 only accesses shadow banking after a random period since t, and banker 2 can use shadow banking right after t. Since banker 1 is unable to choice certain portfolio choices that are available to banker 2, the continuation of banker 1 cannot be larger than banker 2. Given the continuation value function of bankers with logarithm preference and the fact that banker 1 and 2 have the same net worth, we show that $H \geq 0$, for all $\omega \in (0, \bar{\omega}]$.

Next, we show that H also has an upper bound. Notice that in period u the banker who can access shadow banking can save the tax payment $S_u^*\tau_u$. Since bankers' leverage has an upper bound $\bar{s} = 1/\kappa$, the maximum benefit a banker can get should be $W_u\bar{s}\tau$. We next calculate by how much a banker's continuation could increase if she can get the maximum benefit every period compared to the situation that she cannot access shadow banking. Let g_u and κ_u^W denote the growth rate of the banker who cannot access shadow banking and the drop rate of her net worth after a bad shock. Then, the banker's net worth would grow at rate $g_u + \bar{s}\tau$ if she gets the maximum benefit. The law of motion for her net worth is

$$dW_u = W_u\left(\left(g_u+\bar{s}\tau\right)du - \kappa_u^W dN_u\right).$$

In period t, her net worth would be

$$W_0 \exp\left(\int_0^t \left(g_u + \bar{s}\tau\right) du - \kappa_u^W dN_u\right).$$

Since logarithmic agents would only consume ρ fraction of their wealth, the banker's continuation would incrase by

$$\int_0^\infty \exp\left(-\rho t\right) \int_0^t \bar{s}\tau du dt = \frac{\bar{s}\tau}{\rho^2} < \infty.$$

Hence, $H\left(\omega\right) \leq \left(\bar{s}\tau\right)/\rho^2$, for $\omega \in (0, \bar{\omega}]$. ■

Alternative Proof of Proposition 4. We first establish the differential equation that H_t satisfies

$$\left(\rho + \xi + \chi\right) H\left(\omega\right) = f\left(\omega\right) + \omega\mu^\omega H'\left(\omega\right) + \lambda\left(H\left(\omega\left(1 - \kappa^\omega\right)\right) - H\left(\omega\right)\right),$$

where

$$f\left(\omega\right) = \frac{1}{\rho}\left(\begin{array}{c} \left(s\left(\omega\right) + s^*\left(\omega\right) - \hat{s}\left(\omega\right)\right)\left(R\left(\omega\right) - r(\omega) - \tau\left(\omega\right)\right) + s^*\left(\omega\right)\tau\left(\omega\right) \\ +\lambda\left(\ln\left(1 - \left(1 + s\left(\omega\right) + s^*\left(\omega\right)\right)\kappa^Q\left(\omega\right)\right) - \ln\left(1 - \left(1 + \hat{s}\left(\omega\right)\right)\kappa^Q\left(\omega\right)\right)\right) \end{array}\right).$$

Suppose $H\left(\omega\right)$ solves the above differential equation. We look at

$$H\left(\omega_t\right)\exp\left(-\left(\rho + \xi + \chi\right)t\right).$$

Ito's Lemma implies

$$d\left(H\left(\omega_t\right)\exp\left(-\left(\rho + \xi + \chi\right)t\right)\right) =$$

$$\exp\left(-\left(\rho + \xi + \chi\right)t\right)\left(\left(-H\left(\omega_t\right)\left(\rho + \xi + \chi\right) + H'\left(\omega_t\right)\omega_t\mu_t^\omega\right)dt + \left(H\left(\omega_t\left(1 - \kappa_t^\omega\right)\right) - H\left(\omega_t\right)\right)dN_t\right)$$

Given a fixed time T,

$$H(\omega_t)\exp(-(\rho+\xi+\chi)t) =$$
$$E_t\left(\int_t^T \exp(-(\rho+\xi+\chi)u)\, f(\omega_u)\, du\right) + E_t\left(H(\omega_T)\exp(-(\rho+\xi+\chi)T)\right).$$

Since $f(\cdot)$ is bounded, by the dominated convergence theorem,

$$\lim_{T\to\infty} E_t\left(\int_t^T \exp(-(\rho+\xi+\chi)u)\, f(\omega_u)\, du\right)$$

is well defined. Since we focus on the bounded $H(\cdot)$, $E_t\left(H(\omega_T)\exp(-(\rho+\xi+\chi)T)\right)$ converges to zero as $T\to\infty$. Therefore,

$$H(\omega_t) = E_t\left[\int_t^\infty \exp(-(\rho+\xi+\chi)(u-t))\, f(\omega_u)\, du\right]$$

■

Proof of Theorem 1. We will apply the contraction mapping theorem to show the uniqueness of the solution $H(\omega)=0$. First, we define a complete metric space. Since the state variable ω is between 0 and $\bar{\omega}$, we focus on the space $B((0,\bar{\omega}])$ of bounded continuous functions $h:(0,\bar{\omega}]\to R$ under sup norm. Theorem 3.1 in Stokey et al. (1989) implies that $B((0,\bar{\omega}])$ is a complete metric space.

We will use Blackwell's sufficient conditions to show Γ is a contraction mapping. Suppose both h, $\tilde{h}\in B((0,\bar{\omega}])$ and $h(\omega)\geq\tilde{h}(\omega)$, for all $\omega\in(0,\bar{\omega}]$, since

$$\bar{s}^* = (1-\exp(-\rho H))\left(\frac{1}{\kappa^Q}-(1+\tilde{s})\right)$$

all portfolio choices permitted under $\tilde{h}(\omega)$ are feasible under $h(\omega)$. Hence, $\Gamma h\geq\Gamma\tilde{h}$, for all $\omega\in(0,\bar{\omega}]$. Next, we need to show that there exists a positive constant $\beta<1$

such that $\Gamma(h+v) \leq \Gamma h + \beta v$, for all $h \in B((0,\bar{\omega}])$, $v \geq 0$, $\omega \in (0,\bar{\omega}]$. Consider

$$\Gamma(h+v)[\omega] = E_0\left[\int_0^\infty \exp\left(-(\rho+\xi+\chi)u\right) f(\omega_u)\, du \middle| \omega_0 = \omega\right],$$

where

$$f(\omega) = \frac{1}{\rho}\begin{pmatrix} (s(\omega)+s^*(\omega)-\hat{s}(\omega))(R(\omega)-r(\omega)-\tau(\omega))+s^*(\omega)\tau(\omega) \\ +\lambda\left(\ln\left(1-(1+s(\omega)+s^*(\omega))\kappa^Q(\omega)\right)-\ln\left(1-(1+\hat{s}(\omega))\kappa^Q(\omega)\right)\right)\end{pmatrix},$$

$$s^* \leq \bar{s}^*,$$

and

$$\bar{s}^* = (1-\exp(-\rho(h+v)))\left(\frac{1}{\kappa^Q}-(1+\tilde{s})\right).$$

Since the lower bound of h is zero, then

$$\begin{aligned}
\bar{s}^* &= (1-\exp(-\rho(h+v)))\left(\frac{1}{\kappa^Q}-(1+\tilde{s})\right) \\
&\leq (1-\exp(-\rho h)+\rho v)\left(\frac{1}{\kappa^Q}-(1+\tilde{s})\right) \\
&\leq (1-\exp(-\rho h))\left(\frac{1}{\kappa^Q}-(1+\tilde{s})\right)+\frac{\rho v}{\kappa}
\end{aligned}$$

With the assistance of above inequality, we derive that

$$\begin{aligned}
\Gamma(h+v)[\omega] &\leq \Gamma h + E_0\left[\int_0^\infty \exp\left(-(\rho+\xi+\chi)u\right)\frac{v}{\kappa}\tau du \middle| \omega_0=\omega\right] \\
&\leq \Gamma h + \frac{\tau}{(\rho+\xi+\chi)\kappa}v.
\end{aligned}$$

If $\tau < (\rho+\xi+\chi)\kappa$, Γ is a contraction mapping. ■

Table 1.2: Moments[1]

Moment	Model	Target	Source
average Sharpe ratio	34.4%	40%	Wachter (2013)
$\frac{\text{highest Sharpe ratio}}{\text{average Sharpe ratio}}$	15	15	He and Krishnamurthy (2012a)
average $\frac{\text{investment}}{\text{capital}}$ ratio	11.2%	11%	He and Krishnamurthy (2012a)
ratio of securitization at steady state[2]	25.5%	25.1%	ratio of securitization in third quarter of 2006
bankers' overall leverage	2.9	3	He and Krishnamurthy (2012a)
volatility of bankers' wealth growth rate			
in distress periods[3]	35.3%	31.5%	He and Krishnamurthy (2012a)
in non-distress periods	19.1%	17.5%	He and Krishnamurthy (2012a)

[1] We use the density of the stationary distribution to calculate all moments.
[2] The steady state is where $\mu^\omega - \lambda\kappa^\omega = 0$.
[3] The distress periods are those with highest 33% Sharpe ratio.

Table 1.3: Details of Securitization Data

	Outstanding	Securitized
Home Mortgages	FL383165105	FL673065105
Multifamily Residential Mortgages	FL143165405	FL673065405
Commercial Mortgages	FL383165505	FL673065505
Commercial and Industrial Loans[1]	FL253169255 FL263168005 FL263169255	FL673069505
Consumer Credit	FL153166000	FL673066000

[1] Becuase item FL253169255 is not available now, we use the ratio of securitization calculated by Loutskina (2011a) to estimate the outstanding commercial and industrial loans.

1.10 Appendix D: Data and Tables

Securitization. We follow Loutskina (2011a) to compute the ratio of securitization. The difference is that we focus on securitization done by non-agency security issuers. All data are drawn from the "Flow of Funds Accounts of the United States". There are 5 loan categories. The details of items for each category are listed in Table 2.3.

1.11 Appendix E: Figures

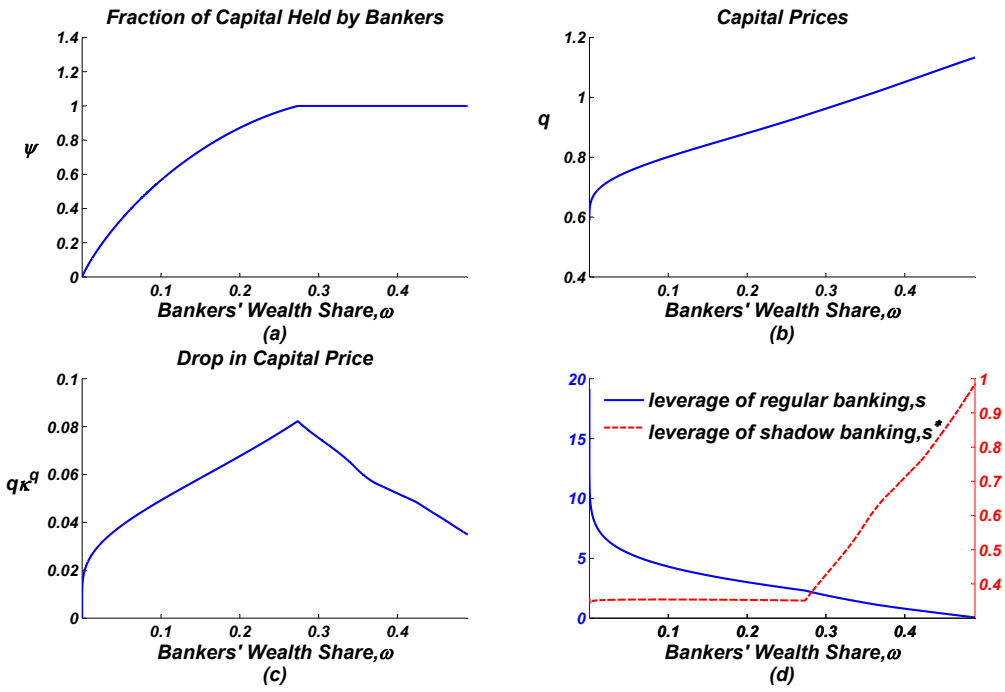


Figure 1.19: ψ, q, $q\kappa^q$, s, and s^* as functions of the state variable ω in the modified model with a constant cost of default $\bar{H} = 2.7709$, which equals the average cost of default in the calibrated model in Subsection 1.3.3. For other parameter values see Section 1.3.3.

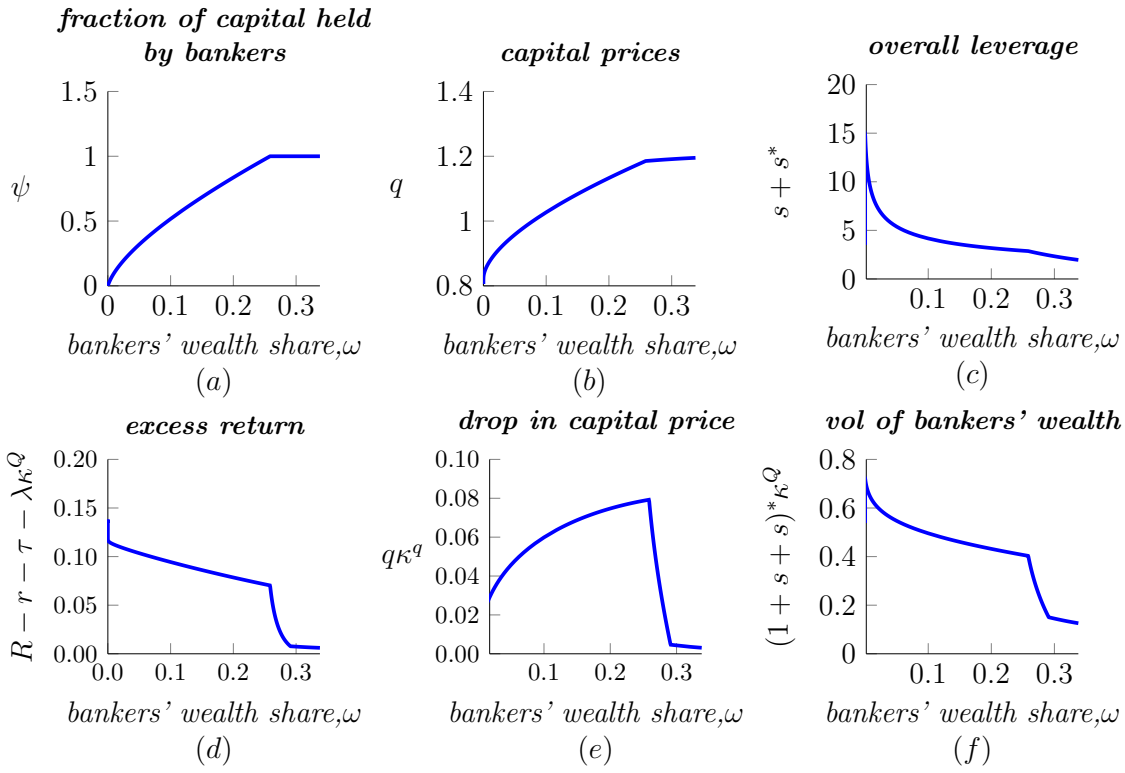


Figure 1.20: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in the "good" equilibrium of the modified model in which households have Epstein-Zin preferences. For the choice of parameter values, see Section 1.5.2.

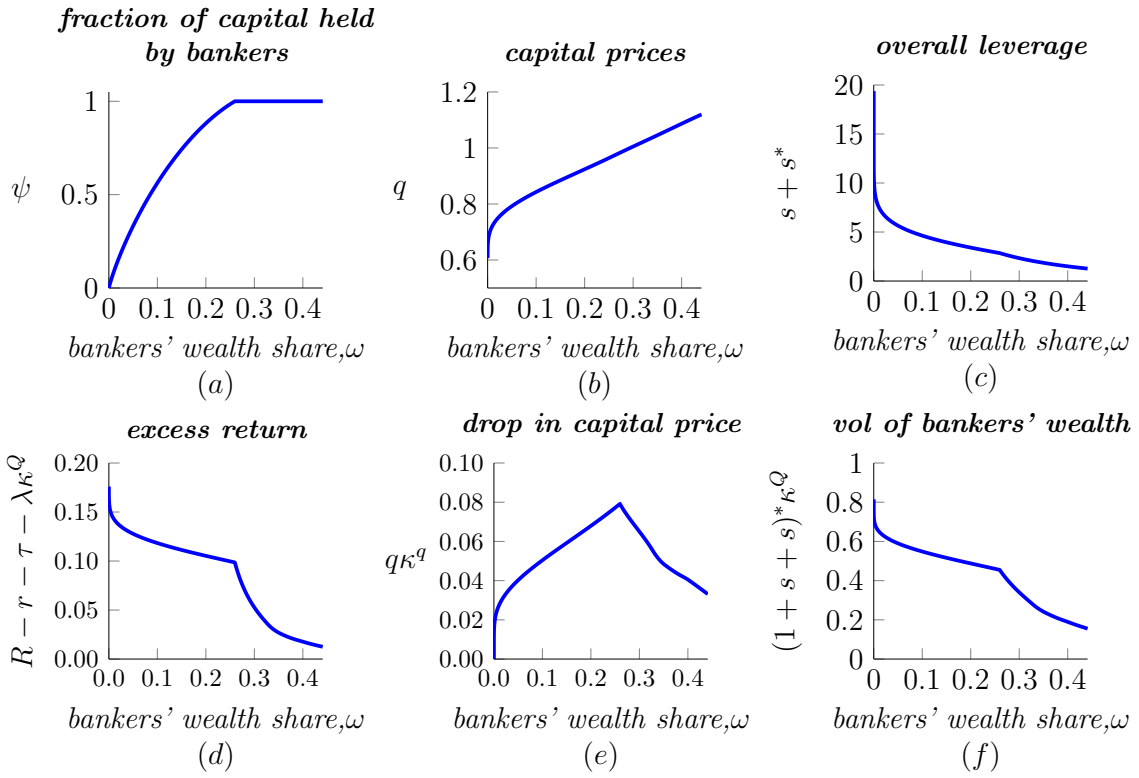


Figure 1.21: $\psi, q, (s+s^*), R-r-\tau-\lambda\kappa^Q, q\kappa^q$, and $\left(1+s+s^*\right)\kappa^Q$ as functions of the state variable ω, i.e., bankers' wealth share, in the "good" equilibrium of the modified model in which the maximum leverage of shadow banking is given by (1.35). For the choice of parameter values, see Section 1.7.

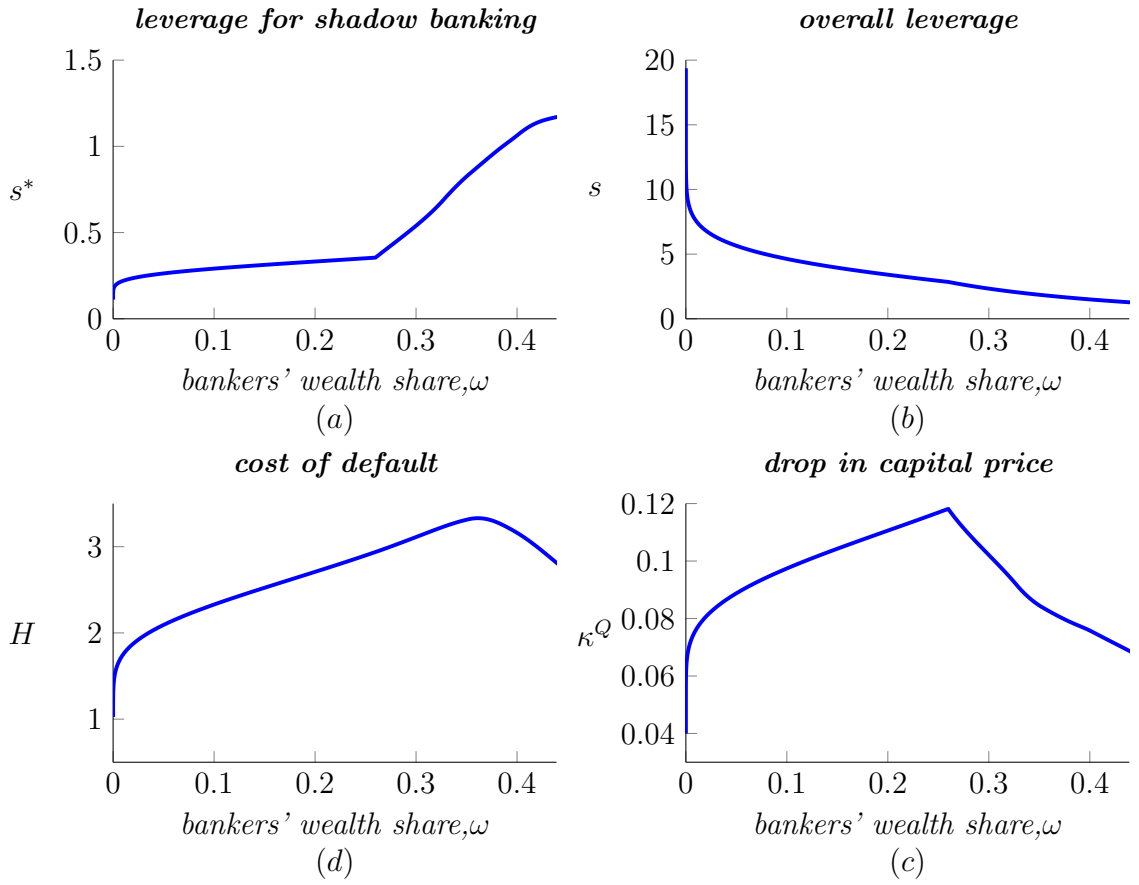


Figure 1.22: Pro-cyclical Shadow Banking
This figure presents the leverage of shadow banking s^*, "the leverage of regular banking" $\tilde{s}$, asset volatility κ^Q, and the cost of default H as functions of the state variable ω (i.e., bankers' wealth share) in the "good" equilibrium of the modified model in which the maximum leverage of shadow banking is given by (1.35). For parameter values, see Section 1.7.

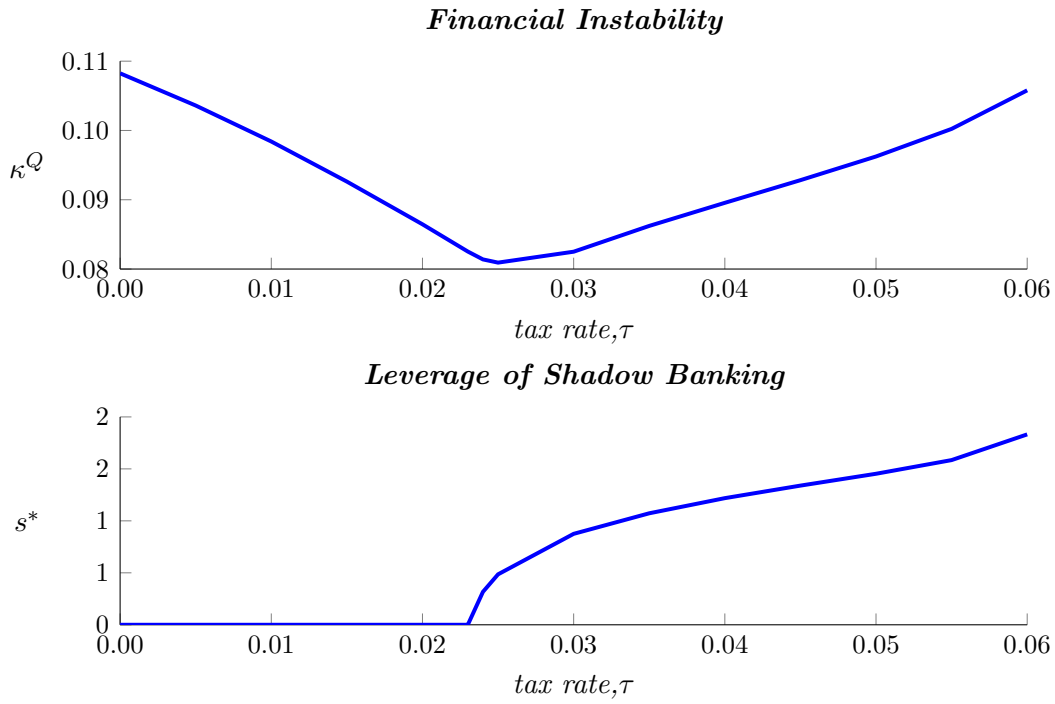


Figure 1.23: This figure shows the investment risk κ^Q (upper panel) and the leverage for shadow banking (lower panel) at the stochastic steady state of different economies with different tax rates τ in the modified model in which the maximum leverage of shadow banking is given by (1.35). The stochastic steady state is the state where $\omega\mu^\omega - \lambda\omega\kappa^\omega = 0$. We assume that the "good" equilibrium prevails if it exists. For the choice of parameter values, see Section 1.7.

Chapter 2

Systemic Run on Shadow Banks: The Minsky Moment

2.1 Introduction

After the 2007-2009 global financial crisis, the rise and fall of the US shadow banking system popularized the "Financial Instability Hypothesis" that Hyman Minsky developed in the 1980s (McCulley, 2009). However, some thirty years later, such an appealing theory for explaining financial crises cannot find its place within the dynamic general equilibrium framework that is widely used in macroeconomic research and policy discussions. Hence, in this chapter, we intend to recast the Financial Instability Hypothesis by analyzing shadow banking within the continuous-time macro-finance framework recently developed by He and Krishnamurthy (2012b) and Brunnermeier and Sannikov (2014).

[1]To follow Pozsar et al. (2010), traditional bank liabilities refer to total liabilities of the commercial banking sector (line 19 of Table L.109). Shadow bank liabilities (netted from overlaps with Table L.109) refer to the sum of total outstanding open market paper (line 1 of Table L.208), total repo liabilities (line 1 of Table L.207), net securities loaned (line 20 of Table L.130), total GSE liabilities and pool securities (lines 21 and 6 of Tables L.124 and L.125, respectively), total liabilities of ABS

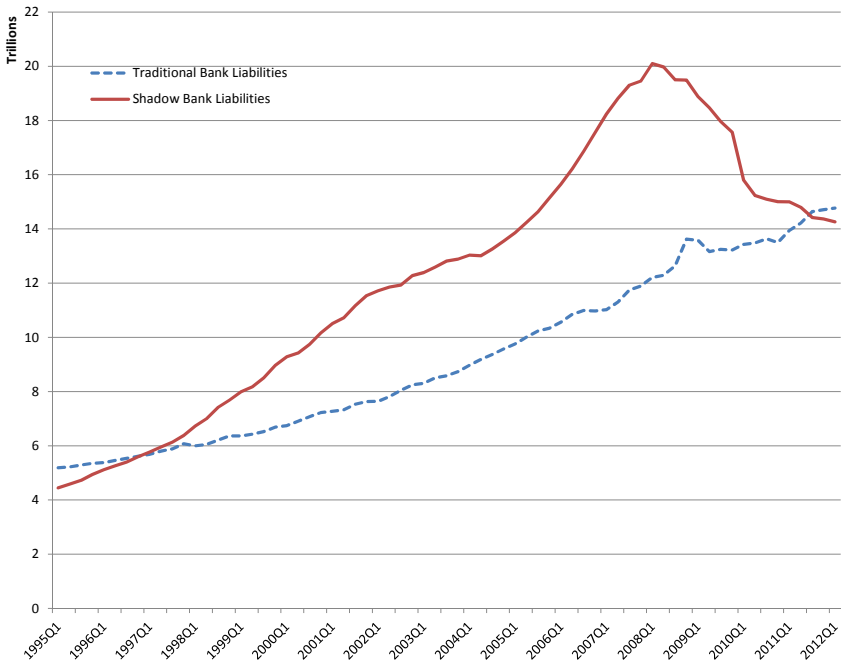


Figure 2.1: The Liability of Traditional Banking (blue dashed line) and the liability of Shadow Banking (red solid line) from the first quarter of 1995 to the first quarter of 2012. *Source*: Board of Governors of the Federal Reserve System, "Flow of Funds Accounts of the United States" [1]

In this chapter, we model shadow banking as off-balance-sheet financing and regular banking as on-balance-sheet financing (Pozsar, Adrian, Ashcraft, and Boesky, 2010).

Although regular banks specialize in screening potential borrowers and originating loans, they bear a managerial cost for holding loans to maturity since their employees monitor debtors' financial conditions and collect proceeds, and also because different regulators require inspection.

By exploiting the specialization in the capital market, regular banks can save the managerial costs of holding loans and free up their balance sheets by moving loans that they originate to their off-balance-sheet vehicles, which are more efficient for warehousing loans and processing proceeds. Such off-balance-sheet vehicles are simply shadow banks in the model I use in this chapter.

issuers (line 11 of Table L.126), and total shares outstanding of money market mutual funds (line 14 of Table L.121).

To enhance credit for their shadow banks, regular banks typically promise to protect creditors of shadow banks from potential losses due to any credit event. However, an enforceability problem exists, as regular banks may or may not fulfill their promises. Consequently, the size of the shadow bank that a regular bank can establish is constrained by its creditors so that the cost of keeping its promise is not too large for the regular bank.

Since the bailout cost is smaller when the asset price is less volatile, the size of shadow banking moves inversely with asset volatility. Thus, shadow banking expands as asset volatility declines during economic booms. However, the shadow banking sector's expansion of credit potentially destabilizes financial markets, as the risk of a systemic run on the shadow banking system increases.

A systemic run on the shadow banking sector could lead to the simultaneous default of all shadow banks. When the economy panics about shadow banking and a large number of shadow banks must liquidate assets and repay creditors, asset prices necessarily depreciate, as regular banks hold these assets reluctantly because of managerial costs (as discussed earlier). In the process, the market liquidity of assets deteriorates, thereby heightening shadow banks' credit risk. If assets become very illiquid and shadow banks must sell them at rather low prices to repay creditors, then shadow banks are more likely to fail, as sponsoring regular banks find it too expensive to bail out shadow banks. In turn, these decisions intensify widespread panic in the economy. This self-fulfilling banking panic eventually leads to the collapse of the shadow banking system, as experienced in 2007 and 2008.

The tractability of this model allows us to apply the global game device to uniquely select an equilibrium, when the self-fulfilling banking panic equilibrium is one of them. More importantly, the equilibrium selection mechanism endogenizes the link between the likelihood of a systemic run on shadow banks and their creditors' exposure to

this systemic risk. Intuitively, the banking panic is more likely to happen if creditors' exposure to shadow banking is larger.

Following the Financial Instability Hypothesis, declining asset volatility during prolonged economic upturns accelerates the growth of the shadow banking sector, which in turn increases creditors' exposure to the systemic risk that the shadow banking system may suddenly collapse. When the size of the shadow banking system crosses the endogenously-determined threshold in this model, the economic system becomes unstable, as the possibility that the shadow banking sector collapses increases.

Within a modern dynamic general equilibrium framework, this model adds values to the Financial Instability Hypothesis in the following aspects. First, this model formally characterizes the condition under which the economy or the financial system becomes unstable. As a result, we are able to quantitatively examine the systemic risk of a real economy using this model. In particular, this calibrated model shows that the chance of a systemic run grew from 0% in 2003 to around 25% in early 2007. Second, our model shows that a healthy shadow bank can go bankrupt if a systemic run occurs. This result helps explain why it is difficult to identify Ponzi financial schemes that destabilize the financial system, and also explains why people find it hard to detect systemic risk. Before a systemic run occurs, the model in this chapter does not include Ponzi finance schemes, as it is due to the endogenous systemic event itself that all healthy shadow banks suddenly collapse.

Related Literature. Broadly speaking, there are two strands of literature on financial instability. The first starts from the seminal works of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1999). These papers point out channels that the credit market could use to amplify the impact of exogenous shocks through adverse feedback loops. The financial instability in this literature takes the form that the financial market magnifies the impact of exogenous aggregate

shocks. From this literature, we borrow the important notion that asset volatility could fluctuate endogenously due to frictions within financial markets (i.e., the endogenous risk in our chapter). However, financial markets in this literature are not an independent source of instability, which only amplifies the impact of exogenous shocks to the economy. In addition, papers in this literature do not focus on why overheated credit booms expose the economy to the risk of a sudden collapse of the credit market.

Bryant (1980) and Diamond and Dybvig (1983) started the second strand of literature, which studies the self-fulfilling banking panic model. The financial instability in these models is that a bank or a financial market might suddenly break down due to the maturity or liquidity mismatch of their debt structures and the strategic complimentarity among creditors. This type of financial instability is the systemic risk featured in this chapter. That said, this literature does not consider how an economy could endogenously evolve into a phase during which the risk of a market-wide panic is high.

Gertler and Kiyotaki (2013), more closely aligned with my present analysis, quantitatively examine the impact of a bank run on the aggregate economy in a dynamic model with financial frictions. The major difference between their paper and our analysis is that they do not attempt to explain how the economy evolves into states where the risk of banking panics is high.[2] As a result, they do not measure the systemic risk of an economy.

This chapter is also related to theoretical studies on shadow banking; these studies include Gennaioli et al. (2013b) and Goodhart et al. (2012). In these papers, the distinction between shadow banking and traditional banking is not based on the differences of their institutional details. This chapter particularly focuses on the financial

[2]For their numerical exercise, they selectively pick values of relevant parameters such that the bank run equilibrium exists right after the economy is hit by a bad shock.

friction associated with off-balance-sheet financing and explains different behaviors of the shadow and traditional banking systems. Also, our explanation of financial crises differs from that provided by Gennaioli et al. (2013b) in two aspects. First, we consider a fully rational model, in which financial crises occur on the equilibrium path; in contrast, in Gennaioli et al. (2013b), financial crises will not happen unless there are optimistic creditors. Second, there is no difference between shocks that cause financial crises and shocks that do not in our model. Rather, the leverage for off-balance-sheet financing and the potential drop in capital price in the event of a banking panic determines the likelihood of a financial crisis occurring. In Gennaioli et al. (2013b), the magnitude of the shock incurring a financial crisis is larger than that of a normal shock. Accordingly, the large size of shadow banking only magnifies the severity of a crisis.

With respect to methodology, this chapter follows the emerging literature that investigate economies with financial frictions in a continuous-time setting. This literature includes Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013, 2012b). The methodology employed in this literature allows for the exact characterization of full equilibrium dynamics. In our model, the tractability allows for explicitly relating the leverage constraint to the endogenous risk. Several papers, such as Adrian and Boyarchenko (2012), Danielsson et al. (2012), and Phelan (2012), also do this by assuming various forms of Value-at-Risk constraints without offering relevant microfoundations. That said, the leverage constraint in this chapter comes from the financial friction that bankers could strategically default on securities generated by off-balance-sheet financing, which is intrinsic to shadow banking. Besides the maximum leverage for off-balance-sheet financing, the microfounded leverage constraint offers an important property–that bankers default if the constraint does not hold–that helps position systemic panic in our model.

The chapter is structured as follows. Section 2.2 establishes the model setup and defines the equilibrium. In Section 2.3, we characterize the equilibrium of the baseline model. Section 2.4 examines the qualitative properties of this model, and Section 2.5 looks at the quantitative performance of this model, showing that it provides early warning signs of a systemic run.

2.2 The Model

In this section, we build a continuous-time macro-finance model with both regular banking and shadow banking. More importantly, it incorporates the possibility of a systemic event that the shadow banking system collapses.

2.2.1 Preferences and Physical Environment

Two types of agents populate in a continuous-time infinite horizon economy: bankers and households. Bankers have logarithmic utility with time discount factor ρ. Households have the Epstein-Zin preferece with the time discount rate ρ, the relative risk-aversion coefficient γ, and the elasticity of intertemporal substitution b^{-1}. Both bankers and households consume a single type of perishable final good. Its total stock at time t is denoted by Y_t (i.e., the GDP of the economy). Final goods also serve as the numéraire.

Both bankers and households are equipped with technologies for producing final goods and physical assets. Bankers have final goods production function $y_t = ak_t$, where y_t is the final good output, a the productivity, and k_t the used physical assets. Bankers are more efficient than households in the sense that households' productivity a^h is less than a. However, both types of agents have the same capital good production skill. In particular, they can invert $\Phi\big(\iota_t\big)$ units of final goods into ι_t units of physical

assets. The holding of physical assets is risky in the sense that the stock of assets will shrink by κ proportion if a macro shock $\{N_t, t \geq 0\}$ hits the economy, which occurs at rate λ. If the investment choice $\{\iota_t, t \geq 0\}$ is the same for all agents, then the law of motion of aggregate capital stock $\{K_t, t \geq 0\}$ is

$$dK_t = K_{t-}\big(\iota_{t-} - \delta\big)dt - K_{t-}\kappa dN_t,$$

We next describe the rate of return for bankers' and households' holding physical assets. To do so, We denote the law of motion of the endogenous asset price process $\{q_t, t \geq 0\}$ by

$$dq_t = q_{t-}\mu^q_{t-}dt - q_{t-}\big((1-\Gamma_t)\kappa^q_t + \Gamma_t\kappa^{q,r}_t\big)dN_t. \tag{2.1}$$

where Γ_t indicates the systemic event that might happen after the Poisson shock hits the economy. If a banker spends one dollar on physical assets in the end of a period denoted by $t-$, then she would have $1/\,q_{t-}$ units. In the next period t, the net output generated by these assets is $\big(a - \Phi(\iota_{t-})\big)/\,q_{t-}$. In addition, she also obtains capital gain $\iota_{t-} - \delta + \mu^q_{t-}$. On the other hand, if the macro shock hits the economy and the systemic event does not occur, then the banker will have $\big(1-\kappa\big)/\,q_{t-}$ units of assets left. Moreover, the remaining assets are only worth $q_{t-}\big(1-\kappa^q_t\big)$. Thus, the total loss to the banker's investment is $\kappa + \big(1-\kappa\big)\kappa^q_t$. In sum, the rate of return for holding physical assets is

$$\underbrace{\left[\frac{a-\Phi(\iota_{t-})}{q_{t-}} + \iota_{t-} - \delta + \mu^q_{t-}\right]}_{\equiv R_{t-}}dt - \big((1-\Gamma_t)\kappa^Q_t + \Gamma_t\kappa^{Q,r}_t\big)dN_t,$$

where $\kappa_t^Q \equiv \kappa + \big(1-\kappa\big)\kappa_t^q$ and $\kappa_t^{Q,r} \equiv \kappa + \big(1-\kappa\big)\kappa_t^{q,r}$. Similarly, the rate of return for households to hold assets is

$$\underbrace{\left[\frac{a^h - \Phi(\iota_{t-}^h)}{q_{t-}} + \iota_{t-}^h - \delta + \mu_{t-}^q\right]}_{\equiv R_{t-}^h} dt - \big((1-\Gamma_t)\kappa_t^Q + \Gamma_t \kappa_t^{Q,r}\big) dN_t.$$

2.2.2 Time Line: Sequential Decisions within a Period

The key insight of this model hinges on the interaction of bankers' sequential movements within a period. In particular, each banker has at most five sequential decisions to make at five stages. To illustrate bankers' decisions more clearly, we split five stages into two groups and introduce them in reverse order. As Figure 2.2 indicates, if the Poisson shock does not hit the economy, the first group of stages will not be reached in the subgame.

Stage 4 and 5: Regular Banking and Shadow Banking

A banker owns a regular bank as its only equity holder and sponsors a shadow bank as its essential beneficiary. At stage 4, regular banks *i*) return both the principal and interest to their creditors (i.e., households), *ii*) recognize losses to their equity holders if there are any, and *iii*) accumulate physical assets. At stage 5, shadow banks pay back principal and interest, "trade" physical assets with their own regular banks, and issue financial assets that regular banks owned by other bankers invest in.

Regular banks need to pay managerial cost τ per dollar physical assets on their books. However, such costs do not apply to any "safe" financial asset on their balance sheets. More specifically, the total managerial cost paid by a regular bank depends on the maximum number of physical assets that it holds at stage 4 and 5.

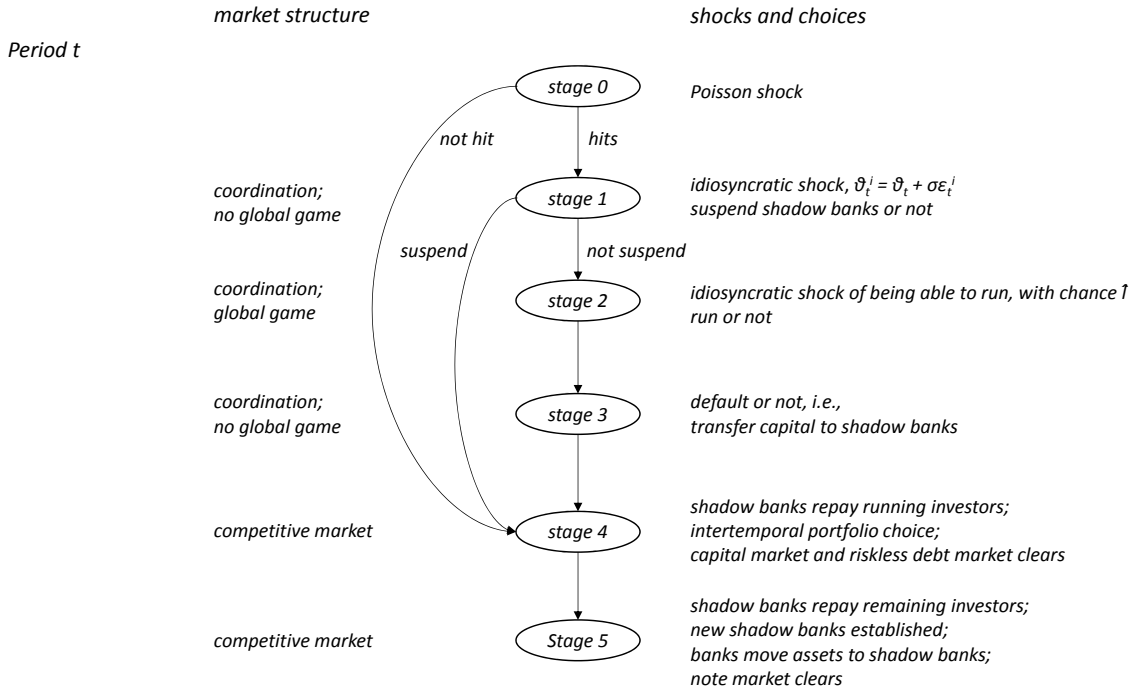


Figure 2.2: Game Tree

Shadow banks, which have zero equity by construction, specialize in warehousing physical assets and do not pay any managerial cost. We name the financial assets that shadow banks issue as "notes." The following assumption specifies the structure of the note market.

Assumption 9

i. A continuum of regular banks can form a union, which offers one-period note contracts $(\bar{s}_t^*, \tilde{r}_t)$ *to any shadow bank, whose sponsoring banker is outside of the union.*

ii. The shadow bank that accepts the contract can borrow up to $\bar{s}_t^*$ *times its sponsor's net worth and pay the principal and interest at rate* $\tilde{r}_t$ *next period.*

iii. If a shadow bank fails, then the union would enforce the punishment scheme that its sponsor pays l_0 *fraction of her net worth to the union every period in the*

future. For instance, the fine stream would be $\{l_0 W_s, s \geq t\}$ *for a banker given that the process of her net worth is* $\{W_s, s \geq t\}$ *since she misbehaves in period* t*.*

iv. The note market is competitive.

Although shadow banks are supposed to pay back creditors at stage 5, creditors can–if they run–have their principal back at stage 4.

Stage 1, 2, and 3: Systemic Run and Strategic Default

At stage 1, banker i receives a private signal $\theta_t^i = \theta_t + \sigma\varepsilon_t^i$. Each banker decides whether to suspend her shadow bank, conditional on the Poisson shock's hit. If a banker suspends her shadow bank, then no creditor can have the principal back at stage 4. In equilibrium, all bankers might suspend their shadow banks simultaneously, which will cause the market-wide shadow bank default. The following assumption specifies what happens after such a systemic event.

Assumption 10

i. If almost all shadow banks default, then the economy falls into a distressed phase where no shadow banking exists.

ii. The economy in the distressed phase switches to the normal phase with the intensity π*.*

iii. The capital gain that occurs when the economy switches back to the normal phase is redistributed across all agents, according to their wealth levels.

Since this systemic event is beyond the control of an individual agent, we consider "safe" financial assets those that guarantee the principal's safety conditional on the systemic event not happening.

At stage 2, each banker receives an idiosyncratic shock about whether she is able to run or not. A banker is able to run with probability $\hat{l}$.[3] This random event is independent across all bankers. At stage 2, bankers, who are able to run, decide whether to let their regular banks run on shadow banks as note investors. If banker i runs, then she pays a utility cost θ_t^i.

At stage 3, the proportion of note investors who run becomes publicly observable, and bankers decide whether to transfer physical assets from their regular banks to their shadow banks. If nearly no bankers suspend their shadow banks at stage 1, then they can renege their early decisions and decide to support their shadow banks at this stage.

2.2.3 Bankers' and Households' Portfolio Choices

To simplify the analysis of bankers' portfolio choice problem, we conjecture that if almost no banker suspends her shadow bank (i.e., no systemic event occurs), then no regular bank runs at stage 2, and all bankers transfer enough physical assets, if it is necessary, at stage 3. We verify this conjecture in the next section.

Suppose a banker's net worth is W_t, which is also the regular bank's equity value. S_t denotes the value of physical assets on her balance sheet at stage 5, which equals $q_t k_t$ given that there are k_t units of physical capital. B_t denotes the value of notes that the regular bank holds. The excess return from the holding of physical assets is

$$S_{t-}\big(R_{t-} - \tau - r_{t-}\big)dt - \big((1-\Gamma_t)S_{t-}\kappa_t^Q + \Gamma_t S_{t-}\kappa_t^{Q,r}\big)dN_t,$$

[3] where $\hat{l} < 1 - \kappa_t^{Q,r}$ for all $t \geq 0$

where r_t is the risk-free rate. The excess return from holding notes issued by shadow banks (i.e., from lending to shadow banks sponsored by other bankers) is

$$B_{t-}\big(\tilde{r}_{t-} - r_{t-}\big)dt - \Gamma_t B_{t-}\kappa_t^{Q,r} dN_t,$$

where $\tilde{r}_t$ is the rate of return for holding notes.

The increase in the banker's net worth is also due to the return from sponsoring the shadow bank. As the beneficiary of the shadow bank, the banker then obtains all excess return, that is

$$S^*_{t-}\big(R_{t-} - \tilde{r}_{t-}\big)dt - (1-\Gamma_t)S^*_{t-}\kappa_t^Q dN_t,$$

where S^*_t is the value of notes issued by the shadow bank. The loss comes from the fact that the banker transfers physical assets to cover the loss of the shadow bank at stage 3, when a Poisson shock hits the economy. Consequently, such a loss will be recognized as the loss to the regular bank's equity value.

In sum, the law of motion for the banker's net worth W_t is

$$\begin{aligned} dW_t &= \big(W_{t-}r_{t-} + S_{t-}\big(R_{t-} - \tau - r_{t-}\big) + B_{t-}\big(\tilde{r}_{t-} - \hat{r}_{t-}\big) + S^*_{t-}\big(R_{t-} - \tilde{r}_{t-}\big)\big)dt \\ &\quad -\big(\big(1-\Gamma_t\big)\big(S_{t-} + S^*_{t-}\big)\kappa^Q_{t-} + \Gamma_t\big(S_{t-} + B_{t-}\big)\kappa_t^{Q,r}\big)dN_t - c_{t-}dt. \end{aligned} \tag{2.2}$$

Taking $\{q_t, r_t, \tilde{r}_t, \bar{s}^*_t, t \geq 0\}$ as given, the banker's problem is to choose $\{c_t, S_t, B_t, S^*_t, \iota_t, t \geq 0\}$ and maximize

$$E\left[\int_0^\infty e^{-\rho t}\ln(c_t)\ dt\right]$$

subject to the dynamic budget constraint (2.2) and the leverage constraint $S^*_t \leq \bar{s}^*_t W_t$.

Households split their wealth between physical assets and riskless debts issued by regular banks. With S_t^h as the value of physical assets that household h has, the wealth W_t^h of the household then evolves according to

$$dW_t^h = W_{t-}^h r_{t-} dt + S_{t-}^h \left(R_{t-}^h - r_{t-}\right) dt - \left(\left(1-\Gamma_t\right) S_{t-}^h \kappa_t^Q + \Gamma_t S_{t-}^h \kappa_t^{Q,r}\right) dN_t - c_{t-}^h dt. \quad (2.3)$$

By choosing $\left\{c_t^h, S_t^h, t \geq 0\right\}$ a household maximizes

$$U_0^h = E_0 \left[\int_0^\infty f\left[c_s^h, U_s^h\right] ds\right],$$

where

$$f\left[c^h, U^h\right] = \frac{1}{1-b}\left\{\frac{\rho\left(c^h\right)^{1-b}}{\left(\left(1-\gamma\right)U^h\right)^{\frac{\gamma-b}{1-\gamma}}} - \rho\left(1-\gamma\right)U^h\right\}$$

and

$$U_t^h = E_t \left[\int_t^\infty f\left[c_s^h, U_s^h\right] ds\right], \text{ for } t > 0,$$

subject to the dynamic budget constraint (2.3).

2.2.4 Equilibrium

We make the following assumption to guarantee that the wealth share of bankers does not become large enough to undo all financial frictions:

Assumption 11 *Each banker exits the economy independently with intensity ξ.*

$\boldsymbol{I} = [0,1]$ and $\boldsymbol{J} = (1,2]$ denote sets of bankers and households, respectively. Individual bankers and households are indexed by $i \in \boldsymbol{I}$ and $j \in \boldsymbol{J}$.

Definition 2 *Given any initial endowments of capital goods $\left\{k_0^i, k_0^j; i \in \boldsymbol{I}, j \in \boldsymbol{J}\right\}$ such that*

$$\int_0^1 k_0^i di + \int_1^2 k_0^j dj = K_0,$$

the equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{N_t, \theta_t, \varepsilon_t^i, t \geq 0, , i \in \boldsymbol{I}\}$*: capital price* $\{q_t, t \geq 0\}$*, risk-free rate* $\{r_t, t \geq 0\}$*, the rate return of notes* $\{\tilde{r}_t, t \geq 0\}$*, the likelihood of the systemic event* $\{\varrho_t, t \geq 0\}$*, wealth* $\left\{W_t^i, W_t^j, t \geq 0\right\}$*, capital holdings* $\left\{k_t^i, k_t^j, t \geq 0\right\}$*, investment decisions* $\{\iota_t^i, t \geq 0\}$*, holdings of financial instruments* $\{B_t^i, t \geq 0\}$*, maximum leverage of shadow banking* $\{\bar{s}_t^*, t \geq 0\}$*, suspension decisions* $\{\gamma_t^i, t \geq 0\}$*, and consumption* $\left\{c_t^i, c_t^j, t \geq 0\right\}$ *of individual banker* $i \in \boldsymbol{I}$ *and individual household* $j \in \boldsymbol{J}$*; such that*

1. $\left\{W_0^i, W_0^j\right\}$ *satisfy* $W_0^i = q_0 k_0^i$ *and* $W_0^j = q_0 k_0^j$*, for* $i \in \boldsymbol{I}$ *and* $j \in \boldsymbol{J}$**;**

2. *bankers solve their problems given* $\{q_t, t \geq 0\}$*,* $\{r_t, t \geq 0\}$*,* $\{\tilde{r}_t, t \geq 0\}$*,* $\{\varrho_t, t \geq 0\}$*, and* $\{\bar{s}_t^*, t \geq 0\}$*;*

3. *households solve their problems given* $\{q_t, t \geq 0\}$ *and* $\{r_t, t \geq 0\}$*;*

4. *markets for consumption goods and capital goods clear for all* $t \geq 0$

$$\begin{aligned} \int_0^1 c_i^i di + \int_1^2 c_t^j dj &= \int_0^1 \left(a - g\left(\iota_t^i\right)\right) k_t^i \, di + \int_1^2 \left(a^h - g\left(\iota_t^j\right)\right) k_t^j \, dj, \quad (2.4) \\ \int_0^1 k_t^i di + \int_1^2 k_t^j dj &= K_t, \\ \text{where } dK &= \left(\int_0^2 \left(\iota_t^i - \delta\right) k_t^i di\right) dt - \kappa K_{t-} dN_t; \end{aligned}$$

5. *Note market clears for all* $t \geq 0$

$$\int_0^1 S_t^{*,i} di = \int_0^1 B_t^i di.$$

Given this definition, the debt market automatically clears by Walras' Law given this definition. Later, we show that bankers' decisions to run play a role off the equilibrium path.

2.3 Systemic Risk and Banking Panic

In this section, we endogenize the systemic event that leads to the shadow banking system shutting down and characterize the likelihood of the systemic event $\{\varrho_t, t \geq 0\}$.

2.3.1 Characterizing Portfolio Choices

Bankers have both standard portfolio choices and strategic choices to make. To characterize their decisions, we begin our analyses with bankers' portfolio choice problem, given the conjecture that no banker defaults in the absence of the systemic event. By using bankers' continuation values, we will verify our conjecture by examining their strategic decisions.

Bankers' Portfolio Choices

ι_t denotes a banker's investment at time t. The expression of R_t implies that optimal level of ι_t maximizes

$$\frac{-\iota_t - 0.5\tau\big(\iota_t - \delta\big)^2}{q_t} + \iota_t.$$

The first-order condition yields that the optimal investment rate is a function of the capital price q_t

$$\iota\big(q_t\big) = \delta + \frac{q_t - 1}{\tau}. \tag{2.5}$$

Households have the same investment function $\iota(\cdot)$ since they have the same technology.

We apply the stochastic control approach to solve for the bankers' optimal consumption and portfolio choices, based on the conjecture that no banker defaults. The following proposition summarizes analytical results:

Proposition 9 *A banker's optimal consumption* $\{c_t, t \geq 0\}$ *and optimal portfolio weights* $\{s_t, b_t, s_t^*, t \geq 0\}$ *satisfy*

$$c_t = \rho W_t, \tag{2.6}$$

$$\max\{\left(s_{t-} + s_{t-}^*\right)\kappa_t^Q, \left(s_{t-} + b_{t-}\right)\kappa_t^{Q,r}\} < 1, \tag{2.7}$$

$$R_{t-} - \tau - r_{t-} \leq \frac{\lambda(1-\varrho_t)\kappa_t^Q}{1-\left(s_{t-} + s_{t-}^*\right)\kappa_t^Q} + \frac{\lambda\varrho_t\kappa_t^{Q,r}}{1-\left(s_{t-} + b_{t-}\right)\kappa_t^{Q,r}}, \ = \ \textit{if } s_{t-} > 1, \tag{2.8}$$

$$\tilde{r}_{t-} - r_{t-} \leq \frac{\lambda\varrho_t\kappa_t^{Q,r}}{1-\left(s_{t-} + b_{t-}\right)\kappa_t^{Q,r}}, \ = \ \textit{if } b_{t-} > 0, \tag{2.9}$$

$$R_{t-} - \tilde{r}_{t-} \geq \frac{\lambda(1-\varrho_t)\kappa_t^Q}{1-\left(s_{t-} + s_{t-}^*\right)\kappa_t^Q}, \ = \ \textit{if } s_{t-}^* < \bar{s}_{t-}^*, \tag{2.10}$$

where $s_t = S_t / W_t$ *and* $s_t^* = S_t^* / W_t$.

Proof. See Appendix. ■

Equation (2.7) guarantees that the banker's net worth is positive after a bad shock to avoid negative infinite utility. Equation (2.8) − (2.10) are standard asset pricing equations.

Notice that the portfolio weight on notes b is indeterminant given equation (2.9) and other first-order conditions. To tackle this problem, we make the following assumption:[4]

Assumption 12 *If bankers are indifferent about holding notes or not, then their portfolio weights on notes are the same.*

[4]This assumption is only necessary for the simple financial regulation setup in this model. The simple setup offers a quantitative advantage because there is only one parameter associated with the financial regulation that needs to be calibrated.

Households' Portfolio Choice

In solving for households' optimal portfolio and consumption strategies, we also apply the stochastic control approach. For the household with net worth W_t^h, her continuation value function is defined as

$$V_t^h = \max_{\left\{c_u^h, S_u^h, u \geq t\right\}} U_t^h.$$

We conjecture that the continuation value takes the functional form

$$V_t^h = V\left(\zeta_t, W_t^h\right) \equiv \frac{\left(\zeta_t W_t^h\right)^{1-\gamma}}{1-\gamma},$$

where ζ_t follows

$$d\zeta_t = \zeta_{t-}\mu_{t-}^{\zeta} dt - \zeta_{t-}\left((1-\Gamma_t)\kappa_t^{\zeta} + \Gamma_t \kappa_t^{\zeta,r}\right) dN_t.$$

As in Di Tella (2012), ζ_t is interpreted as the continuation value multiplier of households' net worth since the function $V\left(\zeta, W^h\right)$ is homogeneous of degree $1-\gamma$ with respect to W^h. Given this conjecture, the Hamilton-Jacobi-Bellman equation of the household's dynamic programming problem is

$$0 = \max_{c^h, S^h} \left\{ f\left(c_t^h, V_t^h\right) + \mathcal{D}^{c^h, S^h} V\left(\zeta_t, W_t^h\right) \right\}, \tag{2.11}$$

where

$$\begin{aligned}
\mathcal{D}^{c^h, S^h} V\left(\zeta_t, W_t^h\right) &= \left(W_{t-}^h r_{t-} + S_{t-}^h\left(R_{t-}^h - r_{t-}\right) - c_{t-}^h\right)\zeta_{t-}^{1-\gamma}\left(W_{t-}^h\right)^{-\gamma} \\
&+ \mu_{t-}^{\zeta}\left(\zeta_{t-} W_{t-}^h\right)^{1-\gamma} + \frac{\lambda(1-\varrho_t)\left(\left(\zeta_{t-} - \zeta_{t-}\kappa_t^{\zeta}\right)\left(W_{t-}^h - S_{t-}^h \kappa_t^Q\right)\right)^{1-\gamma}}{1-\gamma} \\
&+ \frac{\lambda \varrho_t \left(\left(\zeta_{t-} - \zeta_{t-}\kappa_t^{\zeta,r}\right)\left(W_{t-}^h - S_{t-}^h \kappa_t^{Q,r}\right)\right)^{1-\gamma}}{1-\gamma} - \frac{\lambda\left(\zeta_{t-} W_{t-}^h\right)^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

We summarize the key results of the problem in the following proposition:

Proposition 10 *Each household's optimal consumption weight* $\left\{\hat{c}_t^h, t \geq 0\right\}$*, optimal portfolio weight* $\left\{s_t^h, t \geq 0\right\}$*, and the process* $\{\zeta_t, t \geq 0\}$ *satisfy*

$$\left(\hat{c}_t^h\right)^b = \frac{\rho}{\zeta_t^{1-b}}, \tag{2.12}$$

$$R_{t-}^h - r_{t-} \leq \frac{\lambda(1-\Gamma_t)\kappa_t^Q}{\left(1-s_{t-}^h\kappa_t^Q\right)^\gamma}\left(1-\kappa_t^\zeta\right)^{1-\gamma} + \frac{\lambda\Gamma_t\kappa_t^{Q,r}}{\left(1-s_{t-}^h\kappa_t^{Q,r}\right)^\gamma}\left(1-\kappa_t^{\zeta,r}\right)^{1-\gamma}, \;\; = \;\; \textit{if } s_{t-}^h > 0, \tag{2.13}$$

$$\begin{aligned} 0 \;\; = \;\; & \frac{\rho\left(\hat{c}_{t-}^h\right)^{1-b}}{(1-b)\zeta_{t-}^{1-b}} - \frac{\rho}{1-b} + r_{t-} + s_{t-}^h\left(R_{t-}^h - r_{t-}\right) - \hat{c}_{t-}^h + \mu_{t-}^\zeta \\ & + \lambda\left(\frac{\left(1-\varrho_t\right)\left(\left(1-\kappa_t^\zeta\right)\left(1-s_{t-}^h\kappa_t^Q\right)\right)^{1-\gamma}}{1-\gamma} + \frac{\varrho_t\left(\left(1-\kappa_t^{\zeta,r}\right)\left(1-s_{t-}^h\kappa_t^{Q,r}\right)\right)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma}\right), \end{aligned} \tag{2.14}$$

where $\hat{c}_t^h = c_t^h / W_t^h$ *and* $s_t^h = S_t^h / W_t^h$*.*

Proof. See Appendix. ■

The first-order conditions of households have similar interpretations except that the continuation value multiplier ζ_t affects households' consumption and investment decisions, as shown by equation (2.12) and (2.13). By plugging the household's optimal choices into the HJB equation (2.11), we can derive equation (2.14), which specifies the condition that $\{\zeta_t, t \geq 0\}$ should satisfy.

2.3.2 Enforceability Constraint and Inevitable Systemic Risk

By analyzing the the note market's equilibrium, we characterize the maximum leverage of shadow banking and explain why the financial market cannot escape from the systemic risk by itself.

Maximum Leverage of Shadow Banking

Bankers assume the maximum leverage of shadow banking $\{\bar{s}_t^*, t \geq 0\}$ when they solve for optimal portfolio choices at stage 4 and 5 of each period. We next characterize $\{\bar{s}_t^*, t \geq 0\}$ based on the strategic default choices of bankers at stage 3 and the market clearing condition of the note market at stage 5.

Thanks to the logarithm preference, the expected lifetime utility of a banker with net worth W_t at time t is $\ln(W_t)/\rho + h_t$, which will be referred to as the banker's continuation value. The following lemma characterizes $\{h_t, t \geq 0\}$.

Lemma 4 *The law of motion of* $\{h_t, t \geq 0\}$ *satisfies*

$$\begin{aligned}
\rho h_{t-} &= h_{t-}\mu_{t-}^h + \xi\left(\frac{r-\rho}{\rho} - h_{t-}\right) + \log(\rho) \\
&+ \frac{1}{\rho}\big(r_{t-} + s_{t-}\big(R_{t-} - \tau - r_{t-}\big) + b_{t-}(\tilde{r}_{t-} - r_{t-}) + s_{t-}^*\big(R_{t-} - \tilde{r}_{t-}\big) - \rho\big) \\
&+ \lambda\left(\frac{1}{\rho}\ln\left(1 - \left(1 + s_{t-} + s_{t-}^*\right)\kappa_t^Q\right) + h_{t-}\left(1 - \kappa_t^h\right) - h_{t-}\right).
\end{aligned}$$

Furthermore, if a banker defaults at time t, her continuation value is

$$\frac{\ln(W_t)}{\rho} + h_t - \frac{l_0}{\rho(\rho+\xi)}.$$

Proof. See Appendix ■

Given the continuation value that a banker would have if she defaults, we consider the default decision of the banker at stage 3 conditional on the hit of a Poisson shock. Without loss of generality, we only focus on the case that the systemic event does not happen. If the banker does not default, her continuation value will be

$$\frac{\ln\left(W_{t-}\left(1-(s_{t-}+s^*_{t-})\kappa^Q_t\right)\right)}{\rho}+h_t.$$

If she defaults instead, the banker does not bear the loss $s^*_{t-}W_{t-}\kappa^Q_t$. The downside is that she will receive the punishment described in Assumption 9. Hence, the continuation value of the banker would be

$$\frac{\ln\left(W_{t-}\left(1-s_{t-}\kappa^Q_t\right)\right)}{\rho}+h_t-\frac{l_0}{\rho(\rho+\xi)}.$$

Thus, the banker defaults if

$$\frac{\ln\left(W_{t-}\left(1-(s_{t-}+s^*_{t-})\kappa^Q_t\right)\right)}{\rho}<\frac{\ln\left(W_{t-}\left(1-s_{t-}\kappa^Q_t\right)\right)}{\rho}-\frac{l_0}{\rho(\rho+\xi)}.$$

Proposition 11 *The note contract in equilibrium across the time* $\{(\bar{s}^*_t,\tilde{r}_t),t\geq 0\}$ *can be characterized by*

$$\frac{l_0}{\rho+\xi}=\ln\left(1+\frac{\bar{s}^*_{t-}\kappa^Q_t}{1-\left(s_{t-}+\bar{s}^*_{t-}\right)\kappa^Q_t}\right). \tag{2.15}$$

$$\tilde{r}_{t-}=r_{t-}+\frac{\lambda\varrho_t\kappa^{Q,r}_t}{1-\left(s_{t-}+b_{t-}\right)\kappa^{Q,r}_t} \tag{2.16}$$

Proof. Assume that ϱ_t is zero. Suppose that (2.15) does not hold at time $t-$, and the left hand side of (2.15) is strictly less than its right hand side; individual bankers would then default at stage 3 if the Poisson shock hits the economy at time t. Thus, notes become risky assets, and regular banks have to pay managerial cost

τ to hold them. The managerial cost will be transmitted to the required rate of return for holding notes, which neutralizes any advantage of shadow banking. Thus, there would be no demand for any note contract. However, this cannot be true in equilibrium because unions formed by regular banks could be strictly better off by lowering $\bar{s}_t^*$ such that it is risk-free to hold notes and earn a positive excessive return $\tilde{r}_t - r_t$ without taking any risk.

If the left hand side of (2.15) is strictly larger, then unions can always increase their profits by raising $\bar{s}_t^*$ up to the level that equation (2.15) exactly holds. Thus, this cannot be true, either.

Notice that if bankers want to deviate at time $t-$ and choose the leverage of regular banking larger than s_{t-} as specified by (2.8), then note investors at stage 5 can easily refuse to lend to them.

Next, consider the case where the systemic event occurs with some positive probability. In this case, although the systemic event leads to any individual banker's default decision because $\kappa_t^{Q,r} \geq \kappa_t^Q$, the holding of notes is not subject to managerial costs.

Due to the perfect competition among unions, the excessive rate of return $\tilde{r}_{t-} - r_{t-}$ has to equal the expected loss $\lambda \varrho_t \kappa_t^{Q,r}$ factored by the marginal impact on bankers' continuation value $1/\left(1 - (s_{t-} + b_{t-})\kappa_t^{Q,r}\right)$. ■

Proposition (11) implies that when a bad shock hits the economy, any individual banker finds it optimal to default if all other bankers do so because the economy switches to a distressed phase, and the asset price declines by $\kappa_t^{q,r} (> \kappa_t^q)$.

Corollary 2 *The systemic event that all shadow banks default simultaneously is an equilibrium outcome.*

Systemic Risk as the Tragedy of the Commons

We will elaborate how bankers' optimal decisions and the note market clearing lead to the systemic risk. We consider a scenario in which all unions of regular banks offer the same systemic-risk-proof note contract $\{(\tilde{r}_{t,pf}, \bar{s}^*_{t,pf}), t \geq 0\}$, where $\bar{s}^*_{t,pf}$ satisfies

$$\frac{l_0}{\rho + \xi} = \ln \left[1 + \frac{\bar{s}^*_{t-,pf} \kappa_t^{Q,r}}{1 - \left(s_{t-} + \bar{s}^*_{t-,pf} \right) \kappa_t^{Q,r}} \right]. \tag{2.17}$$

Equation (2.17) is such a tight enforceability constraint that individual bankers would not default even, if all other bankers do so when a negative shock comes. Thus, no systemic event occurs in equilibrium if equation (2.17) is true.

However, Proposition 11 implies that the hypothetical scenario is unstable since a union can always propose an alternative contract to zero measure of bankers, such that they could raise more credit for paying a slightly higher interest rate. In particular, the alternative contract $(\tilde{r}'_t, \bar{s}^{*\prime}_t)$ would be such that $\tilde{r}'_t$ is slightly higher than $\tilde{r}_{t-}$ and $\bar{s}^{*\prime}_{t-}$ satisfies

$$\frac{l_0}{\rho + \xi} = \ln \left(1 + \frac{\bar{s}^{*\prime}_{t-} \kappa_t^{Q}}{1 - \left(s'_{t-} + \bar{s}^{*\prime}_{t-} \right) \kappa_t^{Q}} \right). \tag{2.18}$$

Since $\kappa_t^{Q,r} > \kappa_t^{Q}$ implies $\bar{s}^{*\prime}_t > \bar{s}^*_t$, the alternative contract provides more credit. As a result, bankers would accept the new contract if $\tilde{r}'_t$ is slightly higher than $\tilde{r}_t$. For note investors (i.e., regular banks), the alternative contract offers risk-free notes because *i*) the dynamics of the economy do not change if only zero measure of bankers accept the contract; *ii*) no systemic event would happen since equation (2.17) still holds; and *iii*) the enforceability constraint (2.18) guarantees that bankers who accept the alternative contract would not default if a systemic event does not happens. Therefore, it is profitable for the union of regular banks to offer the new contract. However, since bankers think in the same way, they would deviate from the original contract $\left(\tilde{r}_{t,pf}, \bar{s}^*_{t,pf}\right)$, which shows how the systemic risk is caused by bankers' optimal

decisions and the note market clearing. Generally speaking, the optimal decisions of individual bankers fail to internalize the effect that if everyone borrows or lends aggressively, then the economy faces the systemic risk that the shadow banking system could collapse suddenly.

The inevitability of the systemic risk is the tragedy of the commons. If the tight enforceability constraint (2.17) holds in an economy, then no systemic event occurs, which is a common resource. While individual bankers could deplete this common resource and raise more credit, the common resource would be exhausted eventually, and the economy would face systemic risk if all bankers exploited the common resource to their best.

2.3.3 Systemic Run

The market-wide run on shadow banks triggers the systemic event that all shadow banks default simultaneously. In this section, we use the global game method to show that a systemic run is more likely to occur when creditors have larger exposure to shadow banking.

Default Decisions at Stage 3 in the Event of Banking Panic

Bankers at stage 3 can perfectly foresee the price of physical assets and the note contract because there is no uncertainty from stage 3 to stage 5, and all markets are competitive. Therefore, bankers can make their strategic choices at stage 3 based on their continuation values in the beginning of the next period.

The panic of shadow banks' creditors causes the default of a banker at stage 3; we defer the full analysis to the proof of Proposition 12. If shadow banks' creditors run at stage 2 because of a Poisson shock hit, then shadow banks have to sell physical assets

to repay their creditors at stage 4. Accordingly, regular banks absorb these assets and expand their balance sheets at stage 4. Recall that the balance sheet expansion is expensive for regular banks because of managerial costs. Thus, if shadow banks are forced to liquidate too many physical assets by creditors who run, then regular banks will be very reluctant to purchase these assets. As a result, the asset price at stage 4 would be lower than it would be if no creditor runs. Bankers at stage 3 could perfectly predict how much the capital price would drop at stage 4, knowing how many note investors run at stage 2. A large drop in capital price implies a large cost of bailing out shadow banks. If the cost is too large, then bankers would default at stage 3. In sum, bankers will default at stage 3 if they observe that there are too many investors who run on shadow banks.

Proposition 12 derives the threshold $\bar{l}_t$ such that all bankers default at stage 3 if the proportion of running investors is larger than $\bar{l}_t$, where $\bar{l}_t \equiv 1 - \bar{s}_t^*\big(1 - \big(s_{t_-} + \bar{s}_{t_-}^*\big)\kappa_t^Q\big) \Big/ \bar{s}_{t_-}^*$.

Global Games at Stage 2

At stage 2, creditors of each shadow bank play a standard global game. If creditor i runs, then she will suffer the utility loss θ_t and the benefit is to secure the investment principal. If a creditor does not run, then the creditor risks losing a proportion of the principal when the systemic event happens. There is strategic complementarity among creditors of a shadow bank because the loss to creditors who do not run increases with respect to the number of creditors who run and have their investments back at stage 4. Hence, the loss of a failing shadow bank is borne by creditors who fail to run and wait for its liquidation at stage 5.

Global games across all shadow banks are interconnected because the failure of a shadow bank is the choice of its sponsor. A banker's decision to default relies on the

market clearing price of physical assets at stage 4, which in turn is affected by the proportion of running investors in the entire economy at stage 2, according to the analysis in Section 2.3.3.

Next, we characterize a banker's payoff function with respect to her run decision at stage 2. Notice that each regular bank holds notes issued by a continuum of shadow banks. The payoff to the banker depends on decisions made by creditors of all relevant shadow banks. To simplify this analysis, we focus on the symmetric equilibrium in which all bankers use the same switching strategy. Conveniently, a symmetric equilibrium analysis yields the following three variables that are equal: The proportion of creditors who run against a single shadow bank, the proportion of notes that a shadow bank has to pay back early at stage 4, and the proportion of creditors who run in the economy. Hereafter, l_t denotes the fraction of run creditors among those who are able to do so. To explain the payoff function, suppose the proportion of run creditors $l_t\hat{l}_t$ is larger than $\bar{l}_t$ and thus all bankers default at stage 3. Hence, the capital price declines drop by $\kappa_{t_-}^{q,r}$ at stage 4 and 5. The total loss to a shadow bank is $S_{t-}^{*}\kappa_t^{Q,r}$ given its size S_{t-}^{*}. Since $l_t\hat{l}_t$ fraction of notes would be redeemed in full at stage 4, the loss per dollar note that has not been redeemed is $\kappa_t^{Q,r}\Big/\left(1-l_t\hat{l}_t\right)$. Thus, if a banker's regular bank does not run at stage 2, then her net worth at the end of the period would be $W_{t-}\big(1-\big(s_{t-}\kappa_t^{Q,r}+b_{t-}\,\kappa_t^{Q,r}\Big/\left(1-l_t\hat{l}_t\right)\big)\big)$; otherwise, her net worth would be $W_{t_-}\left(1-s_{t_-}\kappa_t^{Q,r}\right)$.

In the second case that $l_t\hat{l}_t<\bar{l}_t$, no banker defaults, and all creditors of shadow banks receive their investment back in full. Thus, by plugging the note investor's net worth into her continuation value function $\ln\left(W_t\right)\big/\,\rho+h_t$, we derive the payoff function for note investor i with respect to her run decision, which is

$$\begin{cases} -\frac{1}{\rho}\ln\left(1-\frac{b_{t-}\kappa_t^{Q,r}}{\left(1-l_t\hat{l}_t\right)\left(1-s_{t-}\kappa_t^{Q,r}\right)}\right)-\theta_t^i, & \text{if } l_t\hat{l}_t\geq\bar{l}_t; \\ -\theta_t^i, & \text{otherwise.} \end{cases} \tag{2.19}$$

The payoff function increases weakly in l_t, which satisfies the property of global strategic complementarities. Additionally, the payoff function also increases weakly in both b_{t-} and $\kappa_t^{Q,r}$, where b_{t-} is bankers' portfolio weight on notes and $\kappa_t^{Q,r}$ is the fraction by which the asset value of a shadow bank drops, conditional with respect to the systemic event. Both terms affect an investor's exposure to the systemic risk.

Two equilibria exist for the subgame starting at stage 2. One equilibrium is about the banking panic, such creditors run on all shadow banks, and bankers default on notes in expectation of shadow banks' early liquidation at stage 4. Thus, we associate the systemic event with the systemic run that creditors of shadow banks initiate at stage 2. In the Appendix, we solve for the multiple equilibria problem at stage 2 by applying the equilibrium selection mechanism developed in the global game literature, see Morris and Shin (2002).

2.3.4 Subgame Perfect Equilibrium

The following proposition defines a subgame perfect equilibrium, which endogenizes the probability that the systemic event would happen:

Proposition 12 *If σ is arbitrarily small, then the model specified in section 2.2 permits a subgame perfect equilibrium for games that start at any period t when the economy is in the normal phase, where shadow banking is active. In the equilibrium,*

at stage 1, bankers suspend their shadow banks if their private signals are lower than θ_t^*; otherwise, they do not suspend;

at stage 2, bankers ask their regular banks to run if their private signals are lower than θ_t^*; otherwise, they do not;

at stage 3, bankers default if $l_t\hat{l}_t > \bar{l}_t$; otherwise, they do not;

at stage 4, bankers and households solve their intertemporal portfolio choice problems;

at stage 5, unions of regular banks offer the same note contract $(\tilde{r}_t, \bar{s}_t^*)$,

where l_t *is the proportion of run investors among those who are able to do so,* θ_t^* *satisfies*

$$\rho\theta_t^* = -\int_{\bar{\hat{l}}}^{1} \ln\left(1 - \frac{b_{t_-}\kappa_{t_-}^{Q,r}}{\left(1-l\hat{l}\right)\left(1-s_{t_-}\kappa_{t_-}^{Q,r}\right)}\right) dl, \tag{2.20}$$

$\bar{l}_t$ *is defined by*

$$\bar{l}_t = 1 - \frac{\bar{s}_t^*}{\bar{s}_{t_-}^*}\left(1 - \left(s_{t_-} + \bar{s}_{t_-}^*\right)\kappa_{t_-}^{Q}\right), \tag{2.21}$$

$\tilde{r}_t$ *satisfies equation* (2.16)*, and* $\bar{s}_t^*$ *is such that equation* (2.15) *holds.*

The proof is deferred to the Appendix. In equilibrium, the systemic event occurs if $\theta_t < \theta_t^*$. Thus,

$$\varrho_t = E_{t-}\left[\mathbf{1}_{(\theta_t<\theta_t^*)}\right]. \tag{2.22}$$

2.3.5 Markov Equilibrium

This model has the property of scale-invariance with respect to K_0. This property implies that the equilibrium can be described by state variables normalized by K_0. Since all endogenous variables such as $\{q_t, t \geq 0\}$ are identical in economies, which are the same up to scale K_0, we use the reciprocal of the total wealth in an economy as the scaling factor.

In this chapter, we focus on a Markov equilibrium with a single state variable ω, which denotes bankers' wealth share in the economy $\left(\int_0^1 W_t^i di \Big/ q_t K_t\right)$. Later, we verify the existence of the Markov equilibrium.

Lemma 5 *The law of motion of ω_t in the Markov equilibrium is*

$$\begin{aligned} d\omega_t &= \omega_{t-}\mu^{\omega}_{t-}dt - \omega_{t-}\kappa^{\omega}_t dN_t, && (2.23)\\ \textit{where } \mu^{\omega}_{t-} &\equiv r_{t-} + s_t\big(R_{t-} - \tau - r_{t-}\big) + s^*_{t-}\big(R_{t-} - r_{t-}\big) - \mu^q_{t-} - \mu^K_{t-} - \rho - \xi, && (2.24)\\ \textit{and } \kappa^{\omega}_t &\equiv \frac{\big(s_{t-} + s^*_{t-} - 1\big)\,\kappa^Q_t}{1 - \kappa^Q_t}. && (2.25) \end{aligned}$$

Proof. See Appendix. ■

In the Markov equilibrium, all stochastic processes of endogenous variables can be expressed as functions of ω_t including q_t. Hence:

$$\mu^q_t = \frac{q'(\omega_t)}{q(\omega_t)}\omega_t\mu^{\omega}_t, \tag{2.26}$$

$$\kappa^q_t = \frac{q(\omega_{t-}) - q\big(\omega_{t-}\big(1-\kappa^{\omega}_t\big)\big)}{q(\omega_{t-})}, \tag{2.27}$$

$$\mu^{\zeta}_t = \frac{\zeta'(\omega_t)}{\zeta(\omega_t)}\omega_t\mu^{\omega}_t, \tag{2.28}$$

$$\kappa^{\zeta}_t = \frac{\zeta(\omega_{t-}) - \zeta\big(\omega_{t-}\big(1-\kappa^{\omega}_t\big)\big)}{\zeta(\omega_{t-})}. \tag{2.29}$$

from Ito's Lemma.

In the Markov equilibrium, the consumption good market clearing condition (2.4) can be simplified as

$$a\psi + \underline{a}\,(1-\psi) - g(\iota) = \Big(\rho\omega + \rho^{\frac{1}{b}}\zeta^{\frac{b-1}{b}}\,(1-\omega)\Big)\,q \tag{2.30}$$

based on the optimal consumption choices of bankers and households, where ψ is the proportion of capital held by bankers.

The following proposition establishes the existence of the Markov equilibrium and characterizes its properties with a system of delay differential equations and their boundary conditions.

Proposition 13 *The Markov equilibrium can be fully characterized by* $q[\omega]$ *and* $\zeta[\omega]$*, both of which are defined over* $(0, \bar{\omega}]$*. In addition,* $q[\omega]$ *and* $\zeta[\omega]$ *solve the system of delay differential equations implicitly defined by* $(2.5)-(2.16)$ *and* $(2.20)-(2.30)$ *with boundary conditions*

$$\begin{aligned} \mu^q[\bar{\omega}] &= \mu^\zeta[\bar{\omega}] = \mu^\omega[\bar{\omega}] = 0, \\ \lim_{\omega\to 0} q[\omega] &= \underline{q} \text{ and } \lim_{\omega\to 0}\zeta[\omega] = \underline{\zeta}, \end{aligned}$$

where $\underline{q}$ *and* $\underline{\zeta}$ *satisfy*

$$\frac{\rho}{1-b} = \frac{\rho\hat{\underline{c}}^{1-b}}{(1-b)\,\underline{\zeta}^{1-b}} + \frac{\underline{a}-\delta}{\underline{q}} + \frac{\left(\underline{q}-1\right)^2}{2\tau\underline{q}} - \hat{\underline{c}} + \lambda\left(\frac{(1-\kappa)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma}\right), \tag{2.31}$$

$$\underline{a} - \delta - \frac{\underline{q}^2-1}{2\tau} = \rho^{\frac{1}{b}}\underline{\zeta}^{\frac{b-1}{b}}\underline{q}. \tag{2.32}$$

Proof. See Appendix for the proof and associated algorithm for solving the system of differential equations. ■

2.4 Qualitative Results

In this section, we highlight several qualitative properties of this model. To do so, we computed a particular example numerically with the parameter choices shown by Table 2.1 in Section 2.5.

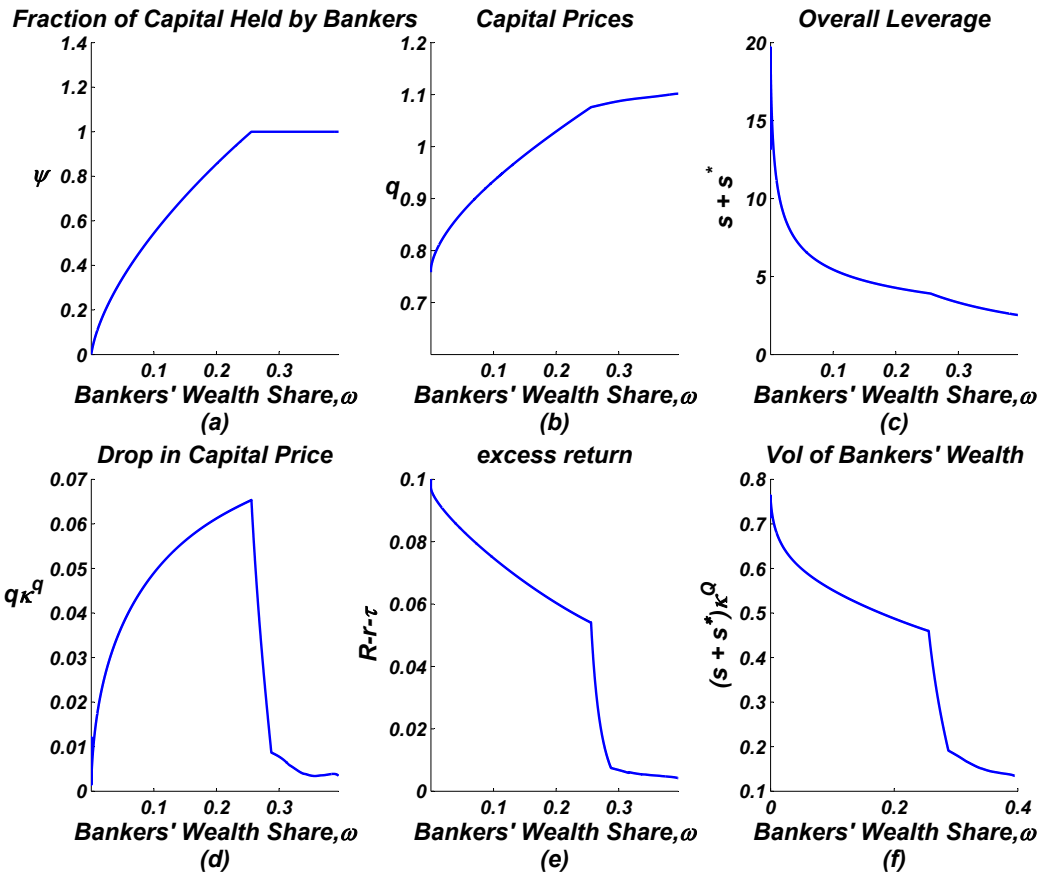


Figure 2.3: $\psi, q, \mu^q, q\kappa^q, \phi\,(=b+s)$, and s^* as functions of the state variable ω(i.e., bankers' wealth share) in equilibrim. For parameter values, see the beginning of Section 2.4.

2.4.1 Capital Misallocation and Endogenous Risk

To facilitate further analyses, we briefly mention several properties of this model that are standard in the continuous-time macro-finance literature, such as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013). We only focus on the phase where bankers are able to raise funds through shadow banking.

Due to the no equity financing setup, there is capital misallocation when bankers' wealth share is low in the economy (Panel a in Figure 2.3). As a result, as bankers own more wealth of the economy, both aggregate productivity and the capital price increase (Panel b in Figure 2.3).

As is the case with standard models that have financial frictions, no equity financing friction gives rise to the endogenous risk in this model. If bad shocks hit the asset side of their balance sheets, then bankers' net worth declines substantially due to the leverage effect. As the result of their risk management, all banks sell physical assets on the market simultaneously. The asset price drops accordingly, which in turn hurts bankers' net worth further and triggers yet another round of loss spiral. Since the overall leverage that bankers can take is mechanically low when their wealth share is large (Panel c in Figure 2.3), the impact of the loss spiral is less significant. On the other hand, when bankers seize only a small fraction of physical assets in the economy, the endogenous risk is small, due to the low selling pressure. Panel d in Figure 2.3 shows the hump-shaped property of the endogenous risk.

As in other continuous-time macro-finance papers, the risk premium and the volatility of productive agents' net worth is counter-cyclical in this model (Panels e and f in Figure 2.3).

2.4.2 Threshold of Triggering a Systemic Run: θ^*.

The value of the threshold variable θ^* depends on relevant endogenous variables $\kappa^{Q,r}$, b, and $\bar{l}$. Figure 2.4 shows that the driving force of high θ^* in economic booms is creditors' large exposure to shadow banking. Further, the decline in the threshold $\bar{l}$ also contributes to high θ^*. Given the note market clearing condition $b = s^*$, it is shadow banking's high leverage that causes the increase in the systemic risk.

θ^* is rather low when the state is between 0.25 and 0.3 for this example. $\kappa^{Q,r}$ has two kinks, which is similar to $q\kappa^q$. The second factor is that the threshold $\bar{l}$ rises sharply between the two kinks, which implies that it requires a large fraction of run creditors to trigger shadow banks' default. The increase in $\bar{l}$ is due to the rise of the leverage for shadow banking between two kinks. Hence, if the economy is around a

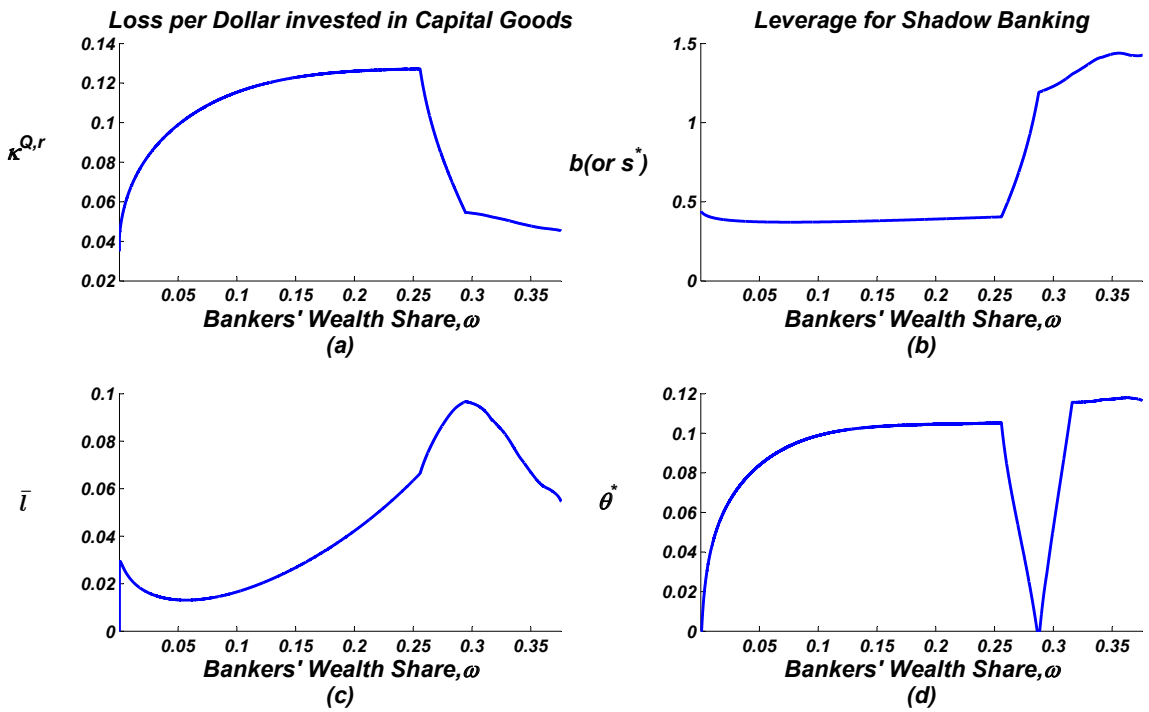


Figure 2.4: This figure shows the $\kappa^{Q,r}$, $b\,(s^*)$, $\bar{l}$, and θ^* as functions of the state variable ω. For parameter values, see Table 2.1 in Section 2.5.

state between two kinks, then a bad shock leads to a substantial drop in the leverage for shadow banking, which results in a large threshold $\bar{l}$ according to equation (2.21).

2.4.3 Dynamics of the State Variable

Panel a in Figure 2.5 shows that the economy typically stays in states where the systemic risk is low (i.e., between $\omega = 0.25$ and $\omega = 0.3$). However, the economy occasionally enters a high systemic risk regime due to shadow banking's expansion, as shown by Panel b in Figure 2.5. If the economy is indeed in the high systemic risk regime, the decline in the state variable ω due to the systemic event is much larger than its drop in the absence of it (see Panel c of Figure 5). This is mainly due to the fact that the asset price drops substantially as the economy drops from the normal phase to the distressed phase given the systemic event (Panel d in Figure 2.5). Notice

that the drop of ω leads to a further decrease in capital price, which in turn hurts bankers' net worth. This adverse feedback loop is drastic, as the asset price is lower in the distressed phase than it is in the normal one.

2.4.4 Heterogeneity of Credit Markets.

Panel b in Figure 2.5 illustrates the different behaviors of regular banking and shadow banking. In states where bankers' wealth share is large, the growth of shadow banking is accompanied by the decline of regular banking.

As Figure 2.5 shows, this model tells a particular story about the crisis of a systemic run. In economic upturns, financial institutions–especially shadow banks–accumulate lots of physical assets, and the economy grows gradually if it is not hit by any bad shock. Meanwhile, asset volatility goes down. This declining volatility then accelerates the expansion of shadow banking (i.e., the substitute of regular banking). However, if the economy is overheated, then the credit boom may go wrong. Specifically, the improper expansion of shadow banking exposes the economy to the risk that the shadow banking system may collapse suddenly.

The instability of shadow banking has two features. First, a normal negative shock could cause a self-fulfilling panic, thereby leading to a credit market breakdown. Second, the collapse of shadow banking is preceded by its expansion in a tranquil period (see Panels a and b of Figure 2.5).

Market Illiquidity and Credit Risk. There is liquidity mismatch in shadow banks' balance sheets (i.e., bankers' hidden balance sheets). Notes are liquid because they are short-term and investors could stop rolling over them at any time they like. Physical assets are less liquid because it is costly to convert them into consumption goods, and the only option left is to sell them in the market. Although the market

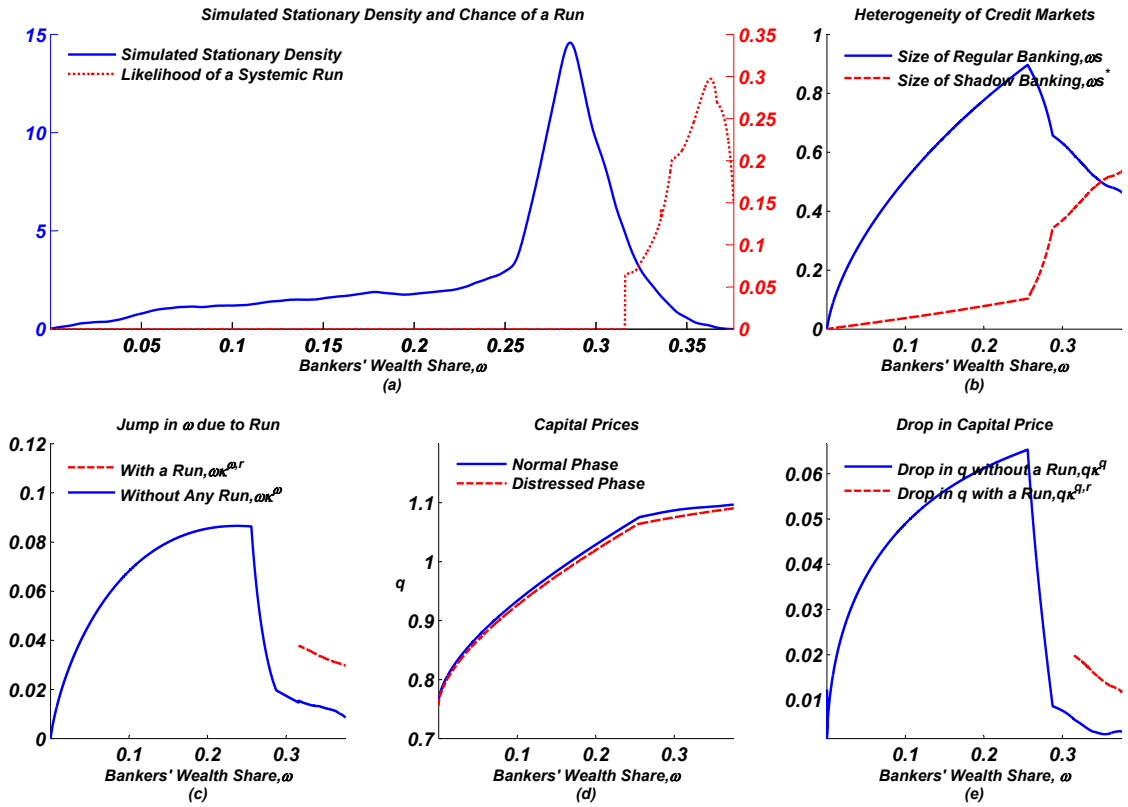


Figure 2.5: Panel (a) shows the estimated density of simulated stationary distribution (solid line) and the likelihood that a systemic run would occur, given a Poisson shock (dashed line); Panel (b) displays riskless debt outstanding (solid line) and note outstanding (dashed line); Panel (c) reports the drop size of the state variable ω if a systemic run does not happen (solid line) and the drop size if a systemic run does occur (dashed line); Panel (d) shows the capital price in the normal phase (solid line) and capital price in the distressed phase (dashed line); Panel (e) reports the drop size in capital price if a systemic run occurs (solid line) and the drop size if a systemic run does not happen. For parameter values, see Table 2.1 in Section 2.5.

for capital goods itself is fully liquid, shadow banks have to sell at a very low price during a market-wide panic because all shadow banks dump physical assets at the same time, and regular banks find it very costly to hold too many physical assets on their balance sheets. As Panel e of Figure 2.5 shows, the capital price drops a lot when the economy enters the distressed phase. This market illiquidity worsens the credit risk of each shadow bank, which in turn intensifies the banking panic.

2.4.5 Endogenous Risk and Systemic Risk

Panels a and e of Figure 2.5 show that the risk of a systemic run elevates as endogenous risk declines. The leverage behavior of shadow banking explains why systemic risk and endogenous risk move in opposite directions. The maximum leverage of shadow banking goes up when the endogenous risk goes down. Thus, as the endogenous risk declines, creditors of shadow banks are increasingly exposed to systemic risk, which in turn increases their incentives to run, as well as increases the likelihood of the banking panic.

The systemic risk of banking panics shares some features with endogenous risk. First, systemic risk is also endogenous. Second, when systemic risk materializes, it also involves the adverse feedback loop and the margin spiral related to reintermediation, the process in which shadow banks fire sell physical assets. However, the impact of an exogenous shock is more profound quantitatively when the systemic risk comes on stage.

There are some qualitative differences between the two types of risks in this model. In contrast to situations that involve endogenous risk, shadow banks fail massively upon realizing systemic risk and the note market closes for a certain period afterwards, which explains why the systemic risk causes more severe consequences. On the other hand, the materialization of systemic risk is much less frequent than that experienced by the endogenous risk; the former shows up only occasionally, while the latter unfolds every time when a negative shock hits. Hence, systemic risk is not as frequently observed as endogenous risk is. Moreover, systemic risk hides in the economy in the sense that its outbreak is preceded by a quiet process, during which time both the endogenous risk declines and the leverage for shadow banking increases smoothly. This observation implies that it might not be possible to gauge the systemic risk

precisely, for there may be a misinterpretation of the connection between endogenous risk observed everyday and the systemic risk that the economy would face occasionally.

2.4.6 Minsky Moments

The qualitative prediction of this model is consistent with Minsky's Financial Instability Hypothesis summarized by Minsky (1992). In this model, the economy is not always in states where the likelihood of the systemic run is positive, which reflects the first theorem of Minsky's hypothesis: that an economy has both stable financing regimes and unstable financing regimes. The economy enters unstable financing regimes in this model because of the expansion of the shadow banking sector and the increase in creditors' exposure to the risk of a systemic run. The blossoming of the shadow banking sector, in turn, is the consequence of either prolonged economic growth or extended periods that did not include any bad shock. The fact that long-term economic growth can endanger the financial system is consonant with the second theorem of Minsky's hypothesis: that the economy shifts from stable regimes to unstable regimes after a long period of economic prosperity.

The Minsky moment in this model is observed when the systemic run on the shadow banking sector occurs. Two factors contributes to the economy-wide failure of shadow banks: the endogenous decline in asset prices, as well as bankers' refusal to offer protection to shadow banks in trouble. These two factors not only reinforce each other but also turn the business of shadow banking into Ponzi finance, as defined by Minsky (1992). The lack of support from their sponsors and the decline in asset prices make the asset value of shadow banks short of their liability value. Moreover, the systemic run on shadow banks disallows these banks to rollover their debts, which further supports the presence of the Ponzi financing in shadow banking.

Different from Minsky's original hypothesis, shadow banking shifts to Ponzi financing in this model only when creditors initiate a market-wide run on shadow banks. This property shows that it is self-fulfilling to market, whether or not a financing business can be classified as Ponzi finance in this model. This result highlights the intrinsic instability of the financial market as well as the difficulty of detecting Ponzi finance, which destabilizes the economy.

2.5 Quantitative Exercises

In this section, we calibrate the model and compare the quantitative performance of this study with that of He and Krishnamurthy (2012a) (hereafter HK), who calibrate a continuous-time macro-finance model and use it to match several empirical facts. In addition, we use this model to assess the likelihood that a systemic run might occur before the Great Recession.

2.5.1 Calibration

The choices of preference parameters ρ, γ, and b and technology parameters δ and τ are consistent with the literature. For instance, Gertler and Kiyotaki (2013) and Gertler and Kiyotaki (2010) use quarterly discount factor $\beta = 0.99 \, (\approx \exp(-0.25 \times 0.04))$ and He and Krishnamurthy (2013) choose $\rho = 4\%$; for γ, Gertler et al. (2012) and He and Krishnamurthy (2013) pick the same value; Gruber (2006) estimates that the elasticity of intertemporal substitution is 2 using non-durable consumption data; for δ, HK, Gertler and Kiyotaki (2010), and Gertler et al. (2012) choose 10% depreciation rate; and $\tau = 3$ follows He and Krishnamurthy (2012a).

Table 2.1: Parameters

Parameter		Choice	Target
Panel: Preference			
ρ	Time discount rate	0.04	Literature
γ	Risk aversion	2	Literature
$1/b$	EIS	2	Literature
Panel: Technology			
a	Productivity of Bankers	0.21	Investment Rate
$\underline{a}$	Productivity of Households	0.1	Price-to-dividend ratio
δ	Depreciation rate	0.1	Literature
τ	Capital adjustment cost	3	Literature
ξ	Bankers exit intensity	0.15	Steady state
Panel: Shocks			
λ	Poisson shock intensity	1	Literature
κ	Poisson shock magnitude	0.035	Consumption volatility
Panel: Frictions			
τ	Managerial cost	0.05	Leverage
l_0	Leverage constraint	1.45	Securitization

We choose $a = 0.21$ to match the average investment-to-capital ratio, which is 11% from 1973 to 2010 in data according to HK, and it is 11.63% in our simulated model. The productivity parameter of households a^h affects the boundary conditions as the state variable $\omega \to 0$. We set $a^h = 0.1$ to target the lowest price dividend ratio 10 in the past 100 years in the US, as in Muir (2013). In this model, $a^h = 0.1$ yields the lowest price-dividend ratio 9.88. We adjust the banker exit rate ξ so that banks' leverage is reasonable around the stochastic steady state (i.e. the state in which $\mu^\omega - \lambda\kappa^\omega = 0$). In the simulation, the leverage around the stochastic steady state is 3.83.

There are two parameters to choose for the Poisson shock. The intensity λ is set to be 1 as in Brunnermeier and Sannikov (2013). Also, $\kappa = 0.035$ is set to match the volatility of consumption and the investment growth rate during the tranquil period, which is discussed in detail later.

For the managerial cost, τ, its value affects the leverage of bankers in the model. A high τ leads to low leverage chosen by regular banks. For the purposes of this study,

the target is the same as it is in Gertler and Kiyotaki (2010): banks' leverage being 4. On the other hand, the leverage for shadow banking financing is affected by the parameter l_0. The value of l_0 is selected such that the likelihood of a systemic run is most likely when the size of shadow banking is comparable to that of regular banking, which is used to finance physical assets.

Further, $\pi = 1/3.4$ is set so that the economy would be in the distressed phase for 3.4 years on average, which is the average year duration of downturn found by Reinhart and Rogoff (2009). Given this average duration, the upper and lower bounds of θ are adjusted such that the unconditional probability of the economy being in the distressed phase is close to 7%.

Finally, $\hat{l}$ involves two regularity problems. The first has to do with the global complementarity property, which can be guaranteed by $\hat{l} < 1 - \kappa^{Q,r}$. The second is also associated with the payoff function that investors have with respect to the run decision. When s is very large, investors could also have a very strong incentive to run, although their exposure to the failure of notes itself is low. This is because the substantial net worth loss due to high s considerably increases the marginal benefit of extra money that a banker could have. Hence, even when investors' exposure to the risk that notes might fail is not large in absolute terms, the marginal benefit of recovering their investments in notes could still be very high. Therefore, the incentive to run could also be very high. This channel looks less sensible in the real world. We attempt to dampen the impact of this channel by choosing a relatively low value of $\hat{l}$.

2.5.2 Nonlinearity and Conditional Moments

To assess the quantitative performance of our model, we compare the nonlinear results of HK and what they find in the data with the results of this study. The nonlinearity in the model refers to the fact that moments of endogenous variables,

such as the volatility of equity growth rate, conditional on that the banking sector is under-capitalized is significantly different than those moments conditional on that the banking sector is well-capitalized. See HK for the detail of the data used in that paper and how the data are processed.

For each simulation, we run the model for 5000 years and only record the data for the last 3000 years. We then simulate the model 2000 times and report the means of moments for simulations. Then, we report these results as well as those from HK in Table 2.2. Following HK, we also classify periods with the top one-third of Sharpe ratio realizations as "financially distressed" periods and others as "financially tranquil" periods.[5] The Sharpe ratio is defined as the ratio of the excess rate of return from holding capital over its expected loss (i.e., $SR=\left(R-r-\tau-\lambda\kappa^{Q}\right)/\left(\lambda\kappa^{Q}\right)$).

Table 2.2: Conditional Moments (in percentage)
This table displays standard deviations and covariances for bankers' net worth (Eq), investment growth (I), and consumption growth (C), and their counterparts in the data and in He and Krishnamurthy (2012a). As in He and Krishnamurthy (2012a), growth rates are defined as annual changes in log value from t to $t+1$. Numbers under column "Data" and "HK" are from the "Data" and "Baseline" column in Table 4 in He and Krishnamurthy (2012a). Numbers in the "Model" column are calculated based on the simulation of the full model, whose details are described in the text. The "Finanacially Distressed Periods" are classified as periods with the highest one-third Sharpe ratio realizations.

Financially Tranquil Periods							
	vol(Eq)	vol(I)	vol(C)	vol(SR)	cov(Eq,I)	cov(Eq,C)	cov(Eq,SR)
Data	17.54	6.61	1.28	12.72	0.07	0.03	-0.14
HK	6.4	4.8	2.4	8.7	0.3	0.1	-0.2
Model	14.37	5.86	1.53	12.22	0.43	-0.05	-0.18
Financially Distressed Periods							
	vol(Eq)	vol(I)	vol(C)	vol(SR)	cov(Eq,I)	cov(Eq,C)	cov(Eq,SR)
Data	31.48	8.05	1.71	60.14	1.31	0.25	-6.81
HK	31.2	5.4	1.8	71.1	0.9	0.0	-13.0
Model	57.78	19.35	4.75	20	5.46	0.32	-3.84

As indicated in Table 2.2, the performance of this model is slightly better than that of HK in terms of volatility terms for the "financially tranquil" periods. For the

[5]In He and Krishnamurthy (2012a), they are named as "distress" and "non-distress" periods. To avoid the confusion with the distressed phase in this chapter, we rename them as "financially distressed" and "financially tranquil" periods.

"financially distressed" periods, the basic nonlinearity property is well preserved. For instance, the volatility of consumption growth in this model is higher in the tranquil periods, which is true in the data but not true in HK. However, the volatility terms of equity growth, investment growth, and consumption growth in this model are larger than their counterparts for this data and HK, mainly due to an assumption in this model's setup: that households also could hold physical assets, although they are less productive. However, households in HK are unproductive. Since bankers sell capital to households in the "financially distressed" periods in this model, the aggregate productivity and, thus, total output are more volatile in those periods. Therefore, the growth rates of equity, investment, and consumption are more volatile in "financially distressed" periods. On the other hand, HK match the volatility of the Sharpe ratio in the data for the "financially distressed", since they calibrate their model such that its highest Sharpe ratio is 15 times the average level, which is consistent with the finding of Gilchrist and Zakrajšek (2012).

2.5.3 Data

We use data that associated with endogenous variables in the model to estimate the underlying state variable of the economy. Specifically, we look at endogenous variables that represent three aspects of the economy: price, quantity, and volatility.

We assess the systemic risk of the Great Recession based on the data of the Sharpe ratio, securitization, and the volatility of the stock market. Following HK, we compute the monthly Sharpe ratio based on the excess bond premium of Gilchrist and Zakrajšek (2012). The securitization data are drawn from the "Flow of Funds Accounts of the United States." Appendix 2.8 contains the construction of the ratio of securitization. We use the Chicago Board Options Exchange Volatility Index (VIX) as the measure of the stock market volatility.

Upper panels of Figure 2.6 summarize the movements of the three variables based on this data. Three plots on the top show that both the Sharpe ratio and VIX index were around their historically low level in early 2007, and the ratio of securitization was well above its trend around the same time. Finally, we use the detrended data about the ratio of securitization to assess the systemic risk.

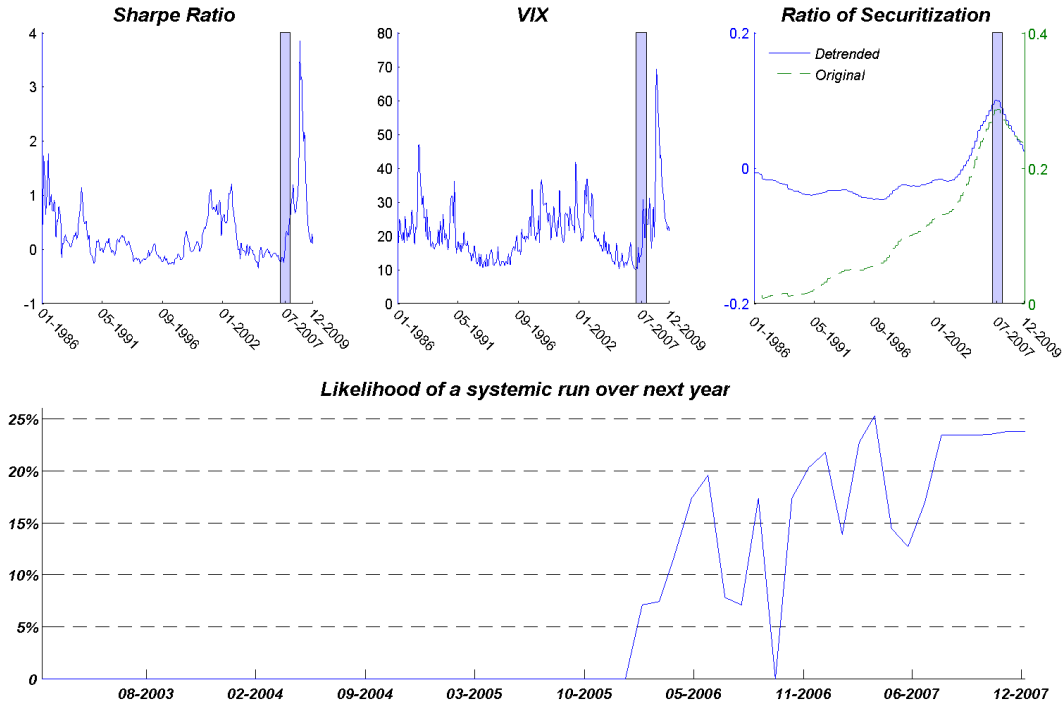


Figure 2.6: In this figure, the upper panels from left to right show monthly Sharpe ratio, the monthly Chicago Board Options Exchange Volatility Index, and the quarterly ratio of securitization done by non-agency Asset-Backed Security issuers over aggregate originated loans from January, 1986 to December, 2009. The lower panel presents the estimated probability that a systemic run would happen over the next year from January, 2003 to January, 2007. We first estimate the underlying state of the economy at a given by using the method described in the current section. Given that the economy starts from the estimated state, we simulate the model for one year 10000 times and report the frequency of the simulation for which a systemic run happens within one year.
Source: See Appendix 2.8

2.5.4 Assessment of the Systemic Risk

We first define endogenous variables that can be linked to available data. The Sharpe ratio is denoted by SR. The volatility of the stock market (Vol) is defined as the volatility of bankers' net worth (i.e., $Vol = (s + s^*)\,\kappa^Q$). The ratio of securitization (RA) has a natural definition (i.e., $RA = s^*/(s + s^*)$). Figure 2.7 shows the behaviors of these endogenous variables in the equilibrium. It is clear that when the systemic risk is high, the Sharpe ratio and stock market volatility tend to be lower than their average level, and the ratio of securitization is higher than its average level.

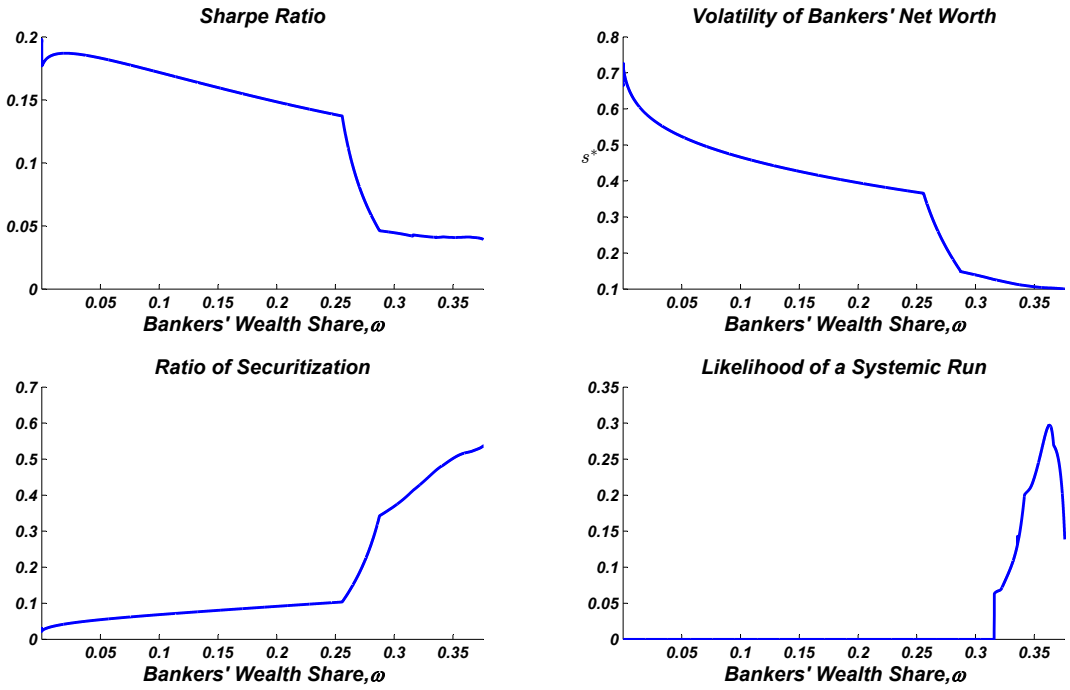


Figure 2.7: This figure shows the Sharpe ratio (the upper left panel), the volatility of the stock market return (the upper right panel), the ratio of securitization (the lower left panel), and the likelihood of a systemic run (conditional on a bad capital quality shock, as depicted in the lower right panel) as functions of the state variable ω in Equilibrium R. The Sharpe ratio is defined as $\left(R - r\tau - \lambda\kappa^Q\right)/\left(\lambda\kappa^Q\right)$, the ratio of securitization as $s^*/(s + s^*)$, and the volatility of stock market return as $(s + s^*)\,\kappa^Q$. For parameter values, see Table 2.1 in Section 2.5.

To find the underlying state of an economy at a given time, we search for a state that minimizes the distance between endogenous variables in the model and their

counterparts in the data. Since the range of some variables is different from their range in the data, such as the Sharpe ratio, we first standardize variables and then calculate the distance. For instance, given (SR_t, RA_t, Vol_t) at time t from the data, to find the corresponding ω_t, we solve the following problem:

$$\max_{\omega} : \left(\widehat{SR}\left[\omega\right] - \widehat{SR_t}\right)^2 + \left(\widehat{RA}\left[\omega\right] - \widehat{RA_t}\right)^2 + \left(\widehat{Vol}\left[\omega\right] - \widehat{Vol_t}\right)^2,$$

where $\hat{x}$ is the standardized x. In doing so, we follow the approach used in Eisfeldt and Muir (2012) and Muir (2013).

To show that this model is able to provide early warning signs, we only use data available up to the end of 2007 for the standardization of variables. Given the corresponding state of an economy, we simulate the model and calculate the probability that a systemic run occurs in the economy over the next few years. The lower panel of Figure 2.6 shows that the risk of a systemic run gradually increased from 0 percent in 2003 to 25 percent in 2007.

2.6 Final Remarks

The idea that the instability of credit causes financial crises can be traced back to nineteenth-century economists such as John Stuart Mill and Alfred Marshall (Hansen, 1951). Oft-cited modern works on this idea include Minsky (1986) and Kindleberger (2000). In addition, recent empirical works based on long-run historical data sets for developed countries, such as Schularick and Taylor (2012) and Jordà et al. (2011), show that the predictive power of a credit boom for a financial crisis is statistically significant. Nevertheless, the importance of the instability of credit for financial stability was undervalued from time to time, reflecting Charles Kindleberger's quote in this chapter's introduction. In short, every time that we tame the credit of a

new form that caused the last financial crisis, we tend to think that theories on the instability of credit become obsolete.

This current model could be extended in several directions. For instance, one could investigate the extent to which a risk of systemic run within the shadow banking sector can be transmitted into the traditional banking sector. Also, one could incorporate the labor market into our model and quantify the impact of a systemic run on shadow banking in the real sectors.

The policy implication of this model is straightforward. According to this model, the government should take active actions to slow down the growth of the shadow banking system and perhaps slow down the economic growth as well if the shadow banking system is overheated, for the risk of systemic run is very high. On the other hand, the government is reluctant to sacrifice economic growth for the sake of financial stability. This model then offers a one-to-one relationship between economic growth and systemic risk such that the government can find an ideal combination based on its specific objectives.

2.7 Appendix A:Proofs

Proof of Proposition 9. We define the continuation value of a banker by

$$J(W_t, \omega_t) = \max_{\{c_u, s_u, b_u, s_u^*, \iota_u, u \geq 0\}} E\left[\int_t^\infty e^{-\rho u} \ln(c_u)\ du\right].$$

T denotes the time when the banker retires. If W_T is positive, her continuation value is

$$\big(\ln(W_T) + r - \rho\big)\big/\,\rho;$$

otherwise, the continuation value would be negative infinity. Given that a banker will retire almost surely, the strategy, which yields negative net worth upon retirement, would be less optimal than the strategy of always holding risk-free debts before retirement. Thus, W_T is positive almost surely.

Next, we show that W_t is positive almost surely. Suppose $t < T$ and there is no bad shock between t and T, then

$$W_T = W_t \exp\left[\int_t^T \left(r_u + s_u\big(R_u - \tau - r_u\big) + b_u\big(\tilde{r}_u - r_u\big) + s_u^*\left(R_u - \tilde{r}_u\right) - \rho_u\right)\, du\right]$$

given any consumption and portfolio choices $\{c_u, s_u, b_u, s_u^*, t \leq u \leq T\}$. Since it has positive probability that there is no bad shock between t and T, W_t must be positive almost surely. Then, we show that (2.7).

We proceed to show (2.8), (2.9), and (2.10). Conjecture that $J\big(W,\omega\big) = \ln(W)/\,\rho + h(\omega)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation

$$\rho J\big(W,\omega\big) = \max_{c,s,b,s^*} \left\{ \begin{array}{c} \ln(c) + W\left(r + s\left(R - \tau - r\right) + s^*\left(R - \tilde{r}\right) + b(\tilde{r} - r) - c\right) J_W + \omega\mu^\omega J_\omega \\ +\lambda\left(J\left(W\left(1 - \left(s + s^*\right)\kappa^Q\right), \omega - \omega\kappa^\omega\right) - J\left(W,\omega\right)\right) \\ +\xi\left(\left(\ln\left(W\right) + r - \rho\right)/\,\rho - J\left(W,\omega\right)\right) \end{array} \right\}.$$

(2.8), (2.9), and (2.10) are just first-order conditions with respect to s, b, and s^*. Furthermore, $h\left[\omega\right]$ has to satisfy

$$\begin{aligned} \rho h\left(\omega\right) \;=\; & \ln\left(\rho\right) + \frac{1}{\rho}\left(r + s\left(R - \tau - r\right) + s^*\left(R - \tilde{r}\right) + b(\tilde{r} - r) - \rho\right) + \omega\mu^\omega h'\left[\omega\right] \\ & +\lambda\left(\frac{1}{\rho}\ln\left(1 - \left(s + s^*\right)\kappa^Q\right) + h\left(\omega\left(1 - \kappa^\omega\right)\right) - h\left(\omega\right)\right) \\ & +\xi\left(\left(r - \rho\right)/\,\rho - h\left(\omega\right)\right). \end{aligned} \tag{2.33}$$

To complete the argument, the transversality condition

$$\lim_{t\to\infty} E\left[\exp\left(-\rho t\right) J\left(W_t, \omega_t\right)\right] = 0.$$

also must hold ■

Proof of Proposition 10. Combining with the transversality condition

$$\lim_{t\to\infty} E\left[\exp\left(-\rho t\right) V\left(W_t^h, \zeta_t\right)\right] = 0,$$

the HJB equation with respect to $V\left(W^h, \zeta\right)$ is sufficient to yield a household's optimal choices at each state ω. Plugging $V\left(\zeta_t, W_t^h\right) \equiv \left(\zeta_t W_t^h\right)^{1-\gamma} \Big/ \left(1-\gamma\right)$ into (2.11), we have

$$0 = \max_{\underline{c},\underline{s}} \left\{ \begin{array}{c} \frac{1}{1-b}\left\{ \frac{\rho\left(c^h\right)^{1-b}}{\left(\zeta W^h\right)^{\gamma-b}} - \rho\left(\zeta W^h\right)^{1-\gamma} \right\} \\ +\left(r + s^h\left(R^h - r\right) - c^h\right)\zeta^{1-\gamma}\left(W^h\right)^{-\gamma} + \mu^\zeta \zeta^{1-\gamma}\left(W^h\right)^{1-\gamma} \\ +\lambda\left(\frac{\left(\zeta\left(1-\kappa^\zeta\right)W^h\left(1-s^h\kappa^Q\right)\right)^{1-\gamma}}{1-\gamma} - \frac{\left(\zeta W^h\right)^{1-\gamma}}{1-\gamma}\right) \end{array} \right\},$$

where To take the first order condition with respect to $\underline{c}$ and $\underline{s}$, we have (2.12) and (2.13). After we plug $\underline{c}$ back to above equation, we derive (2.14). ■

Proof of Lemma 4. Equation (2.33) is equivalent to the equation that $\{h_t, t \geq 0\}$ must satisfy taking into account the Ito's Lemma.

Given the aggregate state variable ω, consider a banker with net worth W, who has just defaulted. Given that she has to pay l fraction of her net worth as fines at each period, her HJB equation is

$$\rho J^a\left(W,\omega\right) = \max \left\{ \begin{array}{c} \ln\left(c\right) + \left(Wr + S\left(R-\tau-r\right) + S^*\left(R-\tilde{r}\right) + B(\tilde{r}-r) - c - l_0 W\right) J_W^a \\ +\omega\mu^\omega J_\omega^a + \lambda\left(J^a\left(W - \left(S+S^*\right)\kappa^Q, \omega - \omega\kappa^\omega\right) - J^a\left(W,\omega\right)\right) \\ +\xi\left(\left(\ln\left(W\right) + r - \rho\right)/\rho - J^a\left(W,\omega\right)\right) \end{array} \right\}.$$

Corresponding $h^a[\omega]$ is the solution of

$$\begin{aligned}\rho h^a(\omega) &= \ln(\rho) + \frac{1}{\rho}\left(r + s(R-\tau-r) + s^*(R-\tilde{r}) + b(\tilde{r}-r) - \rho - l\right) + \omega\mu^\omega h^{a\prime}[\omega] \\ &\quad + \lambda\left(\frac{1}{\rho}\ln\left(1-(s+s^*)\kappa^Q\right) + h^a(\omega(1-\kappa^\omega)) - h^a[\omega]\right) \\ &\quad + \xi\left(\frac{r-\rho}{\rho} - h^a(\omega)\right).\end{aligned}$$

Hence, $H(\omega)$, which is defined as $h(\omega) - h^a(\omega)$ satisfies

$$\rho H(\omega) = \frac{l_0}{\rho} + \omega\mu^\omega H'(\omega) - \xi H(\omega),$$

which has a solution that

$$H(\omega) = \frac{l_0}{\rho(\rho+\xi)}.$$

■

Proof of Proposition 12. We will prove the proposition by using backward induction.

First, we conjecture that, if the note market opens at stage 5, then the note contract in equilibrium at stage 5 is $(\tilde{r}_t, \bar{s}_t^*)$. In addition, we also conjecture that the proportion of run creditors $l_t\hat{l}$ on a single shadow bank is the same as the proportion of notes redeemed by them, and that the proportion of run creditors across different shadow banks is the same. We will verify these conjectures later.

Since there is no uncertainty between stage 4 and 5, bankers solve their intertemporal portfolio choice problems at stage 4 in expectation of the note contract they would have at stage 5. Households also solve their own problems at stage 4. All markets clear at stage 4 except the note market. Hereafter, we only focus on the case that a Poisson shock hits the economy after stage 2. Given the market clearing capital price

and bankers' early decisions, we can fix bankers' continuation value at the beginning of next period.

If all shadow banks fail at stage 5, which can be expected at stage 4, capital price drop by $\kappa_t^{q,r}$ fraction. Consider a banker, whose net worth at the beginning of period t is W_{t-}. If the banker runs at stage 3, her net worth would be $W_{t-}\big(1 - s_{t-}\kappa_t^{Q,r}\big)$; otherwise, her remaining net worth would be $W_{t-}\big(1 - s_{t-}\kappa_t^{Q,r} - b_{t-}\kappa_t^{Q,r} \Big/ \big(1 - l_t\hat{l}\big)\big)$.

Suppose that no shadow bank fails at stage 5. Then, the note market opens at stage 5. Now, let's look at how each bank's balance sheet changes from stage 4 to 5. Consider a banker, whose net worth and portfolio weights at the beginning of this period are W_{t-} and $\big(s_{t-}, s_{t-}^*\big)$. Given the banker's portfolio in the end of stage 5, $\big(s_t, s_t^*\big)$, the value of physical assets that she holds on her balance sheet at stage 4 is $W_t\big(s_t + s_t^*\big) - \big(1 - l_t\hat{l}\big)W_{t-}s_{t-}^*$, where $\big(1 - l_t\hat{l}\big)W_{t-}s_{t-}^*$ is the value of physical assets on the balance sheet of her shadow bank at stage 4. We can rearrange $W_t\big(s_t + s_t^*\big) - \big(1 - l_t\hat{l}\big)W_{t-}s_{t-}^*$ as $W_t s_t + \big(l_t\hat{l} - \bar{l}_t\big)W_{t-}s_{t-}^*$.

If $l_t\hat{l} < \bar{l}_t$, the managerial cost that bankers pay does not exceed the level in the normal case where there is no systemic run. Hence, the capital price drops by κ_{t-}^q fraction as what it is in the normal case. Then, the banker's net worth in the beginning of next period is $W_{t-}\big(1 - s_{t-}\kappa_t^Q - s_{t-}^*\kappa_t^Q\big)$

If $l_t\hat{l} > \bar{l}_t$, all regular banks have to pay higher managerial costs than they do in the normal case. The general equilibrium effect is that they would ask for a higher rate of return for holding physical capital and the capital price has to drop by more than κ_t^q in order to clear the capital market. $\kappa_t^{q\prime}$ denotes the fraction by which the capital price drops. Then, the banker's net worth at the beginning of next period is $W_{t-}\left(1 - s_{t-}\kappa_t^{Q\prime} - s_{t-}^*\kappa_t^{Q\prime}\right)$.

Now, we are ready to discuss bankers' strategic choices at stage 3. If $l_t\hat{l} > \bar{l}_t$, it is impossible to have an equilibrium in which no banker defaults at stage 3 because a

banker's continuation value with default $\ln\left(W_{t-}\left(1-s_{t-}\kappa_t^{Q\prime}\right)\right)\Big/\rho+h_t^a$ is larger than her continuation value without default $\ln\left(W_{t-}\left(1-\left(s_{t-}+s_{t-}^*\right)\kappa_t^{Q\prime}\right)\right)\Big/\rho+h_t$, which is implied by the enforceability constraint (11). Hence, if $l_t\hat{l}>\bar{l}_t$, the Nash equilibrium at stage 3 is that all bankers default. If $l_t\hat{l}<\bar{l}_t$, we focus on the Nash equilibrium that no banker defaults.

Given the result that if $l_t\hat{l}\leq\bar{l}_t$, no banker defaults at stage 3; otherwise, all bankers default at stage 3, we could specify the payoff function for individual bankers

$$\begin{cases} -\frac{1}{\rho}\ln\left(1-\frac{b_{t-}\kappa_t^{Q,r}}{\left(1-l_t\hat{l}_t\right)\left(1-s_{t-}\kappa_t^{Q,r}\right)}\right)-\theta_t^i, & \text{if } l_t\hat{l}_t\geq\bar{l}_t; \\ -\theta_t^i, & \text{otherwise.} \end{cases}$$

The global strategic complementarity is satisfied since the payoff function is increasing in l_t. Proposition 2.1 in Morris and Shin (2002) yields a unique equilibrium at stage 2. In the equilibrium, if bankers receive private signals smaller than l_t^*, they run; otherwise, they do not run, where θ_t^* is defined by

$$\rho\theta_t^*=-\int_{\frac{\bar{l}}{\hat{l}}}^1\ln\left(1-\frac{b_{t-}\kappa_{t-}^{Q,r}}{\left(1-l\hat{l}\right)\left(1-s_{t-}\kappa_{t-}^{Q,r}\right)}\right)dl,$$

Given creditors' switching strategies at stage 2, the fact that the idiosyncratic shock $\hat{l}_t^i$ is independent across bankers implies that the proportion of run creditors is the same as the proportion of notes redeemed by run creditors by the law of large number. By the same reason, the proportion of run creditors is the same across different shadow banks. Thus, we justify our early conjectures.

For the subgame that starts at stage 1, it is straightforward to show that, in an equilibrium, bankers suspend their shadow banks if their private signals are lower than l_t^*; otherwise, they do not suspend. This equilibrium has an advantage over other equilibria that it reduces shadow banks' creditors' exposure to the idiosyncratic

risk that they might not be able to run at stage 2, which in turn reduces the risk premium that these creditors would ask for in the previous period.

Now, let's consider the stage 5 of period $t-$ given that no shadow bank fails. We can use the same argument as what we have used for the proof of Proposition 11 to show that $\left(\tilde{r}_{t-}, \bar{s}^*_{t-}\right)$ is the market clearing note contract. Thus, we complete our proof. ■

Proof of Lemma 5. W_t denotes $\int_0^1 W_t^i di$. In a Markov equilibrium, (2.2), the optimal choice of bankers, and the defaultable debt market clearing imply that

$$\begin{aligned} dW_t &= W_{t-}\big(r_{t-} + s_{t-}\big(R_{t-} - \tau - r_{t-}\big) + b_{t-}\big(\tilde{r}_{t-} - r_{t-}\big) + s^*_{t-}\big(R_{t-} - \tilde{r}_{t-}\big) - \rho - \xi\big)dt \\ &\quad -W_{t-}\big(s_{t-} + s^*_{t-}\big)\kappa^Q_t dN_t \\ &= W_{t-}\big(\big(r_{t-} + s_{t-}\big(R_{t-} - \tau - r_{t-}\big) + s^*_{t-}\big(R_{t-} - r_{t-}\big) - \rho - \xi\big)dt\big) \\ &\quad -W_{t-}\big(s_{t-} + s^*_{t-}\big)\kappa^Q_t dN_t. \end{aligned}$$

Note bankers exit the economy at the intensity ξ. Next, consider the scaling factor $1/\, q_t K_t$.

$$d\big(q_t K_t\big) = q_{t-}K_{t-}\big(\big(\mu^q_{t-} + \mu^K_{t-}\big)dt - \kappa^Q_t dN_t\big),$$

and

$$d\big(\frac{1}{q_t K_t}\big) = \big(\frac{1}{q_{t-}K_{t-}}\big)\big(-\big(\mu^q_{t-} + \mu^K_{t-}\big)dt + \frac{\kappa^Q_t}{1-\kappa^Q_t}dN_t\big).$$

Then,

$$\begin{aligned} d\omega_t &= \omega_{t-}\mu^\omega_{t-}dt - \omega_{t-}\kappa^\omega_t dN_t, \\ \text{where } \mu^\omega_{t-} &\equiv r_{t-} + s_t\big(R_{t-} - \tau - r_{t-}\big) + s^*_{t-}\big(R_{t-} - r_{t-}\big) - \mu^q_{t-} - \mu^K_{t-} - \rho - \xi, \\ \text{and } \kappa^\omega_t &\equiv \frac{\big(s_{t-} + s^*_{t-} - 1\big)\,\kappa^Q_t}{1-\kappa^Q_t}. \end{aligned}$$

■

Proof of Proposition 13. We start with the boundary conditions as ω goes to zero. Given that $\mu^q = \mu^\zeta = \kappa^q = \kappa^\zeta = 0$, (2.31) are implied by (2.12) and (2.14), and (2.32) is the limit of the consumption good market clearing condition (2.30). At the same time, we know the limits of $\left(\mu^\omega, s, s^*, \underline{s}, \kappa^\omega, \kappa^{\omega,r}\right)$.

Next, we show how to advance q and ζ from ω to $\omega+\Delta\omega$. Given ω and $\left(q\left(\omega'\right), \zeta\left(\omega'\right) : 0 < \omega' \leq \omega\right)$ as well as $\left(\mu^\omega, s, s^*, \underline{s}, \kappa^q, \kappa^{q,r}, \kappa^\zeta, \kappa^{\zeta,r}, \kappa^\omega, \kappa^{\omega,r}, \varrho\right)$ at ω, we can calculate $q\left(\omega+\Delta\omega\right)$ and $\zeta\left(\omega+\Delta\omega\right)$ by the following procedure.

First, bankers' Euler equation (2.8) yields the difference between μ^q and r, that is,

$$\mu^q - r = \frac{\lambda(1-\varrho)\kappa^Q}{1-\left(s+s^*\right)\kappa^Q} + \frac{\lambda\varrho\kappa^{Q,r}}{1-\left(s+s^*\right)\kappa^{Q,r}} + \tau - \left(\frac{a-\delta}{q} + \frac{(q-1)^2}{2\tau q}\right),$$

and we also know the sum of r and μ^ζ by households' HJB equation (2.14), that is,

$$\begin{aligned} r+\mu^\zeta &= \frac{\rho}{1-b} - \frac{\rho\hat{\underline{c}}^{1-b}}{(1-b)\,\zeta^{1-b}} + \hat{\underline{c}} \\ &\quad -\underline{s}\left(\underline{R}-r\right) - \lambda(1-\varrho)\frac{\left(\left(1-\kappa^\zeta\right)\left(1-\underline{s}\kappa^Q\right)\right)^{1-\gamma}}{1-\gamma} \\ &\quad -\lambda\varrho\frac{\left(\left(1-\kappa^{\zeta,r}\right)\left(1-\underline{s}\kappa^{Q,r}\right)\right)^{1-\gamma}}{1-\gamma} + \frac{\lambda}{1-\gamma}, \end{aligned}$$

where

$$\underline{R} - r = \frac{\lambda(1-\varrho)\kappa^Q}{\left(1-\underline{s}\kappa^Q\right)^\gamma}\left(1-\kappa^\zeta\right)^{1-\gamma} + \frac{\lambda\varrho\kappa^{Q,r}}{\left(1-\underline{s}\kappa^{Q,r}\right)^\gamma}\left(1-\kappa^{\zeta,r}\right)^{1-\gamma}$$

by households' Euler equation (2.13) and $\hat{\underline{c}} = \rho^{\frac{1}{b}}\zeta^{\frac{b-1}{b}}$. In order to fix $\left(\mu^q, \mu^\zeta, r\right)$ at ω, we resort to the consumption good market clearing condition at $\omega+\Delta\omega$. In particular,

we pick a triple $\left(\mu^q, \mu^\zeta, r\right)$ and solve q and ζ at $\omega + \Delta\omega$ by Ito's Lemma, i.e.,

$$\begin{aligned} q\left(\omega + \Delta\omega\right) &= q\left(\omega\right) + \Delta q = q\left(\omega\right) + \Delta\omega \frac{q\left(\omega\right)\mu^q}{\omega\mu^\omega}, \\ \zeta\left(\omega + \Delta\omega\right) &= \zeta\left(\omega\right) + \Delta\zeta = \zeta\left(\omega\right) + \Delta\omega \frac{\zeta\left(\omega\right)\mu^\zeta}{\omega\mu^\omega}. \end{aligned}$$

Given q and ζ, we can solve for $\left(s, s^*, \underline{s}, \kappa^q, \kappa^{q,r}, \kappa^\zeta, \kappa^{\zeta,r}, \kappa^\omega, \kappa^{\omega,r}, \varrho\right)$ based on Euler equations $(2.8) - (2.10)$ and (2.13), the enforceability constraint (2.15), systemic run related equations $(2.20 - 2.22)$, and Ito's Lemma

$$\begin{aligned} \kappa^q &= 1 - \frac{q\left(\left(\omega + \Delta\omega\right)\left(1 - \kappa^\omega\right)\right)}{q\left(\omega + \Delta\omega\right)}, \\ \kappa^\zeta &= 1 - \frac{\zeta\left(\left(\omega + \Delta\omega\right)\left(1 - \kappa^\omega\right)\right)}{q\left(\omega + \Delta\omega\right)}, \end{aligned}$$

where

$$\kappa^\omega = \frac{\left(s + s^* - 1\right)\kappa^Q}{1 - \kappa^Q}.$$

Given $\left(q, \zeta, s, s^*, \underline{s}\right)$ at $\omega + \Delta\omega$, we check if the market clearing condition

$$a\left(s + s^*\right)\left(\omega + \Delta\omega\right) + \underline{a}\underline{s}\left(1 - \omega - \Delta\omega\right) - \delta - \tfrac{q^2 - 1}{2\tau} = \left(\rho\left(\omega + \Delta\omega\right) + \rho^{\frac{1}{b}}\zeta^{\frac{b-1}{b}}\left(1 - \omega - \Delta\omega\right)\right) q$$

holds. This is how we solve $\left(\mu^q, \mu^\zeta, r\right)$ at ω. Before proceeding to next iteration, we solve for μ^ωwithout knowing $\left(\mu^q, r\right)$ at $\omega + \Delta\omega$ by

$$\begin{aligned} \mu^\omega &= r + s\left(R - \tau - r\right) + s^*\left(R - r\right) - \mu^q - \mu^K - \rho - \xi \\ &= \left(s + s^* - 1\right)\left(\frac{\lambda(1 - \varrho)\kappa^Q}{1 - \left(s + s^*\right)\kappa^Q} + \frac{\lambda\varrho\kappa^{Q,r}}{1 - \left(s + s^*\right)\kappa^{Q,r}}\right) \\ &\quad + \left(1 + s^*\right)\tau + \frac{a - \delta}{q} + \frac{\left(q - 1\right)^2}{2\tau q} - \mu^K - \rho - \xi, \end{aligned}$$

where the second equality comes from bankers' Euler equation at $\omega + \Delta\omega$. Thus, we have computed $\left(\mu^\omega, s, s^*, \underline{s}, \kappa^q, \kappa^\zeta, \kappa^\omega\right)$ and proceed.

Since bankers will exit the economy with a certain probability, the state variable has an endogenous upper bound denoted by $\underline{\omega}$. By definition, $\mu^{\omega} = 0$ at $\underline{\omega}$. By Ito's Lemma, $\mu^{q} = \mu^{\zeta} = 0$ at $\underline{\omega}$.

Thus, we have described the algorithm of solving for the equilibrium and also showed that the equilibrium can be fully characterized by a system of delay differential equations, i.e., the existence of the Markov equilibrium. ■

2.8 Appendix B: Data

We use monthly data to estimate the chance that a systemic run might happen for a given month. When daily data are available , say VIX, we sample the data point in the middle of the month; when only quarterly data are available, say securitization data, we use the variable's value at the corresponding quarter as the substitute for its value at each month.

Sharpe Ratio. To follow He and Krishnamurthy (2012a), when calculating the monthly Sharpe ratio, we subtract "predicted GZ credit spread" from "GZ credit spread" and divide the residual term by the "predicted GZ credit spread". The "predicted GZ credit spread" term that we choose is the one that excludes the effects of term structure and interest rate volatility.

Securitization. We follow Loutskina (2011a) to compute the ratio of securitization. The difference is that we focus on securitization done by non-agency security issuers. All data are drawn from the "Flow of Funds Accounts of the United States". There are 5 loan categories. The details of items for each category are listed in Table 2.3.

Volatility. We use the Chicago Board Options Exchange Volatility Index as the measure of the stock market volatility since January, 1986.

Table 2.3: Details of Securitization Data

	Outstanding	Securitized
Home Mortgages	FL383165105	FL673065105
Multifamily Residential Mortgages	FL143165405	FL673065405
Commercial Mortgages	FL383165505	FL673065505
Commercial and Industrial Loans[1]	FL253169255 FL263168005 FL263169255	FL673069505
Consumer Credit	FL153166000	FL673066000

1 Becuase item FL253169255 is not available now, we use the ratio of securitization calculated by Loutskina (2011a) to estimate the outstanding commercial and industrial loans.

Chapter 3

A Tale of Two Items Sent Off-Balance Sheet: Securitization and C&I Loan Commitment

3.1 Introduction

Via securitization, banks transfer their outstanding loans off their balance sheets and raise liquidity by selling capital markets claims to those loans. Hence, securitization effectively increases the liquidity pool that banks can access. The development of the securitization market in the past two decades substantially increases options that banks could have for managing their liquidity. For instance, Loutskina (2011b) highlights that because of the growth of the securitization market, U.S. banks tend to lower their holding of liquid assets on balance sheets.

In this chapter, we move forward and investigate how securitization influences banks' issuance of commercial and industrial (C&I) loan commitments. As an off-balance-sheet item, the issuance of C&I loan commitments is a critical line of liquidity pro-

vision business that banks conduct. Kashyap et al. (2002) emphasize that if deposit withdraws are imperfectly correlated with loan commitments takedowns, both banking activities can share the cost of maintaining the pool of liquid assets.[1] Furthermore, Gatev and Strahan (2006) find that banks have the advantage in offering liquidity in the form of loan commitments because they experience deposit inflows when the market liquidity declines systematically.

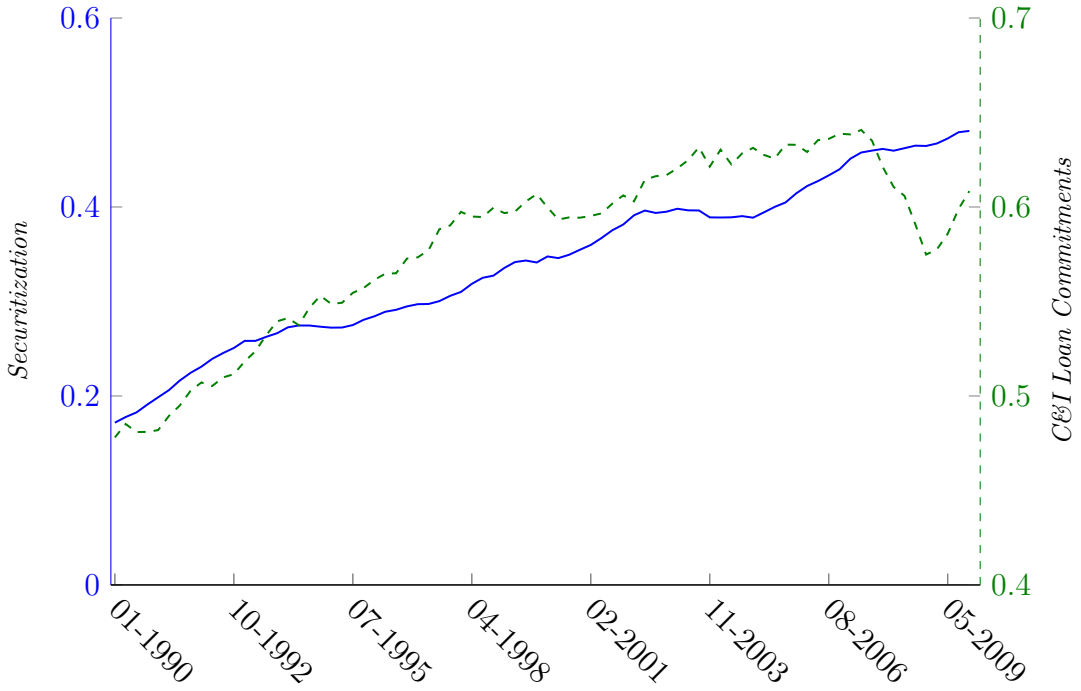


Figure 3.1: Securitization and C&I Loan Commitments. This figure shows the relation between the fraction of outstanding loans in the U.S. economy that has been securitized (solid line, left-hand-side scale) and the U.S. banks' exposure to C&I loan commitments (total C&I loan commitments divided by the sum of C&I loan commitments and C&I loan, dashed line, the right-hand-side scale). Loan data are from the "Flow of Funds Accounts of the United States" and the bank-level loan data are from "Call Reports"

The development of the securitization market lowers the stockpile of liquid assets that a bank has to maintain for accommodating unpredictable liquidity needs of its customers. Similarly, if a bank expects to securitize its outstanding loans more easily, it has freedom to benefit from extending a line of credit to a new customer. At the

[1]Holding liquid assets involves both opportunity costs and agency costs for financial institutions (Flannery, 1994; Myers and Rajan, 1998).

aggregate level, we observe that as the fraction of bank loans that are securitized increases in the U.S. economy, banks also gradually expand their C&I loan commitment businesses (see Figure 3.1). The magnitude of this effect is substantial: as the fraction of bank loans being securitized increased from 20 percent in 1990 to above 40 percent in 2009, the ratio of C&I loan commitments to C&I loans rose from 1 in 1990 to around 1.25 in 2009.

Based on regressions using bank-level data, the first major result of this chapter confirms what Figure 3.1 presents, that is, a bank that finds it easier to convert its existing loans into marketable securities tends to sell more liquidity insurance via C&I loan commitments. This result indicates that it is crucial to include banks' exposure to both securitization market and C&I loan commitment takedowns for explaining U.S. banks' liquidity problem during the 2007-09 financial crisis. Our second set of results shows that the massive takedowns of C&I loan commitments played a more critical role in the first half of the 2007-09 financial crisis and the meltdown of the securitization market dominated during the second half.

In order to discuss identification strategies of our major results, we mention two indexes that we use to measure a bank's potential to securitize its existing loans. The first index, which is proposed by Loutskina (2011b), is the weighted average of the potential to securitize loans of a particular type, where the weight is the portfolio weight of loans of that type and the securitization potential is the fraction of outstanding loans of that type being securitized in the U.S. economy. The second index is a modification of the first one. Notice that the likelihood of securitizing a given type of loans within a period, say a quarter, should depend more on the proportion of loans being securitized within the period rather than the one in the history. Therefore, the second index utilizes flow variables instead of stock variables to measure securitization potentials.

Identification strategies of our first result follow Loutskina (2011b). The most representative identification that we use is the difference-in-difference analysis around the outbreak of the Enron scandal. Right after the Enron collapse, Enron's use of off-balance-sheet financing was widely criticized and the securitization market expected that regulatory agencies would take more stringent rules against off-balance-sheet financing that securitization also heavily uses. Naturally, the securitization market slows down due to the scandal and we find that banks with higher securitization indexes did not extend as many C&I loan commitments after the Enron scandal as they did before that.

The identification strategy of our second set of results is straightforward. We use lagged independent variables to explain U.S. banks' liquidity problem in the 2007-09 financial crisis. As a comparison, we run regressions that include and do not include banks' exposure to the securitization market. Results indicate that it would overestimate the significance of small banks' exposure to C&I loan commitments in the second half of the crisis if banks' exposure to the securitization market is missing.

Our work has two major contributions to the literature. First, it draws a bridge between two lines of research that emphasize two separate off-balance-sheet items. One line of research is about loan commitments or lines of credit (Shockley and Thakor, 1997; Kashyap et al., 2002; Gatev and Strahan, 2006; Gatev et al., 2009; Cornett et al., 2011). The other line of research focuses on securitization (Gorton and Souleles, 2007; Loutskina and Strahan, 2009; Loutskina, 2011b). The second contribution of this chapter is to highlight the potential missing variable problem in the two strands of literature that attempt to explain U.S. banks' liquidity problem in the 2007-09 financial crisis. The first strand of literature emphasizes the liquidity risk of loan commitment takedowns (Ivashina and Scharfstein, 2010; Cornett et al.,

2011; Acharya and Mora, 2015). The second strand of literature concerns the freeze of the securitization market (Acharya et al., 2012, 2013).

3.2 Data and Variable Description

Our bank-level panel data come from the Federal Reserve's Report of Condition and Income that banks file with regulators quarterly ("Call Reports"). Following Kashyap et al. (2002), we aggregate the bank-level data for banks belonging to the same top holders. Therefore, "banks" in the remainder of the chapter mean the top holding companies, and standard errors are clustered at the top holding company level. We compile our data set for all reporting banks over the period from the first quarter of 1990 to the fourth quart of 2009. Banks started reporting their unused loan commitment off-balance sheet items since the first quarter of 1990.

Our data cleaning procedure is the same as that in Loutskina (2011b). In particular, we eliminate bank-quarters in which banks experienced mergers primarily using database that Federal Reserve Bank of Chicago provides. Furthermore, we also exclude observations of acquiring banks before and after mergers. To minimize the contamination of outliers, we drop observations with asset growth over the previous quarter more than 50%, those with total loan growth more than 100%, those with total loan to asset ratio less than 10%, and those with credit card loan share more than 50%. Our ultimate data set has $617,361$ bank-quarter observations. In addition, all items from Call Reports are winsorized at the 1st and 99th percentiles.

Loan-level data are collected from Thomson Reuters' Dealscan database, which primarily covers large syndicated loans. According to Ivashina and Scharfstein (2010), syndicated loans account for at 27% of the C&I loans as reported by Federal Reserve Board in 2007. Our sample includes all USD-denominated loan facilities that were

arranged between January 1, 2005 and December 31, 2009. Each loan facility is manually linked to its bank lenders based on each lender's name in the Dealscan database and the bank's unique RSSD identification.

Measuring Securitization-based Loan Liquidity. Measures of a bank's securitization-based loan liquidity reflect the likelihood that it can securitize its loans. As explained in Loutskina (2011b), banks' loan liquidity is increasing in the proportion of loans that can be securitized easily.

The first measure of loan liquidity that we will use is constructed by Loutskina (2011b). It involves two steps to construct the index of a bank-quarter observation. The first step is to calculate the potential to securitize loans of a given category for the quarter based on the aggregate-level data that comes from "Financial Accounts of the United States". The second step is to derive the weighted average of securitization potential of all loan categories, where weights are the bank's portfolio weights of corresponding loan categories. The categories of loans are *i*) home mortgages, *ii*) multi-family residential mortgages, *iii*) commercial mortgages, *iv*) consumer credit, *v*) commercial and industrial loans, and *vi*) farm mortgages.

$$\begin{aligned}\text{Loan liquidity}_{i,t} \;=\; & \sum_{j=1}^{6}\left(\frac{\text{Economy-wide Securitized Loans of Type } j \text{ at Time } t}{\text{Economy-wide Total Loans Outstanding of Type } j \text{ at Time } t}\right) \\ & \times\big(\text{Share of Type } j \text{ Loans in Bank } i \text{ Portfolio at Time } t\big) \qquad (3.1)\end{aligned}$$

Since the above measure uses stock variables to estimate the securitization potential of each type of loans, it may not capture the contemporaneous movement of the securitization market in a given quarter. To address this issue, we propose an alternative

measure.

$$\text{Loan liquidity}^{\Delta}_{i,t} = \sum_{j=1}^{6}\left(\frac{\Delta\text{Economy-wide Securitized Loans of Type } j \text{ at Time } t}{\text{Economy-wide Total Loans Outstanding of Type } j \text{ at Time } t-1}\right) \times\big(\text{Share of Type } j \text{ Loans in Bank } i \text{ Portfolio at Time } t\big), \quad (3.2)$$

where ΔEconomy-wide Securitized Loans of Type j at time t = Economy-wide Securitized Loans of Type j at time t − Economy-wide Securitized Loans of Type j at time $t-1$. This measure modifies the first step used to construct the original one. In particular, the potential to securitize loans of type j within a quarter t is the growth in the securitized loans of type j in quarter t over the previous period divided by the outstanding of total loans of type j in quart $t-1$. Intuitively, the new measure captures the change in the securitization market that occurs in a given quarter.[2]

Measuring C&I Loan Commitment Exposure. We focus on C&I loan commitment primarily because its drawdowns are blamed for the US banks' lending decline in the literature (Ivashina and Scharfstein, 2010). Since Call Reports do not contain a specific item for C&I loan commitment, we follow Kashyap et al. (2002) to use "other commitment" as the measure of a bank's outstanding C&I loan commitment since it predominantly consists of commitments to issue C&I loans.

In this chapter, we are interested in banks' liquidity management decision about their optimal exposure to the liquidity risk of C&I loan commitment takedowns. However, the amount of C&I loan commitments directly affects the actual C&I loan lending, which is apparently banks' loan portfolio choice. To purge the impact of the portfolio choice, we use the ratio of "other commitments" over the sum of "other commitments" plus C&I loans as the measure of a banks' exposure to the liquidity risk coming from the takedowns of the C&I loan commitments, which is denoted by "C&I comrat".

[2]It is not a perfect measure because we only observe the net change of the outstanding securitized loans. It will be better if we observe the gross amount of loans securitized within a period.

Proxies for Bank Liquidity Problem. Banks suffering liquidity problem can react by cut new lending and/or absorb more deposits by promising higher returns for depositors. To measure the lending of a bank i in quarter t, we sum its total loans and total unused loan commitments so that the takedowns of pre-existing commitments does not count as new lending. The net new lending of a bank i in quarter t is the bank's lending in quarter t minus its lending in quarter $t-1$. Furthermore, we normalize the difference by the sum of the bank's total assets and outstanding commitments.

Besides the new lending at the bank level, we also examine to what degree the liquidity problem for banks is transmitted to individual loan deals made during the 2007-09 financial crisis. In particular, we run regressions of the loan facility amount on both bank and facility characteristics.

Other than new lending, banks also scramble deposits to mitigate the liquidity problem that they suffer during the financial crisis. Thus, we also investigate how the liquidity shortage that banks have experienced during the crisis affected the interest rate promised to depositors.

Bank Characteristics. Other than C&I loan commitments, banks also extend other types of commitments. Most of other loan commitments have a commonality that if they are realized, they will be loans secured by residential or commercial properties. One example is home equity lines. We also construct variable "Real estate comrat" to control for banks' exposure to the liquidity risk along this dimension

$$\text{Real estate comrat}_{it} = \frac{\text{Commitments backed by real estate}}{\text{Commitments backed by real estates} + \text{Loans backed by real estates}}. \tag{3.3}$$

There are a number of bank characteristics that affect banks' optimal exposure to loan commitment takedown. For instance, banks with ample liquid assets such as treasury

Table 3.1: Summary Statistics

The table reports summary statistics for all bank-level variables used in regressions next as well as the number of bank-quarter in the full sample and two subsamples. The small bank subsample includes bottom 75% in total asset size of each quarter; the large bank sample includes the top 5%. The data is from the Federal Reserve's Report of Condition and Income for the period between 1990:I and 2009:IV. We drop observations with asset growth over the previous quarter more than 50%, those with total loan growth more than 100%, those with total loan to asset ratio less than 10%, and those with credit card loan share more than 50%. In addition, all items from Call Reports are winsorized at the 1st and 99th percentiles.

	Full Sample (N = 617,361)				Small Banks (N = 462,992)				Large Banks (N = 30,832)			
	Mean	Q1	Median	Q3	Mean	Q1	Median	Q3	Mean	Q1	Median	Q3
Total assets	815.1	36.9	78.8	185.5	715.9	29.6	54.5	95.0	13689	1293	2401	6316
Loan liquidity	0.237	0.156	0.233	0.308	0.230	0.148	0.224	0.300	0.253	0.187	0.260	0.314
Loan liquidity$^{\Delta}$ (in percentage)	0.642	0.334	0.616	0.941	0.618	0.312	0.589	0.911	0.674	0.402	0.683	0.976
C&I comrat (in percentage)	25.3	11.3	22.6	34.8	22.1	9.1	19.7	30.5	43.2	31.7	44.1	55.5
Real estate comrat (in percentage)	7.57	0.15	4.54	11.2	6.32	0	2.96	9.01	15.7	8.7	15.1	22.2
Δcredit/ (commit + asset) (in percentage)	1.58	-0.51	1.00	2.91	1.47	-0.57	0.98	2.92	2.58	-0.48	0.37	2.36
Interst rate, large time deposits	2.88	1.44	2.59	3.99	2.87	1.44	2.58	3.98	2.91	1.43	2.59	4.03
Interst rate, core deposits	2.27	1.09	1.99	3.24	2.29	1.10	2.01	3.25	2.14	0.97	1.80	3.08
Bank liquidity (percentage of total assets)	31.1	19.9	29.3	40.9	32.3	20.6	30.8	42.5	24.1	18.0	22.1	29.7
Total deposits (percentage of total assets)	82.8	81.9	86.7	89.7	84.8	83.2	87.3	89.9	63.8	63.4	72.7	83.7
Equity capital (percentage of total assets)	10.1	7.89	9.33	11.5	10.7	8.13	9.67	12.0	7.04	6.74	7.44	8.83
Net income (percentage of total assets)	0.567	0.264	0.545	0.908	0.558	0.264	0.544	0.918	0.494	0.180	0.522	0.712
Nonperforming loan (percentage of total loans)	1.38	0.23	0.76	1.76	1.40	0.20	0.75	1.79	1.36	0.39	0.97	2.07
Wholesale funding (percentage of total assets)	14.7	7.11	12.8	20.3	13.8	6.8	12.1	19.0	17.9	8.3	19.4	23.8

securities should easily collect fees by choosing larger exposures to the liquidity risk of commitment drawdowns. Due to this concern, we will control banks' liquid asset holdings, which include US treasury securities, US government agency obligations, federal funds repurchase agreement, securities held to maturity, securities available for sale, and so on. In addition, we also need to control the total amount of deposits on banks' liability side because of the synergy between banks' lending via commitment and their deposit-taking that Kashyap et al. (2002) ememphasize. Other bank characteristic controls that we also incorporate into our regressions are the log of banks' total assets, banks' equity capital, wholesale funds, net incomes, and the ratio of non-performing loans.

Summary Statistics. We collect the summary statistics of major bank-level variables in Table 3.1. Besides the full sample, we also present the summary statistics of both small bank sub-sample and large bank sub-sample. A bank in quarter t is classified as a small bank if its total assets is within the bottom 75% of the bank size distribution in quarter t. Large banks in a quarter are those whose total assets are in top 5% of the distribution in the quarter.

Table 3.1 shows that the average C&I comrat and Real estate comrat are 43.2% and 15.7% for large banks, respectively. These figures are much larger than their counterparts for small banks. Thus, large banks have larger exposures to the loan commitment drawndowns than small banks do. However, they hold less liquid assets than small banks do, even though liquid assets are generally regarded as buffer stocks against liquidity risks. Based on these results, a well educated conjecture is that large banks definitely find a substitute for the liquid assets as the source of liquidity, which is likely to be loans that have high potential of being securitized. As a matter of fact, the securitization-based loan liquidity of small banks is generally lower than that of large banks, as both of the loan liquidity measure show.

On the liability side of banks' balance sheets, we observe that small banks rely more on deposit and equity financing than large banks do. On the other hand, large banks are more involved in the wholesale funding market.

3.3 Empirical Results

In this section, we highlight two sets of empirical results. First, we uncover the intrinsic connection between two liquidity variables that are off banks' balance sheets. By using various identification strategies, we show that banks expand their C&I loan commitment businesses if they find it easier to securitize loans on their balance sheets. Our second set of results concerns the massive liquidity problem that U.S. banks suffered during the 2007-09 financial crisis. Our first set of results indicates the importance of incorporating both of the two liquidity variables as explanatory variables. In particular, we will differentiate their significance of explaining U.S. banks' liquidity problem.

3.3.1 More Loan Liquidity, More C&I Loan Commitments

By securitizing their outstanding loans, banks can enjoy large amounts of liquidity pools, which in turn allows them to make more profits from fees of extending C&I loan commitments. This section presents a highly consistent result that if banks find larger potential to securitize their loans, they tend to issue more C&I loan commitments. The test of this result involves the endogeneity problem that banks choose their loan portfolio and exposure to liquidity risk simultaneously. To avoid this endogeity problem, we apply three identification methods used in Loutskina (2011b). The first approach is to use the lagged explantory variables, the second approach is to run instrumental variable regressions, and the third approach is to exploit the regulatory discontinuity.

Multivariate Analyses

To implement the first two approaches, we regress C&I comrat$_{it}$ on one period lagged variables of Loan liquidity$_{it-1}$ and other bank characteristics

$$\text{C\&I comrat}_{it} = \alpha + \beta \text{Loan liquidity}_{it-1} + \text{Bank characteristics}_{it-1} + \delta_i + \gamma_t + \varepsilon_{it}, \quad (3.4)$$

where δ_i is the bank fixed effect and γ_t the time fixed effect. We expect that β, the coefficient of Loan liquidity$_{it-1}$ should be positive because if banks can easily securitize their loans, equivalently they have more liquid assets and thus they become more capable of providing liquidity on demand and extending their C&I loan commitment businesses.

Table 3.2: C&I loan commitment exposure and securitization-based loan liquidity

This table reports the fixed effects regressions of C&I loan commitment exposure, C&I loan commitment$_{it}$/ (C&I loan$_{it}$ + C&I loan commitment$_{it}$) on securitized based loan liquidity (Loan liquidity$_{it}$ or Loan liquidity$^{\Delta}_{it}$), quarterly time dummies and other bank characteristics including the log of total asset, bank liquidity (liquid asset to total assets ratio), deposit financing (total deposits to total assets ratio), equity financing (equity capital to total assets ratio), net income to total assets ratio, share of non-performing loans, and wholesale financing (wholesale funds to total assets ratio). Specification (1) - (4) show the results of ordinary regressions and (5) - (8) show those of intrumental variable regressions. The instruments for Loan liquidity$_{it}$ and Loan liquidity$^{\Delta}$ are constructed based on equation 3.1 and 3.2 with loan level in quarter t replaced by the average loan level of previous four quarters. Standard errors are clustered at the bank level. Robust t statistics are reported in parentheses. Coefficients denoted *, **, and *** are statistically significantly different from zero at the 10%, 5%, and 1% level, respectively.

	C&I Loan Commitment$_{it}$/ (C&I Loan$_{it}$ + C&I Loan Commitment$_{it}$)							
	Ordinary regression				Instrumental variable regression			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Loan liqudity$_{it-1}$	0.610***	0.457***			0.460***	0.549***		
	(24.15)	(4.01)			(4.04)	(19.25)		
Loan liquidity$_{it-1}$ × size$_{it-1}$		0.0135			0.0131			
		(1.42)			(1.38)			
Loan liqudity$^{\Delta}_{it-1}$			4.884***	5.003***			4.878***	5.024***
			(20.50)	(6.21)			(20.61)	(6.24)
Loan liquidity$^{\Delta}_{it-1}$ × size$_{it-1}$				-0.0102				-0.0124
				(-0.16)				(-0.19)
Size$_{it}$(log of total asset$_{it-1}$)	0.00983***	0.00630*	0.00930***	0.00936***	0.00657**	0.0101***	0.00947***	0.00954***
	(4.09)	(1.89)	(3.80)	(3.82)	(1.97)	(4.20)	(3.87)	(3.90)
Bank liquidity$_{it-1}$	0.0587***	0.0580***	0.0531***	0.0531***	0.0579***	0.0580***	0.0530***	0.0530***
	(7.86)	(7.81)	(7.05)	(7.06)	(7.79)	(7.76)	(7.04)	(7.04)
Deposit financing$_{it-1}$	0.00116	0.00634	-0.00853	-0.00850	0.00602	-0.0000543	-0.00865	-0.00861
	(0.06)	(0.33)	(-0.45)	(-0.45)	(0.31)	(-0.00)	(-0.45)	(-0.45)
Equity capital$_{it-1}$	0.0876***	0.0885***	0.0634*	0.0634*	0.0884***	0.0847**	0.0633*	0.0633*
	(2.65)	(2.67)	(1.92)	(1.92)	(2.66)	(2.56)	(1.91)	(1.91)
Net income$_{it-1}$	0.116*	0.118*	0.111	0.111	0.118*	0.115*	0.111	0.111
	(1.72)	(1.75)	(1.62)	(1.62)	(1.74)	(1.69)	(1.61)	(1.62)
Nonperforming loan$_{it-1}$	-0.265***	-0.265***	-0.267***	-0.268***	-0.265***	-0.267***	-0.267***	-0.267***
	(-9.33)	(-9.33)	(-9.34)	(-9.35)	(-9.32)	(-9.39)	(-9.32)	(-9.34)
Wholesale financing$_{it-1}$	-0.0629***	-0.0626***	-0.0686***	-0.0685***	-0.0624***	-0.0633***	-0.0684***	-0.0683***
	(-5.06)	(-5.01)	(-5.57)	(-5.59)	(-5.00)	(-5.10)	(-5.56)	(-5.57)
Quarter dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Bank dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	576467	576467	576467	576467	575879	575879	575879	575879
Adjusted R^2	0.146	0.146	0.129	0.129	0.122	0.122	0.105	0.105

The instrument for Loan liquidity$_{it}$ is

$$\sum_{j=1}^{6}\left(\frac{\text{Economy-wide Securitized Loans of Type } j \text{ at Time } t}{\text{Economy-wide Total Loans Outstanding of Type } j \text{ at Time } t}\right) \times\Big(\text{Average Share of Type } j \text{ Loans in Bank } i \text{ Portfolio over } t-1 \text{ to } t-4\Big).$$

In constructing the instrument, we use a bank's average portfolio shares to capture the persistent instead of contemporaneous components of the bank's loan liquidity. The instrument for Loan liquidity$^{\Delta}_{it}$ is constructed in the same fashion.

Table 3.2 reports the results of both ordinary regressions (column (1) - (4)) and instrumental variable regressions (column (5) - (8)). We observe a statistically signif-

icant pattern that the securitization-based loan liquidity assists banks with their C&I loan commitment businesses. This pattern consistently holds across eight different econometric specifications. In particular, as the securitization-based loan liquidity increases by 1 percent, the composition of C&I loan commitments (i.e., C&I comrat) rises by around 50 basis points.

In addition to our main results, Table 3.2 also confirms other stylized phenomena of banks' liquidity risk management. Banks, which are larger or hold more liquid assets, are inclined to extend more C&I loan commitment businesses. The stability of equity capital is also helpful for banks to make C&I loan commitments. In contrast, the instability of wholesale funding refrains banks from exposing themselves to the liquidity risk of loan commitment takedowns. For instance, as a bank's equity capital increases by 1 percent, the composition of its C&I loan commitments increases by at least 6 basis points; if a bank's wholesale financing increases by 1 percent, it will lower the composition of its C&I loan commitments by more than 6 basis points. Lastly, banks that suffer more bad loans tend to have less C&I loan commitments in the subsequent quarter. In particular, as the fraction of a bank's non-performing loan increases by 1 percent, it lowers the composition of C&I loan commitments (i.e., C&I comrat) by 26 basis points. However, we do not find any evidence that shows deposit financing or banks' net income help them issue C&I loan commitments.

Regulatory Discontinuity

We next apply the third difference-in-difference identification strategy by exploiting the exogenous regulatory shocks that considerably changed the functioning of the

securitization market. The econometric specification is

$$\begin{aligned} \text{C\&I comrat}_{it} &= \alpha + \beta \text{Loan liquidity}_{it-1} + \beta_{\text{post-shock}} \text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{post-shock}} \\ &\quad + \text{Bank characteristics}_{it-1} + \delta_i + \gamma_t + \varepsilon_{it}. \end{aligned} \tag{3.5}$$

We expect that banks' liquidity risk exposure decisions will become less sensitive to the change in its Loan liquidity$_{it}$ ($\beta_{\text{post-shock}} < 0$) if the regulatory shock affects the securitization market in an adverse way.

We will borrow two unexpected regulatory changes that Loutskina (2011b) has applied in her difference-in-difference analysis. The first regulatory shock is the Enron Scandal. Before Enron collapsed, it had used hundreds of special purpose entities to hide their its debt so that it overstated its equity and understates its liability. The collapse of Enron led the banking sector to expect regulatory agencies to impose additional capital requirement for assets sold to special purpose entities (i.e., securitization). Therefore, the market anticipated that the consequential regulatory change might halt the growth of the securitization market. Given such an expectation, banks should lower their exposure to the liquidity risk of C&I loan commitment drawdowns ($\beta_{\text{post-shock}} < 0$) since their liquidity pool becomes smaller given the same level of securitization-based loan liquidity.

Two years later, in July 2004, bank regulators made it clear that there was no dramatic change to the capital requirement of securitized loans. For instance, the official release states that "[t]he final rule will permanently permit sponsoring banks, bank holding companies, and thrifts (collectively, sponsoring banking organizations) to exclude from their risk-weighted asset base those assets in ABCP programs that are consolidated onto sponsoring banking organizations' balance sheets as a result of Financial Accounting Standards Board Interpretation No.46, Consolidation of Variable

Table 3.3: C&I loan commitment exposure and securitization-based loan liquidity: regulatory discontinuity

This table reports the fixed effects regressions of C&I loan commitment exposure, C&I loan commitment$_{it}$/ (C&I loan$_{it}$ + C&I loan commitment$_{it}$) on securitized based loan liquidity (Loan liquidity$_{it}$ or Loan liquidity$^{\Delta}_{it}$), bank characteristics and their interactions with exogenous event dummies ($\mathbb{1}_{2002:\text{I-II}}$ or $\mathbb{1}_{2004:\text{III-IV}}$). $\mathbb{1}_{2002:\text{I-II}}$ equals one for quarter 2002:I and 2002:II when the Enron scandal was fully revealed and zero otherwise. $\mathbb{1}_{2004:\text{III-IV}}$ equals one for quarter 2004:III and 2004:IV when bank regulators issued securitization-friendly new rules and zero otherwise. Quarterly fixed effect is also controlled in these regressions. Standard errors are clustered at the bank level. Robust t statistics are reported in parentheses. Coefficients denoted *, **, and *** are statistically significantly different from zero at the 10%, 5%, and 1% level, respectively.

	$\frac{\text{C\&I Loan Commitment}_{it}}{\text{C\&I Loan}_{it}+\text{C\&I Loan Commitment}_{it}}$			
	(1)	(2)	(3)	(4)
Loan liqudity$_{it-1}$	0.402***		0.444***	
	(6.36)		(4.86)	
Loan liquidity$_{it-1}$ × $\mathbb{1}_{2002:\text{I-II}}$	-0.0518***			
	(-4.53)			
Loan liquidity$_{it-1}$ × $\mathbb{1}_{2004:\text{III-IV}}$			0.0408***	
			(2.98)	
Loan liqudity$^{\Delta}_{it-1}$		1.919***		0.520
		(5.08)		(1.26)
Loan liquidity$^{\Delta}_{it-1}$ × $\mathbb{1}_{2002:\text{I-II}}$		-1.223***		
		(-4.37)		
Loan liquidity$^{\Delta}_{it-1}$ × $\mathbb{1}_{2004:\text{III-IV}}$				1.105***
				(2.76)
Sample period	2001:III - 2002:II		2004:I - 2004:IV	
Other controls	Yes	Yes	Yes	Yes
Other controls × $\mathbb{1}_{2002:\text{I-II}}$	Yes	Yes		
Other controls × $\mathbb{1}_{2004:\text{III-IV}}$			Yes	Yes
Quarter dummies	Yes	Yes	Yes	Yes
Bank dummies	Yes	Yes	Yes	Yes
Observations	27000	27000	26486	26486
Adjusted R^2	0.021	0.019	0.018	0.015

t statistics in parentheses

* $p < .1$, ** $p < .05$, *** $p < .01$

Interest Entities, as revised (FIN46-R)." Thus, the securitization market prospered again since then (Acharya et al., 2012).

We have done two regulatory discontinuity analyses for two sub-sample periods: one over 2001Q3 to 2002Q2 around the Enron Scandal period and the other over 2004Q1 to 2004Q4 around the regulatory relief period. The indicator variables are $\mathbb{1}_{2002:\text{I-II}}$ for the first and second quarter of 2002, and $\mathbb{1}_{2004:\text{III-IV}}$ for the third and fourth quarter of 2004.

Table 3.3 reports the results of the difference-in-difference analyses. The Enron scandal significantly reduced the responsiveness of the C&I loan commitment decisions to the securitization-based loan liquidity irrespective of what loan liquidity measure we

use. The regression using stock variables to measure securitization-based loan liquidity indicates that the coefficient of Loan liquidity$_{it}$ declines by 12.5 percent because of the scandal. If we use flow variables, the coefficient of Loan liquidity$^{\Delta}_{it}$ drops by more than 50 percent due to the Enron collapse. On the other hand, the issuance of the securitization-friendly rule not only revived this market but also reunited the complementary roles of the C&I loan commitment exposure and the securitization-based loan liquidity.

3.3.2 Securitization and C&I Loan Commitment in the 2007-09 Financial Crisis

Having established the causal relationship that greater potential of securitizing loans leads to more C&I loan commitment issuance, we next examine the role of the two variables in explaining the liquidity problem that U.S. banks experienced during the 2007-09 financial crisis. Our regression results indicate that the liquidity risk of C&I loan commitment takedowns is the main force that caused banks to cut their new lending and raise deposit rates to scramble deposits within the time interval between 2007Q3 and 2008Q2. From 2008Q3 onward, the freeze of the securitization market became the dominant factor that drained U.S. banks' liquidity.

Lending Cut

We base our analyses of how liquidity risk affects banks' new lending on both bank-level and loan-level data, which yield the consistent result mentioned above.

Bank-level. We run several regressions to test what factors affect banks' new lending during the 2007-09 financial crisis. The representative econometric specification is

$$\frac{\Delta\text{Loan Commitment}_{it} + \Delta\text{Loan}_{it}}{(\text{Commitment} + \text{Asset})_{it}} = \alpha + \beta\text{C\&I comrat}_{it-1} + \beta_{\text{crisis1}}\text{C\&I comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}}$$

$$+ \beta_{\text{crisis2}}\text{C\&I comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}} + \zeta\text{Real estate comrat}_{it-1}$$

$$+ \zeta_{\text{crisis1}}\text{Real estate comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}} + \zeta_{\text{crisis2}}\text{Real estate comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}}$$

$$+ \eta\text{Loan liquidity}_{it-1} + \eta_{\text{crisis1}}\text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis1}}$$

$$+ \eta_{\text{crisis2}}\text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis2}} + \text{Bank characteristics}_{it-1} + \delta_i + \gamma_t + \varepsilon_{it},$$

where indicator $\mathbb{1}_{\text{crisis1}}$ equals one for quarters between 2007Q3 and 2008Q2 and indicator $\mathbb{1}_{\text{crisis2}}$ equals one for quarters between 2008Q3 and 2009Q2. As in Acharya and Mora (2015), we divide the two-year window of the 2007-2009 financial crisis into two one-year periods. This division can capture the fact that the overall conditions of the financial market in the two periods are significantly different because of the failure of Lehman and the massive government intervention. The sample period of our regression analyses is from 2005Q1 to 2009Q4. In addition, we run regressions for the large bank and small bank sub-samples. Large banks are those with total assets above one billion; small banks are those with total assets below one billion.

In addition to the representative specification, we run regressions with all terms except Loan liquidity and regressions with term "Loan liquidity$_{it}$" replaced by "Loan liquidity$^{\Delta}_{it}$".

Column (1) - (6) in Table 3.4 report the results of these regressions. The first three columns show that the new lending of large banks are barely hurt by the liquidity drain of loan commitments and the collapse of the securitization market. It is that small banks are mostly affected by the risk of C&I loan commitment takedowns and the freeze of the securitization market during the 2007-09 financial crisis.

Table 3.4: C&I New lending, C&I loan commitment takedowns, and securitization

This table reports the fixed effects regressions of new lendings, $\Delta\text{Loan Commitment}_{it} + \Delta\text{Loan}_{it}/\left(\text{Commitment} + \text{Asset}\right)_{it}$ (specification (1) - (6)) and log of facility amount (specification (7) - (9)) on exposure to C&I loan commitment drawndowns ($\text{C\&I loan commitment}_{it}/\left(\text{C\&I loan}_{it} + \text{C\&I loan commitment}_{it}\right)$), exposure to real estate related loan commitment drawndowns ($\text{real estate related loan commitment}_{it}/\left(\text{real estate related loan}_{it} + \text{real estate related loan commitment}_{it}\right)$), securitization-based loan liquidity ($\text{Loan liquidity}_{it}$ or $\text{Loan liquidity}^{\Delta}_{it}$), bank characteristics and their interactions with event dummies, $\mathbb{1}_{\text{crisis1}}$ and $\mathbb{1}_{\text{crisis2}}$. $\mathbb{1}_{\text{crisis1}}$ equals one for quarters from 2007:III to 2008:II and zero otherwise. $\mathbb{1}_{\text{crisis2}}$ equals one for quarters from 2008:III to 2009:II and zero otherwise. Sample period is from 2005Q1 to 2009Q2. Large banks are those with quarter total asset more than 1 billion, and small banks are those with quarter total asset less than 1 billion. Quarterly fixed effect is also controlled in these regressions. Standard errors are clustered at the bank level. Robust t statistics are reported in parentheses. Coefficients denoted *, **, and *** are statistically significantly different from zero at the 10%, 5%, and 1% level, respectively.

	$\frac{\Delta\text{Loan Commitment}_{it}+\Delta\text{Loan}_{it}}{(\text{Commitment}+\text{Asset})_{it}}$, large banks			$\frac{\Delta\text{Loan Commitment}_{it}+\Delta\text{Loan}_{it}}{(\text{Commitment}+\text{Asset})_{it}}$, small banks			Log of facility amount$_{jit}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
C&I comrat$_{it-1}$	-0.0494***	-0.0650***	-0.0467***	-0.0298***	-0.0323***	-0.0323***	-0.565	-0.503	-0.471
	(-3.08)	(-3.84)	(-2.98)	(-10.93)	(-11.65)	(-11.79)	(-1.32)	(-1.19)	(-1.06)
C&I comrat$_{it-1} \times \mathbb{1}_{\text{crisis1}}$	0.0115	0.0138	0.0104	-0.0122***	-0.00989***	-0.00939***	-1.447***	-1.638***	-1.582***
	(0.72)	(0.86)	(0.66)	(-5.06)	(-3.89)	(-3.70)	(-4.29)	(-4.25)	(-4.38)
C&I comrat$_{it-1} \times \mathbb{1}_{\text{crisis2}}$	-0.0110	-0.0126	-0.0193	-0.00660***	-0.00527**	-0.00293	-0.635	-0.226	-0.342
	(-0.78)	(-0.93)	(-1.33)	(-2.80)	(-2.15)	(-1.20)	(-0.73)	(-0.23)	(-0.40)
Real estate comrat$_{it-1}$	-0.0298	-0.00935	-0.0337	-0.0227***	-0.0205***	-0.0228***	0.0107	-0.206	-0.0831
	(-0.84)	(-0.25)	(-0.94)	(-3.36)	(-2.99)	(-3.37)	(0.02)	(-0.32)	(-0.11)
Real estate comrat$_{it-1} \times \mathbb{1}_{\text{crisis1}}$	0.0129	0.00942	0.0144	-0.00854	-0.00967	-0.00906	-1.284	-0.714	-0.916
	(0.52)	(0.38)	(0.57)	(-1.36)	(-1.54)	(-1.45)	(-1.62)	(-0.67)	(-1.02)
Real estate comrat$_{it-1} \times \mathbb{1}_{\text{crisis2}}$	-0.0229	-0.0165	-0.0115	-0.0247***	-0.0257***	-0.0256***	-0.552	-1.895	-1.437
	(-0.88)	(-0.69)	(-0.47)	(-3.31)	(-3.47)	(-3.46)	(-0.47)	(-1.19)	(-1.12)
Loan liqudity$_{it-1}$		0.335***			0.0575***			-0.605	
		(2.80)			(2.62)			(-0.44)	
Loan liquidity$_{it-1} \times \mathbb{1}_{\text{crisis1}}$		-0.0118			-0.0178***			1.805	
		(-0.31)			(-3.82)			(0.76)	
Loan liquidity$_{it-1} \times \mathbb{1}_{\text{crisis2}}$		0.0229			-0.0135***			-3.353	
		(0.76)			(-2.95)			(-1.38)	
Loan liqudity$^{\Delta}_{it-1}$			-0.499			0.582***			-1.649
			(-0.73)			(5.88)			(-0.07)
Loan liquidity$^{\Delta}_{it-1} \times \mathbb{1}_{\text{crisis1}}$			-0.0284			-0.356***			39.76
			(-0.03)			(-2.58)			(1.16)
Loan liquidity$^{\Delta}_{it-1} \times \mathbb{1}_{\text{crisis2}}$			3.511**			-0.589**			-131.5**
			(2.42)			(-2.48)			(-2.20)
Other controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Other controls × event dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9444	9444	9444	118188	118188	118188	9298	9298	9298
Adjusted R^2	0.297	0.301	0.299	0.138	0.138	0.138	0.513	0.513	0.513

Column (4) - (6) convey two crucial messages. First, the drawdown risk of pre-existing C&I loan commitments significantly prevent small banks from extending credit to firms or households for the first twelve-month period of the 2007-09 financial crisis. Second, the missing variable problem can misguide us about the importance of the loan commitment takedown during the second twelve-month period. If we exclude the loan liquidity factor, the coefficient of "$\text{C\&I comrat}_{it} \times \mathbb{1}_{\text{crisis2}}$" is statistically significant at level 1 percent, which implies that small banks' exposure to the C&I loan commitment drawdown risk during the second half of the 2007-09 financial crisis significantly reduces their new lending. However, if we include loan liquidity measures, the significance of "$\text{C\&I comrat}_{it} \times \mathbb{1}_{\text{crisis2}}$" declines (column (5) of Table 3.4) or just

disappears (column (6) of Table 3.4) depending on which loan liquidity measure we use. The intuition underlying this result is the following. Banks, which expect to enjoy securitization-based loan liquidity, tend to extend more C&I loan commitments. However, when the securitization market suddenly stalled, those banks responded by cutting their new lending. If we overlook this channel, we might interpret the result of column (4) of Table 3.4 as that banks reduce their new lending because of the drawndown risks of their outstanding C& loan commitments.

Overall, the evidence of Table 3.4 shows that during the first twelve-month period of 2007-09 financial crisis, both the liquidity takedown of outstanding C&I loan commitments and the freeze of the securitization market that negatively affected U.S. banks' new loan / loan commitment issuance; however, during the second twelve-month period, it is only the malfunction of the securitization market matters.

Loan-level. We next focus on banks' lending decisions at the intensive margin by extending our analysis to the loan-level data. The general idea of this exercise is that conditional on that a loan has been granted, the lending banks' financial soundness would influence the size of the loan. A bank that struggles to provide liquidity demanded by its customers would definitely limit the amount of money lent to new borrowers. Similarly, as a bank finds it challenging to replace its illiquid outstanding loans with liquid assets, say through securitization, it would also be very reluctant about lending large amounts of money to new customers.

The econometric specification is similar to the previous one. We regress the log of the loan facility indexed by j on both bank characteristics and loan characteristics. Loan facility characteristics include the log of the borrower's sale, the leverage of the borrower, the number of lenders that serve the loan facility, the spread the borrower pays, the maturity of the loan facility, and the indicator of the major loan purpose (corporate purpose, debt repay, working capital, and leveraged buyout).

$$\text{Log of facility amount}_{jit} = \alpha + \beta \text{C\&I comrat}_{it-1} + \beta_{\text{crisis1}} \text{C\&I comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}}$$
$$+ \beta_{\text{crisis2}} \text{C\&I comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}} + \zeta \text{Real estate comrat}_{it-1}$$
$$+ \zeta_{\text{crisis1}} \text{Real estate comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}} + \zeta_{\text{crisis2}} \text{Real estate comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}}$$
$$+ \eta \text{Loan liquidity}_{it-1} + \eta_{\text{crisis1}} \text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis1}}$$
$$+ \eta_{\text{crisis2}} \text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis2}} + \text{Bank characteristics}_{it-1}$$
$$+ \text{Loan characteristics}_{jt-1} + \delta_i + \gamma_t + \varepsilon_{it}.$$

Since the database we use primarily covers syndicated loans, each loan facility in our sample typically have more than one lender. When there are many lenders, some bank's contribution to a loan facility could be very small. We drop this type of banks since their role in arranging loans should not be very significant. Thus, characteristics of banks with small allocation should not significantly affect terms of loans. In particular, we drop loan-bank observations where the bank's contribution to the loan facility is less than 35 percent. Our main results are not sensitive to the threshold.

We find results that are consistent with the conclusions of our analysis based on the bank-level data. Banks with more outstanding C&I loan commitments in the first twelve-month period of the last financial crisis significantly lent less for each new loan deal they made with their borrowers. In particular, Column (7) - (9) in Table 3.4 show that as a bank's composition of C&I loan commitments increases by 1 percent in the second half of 2007, the amount of a loan granted by the bank will decline by around 1.5 percent. However, we do not find such evidence in the second half of 2007-09 financial crisis. On the other hand, we do observe that banks that hypothetically enjoyed higher loan liquidity (as measured by Loan liquidity$^{\Delta}_{it}$) in the second period

of the crisis did not lend as much as they used to do through each loan facility before the crisis.

Scrambling Deposits

During the 2007-09 financial crisis, banks tend to raise deposit rates to absorb deposits and ease the liquidity problem that they suffer, as Acharya and Mora (2015) find. Next, we quantify the importance of different bank characteristics for explaining the rise in the deposit rate during the crisis. The econometric specification is

$$\begin{aligned}
&\text{Deposit rate}_{it} = \alpha + \beta \text{C\&I comrat}_{it-1} + \beta_{\text{crisis1}} \text{C\&I comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}} \\
&+ \beta_{\text{crisis2}} \text{C\&I comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}} + \zeta \text{Real estate comrat}_{it-1} \\
&+ \zeta_{\text{crisis1}} \text{Real estate comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}} + \zeta_{\text{crisis2}} \text{Real estate comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}} \\
&+ \eta \text{Loan liquidity}_{it-1} + \eta_{\text{crisis1}} \text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis1}} \\
&+ \eta_{\text{crisis2}} \text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis2}} + \text{Bank characteristics}_{it-1} + \delta_i + \gamma_t + \varepsilon_{it},
\end{aligned}$$

which is similar to the one for the analysis of the bank-level lending cut. We run regressions for both large bank and small bank subsamples. The sample period is also from 2005Q1 to 2009Q4. Table 3.5 and 3.6 report two sets of results: the first set includes regressions with the rate of large time deposits as dependent variables (Table 3.5), and the second set includes regressions with the rate of core deposits as dependent variables (Table 3.6).

For the deposit rate of large-time deposits, the result is consistent with the finding of our lending cut analysis: during the first half of the 2007-09 financial crisis, if small banks have more pre-existing C&I loan commitments, they tend to raise their deposit rates more in order to attract deposit customers. For instance, if the composition of a small bank's C&I loan commitments increases by 1 percent, it will raise the deposit

Table 3.5: Large time deposit, exposure to C&I loan commitment takedowns, and securitization

This table reports the fixed effects regressions of interest rates paid on large time deposits on the exposure to C&I loan commitment drawndowns ($\text{C\&I loan commitment}_{it}/(\text{C\&I loan}_{it} + \text{C\&I loan commitment}_{it})$), exposure to real estate related loan commitment drawndowns ($\text{real estate related loan commitment}_{it}/(\text{real estate related loan}_{it} + \text{real estate related loan commitment}_{it})$), securitization-based loan liquidity ($\text{Loan liquidity}_{it}$ or $\text{Loan liquidity}^{\Delta}_{it}$), bank characteristics and their interactions with event dummies, $\mathbb{1}_{\text{crisis1}}$ and $\mathbb{1}_{\text{crisis2}}$. $\mathbb{1}_{\text{crisis1}}$ equals one for quarters from 2007:III to 2008:II and zero otherwise. $\mathbb{1}_{\text{crisis2}}$ equals one for quarters from 2008:III to 2009:II and zero otherwise. Sample period is from 2005Q1 to 2009Q2. Large banks are those with quarter total asset more than 1 billion, and small banks are those with quarter total asset less than 1 billion. Quarterly fixed effect is also controlled in these regressions. Standard errors are clustered at the bank level. Robust t statistics are reported in parentheses. Coefficients denoted *, **, and *** are statistically significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Interest rate, large time deposit					
	Large banks			Small banks		
	(1)	(2)	(3)	(4)	(5)	(6)
C&I comrat_{it-1}	-0.250*	-0.282**	-0.251*	-0.0878***	-0.107***	-0.0844***
	(-1.90)	(-2.30)	(-1.88)	(-3.45)	(-4.09)	(-3.28)
C&I $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}}$	-0.0982	-0.00937	-0.0848	0.0594**	0.119***	0.0782***
	(-0.84)	(-0.07)	(-0.68)	(2.17)	(4.16)	(2.73)
C&I $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}}$	-0.00304	0.0314	0.0111	0.0173	0.0147	0.00274
	(-0.02)	(0.18)	(0.07)	(0.57)	(0.46)	(0.09)
Real estate comrat_{it-1}	-0.610**	-0.596*	-0.612**	-0.0779	-0.0711	-0.0819*
	(-2.06)	(-1.93)	(-2.03)	(-1.60)	(-1.45)	(-1.69)
Real estate $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}}$	-0.138	-0.261	-0.147	0.155***	0.124**	0.159***
	(-0.43)	(-0.83)	(-0.45)	(2.65)	(2.11)	(2.71)
Real estate $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}}$	0.133	0.0910	0.109	-0.0436	-0.0433	-0.0314
	(0.20)	(0.14)	(0.17)	(-0.55)	(-0.55)	(-0.40)
$\text{Loan liqudity}_{it-1}$		0.199			0.268*	
		(0.22)			(1.66)	
$\text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis1}}$		-0.843***			-0.440***	
		(-2.65)			(-7.61)	
$\text{Loan liquidity}_{it-1} \times \mathbb{1}_{\text{crisis2}}$		-0.281			0.00437	
		(-0.83)			(0.07)	
$\text{Loan liqudity}^{\Delta}_{it-1}$			-1.494			-1.124
			(-0.21)			(-1.02)
$\text{Loan liquidity}^{\Delta}_{it-1} \times \mathbb{1}_{\text{crisis1}}$			-5.516			-4.988***
			(-0.65)			(-2.94)
$\text{Loan liquidity}^{\Delta}_{it-1} \times \mathbb{1}_{\text{crisis2}}$			-9.987			4.834
			(-0.59)			(1.31)
Other controls	Yes	Yes	Yes	Yes	Yes	Yes
Other controls × event dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9390	9390	9390	117723	117723	117723
Adjusted R^2	0.880	0.880	0.880	0.871	0.872	0.871

t statistics in parentheses

* $p < .1$, ** $p < .05$, *** $p < .01$

rate of its large-time deposit by at least 5 basis points. This result is also consistent to Acharya and Mora (2015).

However, the potential to securitize loans during the same period seems to alleviate the liquidity problem that both large and small banks suffered. During the first half of 2007-09 financial crisis, if a large bank's liquidity index increases by 1 percent, then it can save almost 85 basis points for each dollar deposit that it absorbs. The reason should be that even though the securitization market slowed down considerably during the first half of the crisis, banks obtained certain liquidity if they could securitize loans on their balance sheets. This benefit helped banks to deal with their liquidity shortage and prevent them from raising deposit rates.

The results of core deposit rate regressions complement our previous results. First, the liquidity risk of C&I loan commitment takedowns do not cause any liquidity problem in terms of scrambling for core deposits for either large or small banks during the second half of the 2007-09 financial crisis. Second, banks with securitization-based loan liquidity measures could not offer low deposit rates during the second half of the financial crisis as they did before the crisis. In particular, if there were not the 2007-09 financial crisis, as a large bank's loan liquidity increased by 1 percent from 2008Q3 to 2009Q2, it would save 80 basis points interests for a deposit of one dollar. This result shows that the channel of obtaining liquidity by securitizing loans on banks' balance sheets was considerably blocked due to the market-wide negative expectation about the future of securitization.

3.4 Appendix A: Bank Characteristics

All bank characteristic variables are constructed based on Call Report items.

Table 3.6: Core deposit, exposure to C&I loan commitment takedowns, and securitization

This table reports the fixed effects regressions of interest rates paid on core deposits on the exposure to C&I loan commitment drawndowns ($\text{C\&I loan commitment}_{it}/\left(\text{C\&I loan}_{it} + \text{C\&I loan commitment}_{it}\right)$), exposure to real estate related loan commitment drawndowns ($\text{real estate related loan commitment}_{it}/\left(\text{real estate related loan}_{it} + \text{real estate related loan commitment}_{it}\right)$), securitization-based loan liquidity ($\text{Loan liquidity}_{it}$ or $\text{Loan liquidity}^{\Delta}_{it}$), bank characteristics and their interactions with event dummies, $\mathbb{1}_{\text{crisis2}}$ and $\mathbb{1}_{\text{crisis2}}$. $\mathbb{1}_{\text{crisis1}}$ equals one for quarters from 2007:III to 2008:II and zero otherwise. $\mathbb{1}_{\text{crisis2}}$ equals one for quarters from 2008:III to 2009:II and zero otherwise. Sample period is from 2005Q1 to 2009Q2. Large banks are those with quarter total asset more than 1 billion, and small banks are those with quarter total asset less than 1 billion. Quarterly fixed effect is also controlled in these regressions. Standard errors are clustered at the bank level. Robust t statistics are reported in parentheses. Coefficients denoted *, **, and *** are statistically significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Interest rate, core deposit					
	Large banks			Small banks		
	(1)	(2)	(3)	(4)	(5)	(6)
C&I comrat_{it-1}	-0.146**	-0.0638	-0.0961	-0.0288*	-0.0190	-0.0153
	(-1.99)	(-0.95)	(-1.29)	(-1.83)	(-1.19)	(-0.97)
C&I $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}}$	0.151	0.101	0.0796	-0.00216	0.0154	-0.0295*
	(1.64)	(1.12)	(0.84)	(-0.13)	(0.90)	(-1.74)
C&I $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}}$	0.0738	-0.0455	-0.00460	-0.00458	-0.0624***	-0.0500***
	(0.90)	(-0.57)	(-0.06)	(-0.25)	(-3.29)	(-2.61)
Real estate comrat_{it-1}	-0.0467	-0.145	-0.120	0.0981***	0.0976***	0.102***
	(-0.25)	(-0.76)	(-0.65)	(3.02)	(2.98)	(3.14)
Real estate $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis1}}$	0.331**	0.386**	0.413***	0.123***	0.108***	0.117***
	(2.25)	(2.44)	(2.67)	(3.49)	(3.06)	(3.32)
Real estate $\text{comrat}_{it-1} \times \mathbb{1}_{\text{crisis2}}$	0.0469	0.152	0.133	-0.180***	-0.156***	-0.142***
	(0.27)	(0.86)	(0.76)	(-4.02)	(-3.50)	(-3.21)
Loan liqudity_{it-1}		-1.098*			-0.0563	
		(-1.85)			(-0.55)	
Loan $\text{liquidity}_{it-1} \times \mathbb{1}_{\text{crisis1}}$		0.407*			-0.132***	
		(1.88)			(-4.44)	
Loan $\text{liquidity}_{it-1} \times \mathbb{1}_{\text{crisis2}}$		0.883***			0.405***	
		(5.02)			(12.97)	
Loan $\text{liqudity}^{\Delta}_{it-1}$			-12.56***			-0.714
			(-3.60)			(-1.11)
Loan $\text{liquidity}^{\Delta}_{it-1} \times \mathbb{1}_{\text{crisis1}}$			15.82***			5.779***
			(2.69)			(5.83)
Loan $\text{liquidity}^{\Delta}_{it-1} \times \mathbb{1}_{\text{crisis2}}$			19.71**			19.29***
			(2.05)			(9.53)
Other controls	Yes	Yes	Yes	Yes	Yes	Yes
Other controls × event dummies	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9429	9429	9429	118054	118054	118054
Adjusted R^2	0.875	0.876	0.876	0.900	0.901	0.900

Bank identifier: RSSD9001, the unique identifying number (RSSDID) assigned by the Federal Reserve for all banks.

The highest holding company id: RSSD9348, the RSSDID of the highest holding company. We aggregate balance sheet variables of all banks that have the same highest holding company.

Total assets: RCFD2170, the sum of all asset items.

Total loans: RCFD1400, the gross book value of total loans and leases.

Home mortgages: RCON1430, real estate loans backed by 1-4 family residential properties.

Multi-family residential mortgages: RCON1460, real estate loans backed by residential properties with more than 4 families.

Commercial mortgages: RCON1480, real estate loans backed by non-farm and non-residential properties, such as business and industrial properties, hotels, hospitals and dormitories.

Consumer credit: RCFD1975, loans, not secured by real estate, issued to individuals for family or personal expenditure such as purchasing automobiles and paying medical expenses.

Commercial and industrial loans: The sum of RCFD1600, loans for commercial and industrial purpose to firms and RCFD1590, loans for financing agriculture production before 2000Q4; the sum of RCFD1766, commercial and industrial loans, RCFD1755, acceptances of other banks, and RCFD1590 between 2001Q1 and 2001Q4; the sum of RCFD1766 and RCFD1590.

Farm mortgages: RCON1420, real estate loans backed by farmlands.

Loans secured by real estate: The sum of Home mortgages, Multi-family residential mortgages, and commercial mortgages.

Commercial and industrial (C&I) loan commitments: RCFD3818, commitments used to extend commercial and industrial loans and loans to financial institutions.

Commitments backed by real estate: The sum of RCFD3814, home equity lines, RCFD3816, commitments of loans secured by multifamily residential properties, and RCFD6550, commitments to extend credit for the purpose of financing real estate ventures.

Total unused loan commitment: The sum of C&I loan commitment, commitments backed by real estate, and RCFD3817, commitments related to securities underwriting.

Interest rate, large time deposits (implicit) : To follow Acharya and Mora (2015), RIADA517/ RCONA514 after 1997Q1; RIAD4174/ RCON3345 before 1997Q1, where RIADA517 and RIAD4174 are interest expense on large time deposits and RCONA514 and RCON3345 are quarterly average of large time deposits.

Interest rate, core deposits (implicit): To follow Acharya and Mora (2015),

$$\frac{\text{RIAD4508+RIAD0093(replaced by RIAD4509 + RIAD4511 before 1997Q1)+RIADA518(replaced by RIAD4512 before 1997Q1)}}{\text{RCON3485+RCONB563(RCON3486 + RCON3487 before 2001Q1)+RCONA529(RCON3469 before 1997Q1)}},$$

where the numerator is the interest expense on core deposits and the denominator is the quarterly average core deposits.

Bank liquidity: Liquid assets divided by total assets. The calculation of liquid assets follows Loutskina (2011b). Liquid assets include RCFD0390(total investment securities), RCFD1350(Federal funds and securities purchased under agreement to sell), and RCFD2146(all assets held in trading accounts) before 1993Q4. Between

1994Q1 and 2001Q4, liquid assets are RCFD1350, RCFD1754(held-to-maturity securities) and rcfd1773(total available-for-sale securities). From 2002Q1 to 2009Q4, liquid assets are RCFDB987(Federal funds sold), RCFDB989(securities sold under agreement to resell), RCFD1754, and RCFD1773.

Total deposits: RCFD2200, the sum of total transaction accounts, nontransaction savings deposits, and total time deposits.

Equity capital: RCFD3210, the sum of perpetual preferred stock, common ctock, undivided profits and capital reserves, and others minus net unrealized loss on marketable equity securities.

Net income: RIAD4340.

Nonperforming loans: the sum of RCFD1407 and RCFD1403. RCFD1407 includes loans and lease financing receivables on which payment is due and unpaid for more than 90 days; RCFD1403 covers loans and lease financing receivables that the bank has placed in nonaccrual status.

Wholesale funding: The sum of RCON2604(time deposits of $100,000 or more), RCFD3200(outstanding subordinated notes and debentures), and RCFD2800(Federal Funds repo) before 1994Q1; the sume of RCON2604, RCFD3200, RCFD2800, RCFD3190(money owned to nonrelated commercial banks) between 1994Q2 and 2001Q4; the sum of RCON2604, RCFD3200, RCONB993(federal funds purchased), RCONB995(securities repo), and RCFD3190 since 2002Q1.

3.5 Appendix B: Loan Characteristics

Loan facility-level data are collected from Thomson Reuters' Dealscan database. In this section, we explain how we construct loan characteristic variables based on raw data.

Log of facility amount: The log of "facilityamt", the amount of the facility committed by a facility's lender pool.

Log of a borrower's sale: The log of "sales", the borrower's sales.

Leverage of a borrower: The ratio of "sales" to the log of the entire facility amount under the same package.

Spread of a loan facility: "Allindrawn", the sum of coupon spread and annual fee the borrower has to pay if it draws down one dollar.

Major loan purpose: Loan purposes that we take into account include corporate purpose, debt repay, working capital, and leveraged buyout. Each loan purpose is indicated by a separate dummy variable.

Bibliography

Acharya, Viral V, Gara Afonso, and Anna Kovner (2013), "How do global banks scramble for liquidity? evidence from the asset-backed commercial paper freeze of 2007." Technical report, Federal Reserve Bank of New York Staff Report No.623.

Acharya, Viral V and Nada Mora (2015), "A crisis of banks as liquidity providers." *The Journal of Finance*, 70, 1–43.

Acharya, Viral V, Philipp Schnabl, and Gustavo Suarez (2012), "Securitization without risk transfer." *Journal of Financial Economics*.

Adrian, Tobias and Adam B Ashcraft (2012), "Shadow banking: a review of the literature." Technical report, Federal Reserve Bank of New York.

Adrian, Tobias and Nina Boyarchenko (2012), "Intermediary leverage cycles and financial stability." Technical report, Federal Reserve Bank of New York Staff Report No.567.

Bernanke, Ben and Mark Gertler (1989), "Agency costs, net worth, and business fluctuations." *The American Economic Review*, 14–31.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999), "Chapter 21 the financial accelerator in a quantitative business cycle framework." volume 1, Part C of *Handbook of Macroeconomics*, 1341 – 1393, Elsevier, URL `http://www.sciencedirect.com/science/article/pii/S157400489910034X`.

Berner, Richard (2014), "Financial stability: Progress and challenges." URL `http://www.treasury.gov/initiatives/ofr/news/Documents/Berner_remarks_NYU_Money_Marketeers_10-16-2014_FINAL.pdf`.

Bianchi, Javier (2011), "Overborrowing and systemic externalities in the business cycle." *American Economic Review*, 101, 3400–3426, URL `http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.7.3400`.

Brunnermeier, Markus K and Yuliy Sannikov (2013), "The i theory of money." *Unpublished working paper, Princeton University*.

Brunnermeier, Markus K and Yuliy Sannikov (2014), "A macroeconomic model with a financial sector." *The American Economic Review*, 104, 379–421.

Brunnermeier, Markus Konrad, Andrew Crockett, Charles AE Goodhart, Avinash Persaud, and Hyun Song Shin (2009), "The fundamental principles of financial regulation."

Bryant, John (1980), "A model of reserves, bank runs, and deposit insurance." *Journal of Banking & Finance*, 4, 335–344.

Campbell, John Y and John H Cochrane (1999), "By force of habit: A consumption based explanation of aggregate stock market behaviorv." *Journal of Political Economy*, 107, 205–251.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans (2005), "Nominal rigidities and the dynamic effects of a shock to monetary policy." *Journal of political Economy*, 113, 1–45.

Cornett, Marcia Millon, Jamie John McNutt, Philip E Strahan, and Hassan Tehranian (2011), "Liquidity risk management and credit supply in the financial crisis." *Journal of Financial Economics*, 101, 297–312.

Danielsson, Jon, Hyun Song Shin, and Jean-Pierre Zigrand (2012), "Procyclical leverage and endogenous risk." Technical report, Unpublished Working Paper, Princeton University.

Di Tella, Sebastian (2012), "Uncertainty shocks and balance sheet recessions." Technical report.

Diamond, Douglas W and Philip H Dybvig (1983), "Bank runs, deposit insurance, and liquidity." *The Journal of Political Economy*, 401–419.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl (2014), "A model of monetary policy and risk premia." Technical report, National Bureau of Economic Research.

Eisfeldt, Andrea and Tyler Muir (2012), "Aggregate issuance and savings waves." *manuscript, UCLA*.

Flannery, Mark J (1994), "Debt maturity and the deadweight cost of leverage: Optimally financing banking firms." *The American Economic Review*, 320–331.

Gatev, Evan, Til Schuermann, and Philip E Strahan (2009), "Managing bank liquidity risk: How deposit-loan synergies vary with market conditions." *Review of Financial Studies*, 22, 995–1020.

Gatev, Evan and Philip E Strahan (2006), "Banks' advantage in hedging liquidity risk: Theory and evidence from the commercial paper market." *The Journal of Finance*, 61, 867–892.

Gennaioli, Nicola, Andrei Shleifer, and Robert W Vishny (2013a), "A model of shadow banking." *The Journal of Finance*, 68, 1331–1363.

Gennaioli, Nicola, Andrei Shleifer, and Robert W Vishny (2013b), "A model of shadow banking." *The Journal of Finance*, 68, 1331–1363.

Gertler, Mark and Peter Karadi (2011), "A model of unconventional monetary policy." *Journal of monetary Economics*, 58, 17–34.

Gertler, Mark and Nobuhiro Kiyotaki (2010), "Chapter 11 - financial intermediation and credit policy in business cycle analysis." volume 3 of *Handbook of Monetary Economics*, 547 – 599, Elsevier, URL http://www.sciencedirect.com/science/article/pii/B9780444532381000119.

Gertler, Mark and Nobuhiro Kiyotaki (2013), "Banking, liquidity and bank runs in an infinite horizon economy." *Unpublished Working Paper, Princeton University*.

Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto (2012), "Financial crises, bank risk exposure and government financial policy." *Journal of Monetary Economics*.

Gilchrist, Simon and Egon Zakrajšek (2012), "Credit spreads and business cycle fluctuations." *American Economic Review*, 102, 1692–1720, URL http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.4.1692.

Goodhart, Charles A.E., Anil K Kashyap, Dimitrios P. Tsomocos, and Alexandros P. Vardoulakis (2012), "Financial regulation in general equilibrium." Working Paper 17909, National Bureau of Economic Research, URL http://www.nber.org/papers/w17909.

Gorton, Gary B and Nicholas S Souleles (2007), "Special purpose vehicles and securitization." In *The Risks of Financial Institutions*, 549–602, University of Chicago Press.

Gruber, Jonathan (2006), "A tax-based estimate of the elasticity of intertemporal substitution." Technical report, National Bureau of Economic Research.

Hansen, A.H. (1951), *Business Cycles and National Income*. Norton, URL http://books.google.com/books?id=TuFdAAAAIAAJ.

He, Zhiguo and Arvind Krishnamurthy (2012a), "A macroeconomic framework for quantifying systemic risk." *Fama-Miller Working Paper*, 12–37.

He, Zhiguo. and Arvind. Krishnamurthy (2012b), "A model of capital and crises." *The Review of Economic Studies*, 79, 735–777.

He, Zhiguo and Arvind Krishnamurthy (2013), "Intermediary asset pricing." *American Economic Review*, 103, 732–70, URL http://www.aeaweb.org/articles.php?doi=10.1257/aer.103.2.732.

Huang, Ji (2014), "Systemic run on shadow banks: The minsky moment." working paper, Princeton University, URL http://www.princeton.edu/~jihuang/systemic_run1.pdf.

IMF (2014), "Global financial stability report, october 2014 : Risk taking, liquidity, and shadow banking: Curbing excess while promoting growth." URL https://www.imf.org/external/pubs/ft/gfsr/2014/02/pdf/text.pdf.

Ivashina, Victoria and David Scharfstein (2010), "Bank lending during the financial crisis of 2008." *Journal of Financial economics*, 97, 319–338.

Jordà, Òscar, Moritz HP Schularick, and Alan M Taylor (2011), "When credit bites back: Leverage, business cycles, and crises." Technical report, National Bureau of Economic Research.

Kashyap, Anil K, Raghuram Rajan, and Jeremy C Stein (2002), "Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking." *The Journal of Finance*, 57, 33–73.

Kehoe, Timothy J and David K Levine (1993), "Debt-constrained asset markets." *The Review of Economic Studies*, 60, 865–888.

Kindleberger, C.P. (2000), *Manias, Panics, and Crashes: a History of Financial Crises*. Wiley investment classics, Wiley, URL http://books.google.com/books?id=fHx-kko1GaYC.

Kiyotaki, Nobuhiro and John Moore (1997), "Credit cycles." *Journal of Political Economy*, 105, pp. 211–248, URL http://www.jstor.org/stable/10.1086/262072.

Kocherlakota, Narayana R (1996), "Implications of efficient risk sharing without commitment." *The Review of Economic Studies*, 63, 595–609.

Krishnamurthy, Arvind (2010), "Amplification mechanisms in liquidity crises." *American Economic Journal: Macroeconomics*, 2, 1–30, URL http://www.aeaweb.org/articles.php?doi=10.1257/mac.2.3.1.

Lorenzoni, G. (2008), "Inefficient credit booms." *The Review of Economic Studies*, 75, 809–833.

Loutskina, Elena (2011a), "The role of securitization in bank liquidity and funding management." *Journal of Financial Economics*, 100, 663 – 684, URL http://www.sciencedirect.com/science/article/pii/S0304405X11000389.

Loutskina, Elena (2011b), "The role of securitization in bank liquidity and funding management." *Journal of Financial Economics*, 100, 663–684.

Loutskina, Elena and Philip E Strahan (2009), "Securitization and the declining impact of bank finance on loan supply: Evidence from mortgage originations." *The Journal of Finance*, 64, 861–889.

Luck, Stephan and Paul Schempp (2014), "Banks, shadow banking, and fragility." Working Paper 1726, European Central Bank.

McCulley, Paul (2009), "The shadow banking system and hyman minsky's economic journey."

Minsky, H.P. (1986), *Stabilizing an Unstable Economy*. Number v. 1 in A Twentieth Century Fund report, Yale University Press, URL http://books.google.com/books?id=XTl9QgAACAAJ.

Minsky, Hyman P (1992), "The financial instability hypothesis." *The Jerome Levy Economics Institute Working Paper*.

Moreira, Alan and Alexi Savov (2014), "The macroeconomics of shadow banking." Working Paper 20335, National Bureau of Economic Research, URL http://www.nber.org/papers/w20335.

Morris, Stephen and Hyun Song Shin (2002), "Global games: Theory and applications, advances in economic theory and econometrics: Proceedings of the eighth world congress of the econometric society (m. dewatripont, l. hansen, and s. turnovsky, eds.)."

Muir, Tyler (2013), "Financial crises, risk premia, and the term structure of risky assets." Technical report.

Myers, Stewart C and Raghuram G Rajan (1998), "The paradox of liquidity." *The Quarterly Journal of Economics*, 113, 733–771.

Ordonez, Guillermo (2013), "Sustainable shadow banking." Working Paper 19022, National Bureau of Economic Research, URL http://www.nber.org/papers/w19022.

Phelan, Gregory (2012), "Financial intermediation, leverage, and macroeconomic instability." Working paper, Yale University.

Plantin, Guillaume (2014), "Shadow banking and bank capital regulation." *Review of Financial Studies*, URL http://rfs.oxfordjournals.org/content/early/2014/08/19/rfs.hhu055.abstract.

Pozsar, Z., T. Adrian, A. Ashcraft, and H. Boesky (2010), "Shadow banking." Technical report, Federal Reserve Bank of New York Staff Report No.458.

Reinhart, Carmen M and Kenneth S Rogoff (2009), "The aftermath of financial crises." Technical report, National Bureau of Economic Research.

Schularick, Moritz and Alan M. Taylor (2012), "Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008." *American Economic Review*, 102, 1029–61, URL http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.2.1029.

Shockley, Richard L and Anjan V Thakor (1997), "Bank loan commitment contracts: Data, theory, and tests." *Journal of Money, Credit, and Banking*, 517–534.

Stein, Jeremy C (2012), "Monetary policy as financial stability regulation." *The Quarterly Journal of Economics*, 127, 57–95.

Stokey, Nancy, Robert Lucas, and Edward Prescott (1989), "Recursive methods in economic dynamics." *Cambridge MA*.

Thomas, Jonathan and Tim Worrall (1988), "Self-enforcing wage contracts." *The Review of Economic Studies*, 55, 541–554.

Wachter, Jessica A (2013), "Can time-varying risk of rare disasters explain aggregate stock market volatility?" *The Journal of Finance*, 68, 987–1035.