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# Essays on the Causes and Consequences of Economic Inequality

Lhuillier, Hugo

Princeton, NJ : Princeton University

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# Essays on the Causes and Consequences of Economic Inequality

Hugo Lhuillier

A Dissertation

Presented to the Faculty

of Princeton University

in Candidacy for the Degree

of Doctor of Philosophy

Recommended for Acceptance

by the Department of

Economics

Advisers: Richard Rogerson and Giovanni L. Violante

May 2024

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# Abstract

This dissertation studies the sources and macroeconomic consequences of inequality, across workers, cities, and states of the world. Chapter 1 starts by studying the aggregate consequences of spatial inequality on human capital. While the spatial concentration of talent begets regional disparities, it may benefit aggregate productivity by fostering human capital accumulation. To study this equity-efficiency tradeoff, I propose a theory wherein individuals learn from the other workers in their cities. Learning complementarities determine the existence of a tradeoff. Empirically, I find that workers employed in relatively skill-dense cities experience faster wage growth, and disproportionately so if they are skilled. Once estimated, the model gives rise to a steep tradeoff between aggregate human capital and spatial inequality. As a consequence, I find that large moving vouchers are effective at reducing spatial disparities at the cost of decreased aggregate efficiency.

Chapter 2, joint with Adrien Bilal, then studies what are the implications of the rise of domestic outsourcing on labor market inequality and productivity. Outsourced workers indeed experience large wage declines, yet domestic outsourcing may raise aggregate productivity. To address this question, we contribute a framework in which firms either hire workers in-house, or rent labor services from contractors. More productive firms are more likely to outsource their workers to save on higher labor costs, and they enjoy larger size when doing so. We provide empirical causal evidence in support of these predictions. After structurally estimating the model, we find that the rise in outsourcing in France between 1997 and 2016 lowers low-skill service worker earnings and welfare but raises aggregate output.

Chapter 3, co-authored with Gene Grossman and Elhanan Helpman, finally studies whether firms' supply chains inefficiently expose consumers to global risks. We address this question in a model of production with a critical input and exogenous risks of supply disturbances. We establish that, with constant elasticity of substitution across goods, firms necessarily under-diversify their supply chain. In contrast, when the demand elasticity rises with price, private investments in resilience may be socially excessive.

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To my grandparents:

Reine, Thérèse, André, and Guy,

and to Jean.

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# Chapter 1

# Should I Stay or Should I Grow? How Cities Affect Learning, Inequality and Productivity

## 1.1 Introduction

Spatial inequalities are pervasive across the globe. Cities such as New York, Paris, and Tokyo have become epicenters of talent, leaving remote regions deprived of such individuals. As workers exchange ideas with, collaborate with, and potentially learn from their neighbors, the spatial concentration of skills may have enduring effects on human capital accumulation while at the same time segregating learning opportunities in a few cities. Concerned by such regional disparities, national governments spend a sizable share of their budget every year to rebalance income across space. Yet, little is known about the aggregate effects of these policies on human capital. How does the sorting of workers into cities shape learning? To what extent are learning opportunities spatially concentrated, and how much does their concentration amplify regional disparities? Can policies spur higher productivity in conjunction with lower geographical inequalities?

This chaper offers answers to these questions with four contributions. First, I propose a theory that sheds light on how the concentration of skills in space affects human capital accumulation. Second, I estimate the impact of cities' skill composition on wage growth using French administrative data. Third, I use the estimated model to quantify the extent to which learning opportunities are spatially segregated, and what are the consequences for spatial disparities and aggregate productivity. Fourth, I quantify the tradeoff between spatial inequality and human capital accumulation by evaluating the general equilibrium consequences of a large-scale moving voucher policy.

In the first part of the chaper, I propose a theory of learning across space. The structure is an overlapping generation model with two key features. First, young workers are endowed with heterogeneous skills, cities differ in their productivity, and skill and productivity are complements in production. Second, young workers learn by randomly meeting other workers in their city. The learning depends on which workers are engaged in the interaction. Importantly, I impose minimal restrictions on the learning technology. Skilled individuals may disproportionately learn from other skilled workers. In this case, within-skill learning complementarities dominate between-skill complementarities. On the contrary, between-skill complementarities prevail if the marginal gains from meeting a high-skill are decreasing in workers' own skill. Individuals are forward-looking and freely choose where to work to maximize their lifetime utility.

The sorting of workers in cities shapes the spatial distribution of learning opportunities, which in turn may amplify the spatial concentration of skills. Holding constant future income, high-skill workers sort into productive cities as these locations offer relatively higher wages. Yet, their location decisions also reflect where the local learning opportunities are. When skilled workers agglomerate in productive cities, individuals who learn relatively more from them view these locations as attractive learning centers. The stronger the within-skill learning complementarities, the more skilled workers gain over time from working next to other skilled individuals, and the further the learning motive amplifies the spatial concentration of skills.

Spatial inequality feeds into the aggregate stock of human capital. The agglomeration of high-skill next to each other increases the frequency at which they meet while reducing the opportunity for low-skill to learn from them. Whether this segregation of opportunities enhances aggregate productivity relies on which workers learn the most from the high-skill. The stock of human capital expands with greater spatial inequality if skilled workers learn disproportionately from interacting with their peers. In contrast, the spatial concentration of learning opportunities reduces aggregate productivity if between-skill complementarities prevail.

Spatial policies have the potential to accelerate aggregate human capital accumulation. The competitive allocation is inefficient as workers do not internalize their impacts on others' learning. The marginal social value of interactions hinges on learning complementarities. When high-skill learn relatively more from each other, low-skill crowd out the learning opportunities of productive cities, and the competitive equilibrium features too little spatial skill concentration. The reverse occurs when between-skill complementarities are relatively stronger. Skilled workers do not consider how their location choice helps low-skills learn, and the decentralized equilibrium features too much spatial inequality. Regardless of the complementarities, the optimal allocation spurs human capital accumulation and can be decentralized by place-based policies that vary by skill.

The second part of this chapter develops and structurally estimates a quantitative version of the framework with three main additions. First, workers consume local housing in addition to the freely traded good already present in the theory. Second, workers face migration costs. The migration costs imply that workers' birthplace shapes their lifetime opportunities. Third, I impose a flexible functional form on the learning technology according to which three elasticities govern skill growth: with respect to workers' own skill, the skill of the worker they interact with, and a cross-term that captures learning complementarities.

I estimate the quantitative framework using French administrative data. The estimation is split into two steps. I use the structure of the model to derive estimating equations that

identify a first set of parameters. I then calibrate the remaining parameters in the second step to match salient features of the wage distribution in the aggregate, across cities, and across the lifecycle.

The learning complementarities are transparently estimated. The learning technology is recovered from a model-consistent local projection of future wages on workers' current wages and the average wage of the individuals working in the same city. I find the three elasticities governing the learning technology to be positive. First, relatively skilled workers experience faster skill growth holding constant their interactions. Second, every worker learns more from skilled individuals. Third, the returns to meeting a high skill are relatively larger for skilled workers. Altogether, the learning technology displays stronger within- than between-skill complementarities.

The equation estimating the learning technology allows me to derive clear-cut conditions that guarantee unbiased estimates. Specifically, the drivers of wage growth other than local interactions must be identically and independently distributed across cities and individuals conditional on workers' wages. I use the granularity of the French matched employer-employee data to relax this identification assumption in three ways. First, I control for worker-level characteristics, such as age and tenure, known to shape wage growth. Second, I saturate the local projection with fixed effects to control for unobserved heterogeneity in wage dynamics across cities and firms. In the most restrictive specification, the skill growth elasticities are estimated off variations in average wages across neighborhoods of employment for individuals who live in the same neighborhood and work in the same firm within the same city. Third, to generate quasi-random variations in skill density across space, I instrument present neighborhood wages with variations in the local share of white-collar twenty years ago. Armed with these controls and instrument, I find that neighborhoods with a greater skill density boost the wage growth of the individuals working there, and disproportionately so if they are skilled.

The estimated returns to local interactions are substantial. The average worker experiences

a wage growth 4.5 log points faster over five years when working in Paris (top 10% of the city-growth distribution) compared to when working in Troyes (bottom 10%). These growth gains differ considerably across workers due to the estimated complementarities. A worker in the bottom 10% of the wage distribution sees its wage growth accelerate by 2.8 log points when moving from Troyes to Paris, whereas these dynamic gains are 7.3 log points for individuals in the top 10%. Altogether, the model explains 64% of the between-city wage growth variance.

In the third part of this chapter, I use the estimated model to quantify how much of the spatial differences in human capital accumulation are caused by the segregation of learning opportunities, and what are the implications for spatial disparities and aggregate productivity. To do so, I compare the baseline equilibrium in which cities segment learning interactions with a counterfactual economy in which individuals learn randomly from workers in the entire economy.

I find the spatial segmentation of opportunities, rather than the sorting of fast-learning workers, to explain the vast majority of the learning gaps between cities. This, in turn, engenders spatial learning inequality. Workers born in the 25% poorest French cities have a lifetime skill growth 10% lower than the average French worker. In addition, the concentration of opportunities in a few cities amplifies regional disparities. Productive cities get bigger as workers need to live there to accumulate human capital faster. The spatial concentration of high-skill rises as they benefit more over time from working in skill-dense locations. Combined, spatial wage inequality increases: the between-city wage variance grows from 0.007 to 0.024 when learning occurs within cities.

The confinement of learning opportunities to productive cities also boosts aggregate human capital accumulation. The stock of human capital is 1.2% higher when learning takes place within cities. Learning complementarities explain the majority of this productivity gain: the average skill of the economy would only be 0.5% higher were all workers to learn equally from skilled workers. The learning gains are concentrated at the high end of the skill distribution due to the concentration of these workers in skill-dense cities and the presence

of learning complementarities. In contrast, workers at the bottom of the skill distribution neither gain nor lose from the spatial concentration of learning opportunities.

Combined, the estimated model suggests the existence of a steep tradeoff between spatial inequality and human capital accumulation. In the fourth part of this chapter, I quantify this equity-efficiency tradeoff by evaluating the consequences of a moving voucher policy. Specifically, I study a policy subsidizing every worker born in the 25% poorest French cities to move to the three largest locations.

The moving voucher policy is spatially redistributive. On the one hand, the policy improves the opportunities of treated workers by helping them reallocate to skill-dense locations. When the subsidy covers 80% of the average migration costs, which represents 1.5% of GDP, the probability that treated workers migrate to the three largest French cities rises from 7% to 45%. Treated individuals have an easier access to learning centers, and their lifetime skill growth coincides with that of the average worker. As a result, their welfare increases by 2% on average. On the other hand, the moving vouchers depress the learning of the non-treated. The policy attenuates the spatial concentration of skills by reallocating marginally less skilled workers to productive cities. The local learning opportunities worsen, and workers born there, as well as the other non-treated workers who were migrating to these cities to learn, experience slower human capital accumulation. Combined with the financial cost of the policy, non-treated workers suffer average welfare losses of 2.3%.

While the policy reduces spatial disparities, it acts negatively on aggregate efficiency. The stock of human capital is 0.3% lower when the policy reaches 1.5% of GDP. The general equilibrium effect of the policy more than offset its partial equilibrium gains. Holding the quality of local opportunities constant, human capital would rise by 0.8% as the policy expands the pool of workers with access to fast-learning opportunities. Absent learning complementarities, the general equilibrium feedback would be a third smaller. I conclude that learning complementarities are crucial for the tradeoff between spatial inequality and human capital accumulation.

**Related literature** This chapter primarily adds to two strands of the literature. The first studies how social interactions shape productivity growth. Started by the seminal work of Jovanovic and Rob (1989), and later developed by Lucas (2009a), Lucas and Moll (2014), Perla and Tonetti (2014), and Akcigit et al. (2018), amongst many others, this literature initially abstracted from space. The idea that cities facilitate knowledge diffusion is however not novel and can be traced back to Marshall (1890) and Jacobs (1969).[1] In particular, if interactions spur learning and mainly occur between individuals who work next to each other, then the spatial distribution of talent must matter for human capital accumulation. Two papers have recently quantified this insight.[2] Martellini (2022) finds that local peer effects explain the majority of the faster lifecycle wage growth of large cities in the United States. Crews (2023) builds a dynamic model in which the vibrancy of cities, defined by their size and skill composition, shapes how workers learn. He concludes that relaxing land-use regulations in New York and San Francisco spurs aggregate growth.[3]

I contribute to this literature by characterizing the crucial role played by learning complementarities in shaping how cities affect human capital accumulation. In particular, by not imposing restrictions on learning complementarities, I show that the spatial segmentation of learning opportunities may be detrimental to learning, an insight already noted by Benabou (1993).

The second strand of the literature estimates the dynamic returns to interactions between neighbors and coworkers. Across space, Baum-Snow and Pavan (2012), Roca and Puga (2017), and Eckert et al. (2022), among others, find the wage age-profile of large cities to be relatively steeper. Within the firm, Nix (2020) and Jarosch et al. (2021) document that workers employed in high-wage firms experience faster wage growth. I add to this literature

[1] Since then, a long list of papers have documented how proximity shapes knowledge diffusion (e.g. Jaffe et al., 1993; Audretsch and Feldman, 2004; Moretti, 2021; Atkin et al., 2022; Baum-Snow et al., 2023).

[2] Several other recent papers have studied the impact of space on aggregate growth through trade and technology diffusion, e.g. Lucas (2009b), Alvarez et al. (2013), Sampson (2016), Desmet et al. (2018), Buera and Oberfield (2020), Perla et al. (2021), and Cai et al. (2022).

[3] A related literature focuses on the joint evolution of city sizes and aggregate growth (Rossi-Hansberg and Wright, 2007; Herkenhoff et al., 2018; Hsieh and Moretti, 2019; Duranton and Puga, 2023).

in three ways. First and foremost, I estimate how the returns to local interactions vary across workers. The granularity of the French matched employer-employee allows me in particular to estimate these learning complementarities while addressing some of the endogeneity concerns present in the peer effect literature (Angrist, 2014). Second, I find the effect of cities' skill composition on wage growth to be one order of magnitude larger than that of city size. Third, I show that the neighborhood where individuals work shapes their wage growth even after controlling at which establishment and firm they are employed.

This chapter also connects more broadly with several papers in economic geography. First, it adds to the literature that investigates the sources of spatial agglomeration (for a survey, see Duranton and Puga, 2004; Behrens and Robert-Nicoud, 2015). Two papers in particular have studied theoretically how learning within cities triggers spatial agglomeration. Glaeser (1999) shows that urbanization rises when the ability to learn by imitation increases. More recently, Davis and Dingel (2019) finds that local interactions can provide a motive for sorting even when cities are *ex-ante* similar. I contribute to this literature by quantifying the strength of these agglomeration forces. I find that learning within cities amplifies the skill concentration in space, an empirical pattern documented by Combes et al. (2008), Eeckhout et al. (2014), and Diamond (2016), among others. Second, this chapter adds to the literature that studies optimal spatial policies.[4] Most related are the papers by Rossi-Hansberg et al. (2019) and Fajgelbaum and Gaubert (2020) who study optimal policy in the presence of worker sorting and static production spillovers. Related to these papers, I characterize the optimal spatial policy when spillovers are dynamic.

The rest of this chapter is organized as follows. Section 1.2 presents the theory and characterizes how learning complementarities shape the tradeoff between spatial inequality and human capital accumulation. Section 1.3 turns to the French matched employer-employee to estimate these complementarities. Section 1.4 estimates the rest of the model, and

[4]See Glaeser and Gottlieb (2008) and Kline and Moretti (2014) for reviews of the estimated consequences of place-based policies on productivity.

Section 1.5 quantifies the impact of local interactions on human capital accumulation and agglomeration. Section 1.6 proceeds to study the consequences of spatial policies on spatial inequality, productivity, and welfare. Section 1.7 concludes.

## 1.2 A Theory of Learning across Space

In this section, I shed light on the tradeoff between spatial inequality and human capital accumulation by offering a model of learning across space. The model puts learning complementarities front and center. The remaining ingredients are standard in the quantitative spatial literature (Redding and Rossi-Hansberg, 2017).

### 1.2.1 Setup

I model workers that sort into cities. Every period, a unit mass of young workers enters the labor market. Workers live for two periods and discount the future at rate $\beta$. Denote their first and second life period as young ($y$) and old ($o$). Old workers express no altruism towards future generations. Workers choose in which city to work at any point in time.[5] Cities are indexed by $\ell$, and the total number of cities $L$ in the economy is fixed. I study the steady state of this economy and omit time subscripts.

**Preferences** Individuals consume a single, freely-traded good taken to be the numéraire. I abstract from local consumption for the sake of tractability. I add housing in Section 1.2.6. Individuals hold linear utility over their consumption. They supply inelastically one unit of labor per period, so that wages and incomes are interchangeable. They do not have access to a saving device.

Individuals have idiosyncratic preferences for the city they work in, denoted by $\boldsymbol{\varepsilon} = \{\varepsilon_\ell\}_{\ell=1}^{L}$. These preferences are drawn from independent Gumbel distributions with city- and age-specific location parameters $B_\ell^a$, $a \in \{y, o\}$, and scale parameter $\vartheta^{-1}$. The location parameter

[5] I define a city as a commuting zone in the data and thereby abstract from commuting decisions.

$B_\ell^a$ reflects the amenities offered by city $\ell$ to workers with age $a$. Young workers may prefer cities with a vibrant nightlife whereas old workers may value the opera's quality more, and I impose no relationship between $B_\ell^y$ and $B_\ell^o$. Local amenities are taken as given. Location preferences are independent across the lifecycle.[6]

Finally, consumption and location preferences are perfect substitutes. Hence, the per-period utility of workers with income $y$ and preferences $\boldsymbol{\varepsilon}$ when working in city $\ell$ is $y + \varepsilon_\ell$.

**Human capital** Young workers are endowed with an initial skill $s_y$ drawn from the skill distribution $N^y$. The young skill distribution is a primitive of the model. I suppose that skills are uni-dimensional and that the support of $N^y$ is connected, positive, and bounded, $[\underline{s}_y, \bar{s}_y]$, with $\underline{s}_y \geq 0$ and $\bar{s}_y < \infty$.[7]

Workers learn in their youth. This learning is shaped by the interactions they encounter. If a worker with skill $s_y$ interacts with a partner with skill $s_p$, they obtain new skill $s_o = e\gamma(s_y, s_p)$. The learning technology $\gamma$ encapsulates how workers' human capital accumulation depends on their own skill and that of the other workers. In addition, workers experience an idiosyncratic learning shock, $e$, which captures the other sources of learning (e.g. on-the-job training). These shocks are drawn from the city-independent distribution $F$ with support $\mathbb{R}_+$.[8]

I impose minimal restrictions on the learning technology.

**Assumption 1.1** (Learning technology)**.** *The learning technology $\gamma$ is positive, bounded, and twice differentiable in $(s_y, s_p)$.*

Assumption 1.1 allows my framework to nest prevalent theories of human capital accumulation. First, it encompasses cases where interactions are irrelevant for learning, $\gamma(s_y, s_p) = g(s_y)$ for some function $g$. In this case, workers' capacity to learn may still depend on their own skill, e.g. if skilled workers are better at learning-by-doing (Huggett et al., 2011). Second,

[6] I add migration costs in Section 1.2.6. Migration costs and persistent location preferences are isomorphic up to welfare accounting.

[7] The estimation of the learning technology (Section 1.3) is robust to adding worker fixed effects to account for multi-dimensional skill heterogeneity.

[8] The idiosyncratic shocks are not required for the theory but smooth the aggregate old skill distribution. They help quantitatively to match the wage distribution of old workers.

all workers may learn equally from particular interactions, $\gamma(s_y, s_p) = g(s_y) + h(s_p)$, for $h$ another function. Then, the learning technology does not feature any complementarity. In contrast, the learning complementarities may be stronger within-skill than between-skill. Formally, the learning technology may be supermodular:

$$\frac{\partial^2 \gamma(s_y, s_p)}{\partial s_y \partial s_p} > 0, \quad \forall (s_y, s_p).$$

Supermodularity demands the marginal gains from a skilled interaction to be greater for skilled individuals. This occurs when workers with similar skills are better able to learn from each other (e.g. Jovanovic, 2014). Alternatively, the learning technology may display complementarities that are stronger between than within-skill. Then, the learning technology is submodular,

$$\frac{\partial^2 \gamma(s_y, s_p)}{\partial s_y \partial s_p} < 0, \quad \forall (s_y, s_p),$$

and the marginal gains from skilled interactions are greater for low-skills. This occurs when knowledge diffuses from skilled to relatively less skilled individuals (Lucas and Moll, 2014).

The simple learning technology

$$\gamma(s_y, s_p) = g_1 \times s_y + g_2 \times s_p + g_{12} \times s_y \times s_p \tag{1.1}$$

illustrates well when do within- or between-skill complementarities prevail. While $g_1$ and $g_2$ govern the average impact of workers' and partners' skills on learning, the complementarities are fully determined by $g_{12}$. If $g_{12} > 0$, the learning technology is supermodular, and skilled workers learn relatively more from skilled partners. On the contrary, the learning technology is submodular when $g_{12} < 0$.

Workers interact with one other worker in their youth. Two assumptions govern this interaction. First, interactions are spatially segmented: they occur between individuals

working in the same location. In this section, I also assume for the sake of simplicity that interactions take place between young workers. I relax this assumption in Section 1.2.6. Second, interactions are random. Hence, while workers cannot target specific interactions, they can choose to locate in a particular city to increase their chance of meeting a particular skill-type. The likelihood of an interaction with a skill $s_p$ in city $\ell$ is then given by the local density of this skill, $\pi_\ell^y(s_p)$.

Learning takes time, and the new skill obtained from the interaction can only be used in the old period. The aggregate skill distribution of old workers is denoted by $N^o$. Contrary to the young workers' skill distribution, this is a general equilibrium object that reflects the learning of every young worker.

**Production** Production takes place within cities. Each city has a representative competitive firm with productivity $T_\ell$, which reflects the industries and firms present there. For the equilibrium to be scale invariant, I normalize the dispersion of the idiosyncratic location preferences by the (unweighted) average city TFP, $\vartheta = \theta/\mathbb{E}[T_\ell]$, for $\theta$ the scale-invariant dispersion.

The representative firm produces the tradable good using labor as the sole input. If the firm hires a bundle of skill $\boldsymbol{n}_\ell = \{n_\ell(s)\}_s$, it produces output

$$Y_\ell(\boldsymbol{n}_\ell) = T_\ell \int s n_\ell(s) ds.$$

This production function has two properties. First, cities' TFP and workers' skills are complements: high-skill workers produce relatively more in productive cities than their low-skill counterparts. Second, skills are perfect substitutes. I assume away skill complementarities in production, which has been the focus of the spatial literature thus far (e.g. Diamond, 2016; Giannone, 2022), to isolate the impact of complementarities in the learning process. Appendix A.3.1 shows that the theoretical predictions are robust to adding production complementarities, and so is the estimation of the learning technology in Section 1.3.

The labor markets are segmented by cities and skills. The city-specific zero profit condition pins down wages, $w_\ell(s) = sT_\ell$, which inherit the complementarity between TFP and skill.

**Feasibility** Finally, the spatial allocation of workers must be feasible. The aggregate demand for workers with skill $s$ must be less than or equal to the aggregate supply of that skill:

$$\sum_\ell n_\ell^a(s) \leq n^a(s), \quad \forall\, s,\ \forall\, a \in \{y, o\}, \tag{1.2}$$

where $n_\ell^a(s)$ is the measure of workers with age $a$ and skill $s$ employed in city $\ell$, and $n^a(s) = dN^a(s)$ is the aggregate density of skill $s$ for workers with age $a$. Accordingly, the local density of skill $s$ in city $\ell$ amongst workers with age $a$ is

$$\pi_\ell^a(s) = \frac{n_\ell^a(s)}{N_\ell^a}, \quad \forall\, s, \forall\, \ell,\ \forall\, a, \tag{1.3}$$

for $N_\ell^a = \int n_\ell^a(s)ds$ the mass of workers with age $a$ employed in $\ell$.

### 1.2.2 The Steady State Equilibrium

Three equations govern the steady state equilibrium.[9]

**Old workers** Old workers solve a static location choice problem. They choose a city $\ell$ that maximizes their present utility conditional on their skill $s_o$ and their location preferences $\boldsymbol{\varepsilon}$,

$$V^o(s_o, \boldsymbol{\varepsilon}) = \max_\ell \left\{ w_\ell(s_o) + \varepsilon_\ell \right\}, \quad \forall\, s_o, \forall\, \boldsymbol{\varepsilon}.$$

Workers' idiosyncratic preferences ensure that every skill is present in every location. In

[9]The young spatial allocation is decoupled from that of the old. As a result, the economy is independent from its initial condition and converges to its steady state in one period.

particular, the number of old workers with skill $s$ employed in city $\ell$ is given by

$$n_\ell^o(s_o) = n^o(s_o)\left(\frac{\mathrm{e}^{\vartheta\left(w_\ell(s_o)+B_\ell^o\right)}}{\sum_{\ell'}\mathrm{e}^{\vartheta\left(w_{\ell'}(s_o)+B_{\ell'}^o\right)}}\right), \quad \forall\, s, \forall\, \ell. \tag{1.4}$$

The first term captures the aggregate supply of skill $s_o$ amongst the old and follows from the feasibility condition (1.2). The second term is the probability that skill $s_o$ chooses to work in city $\ell$ and is given by the Gumbel-distributed location preferences.

**Young workers** Young workers solve a dynamic location choice problem,

$$V^y(s_y, \boldsymbol{\varepsilon}) = \max_\ell \left\{ w_\ell(s_y) + \varepsilon_\ell + \beta \int\int \mathcal{V}^o[e\gamma(s_y, s_p)]dF(e)\pi_\ell^y(s_p)ds_p \right\}, \quad \forall\, s_y, \forall\, \boldsymbol{\varepsilon},$$

where $\mathcal{V}^o(s_y) \equiv \mathbb{E}[V^o(s_y, \boldsymbol{\varepsilon}') \mid s_y]$ is the expected utility of an old worker with skill $s_y$ prior to observing their idiosyncratic preferences.[10] Young workers' location decision depends on three considerations. The first two are static and are common to old workers. The third consideration is dynamic and reflects where the best learning opportunities are located. The expected future utility of working in city $\ell$ is indeed given by the weighted average of the gains from particular interactions, $\mathcal{V}^o[e\gamma(s_y, s_p)]$, with weights given by the local likelihood of such interactions, $\pi_\ell^y(s_p)$. A location constitutes an attractive learning center if it provides a large density of individuals from whom workers learn relatively more. When interactions do not shape human capital accumulation, this dynamic consideration disappears.

As for the old, the number of young workers with skill $s$ employed in city $\ell$ follows from the Gumbel location preferences and the feasibility condition (1.2),

$$n_\ell^y(s_y) = n^y(s_y)\left(\frac{\mathrm{e}^{\vartheta\left(w_\ell(s_y)+B_\ell^y+\beta\mathbb{E}[\mathcal{V}^o(E\gamma(s_y,S_\ell^y))]\right)}}{\sum_{\ell'}\mathrm{e}^{\vartheta\left(w_{\ell'}(s_y)+B_{\ell'}^y+\beta\mathbb{E}[\mathcal{V}^o(E\gamma(s_y,S_{\ell'}^y))]\right)}}\right), \quad \forall\, s, \forall\, \ell, \tag{1.5}$$

where $\mathbb{E}[\mathcal{V}^o(E\gamma(s_y, S_\ell^y))] = \int\int \mathcal{V}^o[e\gamma(s_y, s_p)]dF(e)\pi_\ell^y(s_p)ds_p$ is the expected future utility of

[10] Using standard extreme-value algebra, $\mathcal{V}^o(s_y) = c + \log\left(\sum_\ell \mathrm{e}^{\vartheta w_\ell(s_y)}\right)/\vartheta$ for $c$ a constant.

young workers with skill $s_y$ employed in city $\ell$. This expectation is a general equilibrium object that depends on the location choices of the other young workers. When deciding where to work, individuals take the location choice of others as given.

**Skill distribution** The aggregate skill distribution of old workers is a general equilibrium object that reflects the learning experienced by young individuals. Specifically, the fraction of old workers with skill less than $s_o$ is given by

$$N^o(s_o) = \sum_{\ell} \int \int \int \mathbb{1}\left\{e\gamma(s_y, s_p) \leq s_o\right\} dF(e) n_\ell^y(s_y) \pi_\ell^y(s_p) ds_p ds_y, \quad \forall\, s_o. \tag{1.6}$$

The spatial allocation of young workers determines the skill distribution of old workers, and thus the aggregate stock of human capital, by shaping the aggregate probability of two skills meeting.

**Equilibrium** Equations (1.4) to (1.6) describe the steady-state spatial equilibrium. Given the spatial distribution of learning opportunities, young workers choose a city where to work. In equilibrium, their expectations must be consistent with the other workers' location choices. The spatial allocation of young workers determines the aggregate skill distribution of old workers, and the spatial allocation of old workers must be feasible.

**Definition 1.1** (Steady state equilibrium)**.** *A steady state equilibrium is a young and old allocation, $\{n_\ell^y, n_\ell^o\}_\ell$, city-specific skill densities amongst young workers, $\{\pi_\ell^y\}_\ell$, and a skill distribution of old workers, $n^o$, such that:*

1. *Taking the behavior of other young workers as given, the young spatial allocation satisfies* (1.5)*;*
2. *Given the old workers' skill distribution, the old spatial allocation satisfies* (1.4)*;*
3. *The local skill densities are consistent with the location decisions of young workers* (1.3)*;*
4. *The skill distribution of old workers satisfies* (1.6)*.*

The steady state equilibrium boils down to an infinitely-dimensional, non-linear fixed point in the local skill densities. The existence of an equilibrium is guaranteed by the idiosyncratic location preferences which coordinate expectations. Furthermore, as standard in spatial models, there exists a unique steady state to the extent that the dispersion forces are larger than the agglomeration forces (Redding and Rossi-Hansberg, 2017). In the present framework, the larger the scale parameter of the taste shock distributions $\theta^{-1}$, the larger the dispersion forces. Meanwhile, the agglomeration forces emanate from the learning interactions, and the smaller the discount factor $\beta$, the less workers value learning. Hence, when $\theta\beta$ is small enough, there exists a unique steady state.

**Proposition 1.1** (Existence and uniqueness)**.** *A steady state equilibrium exists. If $\theta\beta$ is small enough, there exists a unique steady state.*[11]

I highlight two special types of spatial allocation that may prevail in equilibrium. In the first, the skill densities are the same in every city, and therefore so are the learning opportunities. I refer to this allocation as the symmetric allocation.

**Definition 1.2** (Symmetric allocation)**.** *An allocation is symmetric if the local skill densities are identical, $\pi^a_\ell(s) = n^a(s)$ for all $s$, $\ell$ and $a$.*

Alternatively, productive locations may feature a higher density of skilled workers. In this case, the allocation satisfies stochastic positive assortative matching (SPAM).[12]

**Definition 1.3** (Stochastic positive assortative matching)**.** *An allocation satisfies stochastic positive assortative matching if the local skill densities can be ordered in the first-order stochastic dominance sense by cities' productivity: $\pi^a_\ell \succ_{FOSD} \pi^a_{\ell'} \iff T_\ell > T_{\ell'}$.*

In the absence of restrictions on the learning technology, the steady state equilibrium depends on the entire skill distribution of every city. To gain analytical tractability, I approximate the equilibrium around the point where cities have homogeneous TFP, $\log T_\ell \approx \log \bar{T}$ for all $\ell$ and some $\bar{T}$. When cities share a common TFP, the production complementarities

[11] Appendix A.1 presents the proofs of all propositions.

[12] This definition generalizes the notion of positive assortative matching in the context of many-to-one matching in which every skill is present in every city due to the idiosyncratic location preferences.

vanish. Workers no longer have an exogenous motive to sort across cities, and there exists an equilibrium with a symmetric allocation (Proposition A.1). Proposition 1.1 guarantees that this symmetric equilibrium is unique when $\theta\beta$ is small enough, which I assume to be the case for the rest of this section.[13] Approximating the equilibrium around the homogeneous TFP point thus simplifies how to keep track of the local skill densities.

Sections 1.2.3 and 1.2.4 characterize the approximated equilibrium. Throughout, $\bar{x}$ refers to the value of $x$ in the homogeneous TFP, symmetric equilibrium, and I use capital letters to denote random variables. For instance, $S^y$ is the aggregate skill distribution of young workers, and $S^y_\ell$ the skill distribution of young workers in city $\ell$. I return to the global solution from Section 1.2.5 onward.

### 1.2.3 Local Interactions as a Source of Agglomeration

I first show how the spatial segmentation of learning opportunities affects where individuals choose to work. I start by characterizing the location decisions of old workers to contrast them with those of the young. When between-city differences in TFP are small, (1.4) implies that the number of old workers with skill $s_o$ employed in city $\ell$ is to a first order given by[14]

$$n^o_\ell(s_o) \approx \underbrace{\frac{\bar{n}^o(s_o)}{L}}_{\text{Symmetric}} + \underbrace{\theta \left(\frac{\bar{n}^o(s_o)}{L}\right) \log\left(\frac{T_\ell}{\bar{T}}\right) s_o}_{\text{Sorting for working}}, \quad \forall\, s_o, \forall\, \ell, \tag{1.7}$$

where $\bar{n}^o(s_o)$ is the density of old workers with skill $s_o$ when cities are homogeneous. When cities share a common TFP, old workers locate in cities that maximize their idiosyncratic preferences, and the resulting allocation is symmetric. When some cities are more productive, the local wages are relatively higher, which attracts more workers. Furthermore, the between-

[13] Corollary A.2 also provides a bound on $\theta\beta$ that guarantee uniqueness of the symmetric equilibrium under the assumption that $\gamma(s_y, s_p) = \mathcal{F}(s_y) + \mathcal{G}(s_y)\mathcal{H}(s_p)$ for $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ some functions. For a framework where local interaction alone generate asymmetric equilibria, see Davis and Dingel (2019).

[14] For ease of exposition, I present in the main text the expression for the case of homogeneous local amenities. Appendix A.1 presents the general expressions. Amenity differentials affect city size but not the local skill densities, and as a result, do not affect the qualitative predictions of the model.

city wage gaps are increasing in workers' skills due to the production complementarities. High-skill workers are therefore relatively more present in productive cities, and the old allocation satisfies SPAM.[15] The dispersion in idiosyncratic location preferences, $\theta$, acts as the local labor supply elasticity and modulates the amount of sorting.

The location decisions of young workers differ from those of the old in that they also care about local learning opportunities. Specifically, from (1.5), the measure of young workers with skill $s_y$ in city $\ell$ is to a first order given by

$$n_\ell^y(s_y) \approx \underbrace{\frac{n^y(s_y)}{L}}_{\text{Symmetric}} + \underbrace{\theta\left(\frac{n^y(s_y)}{L}\right)\log\left(\frac{T_\ell}{\bar{\bar{T}}}\right)s_y}_{\text{Sorting for working}} + \underbrace{\theta^2\beta\left(\frac{n^y(s_y)}{L}\right)\log\left(\frac{T_\ell}{\bar{\bar{T}}}\right)\Theta(s_y)}_{\text{Sorting for learning}}, \quad \forall\, s_y, \forall\, \ell.$$

The third, new term captures the learning considerations. Specifically, the function $\Theta(s_y)$ summarizes how much young workers with skill $s_y$ value the learning opportunities of relatively productive cities. When $\Theta(s_y) > 0$, these opportunities are relatively attractive, therefore increasing the supply of $s_y$ in these locations. When $\Theta'(s_y) > 0$, skilled workers disproportionately value productive cities' learning opportunities, intensifying their concentration in space. I define the total willingness to sort of young workers as the sum of the sorting for working and learning motives, $\eta(s_y) \equiv s_y + \theta\beta\Theta(s_y)$.

How much young workers value the learning opportunities of productive cities depends on who works there and how much individuals learn from them,

$$\Theta(s_y) = \int \underbrace{\left(\gamma\left(s_y, s_p\right) - \mathbb{E}\left[\gamma\left(s_y, S^y\right)\right]\right)}_{\substack{\text{Relative learning from} \\ s_p \text{ interaction}}} \underbrace{\left(s_p + \theta\beta\Theta(s_p)\right)}_{\substack{\text{Willingness to} \\ \text{sort of } s_p}} dN^y(s_p), \quad \forall s_y. \tag{1.8}$$

In particular, the probability to meet a skill $s_p$ in a productive city depends on their willingness to work there, and therefore on how they themselves value the local interactions. Formally, $\Theta(s_y)$ is the solution to a infinitely-dimensional fixed point. The first-order approximation

[15] Proposition A.3 shows that SPAM always prevail for old workers for any spatial distribution of TFP.

renders this fixed point linear, which simplifies its characterization.[16] I show in Lemma A.1 that this fixed point has a unique solution when the dispersion forces are large enough.

The willingness to sort of young workers would solely rely on the production complementarities were workers not to learn from their neighbors. The spatial allocation of young workers would then also display stochastic positive assortative matching. The learning motive could *a priori* offset this technological force if low-skill were to value the learning opportunities of productive cities disproportionately. However, a higher density of low-skill workers in productive cities would contradict the empirical evidence on sorting patterns found in this chapter and others (e.g. Combes et al., 2008; Card et al., 2023). I therefore focus my attention to the part of the parameter space that generates a SPAM allocation for young workers.

**Lemma 1.1** (Stochastic positive assortative matching)**.** *If* $\log T_\ell \approx \log \bar{T}$ *for all* $\ell$ *and* $\min_{s_y} \theta\beta\mathbb{C}ov[\gamma_1(s_y, S^y), S^y] \geq -1$*, the young allocation exhibits SPAM,* $\eta'(s_y) \geq 0$ *for all* $s_y$*.*

Lemma 1.1 bounds the relative strength of the between-skill learning complementarities. For instance, when the learning technology takes the form of (1.1), the condition simplifies to $\theta\beta g_{12}\mathbb{V}\text{ar}[S^y] \geq -1$, which imposes a bound on how submodular the learning technology can be.

How does the spatial concentration of skills affect the location decisions of workers when they learn from the other workers in their city? In a SPAM equilibrium, individuals who work in productive cities are more likely to interact with skilled workers. Productive cities are therefore attractive learning centers to workers who disproportionally learn from high-skill. Formally, $\gamma(s_y, \cdot)$ increasing implies $\Theta(s_y) > 0$, and the spatial segmentation of learning opportunities increases the number of young workers with skill $s_y$ in productive cities. Two consequences follow.

First, the learning motive may constitute a source of agglomeration that endogenously increases the size of productive cities. Specifically, if every individual learns relatively more from skilled workers, the size of productive cities is larger in the presence of spatially segmented

[16]These infinite-dimensional linear fixed points are referred to as Fredholm integral equation of the second kind. Similar integral equations can be found in Allen and Arkolakis (2014).

learning interactions as individuals agglomerate there to learn from the local workers.

Second, the learning motive may increase the spatial concentration of skilled workers as they agglomerate in a few cities to learn from each other. To a first order, the average skill of young workers employed in city $\ell$ is

$$\mathbb{E}[S^y_\ell] \approx \underbrace{\mathbb{E}[S^y]}_{\text{Symmetric}} + \underbrace{\theta \log\left(\frac{T_\ell}{\bar{T}}\right) \mathbb{V}\text{ar}[S^y]}_{\text{Sorting for working}} + \underbrace{\theta^2 \beta \log\left(\frac{T_\ell}{\bar{T}}\right) \mathbb{C}\text{ov}[S^y, \Theta(S^y)]}_{\text{Sorting for learning}}, \quad \forall\, \ell, \tag{1.9}$$

where $\mathbb{V}\text{ar}[S^y]$ is the skill variance of young workers and $\mathbb{C}\text{ov}[S^y, \Theta(S^y)]$ is the covariance between workers' skill and how much they value the learning opportunities of productive cities. When the complementarities within-skill dominate those between-skill, young skilled workers have greater incentives to interact with their peers. They therefore place a higher value on the learning opportunities present in productive cities, and the local density of high-skill rises. Greater between-city wage inequality follows (equation (A.14) in Appendix A.1.6). In contrast, if the between-skill complementarities prevail, the learning motive reduces the concentration of skilled workers in productive cities.

I summarize the consequences of the spatial segmentation of learning opportunities on the geographic distribution of economic activity in Proposition 1.2.

**Proposition 1.2** (Spatial inequality)**.** *When* $\log T_\ell \approx \log \bar{T}$ *for all* $\ell$*,* $\gamma(s_y, \cdot)$ *increasing (decreasing) implies* $\Theta(s_y) > 0$ *(*$\Theta(s_y) < 0$*), and* $\gamma$ *supermodular (submodular) implies* $\Theta' > 0$ *(*$\Theta' < 0$*). As a result:*

1. *If* $\gamma(s_y, \cdot)$ *is increasing for all* $s_y$*, (un)productive cities are (smaller) larger when interactions are spatially segmented; the converse holds when* $\gamma(s_y, \cdot)$ *is decreasing;*
2. *If* $\gamma$ *is supermodular, the average skill of young workers in (un)productive cities is (smaller) larger when interactions are spatially segmented, which leads to greater between-city wage variance amongst young workers; the converse holds when* $\gamma$ *is submodular.*

### 1.2.4 The Human Capital Consequences of Local Interactions

Between-city differences in skill growth arise endogenously from the sorting of young workers. To a first order, the expected skill growth of young individuals starting their career with skill $s_y$ in city $\ell_y$ is

$$\mathbb{E}\left[\frac{S^o}{s_y} \mid s_y, \ell_y\right] \approx \mathbb{E}\left[\frac{\bar{S}^o}{s_y} \mid s_y\right] + \theta \log\left(\frac{T_{\ell_y}}{\bar{T}}\right) \mathbb{C}\text{ov}\left[\frac{\gamma(s_y, S^y)}{s_y}, \eta(S^y)\right], \quad \forall\, s_y.$$

In a SPAM equilibrium, interactions with skilled workers are disproportionally present in productive cities. As a result, conditional on learning more from high-skill, individuals employed in productive cities experience faster skill growth.

Do these spatial differences in learning matter for aggregate productivity? In the aggregate, the average skill of old workers is given by

$$\mathbb{E}[S^o] = \sum_{\ell} \int\int\int e\gamma(s_y, s_p) n^y_\ell(s_y) \pi^y_\ell(s_p) dF(e) ds_p ds_y. \tag{1.10}$$

This equilibrium stock of human capital can be contrasted with the average skill of old workers that prevails under the symmetric allocation:

$$\mathbb{E}[\bar{S}^o] = \int\int\int e\gamma(s_y, s_p) n^y(s_y) n^y(s_p) dF(e) ds_p ds_y. \tag{1.11}$$

Equation (1.11) coincides with the stock of human capital were interactions not segmented by cities since the spatial allocation of workers is then irrelevant for human capital accumulation. Comparing (1.10) and (1.11), cities affect aggregate productivity by shaping the aggregate probability of complementarity meetings. Absent learning complementarities, i.e. $\gamma(s_y, s_p) = g(s_y) + h(s_p)$, cities indeed do not shape aggregate productivity, $\mathbb{E}[S^o] = \mathbb{E}[\bar{S}^o]$. In that case, the faster human capital accumulation of individuals working in productive cities is exactly offsetted by the slower learning of workers employed in the other locations.

More generally, cities may amplify or dampen the stock of human capital. To a second

order, (1.10) implies that the average skill of old workers is[17]

$$\mathbb{E}[S^o] \approx \mathbb{E}[\bar{S}^o] + \theta^2 \mathbb{V}\text{ar}[\log T_\ell]\Omega, \tag{1.12}$$

where $\mathbb{V}\text{ar}[\log T_\ell]$ is the variance of cities' TFP. The constant $\Omega$ fully encapsulates the impact of cities on the stock of human capital. To a second order, this stock is larger when interactions are spatially segmented if and only if $\Omega > 0$. This aggregate effect is given by

$$\Omega = \int\int \gamma(s_y, s_p) \left(\eta^y(s_y) - \mathbb{E}[\eta^y(S^y)]\right) \left(\eta^y(s_p) - \mathbb{E}[\eta^y(S^y)]\right) dN^y(s_p) dN^y(s_y).$$

The first term captures how much skill $s_y$ learns from $s_p$. The second and third terms together determine the aggregate probability that they meet, which depends on both skills' willingness to work in productive cities.

Learning complementarities shape the impact of cities on aggregate human capital accumulation. In a SPAM equilibrium, cities increase the chance that workers with similar skills interact. When within-skill complementarities are strong, the marginal gains from skilled interactions increase in workers' skills. In such cases, the agglomeration of skilled individuals in productive locations boosts aggregate human capital accumulation. On the contrary, if the between-skill complementarities dominate, low-skill workers gain relatively more from meeting skilled partners. The spatial segregation of skilled workers in productive cities prevents low-skill workers from learning from them frequently, which lowers aggregate productivity.[18]

**Proposition 1.3** (Human capital accumulation)**.** *Suppose that* $\log T_\ell \approx \log \bar{T}$ *for all* $\ell$*. If* $\gamma$ *is supermodular, the average skill of old workers is larger when interactions are spatially segmented, i.e.* $\Omega > 0$*. Conversely,* $\gamma$ *submodular implies* $\Omega < 0$*.*

[17] Cities do not have a first-order effect on the average skill of old workers as the between-city differences in learning necessarily cancel each other.

[18] Appendix A.1.10 presents the consequences of local interactions on human capital accumulation across the skill distribution. When every worker learns relatively more from skilled interactions, local interactions increase learning inequality across skills by boosting the average learning of skilled workers and dampening that of low-skill individuals.

As a corollary to Proposition 1.3, learning complementarities determine the existence of a tradeoff between spatial inequality and human capital accumulation.[19] In a SPAM equilibrium, larger between-city wage inequality is necessarily associated with a stronger concentration of skilled workers in productive cities and, therefore, a greater segregation of learning opportunities across space. When within-skill learning complementarities prevail, an equity-efficiency tradeoff arises: greater spatial wage inequality boosts the stock of human capital. Such a tradeoff is not present when the between-skill learning complementarities are stronger. Then, a policy that reduces spatial wage inequality also increases the stock of human capital.

**Corollary 1.4** (Equity-efficiency tradeoff)**.** *Suppose that* $\log T_\ell \approx \log \bar{T}$ *for all* $\ell$*. In equilibrium, a greater between-city wage variance amongst young workers leads to a greater (smaller) stock of human capital if* $\gamma$ *is supermodular (submodular).*

Learning complementarities therefore shape the tradeoff between spatial inequality and aggregate productivity. They also determine whether there is too much or too little spatial skill concentration.

### 1.2.5 Optimal Policies

To study whether the spatial allocation of workers is efficient, I solve for the allocation chosen by a utilitarian planner. I assume the planner has homogeneous Pareto weights across skills and cities to focus on the learning externalities rather than redistributive concerns. I further suppose that the planner cannot condition its allocation on workers' idiosyncratic location preferences, either because they are unobserved or because it is not politically feasible. This restriction is standard in models with idiosyncratic location preferences (e.g. Rossi-Hansberg et al., 2019; Fajgelbaum and Gaubert, 2020).

[19] Formally, from (1.12), the equilibrium average skill of old workers can be rewritten $\mathbb{E}[S^o] - \mathbb{E}[\bar{S}^o] \propto \Omega \mathbb{V}\text{ar}[\mathbb{E}(\log W_\ell^y)]$, where $\mathbb{V}\text{ar}[\mathbb{E}(\log W_\ell^y)]$ is the between-city wage variance. To a second-order, the equilibrium is always located on this equity-efficiency loci. In particular, the impact of any policy that affects the spatial allocation of workers on aggregate productivity is given by this technological frontier.

Given these restrictions, the planner chooses a sequence of spatial allocation of workers, $\{n_{t\ell}^{y\star}, n_{t\ell}^{o\star}\}_{t,\ell}$, together with a sequence of consumption allocation across cities and skills, $\{c_{t\ell}^{y\star}, c_{t\ell}^{o\star}\}_{t,\ell}$, to maximize the discounted sum of each cohort's expected lifetime utility,[20]

$$\sum_{\ell} \int \mathbb{E}[V_0^{o\star}(s_o, \boldsymbol{\varepsilon}) \mid s] n_{0\ell}^{o\star}(s_o) ds_o + \sum_{t>0} \beta^t \sum_{\ell} \int \mathbb{E}[V_t^{y\star}(s_y, \boldsymbol{\varepsilon}) \mid s] n_{t\ell}^{y\star}(s_y) ds_y. \tag{1.13}$$

The function $V_0^{o\star}(s_o, \boldsymbol{\varepsilon})$ is the utility of old workers with skill $s_o$ and preferences $\boldsymbol{\varepsilon}$ in the initial period under the planner's allocation. Likewise, $V_t^{y\star}(s_y, \boldsymbol{\varepsilon})$ is the lifetime utility of young workers in cohort $t$. The planner cannot observe workers' idiosyncratic preferences, and thus maximizes workers' expected utility. The consumption allocation must not exceed aggregate output. Meanwhile, the spatial allocation of workers must be consistent with the aggregate supply of each skill and workers' idiosyncratic location preferences. These preferences act as an incentive constraint: to increase the supply of workers in a particular city, the planner must compensate them for their foregone location preferences.[21] I solve first for the global, non-linearized planner's allocation. I focus on the steady state allocation, referred to as the (constrained) optimal allocation.

Old workers do not affect the learning of young workers, and therefore do not exert externalities. As a result, the laissez-faire and the planner allocation coincide.[22]

In contrast, the allocation of young workers need not be efficient. Young workers do not internalize the impact of their location choice on others' learning. As a result, workers' incentives to live in specific cities may fall short of what is socially desirable. Specifically,

[20] By the property of the Gumbel distribution, the expected utility of workers who choose to work in city $\ell$ is identical to the unconditional expected utility, $\mathbb{E}[V_0^{o\star}(s, \boldsymbol{\varepsilon}) \mid s, \ell \succ \ell' \, \forall \ell' \neq \ell] = \mathbb{E}[V_0^{o\star}(s, \boldsymbol{\varepsilon}) \mid s]$.

[21] Appendix A.2.1 describes the full set of constraints faced by the planner.

[22] The efficiency of the old allocation highlights that, absent learning externalities, the planner has no incentives to redistribute resources across cities. Idiosyncratic location preferences alone thus do not create inefficiencies. While this may appear in contradiction with the conclusions of Fajgelbaum and Gaubert (2020), Donald et al. (2023), and Mongey and Waugh (2024), the two sets results are reconciled by noting the absence of local non-traded goods in the present framework. The marginal utilities of consumption are then trivially equalized across space, and the planner does not redistribute across space.

there exists a wedge between the optimal and laissez-faire consumption allocation

$$c_\ell^{y\star}(s_y) - c_\ell^y(s_y) = \tau_\ell^\star(s_y) - \sum_{\ell'} \left( \frac{n_{\ell'}^{y\star}(s_y)}{n^y(s_y)} \right) \tau_{\ell'}^\star(s_y), \quad \forall\, s_y, \forall\, \ell. \tag{1.14}$$

When this wedge is positive, the laissez-faire equilibrium features too few workers with skill $s_y$ in location $\ell$. This wedge is given by the net welfare gains (or losses) brought by a marginal increase in the number of skill $s_y$ in city $\ell$,

$$\tau_\ell^\star(s_y) = \beta \int \int \left( \mathcal{V}^o[e\gamma(\sigma, s_y)] - \mathbb{E}[\mathcal{V}^o(e\gamma(\sigma, S_\ell^{y\star}))] \right) dF(e)\pi_\ell^{y\star}(\sigma)d\sigma, \quad \forall\, s_y, \forall\, \ell, \tag{1.15}$$

where $\mathbb{E}[\mathcal{V}^o(e\gamma(\sigma, S_\ell^{y\star}))] = \int \mathcal{V}^o[e\gamma(\sigma, s_p)]\pi_\ell^{y\star}(s_p)ds_p$ is the expected future utility of working in city $\ell$ for skill $\sigma$. These local welfare gains are positive if workers in $\ell$ learn relatively more from skill $s_y$ than from the average local worker. Hence, the optimal spatial allocation features a greater number of skill $s_y$ in city $\ell$ to the extent that the workers employed in that city learn disproportionately from that skill.

Equation (1.14) implies that the laissez-faire equilibrium is inefficient as long as workers learn from others. Characterizing the spatial misallocation amounts to solving for the non-linear, infinitely dimensional fixed point consisting of (1.14), (1.15), and the local labor supplies. As for the laissez-faire equilibrium, I simplify this fixed point by studying the optimal allocation when the spatial TFP gaps are small, $\log T_\ell \approx \log \bar{T}$. Appendix A.2.3 presents the first-order approximation.

I find learning spillovers within cities to generate spatial misallocation through two channels. First, productive cities may be too large. Conditional on learning from skilled individuals, young workers agglomerate in productive cities (Proposition 1.2.1). In doing so, they forego their idiosyncratic location preferences for less productive locations. However, high-TFP cities do not offer *ex-ante* better learning opportunities. Holding aggregate productivity constant, the planner thus desires to decentralize learning centers.[23] When spatial TFP gaps are small,

[23] In a different setting, Waldfogel (2003) coined this externality the "preference externality".

this is achieved by reallocating workers from high- to low-TFP locations independently of their skills.

Second, the laissez-faire equilibrium may feature too much or too little spatial skill concentration. When within-skill complementarities dominate those between-skill, skilled workers benefit disproportionately their peers. Internalizing these spillovers, the planner increases the frequency of interactions between skilled workers by augmenting their concentration in productive cities. In contrast, relatively low-skill workers experience too few skilled interactions in the laissez-faire equilibrium when the between-skill complementarities prevail. The planner corrects this misallocation by reallocating skilled workers towards low-TFP cities that are abundant in low skills. In both cases, the optimal allocation corrects the learning externalities and increases the stock of human capital.

**Proposition 1.5** (Optimal allocation)**.** *The laissez-faire equilibrium is efficient if interactions do not shape learning,* $\gamma(s, s_p) = g(s)$*. Otherwise, the laissez-faire equilibrium is inefficient. Furthermore, when* $\log T_\ell \approx \log \bar{T}$ *for all* $\ell$*:*

1. *If* $\gamma(s_y, \cdot)$ *is increasing (decreasing) for all* $s_y$*, productive cities are too large (small) in the laissez-faire equilibrium:* $N_\ell^\star < N_\ell \iff T_\ell > \bar{T}$*.*

2. *If* $\gamma$ *is supermodular (submodular), there is too little (too much) young skill concentration in productive cities:* $\mathbb{E}[S_\ell^{y\star}] > \mathbb{E}[S_\ell^y] \iff T_\ell > \bar{T}$*.*

*The optimal allocation implies a larger stock of human capital,* $\mathbb{E}[S^{o\star}] > \mathbb{E}[S^o]$*.*

Which policy instruments can decentralize the optimal allocation? Equation (1.15) implies that efficiency is restored through place-by-skill transfers, $t_\ell^y(s)$. These transfers complement workers' wages, and the income of young workers with skill $s$ employed in city $\ell$ becomes $I_\ell^y(s) = w_\ell(s) + t_\ell^y(s)$. When the spatial TFP gaps are small, the transfers can further be decomposed into two components. First, a city-specific tax that corrects the size distortion found in Proposition 1.5.1. Second, skill-by-city subsidies which generate the optimal amount of spatial skill segregation (Proposition 1.5.2).

Taking stock, the spatial segmentation of learning opportunities may shape geographic disparities inequality, the stock of human capital accumulation, and may call for place-based

policies. To evaluate quantitatively their consequences, I add several extensions to the framework that allow me to bring it to the data.

### 1.2.6 Quantitative model

I add two key elements to the model, housing and migration costs, frequently mentioned to explain the presence of spatial inequality and justify the need for local policies (e.g. Chetty et al., 2014). I also let young workers interact with old partners. I solve for the global, non-linearized solution of the model numerically. Appendix A.4 formally describes the quantitative model and the numerical algorithm.

**Housing** In addition to the freely traded numéraire, households consume a local good sold at (rental) price $p_\ell$. I refer to this good as housing. Households have Cobb-Douglas preferences over the two goods, with $\alpha$ representing the housing expenditure share.[24] In each city, local land-owners supply a stock of housing $H_\ell = \mathcal{H} p_\ell^\delta$, for $\mathcal{H}$ a constant and $\delta$ the housing supply elasticity.

**Migration costs** While workers remain free to change location whenever they please, they incur a migration cost upon moving. I let these migration costs vary across the lifecycle, e.g. if young workers face tighter borrowing constraints or if old workers have accumulated more social capital. I assume that workers are born in the location where their parents lived when they were young; that is, the probability of being born in city $\ell$ is given by $N_\ell^y$.[25] A young worker born in city $\ell$ that moves to city $\ell'$ incurs a utility cost $\kappa_{\ell\ell'}^y$.[26] Likewise, an individual who lived in $\ell$ in their youth and migrates to location $\ell'$ in their old period pays a cost $\kappa_{\ell\ell'}^o$.

[24] For empirical evidence using U.S. data in support of the constant housing expenditure share implied by the Cobb–Douglas functional form, Davis and Ortalo-Magné (2011).

[25] I could alternatively assume that workers' birthplace is given by their parents' location when old. The probability to be born in $\ell$ would be $N_\ell^o$. In the data, I define a young as a worker between 25 and 40 year old. I stick with the first formulation to stick closer to the empirical timing of birth. This will not matter quantitatively since $N_\ell^y \approx N_\ell^o$.

[26] Alternatively, these migration costs can represent auto-correlation in workers' location preferences. The two interpretations are isomorphic from a positive standpoint.

I assume symmetric migration costs, $\kappa^a_{\ell\ell'} = \kappa^a_{\ell'\ell}$ for $a \in \{y, o\}$, and I normalize $\kappa^a_{\ell\ell} = 0$ for every $\ell$ and $a \in \{y, o\}$. The presence of migration costs implies that the expected lifetime utility of young workers now depends on their skill and birthplace. In particular, expected utilities are no longer equalized across space.

**Learning** I let young workers learn from young and old workers. Furthermore, I assume that four elasticities govern the learning technology

$$\log \gamma(s, s_p) = g_0 + g_1 \log s + g_2 \log s_p + g_{12} \log s \log s_p. \tag{1.16}$$

The first parameter, $g_0$, determines the average skill growth. The elasticity $g_1$ governs how workers' own skill shapes their human capital accumulation holding constant the skill of their learning partner. In particular, $g_1 > 1$ implies that skilled workers learn relatively faster. The two other parameters, $g_2$ and $g_{12}$, dictate the returns to interactions. Workers learn relatively more from skilled partners on average when $g_2 > 0$, and particularly so if they are themselves skilled when $g_{12} > 0$.

While parsimonious, the learning technology (1.16) has two advantages. First, it allows for technology that are neither supermodular nor submodular, and therefore for which the theory is ambiguous.[27] Second, Figure A.8 shows that it is not possible to reject empirically its log-linear structure.

**Parametrization** Finally, I suppose that the skill distribution of young workers is log-normal with mean $\mu_y$ and standard deviation $\sigma_y$, and I parametrize the idiosyncratic learning shocks to be log-normally distributed with zero mean and standard deviation $\sigma_e$.[28]

**Relationship to theory** To what extent the results derived in Sections 1.2.3 to 1.2.5 hold in the quantitative model? The introduction of migration costs and housing consumption

[27] The technology (1.16) indeed differs from (1.1) in that it is expressed in logs.

[28] Normalizing the mean of the learning shocks distribution to zero is without loss of generality since it is not separately identified from $g_0$ in the learning technology.

alters the analytical tractability of the model. Instead, I rely on numerical methods. In Section 1.5, I compare the estimated framework with a counterfactual economy in which interactions are not spatially segmented. The numerical results align with the qualitative predictions of Proposition 1.2 and 1.3.

## 1.3 Estimating the Returns to Local Interactions

As highlighted in the previous section, learning complementarities are crucial in determining how the spatial segmentation of learning opportunities affects productivity. In this section, I present evidence on the role of cities' skill composition in shaping workers' wage growth. I argue that these evidence can be used to estimate the learning complementarities.

### 1.3.1 Data

I estimate the model on French matched employer-employee data (DADS).[29] This dataset comes in two formats. The first format is a 4% representative panel that tracks the entire history of individuals in the labor market (DADS panel).[30] The second format is a repeated yearly cross-section covering the universe of employed workers (DADS salariés). Both datasets provide information on workers' wages, the number of hours worked, the location where they work and live, along with demographic information. The data goes back to 1976, but frequent changes of variables make it difficult to use the observations collected prior to 1993.

This dataset has several advantages over other data sources used in the literature. First, its long-panel dimension allows me to measure individual wage growth at a long-term horizon. Second, the cross-sectional format is useful to measure precisely the local skill composition of granular geographical levels. Third, it is one of the few matched employer-employee datasets to provide information on where individuals work (firm, establishment, and neighborhood)

[29]DADS stands for "*Declarations Annuelles de Données Sociales*". The dataset is constructed from employer tax records which are compiled by the French statistical agency (INSEE).

[30]Individuals' employment history is recorded in the dataset if (a) they have at least one employment spell, and (b) they are born in the first fourth days of each quarter.

and live.

I apply the same sample restrictions on both datasets. I focus my analysis on full-time employed workers between 25 and 55 years old. Workers employed in the public sector in France have their wages determined nationally by their tenure, and as such, their wage may not fully reflect their productivity. I therefore keep in the sample workers employed in the private sector, and I exclude the education and health industries due to their large fraction of public servants. I measure workers' wages by their gross yearly salaries divided by the number of hours worked, and I truncate the hourly wage distribution at the 5% and 99.9% percentile to minimize measurement errors in hours worked.[31] Appendix A.5.1 provides further details on the construction of the sample.

Over time, wages grow for two reasons. First, workers experience idiosyncratic changes. Second, the aggregate wage fluctuates.[32] Aggregate trajectories are absent from the framework laid down in Section 1.2. To abstract from them empirically, I normalize the average log wage to zero every year. These trajectories may also differ across space, e.g. for cities impacted by the deindustrialization of the French economy. I therefore focus the study to the 2009-2019 time window. In that decade, aggregate growth was evenly spatially distributed in France (Figure A.2a), guaranteeing that I do not conflate faster lifecycle wage growth with faster aggregate growth.

I define a city as a commuting zone (CZ). A commuting zone is a statistical area defined by the French statistical agency (INSEE). It consists of a collection of contingent municipalities clustered together to reduce the commuting flows across them. There are 297 such areas in metropolitan France, which span the whole territory.[33] Commuting zones are the natural geographical unit when thinking of local labor markets. In particular, their relatively large

[31] In France, between 5% and 10% of the overall workforce is paid at the minimum wage (INSEE, 2021). For these workers, wages are not a valid proxy for their skills. I focus on workers paid above the minimum wage by truncating the left-tail of the wage distribution at the 5%.

[32] Mechanically, the log wage growth of worker $i$ can be decomposed as $\omega_{it+1} - \omega_{it} = \tilde{\omega}_{it+1} - \tilde{\omega}_{it} + \mathbb{E}[\omega_{it+1}] - \mathbb{E}[\omega_{it}]$, for $\omega_{it} = \log w_{it}$ and $\tilde{\omega}_{it} \equiv \omega_{it} - \mathbb{E}[\omega_{it}]$. The first term is the worker-level wage growth and the second is aggregate growth.

[33] Metropolitan France excludes the French overseas regions (Guadeloupe, Réunion, etc.).

size (40,000 employed workers on average) is well-suited to capture all the local interactions that a worker may experience. I use the terms commuting zones and cities interchangeably for the rest of the chapter.

I also use a second, more granular geographical unit for the empirical analysis. Specifically, I define a neighborhood as a municipality ("commune" in French). Municipalities are legal areas that also span the French metropolitan territory. However, they are smaller than commuting zones: in 2019, France was split into 34,839 municipalities with an average size of 1,800 inhabitants and an average area of 15 square kilometers.[34] They compare to ZIP codes in the United States. Figure A.1 displays a map of the French commuting zones alongside the municipalities contained in Paris and Lyon to highlight how granular these spatial units are. I restrict the analysis to metropolitan municipalities with more than 50 employed workers to reduce local measurement errors. I am left with 12,320 municipalities, representing 99% of the French workforce.

### 1.3.2 The Geography of Wage Growth

Fast lifecycle wage growth is very spatially concentrated in France. Figure 1.1a plots the average five-year wage growth at the worker-level by commuting zones.[35] On average, workers under 40 years old experience a wage growth of 10.6 log points every five years. Workers in Paris see their wage increase by 15 log points. Meanwhile, the average wage growth in Lyon, the city in the $75^{\text{th}}$ percentile of the growth distribution and the second biggest French city, is 12 log points, and the growth in Troyes, the city in the $10^{\text{th}}$ percentile of the growth distribution, is 8 log points. In total, half of the cities (20% of the population) offer wage growth below 9 log points.

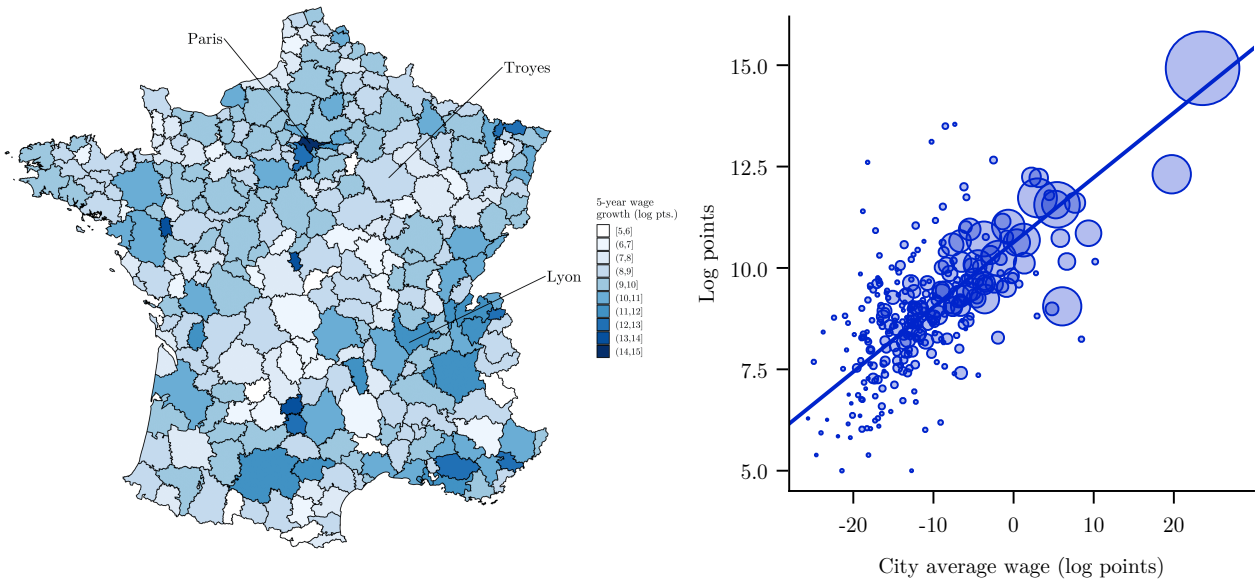
Fast wage growth is concentrated in cities where wages are also relatively high. Figure 1.1b displays the average wage growth at the city level against the city average wage. In the

[34]Municipalities were introduced with the French revolution in 1789. Since the $20^{\text{th}}$ century, the number of municipalities have been rather stable over time. Appendix A.5.2 provides more details on their history and characteristics.

[35]Consistent with the model, these averages are computed on "young" workers under 40 years old.

Figure 1.1: The geography of wage growth

(a) Within-worker wage growth by CZ (b) Within-worker wage growth by CZ wage




Note: left-panel plots the average log wage growth at the five-year horizon by commuting zones for workers under 40 year old. Right-panel plots the average log wage growth at the five-year horizon against the average wage of the commuting zones. The size of the markers is proportional to the city size. In both panels, wage growth is measured at the worker level, i.e. $\log w_{it+1}/w_{it}$.

cross-section, an increase of the city wage by one standard deviation is associated with a wage growth increase by 2 log points at the five-year horizon.[36] I now argue how this pattern can be used to identify learning complementarities.

### 1.3.3 Identification

Given the learning technology (1.16), the next period's skill of a worker $i$ given their present skill, $s_{it}$, and the skill of their learning partner, $s_{p_{it}}$, is

$$\log s_{it+1} = g_0 + g_1 \log s_{it} + g_2 \log s_{p_{it}} + g_{12} \log s_{it} \log s_{p_{it}} + \log e_{it+1}, \tag{1.17}$$

where $e_{it+1}$ is the idiosyncratic learning shock experienced by $i$. These shocks represent, for instance, enrollments in training programs, returns to on-the-job tenure, or strokes of genius.

[36]This relationship is not mechanically caused by high-wage cities diverging away from the other places (Figure A.2) or by housing prices (Figure A.2a).

They are assumed to be identically and independently distributed across space, workers and partners. Through (1.17), the learning technology can therefore be estimated via a local projection of future skills on today's skills, both for $i$ and their partner.

Estimating (1.17) poses two empirical challenges: how to measure skills, and how to observe learning interactions. I use the structure of the model to solve these challenges.

First, in the theory, skills and wages are related through $w_{it} = s_{it}T_{\ell_{it}}$, for $T_{\ell_{it}}$ the TFP of the city where $i$ works. Accordingly, I measure skills as wages deflated by local TFPs, which I obtain through the structural estimation of the model (Section 1.4).[37]

Second, very few datasets track workers' interactions.[38]. However, when interactions are random within cities, the skill density of the location where $i$ works constitutes a valid proxy for their interactions. Specifically, substituting $s_{p_{it}}$ in (1.17) for the average skill in the location where $i$ is employed, $\mathbb{E}_{\ell_{it}}[\log s_{jt}]$, returns

$$\log s_{it+1} = g_0 + g_1 \log s_{it} + g_2 \mathbb{E}_{\ell_{it}}[\log s_{jt}] + g_{12} \log s_{it} \mathbb{E}_{\ell_{it}}[\log s_{jt}] + \log e_{it+1} + \nu_{it}. \tag{1.18}$$

The term $\nu_{it} = (g_2 + g_{12} \log s_{it})(\log s_{p_{it}} - \mathbb{E}_{\ell_{it}}[\log s_{jt}])$ captures the gap between the particular interaction experienced by $i$ and the average interaction in city $\ell$. Random interactions imply that $\nu_{it}$ is orthogonal to $s_{it}$ and $\mathbb{E}_{\ell_{it}}[\log s_{jt}]$.

The learning technology can therefore be estimated from a local projection of future skills on current skills and the average skill of the workers employed in the same city,

$$\log s_{it+1} = \alpha_t + \beta \log s_{it} + \gamma \overline{\log s}_{\ell_{it}} + \delta \log s_{it} \overline{\log s}_{\ell_{it}} + u_{it}. \tag{1.19}$$

In (1.19), $\alpha_t$ is a year fixed effect that controls for aggregate trends in wages, and $\overline{\log s}_\ell$ is the average log skill in city $\ell$. I measure future skills at the five-year horizon to smooth out

[37]The structural estimation of cities' TFP relies on estimates of the learning technology. To solve this fixed point, I iterate on (1.19) and the structural estimation of the model until convergence. In practice, I estimate a small TFP dispersion, so that $\log s_{it} \approx \log w_{it}$.

[38]For recent papers aimed at measuring face-to-face interactions, see Atkin et al. (2022) and Emanuel et al. (2023).

transitory wage shocks while keeping sufficient statistical power.[39] I compute local average skills on all the workers in the sample, but, consistently with the model, I only include young workers under 40 in the regression.

While (1.19) is effectively a linear-in-mean model, it does not suffer from the reflection problem of Manski (1993). The dynamic nature of the underlying learning process indeed rules out the simultaneity issue at the heart of the reflection problem: the average skill of a location is shaped by the interactions that take place there, but those must have happened in the past to have en effect on the skill composition. This argument is formally laid down in Section A.5.3. The two assumptions essential to the identification are summarized in the following proposition.

**Proposition 1.6** (Identification)**.** *Suppose that*

1. *Interactions are random within cities,* $(s_{it}, s_{p_{it}}) \mid \ell_{it} \perp \mathbb{E}_{\ell_{it}}[\log s_{jt}]$,
2. *The idiosyncratic learning shocks are i.i.d. across workers and cities,* $e_{it} \perp (s_{it}, \mathbb{E}_{\ell_{it}}[\log s_{jt}])$.

*Then,* (1.19) *produces unbiased estimates of the learning technology:* $\hat{\beta} = g_1$, $\hat{\gamma} = g_2$ *and* $\hat{\delta} = g_{12}$.

Similar local projections have been used in the learning from coworkers literature. For instance, Nix (2020) and Jarosch et al. (2021) project respectively future wages on the average education within the firm and the average wage within the establishment where workers are employed. My empirical design differs from theirs in two ways. First, it studies the returns to local interactions within cities rather than within firms. Second, it includes an interaction term between workers' wages and the wage of the city where they work to estimate learning complementarities.

### 1.3.4 Endogeneity concerns

The assumptions underlying Proposition 1.6 may not hold empirically. Skilled workers may be able to target interactions with skilled partners. The learning complementarities estimated

[39] Section 1.4.2 describes how the 5-year estimates produced by (1.19) are turned into structural 15-year estimates.

by (1.19) would then be biased upward, $|\delta| > |g_{12}|$. To investigate how large this bias is, I estimate (1.19) separately for small and large cities and more or less unequal locations. To the extent that skilled workers can better target skilled interactions in larger or more unequal cities, the estimated complementarity should be higher in those places.

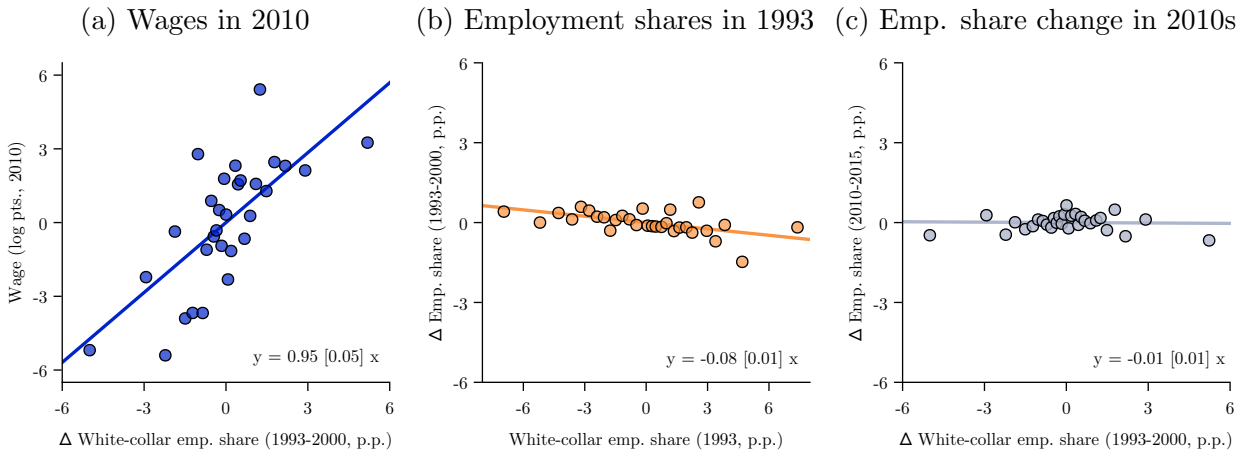
Meanwhile, the distribution of idiosyncratic learning shocks may vary across workers and cities. Young workers may learn faster for reasons other than local interactions while earning lower wages, $e_{it} \not\perp s_{it}$. Young workers may also agglomerate in large productive cities to enjoy the local amenities, $e_{it} \not\perp \mathbb{E}_{\ell_{it}}[\log s_{jt}]$. I alleviate this second set of endogeneity concerns in three ways.

First, I control for as many sources of idiosyncratic learning shocks as my data allows. I include age fixed effects to capture the faster growth of younger workers. To control for long-term contracting and decreasing returns to experience, I add fixed effects for tenure at the job, occupation, industry, and city level. I include a dummy for employer switching to account for its effect on wage growth. Productive firms may have more frequent training programs while offering higher wages. In addition, these training programs may target particular occupations. I include firm-by-occupation fixed effects to account for these firm-specific determinants of human capital accumulation.[40] Finally, establishments may experience long-lasting productivity shocks that are passed through to wages. I account for those by controlling for the growth in average wages at the establishment level. Stacking these controls in the vector $\boldsymbol{X}_{it}$, the exogeneity assumption in Proposition 1.6 becomes $e_{it} \perp (s_{it}, \mathbb{E}_{\ell_{it}}[\log s_{jt}]) \mid \boldsymbol{X}_{it}$.

Second, I introduce a set of granular spatial fixed effects. The distribution of idiosyncratic learning shocks may vary across cities in ways that are complex to control for. Wealthier cities may for instance provide better public goods that influence workers' wage growth. Controlling for unobserved differences in wage growth across cities poses an identification challenge since the skill growth elasticities in (1.19) are also estimated from between-city variations. I slightly depart from the theory to solve this challenge and quantify how neighborhoods' skill

[40]The firm-occupation fixed effects also allow for production complementarities across occupations at the firm level.

Figure 1.2: The change in neighborhoods' white-collar employment share (1993-2000)




Note: panel (a) plots the average wage of neighborhoods in 2010 (y-axis) against the change in their white-collar employment share between 1993 and 2000 (x-axis). Panel (b) plots the change in neighborhoods' white-collar employment share between 1993 and 2000 (y-axis) against their employment share in 1993 (y-axis). Panel (c) plots the change in neighborhoods' white-collar employment share between 2010 and 2019 (y-axis) against the change in employment share between 1993 and 2000. All plots are binned scatter plots with 30 bins. Consistently with (1.20), all variables are first residualized by a city fixed effect and thus normalized to zero. The regression lines are estimated in the full sample of neighborhoods.

composition shapes human capital accumulation.[41] Specifically, I estimate the augmented local projection

$$\log s_{it+1} = \alpha_{t\ell_{it}n^r_{it}} + \beta \log s_{it} + \gamma \overline{\log s}_{n^w_{it}} + \delta \log s_{it} \overline{\log s}_{n^w_{it}} + \boldsymbol{\Psi}' \boldsymbol{X}_{it} + u_{it}. \tag{1.20}$$

In (1.20), $n^r_{it}$ and $n^w_{it}$ are the neighborhoods where worker $i$ resides and works. The parameters $\alpha_{t\ell n^r}$ are year by city of employment by neighborhood of residence fixed effects, which soak unobserved wage growth heterogeneity across cities (e.g. better universities) and across residence (e.g. better transportation network).[42] Meanwhile, $\overline{\log s}_{n^w}$ is the average wage of the workers employed in neighborhood $n^w$, and the skill growth elasticities in (1.20) are estimated by comparing individuals who live in the same neighborhood and work at the same firm and occupation but in different neighborhoods within the same city.[43]

Third and last, I rely on an instrumental variable strategy to generate quasi-random

[41] An alternative would be to include city fixed effect in (1.19) and rely on temporal variations in city wages. Yet, these changes are very likely to be correlated with local productivity shocks, and the estimated skill growth elasticities would then conflate faster human capital accumulation and local aggregate growth.

[42] By comparing workers who live in the same neighborhood and work in the same firm, these fixed effects partially reduce workers' unobserved heterogeneity. Table A.1 further introduces worker fixed effects to soak the remaining unobserved heterogeneity.

[43] I control in addition for the distance between the neighborhood of residence and work.

variations in skill density across neighborhoods. Individuals working in the same location may be exposed to common conditions that affect their present and future wages (Angrist, 2014). To address this endogeneity concern common to the peer effect literature, I instrument the average skill of neighborhoods by the change in their white-collar employment share in the 1990s,

$$Z_n = \frac{\#\text{white-collar}_{n,2000}}{\#\text{workers}_{n,2000}} - \frac{\#\text{white-collar}_{n,1993}}{\#\text{workers}_{n,1993}},$$

where $Z_n$ is neighborhood $n$'s instrument. I define white-collar workers through their occupations. Intuitively, the instrument uses implicit shocks in the 1990s that permanently reallocated high-skill workers across neighborhoods within cities. The exclusion restrictions are that these shocks were orthogonal to the neighborhoods' initial conditions and have faded away twenty years later.

Figure 1.2 argues that the instrument satisfies the relevance and exclusion restrictions. Figure 1.2a displays the average wage of neighborhoods in the 2010s against the instrument. The two are strongly positively correlated, suggesting the relevance of the instrument: an increase by one percentage point of the local share of white collar in the 1990s is associated with 0.95% increase in the average wage of that neighborhood in the 2010s. Figure 1.2b argues that the underlying shocks were uncorrelated with the neighborhoods' initial conditions by showing that the change in the employment share of white collar in the 1990s is barely correlated with their initial employment share. Finally, Figure 1.2c presents suggestive evidence that the shocks in the 1990s are no longer correlated with present changes in the skill composition of neighborhoods by plotting the absence of correlation between the change in the employment share of collars in the 1990s and that in the 2010s.

By isolating random variations in skill density caused by past *changes* in occupation composition, the instrument estimates the effect of neighborhoods' skill composition on wage growth net of time-invariant local characteristics. Over twenty years, changes in

neighborhoods' skill composition may affect local characteristics other than the quality of the learning interactions (e.g. the physical stock of capital), and indirectly through those wage growth. My identification strategy cannot separate the "interaction" channel from these other dynamic forces. However, to the extent that both channels capture the causal impact of the local skill composition on workers' human capital accumulation, I view them as isomorphic.

How can the elasticities estimated by (1.20) be interpreted? The parameters obtained from the local projections (1.19) and (1.20) can be interpreted as reduced-form elasticities. In that case, $\gamma$ measures the average treatment effect of the local skill composition on skill growth, and $\delta$ estimates the heterogeneity in treatment effects across the skill distribution. Interpreting those parameters structurally is challenging since interactions are modeled at the city rather than the neighborhood level. Nevertheless, to the extent that some interactions are happening within neighborhoods, I view the elasticities estimated by (1.20) as informative of the null hypotheses $g_2 = 0$ and $g_{12} = 0$.

### 1.3.5 Results

Table 1.1 presents the estimated skill growth elasticities. Standard errors are clustered at the same level as the spatial skill variations. The first columns presents the estimates produced by the model-consistent local projection (Proposition 1.6). The three elasticities are estimated to be positive and significant at the 0.1%.

First, high-skill workers enjoy faster skill growth ($\hat{\beta} > 1$): a skill increase of one standard deviation boosts skill growth by 0.4 log points over five years for workers in the average city.[44] Workers starting their career with lower skills do not catch up to high-skill individuals – although this effect alone is quantitatively small.[45]

[44]The average skill is normalized to zero, and therefore so is the skill of the average French city. Hence, $\beta - 1$ represents the marginal effect of increasing workers' skill on skill growth for workers in the average French city.

[45]The estimated wage process is therefore non-stationary. This result rests on the time horizon considered. In Figure A.4, I estimate the local projection for time horizons between one and five years. For shorter-term wage growth, I do estimate an auto-correlation parameter $\beta$ close but under the unit root, in accordance with estimates from the literature (e.g. Guvenen, 2009). I take this as indirect evidence that, by considering

Table 1.1: The returns to local interactions

| Dep. variable: 5-year skill | (1) OLS | (2) OLS | (3) OLS | (4) OLS | (5) OLS | (6) 2SLS |
|---|---|---|---|---|---|---|
| Skill ($g_1$) | 1.010 | 1.010 | 1.010 | 0.940 | 0.935 | 0.935 |
| | (0.003) | (0.003) | (0.003) | (0.002) | (0.002) | (0.003) |
| City skill ($g_2$) | 0.107 | 0.095 | 0.111 | 0.081 | | |
| | (0.015) | (0.012) | (0.005) | (0.008) | | |
| Skill × city skill ($g_{12}$) | 0.108 | 0.102 | 0.104 | 0.089 | | |
| | (0.032) | (0.007) | (0.009) | (0.013) | | |
| Log city size | | 0.002 | | | | |
| | | (0.001) | | | | |
| Neighborhood skill | | | | | 0.032 | 0.053 |
| | | | | | (0.008) | (0.021) |
| Skill × neighborhood skill | | | | | 0.071 | 0.077 |
| | | | | | (0.013) | (0.023) |
| Controls | | | | | | |
| Worker-level | · | · | ✓ | ✓ | ✓ | ✓ |
| Establishment growth | · | · | · | ✓ | ✓ | ✓ |
| Distance work-residence | · | · | · | · | ✓ | ✓ |
| Fixed effects | | | | | | |
| Year | ✓ | ✓ | ✓ | ✓ | · | · |
| Occupation × firm | · | · | · | ✓ | ✓ | ✓ |
| Year × residence × city | · | · | · | · | ✓ | ✓ |
| Obs. | 971,401 | 971,401 | 971,401 | 974,401 | 971,401 | 974,401 |
| F-stat. | · | · | · | · | · | 53,539 |

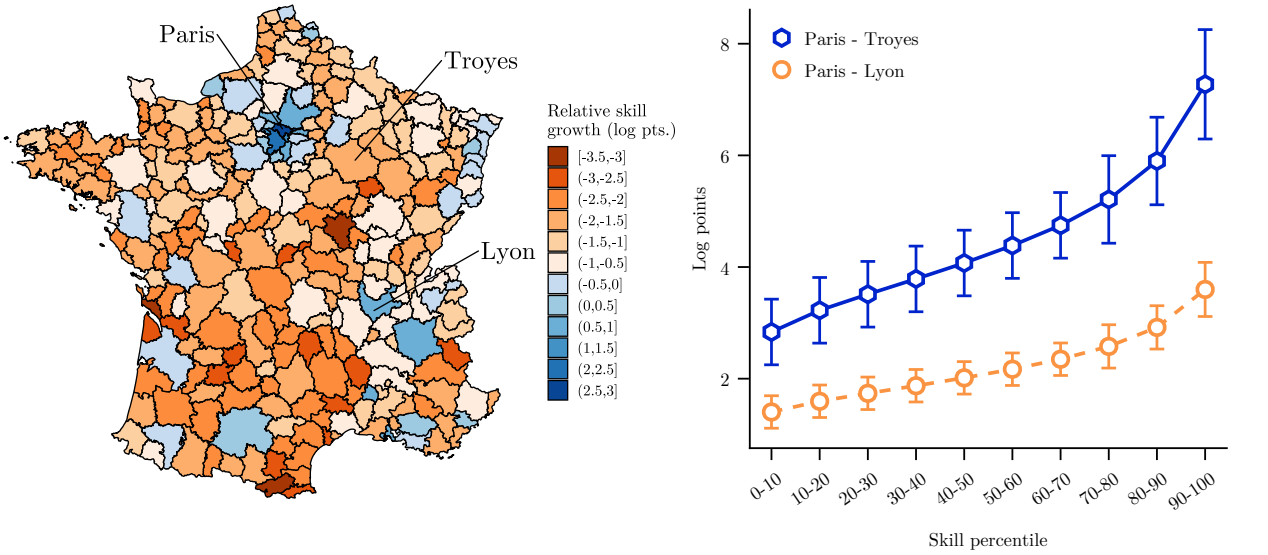
Standard errors clustered at the city level for columns 1 to 4 and at the neighborhood level for columns 5 and 6. Sample includes all workers between 25 and 40 year old. Skills measured as wages deflated by city TFP. Worker-level controls include age, job, occupation, and industry tenure fixed effects, and a dummy for employer switching. Occupations and industry measured at the one-digit level. Establishment growth computed as the growth in average wages, $\mathbb{E}_e[w_{t+1}]/\mathbb{E}_e[w_t]$ for $e$ the establishment. Column 7 instruments for the average neighborhood skill by the change in the white-collar employment share between 1993 and 2000.

Second, individuals employed in skill dense cities experience faster growth ($\hat{\gamma} > 0$). An increase by one standard deviation of the city skill implies an additional growth of 1.6 log points for the average worker. Through the lens of the model, this implies that workers learn on average more from skilled individuals.

longer-term variation, I estimate a learning technology that captures permanent variations in human capital rather than transitory, mean-reverting shocks.

Figure 1.3: Predicted skill growth

(a) Average predicted skill growth by CZ (b) Predicted skill growth by CZ and initial skill




Note: left-panel displays the relative average growth by commuting zone for the average French worker, $\hat{g}_2 \mathbb{E}_{\ell_{it}}[\log s_{it}]$. Right-panel displays the predicted growth difference between Paris and Troyes (orange hexagons) and Paris and Lyon (blue circles) across the wage distribution, $(\hat{g}_2 + \hat{g}_{12} \log s_{it}) (\mathbb{E}_{\ell'}[\log s_{it}] - \mathbb{E}_{\ell}[\log s_{it}])$. Both set of statistics are computed using the estimates of column 1 in Table 1.1.

Third and last, high-skill individuals working in skill dense cities see their skill grow faster than their low-skill counterparts in the same location ($\hat{\delta} > 0$). An increase by one standard deviation of the city average skill implies an additional growth of 1 log point for workers at the 10th percentile of the skill distribution, compared to an increase of 2.6 log points for workers at the 90th percentile. From (1.19), this implies that the learning technology displays strong within-skill complementarities: skilled workers learn relatively more from skilled partners than their low-skill counterparts.

The estimated returns to local interactions are substantial. Figure 1.3a displays the predicted relative skill growth by commuting zones for a worker with the average skill. Migrating from Troyes to Paris triggers a skill growth increase of 4.5 log points per five years for the average French worker, while moving from Lyon to Paris implies skill growth gains of 2.2 log points. Altogether, the predicted skill growth explains 64% of the between-city wage growth variance.

The between-city differences in growth are very heterogeneous across workers due to the

estimated learning complementarities. Figure 1.3b plots the difference in predicted growth between Paris, Lyon and Troyes across the skill distribution. All workers, both low- and high-skill, experience faster growth when employed in Paris relative to the other two cities. High-skill workers benefit however more over time from working in high-skill locations. For instance, the skill growth gap between Paris and Troyes is 2.8 log points for workers in the bottom 10% of the skill distribution. It is 7.3 log points for workers in the top 10%. The same pattern holds between Paris and Lyon, although the gaps are smaller as the two cities have a more similar skill composition.

**Robustness** The remaining columns of Table 1.1 provide robustness exercises to address the endogeneity concerns mentioned in Section 1.3.4.[46] Column 2 investigates the role of large cities in shaping the estimated skill growth elasticities. A vast literature, started by Roca and Puga (2017), finds that larger cities boost workers' wage growth. In the second column, I thus control for city size. In accordance with the previous literature, I also find that workers in larger cities experience faster growth. However, the effect of city size is one order of magnitude smaller than the effect of skill composition: an increase in city size by one standard deviation implies a 0.4 log points increase in skill growth for the average worker.[47] The difference in magnitude justifies why I abstract from city size in the quantitative model.[48]

The third column introduces the worker-level controls, and the fourth adds the occupation-by-firm fixed effects and the control for establishment wage growth. The parameters shaping the returns to local interactions, $g_2$ and $g_{12}$, are smaller than in the baseline. However, the two sets of estimates are not statistically different at the 5%. The same pattern holds when instrumenting workers' wage by their past wage to net out transitory shocks (Table A.1, column 2), or adding worker fixed effect to soak unobserved heterogeneity in abilities to learn

[46] In all of these exercises, I keep the estimate of cities' TFP constant to their baseline values.

[47] Most papers in this literature, to the exception of Eckert et al. (2022), identify the effect of size on growth from spatial correlation between city size and wage growth. In fact, the empirical model of Roca and Puga (2017) coincides with (1.19) under the restriction that $\beta = 1$, $\gamma = 0$, and average skills are replaced by city size.

[48] I also show in Table A.1 (column 1) that the estimated elasticities are not driven by a few large cities, and in Figures A.6 and A.7 that the elasticities are constant across the city size and inequality distributions.

(Table A.1, column 7). I conclude that wage growth heterogeneity across workers and firms does not bias the learning technology estimates.

The fifth column adds the neighborhood of residence by city fixed effects to control for unobserved spatial heterogeneity in wage growth.[49] The last column brings in the instrument variable. The F-statistic for the weak IV test is well above the conventional thresholds (Stock and Yogo, 2002; Andrews et al., 2019). The OLS and 2SLS estimates are not statistically different at the 5%. Furthermore, the two elasticities governing how workers' skill growth depends on the skill composition of where they work, $\gamma$ and $\delta$, remain positive. Comparing workers employed in the same city, in the same firm, at the same occupation, and living in the same neighborhood, individuals working in the relatively skill-dense neighborhood experience faster skill growth – and disproportionately so if these workers are initially relatively skilled.

In summary, Table 1.1 documents that the skill composition of where individuals work shapes their human capital accumulation.[50] In addition, the estimates of the learning technology are robust to controlling for numerous degrees of heterogeneity in wage growth across workers and firms. Finally, allowing for spatial heterogeneity in wage dynamics across cities, I still find that individuals learn relatively more from skilled workers, in particular if they are themselves skilled. I now turn to estimating the remaining parameters of the quantitative model.

## 1.4 Model Estimation

The remainder of the structural estimation is split into three parts. In the first part, I define the model's geography and time horizon, and accordingly externally calibrate the discount factor. In the second part, I use the structure of the model to derive estimating equations

[49]Table A.1 (column 3) further adds city by industry by occupation fixed effects to control for spatial heterogeneity in wage dynamics that differ across occupations and industries and local production complementarities.

[50]Appendix A.5.4 also shows that the skill growth elasticities are increasing in the time horizon considered (Figure A.4), are positive even after controlling for the skill composition of the establishment and firm where workers are employed (Table A.1, column 4), and are positive for all industries although with substantive heterogeneity (Figure A.9).

that identify the first group of parameters without the need for simulation. In the third part, I calibrate the remaining parameters. The estimated parameters are presented in Table A.2.

### 1.4.1 Definitions

I define a location in the model as a commuting zone. Solving the model for the 297 commuting zones is computationally challenging as it requires to solve for the entire skill distribution within each city. Instead, I create 30 city types and restrict all cities within the same type to be homogeneous. Importantly, there remains 297 cities in the model, and learning interactions happen within cities. I define the first 10 types as the 10 largest French cities. To construct the remaining 20 types, I divide France into four geographic districts (North-East, North-West, South-East and South-West). I then subdivide each district into five types using a k-mean algorithm that clusters cities based on variables that proxy for the geographic primitives of the model.[51] Appendix A.6.1 contains details on the variables used in the algorithm, and Figure A.10 plots the resulting city types.

Regarding the model's time horizon, the model features two lifecycle periods spanning workers' career.[52] In the young phase, individuals work and learn, while they only work in the old period. In the data, the wage-age profile plateaus around 40 years old. Accordingly, I set a lifecycle period to fifteen years and map the young period to ages 25 to 40 and the old period to ages between 40 and 55. The model does not provide an explicit moment to estimate the discount factor. I therefore calibrate it externally, take a yearly discount factor of 0.975, and set $\beta = 0.975^{15}$.

[51] While this classification is not without loss of generality, it is likely to have little impact on the counterfactuals since cities within each type are rather homogeneous. With five types, the clustering algorithm captures 83% of the total sum of square residuals (Figure A.11). In addition, clustering cities avoids the over-fitting problem present in spatial granular settings (Dingel and Tintelnot, 2020).

[52] Adding more periods would make the model closer to the learning process estimated in Table 1.1. However, it would also increase its computation complexity without much effects on the counterfactuals.

### 1.4.2 Model Inversion

In the second step, I invert the model to estimate the parameters dictating the housing supply and demand, the migration costs, and the the learning technology as described in Section 1.3. The estimating equations are derived in Appendix A.6.2.

**Housing** From the Cobb-Douglas preferences, $\alpha$ is the housing expenditure share. I read off $\alpha$ from the French national accounts and set $\alpha = 0.2$ (INSEE, 2020). Regarding the housing supply parameters, the housing market clearing conditions relate housing prices to local expenditures through

$$\log p_\ell = \left(\frac{1}{1+\delta}\right) \log\left(\frac{1-\alpha}{\mathcal{H}}\right) + \left(\frac{1}{1+\delta}\right) \log Y_\ell, \quad \forall\, \ell, \tag{1.21}$$

where $p_\ell$ and $Y_\ell$ are the housing rental price and total expenditures in city $\ell$. I obtain $p_\ell$ from the "Carte des Loyers" (Rental Map), which estimates the average monthly rent per meter square at the municipal level.[53] Meanwhile, the expenditures in city $\ell$ are given by the payments to the labor inputs, $Y_\ell = N_\ell \mathbb{E}[W_\ell]$, where $N_\ell$ and $\mathbb{E}[W_\ell]$ are the employment share and average wage of city $\ell$ that I compute from the cross-sectional matched employer-employee dataset. To match the frequency of the rent data, I set wages at the monthly frequency in euros of 2018. Given $\{p_\ell, N_\ell, w_\ell\}_{\ell=1}^{L}$ and $\alpha$, the housing supply parameters, $\mathcal{H}$ and $\delta$, can be recovered from (1.21) by OLS. Figure A.14c compares the rent prices in the data with those in the model. While simple, the model captures 71% of the between-city rent variations.

**Migration costs** The migration costs are recovered non-parametrically from the observed migration flows. Using workers' optimal location decisions and the assumption of symmetric costs, the model predicts that the cost to move from city $l$ to city $\ell$ for workers with age

[53] These rents are residualized and correspond to the rent paid for a typical $40\text{m}^2$ apartment in France. I compute $p_\ell$ at the city-level by taking the population weighted average of the municipal rents.

$a \in \{y, o\}$ is[54]

$$\mathrm{e}^{-2\vartheta\kappa^a_{l\ell}} = \int \left( \frac{n^a_{l\ell}(s)}{n^a_{ll}(s)} \Big/ \frac{n^a_{\ell\ell}(s)}{n^a_{\ell l}(s)} \right) dN^a(s), \quad \forall\, l, \forall\, \ell. \tag{1.22}$$

In equation (1.22), $n^a_{l\ell}(s)$ is the number of workers with age $a$ and skill $s$ who migrates from $l$ to $\ell$. The cost to move between these two cities is thus identified from the size of the bilateral migration flows relative to the number of individuals who do not move. Measuring the right-hand side of (1.22) requires observing workers' skills and migration decisions. I proxy workers' skills with their wage quartile.[55] Consistently with the model, I measure migration decisions differently for young and old workers. For young workers, I compute $n^y_{l\ell}(s)$ as the number of workers under 40 born in city $l$ and currently living in city $\ell$ with skill $s$. For old workers, I compute $n^o_{l\ell}(s)$ as the number of workers with age above 40 that lived in city $l$ 15 years ago and are currently living in city $\ell$ with skill $s$. Both flows are computed from the long-panel.

Figure A.12 plots the estimated migration costs by city of origin. Empirically, 45% of young workers live in cities different from their birthplace, whereas 20% of workers change locations between their youth and their old age. Through the lens of the model, these patterns are rationalized with higher migration costs for old workers. The average migration cost for young workers is 1,255€, which corresponds to 43% of their income. The average migration costs for old workers is 1,594€ (44% of their earnings).[56]

The model predicts substantial heterogeneity in migration probabilities across workers. In particular, skilled workers born in productive cities are relatively more likely to work

[54]These migration costs can alternatively capture correlated location preferences – up to welfare considerations. In this case, (1.22) captures the autocorrelation in location preferences.

[55]Proxying workers' skill with their wage quartile may lead to biased migration costs as wages are the product of workers' skill and city TFP. The biases are likely to be small in practice for two reasons. First, the estimated TFP variance explains less than 2% of the wage variance. Second, (1.22) estimates migration costs off double difference in migration flows, and $n_{l\ell}(s)/n_{\ell\ell}(s)$ partially nets out the biases that emerge from using wages as a proxy for skills.

[56]The estimated migration costs are in utils. The reported estimates are converted to monetary unit through $\kappa^a_{l\ell} P_\ell$. The reported average cost are computed as unweighted average, excluding $\kappa^a_{\ell\ell}$. As such, they do not represent the migration costs *actually* paid by workers.

there, whereas skilled workers born in low-TFP locations are more likely to move out. Appendix A.6.3 provides more details on the migration heterogeneity.

**Learning technology** The learning technology parameters $g_1$, $g_2$ and $g_{12}$ are estimated according to Proposition 1.6. In the model, the young period lasts fifteen years. In the data, I use wage growth at the five-year horizon to have enough statistical power. To get a learning technology with the appropriate time horizon, I estimate (1.19) for three age bins: 25 to 30, 30 to 35 and 35 to 40 (Figure A.5). I then set $g_1 - 1$, $g_2$ and $g_{12}$ as the sum of their respective estimated age-specific skill growth elasticities.

### 1.4.3 Indirect Inference

The remaining 95 parameters govern workers' productivity and location preferences. I calibrate them jointly to match salient features of the wage and city-size distributions. I provide first arguments that justify the identification strategy. I then show numerically how the chosen moments affect the calibration of the model.

**City productivity** I set the productivity of each city to match its average wage in the data. In the model, the average wage of city $\ell$ is given by

$$\mathbb{E}[W_\ell] = T_\ell \int s\pi_\ell(s)ds, \quad \forall\, \ell, \tag{1.23}$$

where $\int s\pi_\ell(s)ds$ is the average skill of the workers employed in $\ell$. The average wage on the left-hand side can be measured from the matched employer-employee dataset. Holding constant the skill distribution, a greater TFP implies higher local wages. However, the average skill in city $\ell$ is not observed, and it is an equilibrium object that depends on the spatial distribution of TFPs. Equation (1.23) therefore cannot be directly inverted to recover $T_\ell$. Nevertheless, given $\{T_\ell\}_{\ell=1}^L$ and the other parameters, the spatial distribution of skills can be recovered in equilibrium from the model. Equation (1.23) then defines a fixed point

over $\{T_\ell\}_{\ell=1}^L$ that can be solved for numerically to obtain cities' TFP without taking a stance on how to measure skills empirically.

**Worker-level productivity** Four parameters govern the distribution of worker-level productivity. The first two, $\mu_y$ and $\sigma_y$, are the mean and the standard deviation of the skill distribution of young workers. The level of workers' skills and cities' TFP is not separately identified. I thus normalize the average log skill to zero, and set $\mu_y$ to ensure this normalization holds. Holding cities' TFP constant, a greater skill dispersion implies a greater wage variance. I therefore calibrate $\sigma_y$ to match the aggregate wage variance of young workers.

The other two parameters implicitly shape the skill distribution of old workers. First, $g_0$ in the learning technology (1.16) dictates the overall amount of human capital accumulation. I set $g_0$ to match the wage ratio between old and young workers. Second, the variance of the idiosyncratic learning shocks, $\sigma_\nu$, influences the dispersion in the skills of old workers. I calibrate $\sigma_\nu$ to match the variance of log wages for old workers.

**Location preferences** The remaining parameters – cities' amenities and the dispersion in the taste shocks – determine workers' location preferences. The higher the amenities a city offers, the more workers want to live there. I set the age-specific local amenities to match the cities' age-specific employment share.[57] The dispersion in idiosyncratic location preferences, $\theta$, governs the amount of sorting that prevails in equilibrium. When $\theta \approx 0$, the idiosyncratic location preferences are very dispersed, and the skill distributions are homogeneous across cities. As a consequence, there are no spatial differences in within-city wage inequality. As $\theta$ increases, skilled workers concentrate in productive cities, and those places become more unequal. I therefore set $\theta$ to match the cross-sectional correlation between cities' average wage and wage variance.

[57] As for their TFP, cities' employment share can theoretically be expressed as a function of the their amenities and a residual (equation (A.39) in Appendix A.6.2). The residual can be computed within the model, and cities' amenities $\{B_\ell^y, B_\ell^o\}_{\ell=1}^L$ are recovered as a solution to a second fixed point. Figure A.13b plots the estimated amenities.

**Sensitivity** How precise is the calibration? Figure A.17 reports how the targeted moments vary as a function of the parameters in the model. Across the board, the targeted moments are sensitive in the parameters they are supposed to identify, with a 1% change in the parameter associated with a change in the targeted moment between 0.5% and 2%. Appendix A.6.4 describes in more detail this sensitivity exercise and reports the sensitivity measure of Andrews et al. (2017).

### 1.4.4 Estimation Results and Over-identification Exercises

The estimated model implies significant sorting across space (Figure A.15). 30% of the workers in the top 10% of the skill distribution choose to work in Paris, and this fraction goes as high as 46% for workers in the top 1%. In contrast, low-skill workers are much less spatially concentrated: only 10% of workers in the bottom half of the skill distribution decide to work in Paris. These differences across skills in willingness to work in productive cities also vary across the lifecycle. In Appendix A.6.5, I show as an over-identification exercise that, in the model and the data, young skilled workers have a relatively higher propensity to work in productive cities than their old counterparts.

Worker sorting explains the majority of the between-city wage disparities. In the model, the between-city wage variance can be decomposed into three terms:

$$\mathbb{V}\text{ar}[\mathbb{E}\left(\log W_\ell\right)] = \mathbb{V}\text{ar}[\log T_\ell] + \mathbb{V}\text{ar}[\mathbb{E}\left(\log S_\ell\right)] + 2\mathbb{C}\text{ov}[\log T_\ell, \mathbb{E}\left(\log S_\ell\right)].$$

The first term captures how much of spatial wage inequality is explained by TFP gaps. I find the dispersion in cities' TFP to rationalizes only 16% of the between-city wage variance – and 1.7% of the aggregate variance. The second and third terms capture the spatial differences in skill composition that emanate from the sorting of workers into cities. Together, they explain the remaining 84% of the between-city wage variance. These estimates align with those of the literature that quantifies the impact of cities on wage inequality through movers

designs (Dauth et al., 2022; Lhuillier, 2022; Card et al., 2023). In Appendix A.6.5, I follow this literature and, as a second over-identification exercise, re-estimate cities' TFP using a two-way fixed effect model à la Abowd et al. (1999a).[58] The newly estimated TFPs can barely be distinguished from the baseline estimates. I conclude that the small dispersion in TFP across space is a robust feature of the French labor market.

Finally, the model explains 64% of the between-city wage growth variance in the data. To what extent does the spatial segmentation of learning opportunities cause these differences in wage growth? And what are the implications for spatial inequality and aggregate productivity? I use the estimated model to answer these questions.

## 1.5 The Tradeoff Between Human Capital and Spatial Inequality

In this section, I compare the baseline equilibrium with a counterfactual economy in which interactions are not segmented by cities. In the counterfactual world, interactions continue to be random, but the likelihood of an interaction is given by the aggregate skill density rather than the skill density of the city where individuals work.[59] I compare the two economies in steady state. Table 1.2 summarizes the key results.

### 1.5.1 Learning within Cities and Spatial Inequality

I find that learning opportunities are very spatially concentrated. Figure 1.4a plots the average skill growth by workplace in the baseline equilibrium (blue circles). On average, workers' skills grow by 15 log points throughout their lifetime. Individuals who work in high-wage cities experience faster human capital accumulation. For instance, workers in Paris

[58]This approach is not used as the baseline as it requires to take a stance on how to empirically measure skills.

[59]The aggregate skill density of the counterfactual economy may in turn differ from the baseline skill distribution to the extent that cities shape human capital accumulation.

Table 1.2: The consequences of spatially segmented learning opportunities

| | No segmentation | Segmentation | Change |
|---|---|---|---|
| Average skill of old workers | 1.231 | 1.246 | 1.21% |
| .. due to learning complementarity | · | · | 58.5% |
| Average wage (€) | 3,210 | 3,296 | 2.68% |
| Between-city wage variance | 0.007 | 0.024 | 0.017 |
| Between-city wage growth variance (log points) | | | |
| .. by workplace | 0.016 | 13.85 | 13.85 |
| .. by birthplace | 0.000 | 3.281 | 3.281 |

Table contains aggregate statistics on inequality and productivity under two different assumption regarding the segmentation of learning interactions. First column corresponds to the case in which interactions are random in the whole economy. Second column is when interactions are fully segmented by cities (baseline, estimated model). The third column is the change between the two equilibrium.

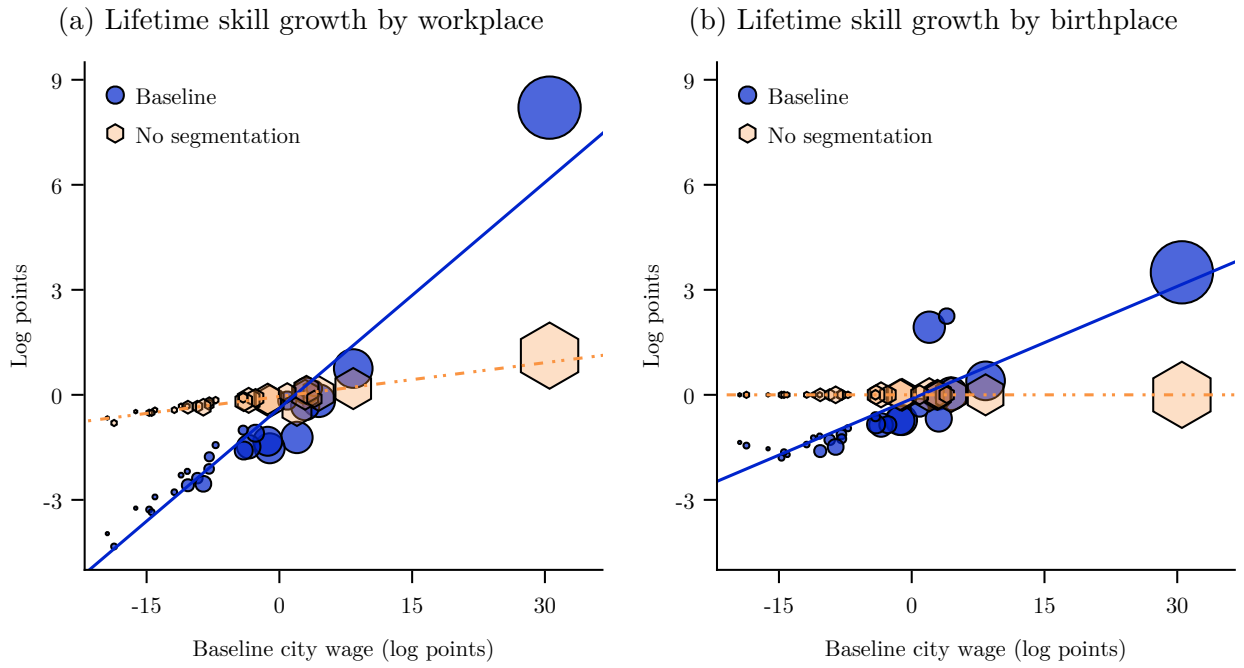
experience a skill growth of 23 log points.

These differences in learning originate both from the sorting of fast-learning workers in productive cities ($g_1 > 1$) and from the concentration of learning opportunities in those locations ($g_2 > 0$ and $g_{12} > 0$). To quantify the relative importance of both channels, the orange hexagons in Figure 1.4a display the average skill growth by workplace were interactions not segmented by cities – holding the spatial allocation of workers constant. In this counterfactual economy, workers in Paris continue to experience on average faster skill growth due to their higher ability to learn. Yet, this sorting channel is quantitatively small as it explains 2% of the model-predicted between-city skill growth variance. I conclude that the segmentation of learning opportunities is the primary driver of the spatial differences in human capital accumulation.

The concentration of learning opportunities in a few cities has two consequences on geographic disparities. First, it engenders learning inequality. Figure 1.4b displays the average skill growth by birthplace against the city's average wage.[60] When workers learn from the individuals around them and face migration costs, workers born in or near skill-dense cities have an easier access to the local learning opportunities and enjoy faster human capital

[60]The model explains 55% of the wage growth variance across birthplaces in the data.

Figure 1.4: Between-city differences in human capital accumulation




Note: panel (a) displays the average skill growth by workplace in the equilibrium with segmented interactions (blue circles) and in the equilibrium without segmented interactions (orange circles) against the baseline city average wage. The blue and orange dashed lines are the (unweighted) fitted lines. The grey line is the empirical (unweighted) fitted line. Panel (b) displays the average skill growth by birthplace. The same nomenclature as in panel (a) applies. The size of the markers is proportional to city size.

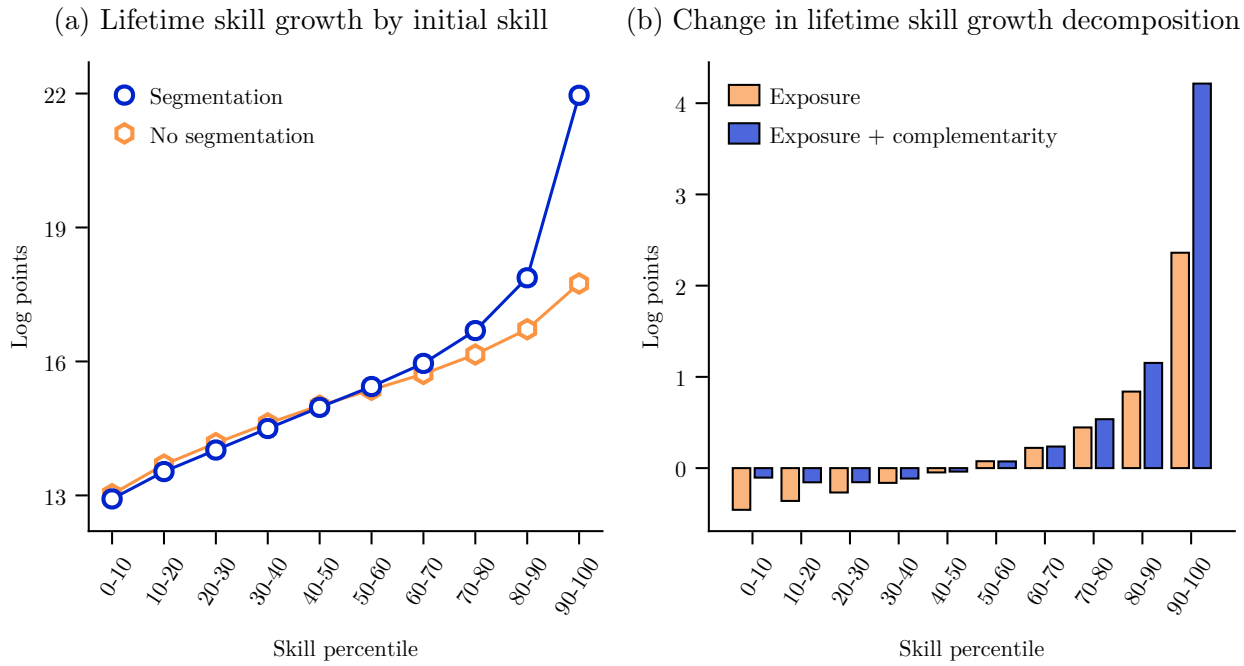
accumulation.[61] This birthplace learning premium is substantial. For instance, workers born in Paris have a lifetime skill growth 5.5 log points higher than workers born in the 25% poorest French cities.[62] The impact of individuals' birthplace on their lifetime learning is exacerbated for middle and high-skill workers who benefit relatively more from meeting other skilled workers (Figure A.21).

Second, the confinement of learning opportunities to productive cities amplifies spatial wage inequality. Proposition 1.2 found that, when individuals learn relatively more from skilled workers, productive cities get bigger as young workers go there to learn. In addition, these opportunities are more tailored to skilled workers, and the spatial concentration of high-skill rises. These agglomeration effects are quantitatively large (Figure A.24): the population of Paris increases from 7% to 14% of the French workforce when workers learn

[61]Workers born near Paris indeed experience relatively faster skill growth, as shown in Figure A.22.

[62]The spatial differences in learning feeds into lifetime income inequality. When opportunities are spatially segmented, the lifetime income gaps between workers born in Paris and those born in the bottom 25% of the wage distribution is 12%, compared to 5% when interactions are not segmented.

Figure 1.5: The aggregate consequences of local interactions on human capital accumulation




Note: panel (a) displays the average skill growth by young skill in the equilibrium with segmented interactions (blue circles) and in the equilibrium without segmented interactions (orange circles). Panel (b) displays the change in skill growth between the baseline economy and the counterfactual economy across the young skill distribution. The total change is decomposed according to (1.24). The blue bars plot the "average exposure" effect, and the orange bars plot the total change.

from the individuals around them, and the propensity of workers in the top 10th percentile of the skill distribution to work there triples from 11% to 32%.[63] The greater agglomeration of skilled workers in a few cities amplifies spatial wage inequality, and the between-city wage variance rises from 0.007 to 0.024.[64]

### 1.5.2 Local Interactions and Human Capital Accumulation

While the concentration of learning opportunities in space generates geographic disparities, it also substantially boosts aggregate productivity. The average skill of old workers is 1.2% higher when individuals learn within cities, and the aggregate average wage rises by by

[63] In the presence of costly migration, local interactions agglomerate both young and old workers in productive cities, as the majority of old workers stay in the locations where they lived when young (Figure A.25a).

[64] The change in between-city wage inequality can be decomposed into a TFP effect, a skill disparity effect, and a sorting effect; specifically, $\Delta\mathbb{V}\text{ar}[\mathbb{E}(\log W_\ell)] = \Delta\mathbb{V}\text{ar}[\log T_\ell] + \Delta\mathbb{V}\text{ar}[\mathbb{E}(\log S_\ell)] + 2\Delta\mathbb{C}\text{ov}[\log T_\ell, \mathbb{E}(\log S_\ell)]$. The TFP effect reflects the greater agglomeration of workers to productive cities and explains only 1% of the total change in between-city wage inequality. The remaining two terms capture the heterogeneity in skill composition across cities, and they explain together the remaining 99%.

2.7%.[65]

The spatial concentration of learning opportunities enhances aggregate productivity by boosting the human capital accumulation of skilled workers without dampening that of other individuals. Figure 1.5a displays the average lifetime skill growth as a function of workers' initial skill when opportunities are spatially segmented (blue) and when they are not (orange). When individuals learn from the other workers in their city, workers in the top 10% of the skill distribution see their skill grow by 22 log points, 4 log points more than their lifetime human capital accumulation when interactions are not spatially segmented. At the other end of the distribution, the segmentation of learning opportunities reduces the lifetime skill growth of workers in the bottom 10% of the skill distribution by 0.1 log points.[66]

Why do high-skill workers gain from the spatial concentration of learning opportunities? Cities shape skill growth through two channels. First, high-skill workers agglomerate relatively more to skill-dense cities, and as result, are more exposed to productive interactions. I refer to this channel as the exposure effect. Second, high-skill workers are better able to capitalize on skilled interactions in the presence of strong within-skill learning complementarities. I coin this force the complementarity effect. Formally, the change in average lifetime skill growth between the baseline and the counterfactual economy can be written

$$\mathbb{E}\left[\log\left(\frac{S^o}{s}\right) \mid s\right] - \mathbb{E}\left[\log\left(\frac{\bar{S}^o}{s}\right) \mid s\right] = \underbrace{g_2 \sum_\ell \left(\frac{n^y_\ell(s)}{n^y(s)}\right) \Delta_\ell}_{\text{Exposure}} + \underbrace{g_{12} \log s \sum_\ell \left(\frac{n^y_\ell(s)}{n^y(s)}\right) \Delta_\ell}_{\text{Complementarity}}, \quad (1.24)$$

where $\bar{x}$ refers to the value of $x$ when interactions are not spatially segmented, and $\Delta_\ell \equiv$

[65]The average wage increases relatively more than stock of human capital as it also takes into account the greater agglomeration of workers in high-TFP cities. Specifically, the change in the average log wage between the no-segmentation and the segmentation economies can be decomposed into a TFP term and a human capital term, $\mathbb{E}[\log \bar{W}] - \mathbb{E}[\log W] = \sum_\ell \log T_\ell \left(\bar{N}_\ell - N_\ell\right) + \mathbb{E}[\log \bar{S}] - \mathbb{E}[\log S]$, where $\bar{x}$ refers to the value of $x$ in the counterfactual economy. In the aggregate, 16% of the wage increase is explained by the larger stock of human capital, and the remaining 84% are due to the greater agglomeration of workers in productive cities.

[66]Learning within cities therefore heightens the difference in human capital accumulation between low- and high-skill. When interactions are spatially segmented, an increase in initial skill by one standard deviation is associated with a lifetime skill growth 1.6 log points larger. In contrast, this skill learning premium flattens to 0.6 log points when individuals learn from every worker.

$\mathbb{E}[\log S_\ell] - \mathbb{E}[\log \bar{S}]$ is the difference between the average skill in city $\ell$ in the baseline equilibrium and the average aggregate skill in the counterfactual equilibrium.[67]

Learning complementarities are essential in amplifying the effect of cities on human capital accumulation. Figure 1.5b plots the skill growth decomposition (1.24). The exposure effect accounts for 56% of the faster learning experienced by workers in the top 10%; the complementarity channel explains the remaining 44% of their learning gains. Meanwhile, the learning complementarities also minimize the learning losses of workers in the bottom half of the skill distribution.[68] By boosting the gains of high-skill and reducing the losses of low-skill, learning complementarities are crucial for the impact of cities on human capital accumulation. Aggregating (1.24) across workers, the complementarity channel explain 59% of the aggregate increase in human capital.

This section therefore suggests that governments face an equity-efficiency tradeoff. On the one hand, the spatial segmentation of learning opportunities improves aggregate productivity. On the other hand, it amplifies spatial wage and learning inequality. As a result, policies aimed at reducing the spatial concentration of skills may have a detrimental effect on the aggregate stock of human capital. I conclude this chapter by quantifying how steep is the equity-efficiency tradeoff.

## 1.6 The Consequences of Spatial Policies

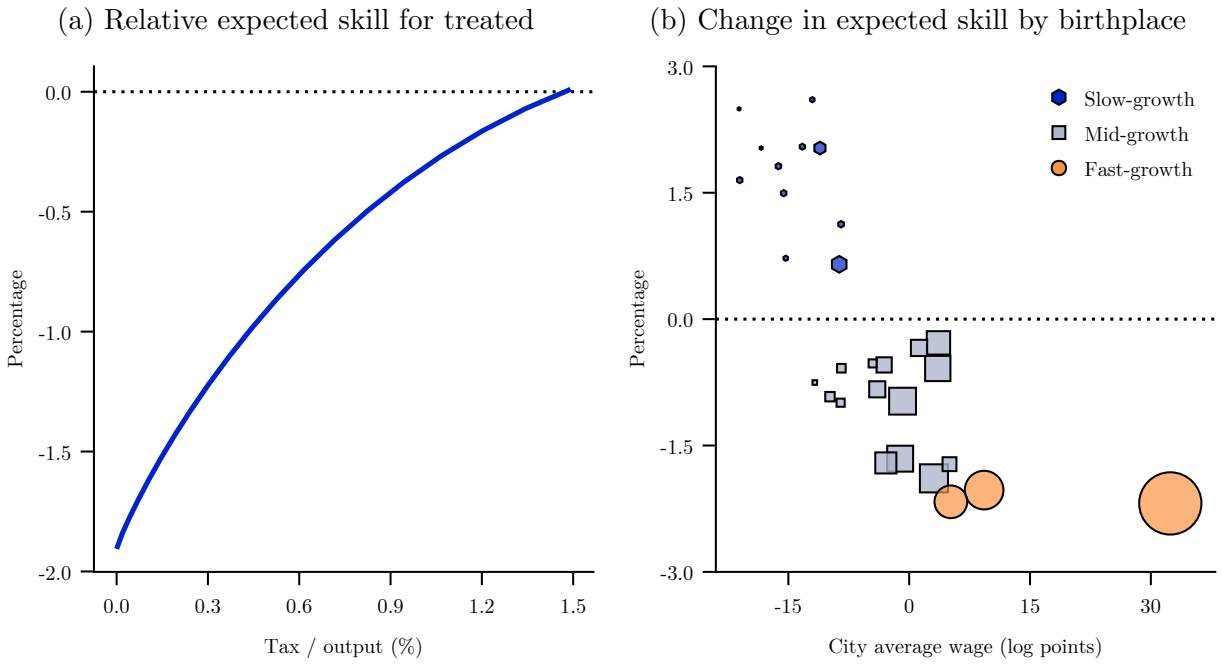
I study the general equilibrium consequences of a moving voucher policy aimed at offering equal learning opportunities to workers born in remote locations (Chetty et al., 2016).[69]

[67] The change in skill growth can further be decomposed into a partial equilibrium effect that holds the average old skill constant, $\mathbb{E}[\log S] - \sum_\ell \left(\frac{n^y_\ell(s)}{n^y(s)}\right) \mathbb{E}[\log S_\ell]$, and a general equilibrium effect, $\mathbb{E}[\log \bar{S}] - \mathbb{E}[\log S]$. The general equilibrium effects alone are however small (see Figure A.23).

[68] In the baseline equilibrium, relatively low-skill workers do not sort much across space (Section 1.4.4). As a result, the average learning opportunity they have access to is similar to the average nationwide opportunity. This, together with the strong within-skill learning complementarities, explain why these workers do not lose much from the spatial segregation of opportunities.

[69] Chetty et al. (2016) evaluates the consequence of the Moving to Opportunity program in the United States, which subsidized children and teenagers born in distressed neighborhoods to move to large, productive places. In my setting, the vouchers are granted to young professionals at the beginning of their career. See

Figure 1.6: The consequences of spatial policies on spatial learning inequality




Note: panel (a) displays the expected old skill of workers born in cities in the bottom 25% of the city growth distribution relative to the nationwide expected old skill against the size of the voucher policy. The size of the policy is measured as the aggregate tax used to finance the policy as a fraction of total GDP. Panel (b) plots the change in expected old skill by birthplace between the baseline equilibrium and the policy equilibrium when the policy reaches 1.5% of GDP. The blue hexagons are the cities in the bottom 25% of the city growth distribution. The orange circles are the three largest French cities (Paris, Lyon and Toulouse). The grey rectangles are the remaining cities. The size of the markers is proportional to the baseline city size.

Specifically, the policy consists of a subsidy granted to young workers born in the 25% poorest locations conditional on moving to the three largest French cities (Paris, Lyon, and Toulouse). Figure 1.8a displays the 185 cities treated by the policy. The policy is self-financed within skill and age group to focus on spatial redistribution.

### 1.6.1 Spatial Inequality

Large moving vouchers have the potential to improve the lifetime human capital of workers born in remote locations. Figure 1.6a plots the average skill of treated workers at the end of their career compared to the average worker as a function of the policy size. Absent the vouchers, treated workers have a human capital 2% lower than the average French worker at the end of their career. As the size of the vouchers increases, it covers a greater share of the migration cost to productive cities. Treated workers have an easier access to the learning

Fogli et al. (2023) for a paper that evaluates the general equilibrium consequences of the MTO program.

opportunities present in these cities, which increases their lifetime human capital. When the voucher reaches 1,030€, which covers 80% of the average migration cost and represents 1.5% of GDP, the probability that treated workers migrate to the three largest French cities rises from 7% to 45%. As a result, workers born in the 25% poorest locations accumulate human capital as much as the average worker.

The policy has heterogeneous treatment effects across workers. Figure 1.6b plots the change in expected old skill by birthplace between the baseline equilibrium and the equilibrium when the policy reaches 1.5% of GDP. Workers born in treated cities (blue hexagons) all experience on average an increase in their human capital.[70] The gains are relatively larger for workers born near Lyon, Paris, and Toulouse for whom the take-up rate of the policy is larger (Figure A.26). Similarly, skilled workers are more likely to opt-in the policy (Figure A.28a). Together with their better capacity to capitalize on the learning opportunities present in productive cities, skilled treated workers benefit relatively more from the policy (Figure A.28b).

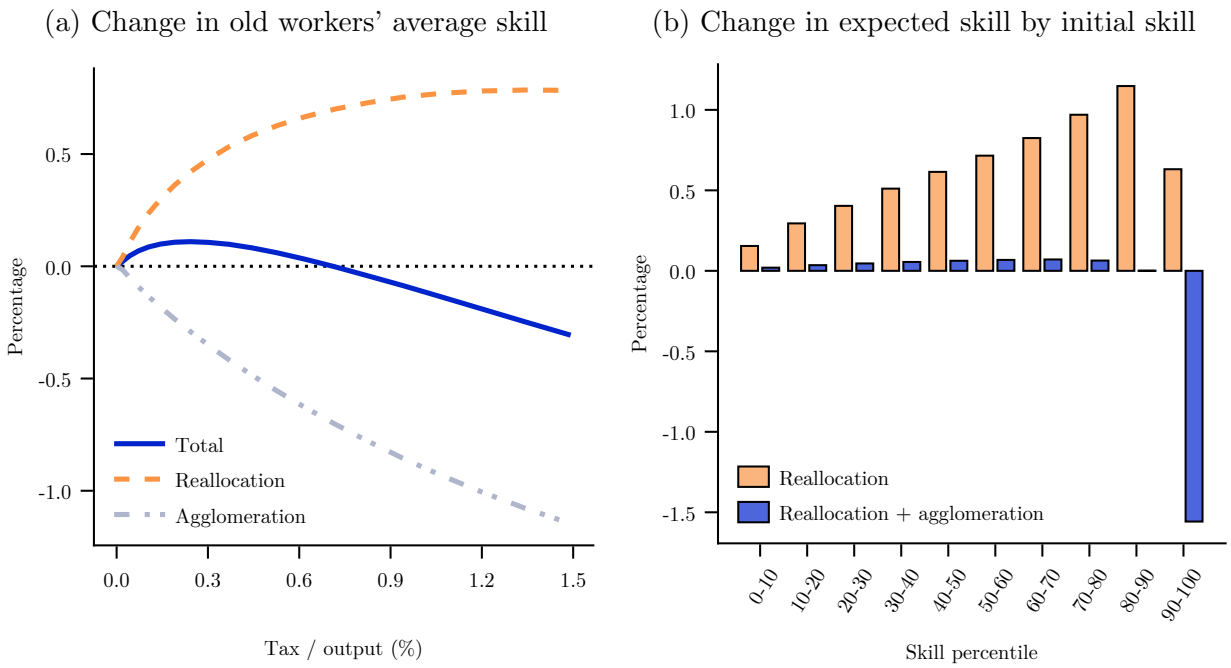
The learning gains experienced by the treated workers come at the expense of slower human capital accumulation for the non-treated. The policy reallocates to productive cities workers that are marginally less skilled, and in doing so, worsens the quality of the local opportunities (Figure A.29). As a result, the policy reduces the learning advantage of workers born in Lyon, Paris and Toulouse (orange circles in Figure 1.6b). The policy also negatively affects the human capital accumulation of non-treated individuals born outside the three largest French cities who used to migrate there to work and learn (grey rectangles).[71]

Combining the effects on the treated and non-treated, the policy reduces spatial learning

[70] The average skill obtained by treated workers at the end of their career increases by 1.8%. Combined with an easier access to high-productivity jobs, the moving voucher policy boosts their lifetime labor income by 5.7%.

[71] The learning losses for the non-treated born outside Lyon, Paris and Toulouse are relatively lower for two reasons. First, not everyone born outside these cities migrates there to work. Second, in general equilibrium, some of these mid-productivity cities experience an increase in their skill density as skilled workers have marginally less incentives to agglomerate in productive locations (Figure A.29a). As such, the learning opportunities offered by these places improve.

Figure 1.7: The consequences of local policies on human capital accumulation




Note: panel (a) displays the average old skill against the size of the policy. The blue solid line is the change in average old skill under the voucher policy. The orange dashed line is the change in average old skill under the quasi-optimal policy. The size of the policy is measured as the aggregate tax used to finance the policy as a fraction of total GDP. Panel (b) and (c) plot the change in expected old skill across the skill distribution between the baseline equilibrium and the policy equilibrium when the policy reaches 1.5% of GDP under the moving voucher and the quasi-optimal policy respectively. The blue bars represent the partial equilibrium, reallocation effect. The orange bars display the total change.

disparities, and the variance of wage growth across birthplaces decreases from 3.6 to 1.9.[72]

### 1.6.2 Human Capital Accumulation

The policy also negatively affects aggregate efficiency. Figure 1.7a displays the change in the average skill of old workers as a function of the size of the policy. When the policy reaches 1.5% of GDP, the stock of human capital is 0.3% lower than in the baseline equilibrium.

The policy negatively impacts aggregate efficiency by reducing the spatial segregation of learning opportunities. The impact of the policy on human capital accumulation can be decomposed into two effects. First, holding the quality of the local interactions constant, the vouchers reallocate workers to skill-dense places and increase the pool of individuals who have access to productive learning opportunities. I refer to this as the reallocation effect. Second,

[72]The policy also reduces spatial wage inequality by decreasing the spatial concentration of skilled workers. The between-city wage variance shrinks from 0.024 to 0.018 when the policy reaches 1.5% of GDP.

in general equilibrium, the policy affects the spatial distribution of skills, and with that the quality of the local opportunities. I name this channel the composition effect. Formally, the percentage change in the average skill of old workers between the policy and the baseline equilibrium can be decomposed into

$$\frac{\mathbb{E}[\bar{S}^o] - \mathbb{E}[S^o]}{\mathbb{E}[S^o]} = \underbrace{\sum_\ell \int\int \left(\frac{\gamma(s,s_p)}{\mathbb{E}[S^o]}\right) \Delta n_\ell^y(s) \pi_\ell(s_p) ds_p ds}_{\text{Reallocation}} + \underbrace{\sum_\ell \int\int \left(\frac{\gamma(s,s_p)}{\mathbb{E}[S^o]}\right) \bar{n}_\ell^y(s) \Delta\pi_\ell(s_p) ds_p ds}_{\text{Composition}},$$

where $\bar{x}$ refers to variable $x$ in the policy equilibrium and $\Delta x \equiv \bar{x} - x$. The dashed orange line and the dotted grey line in Figure 1.7a display the reallocation and composition effects. Abstracting from the composition effect, the moving voucher policy would increase aggregate human capital by 1%. However, the composition effect more than offsets the partial equilibrium impact of the policy, and in net, the policy triggers efficiency losses.

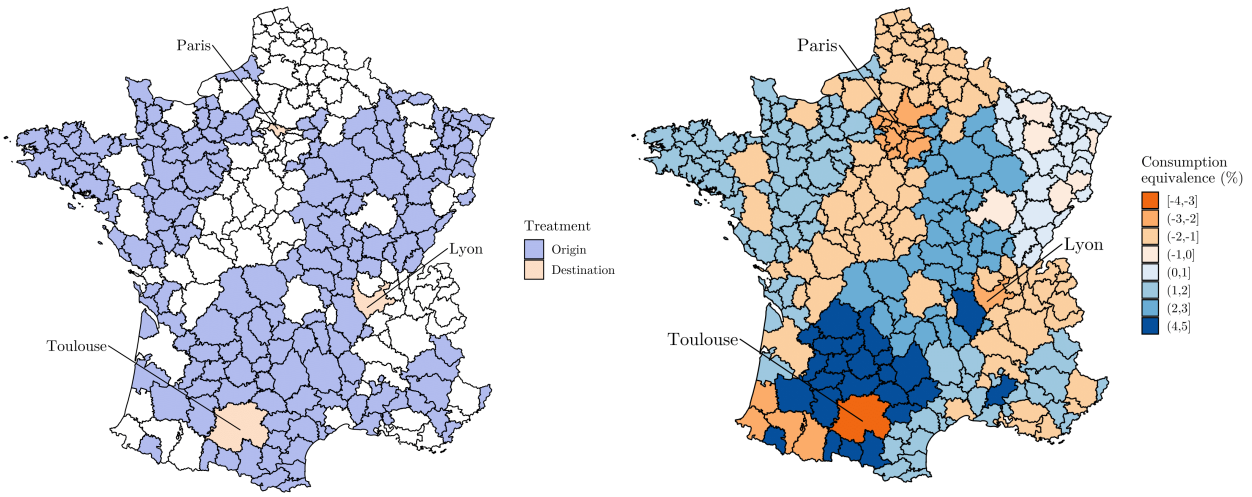
Skilled workers fully bear the learning losses generated by the policy. Figure 1.7b plots the change in workers' expected skill at the end of their career between the policy and the baseline equilibrium across the skill distribution. Holding constant the quality of the local learning opportunities, all workers would on average experience faster human capital accumulation (orange bars). The gains would be the largest for workers between the 50th and 90th percentile of the skill distribution for whom the enhanced access to interactions with skilled workers is most beneficial. However, the general equilibrium effect neutralizes those partial equilibrium gains, and the policy neither fosters nor dampens the learning of the bottom 90% of the skill distribution (blue bars). In contrast, the largest fraction of skilled workers already agglomerates in Lyon, Toulouse and Paris without the moving vouchers. For those workers, the policy only negatively affects the quality of the interactions they experience, thus reducing their lifetime learning.[73]

[73]Learning complementarities are crucial in shaping the efficiency cost of the policy. Were those absent, the

Figure 1.8: The consequences of the moving voucher policy on welfare

(a) Treated cities

(b) Change in welfare by birthplace




Note: panel (a) and (b) compares the welfare difference between the baseline equilibrium and the moving voucher equilibrium when the policy reaches 1.5% of GDP. Panel (a) displays the change in average welfare by birthplace assuming equal weights by skill within birthplaces. Panel (b) plots the change in average welfare by birthplace and initial skill, grouping cities into slow-growth cities (treated), middle-growth cities (non-treated), and fast-growth cities (non-treated but targeted by the policy).

Taking stock, the moving voucher policy reduces geographic learning disparities at the cost of lowered aggregate efficiency, thus facing a substantial equity-efficiency tradeoff. I conclude this section by quantifying the welfare consequences of the policy.

### 1.6.3 Welfare

I measure welfare through the expected lifetime utility of a cohort. I express welfare changes in terms of consumption equivalence (Lucas, 1987; Alvarez and Jermann, 2004). Appendix A.8.1 provides more details on how the consumption equivalence is calculated, as well as a welfare decomposition into different channels.

The policy is spatially redistributive: it generates welfare gains for the treated workers, and welfare losses for the non-treated. Figure 1.8b displays the average welfare effect of the policy by birthplaces.[74] On the one hand, workers born in treated cities enjoy average

learning losses experienced by workers in the top 10% of the skill distribution would be halved (Figure A.27).

[74]The average welfare change by birthplaces are computed under a utilitarian social welfare function that places equal weights on every worker within the city (see Appendix A.8.1 for more details). To complement this analysis, Figure A.31 reports the full distribution of welfare changes.

welfare gains of 2%. Those gains are relatively larger for workers born near Lyon, Paris, or Toulouse, and they are mainly explained by the faster human capital accumulation these workers experience (Figure A.30). On the other hand, workers born in non-treated cities suffer average welfare losses of 2.3%. For workers born outside of Lyon, Paris and Toulouse, these losses are entirely caused by the financial cost of the policy. For the non-treated born in the three largest French cities, the welfare losses are larger, going as high as 3.5% for Toulouse, as the policy negatively affects their lifetime human capital.

In the aggregate, the welfare losses outweigh the gains, and the moving voucher policy generates aggregate welfare losses of 1.3% under a utilitarian social welfare function that places equal weights on skills and cities. Alternatively, a utilitarian planner must put Pareto weights four times larger on the treated workers for the policy to be welfare neutral.

Wrapping up, moving vouchers constitute an effective redistributive policy that substantially reduces spatial learning disparities. However, these vouchers are effective only if they cover a substantial fraction of the migration costs; taking into account the general equilibrium of the policy, moving vouchers reduce aggregate inefficiency and trigger welfare losses for non-treated workers.

## 1.7 Conclusion

This chapter argues that a tradeoff exists between spatial inequality and human capital accumulation. I have shown theoretically that this tradeoff is shaped by the relative strength of within- and between-skill learning complementarities. I have documented that high-skill workers enjoy disproportionately faster wage growth when working in skill-dense cities. I have argued that this pattern reveals the presence of strong within-skill learning complementarities. Using the estimated model, I concluded that the spatial segmentation of learning opportunities increases the aggregate stock of human capital at the cost of amplified geographic disparities.

This equity-efficiency tradeoff matters for spatial policies. On the one hand, local

interactions engender learning spillovers and spatial misallocation. Place-based policies incentivizing a higher spatial concentration of skilled workers may correct the local externalities. On the other hand, the spatial concentration of skills deteriorates the lifetime opportunities of workers born in remote cities. Vouchers that help workers relocate to productive cities have the potential to reduce these inequalities at the cost of aggregate efficiency losses.

Skill complementarities are present everywhere. They shape our productivity. They matter for our preferences for cities. And finally, they define how we learn from each other. All these dimensions are sources of local spillovers. A natural direction along which to expand this research agenda is therefore to study spatial policies in the presence of static and dynamic complementarities. A second promising avenue for this agenda is to study how firms and space interact in shaping human capital accumulation; while firms may create teams that partially internalize the local learning spillovers, they are also constrained by the pool of workers they have access to.

# Chapter 2

# Outsourcing, Inequality and Aggregate Output

Coauthor: Adrien Bilal.

## 2.1 Introduction

Domestic labor outsourcing is fundamentally changing the nature of the labor market. During the last two decades, firms have been increasingly concentrating on core competencies and contracting out a vast array of labor services, such as security guards, food and janitorial services. Workers in these occupations receive much lower wages at contractor firms than at traditional employers: the outsourcing wage penalty (Dube and Kaplan, 2010, Goldschmidt and Schmieder, 2017). This relative wage gap suggests that rising domestic outsourcing redistributes away from workers. At the same time, firms scale up more efficiently by contracting out. Outsourcing may thus generate aggregate productivity and employment gains that benefit workers. Despite the prevalence of outsourcing in the labor market, the tension between its distributional and productivity effects is far from understood. How does outsourcing shape aggregate production and its split between workers and firms?

The answer to this question depends on the fundamental driver of outsourcing. The

*comparative advantage view* holds that contractor firms are more efficient at producing particular labor services because of gains from specialization in production. The comparative advantage view suggests that outsourcing raises aggregate Total Factor Productivity (TFP) as workers are reallocated towards more efficient contractors. Another perspective, the *cost-saving view*, holds that contractor firms simply enable their client firms to sidestep costly hiring and save on labor expenditures. The cost-saving view has more nuanced implications, as the reallocation of workers towards contractor firms may be neutral or even detrimental to aggregate TFP.

In this chapter, we first build a theory of domestic outsourcing that disentangles the comparative advantage view from the cost-saving view. Second, we provide new reduced-form evidence of the distributional and productivity effects of outsourcing that our theory ties together using administrative data from France. Third, we structurally estimate our model and quantify the effects of outsourcing on inequality, rent-sharing and aggregate output.

Specifically, in the first part of the chapter, we contribute a framework to study the emergence of outsourcing. We start with an environment that features three necessary ingredients, but no outsourcing yet. First, goods-producing firms are heterogeneous in productivity and have well-defined boundaries due to decreasing returns to scale in revenue. Second, not all workers are equally exposed to outsourcing. Firms hire workers of different skills who enter as imperfect substitutes in production.

Our third ingredient is key to rationalize the outsourcing wage penalty. Workers search for employment opportunities on and off the job along a wage ladder, and seemingly identical workers earn different wages at different employers. Crucially, as in Burdett and Mortensen (1998), scarce managerial time constrains hiring efforts. Managers have a limited span of control to recruit workers, and wages become an effective hiring tool. As a result, firms face an upward-sloping labor supply curve by skill as they internalize that higher pay attracts and retains more workers from competitors. Rent-sharing between workers and firms arises. We characterize wage and employment distributions in closed form. More productive firms

with a larger target size pay higher wages. Wage inequality and dispersion in labor shares and markdowns across firms emerge in equilibrium.[1]

We then introduce contractor firms in our environment. Contractor firms hire workers in the same frictional labor markets and with the same recruiting technology as goods producers. Thus, they face the same upward-sloping labor supply curve. Instead of producing a consumption good, contractor firms sell labor services of their employees in a competitive labor service market. In the aggregate, contractor firms effectively expand managerial resources available for recruiting.

Consistently with the cost-saving view, goods producers may now outsource and buy labor services at an equilibrium price. In that case, goods producers bypass costly recruiting due to labor market frictions and constraints on managerial time. Instead, they indirectly tap into additional managerial resources by purchasing labor services in the competitive service market. When they outsource, goods producers switch from an upward-sloping labor supply curve to a vertical one. Decreasing returns to scale are thus critical to ensure that firm size remains well-defined in equilibrium.

Consistently with the comparative advantage view, contractor firms may be more productive at generating labor services than goods producers, for instance because of gains from specialization. Contractor firms may also be less productive than goods producers, for instance because of additional capital costs or communication and coordination costs from excess intermediation. The relative productivity of contractor firms captures the net balance between those forces.

Three main implications emerge. First, goods producers select into outsourcing. Productive firms who pay high wages to attract and retain a large workforce have the strongest incentives to outsource and save on labor costs. Less productive firms who pay lower wages prefer to hire in-house and avoid the additional cost of compensating contractor firms.

Second, outsourcing leads to a productivity effect at the firm level. When they outsource,

[1]We use "wages" for concreteness but our theory applies equally to total compensation inclusive of benefits.

goods producers scale up because their marginal cost of labor falls. Revenues and labor demand both rise. All else equal, outsourcing thus increases output and employment.

Third, outsourcing leads to a distributional effect. Its sign crucially hinges on whether the comparative advantage or cost-saving view holds. When contractor firms have a technological advantage, they pay higher wages than goods producers. By contrast, when contractor firms are technologically neutral or even disadvantaged, they pay lower wages than the marginal outsourcing firm.

Our theory is thus uniquely suited to discriminate between the comparative advantage and cost-saving views. When the law of one price fails as in our frictional economy, wages track the marginal product of labor. Thus, the comparative advantage view is incompatible with the outsourcing wage penalty that emerges in the data. By contrast, the cost-saving view in which goods producers outsource to save on labor costs is consistent with the outsourcing wage penalty. In that case, goods producers pass on the cost of labor to contractor firms that remain at a lower point on their labor supply curve.

We strengthen our results in two ways. First, we derive additional testable implications related to the comparative advantage and cost-saving views. Our micro-foundation of firm-specific labor supply curves implies that worker flows across firms are tied to the wage ordering between contractors and goods producers. Second, we show that our results continue to hold once we introduce firm-level idiosyncratic outsourcing costs to confront the predictions of our theory with the richness of the data.

To reach these conclusions, we have required a setup that departs from constant returns to scale and perfect substitution between workers. The wage-posting literature has imposed these assumptions ever since Mortensen and Vishwanath (1991) to retain traction. We overcome this technical challenge with two sufficient conditions. First, the revenue function exhibits a single-crossing property in firm productivity and employment. This condition ensures that more productive firms always prefer to hire more and nests most standard revenue functions. Our second condition consists in a trembling-hand equilibrium refinement

that precludes non-smooth equilibria.

In the second part of the chapter, we test the novel implications of our theory using administrative data from France and rule out alternative mechanisms. We combine matched employer-employee data from employer tax returns, balance sheet records for the universe of firms, firm-level customs data and a firm-level survey that details outsourcing information. We measure outsourcing at the firm level as expenditures on external workers: workers who are not employees of the firm, but are at least partially under the legal authority of the purchasing firm. We identify contractor firms using industry codes. Our main analysis starts in 1996 and stops in 2007 due to a change in data collection procedures. Aggregate expenditures on outsourcing represent 6% of the aggregate wage bill in 1996 before rising to almost 11% in 2007. Extrapolating beyond 2007 suggests that outsourcing may represent between 11% and 20% of aggregate wages by 2016.

We first examine the distributional effect of outsourcing. Consistently with the cost-saving view, we show that contractor firms locate at the bottom of the job ladder along a number of labor market statistics. We start by confirming the outsourcing wage penalty first documented by Dube and Kaplan (2010) and Goldschmidt and Schmieder (2017), but in French data. Contractor firms pay wages that are 14% below other firms after controlling for individual worker heterogeneity. In line with our microfoundation for monopsony power, we show that contractors also hire less from employment than other firms, churn more through workers, and have negative net poaching.

Second, we demonstrate that firms select into outsourcing. We ask whether firms with large value added have a high outsourcing share as in our model. We define the outsourcing share as outsourcing expenditures out of all labor costs including outsourcing. We find a robust correlation between the outsourcing share and value added across and within firms. As highlighted by our extension with idiosyncratic outsourcing costs, a causal interpretation of this relation may be confounded by unobserved shocks such as improvements in Information Technologies (IT) at the firm level. To isolate the effect of a change in revenue productivity,

we instrument for it with shift-share export demand shocks at the firm level: we interact initial exports shares of exporters across destination markets with changes in foreign demand (Hummels et al., 2014). Our identification assumption is that changes in foreign demand are unrelated to the idiosyncratic outsourcing cost of any particular firm (Borusyak et al., 2021). Our Two Stage Least Squares (2SLS) estimate implies that a 10% increase in value added driven by quasi-exogenous foreign demand shocks leads to a 0.33 percentage points (p.p.) rise in the outsourcing share.

Third, we provide evidence for the productivity effect. We show that a decline in the cost of outsourcing leads firms to scale up. Our model indicates that we need to isolate declines in firm-level outsourcing costs from confounding changes in revenue productivity. To that end, we instrument the outsourcing share with shift-share outsourcing shocks using variation in exposure at the firm level: we interact firm-level initial occupation shares with changes in average outsourcing expenditures at the occupation level. Our identification assumption is that initial occupation shares are unrelated to subsequent changes in revenue productivity (Goldsmith-Pinkham et al., 2020). Our 2SLS estimate indicates that a 1 p.p. rise in the outsourcing share driven by quasi-exogenous outsourcing cost shocks leads to a 7.9% rise in value added.

We rule out several alternative mechanisms that could explain our findings. Outsourcing could arise because it provides more flexibility and helps firms insure against demand volatility. We show that more volatile industries use less outsourcing, not more.[2] Firms could outsource to alleviate equity concerns. We show that firms with more unequal pay structures outsource less, not more.[3] Selection into outsourcing could emerge in part because of union wage-setting or size-dependent policies. With a regression discontinuity design, we show that the major threshold (50 employees) for union participation and policies in France indeed generates bunching, but no detectable jump in outsourcing.

[2]At the firm level, more volatility is positively correlated with outsourcing, but the relationship is three times weaker than the effect of firm scale that we highlight.

[3]Our results are consistent with equity concerns (e.g. Card et al., 2012, Breza et al., 2017) since equity concerns primarily bind for workers within the same occupation, rather than across occupations.

In the third part of the chapter, we develop and structurally estimate a quantitative version of the framework. The main addition is a flexible curvature in the recruiting cost function of goods producers and contractor firms. This extension captures that firms may expand their in-house human resources department at some cost and hire in-house without raising wages too rapidly. Our cost-saving channel of outsourcing then operates as long as there is some curvature in this cost function. We also let contractors have a comparative advantage in recruiting to match their relative size together with the outsourcing wage penalty. We use three skill types. High skill and core low skill workers are shielded from outsourcing. Only low skill service workers are exposed to outsourcing.

We estimate the model with a Method of Simulated Moments (MSM) estimator and target cross-sectional moments only. We infer from outsourcing expenditure and employment shares that workers at contractor firms are 35% less efficient than in-house workers. This result falsifies the comparative advantage view but is consistent with communication and monitoring frictions lowering the efficiency of outsourced relative to their in-house counterparts. We also infer that contractors are twice as efficient at recruiting workers than goods producers from their relative size. We interpret this result as the core activity of contractors being to screen and recruit workers. Thus, the data favors a view in which contractor specialize in recruiting activities rather than in production activities.

We support our inference by confirming that our estimated model matches non-targeted moments. It reproduces our reduced-form within-firm estimates for selection into outsourcing and the productivity effect. It also replicates residual wage inequality and differences in worker flows between contractors and goods producers. The distribution of firm-level labor shares is a key metric of rent-sharing and lines up in the model and in the data.

We then quantify the race between the productivity and the distributional effects of outsourcing in the aggregate. Our main counterfactual changes both the demand and supply of outsourcing such that outsourcing expenditures track the rise in France between 1997 and

2016.[4] We estimate demand and supply shocks by matching industry-level panel regressions of value added, wages and employment onto the outsourcing share using data from 1997 to 2007. Supply shocks such as improvements in the recruiting technology of contractor firms rationalize simultaneous employment increases and wage declines. Demand shocks additionnally rationalize rising value added. We interpret this combination of demand and supply shocks as improvements in information technology that facilitate standardization and monitoring of tasks between goods producers and contractor firms.

Our first result is that low skill service workers are worse off because of outsourcing. We focus on changes in expected earnings as they coincide with welfare in our environment. We find that expected earnings of low skill service workers fall by nearly 1.5% by 2016. We unpack this result into partial and general equilibrium channels.

The first, partial equilbrium channel is that service workers are reallocated towards contractor firms, where their employment share rises by 21 p.p. by 2007. Despite being non-targeted, this large reallocation tracks the data reasonably well (16 p.p.). Together with the 14% outsourcing wage penalty, this reallocation drives a 1.5% decline in expected earnings for low skill service workers by 2007. By 2016, earnings fall by 2.8%.

General equilibrium channels partly offset this decline. The productivity effect materalizes as substantial employment gains for low skill service workers, leading to a 5% rise in earnings by 2016. Further, as contractors expand, competition at the bottom of the job ladder increases and pushes up wages at low-paying firms. Yet, outsourcing rarefies high-paying job opportunities, and expected earnings decline by another 3.8%. Our decomposition highlights that reduced-form approaches that only pick up the first, partial equilibrium impact miss key general equilibrium adjustments and overstate welfare losses of service workers.

In the aggregate however, the economy benefits from outsourcing. Output rises by 6.1% due to the productivity effect. This increase is entirely driven by employment gains. Perhaps surprisingly, Total Factor Productivity (TFP) declines by 2%. We use an exact TFP

[4]We use 15% as our 2016 target as it lies in the middle of the range from our extrapolation.

decomposition to understand this reduction. In the aggregate, outsourcing increases TFP by 7% holding effective labor fixed: outsourcing increases allocative efficiency among goods producers by increasing labor demand at firms that were most constrained by labor market frictions and had a high marginal product of labor. Yet, consistently with the cost-saving view, outsourcing also lowers TFP by 9% by reallocating workers towards less efficient contractors. By contrast, the comparative advantage view would imply that both effects are positive, crucially overstating aggregate TFP gains.

Our environment is uniquely equipped to connect the welfare and productive effects of outsourcing to labor market sorting and inequality (Song et al., 2018). We find that outsourcing leads to rising labor market sorting but relatively stable wage inequality. The reallocation of low skill service workers towards low-paying contractors increases labor market sorting. The correlation between worker and firm wage premia rises by 23 p.p. in the model. Despite this increase, the pro-competitive effects of contractors raise goods producer wages for service workers at the bottom of the job ladder by over 10%. Thus, the wage gap between low skill service and high skill workers conditional on the same employer shrinks. This general equilibrium feedback offsets the impact of sorting on wage inequality which remains stable.

Inequality between workers and firm shareholders rises, however, as the share of rents going to workers shrinks. In our monopsony environment, the share of the marginal product of labor that workers receive—the markdown—is determined in equilibrium.[5] Markdowns fall for service workers as the highest paying firms switch to outsourcing. In principle, core and high skill workers could still benefit from outsourcing because of complementarities in production and the increase in service labor demand. Nevertheless, despite an increase in their marginal product of labor, core and high skill workers do not gain from outsourcing because their markdowns fall too. As a result, firm profits rise by 2%. Our results highlight the key role of equilibrium adjustments in markdowns in the transmission of productive gains from outsourcing to workers.

[5]We specify the close connection between markdowns and firm-level labor shares in Section 2.6.3.

We conclude this chapter by asking whether simple labor market policies can mitigate expected earnings losses due to worker reallocation while maintaining most of the employment and output gains. A 5.5% increase in the minimum wage between 1997 and 2016 ensures that expected earnings of low skill service workers remain constant. Output gains are nearly identical to the baseline increase. Thus, minimum wage reforms can ensure that outsourcing benefits low skill service workers while preserving the productivity effects of outsourcing.

This chapter relates to several strands of literature. The first is the rapidly expanding empirical literature that studies the distributional and productivity effects of outsourcing. Dube and Kaplan (2010), Goldschmidt and Schmieder (2017), Dorn et al. (2018) and Drenik et al. (2020) document that domestic outsourcing is on the rise and that outsourced workers experience wage declines in the U.S., Germany and Argentina, respectively. Abraham and Taylor (1996) provide an early discussion of the cost-saving and comparative advantage views.[6] Segal and Sullivan (1997), Katz and Krueger (2017) and Katz and Krueger (2019) document a rise in alternative work arrangements in the U.S.[7] Bertrand et al. (2020) show that an increase in the supply of contract labor helps Indian firms scale up, and Bostanci (2021) highlights the tension between trade secret protection and the productivity effect of outsourcing. Munoz (2022) documents the related role of posted workers, although their employment share of 0.5% remains well below that of domestic contractors of 5-9%. We contribute to this literature by providing a unified general equilibrium theory of outsourcing in which we disentangle the comparative advantage from the cost-saving views, tie the productivity and distributional effects together, and analyze the trade-off between both forces in the aggregate.[8]

Second, this chapter relates to the outsourcing literature that focuses on the make-or-buy

[6]We contrast our results for selection into outsourcing and demand volatility with the ones in Abraham and Taylor (1996) in Sections 2.4.4 and 2.4.6 respectively.

[7]Giannoni and Mertens (2019) emphasize the impact of outsourcing on the labor share in the U.S. Bergeaud et al. (2020) highlight that internet broadband expansion leads firms to concentrate on their core activities in France. Relatedly, LeMoigne (2020) highlights that the consequences of fragmentation events for workers resemble those of outsourcing events. See Handwerker (2021) for a similar idea in the U.S. and Bernhardt et al. (2016) for a review of the earlier literature on outsourcing in the U.S.

[8]Since this paper was first circulated, two papers subsequently emphasized related points. Spitze (2022) studies wages and benefits for low and high skill outsourced workers in the U.S. Felix and Wong (2021) provide evidence for the productivity effect in Brazil.

choice that firms face (Grossman and Hart, 1986, Hart and Moore, 1990, Grossman and Helpman, 2002). Our theory defines the boundary of the firm in product markets through decreasing returns, but requires that goods producers cannot take ownership of contractor firms as a whole. By abstracting from the particular frictions at work in ownership markets, our theory uniquely delivers specific interpretations of the labor market consequences of domestic outsourcing that have been recently documented, together with aggregate efficiency implications.

Third, this chapter connects to the literature studying how labor market frictions give rise to factor price dispersion and misallocation. We contribute to the wage-posting monopsony tradition (Burdett and Mortensen, 1998, Berg and Ridder, 1998, Bontemps et al., 2000, Engbom and Moser, 2021, Heise and Porzio, 2022) by providing sufficient conditions to depart from constant returns to scale and perfect substitutability between workers in production which has remained an open problem since at least Mortensen and Vishwanath (1991). As in Gouin-Bonenfant (2022), our monopsony environment delivers dispersion in markdowns and labor shares across firms that is consistent with evidence in Autor et al. (2020).[9]

Finally, this chapter relates to the literature on trade in intermediate inputs and international offshoring (Feenstra and Hanson, 1999, Antràs, 2003, Grossman and Helpman, 2005, Grossman and Rossi-Hansberg, 2008, Acemoglu et al., 2015, Antràs et al., 2017). When firms trade intermediate inputs, they contract on a physical good. When firms outsource domestically, they contract on the flow of services of a worker, thereby leading to distinct implications for wage inequality. When firms offshore internationally, they take advantage of lower wages in other countries. Domestic outsourcing reflects similar forces, but requires first to break the law of one price in the domestic labor market.

[9]A recent influential literature has developed models based on extreme-value distributed compensating differentials (Card et al., 2018, Sorkin, 2018, Berger et al., 2022, Lamadon et al., 2022). These frameworks are tractable and relevant to study market-level effects of inequality and mobility. This tractability comes with two caveats that are not well-suited to study outsourcing. These models imply welfare equalization across firms within markets, making the connection between inequality and welfare difficult. They also generate constant within-market markdowns and labor shares under standard functional forms, making it difficult to rationalize any dispersion in labor shares across firms. We show that this dispersion is key to understand the impact of outsourcing on inequality in Section 2.6.3. See Online Appendix B.6.2 for a detailed discussion.

The rest of this chapter is organized as follows. Section 2.2 lays out the basic framework without outsourcing. Section 2.3 introduces outsourcing in the economy. Section 2.4 details the reduced-form results supporting our theory. Section 2.5 lays out the quantitative extensions of the model and the structural estimation. Section 2.6 presents our counterfactuals. The last section concludes. Proofs and further details can be found in the Appendix and the Online Appendix.

## 2.2 A theory of wage premia with large firms

### 2.2.1 Setup

Time is continuous, and we focus on a steady-state equilibrium. There is a unit measure of workers. Each worker is characterized by its exogenous and permanent skill type $s \geq 0$. Types are distributed in the population according to the measure $m_s ds$ with respect to a base measure denoted by $ds$.[10] Workers have linear preferences in income, inelastically provide one unit of labor per time period, and discount future utility at rate $r$. They can be either employed or unemployed, in which case they earn skill-specific unemployment benefits $b_s$.

A measure $M^G > 0$ of goods-producing firms populates the economy. Firms are indexed by productivity $z$ with support $[\underline{z}, \overline{z}]$. The corresponding cumulative distribution function $\Gamma$ admits a finite and continuous density. For simplicity, $\underline{z}$ is large enough relative to $\sup_s b_s$ so that all matches are viable. A firm with productivity $z$ that hires a measure $n_s$ of workers of each skill $s$ generates revenue $R(z, \boldsymbol{n})$, where $\boldsymbol{n} = \{n_s\}_s$ denotes the vector of employment across worker types. $R$ is twice continuously differentiable and increasing in each argument.

Labor markets are segmented by skill $s$. Unemployed workers of skill $s$ sample wage offers randomly at Poisson intensity $\lambda_s^U$. Employed workers of skill $s$ sample wage offers with intensity $\lambda_s^E \leq \lambda_s^U$ from the same distribution. Employed workers can break their current

[10] This notation allows us to capture both continuous and discrete type distributions without loss of generality.

contract to accept a new wage offer. Existing matches are destroyed at Poisson rate $\delta_s$.

Firms optimally post wage offers in every skill-specific labor market to attract and retain workers. As in Burdett and Mortensen (1998), firms commit to a single, fixed, and non-state-contingent wage by skill. Wages cannot be renegotiated throughout employment spells. Every firm is endowed with a unit measure of managerial time to devote to recruiting activities ("vacancies") for every skill $s$ to which they attach the same skill-specific wage offer.

### 2.2.2 The labor supply curve

To understand the skill-specific labor supply curve faced by each firm, we must first characterize the job search behavior of workers. This subsection follows closely Burdett and Mortensen (1998). Given the equilibrium distribution of wage offers for skill $s$, denoted $F_s(w)$, the value of unemployment and the value of being employed at a given wage $w$ satisfy:

$$\begin{aligned} rU_s &= b + \lambda_s^U \int \max\{V_s(w) - U_s, 0\} dF_s(w) \\ rV_s(w) &= w + \lambda_s^E \int \max\{V_s(w') - V_s(w), 0\} dF_s(w') + \delta_s(U_s - V_s(w)). \end{aligned}$$

The value of being employed at wage $w$, $V_s(w)$, is increasing with the wage $w$, so that workers behave as income maximizers: they always accept higher wage offers while employed.

The movement of workers up the job ladder determines the skill-specific labor supply curve faced by each firm. To characterize it, we solve for the equilibrium distribution of wages of employed workers $G_s(w)$. By equating inflows and outflows of workers in each wage interval, we relate the wage offer distribution $F_s(w)$ to the wage distribution of employed workers $G_s(w)$ in Appendix B.1.1. We obtain

$$G_s(w) = \frac{F_s(w)}{1 + k_s(1 - F_s(w))}, \qquad k_s = \frac{\lambda_s^E}{\delta_s}. \tag{2.1}$$

From equation (2.1) we characterize the number $N_s(w)$ of employed workers per wage offer

$w$ for every skill $s$:

$$N_s(w) = \frac{(1+k_s)e_s}{\big(1+k_s(1-F_s(w))\big)\big(1+k_s(1-F_s(w^-))\big)}, \tag{2.2}$$

where $e_s = \frac{\lambda_s^U m_s}{\delta_s + \lambda_s^U}$ is the measure of employed workers of skill $s$, and $F_s(w^-)$ denotes the left-limit of $F_s$ at $w$.

Crucially, the labor supply curve $N_s(w)$ is non-decreasing in the wage $w$, with a slope that depends on the equilibrium distribution of wage offers in the economy, $F_s(w)$. We turn to the decision problem of firms to characterize this distribution.

### 2.2.3 Wage and employment distributions

The number $n_s(w)$ of workers per firm posting wage $w$ is simply related to the number of workers employed at every wage by $n_s(w) = N_s(w)/M^G$ since firms have a unit measure of managerial time to devote to recruiting. When the discount rate is low enough, firms choose their wage offers $\{w_s(z)\}_s$ to maximize their flow profits.[11] Firms internalize their skill-specific labor supply curves: they attract and retain more workers if they pay more. Flow profits are given by:

$$\begin{aligned} \pi(z) \quad &= \quad \max_{\{w_s, n_s\}_s} R\big(z, \{n_s\}_s\big) - \int w_s n_s ds \\ &\text{s.t.} \quad n_s \leq n_s(w_s) = \frac{N_s(w_s)}{M^G}. \end{aligned} \tag{2.3}$$

Unless the distribution $F_s(w)$ can be characterized more precisely, the problem in equation (2.3) is intractable in general equilibrium. The wage-posting literature—from Burdett and Mortensen (1998) to Engbom and Moser (2021)—has leveraged a key simplifying assumption to make progress. Under constant returns and perfect substitutability of workers in production, $R(z, \boldsymbol{n}) = z \int n_s ds$, the problem (2.3) can be split at the match level. Once decoupled across

[11] We derive the formulation in equation (2.3) from the dynamic problem of the firm in Online Appendix B.6.1.

matches, it is straightforward to see that (2.3) exhibits a single-crossing property. This structure implies that wages are increasing in productivity $z$, which in turn allows to solve for the distribution of wage offers in terms of the equilibrium wage policy and the exogenous productivity distribution, $F_s(w_s(z)) = \Gamma(z)$.

Outsourcing however requires a well-defined boundary of the firm as well as possible interactions between workers in production. Handling these features has remained an open problem since Mortensen and Vishwanath (1991) who pointed out that the usual arguments for monotone wages and uniqueness do not apply. We overcome the challenges that come with this departure from linearity with two sufficient conditions. Our first and main sufficient condition imposes minimal structure on the revenue function $R$ that lets us rank wages by firm productivity.

**Assumption 2.1.** $(z, \boldsymbol{n}) \mapsto R(z, \boldsymbol{n})$ *is strictly supermodular in all its arguments.*

Given that $R$ is twice continuously differentiable, Assumption 2.1 is equivalent to imposing strictly positive cross-derivatives between all arguments. It amounts to a form of complementarity between productivity and every labor type, as well as between any two types of labor. Assumption 2.1 ensures that more productive firms prefer to hire more workers of every type.

Importantly, the complementarities built in stand in productivity and employment levels, as opposed to the usual notion of complementarity between worker types that stands in proportions. Our supermodularity assumption is thus compatible with a wide class of revenue functions and allows for workers to be complements or substitutes in production in the usual sense.

For instance, consider the revenue function $R(z, \boldsymbol{n}) = z \left( \int (a_s n_s)^{1-\frac{1}{\eta}} ds \right)^{\frac{\eta(\sigma-1)}{\sigma(\eta-1)}}$. Such a revenue function arises when workers have CES demand over $M^G$ differentiated varieties with elasticity of substitution $\sigma > 1$, and firms produce with a CES production function with elasticity of substitution $\eta$ between skills. This revenue function also arises if there are technological decreasing returns to scale in production. Supermodularity then requires $\sigma > \eta$.

By comparing the curvature in the revenue function to the substitutability between worker types, this condition ensures that the marginal revenue gain from rising employment of one skill type does not incentivize the firm to lower employment of another skill type. Since typical estimates of $\sigma$ lie above 3 to 5, while most estimates of $\eta$ lie below 2, the condition for supermodularity is compatible with standard parametrizations.

We impose Assumption 2.1 in the remainder of this chapter. We first show that Assumption 2.1 guarantees existence and uniqueness of an equilibrium with a simple structure in the class of equilibria with continuous wage distributions. We then discuss how our second sufficient condition—an equilibrium refinement concept—ensures uniqueness among all possible equilibria.

Proposition 2.1 shows that more productive firms post higher wages.

**Proposition 2.1** (Wage ranking)**.** *Consider any equilibrium with continuous wage offer distributions $\{F_s(w)\}_s$. Wages $w_s(z)$ are strictly increasing with firm productivity $z$. The wage function is continuous in $z$. The wage offer distributions satisfy $F_s[w_s(z)] = \Gamma(z)$ for all $s$.*

*Proof.* See Appendix B.1.3. □

With Proposition 2.1 at hand, the distribution of workers across firms is fully determined.

**Proposition 2.2** (Employment distribution)**.** *Consider any equilibrium with continuous wage offer distributions $\{F_s(w)\}_s$. The number of workers of skill $s$ hired by firm $z$ is given by*

$$n_s(z) \equiv n_s(w_s(z)) = \frac{(1+k_s)e_s}{M^G[1+k_s(1-\Gamma(z))]^2}.$$

*Proof.* See Appendix B.1.4. □

Firm size in Proposition 2.2 depends only on the ranking of firms, $\Gamma(z)$, because firm size is fully determined by worker flows up the job ladder.[12] Building on Propositions 2.1 and 2.2, we solve explicitly for the wage distribution.

[12]We let firms choose how much recruiting effort to exert—or equivalently, how many vacancies to post—in Section 2.5, so that firm size also reflects the marginal product of labor.

**Proposition 2.3** (Wage distribution)**.** *Consider any equilibrium with continuous wage offer distributions* $\{F_s(w)\}_s$*. Wages are given by*

$$w_s(z) = \underline{w}_s \frac{n_s(\underline{z})}{n_s(z)} + \int_{\underline{z}}^{z} \frac{\partial R}{\partial n_s}\big(z, \boldsymbol{n}(x)\big) \frac{n_s'(x)dx}{n_s(z)}.$$

*Proof.* See Appendix B.1.5. □

Proposition 2.3 captures the logic of the job ladder. Productive firms raise their wages to poach workers from lower-productivity firms in order to attain their target size. The equilibrium value of a worker to these lower-productivity firms is given by their marginal product of labor $\frac{\partial R}{\partial n_s}$. Competitive wage pressure for a firm with productivity $z$ then builds up from below. Wages at a firm with productivity $z$ are pushed up, starting from the reservation wage $\underline{w}_s$ given in Appendix B.1.2, and integrating up to productivity $z$. The resulting wage function is a weighted average of the marginal product of labor at lower-ranked firms. Proposition 2.3 nests the wage equations with linear revenue in e.g. Burdett and Mortensen (1998) and Engbom and Moser (2021).

Proposition 2.3 characterizes wages having assumed that the wage offer distribution is continuous. Propositions 2.1 to 2.3 imply that the wage offer distribution that results from the choices of firms is consistent with continuity. Hence, we have guessed and verified existence of an equilibrium with a smooth wage offer distribution. As shown by Mortensen and Vishwanath (1991), wage-posting models with decreasing returns to scale can however exhibit multiple coordination equilibria due to the emergence of mass points.[13]

Our second sufficient condition is a trembling-hand equilibrium refinement concept to overcome equilibrium multiplicity. If firms make small mistakes in their wage-setting policy, no mass point can arise. When dispersion in mistakes vanishes asymptotically so that we recover the maximization problem in (2.3), the only equilibrium that survives is the one with a smooth wage distribution. We formalize our trembling-hand refinement in Appendix B.1.6

[13] If a positive measure of firms coordinates on exactly the same wage, it may be optimal for other firms to post that same wage since deviating away from that mass point would imply too large a change in size given decreasing returns to production. Thus, equilibria with a smooth wage distribution may in principle co-exist with equilibria with mass points.

and call it Assumption B.1.

**Proposition 2.4** (Existence and uniqueness)**.** *There exists a unique equilibrium among equilibria with a continuous wage offer distribution, described in Propositions 2.1-2.3. Under Assumption B.1, this equilibrium is unique among all possible equilibria.*

*Proof.* See Appendix B.1.6. □

## 2.3 A theory of outsourcing

Having characterized the emergence of wage premia across firms in our baseline economy, we enrich our basic environment with contractor firms that provide outsourcing services and describe their impact on the economy.

### 2.3.1 Contractor firms

We introduce a measure $M_s^C \geq 0$ of identical contractor firms in each skill market $s$. To make the distinction clear, we now call firms that produce a consumption good 'goods producers.'

Contractor firms hire workers in the same frictional labor markets as goods producers. They also post wages, and do so with the same recruiting technology as goods producers: every contractor firm is endowed with a unit measure of managerial time.

A given contractor firm hires in a single skill market $s$. Instead of producting a consumption good, contractor firms produce labor services with workers. Contractor firms sell labor services at price $p_s$ in perfectly competitive rental markets. We endow contractors with constant returns to scale in production to make their production technology as close as possible to goods producers.[14] Consistently with the comparative advantage view, contractors may be either better or worse than goods producers at producing labor services. Contractors have comparative advantage $\tau_s \lesseqgtr 1$ relative to goods producers.

[14]Goods producers effectively transform workers in labor services in-house with constant returns, and then combine labor services to produce output with possibly decreasing returns.

Contractor thus solve the profit-maximization problem:

$$\pi_s^C = \max_w \big(\tau_s p_s - w\big) n_s(w). \tag{2.4}$$

We propose three micro-foundations for our measure of comparative advantage $\tau_s$, detailed in Appendix B.1.7. Regardless of the micro-foundation, the comparative advantage $\tau_s$ is an exogenous parameter that captures how costly it is to outsource workers. When contractors are weakly worse than goods producers $\tau_s \leq 1$, our first micro-foundation has $\tau_s$ reflect either a productivity wedge or the inverse of an iceberg trade cost between contractor firms and goods producers. This productivity wedge or trade cost captures the idea that communication, monitoring and coordination between the goods producer and outsourced workers may be more difficult when workers are employees of another firm. As a result, some efficiency units of labor are lost. In our second micro-foundation, contractor firms combine a small amount of capital and labor according to a Cobb-Douglas production function. $\tau_s$ then simply encapsulates the price of capital. Third, $1/\tau_s$ may be interpreted as a markup charged by contractor firms. When contractors are weakly better than goods producers $\tau_s \geq 1$, $\tau_s$ reflects increasing returns to scale capturing that contractors acquire an advantage by specializing in certain labor services. Whether $\tau_s$ is above or below 1 encapsulates the net balance between those forces.

### 2.3.2 Goods producers and outsourcing

Consistently with the cost-saving view, traditional firms now face an additional possibility: rent labor services from contractors. They may still hire workers in-house in a frictional labor market. Their decision problem becomes

$$\begin{aligned} \pi(z) \quad = \quad & \max_{\substack{\{n_s\}_s, \{w_s\}_s, \\ \{o_s\}_s \in \{0,1\}^S}} R\big(z, \{n_s\}_s\big) - \int \Big[(1-o_s)w_s + o_s p_s\Big] n_s ds \\ \text{s.t.} \quad & n_s \leq n_s(w_s) \text{ if } o_s = 0. \end{aligned} \tag{2.5}$$

The indicators $o_s \in \{0, 1\}$ specify whether a goods producer outsources skill $s$. $n_s$ denotes in-house labor if $o_s = 0$, and denotes outsourced labor if $o_s = 1$.[15] For their problem to be well-defined, we require that $R$ is strictly concave in $\boldsymbol{n}$.

If the goods producer hires in-house ($o_s = 0$), it effectively faces an upward-sloping labor supply curve embedded in the function of $n_s(w)$. Thus, a highly productive goods producer with a large target size $n_s$ moves up its labor supply curve and pays high wages in-house. In contrast, if the goods producer outsources ($o_s = 1$), it faces a vertical labor supply curve at price $p_s$. In that case, outsourcing is more advantageous due to the upward-sloping labor supply curve $n_s(w)$: goods producers have a strong incentive to outsourcing and save on costs.

However, when a goods producer is unproductive and targets a small size $n_s$, it moves down its supply curve. The price of outsourcing $p_s$ then exceeds in-house wages since it reflects both the wage paid to employees of contractor firms as well as compensation to contractors. Outsourcing is then less attractive than hiring in-house. We formalize this discussion in Proposition 2.5 below. In what follows we always consider an equilibrium in which there is some outsourcing.

**Proposition 2.5** (Selection into outsourcing)**.** *There exists a threshold productivity $\hat{z}_s$, such that outsourcing of skill $s$ occurs if and only if $z \geq \hat{z}_s$.*

*Proof.* See Appendix B.1.8. □

Proposition 2.5 states that only the most productive goods producers in the economy outsource, precisely because they otherwise have the highest labor costs. When productivity crosses the outsourcing threshold productivity $\hat{z}_s$, goods producers switch from a convex to a linear cost curve. As opposed to most models of firm selection that require fixed costs (Melitz, 2003), selection into outsourcing arises because of differences in the shape of two possible variable cost functions. Switching between cost functions lets firms expand their activities, as described in Proposition 2.6 below.

[15] For simplicity of exposition, we require that goods producer do not mix in-house and outsourced employment for a given skill $s$. None of our results are materially affected if we lift this restriction.

**Proposition 2.6** (Productivity effect)**.** *Revenues and labor demand increase upon outsourcing:* $R(\hat{z}_s^+, \boldsymbol{n}^*(\hat{z}_s^+)) > R(\hat{z}_s^-, \boldsymbol{n}^*(\hat{z}_s^-))$ *and* $n_s^*(\hat{z}_s^+) > n_s^*(\hat{z}_s^-)$.

*Proof.* See Appendix B.1.9. □

Proposition 2.6 encapsulates the productivity effect of outsourcing. Outsourcing benefits goods producers by letting them expand up to their preferred scale. Combined with Proposition 2.5, these observations imply that outsourcing effectively reallocates labor to the most productive firms in the economy. Those firms were precisely under-sized absent outsourcing due to constraints on managerial time. Outsourcing helps firms bypass those managerial constraints and improves the allocation of labor in the economy.

However, outsourcing also changes the wage structure in the labor market, with distinct implications for inequality. How the wage structure reacts to outsourcing in turn depends on whether contractors have a comparative advantage or disadvantage.

**Proposition 2.7** (Distributional effect)**.**

*(a) When contractors do not have a comparative advantage* $\tau_s \leq 1$*, they pay lower wages than the marginal goods producer: for any contractor wage* $w_s^{\text{cont.}}$*,* $w_s^{\text{cont.}} < w_s(\hat{z}_s) \leq p_s$*. If, in addition, contractors face free entry* $\pi^C = 0$*, they all pay the reservation wage* $\underline{w}_s$*.*

*(b) Suppose that there is a single worker type and that* $0 < \underline{\rho} \leq -\frac{nR_{nn}}{R_n} \leq \overline{\rho} < 1$*. When contractors have a high enough comparative advantage* $\tau_s \gg 1$*, they pay higher wages than the marginal goods producer* $w_s^{\text{cont.}} > w_s(\hat{z}_s)$*, with* $w_s(\hat{z}_s) \leq p_s$*.*

*Proof.* See Appendix B.1.10. □

To understand the forces at work in Proposition 2.7, consider the neutral case $\tau_s = 1$ first. In-house firms at $\hat{z}_s$ equate the marginal product of labor to their marginal cost, that in turn encapsulates the upward-sloping labor supply curve. By contrast, outsourcing firms at $\hat{z}_s$ equate the marginal product of labor to the price of labor services $p_s$ on a vertical labor supply curve. Thus, outsourcing lowers the marginal product of labor of firms down to $p_s$. But the price of outsourcing services $p_s$ is also the marginal product of labor for contractor firms. Hence, the marginal product of labor at contractors firms is below the marginal product of labor at in-house goods producers at $\hat{z}_s$.

As described in Proposition 2.3, wages track the marginal product of labor. Although this co-movement includes a non-constant monopsony markdown, comparing marginal products of labor is enough to compare wages. We conclude that contractor firms pay lower wages than in-house firms that would also outsource at the margin.

Next consider the strict technological disadvantage $\tau_s < 1$. Technological disadvantage only reduces the marginal product of labor of contractors relative to the marginal outsourcing firm at $\hat{z}_s$. This technological disadvantage further widens the gap between contractor wages and in-house marginal firms. When contractors instead have a large enough comparative advantage $\tau_s \gg 1$, their marginal product outstrips that of goods producers and so they pay higher wages.

This basic result captures the distributional effect of outsourcing. When a goods producer at $\hat{z}_s$ decides to outsource for idiosyncratic reasons, and its workers transition to contractor firms, they experience a discrete wage change. In the weakly neutral case $\tau_s \leq 1$, this wage change is a wage penalty, as in Goldschmidt and Schmieder (2017). In the aggregate, the labor reallocation that follows a rise in outsourcing depresses earnings by moving workers away from the highest-paying goods producers and towards low-paying contractor firms.

The magnitude of earning losses depends on the exact wages paid by contractors. Although we empirically measure and structurally match the outsourcing wage penalty in the following sections, we can also make progress by imposing additional structure. A natural although perhaps strong assumption is that competition between contractors exhausts all rents—it may be easier to replicate a contracting agency than a blueprint for a particular consumption good. When this stiff competition materializes as a free-entry condition, contractors can only sustain their business by paying the lowest wage in the economy, the reservation wage $\underline{w}_s$. In that case, the outsourcing wage penalty is maximal.

Regardless of comparative advantage and entry, outsourcing always removes the best-paying opportunities in the labor market. Because no goods producer finds it profitable to hire a worker above the price of outsourcing, in-house wages are capped by $p_s$.

Our micro-foundation of monopsony power and the labor supply curve ties together wages and worker flows and delivers additional testable implications. Labor market frictions imply that workers flow towards high-paying firms. Thus, the wage ordering between contractors and goods producers from Proposition 2.7 translates immediately into an ordering for several other labor market statistics. We define the fraction of hires from employment of any firm paying wage $w$ as

$$\mathrm{HE}_s(w) = \frac{q_s(1-\phi_s)G_s(w)}{q_s\big[\phi_s + (1-\phi_s)G_s(w)\big]}, \tag{2.6}$$

where $\phi_s = \frac{\lambda_s^U u_s}{\lambda_s^U u_s + \lambda_s^E(1-u_s)}$ is the aggregate fraction of hires from unemployment, and $q_s$ denotes the vacancy fill rate. The churn rate—or total separation rate—of a firm paying $w$ is the sum of separations to unemployment and employment:

$$\mathrm{Churn}_s(w) = \frac{\big[\delta_s + \lambda_s^E(1-F_s(w))\big]n_s(w)}{n_s(w)}. \tag{2.7}$$

Churn measures how much firm $w$ turns its workers over to maintain a stable size. We also define net poaching of a firm paying wage $w$ as the difference between the hire rate from employment and the quit rate to employment:

$$\mathrm{NP}_s(w) = \frac{q_s(1-\phi_s)G_s(w) - \lambda_s^E(1-F_s(w))n_s(w)}{n_s(w)}. \tag{2.8}$$

Net poaching is a commonly used revealed preference statistic to assess how attractive a firm appears to workers (Sorkin, 2018, Haltiwanger et al., 2018, Bilal et al., 2022). Proposition 2.8 characterizes how contractors compare to goods producers along all those labor market statistics.

**Proposition 2.8** (Labor market statistics of contractors)**.**

*(a) When contractors do not have a comparative advantage $\tau_s \leq 1$, they hire less from employment, have higher churn and lower net poaching than the marginal goods producer:* $\mathrm{HE}_s^{\text{cont.}} < \mathrm{HE}_s(\hat{z}_s)$, $\mathrm{Churn}_s^{\text{cont.}} > \mathrm{Churn}_s(\hat{z}_s)$ *and* $\mathrm{NP}_s^{\text{cont.}} < \mathrm{NP}_s(\hat{z}_s)$.

*(b) Suppose that there is a single worker type and that* $0 < \underline{\rho} \leq -\frac{nR_{nn}}{R_n} \leq \overline{\rho} < 1$*. When contractors have a high enough comparative advantage* $\tau_s \gg 1$*, they hire more from employment, have lower churn and higher net poaching than the marginal goods producer:* $\text{HE}_s^{\text{cont.}} > \text{HE}_s(\hat{z}_s)$*,* $\text{Churn}_s^{\text{cont.}} < \text{Churn}_s(\hat{z}_s)$ *and* $\text{NP}_s^{\text{cont.}} > \text{NP}_s(\hat{z}_s)$*.*

*Proof.* See Appendix B.1.11. □

Proposition 2.7 ranks wages of contractors and wages of the marginal goods producer, and so our labor market statistics are also ranked. In the weakly neutral case, contractors hire less from employment than the marginal firm, have higher churn, and have lower net poaching. The comparative advantage case reverses these predictions. All these comparisons are the outcome of the job ladder that emerges in equilibrium.

Finally, the ranking between goods producers and contractors on the job ladder translates into size differences. In the weakly neutral case $\tau_s \leq 1$, contractors pay less than the marginal goods producer. Because both face the same labor supply curve (2.2), contractors have fewer employees than the marginal goods producer. In the comparative advantage case $\tau_s \gg 1$, contractors have more employees than the marginal goods producer. We return to this comparison in Section 2.5, where we enrich our environment to jointly match wage and size differences between contractors and goods producers.

Together, Propositions 2.5 to 2.8 characterize the key tension between the productivity and distributional effects of outsourcing. The next section describes how these forces determine the equilibrium wage distribution.

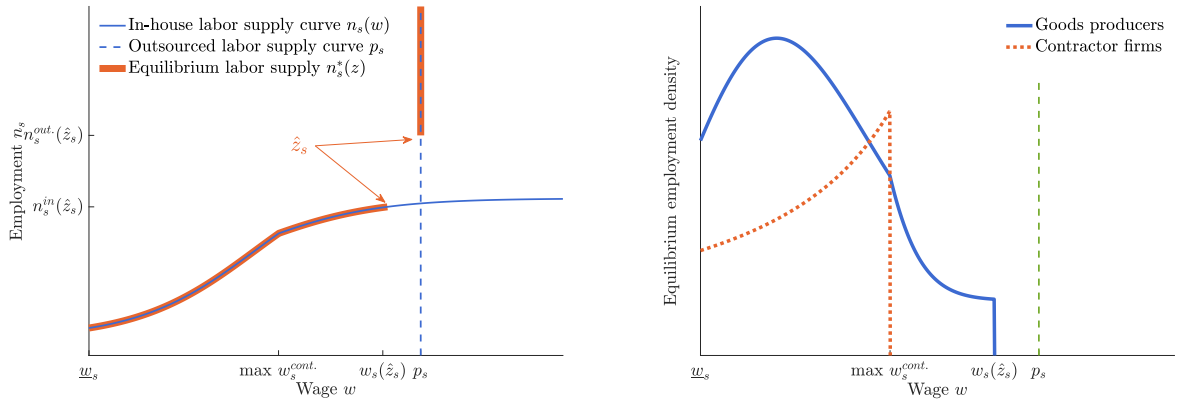
### 2.3.3 Outsourcing equilibrium

To close the description of our economy, we determine the price of outsourced labor services $p_s$ for each skill $s$. It is pinned down by the market clearing condition:

$$M^G \int_{\hat{z}_s}^{\overline{z}} n_s^*(z) d\Gamma(z) = \tau_s M^C \int n_s^{\text{cont.}}(w) dF_s(w). \tag{2.9}$$

Figure 2.1: Labor supply and wage distributions in equilibrium.

(a) Labor supply. (b) Wage distributions.




Note: Panel (a): in-house, outsourced and equilibrium labor supply curves of goods producers. Panel (b): equilibrium wage distributions of contractors and goods producers.

With some outsourcing, the equilibrium has several regions. For brevity, we describe the simplest structure of the equilibrium in this section, and provide a full description in Appendix B.1.12. We focus on the empirically relevant weakly neutral case $\tau_s \leq 1$ and when some goods producers and some contractors operate at the reservation wage. Figure 2.1 describes the equilibrium.

In the first, low wage region, goods producers operate in-house. Their labor supply coincides with the in-house supply curve as shown in Figure 2.1a. In this region, goods producers compete with contractor firms as shown in Figure 2.1b. The wage distribution mixes both types of firms.

The second region arises once wages reach the maximal value that contractors pay, $\max w_s^{\text{cont.}}$. Contractors no longer hire, and only highly productive goods producers who pay high wages operate. In this second region, goods producers do not compete with contractor firms. A given wage increase does not attract as many workers as in the low-wage region, and so the labor supply curve and the wage distribution may have a kink at $\max w_s^{\text{cont.}}$, as shown in panels (a) and (b).

Once goods producers become productive enough, they find outsourcing preferable to

hiring in-house. This change happens at productivity $\hat{z}$ and wage $w_s(\hat{z})$. Proposition 2.6 ensures that employment jumps up as goods producers switch from the upward-sloping labor supply curve to the vertical one depicted in panel (a). Proposition 2.7 does not restrict the size of the gap between the maximal in-house wage and the outsourcing price beyond $w_s(\hat{z}) \leq p_s$. Thus, we represent the equilibrium with a gap between both prices and quantities. When goods producers no longer operate, the employment density drops to zero in panel (b), as there are no firms left to hire workers at higher wages than $w_s(\hat{z})$.

### 2.3.4 An empirical model of outsourcing

Not only do Propositions 2.5 to 2.7 characterize the productivity and distributional effects of outsourcing, they also provide implications that we test in Section 2.4 below. To connect our model with the richness of the data and emphasize possible identification challenges, we supplement our theory with additional heterogeneity.

In addition to productivity $z$, goods producers now face idiosyncratic outsourcing cost shocks $\boldsymbol{\varepsilon} \equiv \{\varepsilon_s\}_s$, with $1 \leq \varepsilon_s \leq \overline{\varepsilon}_s$ for all $s$. These shocks may be arbitrarily correlated with productivity $z$. They capture the idea that some managers are particularly apt at harnessing outsourcing, or that the specific production process of a given firm is well-suited for outsourcing. When deciding to hire in-house or to outsource, goods producers then solve

$$\begin{aligned} \pi(z,\boldsymbol{\varepsilon}) \quad = \quad & \max_{\substack{\{n_s\}_s,\{w_s\}_s,\\ \{o_s\}_s \in \{0,1\}^S}} R\big(z,\{n_s\}_s\big) - \int \Big[(1-o_s)w_s + o_s p_s \varepsilon_s\Big] n_s ds \qquad (2.10) \\ \text{s.t.} \quad & n_s \leq n_s(w_s) \text{ if } o_s = 0. \end{aligned}$$

The only difference between the decision problem (2.10) and the decision problem (2.5) is that idiosyncratic outsourcing cost shocks $\varepsilon_s$ introduce a randomization of the effective price of outsourcing across firms. As a consequence, our results remain unchanged after conditioning on outsourcing costs.

**Corollary 2.9** (Selection into outsourcing with cost shocks)**.** *There exists a threshold productivity function $\hat{z}_s(\boldsymbol{\varepsilon})$, such that outsourcing occurs if and only if $z \geq \hat{z}_s(\boldsymbol{\varepsilon})$. The threshold $\hat{z}_s(\boldsymbol{\varepsilon})$ is increasing in every $\varepsilon_s$. Outsourcing expenditures $\mathcal{E}(z,\boldsymbol{\varepsilon}) = \int p_s \varepsilon_s o_s^*(z,\boldsymbol{\varepsilon}) n_s^*(z,\boldsymbol{\varepsilon}) ds$ are increasing in $z$.*

*Proof.* See Appendix B.1.13. □

Firms face the same outsourcing decision problem as in Section 2.3.2 conditional on outsourcing costs $\varepsilon_s$. Thus, they still select into outsourcing. When the firm faces higher costs $\varepsilon_s$, it takes a higher productivity $z$ to reap the benefits from outsourcing, and so the outsourcing threshold $\hat{z}_s$ is higher. Conditional on outsourcing costs, outsourcing expenditures rise with firm productivity due to positive selection. Similarly, Corollary 2.10 indicates that the productivity effect arises conditional on outsourcing cost shocks.

**Corollary 2.10** (Productivity effect with cost shocks)**.** *Conditional on a vector of outsourcing cost shocks $\boldsymbol{\varepsilon}$, revenues and labor demand increase upon outsourcing: $R(\hat{z}_s(\boldsymbol{\varepsilon})^+, \boldsymbol{\varepsilon}, \boldsymbol{n}^*(\hat{z}_s(\boldsymbol{\varepsilon})^+)) > R(\hat{z}_s(\boldsymbol{\varepsilon})^-, \boldsymbol{\varepsilon}, \boldsymbol{n}^*(\hat{z}_s(\boldsymbol{\varepsilon})^-))$ and $n_s^*(\hat{z}_s(\boldsymbol{\varepsilon})^+, \boldsymbol{\varepsilon}) > n_s^*(\hat{z}_s(\boldsymbol{\varepsilon})^-, \boldsymbol{\varepsilon})$.*

*Proof.* See Appendix B.1.14. □

The same logic ensures that the distributional effect emerges in an economy with idiosyncratic outsourcing costs. The marginal product of labor in-house remains above the marginal product of labor when outsourcing, which in turn is higher than the marginal product of labor at contractors. This ranking of marginal products of labor translates into a ranking of wages and other labor market statistics along the job ladder.

**Corollary 2.11** (Distributional effect with cost shocks)**.**

*(a) When contractors do not have a comparative advantage $\tau_s \leq 1$, they pay lower wages than all marginal goods producers: for any contractor wage $w_s^{\text{cont.}}$, $w_s^{\text{cont.}} < w_s(\hat{z}_s(\boldsymbol{\varepsilon}))$. Contractors also hire less from employment, have higher churn and lower net poaching than all marginal goods producers.*

*(b) Suppose that there is a single worker type and that $0 < \underline{\rho} \leq -\frac{nR_{nn}}{R_n} \leq \overline{\rho} < 1$. When contractors have a high enough comparative advantage $\tau_s \gg 1$, they pay higher wages than all marginal goods producers $w_s^{\text{cont.}} > w_s(\hat{z}_s(\boldsymbol{\varepsilon}))$. They also hire more from employment, have lower churn and higher net poaching than all marginal goods producers.*

*Proof.* See Appendix B.1.15. □

Equipped with Corollaries 2.9 to 2.11, we are in a position to connect our theory with the data.

### 2.3.5 The comparative advantage and cost-saving views

The first link between our theory and data is the outsourcing wage penalty. Our theory is uniquely suited to use the outsourcing wage penalty to disentangle the comparative advantage view from the cost-saving view.

Consistently with the comparative advantage view, our model nests the possibility that firms outsource because contractor firms have a comparative advantage in producing services of a particular type when $\tau_s > 1$, rather than to save on costs per worker. In this case, Proposition 2.7 indicates that contractor firms should pay higher wages than goods producers, not lower ones. This implication is at odds with the outsourcing wage penalty documented in Dube and Kaplan (2010), Goldschmidt and Schmieder (2017), Dorn et al. (2018) and Drenik et al. (2020) for the U.S., Germany and Argentina, as well as our results in Section 2.4.3 for France. According to the comparative advantage view, contractors should also hire more from employment, have less churn and higher net poaching. Section 2.4.3 indicates that all these predictions are at odds with the data.

By contrast, Proposition 2.7 reveals that the cost-saving view—whereby goods producers outsource simply to save on labor costs rather than leverage productive comparative advantage—is entirely consistent with the outsourcing wage penalty. It is also consistent with our results on hiring, churn and net poaching in Section 2.4.3. We conclude that the cost-saving view is a more likely account of outsourcing than the comparative advantage view.

We now connect our novel implications with the data.

## 2.4 Reduced-form evidence

This section lays out how we test our theory of outsourcing. We first describe our data. Second, we discuss aggregate trends in outsourcing in France. Then, we test our three main predictions: the distributional effect, selection into outsourcing, and the productivity effect. Finally, we rule out alternative explanations for outsourcing. We provide more details in Online Appendix B.5.

### 2.4.1 Data

We use a combination of administrative and survey data for France between 1996 and 2007. Our first data source is the near-universe of annual tax records of French firms (*Fichier Complet Unifié de Suse*, FICUS) that report balance sheet and income statement information. We use industry codes to identify contractor firms. We observe employment, payroll, sales and purchases of intermediate inputs, from which we construct value added. However, this dataset does not detail intermediate inputs finely enough to isolate outsourcing expenditures on the buyer side.

Our second data source is a large annual firm-level survey that details purchases of intermediates at the firm level (*Enquête Annuelle d'Entreprise*, EAE). Firms report expenditures on 'external workers.' External workers are employees of another firm, but that fall under a contracting agreement with the surveyed firm and are at least partially under the authority of the surveyed firm. We use expenditures on external workers as our measure of expenditures on outsourced workers. Firms remain in the survey once they enter, leading to a panel structure.

Our third data source consists of employer tax records that cover labor market outcomes for French workers (*Déclaration Annuelle de Données Sociales*, DADS). We use repeated cross-sections with the universe of French workers to construct employment and wages at the firm-occupation-year level (DADS *Postes*). We also use a 4% representative panel to study

the wage penalty of outsourcing (DADS *Panel*).

Our fourth data source are customs records for the universe of trade transactions (*Donnés de Douanes*). We observe exports at the product-country-firm-year level. We use this data to construct export demand shocks at the firm-level and exploit variation in firm scale.

We link these four data sources together using a common firm tax identifier. For our main empirical exercises at the firm level, we aggregate years into three periods 1997-1999, 2000-2002, 2003-2007 and keep only firms with at least ten in-house employees to limit measurement error in outsourcing expenditures. We stop our main analysis in 2007 because of a large change in classification that prevents us from reliably measuring outsourcing expenditures directly in subsequent years. Our final sample consists of 173,547 firm-periods.[16]

### 2.4.2 Aggregate trends in outsourcing

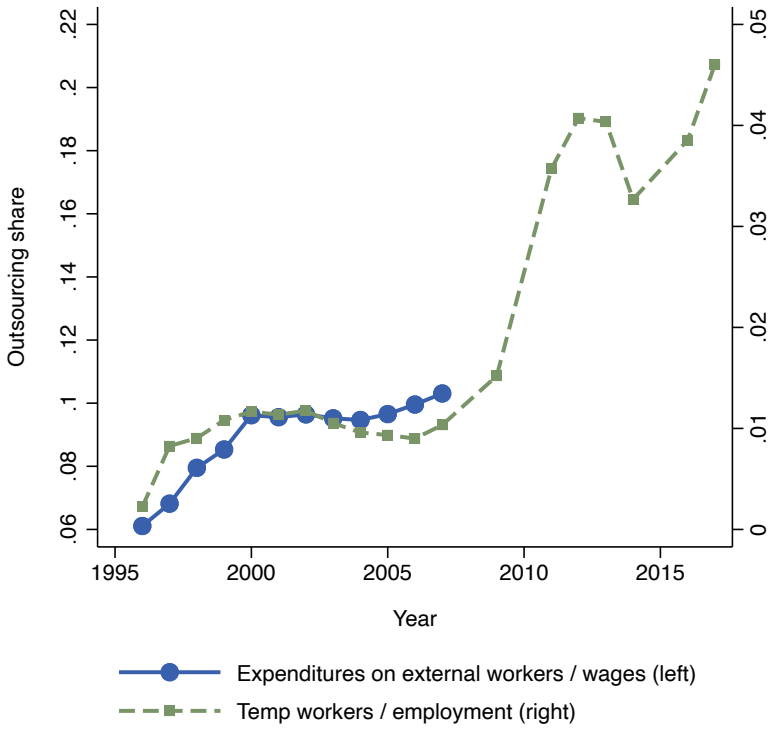
We start by asking by how much did outsourcing rise in France. Figure 2.2 shows that outsourcing expenditures as a fraction of the aggregate wage bill almost double in the decade that we study: they increase from 6% in 1996 to over 10% in 2007. To infer whether the upward trend in outsourcing continues past 2007, we use a subgategory of outsourcing that we can reliably measure past 2008: temporary workers. Of course, extrapolating the trend in overall outsourcing expenditures using the more restrictive group of temporary workers requires strong proportionality assumptions. But to the extent that outsourcing expenditures track the rise in the use of temporary work, Figure 2.2 reveals that outsourcing may account between 10% and 20% of the aggregate wage bill in France by 2016.[17]

This rise in aggregate outsourcing expenditures also translates into an increase in the

[16]France harmonized its data collection procedures and classification in 2008 with the rest of the European Union. The change in industry codes prevents us from identifying contractor firms after 2008. In addition, the EAE survey was discontinued and replaced by the *Enquête Sectorielle Annuelle* (ESA). The ESA includes questions about outsourcing but the response rate is substantially lower than in the EAE for firm-level expenditures, leading to severe measurement difficulties.

[17]That outsourcing represents 20% of France's wage bill by 2016 is likely an overestimate because two regulatory changes directly impact temporary work relative to general outsourcing. In 2005 temporary work agencies are allowed to also help their workers transition into permanent work contracts at their clients. In 2009 governmental institutions are allowed to hire temporary workers.

Figure 2.2: Outsourcing expenditure share and temporary work share.



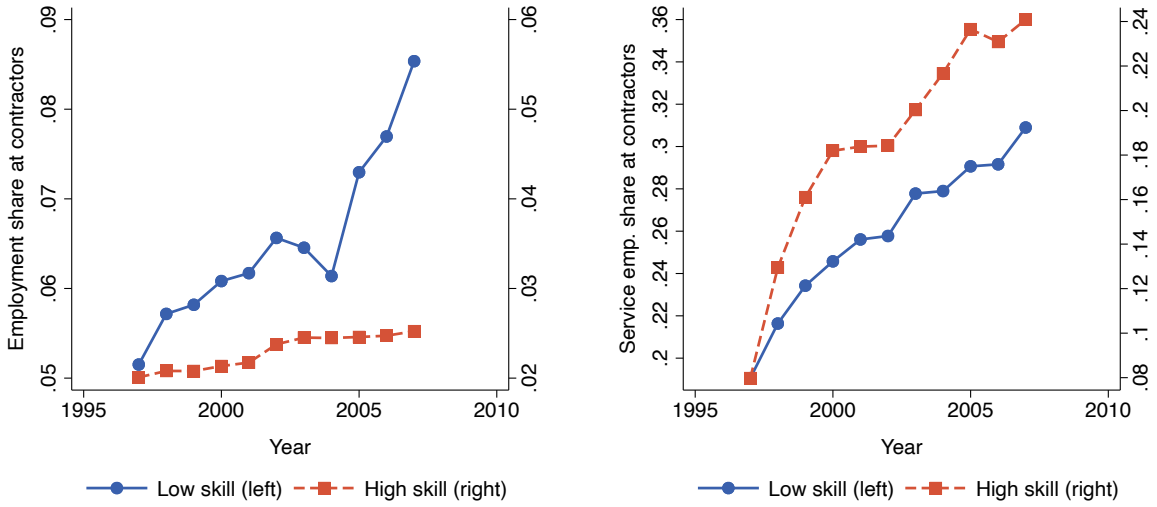

Note: Solid blue line: aggregate outsourcing share, computed as aggregate expenditures on external workers relative to the aggregate in-house wage bill. Dashed green line: temporary work share, computed as total temporary workers at firms with more than 100 in-house employees, relative to all employment at firms with more than 100 in-house employees.

employment share of contractor firms in the aggregate. To complement our expenditure-side data, we follow Goldschmidt and Schmieder (2017) and rely on industry and occupation codes to detect contractor firms and service workers at contractor firms. Figure 2.3a shows that the fraction of low skill workers employed at contractor firms rises from 5% in 1996 to 9% in 2007. This 4 p.p. increase in the outsourcing employment share coincides almost exactly with the increase in the outsourcing expenditure share. Figure 2.3b also reveals that the increase in the employment outsourcing share is driven specifically by service workers reallocating towards contractor firms over time. The fraction of low skill service workers employed at contractors rises by 16 p.p. over the decade we study.[18]

[18]Our employment-based and expenditure-based measures complement each other. Our employment-based measure may miss any firm that is a contractor firm, but does not fall into our specific industry codes. Our measure using outsourcing expenditures is not subject to this limitation.

Figure 2.3: Outsourcing employment share.

(a) Share of employment at contractors. (b) Share of service employment at contractors.




Note: Low skill contractor firm defined by industry codes specifically labeling firms as providing food, security, cleaning or general administrative services to other firms. High skill contractor firm defined by industry codes specifically labeling firms as providing accounting, law or consulting services to other firms.

Next, we compare salient features of firms that rely on outsourcing to firms that do not. Consistently with Propositions 2.5 and 2.6, Table B.1 in Appendix B.2.1 shows that firms that outsource are larger, sell more and have higher value added than firms that do not outsource. Table B.1 in Appendix B.2.1 also provides examples of 3-digit industries that outsource most and least in France. The industry that outsources the most in France is Business supplies and equipment trade. Firms in this industry mostly place orders on behalf of their client companies and outsource delivery and installation. Similarly, the Telecommunications industry manages the grid but outsources a substantial fraction of its maintenance and installation. By contrast, Manufaturing of terracota ceramics requires specific knowledge and thus does not outsource workers. Similarly, Tranportation of goods and individuals into space requires highly specialized knowledge and is subject to strict security measures, and thus does not rely on outsourced workers.

### 2.4.3 The distributional effect

We start by testing the core predictions of our theory relating to the distributional effect of outsourcing. We propose two exercises following our results in Corollary 2.11. Our first step is to verify that there is indeed an upward-sloping labor supply curve for service workers—the premise upon which the entire distributional effect is built. Second, we show that contractor firms locate at the bottom of this job ladder: they pay lower wages, hire less from employment, have higher churn and lower net poaching.[19]

**The upward-sloping labor supply curve.** We start by verifying that larger firms pay service workers more and thus locate at the top of the job ladder. Following the large literature establishing the existence of a firm wage size premium, we leverage our panel data to project log wages at the worker level on the size of their employer. We control for worker fixed effects to absorb worker-level heterogeneity that may be correlated with firm scale. Following Goldschmidt and Schmieder (2017), we restrict attention to wages of workers who are in service occupations: food, security, cleaning or administrative services. We run a fixed-effect regression of the form:

$$\log w_{i,t} = \varphi_i + \phi \log \text{Total employment}_{J(i,t)} + \upsilon_{i,t}. \tag{2.11}$$

$i$ indexes workers, $J(i,t)$ the employer of worker $i$ in quarter $t$, and $\upsilon_{i,t}$ is a mean-zero residual. $\log w_{i,t}$ denotes the log wage, and $\varphi_i$ is a worker fixed effect.

We find that larger firms indeed pay their service workers more after controlling for worker fixed effects $\varphi_i$. We display our estimates in Table B.5 in Appendix B.2.3. Firms with 1,000 employees pay wages that are on average 9.6% higher than firms with 10 employees. We also report that wages rise with firm value added and with the overall average wage at the firm, including non-service workers. Together, these results point to an upward-sloping labor

[19] Importantly, health and retirement benefits are not tied to employers in France as they are in the U.S., and so wages are the main dimension of worker compensation in our context.

supply curve for service workers consistent with our theory.

**Contractors on the job ladder.** Second, we test whether contractor firms locate at the bottom of the job ladder. We identify contractor firms using the industry codes as in Section 2.4.2. Our goal is to measure the wage premium paid by contractors controlling for unobserved heterogeneity at the worker level. We do so with a two-way fixed effects regression as in Abowd et al. (1999b), henceforth AKM:

$$\log w_{i,t} = \varphi_i + \psi_{J(i,t)} + \eta_{i,t}. \tag{2.12}$$

where $\psi_{J(i,t)}$ a firm fixed effect. Workers who move between firms identify separately worker and firm fixed effects if worker mobility is conditionally random (Card et al., 2013).

To limit well-known econometric difficulties linked to limited mobility bias, we follow Bonhomme et al. (2019) and group workers and firms each in 50 equally populated groups based on the unconditional mean worker and mean firm wage. We then estimate equation (2.12) with OLS at the group level. Our results are virtually identical when varying the number of groups between 10 and 200.

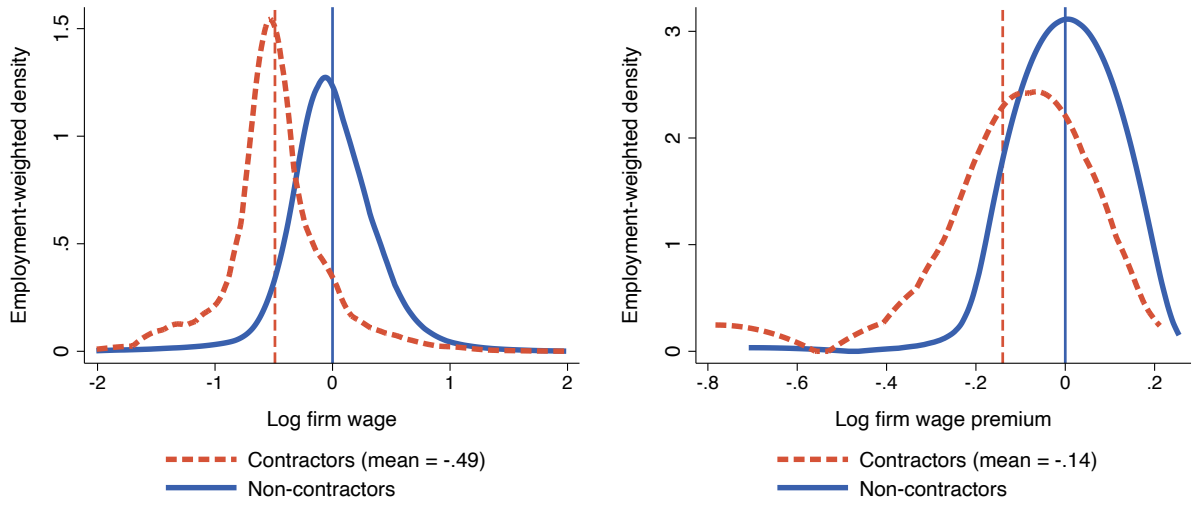
Without controlling for worker composition, contractors pay wages that are almost 50% below wages of non-contractor firms. Figure 2.4a displays the distribution of average wages at contractors and non-contractors with a kernel density plot. We show that controlling for worker composition is key to not overstating the contractor wage penalty in Figure 2.4b. The penalty drops to 14% on average after removing worker fixed effects with specification (2.12). We also estimate the standard deviation of firm effects to be 0.14. Thus, contractors pay on average one standard deviation below the average non-contractor firm. Our results are consistent with Goldschmidt and Schmieder (2017), Dorn et al. (2018) and Drenik et al. (2020) who also find a substantial outsourcing wage penalty driven by the loss of firm wage premia.

Guided by our theory, we also propose novel measures indicating that contractors rank

Figure 2.4: Contractors and non-contractors on the job ladder.

(a) Raw wages.

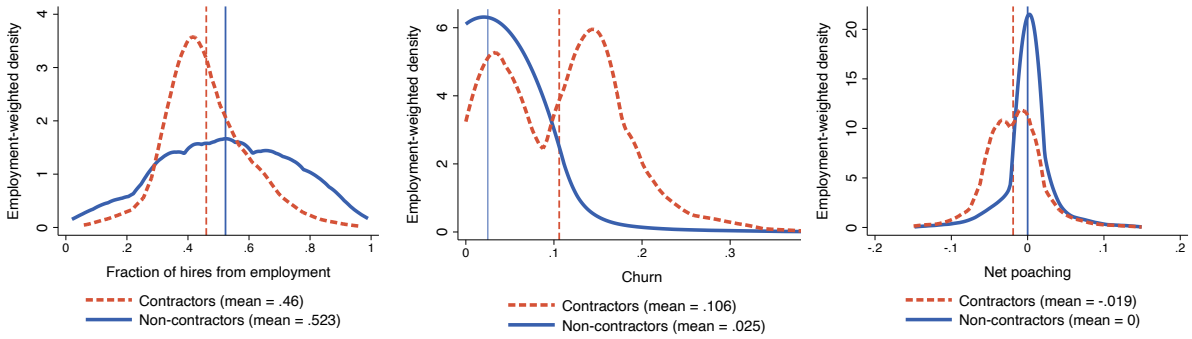
(b) Firm premia.




(c) Hires from employment.

(d) Churn.

(e) Net poaching.




Note: Kernel density plots of labor market statistics of contractor firms (dashed orange) vs. non-contractor firms (solid blue). Vertical lines depict means. Raw wages: firm-level mean wages. Firm premia: firm fixed effect from the AKM regression (2.12). Fraction hires from employment: hires from employment divided by total hires as in equation (2.6). Churn: total separations divided by employment as in (2.7). Net poaching: hires from employment net of quits to employment divided by total employment as in (2.8).

towards the bottom of the job ladder along other key labor market statistics. Contractor hire less from employment than goods producers as we show in Figure 2.4c. The fraction of hires from employment is 6 p.p. lower at contractors. Figure 2.4d indicates that churn at contractors is 8 p.p. above that at other firms. Finally, contractors tend to lose their workers to other firms as shown in Figure 2.4e. Net poaching at contractors is 2 p.p. below net poaching at non-contractors. This difference is substantial and corresponds to net poaching differences between young and old firms in the U.S. (Bilal et al., 2022). Together, these results indicate that contractors indeed locate at the bottom of the job ladder.

### 2.4.4 Selection into outsourcing

Having established the outsourcing wage penalty in France, we test the novel, firm-level predictions of our theory of oursourcing that links workers and firms in equilibrium. We start by testing selection into outsourcing. Corollary 2.9 indicates that when productivity $z$ rises conditional on outsourcing costs $\{\varepsilon_s\}_s$, firms spend relatively more on outsourcing: $\mathcal{E}^*(z, \boldsymbol{\varepsilon})$ increases. To contrast our theory with the data, we must make three measurement choices and address several identification challenges.

First, what is productivity $z$? From our model's perspective, revenue productivity is the driver of outsourcing decisions rather than physical productivity. Still, we do not observe revenue productivity. We leverage that our model ties productivity $z$ with revenues $R^*(z, \{\varepsilon_s\}_s)$: more productive firms sell more. To test selection into outsourcing, we thus use revenues as our main independent variable.

Second, what are revenues? Our model does not have intermediate inputs other than outsourcing. Thus, the empirical counterpart of revenues in our model is value added complemented with outsourcing expenditures which we measure in our data. We define revenues of firm $f$ in time period $t$ as value added $\text{VA}_{ft}$ plus outsourcing expenditures measured as expenditures on external workers $\mathcal{E}_{ft}$, $R_{ft} = \text{VA}_{ft} + \mathcal{E}_{ft}$. For brevity and when unambiguous, we will sometimes refer to our measure of revenues simply as value added. We

replicate all our results using instead value added $\text{VA}_{ft}$ in Appendix B.2.1 and show that they remain virtually unchanged.

Third, what are outsourcing expenditures? We use log expenditures on external workers $\log \mathcal{E}_{ft}$ as our main measure of outsourcing. We also use the outsourcing share $S_{ft}$ of firm $f$ in time period $t$, defined as its expenditures on external workers $\mathcal{E}_{ft}$ divided by the sum of its expenditures on labor $\mathcal{W}_{ft} + \mathcal{E}_{ft}$, where $\mathcal{W}_{ft}$ denotes gross payroll: $S_{ft} = \frac{\mathcal{E}_{ft}}{\mathcal{W}_{ft}+\mathcal{E}_{ft}}$.

Log expenditures connect directly to Corollary 2.9, which predicts that $\frac{\partial \log \mathcal{E}}{\partial z} > 0$. However, there may be reasons beyond our model that drive a mechanical correlation between firm scale and outsourcing expenditures. Therefore, we also use the outsourcing share as an alternative dependent variable. Using the outsourcing share is a more demanding test of our theory as our model does not guarantee that the outsourcing share should always be increasing in productivity.[20]

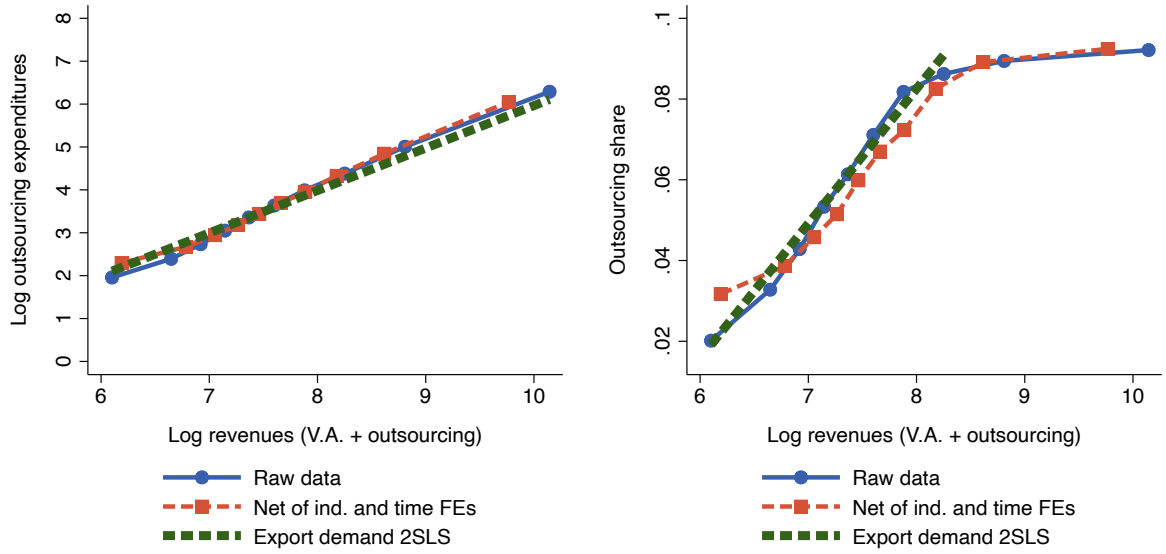
Having established our measurement strategy, we turn to our identification strategy. Selection into outsourcing translates as $\frac{\partial \log \mathcal{E}(z,\boldsymbol{\varepsilon})}{\partial z} > 0$ and, under our stricter test, $\frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial z} > 0$. Since we do not measure revenue productivity directly, we seek to identify whether $\frac{\partial \log \mathcal{E}(z,\boldsymbol{\varepsilon})}{\partial z} \Big/ \frac{\partial \log R(z,\boldsymbol{\varepsilon})}{\partial z} > 0$ and $\frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial z} \Big/ \frac{\partial \log R(z,\boldsymbol{\varepsilon})}{\partial z} > 0$. Identification crucially depends on the co-movement between productivity $z$ and outsourcing costs $\varepsilon_s$ across firms. The starkest identification assumption is that there is no dispersion in outsourcing costs $\varepsilon_s$ across firms. In this case, we may simply correlate revenues $\log R_{ft}$ with outsourcing expenditures $\log \mathcal{E}_{ft}$ or the outsourcing share $S_{ft}$ across firms to test whether there is selection into outsourcing.

Figure 2.5 plots outsourcing by decile of revenues. Figure 2.5a reports how outsourcing expenditures change with revenues. Consistently with Table B.1, Figure 2.5 reveals that in the raw data, high revenue firms outsource more. The relationship is nearly log-linear for outsourcing expenditures. Figure 2.5b reports the outsourcing share by revenues. A firm in

[20]For instance, if the revenue function is non-homothetic and firms increasingly rely on high skill workers as they expand, in-house payroll can rise faster than outsourcing expenditures. In that case, the outsourcing share decreases with productivity. Figure B.3 in Appendix B.2.1 suggests that skill deepening may indeed be affecting the outsourcing share at the far end of the productivity distribution, although our estimates are noisy.

Figure 2.5: Outsourcing by revenues.

(a) Outsourcing expenditures. (b) Outsourcing share.




Note: Solid blue line: raw data. Dashed orange line: after removing 3-digit industry and time period fixed effects from log outsourcing expenditures, the outsourcing share and log value added. Thick dashed green line: 2SLS estimate using the export demand shift-share instrument in equation (2.14). Panel (a): log outsourcing expenditures. Panel (b): outsourcing share.

the first decile of revenue spends 2% of its labor costs on outsourced labor, while a firm in the tenth decile of revenue spends over 8%. This upward-sloping relationship is not an artifact of industry composition or time trends, as shown by the residualized relationship. Under the assumption of no outsourcing costs $\varepsilon_s$, the slopes in both panels of Figure 2.5 support the view that firms select into outsourcing.

Of course, the relationship depicted in Figure 2.5 could be the result of variation in outsourcing costs across firms. Perhaps the particular production process of some firms lends itself well to outsourcing, while other firms find it more difficult to outsource. Since revenues $R(z,\boldsymbol{\varepsilon})$, outsourcing expenditures $\mathcal{E}(z,\boldsymbol{\varepsilon})$ and the outsourcing share $S(z,\boldsymbol{\varepsilon})$ are all likely decreasing in $\varepsilon_s$, the unconditional correlation between both may be positive even if outsourcing expenditures and the outsourcing share are in fact independent from productivity $z$. In that case, selection into outsourcing based on firm productivity would be spuriously identified. This threat to identification is particularly problematic when productivity $z$ and outsourcing costs $\varepsilon_s$ are correlated across firms. For instance, if the CEO of a firm is

particularly skilled at harnessing the advantages of information technology, it may improve overall productivity at the firm at the same time as making outsourcing easier for reasons unrelated to selection into outsourcing.

To circumvent this difficulty we focus next on the conditional correlation between revenues $R_{ft}$, outsourcing expenditures $\mathcal{E}_{ft}$ and the outsourcing share $S_{ft}$ within firms. This conditional correlation identifies selection into outsourcing if outsourcing costs are constant within firms. Under this assumption, productivity $z$ and outsourcing costs $\varepsilon_s$ may be arbitrarily correlated across firms in levels, but not in changes. For instance, a CEO may be particularly skilled at reorganizing its firm to outsource, but their ability may not vary over time. Up to a first-order approximation, the linear regression

$$Y_{ft} = \alpha_t + \beta_f + \gamma \log R_{ft} + \eta_{ft}, \quad Y_{ft} \in \{\log \mathcal{E}_{ft}, S_{ft}\}, \tag{2.13}$$

then identifies selection into outsourcing. $\alpha_t$ is a time period fixed effect, $\beta_f$ a firm fixed effect, and $\eta_{ft}$ a mean zero residual. The firm fixed effect $\beta_f$ absorbs the co-movement between long-run productivity and outsourcing costs across firms. The coefficient $\gamma$ captures selection into outsourcing. It is identified by within-firm changes in revenues and outsourcing driven by shocks to revenue productivity.

We report the regression version of our results without and with firm fixed effects in Tables B.2 and B.3 in Appendix B.2.1. Table B.2 displays our results for outsourcing expenditures, and Table B.3 reports our results for the outsourcing share. In both tables, columns (1-2) show the regression analog of Figure 2.5. A 10 log points increase in revenues is associated with a 11 log points increase in log outsourcing expenditures and a 0.18 p.p. increase in the outsourcing share. The estimate for outsourcing expenditures slightly shrinks to 8 log points when we we focus on within-firm variation in column (3) by including firm fixed effects. For the outsourcing share, the estimate rises to 0.26 p.p. within firms.

An additional threat to identification arises when outsourcing costs vary over time at

the firm level. In that case, the estimate of $\gamma$ in the OLS regression (2.13) may simply capture the joint dependence of revenues and the outsourcing share on outsourcing costs. This threat is even more problematic when changes in outsourcing costs are also correlated with changes in revenue productivity. Returning to our example, the ability of the CEO to improve productivity and make outsourcing easier may vary over time. In this case, changes in productivity and outsourcing costs are correlated even within firm, and the OLS regression (2.13) is inadequate to identify selection into outsourcing.

We turn to an instrumental variable strategy to address this identification challenge. We identify shocks to revenue productivity $z$ that are plausibly unrelated to outsourcing cost shocks $\varepsilon_s$. We consider foreign export demand shocks. We exploit the granularity of our customs data and follow Hummels et al. (2014). We first construct firm-level export shares in the first time period, $\pi_{f,t_0,j}$, across 4-digit industry-country pairs $j$. We then interact those shares with export demand growth $\Delta \log X_{j,t,-f}$ in industry-country pair $j$ between time periods $t_0$ and $t$, excluding firm $f$'s exports. The instrument $Z_{f,t}$ for revenues $R_{ft}$ is thus defined as a shift-share:

$$Z_{f,t} = \sum_j \pi_{f,t_0,j} \ \Delta \log X_{j,t,-f}. \tag{2.14}$$

The identifying variation in the instrument follows from changes in foreign export demand. Consider firm $f$ that exports luxury handbags to South Korea in the initial period. If South Korean demand for luxury handbags subsequently grows, firm $f$ will face an increase in demand. Our exclusion restriction is that this rise in firm $f$'s demand that follows from South Korea's higher demand for luxury handbags is unrelated to firm $f$'s own ability to outsource (Borusyak et al., 2021). In that case, foreign export demand shocks raise revenue productivity and $Z_{f,t}$ is a valid instrument for changes in firm revenues. We display the distribution of shares $\pi_{f,t_0,j}$, changes in export demand $\Delta \log X_{j,t,-f}$ and the resulting instrument $Z_{f,t}$ in Figure B.2, Appendix B.2.1. Up to a first-order approximation, the

2SLS estimate of $\gamma$ in equation (2.13) then captures the ratio of average partial derivatives $\mathbb{E}\left[\frac{\partial \log \mathcal{E}(z,\boldsymbol{\varepsilon})}{\partial z}\right] / \mathbb{E}\left[\frac{\partial \log R(z,\boldsymbol{\varepsilon})}{\partial z}\right] > 0$ and $\mathbb{E}\left[\frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial z}\right] / \mathbb{E}\left[\frac{\partial \log R(z,\boldsymbol{\varepsilon})}{\partial z}\right] > 0$, and thus selection into outsourcing.

The main concern for identification under our instrumental variable strategy arises if firms that happen to have large export exposure to markets that grow are also firms who experience systematic reductions in their costs of outsourcing. This type of correlation would arise if managers are particularly foresighted and invest ahead of time in export destinations that will grow in the future, while subsequently restructuring the firm to make it more amenable to outsourcing. While possible in principle, the systematic prevalence of such situations is unlikely in practice given the large fixed costs typically associated with exporting to new markets

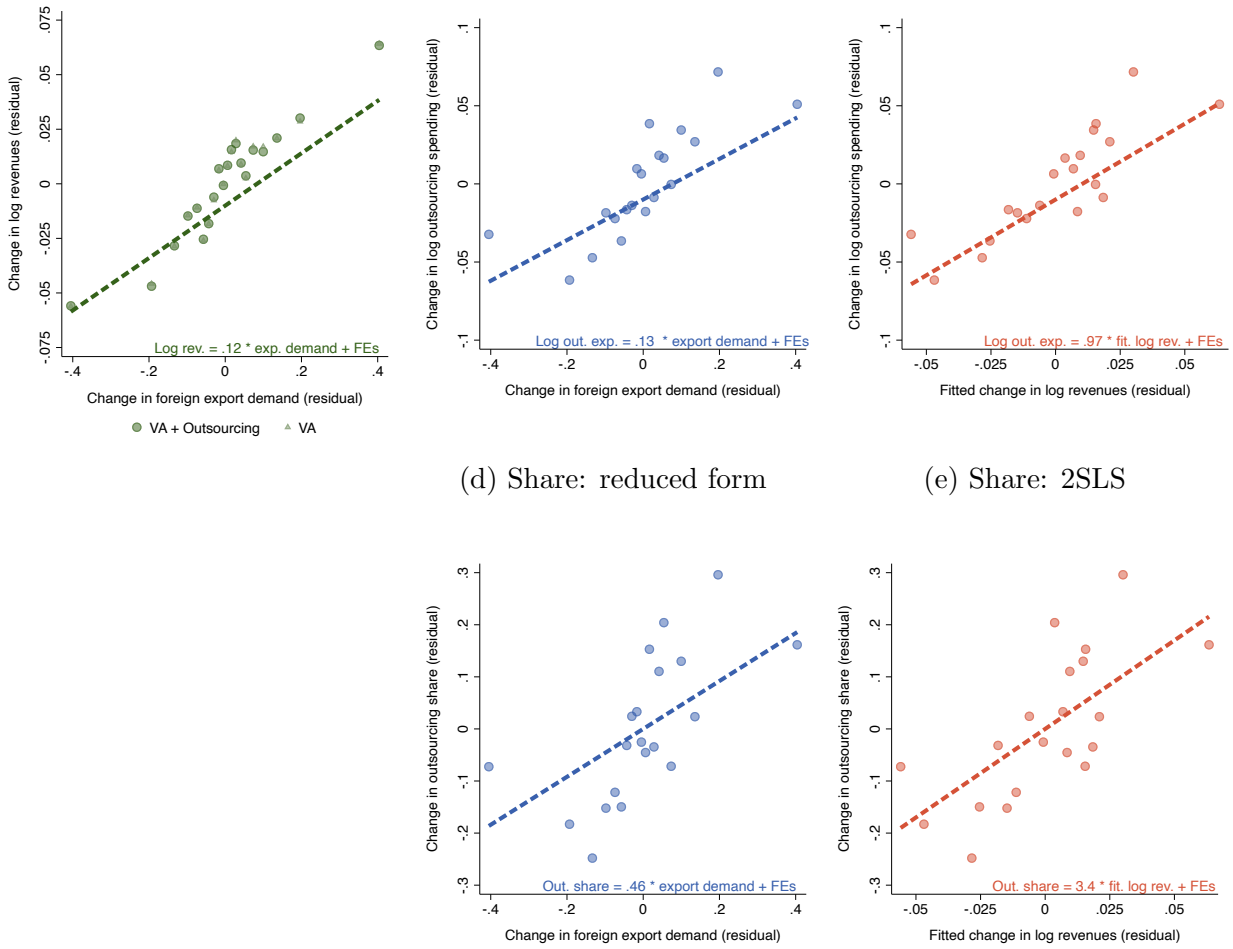
Figure 2.6 represents our instrumental variable strategy graphically. Panel 2.6a shows a strong first stage: export demand growth predicts value added growth, with an F-statistic of 332 well above conventional thresholds for weak instruments (Stock and Yogo, 2005). Panels 2.6b and 2.6d reveal that export demand growth also leads to growth in firm-level outsourcing expenditures and outsourcing shares. As a result, panels 2.6c and 2.6e uncover a positive relationship between revenue productivity and outsourcing.

We collect our estimates using our full firm-level sample in Tables B.2 and B.3, Appendix B.2.1. A 10% increase in value added driven by export demand shocks leads outsourcing expenditure to rise by 10% and the outsourcing share to rise by 0.33 p.p.[21] All our estimates are economically and statistically significant at the 0.1% level. Since our instrument only affects exporters, we also confirm that exporters exhibit a similar within-firm OLS relationship between revenues and outsourcing relative to all firms in column (5) in Tables B.2 and B.3, Appendix B.2.1. The thick dashed green lines in Figure 2.5 depict our 2SLS estimates graphically and show that they are quantitatively comparable to the slope of the cross-sectional relationship.

[21] In panel 2.6e, the coefficient is 0.34 p.p. when we run the regression at the bin level rather than at the firm level.

Figure 2.6: Selection into outsourcing: Instrumental variable approach.

(a) First stage (b) Expenditures: reduced form (c) Expenditures: 2SLS




(d) Share: reduced form (e) Share: 2SLS




Note: Bin-scatterplot of the first stage 2.6a, reduced form 2.6b and two stage least square 2.6c estimates for selection into outsourcing with log outsourcing expenditures as dependent variable. Bin-scatterplot of the reduced form 2.6d and two stage least square 2.6e estimates for selection into outsourcing with the outsourcing share as dependent variable. Panels include regression lines for 20 bins. Coefficients may differ from full sample regression coefficients reported in Tables B.2 and B.3, Appendix B.2.1.

We verify the robustness of our results with other metrics of firm performance. Figure B.1 in Appendix B.2.1 together with column (6) of Tables B.2 and B.3 indicate that our OLS and 2SLS estimates are virtually identical when we use value added without including outsourcing expenditures as our main measure of revenues. We also use employment and value added per worker as alternative measures of firm performance. Columns (7-8) of Tables B.2 and B.3 in Appendix B.2.1 all point to an economically meaningful and statistically significant effect of firm scale on outsourcing regardless of the particular metric we use. We conclude that firms select into outsourcing.[22]

### 2.4.5 The productivity effect

We now test the third core prediction of our theory: the productivity effect of outsourcing. Corollary 2.10 indicates that when outsourcing costs $\varepsilon_s$ decline so much that firms outsource, they expand: $R(z, \boldsymbol{\varepsilon})$ rises. To confront this prediction with the data, we face similar measurement and identification challenges to those described in Section 2.4.4.

We use similar variable definitions as in the previous section. Our main dependent variable is value added at the firm and period level, $\text{VA}_{ft}$. The independent variable is the outsourcing share $S_{ft}$. We seek to identify whether $\frac{\partial \log R(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_s)} > 0$. Since we do not measure outsourcing costs directly, we ask whether $\frac{\partial \log R(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_s)} \Big/ \frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_s)} > 0$.

As in Section 2.4.4, we could use a conditional correlation between revenues and the outsourcing share to identify the productivity effect if revenue productivity $z$ was constant within firms. In that case, we could run a regression

$$\log \text{VA}_{ft} = \alpha'_t + \beta'_f + \gamma' S_{ft} + \varepsilon'_{ft} \tag{2.15}$$

[22] Abraham and Taylor (1996) find that establishments with larger employment are less likely to outsource. Several factors may explain the difference between our findings and theirs. First, they consider establishment-level data, not firm-level data. Second, the minimum size for establishments to be included in their sample ranges between 20 and 100 employees. Figure B.6b reveals that most of the relevant variation is concentrated below 50 employees. Third, they use cross-sectional survey data with 2,700 establishments, while we have administrative panel data with 173,547 firm-periods.

and use the OLS estimate of $\gamma'$ to identify the productivity effect. Of course, the identification assumption we need is inconsistent with those in Section 2.4.4 and is unlikely to hold. In addition, changes in outsourcing costs may be correlated with changes in revenue productivity within firm. The same CEO example illustrates the main threat to identification. The ability of a CEO to improve productivity and make outsourcing easier may vary over time. Finally, the outsourcing share is likely to contain more measurement error than value added at the firm level. Measurement error would bias the OLS estimate of $\gamma'$ towards zero.[23]

We implement an instrumental variable strategy to address these identification challenges. We leverage that firms are differentially exposed to service occupations $o$: food, security, cleaning or general administrative occupations. Our first step is to construct average outsourcing expenditures on occupation $o$, denoted by $\Omega_{o,t,-f}$. We interact initial payroll shares $\omega_{f,o,t_0}$ with firm-level outsourcing expenditures $\mathcal{E}_{f,t}$. We then sum across firms to obtain $\Omega_{o,t,-f} = \frac{1}{N_{-f}} \sum_{f' \neq f} \omega_{f',o,t_0} \mathcal{E}_{f',t}$.

Next, we construct a predicted outsourcing share for firm $f$ by interacting initial payroll shares of firm $f$ with average outsourcing expenditures in the same occupations $\Omega_{o,t,-f}$:

$$\widehat{S}_{f,t} = \frac{\sum_o \omega_{f,o,t_0} \Omega_{o,t,-f}}{\mathcal{W}_{f,t_0} + \mathcal{E}_{f,t_0}}.$$

We then instrument the change in the outsourcing share of any firm using changes in the predicted outsourcing share:[24]

$$Z'_{f,t} = \Delta \widehat{S}_{f,t}$$

The initial exposure of firms to different occupations generates the identifying variation in

[23]We obtain value added from the FICUS data which is based on administrative tax returns. By contrast, outsourcing expenditures come from the EAE survey and correspond to much smaller income statement categories.

[24]The first stage associated with our instrument is positive when $\mathbb{E}\left[\frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_s)}\right] > 0$, i.e. if a decline in outsourcing costs lead firms to spend more on outsourcing. We show that this is the case in Appendix B.1.16 for the setup from Section 2.5. Ultimately however, the sign of the first stage is an empirical question.

the instrument. For instance, firm $f$ that produces luxury handbags also needs to hire many security guards in-house in the initial period to secure its warehouses. Over time, average outsourcing expenditures on security guards are rising, revealing economy-wide declines in outsourcing costs specifically for security guards. Our instrument infers that firm $f$ is particularly exposed to these costs declines, and thus should experience a substantial rise in its outsourcing share. We interpret this differential exposure as idiosyncratic changes in outsourcing costs $\varepsilon_s$.

Our exclusion restriction is that the resulting decline in idiosyncratic outsourcing costs is unrelated to changes in revenue productivity (Goldsmith-Pinkham et al., 2020). We display the distribution of shares $\omega_{f,o,t_0}$, changes in outsourcing expenditures $\Delta\Omega_{o,t,-f}$ and the resulting instrument $Z'_{f,t}$ in Figure B.4 in Appendix B.2.2. Up to a first-order approximation, the 2SLS estimate of $\gamma'$ in (2.15) identifies the ratio of average partial derivatives $\mathbb{E}\left[\frac{\partial R(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_s)}\right] / \mathbb{E}\left[\frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_s)}\right]$ and thus the productivity effect of outsourcing.

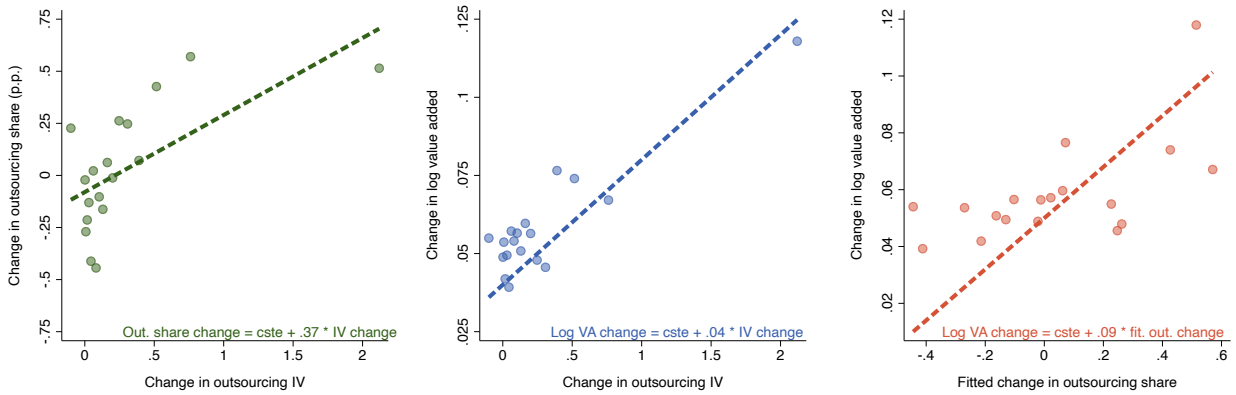
The main threat to identification arises if firms with many security guards in the initial period are also those that experience subsequent productivity growth. Once more, a particularly foresighted manager may have chosen a production structure that heavily relies on security guards, anticipating future outsourcing cost declines. If this manager is also particularly talented and makes its firm expand for reasons unrelated to outsourcing, our exclusion restriction is violated. While the pervasive adoption of such clear-sighted managerial practices would certainly be beneficial to the French economy, predicting which particular occupations will experience outsourcing cost declines in the future and distorting the production process of the firm towards them ex-ante is a high bar.

Figure 2.7 represents our instrumental variable strategy graphically. Panel 2.7a reveals a strong and positive first stage. Panel 2.7b reveals that growth in the predicted outsourcing share also leads to growth in firm-level value added. As a result, panel 2.7c uncovers a positive relationship between outsourcing and value added.

We collect our estimates using our full firm-level sample in Table B.4, Appendix B.2.2.

Figure 2.7: Productivity effect: Instrumental variable approach.

(a) First stage. (b) Reduced form. (c) 2SLS.




Note: Bin-scatterplot of the first stage 2.7a, reduced form 2.7b and two stage least square 2.7c estimates for selection into outsourcing. Panels include regression lines for 20 bins. Coefficients may differ from full sample regression coefficients reported in Table B.4, Appendix B.2.2.

The first stage F-statistic is 23.8. A 1 p.p. increase in the outsourcing share driven by outsourcing cost shocks leads value added to rise by 7.9%. This point estimate is economically and statistically significant at the 0.1% level. We conclude that outsourcing has a positive productivity effect at the firm level: firms that outsource produce more.[25]

### 2.4.6 Alternative explanations

In principle, mechanisms that we do not emphasize through our theory may also lead firms to outsource. In this final subsection, we rule out three prominent alternative explanations as key drivers of outsourcing in France.

**Volatility.** The first explanation is that firms outsource because they value the associated flexibility when demand is volatile. This explanation conflicts with the data along two dimensions. First, we should observe a negative relationship between firm scale and outsourcing since small firms are more volatile. Figure B.5a in Appendix B.2.4 indicates precisely the opposite. Second, Figure B.5b in Appendix B.2.4 shows that industries with higher value

[25]We cannot test directly the productivity for labor demand because we only observe outsourcing expenditures rather than quantities at the firm level.

added volatility actually rely less on outsourcing, not more. At the firm level however, we find some evidence in favor of a modest relationship between volatility and outsourcing, consistently with Abraham and Taylor (1996). Table B.6 in Appendix B.2.4 reveals that firm-level volatility is positively associated with outsourcing. Importantly however, the standardized coefficient is two to three times smaller than for firm scale. Thus, volatility matters substantially less than firm scale. We conclude that the primary reason that firms outsource is unlikely to be workforce flexibility.

**Equity.** Second, the upward-sloping labor supply curve faced by firms could be partly due to equity concerns rather than scarce managerial time. Our theory applies equally well if the labor supply curve is partly generated by equity concerns, but we still investigate whether equity concerns per se lead to outsourcing.

The equity motive for outsourcing implies that firms with more unequal pay structures then have the strongest incentives to outsource. This mechanism turns out to be at odds with the data. Table B.7 in Appendix B.2.4 indicates that, if anything, firms with more unequal pay structures outsource less, not more. Our results hold when within-firm inequality is measured as the standard deviation of log wages or the $90^{\text{th}}$ to $10^{\text{th}}$ percentile ratio. They also hold conditional on log revenues and in changes over time.

Our results are consistent with the growing literature that documents equity concerns (e.g. Card et al., 2012, Breza et al., 2017). Indeed, this strand of work finds that equity concerns are primarily binding across workers within the same occupation, rather than across occupations or worker performance categories. We conclude that equity concerns are not a dominant force pushing firms to outsource.

**Unions and size-based regulations.** Third, the upward-sloping labor supply curve could also be the result of union wage-setting or size-based labor market regulations. This concern is particularly relevant in France where firms with more than 50 employees face a number of legal

obligations that may increase the cost of labor, including accepting a union delegate.[26] Our theory applies equally well if the labor supply curve is partly generated by union wage-setting or size-based regulations. Yet, we ask whether these institutions in France are likely to increase the cost of labor so much as to lead firms to outsource.

We use a regression discontinuity design to test the impact of unions and size-based regulations on outsourcing. Figure B.6a in Appendix B.2.4 shows that French firms indeed bunch at the 50 employees threshold. However, Figure B.6b in Appendix B.2.4 shows that there is no statistically significant nor economically meaningful discontinuity in the outsourcing share around the 50 employees threshold. If anything, consistently with Figure 2.5b, the outsourcing-size gradient seems to flatten above 50 employees. We conclude that union wage-setting or size-based regulations do not impact outsourcing behavior significantly in France.[27]

Having proposed reduced-form evidence supporting the key predictions of our theory, we turn to our general equilibrium quantitative exercises.

## 2.5 Extended model and estimation

To structurally evaluate the impact of outsourcing on the economy, we first enrich our environment along several dimensions. Next, we discuss the estimation strategy.

[26]They must (i) form a committee that represents the interests of the employees to the management of the firm (*"comité d'entreprise"*), (ii) form a committee that monitors health and safety at work (*"comité d'hygiène, de sécurité et des conditions de travail"*), (iii) ratify an agreement that specifies what share of profits employees receive (*"accord de participation"*), (iv) maintain a monthly record of all hires and separations with the French administration (*"déclaration des mouvements de main-d'oeuvre"*), (v) establish a workforce-saving plan if they lay off more than 10 employees within a month (*"plan de sauvegarde de l'emploi"*), (vi) accept a union delegate, in which case annual wage bargaining takes place (*"délégué syndical"*), (vii) establish a plan to hire late-career employees (*"plan de génération et plan senior"*).

[27]This result contrasts with Bertrand et al. (2020) who find substantial effects of firing restrictions on outsourcing in India. In addition to differences in the overall economic and institutional environment, the practical magnitude of firing costs imposed by size-based regulations may differ between both countries.

### 2.5.1 Quantitative setup

To capture the idea that goods producers may expand their human resource departments to hire more in-house, we let goods producers exert any recruiting effort—or post any number of vacancies—$v$ in each market $s$. This recruiting effort comes at a convex cost $c_s(v) = c_{0s}v^{1+\gamma}$ for $\gamma > 0$. When $\gamma \to +\infty$ we recover the model of Section 2.3. The number of workers a goods producer attracts and retains now reflects its vacancy share:

$$n_s(w, v) = \frac{(1 + k_s)e_s}{\big(1 + k_s(1 - F_s(w))\big)^2} \cdot \frac{v}{V_s}, \tag{2.16}$$

where the equilibrium number of vacancies in market $s$ satisfies

$$V_s = \mathcal{V}_s + M^G \sum_{\boldsymbol{o}:o_s=0} \int v_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o})\Omega(\boldsymbol{o}|z, \boldsymbol{\varepsilon})\Gamma(dz, d\boldsymbol{\varepsilon}). \tag{2.17}$$

$\mathcal{V}_s$ is the measure of vacancies posted by contractor firms. $v_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o})$ denotes the number of vacancies posted by a firm with productivity $z$, outsoucing costs $\boldsymbol{\varepsilon}$ and decision $\boldsymbol{o} = \{o_s\}_s \in \{0, 1\}^S$. $\Omega(\boldsymbol{o}|z, \boldsymbol{\varepsilon})$ denotes the share of firms with productivity $z$ and outsoucing costs $\boldsymbol{\varepsilon}$ that choose outsourcing bundle $\boldsymbol{o}$. $\Gamma(z, \boldsymbol{\varepsilon})$ is the joint cumulative distribution function of $(z, \boldsymbol{\varepsilon} \equiv \{\varepsilon_s\}_s)$.

In equilibrium, the number of workers that goods producer $z$ attracts and retains is no longer a function of $\Gamma(z, \boldsymbol{\varepsilon})$ alone because goods producers with different outsourcing bundles $\boldsymbol{o}$ post different wages at the same productivity $z$. Instead, this number of workers is a function of $\Upsilon_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}) = F_s(w_s(z, \boldsymbol{\varepsilon}, \boldsymbol{o}))$, where the wage offer distribution $F_s$ is given in equation (B.23), Online Appendix B.7.2.

We impose a Cobb-Douglas revenue function nested in a decreasing returns upper tier:

$$R\big(z, \{n_s\}_s\big) = \left(z \prod_{s=1}^{S} n_s^{a_s}\right)^{\rho}, \quad \sum_{s=1}^{S} a_s = 1. \tag{2.18}$$

To relate our results to labor shares as simply as possible, we interpret decreasing returns to

scale as arising from an underlying constant returns, Cobb-Douglas production function that uses capital and labor of different skills: $R\big(z,\{n_s\}_s\big) = k^{1-\rho}\left(z\prod_{s=1}^{S} n_s^{a_s}\right)^{\rho}$. For simplicity, we assume that goods producers require exactly one unit of capital to operate, leading to the revenue function in equation (2.18) when $k = 1$. We denote by $\eta$ the cost of this unit of capital.

Goods producers then solve:

$$\begin{aligned}\pi(z,\{\varepsilon_s\}_s) \quad &= \quad \max_{\substack{\{n_s\}_s,\{v_s\}_s,\\ \{w_s\}_s,\{o_s\}_s}} R\big(z,\{n_s\}_s\big) - \sum_{s=1}^{S}\Big\{\big[(1-o_s)w_s + o_s p_s\varepsilon_s\big]n_s + (1-o_s)c_s(v_s)\Big\} - \eta \\ &\text{s.t.} \quad n_s = n_s(w_s,v_s) \text{ as per (2.16) if } o_s = 0. \end{aligned} \tag{2.19}$$

We specify a Cobb-Douglas matching function:

$$\mathcal{M}_s = \mu_s(m_s(u_s + \zeta_s(1-u_s))^{\xi}V_s^{1-\xi}. \tag{2.20}$$

$\zeta_s$ is the relative search intensity of employed workers: $\lambda_s^E = \zeta_s\lambda_s^U = \frac{\zeta_s\mathcal{M}_s}{m_s(u_s+\zeta_s(1-u_s))}$. $\mu_s$ is the matching efficiency in market $s$.

The joint distribution of productivity and outsourcing costs $(z,\boldsymbol{\varepsilon})$ is lognormal with respective standard deviations $\nu,\sigma$ and correlation $\iota$. We normalize the log means to zero as they are not separately identified from $\{\tau_s\}_s$ and $\{b_s\}_s$. We interpret $\varepsilon_s$ as an iceberg trade cost.

To capture the idea that contractors may specialize in screening and recruiting activities, we endow them with a possible comparative advantage in hiring. Their recruiting cost function is $c^C(v) = c_{0s}(\bar{c}^C)^{\gamma}v^{1+\gamma}$. The relative marginal cost $\bar{c}^C \lesseqgtr 1$ lets the model match the outsourcing wage penalty together with size differences between contractors and goods producers by shifting the labor supply curve of contractors.

We solve the model for three skill types $S = 3$. To focus on low skill outsourcing, we impose that high skill workers ($s = 3$) are never outsourced: $\tau_3 = 0$. We also impose that

only one type of low skill workers can be outsourced, $\tau_1 > 0, \tau_2 = 0$. We interpret the low skill workers who cannot be outsourced as "core" workers ($s = 2$), and those that can as "service" workers ($s = 1$). Core and service low skill workers share the same fundamentals $\delta_s, \zeta_s$. We relegate additional derivations to Online Appendix B.7.1 and computation details to Online Appendix B.7.4.

### 2.5.2 Estimation strategy

We set a quarterly frequency. We define low and high skill worker groups as revealed by the occupations of workers.[28] Non-employment is our primary measure of 'unemployment' in the model to capture steady-state flows into employment from individuals reported out of the labor force.

We estimate the model in three steps. We estimate a first group of parameters that can be directly mapped to data. Second, we set one parameter to an external value. In the third step, we estimate the remaining parameters jointly with a MSM estimator. To be consistent with our use of the full sample between 1997 and 2007 to construct most moments, we interpret our baseline parameter estimates as reflective of the French economy at the 2002 midpoint.

First, we identify the parameters $\{\delta_s, \zeta_s\}_{s=1}^3$ from labor market flows. The time-aggregated employment-to-non-employment transition rate $\text{EN}_s$ is equal to the job losing rate parameter $\delta_s$. The non-employment-to-employment transition rate $\text{NE}_s$ is equal to the endogenous offer rate $\lambda_s^U$ from non-employment. Online Appendix B.7.6 relates the employment-to-employment transition rate $\text{EE}_s$ to the arrival rate $\lambda_s^E = k_s \delta_s$:

$$\frac{\text{EE}_s}{\text{EN}_s} = \frac{(1 + k_s)\log(1 + k_s) - k_s}{k_s}.$$

[28] We rank 2-digit occupations by their average wage. We compute the mean predicted wage of a worker by interacting the mean occupational wages and the time spent by the worker in each occupation. A worker is low skill if their predicted wage is below median, and high skill if above.

We recover $\zeta_s = \lambda_s^E / \lambda_s^U$.

Second, we set the elasticity of the matching function $\xi$ to a common value of $\xi = 0.5$ since our data does not let us estimate it credibly (Petrongolo and Pissarides, 2001).

Third, we jointly estimate the remaining parameters $\{\mu_s\}_{s=1}^3, \{b_s\}_{s=1}^3, \{a_s\}_{s=2}^3, \rho, \gamma, \tau_1, \nu, \sigma, \iota, m_1, M^G, \eta, \overline{c}^C$ by MSM. While the parameters are jointly identified, we provide an intuitive argument for identification. We confirm our argument numerically in Figure B.7 in Appendix B.3.

Equation (2.20) reveals that the matching function efficiency for skill $s$, $\mu_s$, has a direct impact on the non-employment-to-employment transition rates $\text{NE}_s$ which we target for each skill. By shifting unemployment benefits conditional on wages, the parameters $b_s$ affects the replacement rate for each skill type which we target. We inform the relative demand for high skill workers $a_3$ using the skill premium. We identify the relative demand $a_2$ for service vis-à-vis core low skill workers imposing that they earn identical average wages. The curvature in the revenue function $\rho$ shifts the aggregate labor share.

We identify the measure of goods producers $M^G$ with their average employment, and the measure of contractors $M^C$ with their employment relative to goods producers. The capital fixed cost $\eta$ affects the fraction of firms with low employment. The parameters $\gamma$ and $\nu$ jointly determine the dispersion in size and value added. When there is less curvature in recruiting costs, productive firms can hire more and dispersion in firm size increases. Conditional on size, the dispersion in productivity raises dispersion in value added. Thus, we target the standard deviation of log firm size to inform $\gamma$, and the standard deviation of log value added to inform $\nu$.

The outsourcing cost $\tau_1$ determines how much outsourcing there is in the economy by directly affecting the returns to outsourcing. Thus, we target aggregate outsourcing expenditures as a fraction of wages. We determine the measure of service low skill service workers $m_1$ by matching the employment outsourcing share for service workers. The dispersion in outsourcing costs $\sigma$ determines the fraction of goods producers who have an outsourcing

share below 10%. To inform the correlation between productivity and outsourcing costs $\iota$, we target the OLS relationship between the outsourcing share and value added. Importantly, we use a cross-sectional moment rather than a within-firm moment. We use the recruiting comparative advantage of contractors $\bar{c}^C$ to match the outsourcing wage penalty jointly with contractor firm size. We provide additional details in Online Appendix B.7.5.

### 2.5.3 Estimation results and identification

Table B.8 in Appendix B.3 summarizes our estimation results. Our estimates fall within conventional ranges found in the literature. The revenue function curvature parameter is $\rho = 0.92$. Our estimate of $a_3$ implies that the marginal product of labor for high skill workers is two and a half times as large as for low skill workers at equal employment shares. The curvature in the vacancy cost $\gamma = 10.1$ is towards the high end of values reported in the literature. Low employment dispersion relative to value added in the data together with $\rho$ being close to 1 requires a high degree of recruiting cost curvature. We find that 37% of workers in the French economy are exposed to outsourcing.

Our estimate of $\tau_1$ implies that outsourced workers are 35% less productive than in-house workers. By jointly targeting outsourcing expenditure and employment shares, the estimation requires that $\tau_1$ and the outsourcing price $p_1$ move in opposite directions. Increasing $\tau_1$ towards 1 implies that the outsourcing price $p_1$ falls. Demand rises more than one-for-one because of decreasing returns to scale. In general equilibrium, this excess demand is incompatible with the outsourcing employment share. If workers at contractors were as efficient as in-house employees, we should observe a higher outsourcing employment share relative to the expenditure share. Thus, we infer that the data favor the cost-saving view relative to the comparative advantage view.

We find that contractors, however, are more efficient than goods producers at recruiting workers. The marginal recruiting cost of contractors is 55% of the marginal recruiting cost of goods producers. This difference in marginal recruiting costs is necessary to rationalize that

contractors are larger than goods producers despite paying lower wages.[29] Thus, the data favors a view in which contractors specialize in screening and recruiting activities rather than production activities.

How well are parameters identified? To answer this question, Figure B.7 in Appendix B.3 plots both the simulated moment (univariate identification) and the loss function (multivariate identification) as we vary the parameter close to its estimated value. Most of the parameters are well identified locally. Moment deviations are steep functions of parameter deviations, and the loss function is peaked around 0.[30] Overall, Figure B.7 confirms our identification argument and supports our estimation strategy.

### 2.5.4 Over-identification

We propose three over-identification checks that relate to the three main predictions of our theory. The first relates to selection into outsourcing. The second relates to the productivity effect. The third relates to the distributional effect.

We start by verifying whether the estimated model accounts for selection into outsourcing. We have targeted the cross-sectional OLS coefficient in column (2) from Table B.3, Appendix B.2.1 to inform $\sigma$. However, focusing on within-firm changes (column 3) as well as instrumenting for firm revenue productivity (column 6) affects this coefficient. While these are non-targeted moments, can the estimated model rationalize these differences?

The solid blue line in Figure 2.8a displays the model equivalent of the coefficient in column (3), from the following experiment. Consider a one standard deviation shock to revenue productivity $z$. We estimate that $z$ and $\varepsilon$ are positively correlated—more productive firms face larger outsourcing costs. This positive correlation implies that the increase in $z$ is also associated with an average increase in $\varepsilon$. For every firm, we compute the change in the

[29]The need for such a shift in the labor supply curve of contractors to rationalize the size gap is not specific to our environment. It would be necessary in any model with a firm-specific labor supply curve to generate wage premia.

[30]Lest we simulate the model on a nineteen-dimensional hypercube, we cannot however guarantee global identification.

outsourcing share following the joint change in $(z, \varepsilon)$ and project it on the associated change in value added. We then display the resulting OLS coefficient in the model by decile of initial value added. We cannot aggregate these OLS effects in the model because we cannot reliably estimate the incidence of shocks firm by firm. Nevertheless, our empirical estimate of 1.82 lies in the middle of the OLS effects in the model that range from 0.06 to 4.15.

In the model, the within-firm OLS coefficient conflates the change in revenue productivity $z$ with the associated change in outsourcing costs $\varepsilon$. We mimick the instrumental variable strategy from Table B.3, column (6) as follows. We interpret the export demand instrument as removing the increase in $\varepsilon$ associated with the increase in $z$. We then only shift $z$ to compute the change in the outsourcing share and value added instead of the joint shift in $(z, \varepsilon)$. The dashed orange line displays our results. Consistently with the data, the model counterpart of the 2SLS estimate is much larger than the OLS estimate. This ordering occurs because of the positive correlation between $z$ and $\varepsilon$. When revenue productivity $z$ rises alone, firms are more inclined to increase outsourcing than when the cost of outsourcing increases simultaneously. Quantitatively, the magnitude of the 2SLS coefficient lies between 0.36 and 9.25 depending on the incidence of shocks in the model, consistently with our point estimate of 3.41.

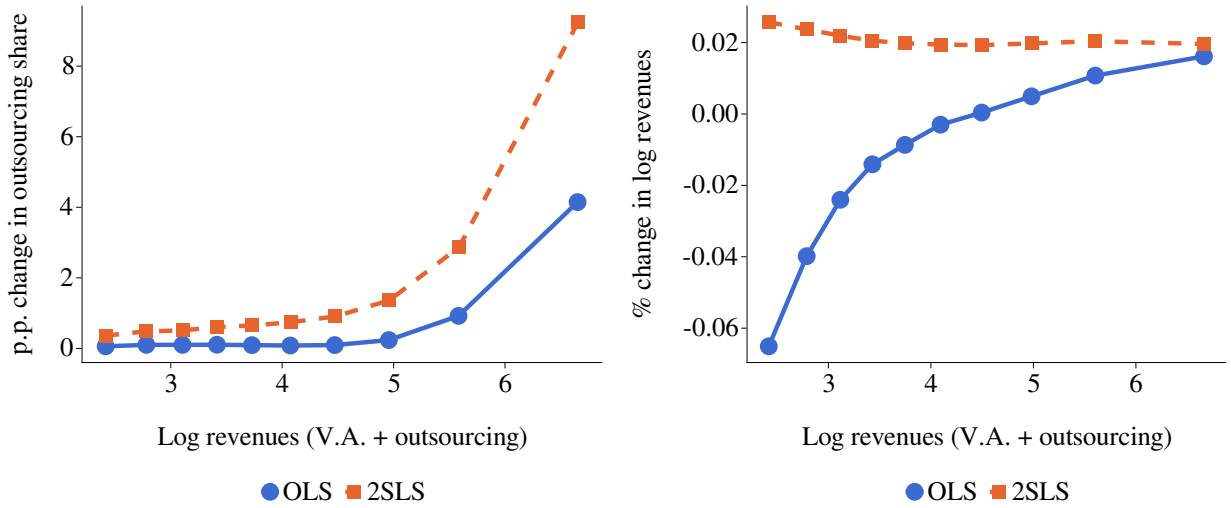
Our second over-identification exercise asks whether the estimated model accounts for the untargeted productivity effect. We mimick the OLS and 2SLS coefficients from Table B.4 similarly to selection into outsourcing. We consider a negative one standard deviation shock to the idiosyncratic outsourcing cost $\varepsilon_s$ of the firm. We then decrease revenue productivity $z$ accordingly for the OLS coefficient, or leave it unchanged for the 2SLS coefficient.

Figure 2.8b displays the within-firm model counterparts of the OLS and 2SLS coefficients from Table B.4, columns (1) and (3), by initial decile of firm value added. In the model, the OLS coefficient ranges from -0.07 to 0.01 depending on the incidence of shocks, consistently with our point estimate that is near zero. The model struggles to generate a high enough 2SLS productivity effect, which remains between 0.02 and 0.03. By contrast, our empirical

Figure 2.8: Selection into outsourcing and the productivity effect in the estimated model.

(a) Selection into outsourcing.

(b) Productivity effect.




Note: Panel (a): OLS (solid blue) and 2SLS (dashed orange) coefficients in the estimated model. OLS coefficient computed by projecting the change in the outsourcing share on the change in log value added, following a one standard deviation $\Delta z$ increase in $z$ and the corresponding change in $\Delta\varepsilon = \frac{\iota\sigma}{\nu}\Delta z$. 2SLS coefficient computed by only increasing $z$ by one standard deviation. Panel (b): OLS (solid blue) and 2SLS (dashed orange) coefficients in the estimated model. OLS coefficient computed by projecting the change in log value added on the change in the outsourcing share, following a one standard deviation $\Delta\varepsilon$ decrease in $\varepsilon$ and the corresponding change in $\Delta z = \frac{\iota\nu}{\sigma}\Delta\varepsilon$. 2SLS coefficient computed by only increasing $\varepsilon$ by one standard deviation.

estimate is close to 0.08. This limitation implies that the model will likely under-predict the employment response from outsourcing in the aggregate.

Taking stock, the estimated model is quantitatively consistent with three out of four of these non-targeted moments, and qualitatively consistent with the fourth. Hence, we conclude that the estimated model provides an empirically plausible account of selection into outsourcing and the productivity effect.

Our third over-identification exercise verifies whether the estimated model accounts for the distributional effects of outsourcing through the job ladder in the model. The estimation targets the outsourcing wage penalty, but does not target wage dispersion. Instead, wage dispersion emerges endogenously from productivity dispersion and worker flows, that are both disciplined with data. Wage dispersion is linked to other non-targeted labor market statistics such as the employment hire share or net poaching through the job ladder structure of the model.

Table 2.1: Wage dispersion and labor market statistics in the model and the data.

| | Data | | | Model | | |
|---|---|---|---|---|---|---|
| | A. Aggregates | | | | | |
| St. dev. firm premia | | | | | | |
| AKM firm premia | 0.14 | | | 0.17 | | |
| Within-skill wage st. dev. | | | | 0.16 | | |
| | B. By producer type | | | | | |
| | Contractors | Goods prod. | Diff. | Contractors | Goods prod. | Diff. |
| Firm premia | -0.14 | 0.00 | -0.14 | -0.13 | 0.00 | -0.13 |
| Frac. hires from emp. | 0.46 | 0.52 | -0.06 | 0.44 | 0.56 | -0.12 |
| Churn | 0.11 | 0.03 | 0.08 | 0.10 | 0.08 | 0.02 |
| Net poaching | -0.02 | 0.00 | -0.02 | -0.01 | 0.00 | -0.02 |

Table 2.1 indicates that the model matches these non-targeted moments well. To estimate firm wage premia consistently with our theory, we run equation (2.12) similarly to Section 2.4.3. Using the AKM decomposition (2.12), we estimate the within-skill standard deviation of firm wage premia to be 0.14 in the data. When we run the same AKM decomposition in our estimated model, the within-skill standard deviation of log wage premia is 0.17. It is close to the within-skill standard deviation of wages (0.16) because workers only differ by skill in the model.

In the data, contractors hire 46% of their workers from employment, while goods producers hire 52% of their workers from employment. These hiring patterns are closely mirrored in the model: contractors hire 45% of their workers from employment, while goods producers hire 56% of their workers from employment. Contractors have relative net poaching of -2% in the model and in the data. Contractors also have more churn in the model, qualitatively consistent with the data. We conclude that our job ladder structure provides a consistent account of wage dispersion and systematic labor market differences between contractors and goods producers. Having validated the core structure of the model, we next estimate the shocks to outsourcing that drive our counterfactuals.

### 2.5.5 Outsourcing demand and supply shocks

Our counterfactual exercises assess the impact of outsourcing on wages and employment. As in any supply and demand framework, outsourcing demand and supply shocks are expected to have different effects on wages and employment. Therefore, we consider both types of shocks. Supply shocks are changes in the productive and recruiting comparative advantage of contractors $\tau_1, \bar{c}^C$, and the measure of contractors $M^C$. Demand shocks are changes in the correlation between productivity and outsourcing costs $\iota$.

We disentangle demand and supply shocks by matching cross-industry regressions of wages and employment, value added and contractor size on the outsourcing share. We report our results in columns (1-4) of Table 2.2. We use labor shares and accounting profits in columns (5-6) as over-identification checks. We also report identification plots for each shock in Figure B.8, Appendix B.3. We discipline the overall magnitude of our shocks by replicating the increase in the aggregate expenditure share on outsourcing from Figure 2.2 between 1997 and 2007. Since our baseline parameter estimates represent the 2002 French economy, we allow $\tau_1, \bar{c}^C, M^C$ and $\iota$ to either increase or decrease in fixed proportions to match the aggregate expenditure share in 1997 and 2007.

We find evidence of demand shocks as industries with stronger increases in the outsourcing share experience a rise in value added. We report our estimates in Table B.9, Appendix B.3. The 0.04 correlation between industry-level value added changes and outsourcing changes implies that the correlation between productivity and outsourcing costs drops from 0.17 to 0.11 between 1997 and 2007, driving up demand for outsourcing.

We also infer strong supply shocks. The 0.06 cross-sectional positive correlation with employment implies that contractors experience strong improvements in their recruiting comparative advantage. $\bar{c}^C$ drops from 1.03 in 1997 to 0.31 in 2007. Our baseline estimate for 2002, 0.55, lies in the middle of these values. The 16% decline in contractor size requires that more than twice as many contractors operate in 2007 relative to 1997 given the improvement in recruiting technology. To fully match the 0.02 decline in wages, we also infer a relative

Table 2.2: Industry-level correlations between changes in outsourcing share and changes in wages, employment, value added, profits and the labor share.

| | (1) Wage | (2) Employment | (3) Value added | (4) Contractor size | (5) Profits | (6) Labor share |
|---|---|---|---|---|---|---|
| Out. share (data) | -0.02 | 0.06 | 0.04 | -0.16 | 0.06 | 0.00 |
| Out. share (model) | -0.02 | 0.06 | 0.04 | -0.15 | 0.05 | 0.00 |
| Obs. | 1150 | 1150 | 1150 | 1150 | 1150 | 1150 |

Linear regressions of changes in log mean wage, log employment, log value added, log contractor size, log residual profits and labor share on changes in the outsourcing share at the commuting zone level. Changes defined between 1996, 2001 and 2007. Residual profits defined as value added net of wage bill. Coefficients normalized to represent the effect of a 5 p.p. increase in the outsourcing share. Includes year dummies.

decline in the productive comparative advantage of contractors, which falls from 0.78 to 0.54.[31] Columns (5-6) in Table 2.2 show that the model also matches non-targeted industry-level changes. It matches the weak correlation between outsourcing and the labor share as well as the strong correlation between accounting profits and outsourcing.

## 2.6 The impact of outsourcing on inequality and output

We are now ready to investigate the impact of the rise in outsourcing on inequality and aggregate output. We compare steady-states of the estimated model, and interpret our results as the effect of outsourcing on the French labor market in the decades between 1997 and 2016. We index our counterfactuals by the aggregate outsourcing expenditure share.

As emphasized in Section 2.4.2, our firm-level expenditure data stops in 2008. We use the model to ask what is the effect of outsourcing on the French labor market by 2016. We choose the midpoint between the 2007 outsourcing expenditure share and our extrapolation in Figure 2.3a to remain conservative. To represent the 2016 economy in our model, we use demand and supply shocks in the same proportions as between 1997 and 2007, and their overall magnitude is set to match the rise in the outsourcing expenditure share. In our figures,

[31] Our model is isomorphic to one in which there is free entry of contractors, and the entry cost varies over time. Thus, the rise in the measure of contractors is consistent with a decline in contractor entry costs together with an improvement in their recruiting comparative advantage. In a broader framework in which contractors would be heterogeneous in $\tau_1$, one could also rationalize the decline in $\tau_1$ through adverse selection following the decline in entry costs. For simplicity, we instead directly estimate the decline in productive comparative advantage $\tau_1$ required to match the data.

we represent the 2007 counterfactual economy with a vertical dotted line.

### 2.6.1 The impact of outsourcing on service workers

We start with the impact of outsourcing on wages and earnings of service workers, displayed in Figure 2.9. The fraction of service workers employed at contractor firms rises by 21 p.p. by 2007, as shown in panel (a). This increase closely tracks the one observed in the data in Figure 2.3a although it is non-targeted. Thus, the link between outsourcing expenditures and outsourcing employment in the aggregate coincides in the model and in the data. By 2016, the fraction of low skill service employment at contractors has risen by 37 p.p.

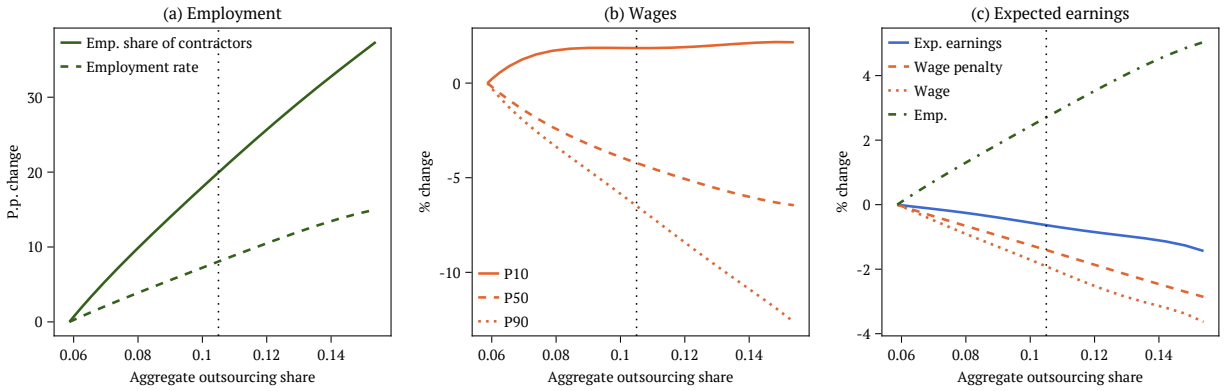
The reallocation of low skill services workers towards contractors preludes to adverse impacts on earnings because of the outsourcing wage penalty. Yet, low skill service workers also experience strong employment gains because outsourcing lowers the effective cost of labor in the aggregate. Panel (a) reveals that outsourcing lowers their non-employment rate by 14.9 p.p., down from 25 p.p in 1997.

Wages of low skill service workers respond ambiguously in general equilibrium as shown in panel (b). Rising outsourcing implies tougher competition for workers at the bottom of the job ladder where contractors operate. Therefore, wages in the bottom decile rise by 2.1%. At the same time, outsourcing removes the highest paying jobs from the labor market. In addition, the reallocation towards low-paying contractor firms drags down the entire wage distribution. Thus, median wages drop by 6.6%, and wages in the top decile fall by 12.6%.

Expected earnings of low skill service workers thus reflect several possibly offsetting forces. We present an exact decomposition of expected earnings in Appendix B.1.17. The reallocation towards contractor firms lowers earnings because of the outsourcing wage penalty. The general equilibrium response of employment raises earnings by pulling more workers out of non-employment. General equilibrium changes in the wage distribution of employed workers are mostly detrimental to earnings, except at the bottom of the job ladder.

Quantitatively, panel (c) shows that the reallocation of low skill service workers towards

Figure 2.9: The effects of outsourcing on wages, earnings and welfare of service workers.



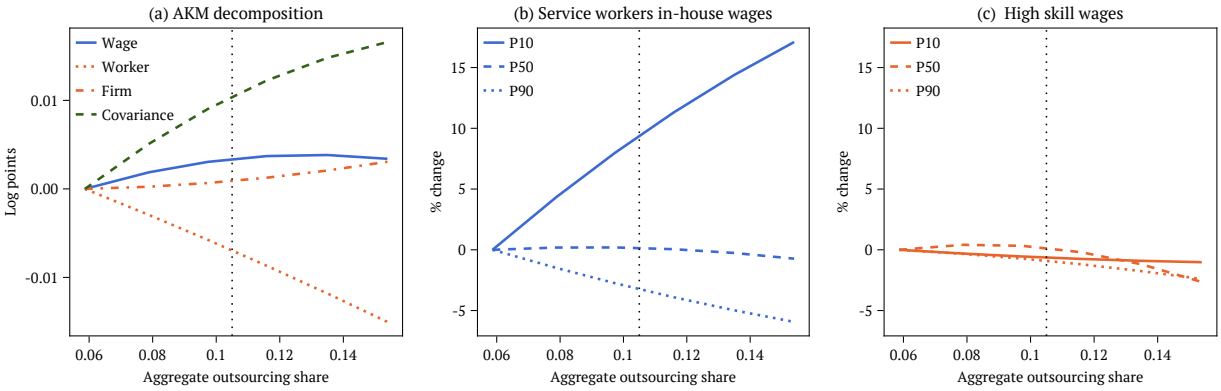

Note: Demand and supply shocks calibrated to match the rise in outsourcing expenditure share in France from 1997 to 2007. All outcomes are shown as a function of aggregate outsourcing expenditures relative to the aggregate wage bill on the x-axis. Leftmost point corresponds to 1997 economy. Vertical dotted line corresponds to counterfactual 2007 economy. Rightmost point corresponds to counterfactual 2016 economy. Panel (c): expected earnings and decomposition as in Appendix B.1.17.

contractor firms results in 2.8% earnings losses in expected earnings by 2016 once combined with the 14% outsourcing wage penalty. The general equilibrium response of the wage distribution in panel (b) aggregates to a further 3.8% decline in earnings. The rise in low skill service employment is the strongest general equilibrium feedback. As outsourcing rises, effective labor market frictions decline and so does the cost of labor. Employment of low skill service workers rises by 13%. Panel (c) shows that it improves expected earnings by 5% by 2016.

Once we aggregate these three channels, we find that low skill service workers ultimately experience a net 1.5% decline in expected earnings. Employment gains are dominated by the partial and general equilibrium declines in wages. We show in Appendix B.1.17 that expected earnings coincide with welfare given our assumption of a small discount rate. Thus, low skill service workers lose from outsourcing.

The decomposition in panel (c) highlights that general equilibrium effects are critical to evaluate the distributional impact of outsourcing on the economy. The partial equilibrium impact alone would overstate welfare losses of low skill service workers and miss important general equilibrium margins of ajustment. Our model lets us quantify these margins and aggregate them into welfare.

Figure 2.10: The impact of outsourcing on labor market sorting and wage inequality.




Note: Decomposition of log wages into worker and firm effects following equation (2.21) in the model. Panel (a): variance decomposition of log wages following equation (2.22). Panel (b): low skill service wages at goods producers. Panel (c) high skill wages.

### 2.6.2 The impact of outsourcing on sorting and wage inequality

Are welfare losses for service workers accompanied by sizeable changes in wage inequality? A rapidly expanding literature on inequality and labor market sorting hypothesizes that the outsourcing-induced reallocation of low skill workers to low paying firms as in Section 2.6.1 increases labor market sorting and inequality (see for instance Goldschmidt and Schmieder (2017) and Song et al., 2018).

We evaluate this hypothesis with our structural model. To stay as close as possible to previous work, we run an AKM regression in our estimated model similarly to Section 2.4.3:

$$\log w_s(z) = \alpha_s + \varphi(z) + \varepsilon_s(z). \tag{2.21}$$

Our main measure of labor market sorting is the correlation between worker and firm effects, $\mathbb{C}\text{orr}[\alpha_s, \varphi(z)]$. The conditional random mobility assumption necessary for identification is satisfied in our environment, although the wage function is not exactly log-linear. Nevertheless, we obtain a close fit between the estimated firm effects and actual wages in the model, as shown in Figure B.9, Appendix B.4.

Perhaps surprisingly, we find that outsourcing has a limited impact on wage inequality in general equilibrium. Panel (a) in Figure 2.10 shows that the variance of log wages barely

rises between our model 1997 and 2016 economies. In light of the conventional wisdom that outsourcing should increase wage inequality, we use the AKM decomposition in equation (2.21) to perform an exact employment-weighted variance decomposition and understand the limited impact of outsourcing on inequality:

$$\mathbb{V}\text{ar}[\log w_s(z)] = \mathbb{V}\text{ar}[\alpha_s] + \mathbb{V}\text{ar}[\log \varphi(z)] + 2\mathbb{C}\text{ov}[\alpha_s, \varphi(z)] + \mathbb{V}\text{ar}[\varepsilon_s(z)]. \tag{2.22}$$

We find that outsourcing strongly increases labor market sorting. Panel (a) reveals that the covariance between the worker component of wages, $\alpha_s$, and the firm component of wages, $\varphi(z)$, increases by nearly 0.015 between 1997 and 2016. This rise is substantial given that we only have three worker types. Our results are consistent with the hypotheses in Goldschmidt and Schmieder (2017) and Song et al. (2018). The correlation between worker and firm effects increases by 23.3 p.p. by 2016, indicative of a substantial rise in labor market sorting.

However, our general equilibrium environment uncovers a force that offsets the rise in labor market sorting: the variance of worker effects declines so much with outsourcing that it offsets the rise in the covariance between worker and firm effects. To understand this decline, three observations are necessary.

The first observation is that the AKM decomposition (2.21) identifies worker effects $\alpha_s$ by comparing wages of different workers employed at the same firm. Thus, low skill service employees at goods producers identify worker effects because they are the only ones who can be compared to other workers within the same firm. Only low skill service workers are employed at contractors. To understand the decline in the variance of worker effects, we thus need to understand how wages of low skill service and high skill workers change at goods producers.

The second observation is that outsourcing leads to pro-competitive effects at the botton of the job ladder. Wages of low skill service workers employed at goods producers respond more strongly than wages of all low skill service workers. Panel (b) of Figure 2.10 indicates

that wages at low-paying goods producers rise by over 15%. Median wages remain constant, while wages at the top decline by 5%. Compared to wage changes at all firms in Figure 2.9(b), these changes are more favorable to service workers because they net out reallocation towards contractors.

The third observation is that wages of high skill workers remain largely stable as outsourcing rises. Panel (c) of Figure 2.10 shows that wages in the bottom decile, at the median and in the top decile for high skill workers do not change as outsourcing rises. That wages of high skill workers remain stable despite the complementarity between skills in production and the expansion of low skill service employment at large productive firms (the productivity effect) is related to rent-sharing. We return to this point in the next section.

These three observations imply that the AKM decomposition (2.21) infers that low skill service workers are better paid compared to their high skill coworkers in 2016 relative to 1997.[32] Thus, the variance of worker effects declines, and wage inequality remains stable despite a substantial rise in labor market sorting.

Our results thus caution against inferring that a rise in labor market sorting necessarily implies a rise in wage inequality. Our environment highlights that outsourcing causes both rising sorting and pro-competitive effects. Wage inequality remains largely constant as both forces offset each other.

### 2.6.3 The impact of outsourcing on rent-sharing

Labor market sorting is only one aspect of inequality. Sorting captures differences in labor market opporunities between different workers. Our environment also captures how the marginal product of labor is split between workers and firms, which we define as rent-sharing. We now investigate how outsourcing changes rent-sharing at goods producers who hire in-house.

Our main measure of rent-sharing is the markdown paid by goods producers. The

[32]This implication would continue to hold in a richer environment in which contractors also employ high skill workers, as long as they employ relatively less high skill workers than goods producers

markdown is defined as the ratio of wage to the marginal product of labor: $\text{markdown}_s(z) = w_s(z)/\text{MPL}_s(z)$. A markdown close to 0 implies that workers recover a small fraction of the rents from production. A markdown close to 1 implies that workers obtain nearly all the rents from production.

A related measure of rent-sharing is the firm-level labor share:

$$\text{LS}(z) = \sum_{\text{in-house } s} \frac{n_s^*(z) w_s(z)}{\text{VA}(z)} = \sum_{\text{in-house } s} \text{markdown}_s(z) \frac{n_s^*(z) \text{MPL}_s(z)}{\text{VA}(z)}. \tag{2.23}$$

Simple accounting ensures that the labor share is a weighted average of markdowns. These weights need not add up to unity in presence of decreasing returns to scale. Markdowns are the conceptually exact metric of rent-sharing, but they are difficult to estimate at the firm-by-skill level. The firm-level labor share is a coarser measure of rent-sharing but is easier to measure.

Our micro-foundation for labor market power delivers dispersion in markdowns and labor shares across firms. We cannot measure markdowns directly, but we can contrast our model with the data for labor shares. Panel (a) in Figure 2.11 displays the distribution of labor shares across firms. Despite being entirely non-targeted, the distribution of labor shares in the model is close to the data although not entirely right-skewed enough.

Our environment also captures the co-movement of labor shares with wages and value added. Panel (b) shows that labor shares fall with firm wages, with a close fit between model and data except perhaps at the very top. Similarly, panel (c) indicates that labor shares decline with firm value added in both model and data. These results suggest that workers extract a smaller fraction of rents when working at highly productive large employers, as highlighted in Autor et al. (2020) and Gouin-Bonenfant (2022).[33]

We unpack how different skill groups contribute to rent-sharing by investigating markdowns in our economy. Panel (a) in Figure 2.12 splits the average wage paid by goods producers

[33]By contrast, models of wage dispersion based on compensating differentials do not generate within-market between-firm dispersion in labor shares or markdowns with standard revenue functions. See Online Appendix B.6.2 for details.

Figure 2.11: Rent-sharing in the labor market.




Note: Panel (a): employment-weighted distribution of labor shares. Beta distribution fitted to both model and data for comparability. Panel (b): Labor share by average wage. Panel (c): Labor share by value added. Only goods producers. Results with all firms in Figure B.10, Appendix B.4.

to service workers into the marginal product of labor and the markdown according to the identity

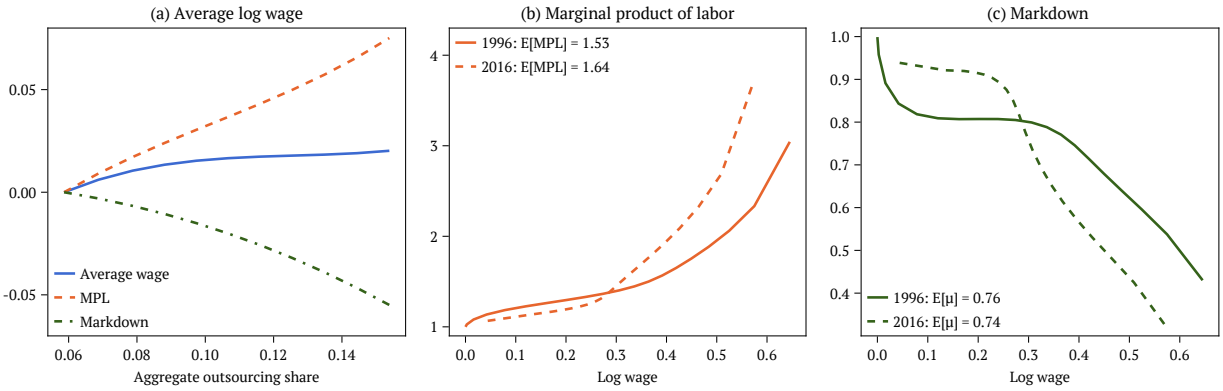
$$\mathbb{E}[\log w_s(z)] = \mathbb{E}[\log \text{MPL}_s(z)] + \mathbb{E}[\log \text{markdown}_s(z)] \tag{2.24}$$

for service workers $s = 1$.

Among goods producers, the marginal product of labor rises with outsourcing. Panel (b) shows that as service workers reallocate towards contractors, goods producers employ fewer service workers and the marginal product of labor increases. This rise is particularly pronounced at the top of the job ladder that even fewer workers ever reach. Figure B.11 in Appendix B.4 confirms that the marginal product of labor only rises at goods producers who hire in-house. When we include include goods producers who outsource, the marginal product of labor instead falls on average at highly productive firms, as firms switch from a upward-sloping to a vertical labor supply curve.

The rise in the marginal product of labor is offset by a strong equilibrium decline in the average markdown at goods producers. This decline results from the clockwise titlt in the markdown distribution depicted in panel (c). Markdowns at goods producers rise at low-paying firms. As contractors compete with goods producers for workers in the lower rungs

Figure 2.12: The impact of outsourcing on rent-sharing for service workers.




Note: Panel (a): decomposition of mean log wage for service workers at goods producers according to the identity (2.24). Panel (b): distribution of the marginal product of labor for service workers at goods producers who hire in-house. Panel (c): distribution of markdowns for service workers at goods producers who hire in-house. See Figure B.12 in Appendix B.4 for results with all firms.

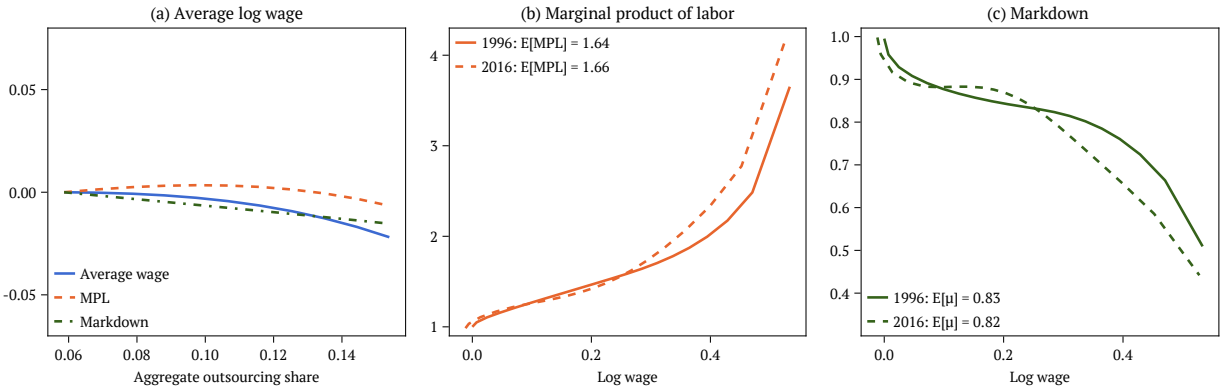
of the job ladder, workers are able to extract a larger fraction of the rents from production. Given the option to cheaply outsource however, high-paying goods producers only find it profitable to hire in-house at low markdowns. This selection implies that markdowns fall at the higher end of the wage distribution. On net, the average markdown drops and workers do not recoup the increase in the marginal product of labor. Our results highlight that both reallocation and changes in rent-sharing are key to understand how outsourcing impacts wages in general equilibrium.

The impact of outsourcing low skill service workers spills over to high skill workers through the supermodularity condition in Assumption 2.1. When employment of service workers rises at goods producers, so does the marginal product of labor for core low skill and high skill workers. Thus, our result that wages of high skill workers are largely stable as outsourcing rises (Figure 2.10) is surprising at first sight.

A more detailed investigation reveals that the marginal product of labor and markdowns also move in opposite directions for high skill workers.[34] Panel (a) in Figure 2.13 indicates that the average marginal product of labor rises moderately with outsourcing for high skill workers. Yet, the average markdown simultaneously drops, implying slightly declining wages for high skill workers.

[34]Low skill core workers display similar patterns to high skill workers.

Figure 2.13: The impact of outsourcing on rent-sharing for high skill workers.




Note: Panel (a): decomposition of mean log wage for high skill workers at goods producers according to the identity (2.24). Panel (b): distribution of the marginal product of labor for high skill workers at goods producers. Panel (c): distribution of markdowns for high skill workers at goods producers.

The marginal product of labor indeed rises only at the top of the job ladder as shown in panel (b), where firms are outsourcing service workers. Yet, markdowns further decline because of the productivity effect. Outsourcing is akin to an increase in revenue productivity from the perspective of high skill workers. Thus, goods producers that outsource more move rightwards in the markdown distribution where markdowns are lower in equilibrium (Figure 2.11). Similarly to service workers, offsetting movements in markdowns and the marginal product of labor shape high skill wages in general equilibrium.

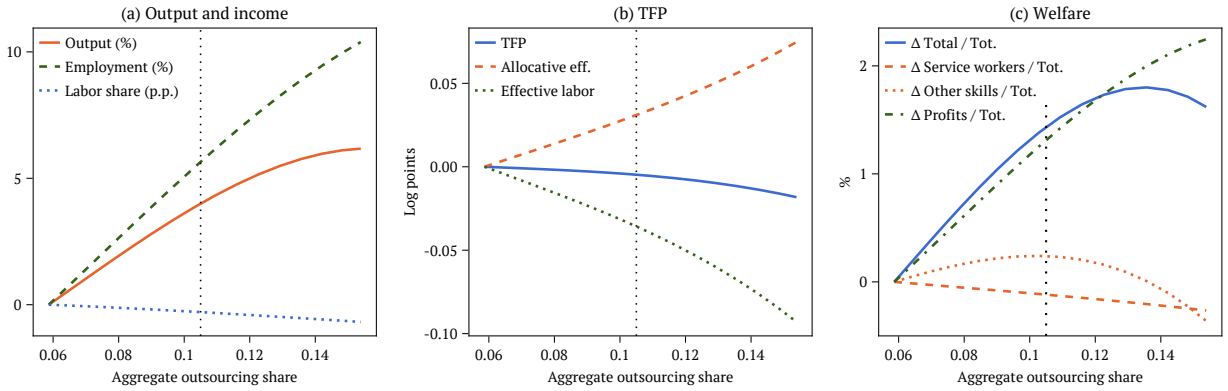
### 2.6.4 The impact of outsourcing on output and TFP

Despite the adverse effect of outsourcing on the welfare of low skill service workers and its subtle implications for sorting and rent-sharing, outsourcing may generate output and productivity gains. We describe the impact of outsourcing on aggregate output in Figure 2.14, panel (a).

We find that aggregate output rises by 6.1%, in line with strong productivity effects of outsourcing. This increase in output could in principle be driven by extensive margin adjustments—a rise in employment—or by intensive margin adjustments—a rise in TFP.

Output gains are entirely driven by the 10.3% increase in aggregate employment, in turn largely due to low skill service workers. In fact, our results indicate that outsourcing has a

Figure 2.14: The impact of outsourcing on aggregate output and TFP.




Note: Demand and supply shocks calibrated to match the rise in outsourcing expenditure share in France from 1997 to 2007. All outcomes are shown as a function of aggregate outsourcing expenditures relative to the aggregate wage bill on the x-axis. Leftmost point corresponds to 1997 economy. Vertical dotted line corresponds to counterfactual 2007 economy. Rightmost point corresponds to counterfactual 2016 economy. Panel (b): TFP calculated as in equation (2.25).

negative 2% impact on aggregate TFP. Panel (b) reveals that this decline in turn masks two offsetting effects captured by an exact decomposition.

We define two labor aggregators consistent with our production function. The first one is simply the Cobb-Douglas labor aggregator that correponds to the revenue function: $\widehat{N} = \left(\prod_{s=1}^{3} \overline{N}_s^{a_s}\right)^{\rho}$, where $\overline{N}_s$ denotes total employment of skill $s$. The second aggregator is $\widetilde{N} = \left(\overline{N}_1^G + \tau_1 \overline{N}_1^C\right)^{\rho a_1} \left(\prod_{s=2}^{3} \overline{N}_s^{a_s}\right)^{\rho}$ where $\overline{N}_1^G, \overline{N}_1^C$ denote aggregate employment of service workers at goods producers and contractors, respectively. $\widetilde{N}$ captures the idea that effective labor of low skill service workers is lower when more of them work at contractors if $\tau_1 < 1$. Changes in the ratio $\widetilde{N}/\widehat{N}$ thus encode changes in effective labor. Our exact TFP decomposition reads:

$$\Delta \log \text{TFP} = \underbrace{\Delta \log \frac{M^G \mathbb{E}[R(z, \boldsymbol{n})]}{\widetilde{N}}}_{\substack{\text{Allocative efficiency} \\ \text{given effective labor}}} + \underbrace{\Delta \log \frac{\widetilde{N}}{\widehat{N}}}_{\text{Effective labor}} . \tag{2.25}$$

Consistently with the idea that outsourcing reallocates labor towards highly productive firms that were constrained by labor market frictions, allocative efficiency rises by 7%. But the reallocation of low skill service workers towards less productive contractors drags TFP down by nearly 9%. Crucially, the comparative advantage view would predict a rise in

effective labor. By contrast, the cost-saving view reveals that allocative TFP gains from outsourcing are muted by a strong reduction in effective labor.

Perhaps surprisingly, we find that outsourcing has little effect on the labor share. This result follows from two observations. First, high skill and core low skill workers workers experience modest wage and employment changes. Thus, they do not contribute strongly to changing the labor share through either changes in markdowns or weights as per the decomposition (2.23). Second, markdowns and weights move in opposing directions for low skill service workers. Markdowns decline (panel (a) in Figure 2.12). However, service workers are also reallocated towards contractors. Contractors locate at the bottom of the job ladder and thus have high labor shares. These two effects cancel out in the aggregate.[35] As a result, panel (c) reveals that all worker types experience weak or negative welfare changes, while profits rise and drive up aggregate welfare.

### 2.6.5 The impact of outsourcing with a binding minimum wage

Our analysis finds that low skill service workers ultimately lose from outsourcing. A natural question is whether standard labor market policy instruments can shield low skill service workers from the outsourcing wage penalty while maintaining most employment and output gains. We focus on the minimum wage as our main policy instrument. The minimum wage is a natural candidate since it maintains wages at any desired level. The counterpart is that the minimum wage may push up the price of labor so much that it deters vacancy creation and reduces employment, ultimately lowering output. We use our estimated model to explore whether outsourcing can benefit workers and firms equally with a simple minimum wage reform.

We conduct a minimum wage experiment, in conjunction with the same outsourcing demand and supply shocks as in Sections 2.6.1 to 2.6.4. The minimum wage is only binding

[35] Autor et al. (2020) find within-firm declines of the labor share, but reallocation towards firms with low labor shares. In our environment, outsourcing has opposite implications and can have neutral effects on the labor share.

Table 2.3: Outsourcing versus minimum wage and employment policies.

| | Outsourcing | Out. with min. wage | Min. wage |
|---|---|---|---|
| Minimum wage (%) | | 5.50 | 5.50 |
| Low-skill service non-emp. rate (p.p.) | -14.9 | -13.8 | 0.37 |
| Average wage (%) | | | |
| Low skill service | -6.81 | -5.07 | 1.42 |
| Other skills | -2.13 | -1.42 | 0.51 |
| AKM correlation (p.p.) | 23.3 | 22.9 | -0.31 |
| Worker expected earnings (%) | | | |
| Low skill service | -1.49 | 0.00 | 0.99 |
| Other skills | -0.67 | -0.09 | 0.33 |
| Outsourcing (expenditure share, p.p.) | 9.52 | 8.95 | 0.02 |
| Value added (%) | 6.09 | 6.51 | 0.15 |
| TFP (%) | -1.76 | -1.01 | 0.49 |
| Profits (%) | 8.51 | 8.42 | -0.32 |
| Total welfare (%) | 1.57 | 2.13 | 0.29 |

for low skill service workers in all experiments. We increase the minimum wage by 5.5%, a value chosen to ensure constant expected earnings for low skill service workers between 1997 and 2016.

Table 2.3 summarizes our results. The first column reports results for the case without any minimum wage, and coincides with the results in Sections 2.6.1 to 2.6.4. The second column shows that a 5.5% minimum wage increase moderately attenuates the employment gains from outsourcing from 14.9 p.p. to 13.8 p.p as it raises the cost of labor. The outsourcing expenditure share remains virtually unchanged, because outsourcing depends on wages at goods producers relative to wages at contractors. By contrast, the minimum wage pushes up all wages in the economy. For comparison, the third column reports the impact of the minimum wage increase without outsourcing demand and supply shocks.

Perhaps surprisingly, output increases by a modest but additional 0.41% under the minimum wage raise. This additional increase in output is due to a 0.77% relative improvement in aggregate TFP: the minimum wage increases alleviates well-known composition externalities

whereby low productivity firms hire too many workers in equilibrium (Acemoglu, 2001, Bilal, 2021). The reduction in these composition externalities interacts with outsourcing, as TFP gains surpass those with only the minimum wage increase and no outsourcing in the third column. Outsourcing exacerbates composition externalities, since contractors operate at the bottom of the job ladder and also post excess vacancies.

Profits increase by less with the minimum wage hike, but moderately enough that aggregate welfare rises. Overall, we conclude that a moderate increase in the minimum wage can ensure that outsourcing benefits workers through both employment and wage gains, at moderate costs for the shareholders of firms.

## 2.7 Conclusion

This chapter started with a theory of domestic outsourcing. We have argued that it is useful to conceptualize the outsourcing decision of firms in the context of frictional labor markets. Monopsony power and firm wage premia emerge in equilibrium. More productive firms are more likely to outsource. Outsourcing raises output at the firm level. Contractors endogenously locate at the bottom of the job ladder, implying that outsourced workers receive lower wages. Together, these observations characterize the tension between productivity enhancements and redistribution away from workers that is tied to outsourcing. Using firm-level instruments for outsourcing and revenue productivity, we have proposed new reduced-form evidence that confirms the productivity and redistributive effects of outsourcing. Finally, equipped with a structurally estimated model, we have shown that outsourcing deteriorates labor market prospects for low skill service workers. Accompanied by a minimum wage increase, outsourcing can benefit both workers and firms.

There are at least four natural directions along which to expand this research agenda. First, identifying explicit make-or-buy frictions and integrating them with our labor market theory of outsourcing may lead to novel policy implications. Second, the comparative advantage and

cost-saving views can in principle be contrasted with the data for high skill workers for whom an outsourcing wage premium may arise. Third, our environment with outsourcing could be adapted to study the implications of the gig economy for inequality and output. Fourth, due to its tractability under parsimonious assumptions, our framework is naturally equipped to study questions with an efficiency-equity trade-off that involve wages and scale-biased aggregate transformations, such as trade liberalizations, automation or the rise of artificial intelligence.

# Chapter 3

# Supply Chain Resilience: Should Policy Promote International Diversification or Reshoring?

Coauthors: Gene Grossman and Elhanan Helpman.

## 3.1 Introduction

> The United States needs resilient, diverse, and secure supply chains to ensure our economic prosperity and national security. Pandemics and other biological threats, cyber-attacks, climate shocks and extreme weather events, terrorist attacks, geopolitical and economic competition, and other conditions can reduce critical manufacturing capacity and the availability and integrity of critical goods, products, and services. Resilient American supply chains will revitalize and rebuild domestic manufacturing capacity, maintain America's competitive edge in research and development, and create well-paying jobs.
>
> Joseph R. Biden, Jr., Executive Order on America's Supply Chains, 02/24/21

Supply chain disruptions have become the new normal. The Great East Japan Earthquake of 2011 and the massive tsunami that it triggered brought such events to the attention of economists. Since then, hardly a month passes without news of a fresh disturbance. The pace of disruptions has quickened with the advent of the COVID-19 pandemic, and now we hear regularly of supply chain breakdowns in industries as disparate as automobiles, dishwashers, plastics, copper wire, lumber, pork, and toilet paper.

Disruptions have a myriad of causes. They result from natural disasters, geopolitical disputes, transportation failures, cyber-attacks, fires, power outages, labor shortages, human error and pandemic lockdowns. McKinsey Global Institute (2020), which recently conducted a series of interviews with supply chain experts, reports that disruptions lasting one to two weeks happen to a given company on average every second year, while those lasting one to two months occur every 3.7 years. The disruptions impose significant costs, presenting firms with expected losses per decade that averaged 42% of their annual pre-tax earnings.

Many commentators associate the increasing frequency and severity of supply chain disruptions with the perils of globalization.[1] *Global* supply chains leave firms exposed to risks in multiple countries, some quite distant from where consumption takes place. This perceived connection between supply shortages and international trade, in turn, has sparked soul searching amongst policy makers and a call to action in the broader public. If costly shocks reflect concentration of input supplies, wouldn't it be sensible for governments to encourage firms to diversify their international sourcing? And if distance from suppliers intensifies the risk of disruption, wouldn't it be better to bring some parts of the supply chains closer to home? The preamble to President Biden's Executive Order suggests that "*diverse* and secure supply chains" are a prerequisite for economic prosperity and that "resilient American supply chains" will rest on "rebuil[t] *domestic* manufacturing capacity [emphasis added]."

Little is known about the efficacy of policies aimed at global supply chain management in

[1]See, for example, Shih (2020b) and Shih (2020a), Iakovou and White (2020), and Baldwin and Freeman (2022).

an environment with recurring disturbances. Disruptions generate input shortages that can give rise to price spikes or even outright unavailability of downstream products. Consumers suffer from their hampered ability to purchase the products they covet. To the extent that households forfeit consumer surplus in the face of supply chain disruptions, governments may have reason to enact policies that curtail their occurrence. But production impediments impact not only consumers' surplus, but also firms' bottom lines. The question for governments is not whether shortages adversely affect households, but whether firms' private incentives to avoid such shortages fall short of (or exceed) what is socially desirable.

In this chapter, we propose a bare-bones framework to evaluate policies that can alter the organization of global supply chains. We abstract from all complexity in the production process by assuming that home firms manufacture unique varieties of nontraded differentiated products using a single, critical input. Firms face a choice of whether to procure their inputs at home or abroad. Sourcing from a foreign supplier is tempting, because production costs are assumed to be cheaper there. But foreign sourcing may be riskier than local sourcing, for geopolitical, logistical or other reasons. Thus, firms may face a tradeoff between the lower costs of offshoring and the greater safety of onshoring. They might also invest in resilience by establishing multiple supply relationships. Redundancy is costly, but it allows a firm to be active in more states of the world.

Focusing on welfare in the country where the final goods are consumed, we identify three potential distortions in private sourcing decisions. First, firms typically do not capture all of the surplus generated by the availability of their product. This *consumer-surplus externality* suggests that too many firms may be ready to accept the extra risk of foreign sourcing in exchange for lower expected costs and that too few firms may bear the extra costs of diversification. Meanwhile, when firms manage to avoid disruptions, some of their profits come at the expense of competitors that are also viable in the same state of the world. This *business-stealing externality* tends to lead firms to *overweight* the resilience of their networks. Finally, a *consumption distortion* arises when prices of differentiated products

exceed marginal costs while other goods are priced competitively. In a setting with potential supply disruptions and positive markups, government policy might be directed to encourage product availability in states of the world when markups would otherwise be especially high.

Inasmuch as the social cost of supply chain disruptions stems from loss of consumer surplus, the form of consumer preferences plays a crucial role in our policy analysis. It has become commonplace to use CES preferences in trade models with endogenous entry, but the very special properties of these preferences have been recognized since the seminal work by Dixit and Stiglitz (1977). In many contexts with CES preferences, the consumer-surplus externality from extra product variety happens to exactly offset the business-stealing externality from extra competition. These considerations apply as much to investments in "resilience" as they do to investments in entry, so it is important for understanding the efficient organization of global supply chains to allow for more flexible forms of demand. To this end, we follow Matsuyama and Ushchev (2017) and Matsuyama and Ushchev (2020b) in adopting a broader class of preferences that are *Homothetic with a Single Aggregator* (HSA). With HSA preferences, the demand for any variety depends on its price relative to a (common) aggregator of all prices. The CES utility function is a member of this class, but, more generally, HSA preferences allow the demand elasticity to increase with price. This property of demand, which characterizes many consumer goods, has been termed "Marshall's Second Law of Demand" (or MSLD). When it applies, the consumer-surplus and business-stealing externalities do not cancel.

We find that, with CES preferences for the differentiated products and optimal subsidies to address consumption distortions arising from markup pricing, the planner need not influence incentives for diversification, nor those for sourcing at home versus abroad. But with more general forms of HSA preferences that obey MSLD, the planner requires not only consumption subsidies, but also a policy to discourage diversification and another to alter the incentives to form chains at home versus abroad. The optimal diversification tax corrects for the business-stealing externality, which generally exceeds the consumer-surplus externality under

MSLD. The optimal policy to influence offshoring versus onshoring reflects that, with a non-constant elasticity of substitution, the benefits from greater competition differ across states of the world.

Turning to the second-best policy problem that arises when consumption subsidies are infeasible, we find that CES preferences dictate a subsidy for diversification as the only necessary supply chain policy. These subsidies are a second-best response to the distortion from markup pricing. However, with more general HSA preferences, the second-best policy must also take into account the dominance of the business-stealing externality relative to the consumer-surplus externality, as well as the different distortions from markup pricing that emerge in different states of the world. For example, if production costs abroad are nearly the same as those at home but the risk of a supply disruption is higher, there will be reduced product availability and less competition in states of the world when procurement from foreign suppliers is disrupted than in states where disturbances happen locally. Price-cost markups will be larger in the former states absent government policy, and second-best policy will tilt supply chain formation in favor of offshoring. On the other hand, if the two countries are relatively similar in their risks but differ greatly in costs, a tax on offshoring or a subsidy for onshoring may be indicated.

This chapter fits into an earlier literature on trade disruptions in a neoclassical setting. Much of this previous work addressed optimal policy responses to potential trade embargoes. Mayer (1977) showed that production subsidies are an optimal response to threats of trade interruption in the presence of costly adjustment. Bhagwati and Srinivasan (1976) made the likelihood of a disruption a function of the volume of trade and elucidated an efficiency role for tariffs to give agents an incentive to internalize the externality arising from their effect on the probability of a trade restriction. Arad and Hillman (1979) extended these earlier papers to allow for learning-by-doing in the production of a good that might later be subject to an embargo. Bergström et al. (1985) developed an infinite-horizon model to study the potential role of inventories to mitigate the threat of embargo. Perhaps the most sophisticated of

these early studies was that by Cheng (1989), who considered recurrent embargo threats as a stationary Markov process that plays out with constraints on the speed of intersectoral reallocation.

The main difference between our work and this earlier literature stems from our treatment of the endogenous availability of differentiated products. With perfect competition and homogeneous goods, aggregate *quantities* matter for welfare but the availability of a particular firm's offering does not. If a disruption causes some import good to be unavailable, there is no harm to consumers beyond the higher price of the domestic (perfect) substitutes. Of course, higher sticker prices play a role in a world with differentiated products, but there is also a direct harm to consumers from a particular variety not being available for purchase. For this reason, we believe that endogenous determination of the set of available products should feature prominently when evaluating policy toward supply-chain security.

This chapter also relates to a literature on distortions in the entry process, which began with the seminal paper by Dixit and Stiglitz (1977) and includes more recent contributions by Bilbiie et al. (2012), Matsuyama and Ushchev (2020b), Baqaee and Farhi (2020), among others. This literature focuses on entry of new firms into an industry in settings with imperfect competition. Bilbiie et al. (2012) study pricing and entry over the business cycle, focusing on intertemporal variation in markups and their relation to intertemporal marginal rates of substitutions. Baqaee and Farhi (2020) decompose the welfare losses from inefficient pricing and entry by calculating second-order approximations around an efficient equilibrium, while mostly treating markups as given. Matsuyama and Ushchev (2020b) analyze the inefficiencies that arise in a one-sector model with endogenous entry and endogenous markups. They introduce HSA preferences to allow for non-constant markups and Marshall's Second Law of Demand, an original approach that proves very useful in our context as well. This chapter differs from this literature inasmuch as we do not consider new entry into an industry, but rather the organization of supply chains that determines product availability in different states of the world. Unlike Baqaee and Farhi (2020), we derive exact wedges that describe

the gaps between social valuations and private valuation and use these wedges to characterize first-best and second-best policies. Our model is designed to address inefficiencies in *global* value chains in the face of supply disruptions and so we emphasize the asymmetries in cost and risk that often characterize international trade in intermediate goods.[2]

The remainder of this chapter is organized as follows. In Section 3.2, we develop our model of risky supply chains and describe the *laissez-faire* equilibrium. Then, in Section 3.3, we pose and solve the social planner's problem that arises when she has access to state-and-product specific consumption subsidies to offset the inefficiencies caused by monopoly pricing. We characterize the "wedges" between social and private incentives for forming supply relationships in each country relative to an alternative of diversified sourcing. Armed with this understanding of the sources of inefficiency in supply-chain formation, we turn in Section 3.4 to the more realistic policy problem that arises when consumption subsidies are infeasible. We characterize the supply chain policies that achieve a constrained optimum for arbitrary cost and risk parameters when preferences take the CES form. For more general, HSA preferences, we derive analytical results that apply when cost and risk parameters are not very different in the two countries. We then turn to numerical simulations in Section 3.5 to extend our insights to asymmetric cost and risk parameters. Section 3.6 concludes. The appendix contains further details, proofs, and additional numerical simulations.

## 3.2 A Simple Model of Risky Supply Chains

### 3.2.1 Supply Relationships

The home economy can produce a homogeneous, numeraire good and potentially a unit measure of nontraded differentiated consumer products. Revenues from sales of the numeraire good amount to $\bar{Y}$, all of which is paid to workers as labor income. Production of differentiated

[2]A tangentially related paper is Elliott et al. (2022), who study networks with idiosyncratic probabilities of breakup that depend on investment choices. They do not entertain aggregate shocks as here. Rather, they are interested in the propogation of idiosyncratic shocks up and down the supply chain.

products requires no labor. Rather, firm $\omega$ in this industry converts a single, customized critical input into the final good $\omega$ using the linear production technology,

$$x(\omega) = m(\omega),$$

where $x(\omega)$ is output of good $\omega$ and $m(\omega)$ is the quantity of the customized input.[3] If the firm has established a supplier relationship in country $i$ and if that supply chain is operative, then the firm can procure the customized inputs at a cost of $q_i$ per unit, $i \in \{H, F\}$, where the subscripts denote "home" and "foreign," respectively, and we assume that $q_F < q_H$ to capture the motive for the internationalization of the value chain.[4]

To form any supply relationship, a firm must bear a sunk investment cost, $k$, in units of the numeraire good. This cost represents the up-front outlays associated with searching for a partner, negotiating a contract, and designing a suitable input. It captures the costliness of resilience, inasmuch as firms that form multiple supply relationships bear extra expenses compared to those whose supply chain is more streamlined.

Once a supply relationship has been established, it is subject to two possible "disruption shocks." With probability $1 - \rho \geq 0$, any particular supply chain breaks down for exogenous and idiosyncratic reasons, which might be a failure of the prototype input, a strike in the supplier factory, a localized weather event in the location where the input would be produced, or anything else that happens independently of all other supply relationships. In any of these circumstances, the downstream firm loses the ability to purchase its input from the particular supplier for the length of the period captured by our model.[5] In the complementary event, with probability $0 < \rho \leq 1$, no idiosyncratic supply disruption occurs and the firm can buy as

[3]Note that the "producer" of the final good might actually be a retailer and the "input" might be a consumer product.

[4]The constant cost of procurement fits best with a market structure in which the downstream firm is vertically integrated with its upstream subsidiaries. Alternatively, we can imagine a situation in which the downstream firm has all the bargaining power in its relationship with arms-length suppliers and the marginal cost of the input is constant.

[5]We treat all disruptions as catastrophic; when they occur, they eliminate all input supply from the affected source. Alternatively, we could allow for less severe shocks that limit supply to some positive quantity or that raise the cost of purchases above $q_i$.

much as it wants from the particular supplier provided that the latter is located in a country that is "open for business."[6] However, with probability $1-\gamma_i$ a country-wide shock disrupts all chains with suppliers in country $i$. These shocks, which we assume to be independent across countries (to simplify the expressions, but with no substantive importance), represent events such as epidemics, political conflicts between national governments, or failures of the national transportation system. The relative safety of the home country is captured by the assumption that $\gamma_H > \gamma_F$.

The realizations of the two country-wide shocks generate four possible aggregate states of the world that we denote by $J \in \{H, F, B, N\}$. In state $H$, which occurs with probability $\delta^H = \gamma_H (1-\gamma_F)$, foreign sources of supply are unavailable, but a firm that has a supplier in the home country can still operate, conditional on its relationship there surviving any idiosyncratic shock. In state $F$, which happens with probability $\delta^F = \gamma_F (1-\gamma_H)$, a common shock hits home input suppliers and only those downstream firms with supply relationships abroad might operate. In state $B$, no broad-based disruptions occur and every downstream firm that avoids idiosyncratic shocks to some of its supply relationships will produce positive output. This state arises with probability $\delta^B = \gamma_H \gamma_F$. Finally, with the residual probability $\delta^N = (1-\gamma_H)(1-\gamma_F)$, both countries suffer adverse shocks and no production or consumption of differentiated products takes place.

### 3.2.2 Preferences and Demand

There is a unit mass of identical consumers in the home country. The representative consumer is risk neutral and holds quasi-linear preferences over consumption of the homogeneous good, $Y$, and consumption of an aggregate index of differentiated products, $X$. We represent her

[6] Our analysis could be conducted without the idiosyncratic shocks, i.e., with $\rho = 1$. However, $\rho < 1$ provides an additional incentive for diversification, and it implies that not all firms will be able to operate even when neither country suffers a broad-based disruption. Moreover, the number of active firms in the state of the world when neither source country is fully disrupted will depend on the sourcing strategies adopted by the firms, which then becomes a consideration in the choice of policy.

(cardinal) utility as

$$V(X,Y) = Y + U(X), \tag{3.1}$$

where $U(\cdot)$ has a constant elasticity $\varepsilon > 1$;[7] i.e.,

$$U(X) = \frac{\varepsilon}{\varepsilon - 1}\left(X^{\frac{\varepsilon-1}{\varepsilon}} - 1\right) \qquad \text{for } \varepsilon > 1.$$

Each consumer maximizes utility in any state of the world subject to a standard budget constraint, $Y + \int_{\omega\in\Omega} p(\omega)\, x(\omega)\, d\omega = I$, where $p(\omega)$ is the price and $x(\omega)$ the quantity purchased of variety $\omega$, $\Omega$ is the set of varieties available in the relevant state, and $I$ is income. When consumers have sufficient income, the constant elasticity of $U(X)$ gives rise to a constant-elasticity demand for differentiated products,

$$X = P^{-\varepsilon}, \tag{3.2}$$

where $P$ is the real price index dual to $U$. The consumer spends $PX = P^{1-\varepsilon}$ on differentiated products and devotes residual spending of $I - P^{1-\varepsilon}$ to the homogeneous good.

Following Matsuyama and Ushchev (2017) and Matsuyama and Ushchev (2020b), we assume that preferences for the bundle of differentiated products belong to a class they aptly term *Homothetic with a Single Aggregator.* Homotheticity implies that the consumption index $X$ is a linearly homogenous function of consumption of the individual varieties $\{x(\omega)\}_{\omega\in\Omega}$. A single aggregator, $A$, which is a linearly homogenous function of the set of prices $\{p(\omega)\}_{\omega\in\Omega}$, guides the substitution between a particular variety $\omega$ and all other varieties. More formally, HSA preferences require the existence of a price aggregator $A$ and market-share function

[7] A unitary elasticity can be treated as a limiting case as $\varepsilon \to 1$. We cannot allow $\varepsilon = 1$, because then $V \to -\infty$ in the state of the world when all firms face supply disruptions in both countries. We could introduce a backup technology such that firms can produce their own critical inputs at some high cost $\bar{q} > q_H$ and then we could entertain $\varepsilon = 1$ and even $\varepsilon < 1$. But introducing the additional parameter $\bar{q}$ complicates the expressions without providing additional insights. Accordingly, we choose to restrict the range of the demand elasticity, in keeping with the empricial evidence provided by Fajgelbaum et al. (2020).

$s\left[p\left(\omega\right)/A\right]$ that is non-negative for all relative prices such that

$$\frac{d\log P}{d\log p\left(\omega\right)}=s\left[z\left(\omega\right)\right] \tag{3.3}$$

and

$$\int_{\omega\in\Omega}s\left[z\left(\omega\right)\right]d\omega=1, \tag{3.4}$$

where $z\left(\omega\right):=p\left(\omega\right)/A$ is the price of variety $\omega$ relative to the price aggregator. Equation (3.3) expresses the demand for any variety $\omega$ in implicit form; the substantive assumption is that this demand depends only on the price of that variety relative to a *common* aggregator. Equation (3.4) stipulates that the market shares sum to one.

We place some mild restrictions on the market-share function, $s\left(z\right)$. First, we impose

**Assumption 3.1.** *The market-share function $s\left(z\right)$ is strictly decreasing when positive, with $lim_{z\rightarrow\bar{z}}s\left(z\right)=0$, for $\bar{z}\equiv\inf\left\{z>0|\,s\left(z\right)=0\right\}$.*

The assumption that $s\left(z\right)$ is strictly decreasing ensures that all varieties in $X$ are gross substitutes. It admits both the case when $\bar{z}<\infty$, so that demand "chokes" at some finite relative price, and the case $\bar{z}=\infty$, when positive quantities are demanded at any finite price. We also assume that the aggregator $A$ is well defined for any measure of firms.[8]

Equation (3.3) implies that the elasticity of substitution between any two goods with equal prices is a function of the common relative price, and is given by

$$\sigma\left(z\right)=1-\frac{zs'\left(z\right)}{s\left(z\right)}>1.$$

We further adopt

**Assumption 3.2.** *(i) Either $\sigma\left(z\right)=\sigma$ or $\sigma'\left(z\right)>0$ for all $z\in\left(0,\bar{z}\right)$ and (ii) $\sigma\left(z\right)>\varepsilon$ in the neighborhood of the equilibrium and the social optimum.*

[8]Matsuyama and Ushchev (2020a) and Matsuyama and Ushchev (2022) discuss various ways to ensure the existence of a well-defined aggregator, such as, for example, limiting the size of the market relative to the fixed cost of entry or assuming that $s\left(z\right)$ is large enough for the lowest feasible value of $z$.

The first part of Assumption 3.2 allows for the case of *Symmetric CES* preferences, where $s(z) = \alpha z^{1-\sigma}$, $\alpha > 0$. For all other HSA preferences, we impose *Marshall's Second Law of Demand (MSLD)*, namely that the demand for a good becomes more elastic as its price rises.[9] For example, the *Symmetric Translog* preferences, developed by Feenstra (2003), drawing on Diewert (1974), satisfy MSLD. These preferences can be represented by a market-share function $s(z) = -\theta \log z$, $z \in (0, 1)$, $\theta > 0$. Then $\sigma(z) = 1 - 1/\log z$.

The second part of Assumption 3.2 ensures that the demand for any variety $\omega$ increases when the aggregate price of competing brands rises. For some market-share functions, this assumption might be satisfied at all values of $z \in (0, \bar{z})$. For others, we would need to verify that it is satisfied *ex post*, i.e., after solving for the equilibrium.

Finally, we note for future reference the relationship between the price index $P$ and the demand aggregator $A$ that applies for any HSA preferences. Matsuyama and Ushchev (2020b) prove that

$$\log P = C + \log A - \int_{\omega \in \Omega} \int_{p(\omega)/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta d\omega, \tag{3.5}$$

where $C$ is a constant.

### 3.2.3 Profit Maximization in State $J$

Once the state of the world has been realized, the surviving producers purchase inputs and set prices to maximize profits taking into account the aggregate demand for differentiated profits (summarized by $P^J$) and the competition they face (summarized by $A^J$). The firm producing variety $\omega$ maximizes its profits in state $J$ by procuring its inputs from its least-cost, viable supplier and by marking up its price relative to that cost. Specifically, a firm that pays

[9] Zhelobodko et al. (2012) and Mrázová and Neary (2017) introduce MSLD by assuming what they refer to as "increasing relative love of variety" and "sub-convex" demand, respectively. In each case, their preferences are directly explicitly additive (DEA), which rules out homotheticity; see Matsuyama and Ushchev (2017). One advantage of the HSA class of utility functions, relative to DEA and many others used in the literature, is that it allows for a non-constant elasticity of substituion without violating homotheticity. See Matsuyama (2023) for further discussion.

$q_K$ for its inputs in state $J$ solves

$$p^{J,K}(\omega) = \arg\max_{\mathsf{p}} \left(P^J\right)^{1-\varepsilon} s\left(\frac{\mathsf{p}}{A^J}\right) \mathsf{p}^{-1}\left[\mathsf{p} - q_K\right] \text{ for } J \in \{H, F, B\}, \ K \in \{H, F\},$$

taking the state-contingent price index $P^J$ and the state-contingent aggregator $A^J$ as given. Profit maximization requires

$$p^{J,K}(\omega) = \frac{\sigma\left[z^{J,K}(\omega)\right]}{\sigma\left[z^{J,K}(\omega)\right] - 1} q_K \tag{3.6}$$

and yields operating profits

$$\pi^{J,K}(\omega) = \frac{s\left[z^{J,K}(\omega)\right]}{\sigma\left[z^{J,K}(\omega)\right]} \left(P^J\right)^{1-\varepsilon}, \tag{3.7}$$

where $z^{J,K}(\omega) := p^{J,K}(\omega)/A^J$.[10] Notice that the price of any variety might vary across states of the world and with the source of its inputs. The markup reflects the elasticity of demand, as usual, but the latter is not constant; rather, it reflects the cost and availability of inputs. Markups will be higher in states of nature with dampened competition due to supply disruptions than in states with more ample competition. Similarly, markups will be higher for goods produced with high cost (domestic) inputs than goods produced with cheaper (foreign) inputs.

### 3.2.4 Supply Chain Management

The identical households collectively own the unit measure of downstream (potential) producers. Since the quasi-linear utility represented by (3.1) implies that these households are risk neutral with respect to income shocks, the firms make their ex-ante investment to maximize expected profits. We allow firms to choose among three modes of organization (plus exit). Strategy $h$ entails investment in a single supply relationship in the home country in the hope

[10] Equations (3.6) and (3.7) reduce to the familiar pricing and profit expressions for the CES case, where $\sigma$ is constant and $s(z) = \alpha z^{1-\sigma}$.

of "onshoring." Strategy $f$ entails investment in a single relationship in the foreign country in the hope of "offshoring." Strategy $b$ (for "both") involves diversification, i.e., investment in supply relationships in both places with the intention of sourcing from the low-cost foreign supplier if that is possible, and from the higher-cost domestic supplier if that is possible and the low-cost foreign option is not available.[11]

Firms calculate expected profits with rational expectations about prices, sales, and costs in each state of the world, in view of the fraction of their competitors that pursues each strategy in equilibrium. Let $\mu_j$ be the fraction of firms that opt for strategy $j$, $j \in \{h, f, b\}$, with $\sum_j \mu_j \leq 1$. In state $H$, when all foreign sources of supply are disrupted, only those firms that have chosen strategy $h$ or strategy $b$ might operate, and among those, only the ones that avoid an idiosyncratic shock to their home supplier. Each such firm faces competition from $\rho(\mu_h + \mu_b)$ others, all of which have a unit cost of $q_H$. Analogously, in state $F$, it is the firms that pursued strategy $f$ or strategy $b$ that might produce. Again, only a fraction $\rho$ can do so, because the others suffer relationship-specific supply disturbances. It follows that in state $F$, an active firm competes with $\rho(\mu_f + \mu_b)$ others, each of which has a unit cost of $q_F$. Recognizing that all firms operating in state $H$ have common costs $q_H$ and those operating in state $F$ have common costs $q_F$, we can use (3.6) and (3.7) to calculate the values of $\pi^H(\omega)$ and $\pi^F(\omega)$ that accrue to all firms operating in those states.[12]

A firm's profit in state $B$ in which supply chains in both countries are active is slightly more complicated. In this state, firms that adopt either strategy $f$ or strategy $b$ anticipate a cost of $q_F$ with probability $\rho$. Those that diversify by choosing strategy $b$ anticipate that they will rely on their backup supplier, at the higher cost $q_H$, with probability $\rho(1-\rho)$. Meanwhile, firms that pursue strategy $h$ also produce at $q_H$, but with probability $\rho$. It follows

[11] In principle, a firm that diversifies may choose to invest in multiple supply relationships in the same country. To avoid a taxonomy, we do not consider this possibility here; it will not be an attractive option for $\rho$ close to one.

[12] Recognizing that all firms operating in state $H$ source their inputs from $H$, and that all firms operating in state $F$ source their inputs from $F$, we henceforth omit the superscript $K$ for any variable $\xi^{J,K}$ that applies when $J = H$ or $F$. For example, we use $\pi^H$ in place of $\pi^{H,H}$, $z^H$ in place of $z^{H,H}$, etc. Also, with some abuse of notation, we write $\xi^J(\mu)$ for the variable $\xi^J(\omega)$, when the value of $\xi^J$ is common to all firms operating in state $J$ and that value depends on the vector $\mu := (\mu_h, \mu_f, \mu_b)$.

that all firms anticipate competition in state $B$ from $(\mu_f + \mu_b)\,\rho$ others producing at cost $q_F$ and from $\mu_h\rho + \mu_b\rho\,(1-\rho)$ others producing at cost $q_H$.

Let $\Pi_j$ be the expected profit that a firm can earn by pursuing strategy $j$, $j \in \{h, f, b\}$. Recalling that $\delta^J$ is the probability of state $J$, $J \in \{H, F, B\}$, we have using (3.6) and (3.7) that

$$\Pi_h = \Pi_h(\boldsymbol{\mu}) := \delta^H \frac{s\left[z^H(\boldsymbol{\mu})\right]}{\sigma\left[z^H(\boldsymbol{\mu})\right]} P^H\left[z^H(\boldsymbol{\mu})\right]^{1-\varepsilon}\rho + \delta^B \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu})^{1-\varepsilon}\rho - k, \tag{3.8}$$

$$\Pi_f = \Pi_f(\boldsymbol{\mu}) := \delta^F \frac{s\left[z^F(\boldsymbol{\mu})\right]}{\sigma\left[z^F(\boldsymbol{\mu})\right]} P^F\left[z^F(\boldsymbol{\mu})\right]^{1-\varepsilon}\rho + \delta^B \frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu})^{1-\varepsilon}\rho - k, \tag{3.9}$$

and

$$\begin{aligned}\Pi_b = \Pi_b(\boldsymbol{\mu}) := &\sum_{J=H,F} \delta^J \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon}\rho \\ &+ \delta^B \left\{ \frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} + \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]}(1-\rho) \right\} P^B(\boldsymbol{\mu})^{1-\varepsilon}\rho - 2k.\end{aligned} \tag{3.10}$$

In equilibrium, if one strategy $j$ dominates the other two, all active firms will make that choice, and so $\mu_\ell = 0$ for $\ell \neq j$. If two strategies yield positive and equally high expected profits and higher than the third, then these two will have positive fractions in equilibrium, while the third will find no takers. The fractions will be such as to generate indifference. Finally, if there exist $\mu_h > 0$, $\mu_f > 0$ and $\mu_b > 0$ such that $\Pi_h = \Pi_f = \Pi_b > 0$, then the equilibrium will exhibit a positive number of firms pursuing each of the available strategies.[13]

### 3.2.5 Welfare

We adopt expected indirect utility as our welfare metric, weighting utility in each aggregate state by the likelihood of that state. Indirect utility comprises labor income, profits, tax revenues (if any) and consumer surplus.

[13]Since firms have the option to exit, it is possible that $\sum_j \mu_j < 1$, in which case all firms that do not exit make zero expected profits net of the fixed costs.

Expected welfare reflects the fractions of firms that choose each organizational mode, outcomes that can be influenced by government policy. When the fraction of firms that adopt strategies $h$, $f$ and $b$ are, respectively $\mu_h$, $\mu_f$, and $\mu_b$, aggregate expected profits (net of any subsidies or taxes received or paid by firms in recognition of their supply chain choices) amount to $\sum_{j=h,f,b} \mu_j \Pi_j(\boldsymbol{\mu})$. Consumer surplus in state $J$ is given by $\frac{1}{\varepsilon-1} P^J(\boldsymbol{\mu})^{1-\varepsilon}$ for $J \in \{H, F, B\}$. In the event that both countries are hit with supply disruptions, consumption of all differentiated products is zero ($X = 0$) and so consumer surplus vanishes. Therefore,

$$W(\boldsymbol{\mu}) = \bar{Y} + \sum_{J=H,F,B} \delta^J T^J(\boldsymbol{\mu}) + \sum_{j=xh,f,b} \mu_j \Pi_j(\boldsymbol{\mu}) + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J P^J(\boldsymbol{\mu})^{1-\varepsilon} \quad , \tag{3.11}$$

where $\bar{Y}$ is the (fixed) labor income from producing the numeraire good, $T^J(\boldsymbol{\mu})$ is tax revenues collected by the government and rebated to households in state $J$ (possibly zero or negative) beyond what is paid to or collected from firms in connection with their supply chain choices.

### 3.2.6 The *Laissez-Faire* Equilibrium

In the absence of any government policy, we can use the fact that all strategies used in equilibrium maximize expected profits and the fact that product markets must clear in every state to solve for $\boldsymbol{\mu}$, as well as the state-contingent aggregators $A^H$, $A^F$, and $A^B$. Market shares must sum to one in every state; i.e.,

$$\rho(\mu_h + \mu_b)\, s\left[z^H(\boldsymbol{\mu})\right] = 1, \tag{3.12}$$

$$\rho(\mu_f + \mu_b)\, s\left[z^F(\boldsymbol{\mu})\right] = 1 \tag{3.13}$$

and

$$\rho(\mu_f + \mu_b)\, s\left[z^{B,F}(\boldsymbol{\mu})\right] + \rho\left[\mu_h + (1-\rho)\mu_f\right] s\left[z^{B,H}(\boldsymbol{\mu})\right] = 1. \tag{3.14}$$

In state $B$, we need one more equation to solve for prices, which comes from comparing the

optimal prices (3.6) for firms that source their inputs in $H$ versus $F$. This gives

$$\frac{z^{B,F}(\boldsymbol{\mu})}{z^{B,H}(\boldsymbol{\mu})} = \frac{\frac{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma[z^{B,F}(\boldsymbol{\mu})]-1}}{\frac{\sigma[z^{B,H}(\boldsymbol{\mu})]}{\sigma[z^{B,H}(\boldsymbol{\mu})]-1}} \frac{q_F}{q_H} .$$

Further details for computing the *laissez-faire* equilibrium can be found in the appendix.[14]

Figure 3.1 illustrates the fraction of firms that choose each organizational form for different values of $k$. The figure is drawn for the case in which production costs are roughly similar ($q_H \approx q_F$), but sourcing from the home country is significantly safer than sourcing from abroad ($\gamma_H \gg \gamma_F$). For $k < k_1$ in the figure, the fixed cost of a sourcing relationship is sufficiently small that all firms find it worthwhile to invest in resilience, so $\mu_b = 1$ and $\mu_h = \mu_f = 0$. Next comes a range of fixed costs $k \in (k_1, k_2)$ for which some firms are diversified, and others source only from the safer country $H$. For higher fixed costs such that $k \in (k_2, k_3)$ in the figure, each of the three strategies is deployed in equilibrium by some positive number of firms. When $k$ surpasses $k_3$, it is no longer profitable for any firm to diversify; $\mu_b = 0$. Then, for $k \in (k_3, k_4)$, a marginal change in the fixed cost has no effect on the relative profitability of strategy $h$ versus strategy $f$.[15] But once $k$ increases beyond $k_4$, the expected operating profits are not sufficient for the full unit measure of firms to cover fixed costs; some firms exit, so that $\mu_h + \mu_f < 0$ and $\Pi_h = \Pi_f = 0$. The total number of firms might remain positive but continue to fall with further increases in $k$, or else $\mu_f$ and then $\mu_h$ might reach zero for some finite values of $k > k_4$.[16]

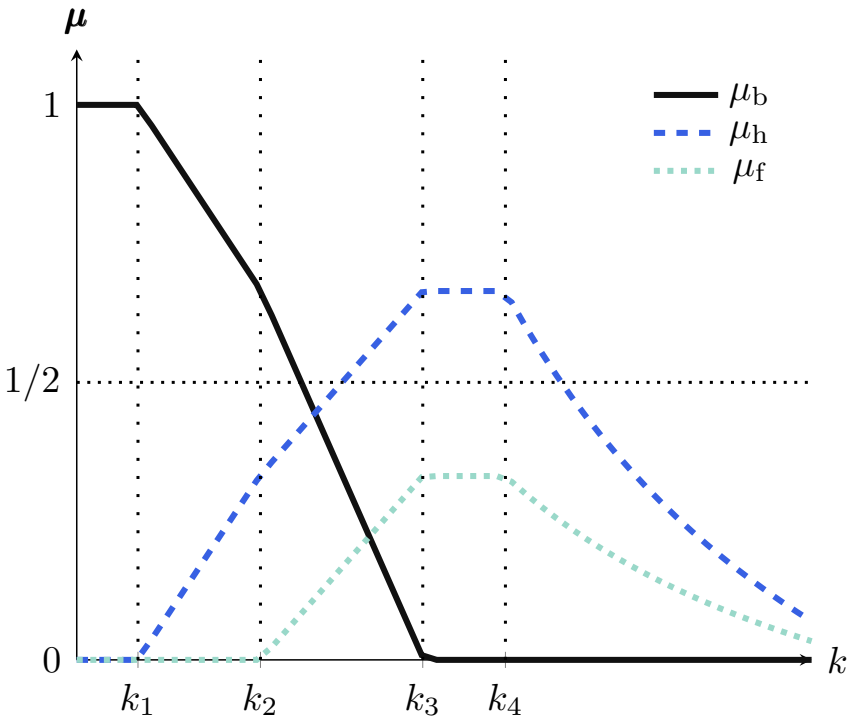
Figures analogous to Figure 3.1 can be drawn for other configurations of cost and risk parameters. In what follows, we focus exclusively on circumstances that give rise to the use

[14]Section numbers in the appendix correspond to the sections in the main text. Henceforth, when we refer to the appendix for formal arguments, we will only note the section number when it is an exception to this general rule.

[15]Meanwhile, for $k \in [k_3, k_4]$, a greater fraction of firms sources their input from the safer country, $\mu_f < 0.5 < \mu_h$, since $\gamma_H \gg \gamma_F$ while $q_H \approx q_F$.

[16]For some preferences, such as symmetric CES, operating profits in a state approach infinity as the number of varieties available in the state approaches zero, in which case some (shrinking number) of firms will invest in supply relationships in each state of the world. For other preferences, the potential operating profits per variety may be bounded. Then, for $k$ large enough, all firms exit the industry.

Figure 3.1: Supply Chain Outcomes for $\gamma_H \gg \gamma_F$, $q_H \approx q_F$




of all available strategies in equilibrium, i.e., $\mu_h > 0$, $\mu_f > 0$, and $\mu_b = 1 - \mu_h - \mu_f > 0$. This outcome seems to resemble most closely what we observe in reality.[17]

## 3.3 The Unconstrained Social Optimum

Do private incentives for firms to invest in safe and resilient supply relationships align with social incentives in the absence of government policy? If the answer is no, can the social planner intervene to restore social efficiency? The answer to the second question may depend on the set of policy instruments that the government has at its disposal to influence resource allocation. As is well known, the markup pricing reflected in equation (3.6) creates a wedge between social and private incentives to consume differentiated products relative to the homogeneous good, because consumers face a price in excess of marginal cost for the former, but a price equal to marginal cost for the latter. As in other contexts (see, for example Dhingra

[17]The cases in which some strategies are not used by any firms could be analyzed similarly, but do not generate any interesting insights beyond what we recount below.

and Morrow, 2019; Campolmi et al., 2018), this distortion can be eliminated, in principle, by an optimal set of consumption subsidies. In practice, such subsidies are difficult to implement and rarely observed; in our setting with non-constant markups, the subsidies must vary with both the state of nature ($H, F$, or $B$) and with the source of the inputs embodied in the final good.[18] Nonetheless, it is instructive to begin our analysis under the assumption that optimal subsidies are feasible, to focus squarely on the wedges between private and social incentives for supply chain formation. In this section, we study the unconstrained (or "first-best") planner's problem, leaving the more realistic, constrained (or "second-best") problem for the next section. By assuming away the distortions caused by markup pricing, we are able to develop intuition for whether and when the incentives firms face to invest in safety and resilience are excessive, insufficient or appropriate.

To characterize the optimal supply chain policies, we define the wedge between private and social incentives to pursue strategy $j$ relative to strategy $b$ when the existing mix of strategies is $\boldsymbol{\mu}$ as

$$\tilde{w}_j(\boldsymbol{\mu}) := \left[\tilde{\Pi}_j(\boldsymbol{\mu}) - \tilde{\Pi}_b(\boldsymbol{\mu})\right] - \frac{d\tilde{W}(\boldsymbol{\mu})}{d\mu_j}, \quad j \in \{h, f\}, \tag{3.15}$$

where tildes indicate relationships that apply with optimal consumption subsidies in place and

$$\frac{d\tilde{W}(\boldsymbol{\mu})}{d\mu_j} := \frac{\partial\tilde{W}(\mu_h, \mu_f, \mu_b)}{\partial\mu_j} - \frac{\partial\tilde{W}(\mu_h, \mu_f, \mu_b)}{\partial\mu_b}, \quad j \in \{h, f\}$$

is the marginal change in welfare from a small change in $\mu_j$ at the expense of $\mu_b$; i.e., $d\mu_j = -d\mu_b$.[19]

[18] In state $B$, some final producers source from suppliers in the foreign country at unit cost $q_F$ while others source from the home country at the higher cost $q_H$. The subsidies needed to ensure that consumers see relative prices equal to relative marginal costs will vary, therefore, with the sourcing of the inputs, despite the fact that all final goods enter demand symmetrically.

[19] We henceforth use the notation of total derivaties, $dG(\boldsymbol{\mu})/d\mu_j$, for $j \in \{h, f\}$, to denote the marginal change in the function $G(\cdot)$ with respect to $\mu_j$, taking into account that $\mu_b = 1 - \mu_h - \mu_f$.

At an (interior) first-best allocation $\boldsymbol{\mu}^o$, the first-order conditions for welfare maximization require $d\tilde{W}_j\left(\boldsymbol{\mu}^o\right)/d\mu_j = 0$ for $j \in \{h, f\}$. Therefore, the first best can be achieved by a set of subsidies that satisfy

$$\tilde{w}_j\left(\boldsymbol{\mu}^o\right) = \tilde{\Pi}_j\left(\boldsymbol{\mu}^o\right) - \tilde{\Pi}_b\left(\boldsymbol{\mu}^o\right) = \varphi_b - \varphi_j, \quad j \in \{h, f\}, \tag{3.16}$$

where $\varphi_j$ is a subsidy (possibly negative) paid unconditionally to any firm that pursues strategy $j$ and $\varphi_b$ is a subsidy (possibly negative) paid to a firm that diversifies its sourcing options. The optimal supply chain policies offset the wedges that remain (if any) when only consumption subsidies are applied.

We have given the social planner three policy instruments to counteract two wedges. Clearly, she has a degree of freedom in her policy choices. The first best can be achieved with a continuum of combinations of subsidies/taxes, including ones that eschew the use of one instrument entirely.

In the appendix, we compute the two wedges, (C.26) and (C.27), which yields

$$\tilde{w}_f^o := \tilde{w}_f\left(\boldsymbol{\mu}^o\right) = \delta^H \Phi\left[z\left(\mu_f^o\right)\right] \tilde{P}^H\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho + \delta^B \Phi\left[z^{B,H}\left(\boldsymbol{\mu}^o\right)\right] \tilde{P}^B\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho\left(1-\rho\right) \tag{3.17}$$

and

$$\begin{aligned}\tilde{w}_h^o := \tilde{w}_h\left(\boldsymbol{\mu}^o\right) = {} & \delta^F \Phi\left[z\left(\mu_h^o\right)\right] \tilde{P}^F\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho + \delta^B \Phi\left[z^{B,H}\left(\boldsymbol{\mu}^o\right)\right] \tilde{P}^B\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho\left(1-\rho\right) \\ & + \delta^B \left\{\Phi\left[z^{B,F}\left(\boldsymbol{\mu}^o\right)\right] - \Phi\left[z^{B,H}\left(\boldsymbol{\mu}^o\right)\right]\right\} \tilde{P}^B\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho\,,\end{aligned} \tag{3.18}$$

where

$$\Phi\left(z\right) := \int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta - \frac{s\left(z\right)}{\sigma\left(z\right) - 1}.$$

Assumption 3.2(i) specifies that $\sigma'\left(z\right) > 0$ for all $z \in \left(0, \bar{z}\right)$ (i.e., Marshall's Second

Law of Demand) or else $\sigma$ is constant for all $z \in (0, \bar{z})$. Taking the latter case first, it is straightforward to see that a constant elasticity of substitution implies $\Phi(z) = 0$ for all $z \in (0, \bar{z})$. This in turn implies $\tilde{w}_f^o = \tilde{w}_h^o = 0$; both wedges are zero in the market equilibrium when (only) optimal consumption subsidies are applied.

In contrast, when the elasticity of substitution rises with the relative price, we show in (25) in the appendix that $\Phi(z) < 0$ for all $z \in (0, \bar{z})$. Then (3.17) implies that $\tilde{w}_f^o < 0$, because both terms on the right-hand side are negative. As for strategy $h$, we have that $\tilde{z}^{B,H}(\boldsymbol{\mu}^o) > \tilde{z}^{B,F}(\boldsymbol{\mu}^o)$, because efficient relative prices are equal to relative marginal costs, and $q_H > q_F$. Together with $\Phi'(z) > 0$, (3.18) implies that $\tilde{w}_h^o < 0$ as well.

The negative wedges imply that, with only consumption subsidies but no subsidies or taxes to influence supply chain formation, firms have excessive incentives for diversification; that is, converting a firm with an exclusive relationship in either country to one that is diversified will reduce aggregate welfare. To interpret this finding, note that firms' investments in supply chains determine the number of differentiated products available in each state of the world, as well as (with MSLD) their markups and prices. Greater diversification is like additional entry in states $H$ and $F$, because more firms survive the country-specific supply disruptions. At the same time, it generates greater competition in state $B$ to the extent that idiosyncratic shocks ($\rho < 1$) impede product availability in that state.

When a firm chooses its investment strategy and thereby affects the number of varieties available in different states, it conveys two externalities. On the one hand, consumers love variety and they reap consumer surplus from greater availability at a given price. Firms do not take account of this positive effect of their product's availability on consumer surplus when forming their supply chains. On the other hand, more variety spells less demand and less profits for any particular product at given prices. Firms do not take account of this adverse effect of their product's availability on the profits earned by others. The sign of the wedge at the optimal allocation reflects the relative sizes of these two countervailing externalities.

Now consider, for example, how a change in the number of products $n^H$ available in state $H$ affects the gap between the price index, $P^H$, and the demand aggregator, $A^H$. The former fully captures the effect of product availability on consumer surplus whereas the effect on aggregate profits also depends on the latter. Using (3.5), (3.6) and (3.12) we calculate

$$\frac{1}{P^H}\frac{dP^H}{dn^H} - \frac{1}{A^H}\frac{dA^H}{dn^H} = \frac{s\left(z^H\right)}{\sigma\left(z^H\right)-1} - \int_{z^H}^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta = -\Phi\left(z^H\right).$$

When preferences satisfy MSLD, an extra variety in state $H$ reduces the price index for that state by proportionately less than it does the demand aggregator. Thus, the positive consumer-surplus externality from added availability in state $H$ falls short of the negative business-stealing externality. In such circumstances, the private incentives for resilience exceed the social incentives.[20]

Similar forces are at work with respect to a firm's choice between strategy $h$ and strategy $b$. An increase in $\mu_b$ at the expense of $\mu_h$ means greater availability in state $F$ (when the home suppliers are disrupted) and in state $B$ (when diversified firms stand a better chance of avoiding the disruption from idiosyncratic shocks). However, an increase in $\mu_b$ at the expense of $\mu_h$ has a further effect on the wedge $\tilde{w}_h$, as represented by the third term on the right-hand side of (3.18). When $\mu_b$ rises and $\mu_h$ falls, the extra products that become available in state $B$ are low-cost goods, whereas when $\mu_b$ rises and $\mu_f$ falls, the marginal products are high-cost goods. Extra low-cost goods take a greater toll on the profits of competitors than do extra high-cost goods, which adds the additional negative term to the wedge $\tilde{w}_h^o$.

We turn now to the policies that the planner can introduce, alongside the optimal, state-and-product-contingent consumption subsidies, to implement the first best. Let us begin with the limiting case of symmetric, CES preferences. With $\tilde{w}_f^o = \tilde{w}_h^o = 0$, the optimum can be achieved without any intervention in supply chain formation whatsoever; i.e., $\varphi_h = \varphi_f = \varphi_b = 0$. This is because, with CES preferences, the price index $P$ that (inversely)

[20]This result echoes that in Matsuyama and Ushchev (2020b) that there is excessive entry under MSLD in a one-sector model of monopolistic competition and no supply shocks.

measures welfare is proportional to the demand aggregator $A$. Then the external effects of a product's availability on consumers and competitors are equal in magnitude and opposite in sign. Once optimal consumption subsidies are in place to counter the distortion created by markup pricing, there is no need for further government policy to influence the number of products available in any state.

Next consider the special case of symmetric translog preferences. We have seen that, for all HSA preferences that obey MSLD, the government must discourage investments in resilience. But, for the translog case, we can say more. Using $s(z) = -\theta \log z$, we find $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \frac{1}{2} \frac{s(z)}{\sigma(z)-1}$ for all $z \in (0,1)$. Then using the planner's first-order conditions for the optimal choice of $\mu_h$ and $\mu_f$, along with (C.11):(C.13) in the appendix, we find that $\tilde{w}_f^o = \tilde{w}_h^o = -k$; see Appendix Lemma C.3. The two wedges coincide for all values of the cost and risk parameters and they are equal in absolute value to the fixed cost of forming a supply relationship. In the translog case, the planner can achieve the first best by combining the optimal consumption subsidies with a tax on diversification; $\varphi_b = -k$, with $\varphi_h = \varphi_f = 0$. Alternatively, she can leave diversified firms to face the private cost of their supply chains ($\varphi_b = 0$), while subsidizing firms that form exclusive supply relationships to the full extent of their investment costs ($\varphi_h = \varphi_f = k$). In either case, she has no reason to favor onshore investments relative to offshore investments.

These surprising results reflect a special property of symmetric translog preferences, namely that the ratio of $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$ to $\frac{s(z)}{\sigma(z)-1}$ is independent of price and always equal to one half. Since the consumer surplus loss from removing a variety in some state of the world is proportional to $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$, while the loss in operating profits for the firm that does not produce its variety is proportional to $\frac{s(z)}{\sigma(z)-1}$, translog preferences imply that the consumer surplus loss from switching a firm from having two suppliers to one is exactly half of the loss in operating profits. But the wedge $\tilde{w}_j^o$ is equal to the difference in total expected profits per (3.16), which in turn is equal to the fixed cost of an extra relationship minus the loss in operating profits. Finally, the first-order condition for maximizing $\tilde{W}(\boldsymbol{\mu})$ dictates that the

marginal loss of consumer surplus from switching a firm from strategy $b$ to strategy $j$ should match the cost of an extra supplier, $k$.

With more general HSA preferences, the incentives created by optimal supply chain policy are not neutral with respect to the location of firms' input suppliers. The optimal policies favor onshore relationships relative to offshore relationships if $|\tilde{w}_h^o| > |\tilde{w}_f^o|$ and offshore relationships relative to onshore relationships if the ranking of the two wedges is reversed.

To shed further light on the desired national bias in first-best supply chain policy, we examine the limiting case when production costs are nearly the same, $q_H \approx q_F$. Then we find in (C.31) in the appendix that

$$|\tilde{w}_h^o| - |\tilde{w}_f^o| \propto \Psi\left[z^H\left(\boldsymbol{\mu}^o\right)\right] - \Psi\left[z^F\left(\boldsymbol{\mu}^o\right)\right]$$

where $\Psi(z) := \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta / \left[\frac{s(z)}{\sigma(z)-1}\right]$. In the appendix, we also show that $z^H\left(\boldsymbol{\mu}^o\right) < z^F\left(\boldsymbol{\mu}^o\right)$. It follows that, with nearly equal costs, the government should encourage the less risky investments at home relative to the more risky investments abroad if $\Psi(z)$ is a decreasing function, and the reverse if it is an increasing function; see Lemma C.5 in the appendix.

When $q_H \gg q_F$, this simple reasoning does not apply, because the planner's preference for home sourcing on safety grounds is counteracted by her preference for foreign sourcing on cost grounds, so that $\mu_h^o \lesseqgtr \mu_f^o$. Moreover, with unequal costs, an increase $\mu_b$ at the expense of $\mu_h$ makes a greater contribution to consumer surplus in state $B$ while taking a greater toll on rivals' profits than does an increase in $\mu_b$ at the expense of $\mu_f$; although both increase product availability in state $B$ by similar amounts, the former spells greater availability of low-cost products, whereas the latter generates greater availability of high-cost products.

We summarize our findings about first-best supply-chain policy when all three strategies are used in

**Proposition 3.1.** *Under Assumptions 3.1 and 3.2, the unconstrained planner uses consumption subsidies to undo the markup distortion for each good in each state of nature. With symmetric CES preferences, a hands-off policy with respect to supply chain formation*

*($\varphi_h = \varphi_f = \varphi_b = 0$) achieves the first best. Under MSLD, the planner encourages single sourcing relationships relative to diversification. With symmetric translog preferences, the planner can achieve the first best with a tax on diversification of size $k$. More generally, if cost differences are small ($q_H \approx q_F$), the planner encourages onshore sourcing relative to offshore sourcing if $\Psi'(z) > 0$ for all $z \in (0, \bar{z})$ and encourages offshore sourcing relative to onshore sourcing if $\Psi'(z) < 0$ for all $z \in (0, \bar{z})$. For larger cost differences, the national bias in optimal sourcing policy hinges not only on the sign of $\Psi'(z)$, but also on the magnitudes of the cross-country cost and risk differences.*

## 3.4 The Constrained Social Optimum

In the previous section, we characterized the first-best allocation of resources when firms in a monopolistically competitive industry face potential supply chain disruptions. We noted that attainment of the first best requires not only that government use policies to offset distortions in firms' incentives for forming supply relationships, but also a policy to counter the distortion created by monopoly pricing of differentiated products alongside the competitive pricing of goods elsewhere in the economy. As we discussed, the requisite consumption subsidies are rarely implemented in practice. In our context, not only would they need to be adjusted in response to realized disruptions, but they would also need to vary across otherwise symmetric products that differ only in the sourcing of their critical inputs. Nonetheless, by allowing for optimal consumption subsidies, we were able to lay bare the wedges between private and social incentives for supply diversification and for onshoring versus offshoring.

In this section, we consider the second-best problem that confronts a welfare-maximizing government that lacks the ability to implement state-contingent and sourcing-contingent consumption subsidies. We grant the policy maker only taxes or subsidies to encourage or discourage supply chain resilience and to influence whether sourcing partnerships are formed at home or abroad. As with the unconstrained optimum, the constrained social optimum can be achieved by a continuum of combinations of subsidies or taxes for the formation of home relationships, foreign relationships, and multiple relationships and, indeed, any two of these

three instruments will suffice.

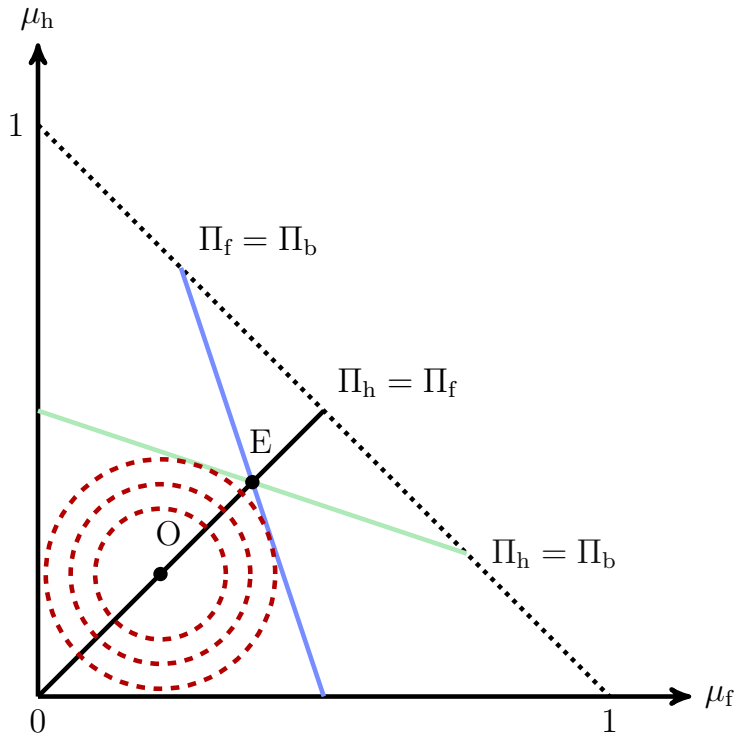
Let us begin with the limiting case in which the home and foreign countries are symmetric in terms of both input costs and disruption risks; i.e., $q_H \approx q_F$ and $\gamma_H \approx \gamma_F$. In Figure 3.2, we illustrate a *laissez-faire* equilibrium at $E$ for a typical case in which all three strategies are employed by positive measures of firms. In such circumstances, we can use the equilibrium relationship $\mu_b = 1 - \mu_h - \mu_f$ to project the three-dimensional space $(\mu_f, \mu_h, \mu_b)$ onto two dimensions. Accordingly, the figure shows $\mu_h$ and $\mu_f$ on the vertical and horizontal axes, and the equilibrium falls inside the unit simplex.

The curve labeled $\Pi_h = \Pi_f$ represents combinations of $\mu_h$ and $\mu_f$ such that a strategy of forming a single supply relationship at home yields the same expected profit as that of forming a single supply relationship abroad. With similar production costs, the profits from strategies $h$ and $f$ are the same in state $B$. Hence, when choosing between these two strategies, firms compare realized profits in states $H$ and $F$. Profits in state $H$ are declining in the number of firms $n^H(\boldsymbol{\mu}) = \rho(1 - \mu_f)$ that are active in that state. Similarly, profits in state $F$ are declining in $n^F(\boldsymbol{\mu}) = \rho(1 - \mu_h)$. It follows that firms will be indifferent between $h$ and $f$ if only if the expected competition in the two states is the same, i.e., $\mu_h = \mu_f$. Thus, we represent the $\Pi_h = \Pi_f$ curve by a 45° ray from the origin.

The downward sloping curve labelled $\Pi_h = \Pi_b$ shows combinations of $\mu_h$ and $\mu_f$ for which investing in a single relationship at home yields the same expected profits as a strategy of diversification. The downward slope of the curve can be understood as follows. Starting from a point on the curve suppose we raise $\mu_h$ and reduce $\mu_b$ so that $\mu_f$ remains constant. This does not affect the number of firms $n^H$ active in state $H$. But it decreases the number of firms active in state $F$ and also in state $B$, since $n^B(\boldsymbol{\mu}) = \rho + \rho(1 - \rho)\mu_b$. Therefore, $\pi^F$ and $\pi^B$ both rise, leaving a (positive) gap between $\Pi_b$ and $\Pi_h$.[21] Now consider a fall in $\mu_f$ accompanied by an offsetting rise in $\mu_b$ that leaves $\mu_h$ unchanged. This change in composition

[21]The effect on expected profits of a diversified firm relative to a home-only firm conditional on state $B$ are equal and opposite for a given increase in $\mu_h$ and comparable decrease in $\mu_f$. But the increase in $\mu_h$ (and accompanying decrease in $\mu_b$) gives an added boost to the relative profitability of diversification, because it raises the expected profits for a $b$ firm if state $F$ arises.

Figure 3.2: Equilibrium and Constrained Optimum for the Symmetric Case




has no affect on the number of firms active in state $F$, but increases the number that are active in state $H$. However, the more intense competition in state $H$ does not affect the *relative* attractiveness of strategy $h$ versus $b$, because each of these strategies yields operating profits $\pi^H$ with probability $\rho$ in that state. Meanwhile, competition also intensifies in state $B$. With more firms active, $\pi^B$ falls, which depresses expected profits more for diversified firm than for firms that have only a home supplier, because the diversified firms are more likely to survive. Thus, a decrease in $\mu_f$ offset by an increase in $\mu_b$ reduces $\Pi_b$ relative to $\Pi_h$. It follows that a decrease in $\mu_f$ is needed to offset the effects of an increase in $\mu_h$ if strategies $h$ and $b$ are to remain equally profitable. We note further that the $\Pi_h = \Pi_b$ curve must have a slope less than one in absolute value.[22] By an analogous argument, the curve $\Pi_f = \Pi_b$ also

[22]The effect on expected profits of a diversified firm relative to a home-only firm conditional on state $B$ are equal and opposite for a given increase in $\mu_h$ and comparable decrease in $\mu_f$. But the increase in $\mu_h$ (and accompanying decrease in $\mu_b$) gives an added boost to the relative profitability of diversification, because it raises the expected profits for a $b$ firm if state $F$ arises.

slopes downward in the figure, with a slope greater than one in absolute value.

In equilibrium, if all strategies are used, all must yield equal profits. So, the equilibrium is represented by the point $E$ in Figure 3.2. The figure also illustrates a constrained optimum at $O$. The constrained optimum maximizes $W$ over the choice of $\boldsymbol{\mu}$ in the presence of monopoly pricing of differentiated products. In the appendix, we show that the first-order conditions for a constrained maximum are satisfied when $\mu_h = \mu_f$.[23] The figure depicts some iso-welfare loci for successively lower levels of expected welfare as we move away from $O$. These curves are symmetric about the 45-degree line, thanks to the symmetry across countries.

It should be clear that, if $O$ falls on the 45° line, the constrained optimum can be achieved with a tax or subsidy on diversification alone, with $\varphi_f = \varphi_h = 0$. Such a policy shifts the equilibrium along the $\Pi_h = \Pi_f$ curve and thereby preserves the equality between $\mu_h$ and $\mu_f$. What remains to be addressed is whether the government should encourage diversification with a subsidy for firms that form multiple relationships ($\varphi_b > 0$) or whether it should discourage diversification with a tax ($\varphi_b < 0$) on such firms. This amounts to the same question as to whether point $O$ lies to the southwest of $E$ along $\Pi_h = \Pi_f$ or whether it lies instead to the northeast of $E$.

We can answer these questions formally using methods similar to the ones we applied in Section 3.3. We begin with the planner's objective in (3.11). The wedge between social and private incentives for diversification is given by

$$w_j^* := \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) - \frac{dW(\boldsymbol{\mu}^*)}{d\mu_j}, \qquad j \in \{h, f\},$$

where $\boldsymbol{\mu}^*$ represents the allocation in the constrained optimum and recall that $dG(\boldsymbol{\mu})/d\mu_j$ denotes the variation in any function $G(\boldsymbol{\mu})$ for $d\mu_j = -d\mu_b > 0$. Using the first-order

[23]This statement is valid for all HSA preferences. In the appendix, we also show that the welfare function $W(\boldsymbol{\mu})$ is globally concave when $X$ takes a CES form and that the constrained optimum must have $\mu_h = \mu_f$ when preferences take the symmetric translog form. Evaluating the second-order conditions for more general HSA preferences is challenging, but it seems compelling that the planner would want equal numbers of firms with single relationships at home and abroad.

condition for the second-best allocation, this wedge is given by

$$w_j^* = -\sum_{i=h,f,b} \mu_i \frac{d\Pi_i\left(\boldsymbol{\mu}^*\right)}{d\mu_j} - \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \frac{d\left[P^J\left(\boldsymbol{\mu}^*\right)^{1-\varepsilon}\right]}{d\mu_j}, \quad j \in \{h, f\}. \tag{3.19}$$

The first term on the right-hand side of (3.19) represents the business-stealing externality; i.e., the change in other firms' profits that results from shifting a marginal firm from diversified sourcing to sole sourcing in country $j$. The second term represents the consumer-surplus externality; i.e., the change in consumer-surplus that results from reduced product availability and higher prices in the three states. The difference from the analogous expressions in Section 3.3 reflects the fact that the constrained policy maker needs to take account not only of the *direct* effects on profits and consumer surplus of changing the numbers of firms in each state (holding prices constant), but also the *indirect* effects on profits and consumer surplus that come from marginal adjustments in the markups. The unconstrained planner can ignore these latter effects when deciding $\boldsymbol{\mu}^o$, because the choice of optimal consumption subsidies ensures that the induced changes in markups have a negligible effect on aggregate utility.

In the appendix, we provide a general formula for the wedge $w_j^*$ for an arbitrary share function $s\left(z\right)$ that satisfies Assumptions 3.1 and 3.2. Then we turn to the symmetric case depicted in Figure 3.2, where $w_h^* = w_f^* = w^*$. Point $E$ lies above point $O$ whenever $w^* > 0$ and below point $O$ whenever $w^* < 0$.

The results for our two special cases of HSA preferences are instructive. First, with *symmetric CES preferences*, we find that $w^* > 0$ and thus $\varphi_b^* > 0$ for all $\sigma > 1$.[24] Recall that with optimal consumption subsidies in place, the optimal policy has $\varphi_b^o = 0$, because the extra consumer surplus generated by adding firms in a given state exactly matches the loss in aggregate profits. Now, with consumption subsidies unavailable to the policy maker, the monopoly pricing of differentiated products generates too little consumption of these goods relative to the numeraire good in the *laissez-faire* equilibrium. A subsidy for

[24] Equilvalently, the planner can set $\varphi_b^* = 0$ and $\varphi_h^* = \varphi_f^* < 0$

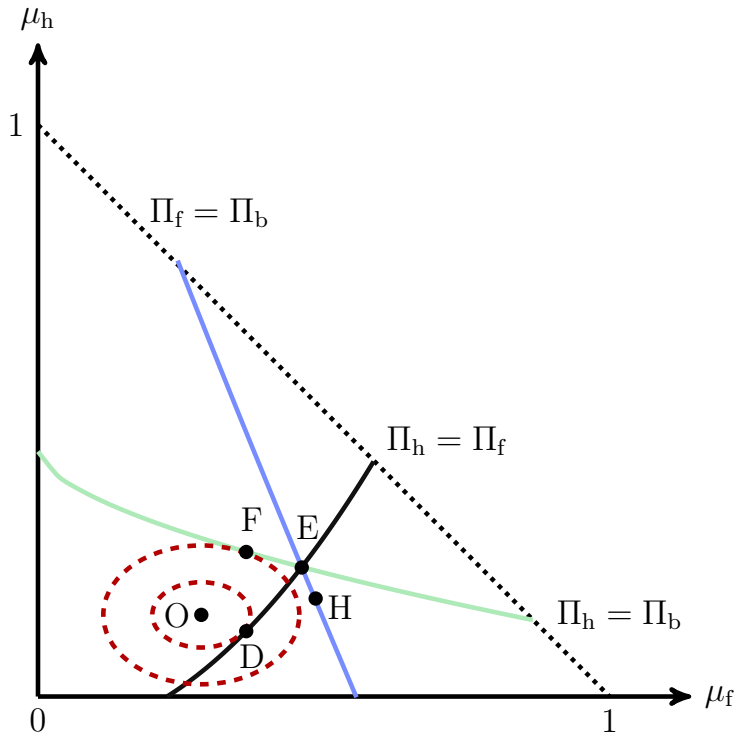
diversification increases the number of available products in every state, which reduces $P^J$ for all $J \in \{H, F, B\}$, even though prices of marketed products do not change. The fall in the price index in state $J$ stimulates consumption of differentiated products in that state, thereby mitigating the consumption distortion.

Second, with *symmetric translog preferences*, we show that $w^* > 0$ if $\varepsilon > \frac{\theta\rho(2-\rho)[1+\theta\rho(2-\rho)]}{1+3\theta\rho(2-\rho)}$ and $w^* < 0$ if $\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)}$; see Lemma C.7. Recall that a tax on diversification is needed to align social and private incentives for supply chain formation for any HSA preferences other than CES when a consumption subsidy is available to correct the distortion otherwise generated by markup pricing. In the absence of consumption subsidies, the tendency for firms to overinvest in resilience continues to figure in the policy maker's calculus, because the business-stealing externality is large relative to the consumer-surplus externality. However, the distortion arising from monopoly pricing points in the opposite direction; there is too little consumption of differentiated products relative to the numeraire good and a subsidy for diversification would boost consumption of these goods. When demand for differentiated goods is highly elastic, the distortion from monopoly pricing looms large and the planner's imperative to encourage consumption of these goods outweighs her concern about firms' excessive investments in resilience, much as with CES preferences. In contrast, when demand for differentiated products is not so elastic, the welfare effects of the consumption distortion are muted and the planner acts to dampen firms' excessive incentive to be present in the market.

Let us return now to the case in which input costs are lower abroad than at home ($q_H > q_F$) but foreign sourcing entails greater risk of disruption than home sourcing ($\gamma_H > \gamma_F$). Figure 3.3 depicts the *laissez-faire* equilibrium and the constrained optimum in such a setting for general HSA preferences. Again we consider fixed costs of sourcing relationships in the range that a positive measure of firms chooses each of the available investment strategies.

We observe first that, for general HSA preferences, the constrained optimum, $O$, need not fall on any of the three equiprofitability curves. This means that, generically, the government

Figure 3.3: Equilibrium and Constrained Optimum for the Asymmetric Case




cannot achieve the constrained optimum with a single policy instrument. For the case illustrated in Figure 3.3, a subsidy for diversification improves welfare relative to $E$, but since such a policy preserves $\Pi_h = \Pi_f$, such a policy can achieve at best the utility associated with the iso-welfare curve through point $D$. A tax to discourage sourcing abroad ($\varphi_f < 0$) shifts the equilibrium to the left along the $\Pi_h = \Pi_b$ curve, but at best can achieve the utility associated with the iso-welfare curve through point $F$. Finally, a tax on onshore relationships with a single partner ($\varphi_h < 0$) can be used to achieve point $H$. For the scenario depicted in the figure, the constrained optimum could be achieved with a combination of a subsidy for diversification and a tax on sole-sourcing offshore or with a subsidy to diversification and a subsidy for sole-sourcing at home.

Although the constrained optimum can be characterized for particular preferences and parameters, in general the wedges $w_h^*$ and $w_f^*$ that dictate the second-best policy combinations

can take any sign and a range of relative magnitudes. The reason for this reflects the complexity of the planner's constrained maximization problem. While the planner faces a general tradeoff between alleviating the markup distortion by supporting greater resilience and mitigating the business-stealing externality, the size of this tradeoff will vary across states of the world and the planner cannot separately address the tradeoff state by state. For example, a policy that encourages greater diversification at the expense of sole-sourcing offshore generates an increase in the number of products available both in state $H$ and in state $B$. As a result, the optimal policy is dictated by some weighted average of the tradeoff between consumption distortion and business stealing externality in each state of the world, and this depends on the exact form of preferences, the various preference parameters, and the sizes of the cross-country differences in costs and riskiness.

In the next section, we resort to numerical methods to explore some of these tradeoffs. Before that, we return briefly to the special case of symmetric CES preferences, for which a strong characterization of the second-best policies is possible even with asymmetric costs and risks. With CES preferences, the price index plays a dual role as both welfare metric and demand aggregator. We show in the appendix that this exceptional feature of the CES implies that the constrained optimum is characterized by $\Pi_h = \Pi_f$, much like the *laissez-faire* equilibrium. That is, the planner has no reason to tilt supplier relationships toward one location or the other. This means that the second best can be achieved with a single policy instrument, namely a tax or subsidy for diversification. However, as we also show in the appendix, point $O$ always lies *below* point $E$ on the $\Pi_h = \Pi_f$ curve, so, with CES preferences, it is always desirable for the government to promote resiliency with a subsidy to strategy $b$ for all values of the cost and risk parameters. The explanation is the same as in the symmetric case; with CES preferences, the consumer-surplus externality and the business-stealing externality exactly offset one another *in every state of the world*. What remains are the distortions that result from the fixed and positive markup of consumer prices over marginal costs. The constrained policy maker who cannot eliminate the consumption

distortions directly can instead partially alleviate the distortion by promoting greater product availability in all states of the world.

We summarize our analytical findings about second-best supply-chain policy in

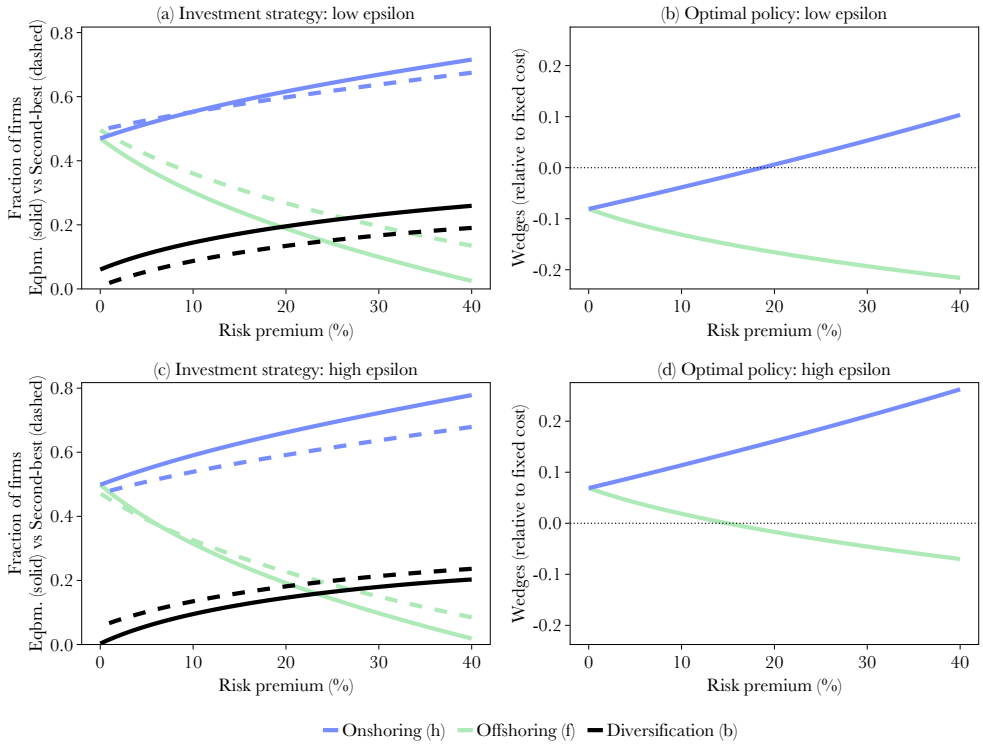
**Proposition 3.2.** *Suppose that consumption subsidies are infeasible. If consumers have symmetric CES preferences, the planner can achieve a constrained optimum with a single policy instrument, namely a subsidy for diversification ($\varphi_b > 0$). If consumers have symmetric translog preferences and the input costs and disruption risks in the two countries are symmetric, the constrained optimum can again be achieved with a single policy, which must be a tax on diversification if $\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)}$ and a subsidy for diversification if $\varepsilon > \frac{\theta\rho(2-\rho)[1+\theta\rho(2-\rho)]}{1+3\theta\rho(2-\rho)}$. In other circumstances, two policy instruments are generally needed to alter both the incentives for diversification and the incentives for sourcing at home versus abroad.*

## 3.5 Numerical Exploration of the Constrained Optimum

When monopolistically-competitive firms form their supply chains with an eye to potential disruptions, the market equilibrium features several sources of inefficiency. The consumer-surplus externality associated with product availability suggests underinvestment in resilience, whereas the business-stealing externality implies just the opposite. Meanwhile, monopoly pricing generates insufficient consumption of differentiated products relative to the numeraire good in realistic situations when fiscal policies cannot be used to align relative prices with relative marginal costs. We have been able to characterize the policy imperatives that these distortions create under CES preferences and, with more general HSA preferences, when the home and foreign suppliers are similar with respect to costs and risks. Armed with our understanding of the nature of the distortions, we turn now to numerical methods to explore the constrained optimal policies in situations when costs and risks differ in the two countries. To this end, we henceforth assume that preferences take the symmetric translog form.

Figure 3.4 depicts the constrained optimal fractions of firms (on the left) and the policy wedges at the second-best allocation (on the right) for two different values of $\varepsilon$, the elasticity

Figure 3.4: Second-Best Policies: Risk Differences Across Locations




Note: Baseline simulation has $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Low and high epsilon correspond to $\varepsilon = 1.2$ and $\varepsilon = 1.7$ respectively. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.13$ for $\varepsilon = 1.2$ and $k = 0.37$ for $\varepsilon = 1.7$. The risk premium is computed as $-(\gamma_F - \gamma_H)/\gamma_H$, where we keep $\gamma_H$ constant at its baseline value.

of demand for differentiated products. The figure is drawn for the case when production costs are similar in the two countries ($q_H = q_F$), but we show in the appendix (see Figures C.1 and C.2) that qualitatively similar patterns emerge when costs differ. Figure 3.4 illustrates the comparative statics of the equilibrium, constrained optimum and optimal supply-chain policies with respect to variation in the cross-country risk differential.

In panels (a) and (b), we see the outcomes for a relatively low value of $\varepsilon$, namely $\varepsilon = 1.2$. Panel (a) shows the fraction of firms that choose each of the investment strategies in the laissez-faire equilibrium (solid curves) and in the constrained optimum (dashed curves). When $\gamma_H = \gamma_F$, at the left side of the panel, the market equilibrium features excess investment in resilience ($\mu_b > \mu_b^*$) and insufficient investment in exclusive supply relationships

($\mu_h^* = \mu_f^* = \mu^* > \mu$). These numerical outcomes mirror the theoretical results from Section 3.4. They reflect the fact that, under MSLD, the business-stealing effect dominates the consumer surplus effect. Moreover, with $\varepsilon$ relatively small, the consumption distortion caused by markup pricing is not too severe. In panel (b), we see that $w_h^* = w_f^* = w^* < 0$, so the constrained optimum can be achieved either with a tax on firms that diversify, or with equal subsidies to firms that invest in exclusive supply relationships either at home or abroad. As we increase the risk of disruption in $F$—so that the "safety premium" in the home country rises—both the competitive equilibrium and the constrained optimum are characterized by greater fractions of diversified firms and greater fractions of firms that form relationships only onshore. These findings are intuitive, but what is less obvious is what happens to the wedges between social and private incentives. In panel (b) we see that $w_h^*$ rises while $w_f^*$ falls. This implies, for example, that second-best subsidies for exclusive offshore relationships grow ($\varphi_f^*$) while subsidies for sourcing relationships at home ($\varphi_h^*$) shrink, if the planner eschews taxes or subsidies on diversification ($\varphi_b^* = 0$).

How do we understand this finding? In panel (a), we see that the fraction of firms with sourcing relationships exclusively in the home country rises above the fraction with sourcing relationships exclusively offshore. The increase in product diversity and in competition in state $H$ relative to state $F$ generates a decline in the price index $P^H$ relative to $P^F$. But the monopoly distortion is more severe when the price index is high, so the consumption shortfall is greater in state $F$ than in state $H$. The planner wishes to combat the higher price index in state $F$ with a policy that tilts sourcing toward the foreign country.

As the foreign country becomes even riskier, the planner continues to discourage diversification; we continue to find the second-best fraction of diversified firms, $\mu_b^*$, below the free-market level. But the social cost of the market's misallocation between home sourcing and foreign sourcing also grows, and so the gap between the two wedges $w_h^*$ and $w_f^*$ widens. At some risk differential close to 20% in the figure, the planner's desire to shift the location of exclusive sourcing from the home country to the foreign country implies a second-best

*tax* on onshore relationships, combined with an even larger subsidy for investing in a single relationship abroad.[25]

The situation is similar for larger values of $\varepsilon$, such as depicted in panels (c) and (d) of Figure 3.4, except in one important respect. With a more elastic demand for differentiated products, the misallocation generated by markup pricing weighs more heavily in the planner's calculus compared to the net effect of the business-stealing and consumer-surplus externalities. The optimal policy in the symmetric environment entails a net subsidy to diversification, which can be achieved with $\varphi_b > 0 = \varphi_h = \varphi_f$ or with $\varphi_h = \varphi_f < 0 = \varphi_b$. As the risk differential grows, the planner once again tilts policy in favor of exclusive sourcing relationships abroad, to offset the increasingly deleterious effects of under-consumption of differentiated products when foreign supply is disrupted. For a sufficiently great probability of supply disruption in the foreign country, the planner subsidizes strategy $f$, while still ensuring that the net effect of supply chain policy is to induce more diversification and greater resilience.

Notice too the scale of the optimal subsidies. Recall from Section 3.3 that, when able to implement the optimal, state-and-product-contingent consumption subsidies, the planner taxes diversification (or subsidizes the two strategies involving exclusive relationships) at 100% of the fixed cost $k$, regardless of the configuration of cost and risk parameters.[26] In the second best described here, the impetus to tax diversification in order to dampen incentives for business stealing is offset by an urge to subsidize diversification to stimulate consumption of differentiated goods. The offsetting forces result in second-best policies that are an order of magnitude smaller than in the first best.

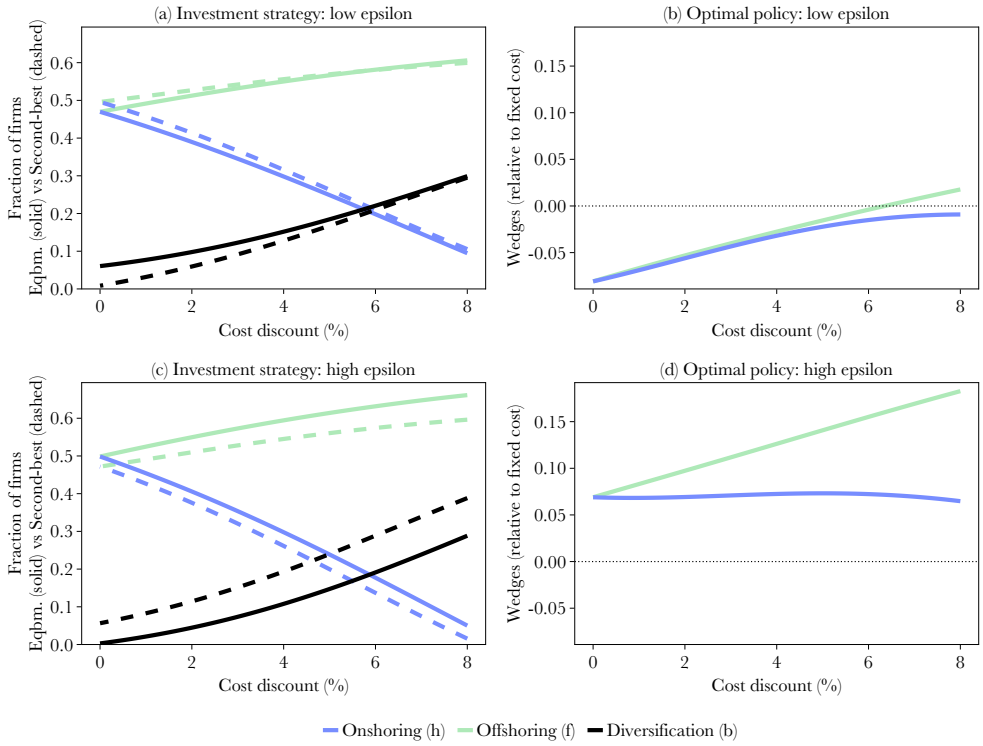
Figure 3.5 depicts the comparative statics with respect to foreign production costs, holding riskiness in the two locations constant (and, in this figure, equal to one another).[27] The symmetric equilibrium again requires a second-best tax on diversification when $\varepsilon$ is small

[25]Alternatvely, the planner can achieve the same allocation with a subsidy for diversification and an even larger subsidy for offshore relationships (and $\varphi_h = 0$), so that $\mu_f$ grows at the expense of both $\mu_h$ and $\mu_b$.

[26]This statement applies to situations, as here, with symmetric translog preferences

[27]The parameters used for this figure are the same as for Figure 3.4, so the baseline (symmetric) outcome is the same at the left-most point in both figures.

Figure 3.5: Second-Best Policies: Cost Differences Across Locations




Note: Baseline simulation has $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Low and high epsilon correspond to $\varepsilon = 1.2$ and $\varepsilon = 1.7$ respectively. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.13$ for $\varepsilon = 1.2$ and $k = 0.37$ for $\varepsilon = 1.7$. The cost discount is computed as $-(q_F - q_H)/q_H$, where we keep $q_H$ constant at its baseline value.

(top panels) and a subsidy when $\varepsilon$ is larger (bottom panels). A fall in the cost of producing inputs abroad, which expands the cost discount in the offshore location, reduces the price index in state $F$ relative to that in state $H$.[28] Thus, the social benefit from promoting consumption in state $H$ comes to exceed that in state $F$. In panel (b), the planner alters the composition of exclusive supply relationships by offering a larger subsidy for onshore sourcing than for offshore sourcing (or, equivalently, a subsidy for onshoring combined with a tax on diversification). For a high enough cost discount, the wedge for offshore relationships actually turns positive. In panel (c), with more elastic demand, the planner encourages resilience at

[28]In this case there are two reasons: more firms choose strategy $f$ than strategy $h$ and hence the market is more competitive in state $F$ than in state $H$; and products containing inputs produced in $F$ bear a lower cost than those produced in $H$.

the expense of exclusive relationships at home and abroad. As seen in panel (d), the taxes on single relationships that are used to encourage diversification diverge; onshore relationships face a lower tax than their offshore counterparts (or else the planner can subsidize both diversification and onshoring). This policy combination reflects the fact that consumption distortion is more harmful in state $H$ than in state $F$.

When both cost and risk parameters differ in the two possible locations for producing inputs, the ranking of the price index in states $H$ and $F$ is less clear-cut. Greater risk of supply disruption in the foreign country discourages private investment there, contributing to a relatively higher price index in state $F$. But lower production costs offshore raises the relative attractiveness of strategy $f$ compared to strategy $h$, and it also has a direct effect on comparative prices in the two states reflecting the relatively lower foreign unit cost. The relative size of the wedges, $w_h^*$ versus $w_f^*$, and thus the net effect on the incentives for exclusive offshoring versus exclusive onshoring, hinges on the relative strength of these forces.

## 3.6 Conclusion

Global supply chain disruptions are increasingly salient and often costly. Many commentators have been quick to conclude that governments ought to be doing something to promote greater market resilience. But the welfare-theoretic calculus around government intervention is rather subtle. Private actors have a clear self-interest in taking measures to avoid disruptions to their production processes. Only when the private incentives for resilience fall short of the social benefits will government encouragement be warranted. Pointing in that direction is the observation that consumers capture part of the surplus created by the ongoing availability of firms' products. But firms also have an incentive to be in a position to reap extra profits when their rivals are suffering. The temptation for "business stealing" suggests that excess resilience is also a possible market outcome.

Surprisingly little research has addressed the desirability of government policy to promote

resilience or to encourage sourcing from safer locations. In this chapter, we have taken a first step. We have proposed a simple framework in which the supply of any product requires the availability of a critical input. Idiosyncratic and aggregate shocks can disrupt firms' relationships with their suppliers. Firms face the choice of where to develop a relationship and whether to protect their operation with backup sources of supply. We study the simplest case of two potential supply sources, one at home and one abroad and focus on a situation where domestic sourcing is costlier than sourcing abroad, but also less risky.

Since consumer gains from product availability reflect their preferences, the form of demand plays a critical role in the policy calculus. The CES demand system is popular and tractable for analysis such as ours. But it also introduces restrictions that color the findings. We allow for a CES utility function, but also for a broader class of preferences that Matsuyama and Ushchev (2017) and Matsuyama and Ushchev (2020b) have developed and termed Homothetic with a Single Aggregator. The more general preferences admit non-constant markups and, in particular, application of Marshall's Second Law of Demand.

Our analysis yields several broad lessons. First, the government generally needs at least two supply-chain policies to achieve efficient sourcing (first or second best). One instrument regulates the margin between sourcing from one location or two. The other guides the choice between sourcing at home and abroad. For example, the government might subsidize or tax supply-chain diversification, while subsidizing or taxing firms that source only at home. Or it might subsidize or tax diversification, while subsidizing or taxing offshoring.

When preferences take the CES form, the first best can be achieved simply with a state-independent consumption subsidy and with no interference in supply chain organization. The second best requires a subsidy to diversification but no bias for home versus foreign sourcing. But with more general forms of HSA preferences that obey Marshall's Second Law of Demand, the planner requires state-and-product-specific consumption subsidies to achieve the first best, along with a tax on firms that diversify, and a policy that tilts sourcing to one country or the other depending on the relative sizes of the consumer-surplus externality and

the business-stealing externality. In the second best, the optimal policies are qualitatively similar to the ones that achieve the first best if the elasticity of demand for differentiated products is small, but a large demand elasticity tilts the policy toward smaller taxes or even subsidies for diversification.

Needless to say, there are many ways that our analysis could be enriched. For example, we could introduce a richer technology with potential substitution between manufactured inputs and primary factors of production. We could entertain more complex supply chains, with multiple inputs and with a sequencing of them such that some inputs enter the production process upstream from others. We could allow for dynamics, which would render inventories an additional tool for firms to invest in resilience and give governments additional policy instruments such as stockpiling supplies or allowing accelerated depreciation of inventory costs. We could introduce political-economy considerations that might drive a wedge between the parameters that capture the risk aversion of managers versus that of policy makers. We see all of these potential extensions as worthwhile and germane to the ultimate policy assessment. We believe that our simpler setting suggests a way to pose the question and that our analysis provides a "proof of concept."

# Appendix A

# Should I Stay or Should I Grow? How Cities Affect Learning, Inequality and Productivity

## A.1 Theory

### A.1.1 Proof of Proposition 1.1

**Existence** The problem of the young is decoupled from that of the old. The old allocation in turn depends on the aggregate old skill distribution, which follows from the young's allocation. To prove the existence of an equilibrium, I impose mild conditions on the distribution of the idiosyncratic learning shocks.

**Assumption A.1** (Idiosyncratic learning shocks)**.** *Suppose that (i) the support of $F$ is $\mathbb{R}_+$, and (ii) for any function $h : \mathbb{R}_+ \mapsto \mathbb{R}_+$, $\int h(ex)dF(e) < \infty$ for all $x < \infty$.*

I start from the young spatial allocation. Existence of a solution to (1.5) is essentially given by Schauder fixed point theorem. I drop the $y$ superscript for notational simplicity. Let $X \equiv [\underline{s}, \bar{s}]^L \subseteq \mathbb{R}_+^L$. Suppose for simplicity that $\underline{s} > 0$, $\bar{s} < \infty$, and $n(s) < \infty$ for all

$s \in X$. Then, $X$ is a compact convex subset of $\mathbb{R}^L_+$. Define $C_b(X,Y)$ as the set of bounded continuous functions that maps $X$ into $Y \subset \mathbb{R}^+_L$, where $Y = [\underline{n}, \bar{n}]^L$ is also a compact convex subset of $\mathbb{R}^L_+$ with $\underline{n} > 0$ and $\bar{n} < \infty$. When equipped with the sup norm, the vector space $C_b(X,Y)$ is a Banach space. Define the operator $\mathcal{M} : C_b(X,Y) \to Q$ by

$$\mathcal{M}[\boldsymbol{\eta}](\boldsymbol{s}) = \left\{ n(s_\ell) \left( \frac{\mathrm{e}^{\vartheta\left(s_\ell T_\ell + B^y_\ell \beta + O(s_\ell, \eta_\ell)\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(s_\ell T_{\ell'} + B^y_{\ell'} + \beta O(s_\ell, \eta_{\ell'})\right)}} \right) \right\}_{\ell=1}^{L}$$

for $\boldsymbol{\eta} \in C_b(X,Y)$, where

$$O(s,\eta) = \int \left( \int \mathbb{E}(V^o[e\gamma(s,s_p), \boldsymbol{\varepsilon}]) dF(e) \right) \left( \frac{\eta(s_p)}{\int \eta(\sigma) d\sigma} \right) ds_p.$$

The second part of Assumption A.1 guarantees that $O(s,\eta)$ exists. For any $\boldsymbol{\eta} \in C_b(X,Y)$, we know that $\mathcal{M}[\boldsymbol{\eta}](\boldsymbol{s})$ is bounded. Furthermore, Assumption 1.1 implies that $s \to O(s,n)$ is continuous, and therefore so is $\boldsymbol{s} \to \mathcal{M}[\boldsymbol{\eta}](\boldsymbol{s})$. Hence, $\mathcal{M} : C_b(X,Y) \to C_b(X,Y)$. Finally, $O(s,\eta)$ is continuous in $\eta$ since $\eta_\ell(s) > 0$ and $\eta_\ell(s) < \infty$ for all $s \in [\underline{s}, \bar{s}]$, all $\ell \in \{1, 2, \ldots, L\}$, and any $\boldsymbol{\eta} \in C_b(X,Y)$. It automatically follows that $\mathcal{M}$ is a continuous operator, and Schauder fixed point theorem implies that there exists at least one $\eta^\star \in C_b(X,Y)$ such that $\mathcal{M}[\boldsymbol{\eta}^\star] = \eta^\star$.

Turning to the old allocation, (1.4) has a solution for all $s$ as long as $n^o(s)$ has a solution. Using the first part of Assumption A.1, the old skill distribution (1.6) can be rewritten

$$N^o(s^o) = \sum_\ell \int \int n^y_\ell(s_y) \pi^y_\ell(s_p) F\left( \frac{s_o}{\gamma(s_y, s_p)} \right) ds_p ds_y. \tag{A.1}$$

The two integrals always exist. Furthermore, $N^o$ is continuous differentiable with $\lim_{s \to 0^+} N^o(s) = 0$ and $\lim_{s \to \infty} N^o(s) = 1$, and $n^o(s)$ exists. □

**Uniqueness** Equation (A.1) implies that, if the spatial allocation of young workers is unique, then so is $N^o$ and therefore the equilibrium. Uniqueness of the young spatial allocation is

proven by Banach fixed point theorem. I have already shown that $\mathcal{M}$ maps $C_b(X,Y)$ onto itself. It therefore only remain to prove that $\mathcal{M}$ is a contraction. For any $\boldsymbol{\eta}, \boldsymbol{\eta}' \in C_b(X,Y)$, we want to show

$$\begin{aligned}\|\mathcal{M}_\ell[\boldsymbol{\eta}](\boldsymbol{s}) - \mathcal{M}_\ell[\boldsymbol{\eta}'](\boldsymbol{s})\| &= \left\| n(s_\ell) \left[ \left( \frac{\mathrm{e}^{\vartheta\left(s_\ell T_\ell + B^y_\ell + \beta O(s_\ell, \eta_\ell)\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(s_\ell T_{\ell'} + B^y_{\ell'} + \beta O(s_\ell, \eta_{\ell'})\right)}} \right) - \left( \frac{\mathrm{e}^{\vartheta\left(s_\ell T_\ell + B^y_\ell + \beta O(s_\ell, \eta'_\ell)\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(s_\ell T_{\ell'} + B^y_{\ell'} + \beta O(s_\ell, \eta'_{\ell'})\right)}} \right) \right] \right\| \\ &\leq K \|\eta_\ell(s_\ell) - \eta'_\ell(s_\ell)\|,\end{aligned}$$

for $K \in [0,1)$. The term $\|\eta_\ell(s_\ell) - \eta'_\ell(s_\ell)\|$ is independent of $\vartheta\beta$. Meanwhile, the difference in the second equality is bounded, decreasing in $\vartheta\beta$, and converging to zero as $\vartheta\beta = 0$. Hence, for any $\boldsymbol{\eta}$ and $\boldsymbol{\eta}'$ with $\|\eta_\ell(s_\ell) - \eta'_\ell(s_\ell)\| > 0$, there always exist a $\vartheta\beta$ small enough such that the inequality holds. It follows that, for $\vartheta\beta$ small enough, $\mathcal{M}$ is a contraction, and the spatial allocation of young workers is unique. □

### A.1.2 Uniqueness and Stability of the Symmetric Equilibrium

**Proposition A.1** (Stability and uniqueness)**.** *Suppose that $T_\ell = \bar{T}$ for all $\ell$. Then, a symmetric equilibrium exists. If $\vartheta\beta$ is small, this equilibrium is unique.*

*Proof.* Existence is shown by guessing and verifying that $\pi^y_\ell = n^y$ is an equilibrium. Uniqueness follows from Proposition 1.1. □

The corollary below derives a bound on $\vartheta\beta$ that ensure the local stability and uniqueness of the symmetric equilibrium under a functional form restriction on the learning technology. For that, define the relative amenity of city $\ell$

$$\mathcal{B}^a_\ell \equiv \frac{\mathrm{e}^{\vartheta B^a_\ell}}{\sum_{\ell'} \mathrm{e}^{\vartheta B^a_{\ell'}}}, \quad a \in \{y, o\}. \tag{A.2}$$

**Corollary A.2.** *Suppose that $T_\ell = \bar{T}$ for all $\ell$, and $\gamma(s,s') = \mathcal{F}(s) + \mathcal{G}(s)\mathcal{H}(s')$ for $\mathcal{F}$, $\mathcal{G}$ and*

$\mathcal{H}$ *some functions. The symmetric equilibrium is stable if*

$$\theta\beta \max_{\ell} \max \left\{1 - \mathcal{B}_\ell^y, \mathcal{B}_\ell^y\right\} \left|\mathbb{C}ov[\mathcal{G}(S^y), \mathcal{H}(S^y)]\right| < 1,$$

*and it is unique if $L$ is large and*

$$4\theta\beta \left(\max_s \mathcal{G}(s) - \min_s \mathcal{G}(s)\right) \left(\max_s \mathcal{H}(s) - \min_s \mathcal{H}(s)\right) < 1.$$

*Proof.* For simplicity, I do the proof assuming away the idiosyncratic learning shocks. Under $\boldsymbol{T} = \bar{T}$, the expected utility of old workers is $\mathcal{V}^o(s) = s\bar{T} + c$ for some constant $c$. Since the rest of the equilibrium is fully determined by the young spatial allocation, I omit the $y$ superscript. Given the old utility, the expected future utility of young workers employed in city $\ell$ is $O_\ell(s) = \bar{T}\mathcal{F}(s) + \bar{T}\mathcal{G}(s)\mathbb{E}[\mathcal{H}(S_\ell)]$. Let $\mu_\ell \equiv \mathbb{E}[\mathcal{H}(S_\ell)]$ and $\mu \equiv \mathbb{E}[\mathcal{H}(S)]$. The spatial allocation of young workers is given by

$$n_\ell(s) = n(s) \left(\frac{\mathrm{e}^{\vartheta B_\ell + \theta\beta\mathcal{G}(s)\mu_\ell}}{\sum_{\ell'} \mathrm{e}^{\vartheta B_{\ell'} + \theta\beta\mathcal{G}(s)\mu_{\ell'}}}\right).$$

Note that only the $\boldsymbol{\mu} \equiv \{\mu_\ell\}_\ell$ matters to pin down the young allocation. These local averages are in turn given by

$$\mu_\ell = \frac{\int \mathcal{H}(s) \left(\frac{\mathrm{e}^{\theta\beta\mathcal{G}(s)\mu_\ell}}{\sum_{\ell'} \mathrm{e}^{\vartheta B_{\ell'} + \theta\beta\mathcal{G}(s)\mu_\ell}}\right) dN(s)}{\int \left(\frac{\mathrm{e}^{\theta\beta\mathcal{G}(s')\mu_\ell}}{\sum_{\ell'} \mathrm{e}^{\vartheta B_{\ell'} + \theta\beta\mathcal{G}(s')\mu_{\ell'}}}\right) dN(s')}.$$

Hence, we have reduced the problem to a $L$-dimensional fixed point. Let $\mathcal{M}$ be the operator that maps $\boldsymbol{\mu}$ onto itself as defined by the above equation. Note that Assumption 1.1 requires $\mathcal{H}(s)$ to be bounded above and below. Let $\underline{h} \equiv \min_s \mathcal{H}(s)$ and $\bar{h} \equiv \max_s \mathcal{H}(s)$. Then, we know that $\mathcal{M} : [\underline{h}, \bar{h}]^L \mapsto [\underline{h}, \bar{h}]^L$. Clearly, $\mu_\ell = \mu$ is a solution to this fixed point. A sufficient condition for the symmetric equilibrium to be stable is $|\partial_{\mu_{\ell'}} \mathcal{M}_\ell(\boldsymbol{\mu})| < 1$ for all pair $(\ell, \ell')$ and $\boldsymbol{\mu} = \{\mu\}_{\ell=1}^L$. A sufficient condition for the symmetric equilibrium to be unique is $|\partial_{\mu_{\ell'}} \mathcal{M}_\ell(\boldsymbol{\mu})| < 1$ for all pair $(\ell, \ell')$ and all $\boldsymbol{\mu}$, in which case $\mathcal{M}$ is a contraction.[1] For any $\boldsymbol{\mu}$,

[1] To see this, note that the condition implies $\|D\mathcal{M}(\boldsymbol{\mu})\| \leq K < 1$. Then, take two points $\boldsymbol{\mu}$ and $\boldsymbol{\mu}'$ in

the partial derivative of $\mathcal{M}_\ell$ w.r.t. $\mu_\ell$ is

$$\frac{\partial \mathcal{M}_\ell(\boldsymbol{\mu})}{\partial \mu_\ell} = \theta\beta \int (\mathcal{H}(s) - \mu_\ell)\, \mathcal{G}(s)\pi_\ell(s) \left(1 - \frac{n_\ell(s)}{n(s)}\right) ds,$$

and, for $\ell' \neq \ell$, it is

$$\frac{\partial \mathcal{M}_\ell(\boldsymbol{\mu})}{\partial \mu_{\ell'}} = -\theta\beta \int (\mathcal{H}(s) - \mu_\ell)\mathcal{G}(s)\pi_\ell(s) \left(\frac{n_{\ell'}(s)}{n(s)}\right) ds.$$

**Stability**. When the equilibrium is symmetric, $\boldsymbol{\mu} = \{\mu\}_\ell$. The derivatives simplify to

$$\frac{\partial \mathcal{M}_\ell(\boldsymbol{\mu})}{\partial \mu_\ell} = \theta\beta\,(1 - \mathcal{B}_\ell) \int (\mathcal{H}(s) - \mu)\, \mathcal{G}(s)n(s)ds = \theta\beta\,(1 - \mathcal{B}_\ell)\, \mathbb{C}\text{ov}[\mathcal{G}(S), \mathcal{H}(S),]$$

and

$$\frac{\partial \mathcal{M}_\ell(\boldsymbol{\mu})}{\partial \mu_{\ell'}} = -\theta\beta\mathcal{B}_{\ell'}\mathbb{C}\text{ov}[\mathcal{G}(S), \mathcal{H}(S)]$$

for $\ell' \neq \ell$. Imposing $|\partial_{\mu_{\ell'}} F_\ell(\boldsymbol{\mu})| < 1$ yields the first condition of Proposition A.1.

**Uniqueness**. For uniqueness, the above condition is necessary but not sufficient. To derive a sufficient condition, suppose that there are many cities. Since $\mathcal{S}^y$ is bounded, we know that $|\mu_\ell - \mu_{\ell'}| \leq \bar{s} - \underline{s} < \infty$ for all $(\ell, \ell')$. That is, no city will ever have learning opportunities so attractive as to capture the entire employment share. As a consequence, if amenities are independent of the number of cities, then $\lim_{L\to\infty} \mathcal{B}_\ell \to 0^+$ for all $\ell$. Hence, for $L$ large, the

$[\underline{s}, \bar{s}]^L$. Define $\Delta(\lambda) \equiv \lambda\boldsymbol{\mu} + (1-\lambda)\boldsymbol{\mu}'$, for $\lambda \in [0,1]$. Clearly, $\partial_\lambda \Delta(\lambda) = \boldsymbol{\mu} - \boldsymbol{\mu}' \perp \lambda$. Hence, it follows that

$$\mathcal{M}(\boldsymbol{\mu}) - \mathcal{M}(\boldsymbol{\mu}') = \mathcal{M}[\Delta(1)] - \mathcal{M}[\Delta(0)] = \int_0^1 \frac{d\mathcal{M}[\Delta(\lambda)]}{d\lambda} d\lambda = \int_0^1 d\mathcal{M}[\Delta(\lambda)] \frac{\partial \Delta(\lambda)}{\partial \lambda} d\lambda,$$

where the second equality is from the fundamental theorem of calculus and the third from the chain rule. Thus, applying the norm $\|\cdot\|$ on both sides,

$$\|\mathcal{M}(\boldsymbol{\mu}) - \mathcal{M}(\boldsymbol{\mu}')\| = \| \int_0^1 d\mathcal{M}[\Delta(\lambda)] \frac{\partial \Delta(\lambda)}{\partial \lambda} d\lambda \| \leq \int_0^1 \|d\mathcal{M}[\Delta(\lambda)]\| \| \frac{\partial \Delta(\lambda)}{\partial \lambda} \| d\lambda \leq \int_0^1 K \|\boldsymbol{\mu} - \boldsymbol{\mu}'\| d\lambda = K \|\boldsymbol{\mu} - \boldsymbol{\mu}'\|,$$

where the first inequality is from the triangle inequality. Since $K < 1$, $\mathcal{M}$ is a contraction.

partial derivatives rewrite

$$\frac{\partial \mathcal{M}_\ell(\boldsymbol{\mu})}{\partial \mu_\ell} \approx \theta\beta \int (\mathcal{H}(s) - \mu_\ell)\, \mathcal{G}(s)\pi_\ell(s)ds = \vartheta\beta \mathbb{C}\text{ov}[\mathcal{G}(S_\ell), \mathcal{H}(S_\ell)]$$

and $\partial_{\mu_{\ell'}} \mathcal{M}_\ell(\boldsymbol{\mu}) \approx 0$. If we knew the local distributions $\{\pi_\ell\}$, then uniqueness would be guaranteed by

$$\theta\beta \left|\mathbb{C}\text{ov}[\mathcal{G}(S_\ell), \mathcal{H}(S_\ell)]\right| < 1.$$

The local distributions are equilibrium objects. Yet, it is possible to bound the covariance since we know that the $\mathcal{G}(S_\ell)$ and $\mathcal{H}(S_\ell)]$ are bounded. This yields the second condition of Proposition A.1. □

### A.1.3 The Sorting of Old Workers

**Proposition A.3** (Sorting)**.** *The old allocation satisfies SPAM.*

*Proof.* For the entire proof I omit the old superscript for notational simplicity. I first derive an expression for the difference in density between $\ell$ and $\ell'$:

$$\pi_\ell(s) - \pi_{\ell'}(s) = \left(\frac{N_{\ell'}}{N_\ell}\right) \pi_{\ell'}(s) \mathrm{e}^{\vartheta(B_\ell - B_{\ell'})} \left(\mathrm{e}^{\vartheta s(T_\ell - T_{\ell'})} - \mathbb{E}\left[\mathrm{e}^{\vartheta S_{\ell'}(T_\ell - T_{\ell'})}\right]\right). \tag{A.3}$$

This follows from the definition of $\pi_\ell(s)$, $N_\ell$ and (1.4). Let $f : \mathcal{S}^o \mapsto \mathbb{R}$ be any measurable function. Using (A.3), the difference in the $g$-transformed mean skill between city $\ell$ and $\ell'$ is

$$\mathbb{E}[g(S_\ell)] - \mathbb{E}[g(S_{\ell'})] = \left(\frac{N^o_{\ell'}}{N^o_\ell}\right) \mathrm{e}^{\vartheta(B_\ell - B_{\ell'})} \mathbb{C}\text{ov}\left(g(S_{\ell'}), \mathrm{e}^{\vartheta S_{\ell'}(T_\ell - T_{\ell'})}\right).$$

Take any non-decreasing function $g$. Then, the above expression implies $\mathbb{E}[g(S_\ell)] > \mathbb{E}[g(S_{\ell'})]$ iff $T_\ell > T_{\ell'}$. That is, the local skill densities of old workers can be FOSD-ordered by $T_\ell$. □

### A.1.4 The First Order Approximation – general technology

**Old workers** Linearizing the old allocation is straightforward. From (1.4), we know that

$$\left.\frac{\partial n_\ell^o(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \mathcal{B}_\ell^o \left( \left.\frac{\partial n^o(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} + \theta s \bar{n}^o(s) I_{\ell\ell'} \right),$$

for $I_{\ell\ell'}^o \equiv \mathbb{1}\{\ell' = \ell\} - \mathcal{B}_{\ell'}^o$ and $\mathcal{B}_\ell^o$ defined in (A.2). The first term is the GE effect of increasing the TFP of city $\ell'$ on the density of skill $s$. We can guess that this term will be nil. The second term is the reallocation of workers over space. Hence, to a first order, the spatial allocation of old workers is

$$n_\ell^o(s) \approx \bar{n}^o(s)\mathcal{B}_\ell^o + \theta \bar{n}^o(s)\mathcal{B}_\ell^o \log\left(\frac{T_\ell}{\bar{T}}\right) s, \tag{A.4}$$

where

$$\mathcal{T}_\ell^o \equiv \log\left(\frac{T_\ell}{\bar{T}}\right) - \sum_l \mathcal{B}_l^o \log\left(\frac{T_l}{\bar{T}}\right). \tag{A.5}$$

When $B_\ell^o = B^o$ for all $\ell$ and $\mathbb{E}[\log T_\ell] = \log \bar{T}$, this boils down to (1.7).

**Young workers** For the young, given (1.5), we have

$$\left.\frac{\partial n_\ell^y(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \vartheta \mathcal{B}_\ell^y n^y(s) \left[ \bar{T} s I_{\ell\ell'}^y + \beta \left( \frac{\partial O_\ell}{\partial \log T_{\ell'}} - \sum_l \mathcal{B}_l^y \frac{\partial O_l(s)}{\partial \log T_{\ell'}} \right) \right].$$

The first term is identical to the old allocation. The second term captures how the learning opportunities varies with $\boldsymbol{T}$. Recall that

$$O_\ell(s) = \int\int \mathcal{V}^o[e\gamma(s, s_p)]\pi_\ell^y(s_p) dF(e) ds_p.$$

When $\boldsymbol{T} = \bar{T}$, $\mathcal{V}^o(s) = \bar{T}s + c$ for some constant $c$. In addition, $\partial_{T_\ell}\mathcal{V}^o(s)|_{\boldsymbol{T}=\bar{T}} = \bar{T}s\mathcal{B}^o_\ell$. Hence,

$$\left.\frac{\partial O_\ell(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \bar{T}\mathcal{B}^o_{\ell'}\int \gamma(s,s_p)n^y(s_p)ds_p + \bar{T}\int \gamma(s,s_p)\frac{\partial \pi^y_\ell(s_p)}{\partial \log T_{\ell'}}ds_p + c,$$

where I have redefined $\gamma = \gamma\int edF(e)$ wlog.[2] Using (1.2), it follows that

$$\begin{aligned}\left.\frac{\partial O_\ell}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} - \sum_l \mathcal{B}^y_l \left.\frac{\partial O_l}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} &= \bar{T}\int \left.\frac{\partial \pi^y_\ell(s_p)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} \gamma(s,s_p)ds_p \\ &= \frac{\bar{T}}{\mathcal{B}^y_\ell}\int \left.\frac{\partial n^y_\ell(s_p)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} \left(\gamma(s,s_p) - \mathbb{E}[\gamma(s,\mathcal{S}^y)]\right)ds_p,\end{aligned}$$

where the second equality follows from the expression for $\partial_{T_{\ell'}}\pi^y_\ell(s_p)$. Plugged in the initial expression, we obtain

$$\left.\frac{\partial n^y_\ell(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \theta n^y(s)\left(\mathcal{B}^y_\ell s I^y_{\ell\ell'} + \beta\int \frac{\partial n^y_\ell(s_p)}{\partial T_{\ell'}}\left(\gamma(s,s_p) - \mathbb{E}[\gamma(s,S^y)]\right)ds_p\right).$$

The above expression constitutes a fixed point over the functions $\partial_{T_{\ell'}}n^y_\ell(s)$. I solve this fixed point in two steps. First, I guess and verify the general form of the solution:

$$\left.\frac{\partial n^y_\ell(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \theta n^y(s)\mathcal{B}^y_\ell I^y_{\ell\ell'}\eta(s), \tag{A.6}$$

for $\eta$ some city-independent function. From this guess, we also know that the partial derivative of the local skill density is

$$\left.\frac{\partial \pi^y_\ell(s)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \theta n^y(s)I^y_{\ell\ell'}\left(\eta(s) - \mathbb{E}[\eta(S^y)]\right). \tag{A.7}$$

[2]Clearly the mean of the learning shock and the average learning $\gamma$ cannot be separately identified from this theory. From the estimation, I assume that $e$ is log-normally distributed and I normalize the mean of $e$ to zero.

Second, plugging the guess in the expression for $\partial_{T_{\ell'}} n^y_\ell(s)$ verifies it and yields the implicit expression for $\eta$:

$$\eta(s) = s + \theta\beta \int \left(\gamma(s, s_p) - \mathbb{E}[\gamma(s, S^y)]\right) n^y(s_p)\eta(s_p)ds_p = s + \theta\beta\Theta(s),$$

where $\Theta(s)$ is defined in (1.8). The above expression is a Fredholm integral equation of the second type. Standard results can be used to show that a solution exists when the dispersion forces, summarized here by $1/\theta$, are large enough.

**Lemma A.1** (Existence and uniqueness)**.** *Suppose that*

$$\theta\beta \left( \int \int |\gamma(s, s_p) - \mathbb{E}[\gamma(s, S)]|^2 dN(s_p)dN(s) \right)^{1/2} < 1.$$

*Then, a solution to* (1.8) *exists, is unique, and is continuous and bounded in* $[\underline{s}, \bar{s}]$*.*

*Proof.* I omit the $y$ superscript for simplicity. Define $K(s, s_p) \equiv \gamma(s, s_p) - \mathbb{E}[\gamma(s, S)]$ so as to rewrite (1.8) as

$$\eta(s) = s + \theta\beta \int K(s, s_p)\eta(s_p)dN(s).$$

This constitutes a Fredholm equation of the second kind. I restrict my attention to solutions to (1.8) that belong to $L_2(dN, [\underline{s}, \bar{s}])$. Under Assumption 1.1 and $\bar{s} < \infty$, we have[3]

$$\int |K(s, s_p)|^2 dN(s_p) \leq C \quad \text{and} \quad \int |s|^2 dN(s) < \infty,$$

for $C$ some constant. In addition, $K$ is globally continuous with respect to $s$. It follows that any solution to (1.8) is bounded and continuous in $[\underline{s}, \bar{s}]$ (Zabreĭko, 1975, Chapter 2, Theorems 1.7 and 1.8).

Uniqueness of a solution to (1.8) implies existence (Zabreĭko, 1975, Theorem 1.2). Equation

[3]As mentioned in the main text, $\bar{s} = \infty$ is possible as long as $\int |s|^2 dN(s) < \infty$ is satisfied.

(1.8) has a unique solution when $\theta\beta < |\bar{\lambda}|$, where $\bar{\lambda}$ is the characteristic value of $K$ with smallest magnitude. This characteristic value can be solved for when imposing a separability condition on $\gamma$ (see Section A.1.5). Without this restriction, solving for $|\bar{\lambda}|$ is non-trivial. However, if the Neumann series

$$\eta^f(s) = f(s) + \sum_{j=1}^{\infty} (\theta\beta)^j \int K_j(x,t) t dN(t) \tag{A.8}$$

converges for any function $f$ satisfying $\int |f(s)|^2 dN(s) < \infty$, where $K_j$ is the $j$-th iterated kernel of $K$, then $\theta\beta < |\bar{\lambda}|$ (e.g. Zabreĭko, 1975, Chapter 2). The inequality $\theta\beta\|K\| < 1$ provides a sufficient condition for (A.8) to converge. □

From there, the young spatial allocation is given to a first order by

$$n_\ell^y(s) \approx n^y(s)\mathcal{B}_\ell^y + \theta n^y(s)\mathcal{B}_\ell^y\mathcal{T}_\ell^y s + \theta^2\beta n^y(s)\mathcal{B}_\ell^y\mathcal{T}_\ell^y\Theta(s), \tag{A.9}$$

where $\mathcal{T}_\ell^y$ is defined analogously to (A.5). The within-city young skill distribution is

$$\pi_\ell^y(s) \approx n^y(s) + \theta n^y(s)\mathcal{T}_\ell^y\left(s - \mathbb{E}[S^y]\right) + \theta^2\beta n^y(s)\mathcal{T}_\ell^y\left(\Theta(s) - \mathbb{E}[\Theta(S^y)]\right). \tag{A.10}$$

Accordingly, the young average skill in city $\ell$ is

$$\mathbb{E}[S_\ell^y] \approx \mathbb{E}[S^y] + \theta\mathcal{T}_\ell^y\mathbb{V}\mathrm{ar}[S^y] + \theta^2\beta\mathcal{T}_\ell^y\mathbb{C}\mathrm{ov}[S^y, \Theta(S^y)],$$

as claimed in (1.9).

**Aggregates** I have argued earlier that, to a first order, there is no aggregate effect of cities on the old skill distribution, $\partial_{T_\ell} N^o(s) = 0$. Differentiating (1.6) w.r.t. $T_{\ell'}$ yields

$$\left.\frac{\partial N^o(s_o)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} = \mathcal{B}_\ell^y \int\int \sum_\ell \left.\frac{\partial \pi_\ell^y(s_p)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} F\left(\frac{s_o}{\gamma(s_y, s_p)}\right) ds_p dN^y(s_y) + \int\int \sum_\ell \left.\frac{\partial n_\ell^y(s_y)}{\partial \log T_{\ell'}}\right|_{\boldsymbol{T}=\bar{T}} F\left(\frac{s_o}{\gamma(s_y, s_p)}\right) dN^y(s_p) ds_y = 0,$$

where the second equality follows from (1.2).

### A.1.5 The First Order Approximation – example

Suppose that the learning technology reads:

$$\gamma(s, s_p) = \mathcal{F}(s) + \mathcal{G}(s)\mathcal{H}(s_p),$$

for $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ functions that satisfy Assumption 1.1. This class of learning technology contains (1.1). The integral equation simplifies to

$$\eta(s) = s + \theta\beta\mathcal{G}(s) \int \left(\mathcal{H}(s_p) - \mathbb{E}[\mathcal{H}(S^p)]\right) \eta(s_p) dN^y(s_p). \tag{A.11}$$

Solving for this fixed point amounts to solving for the integral on the right-hand side. Multiplying both sides by $(\mathcal{H}(s) - \mathbb{E}[\mathcal{H}(s)])n^y(s)$, integrating over $s$ and re-arranging, we get

$$(1 - \theta\beta\mathbb{C}\text{ov}[\mathcal{G}(S^y), \mathcal{H}(S^y)]) \int (\mathcal{H}(s) - \mathbb{E}[\mathcal{H}(S^y)])\eta(s) dN^y(s) = \mathbb{C}\text{ov}[S^y, \mathcal{H}(S^y)].$$

From here, it is easy to show that the characteristic root of (A.11) is $|\bar{\lambda}| = |1/\mathbb{C}\text{ov}[\mathcal{G}(S^y), \mathcal{H}(S^y)]|$. Hence, a solution to (A.11) exists and is unique when $\theta\beta|\mathbb{C}\text{ov}[\mathcal{G}(S^y), \mathcal{H}(S^y)]| \leq 1$. This condition is tighter than the one laid out in Lemma A.1. When it is satisfied, we can plug back

the expression for the integral into (A.11) to obtain a closed-form expression for $\Theta$:

$$\Theta(s) = \mathcal{G}(s) \left( \frac{\mathbb{C}\text{ov}[S^y, \mathcal{H}(S^y)]}{1 - \theta\beta\mathbb{C}\text{ov}[\mathcal{G}(S^y), \mathcal{H}(S^y)]} \right).$$

If the covariance between $S^y$ and $\mathcal{H}(S^y)$ is positive, so that, on average, workers learn more from skilled individuals, then $\Theta$ inherits the property of $\mathcal{G}(s)$. If $\mathcal{G}(s) > 0$, then worker $s$ learns more from skilled partner, and $\Theta(s) > 0$. If $\mathcal{G}'(s) > 0$, then so is $\Theta(s)$. Furthermore, $\Theta$ is independent from $\mathcal{F}(s)$. Using $\mathcal{G}(s) = g_2 + g_{12}s$ and $\mathcal{H}(s_p) = s_p$ yields

$$\Theta(s) = \Theta\left(g_2 + g_{12}s\right) \quad \text{where} \quad \Theta = \frac{\mathbb{V}\text{ar}[S^y]}{1 - \theta\beta g_{12}\mathbb{V}\text{ar}[S^y]},$$

as claimed in the main text.

### A.1.6 The Second Order Approximation

The second order approximations are implemented assuming identical amenities for simplicity.[4]

**Old skill** The expected old skill of a worker with initial skill $s$ reads

$$\mathbb{E}[S^o \mid s] = \sum_\ell \left( \frac{n^y_\ell(s)}{n^y(s)} \right) \mathbb{E}[S^o_\ell \mid s] = \sum_\ell \left( \frac{n^y_\ell(s)}{n^y(s)} \right) \int \gamma(s_y, s_p)\pi^y_\ell(s_p)ds_p,$$

while the average skill of old workers is

$$\mathbb{E}[S^o] = \int \mathbb{E}[S^o \mid s]dN^y(s).$$

[4]The aggregate effects on productivity are due to the spatial variations in local skill density, see (A.10). The local skill densities are independent of city amenity. Adding city amenity would therefore not change the conclusion.

In both expressions I have subsumed the expected learning shock in $\gamma$ wlog. The partial derivative of the expected skill for workers with skill $s$ w.r.t. the TFP of city $\ell$ is

$$\frac{\partial \mathbb{E}[S^o \mid s]}{\partial \log T_l} = \frac{1}{n^y(s)} \left( \sum_\ell \int \gamma(s,s') \left( \frac{\partial n^y_\ell(s)}{\partial \log T_l} \right) \pi^y_\ell(s') ds' + \sum_\ell \int \gamma(s,s') n^y_\ell(s) \left( \frac{\partial \pi^y_\ell(s')}{\partial \log T_l} \right) ds' \right).$$

By a similar argument as for $N^o$, there is no first order effect around $\boldsymbol{T} = \bar{T}$. Hence, moving to the second order, differentiate the above expression w.r.t. $T_{l'}$ to get

$$\begin{aligned} \left. \frac{\partial^2 \mathbb{E}\left[S^o \mid s\right]}{\partial \log T_l \partial \log T_{l'}} \right|_{\boldsymbol{T}=1} = \frac{1}{n^y(s)} \int \gamma(s,s') \underbrace{\left( \sum_\ell \partial^2_{ll'} n^y_\ell(s) \right)}_{=0} n^y(s') ds' + \\ \frac{1}{n^y(s)} \frac{1}{L} \int \gamma(s,s') n^y(s) \left( \sum_\ell \partial^2_{ll'} \pi^y_\ell(s') \right) ds' + \\ \frac{1}{n^y(s)} \int \gamma(s,s') \left( \sum_\ell \partial_l n^y_\ell(s) \partial_{l'} \pi^y_\ell(s') \right) ds' + \\ \frac{1}{n^y(s)} \int \gamma(s,s') \left( \sum_\ell \partial_{l'} n^y_\ell(s) \partial_l \pi^y_\ell(s') \right) ds', \end{aligned}$$

where I am using the notation $\partial_l f(x) = |\partial f(x)/\partial \log T_l|_{\boldsymbol{T}=\bar{T}}$, and the equality on the first line follows from (1.2). Meanwhile, the second-order derivative of $\pi^y_\ell(s)$ evaluated at $\boldsymbol{T} = \bar{T}$ and summed across cities read

$$\sum_\ell \left. \frac{\partial^2 \pi^y_\ell(s)}{\partial \log T_l \partial \log T_{l'}} \right|_{\boldsymbol{T}=\bar{T}} = -L \sum_\ell \left( \left. \frac{\partial \pi^y_\ell(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \left. \frac{\partial N_\ell}{\partial \log T_{l'}} \right|_{\boldsymbol{T}=\bar{T}} + \left. \frac{\partial \pi^y_\ell(s)}{\partial \log T_{l'}} \right|_{\boldsymbol{T}=\bar{T}} \left. \frac{\partial N_\ell}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \right).$$

Plugged in the expression for the aggregate change in old skill, we therefore obtain

$$\left. \frac{\partial^2 \mathbb{E}[S^o \mid s]}{\partial \log T_l \partial \log T_{l'}} \right|_{\boldsymbol{T}=\bar{T}} = \frac{1}{n^y(s) L} \int \gamma(s,s') \sum_\ell \left( \left. \frac{\partial \pi^y_\ell(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \left. \frac{\partial \pi^y_\ell(s')}{\partial \log T_{l'}} \right|_{\boldsymbol{T}=\bar{T}} + \left. \frac{\partial \pi^y_\ell(s)}{\partial \log T_{l'}} \right|_{\boldsymbol{T}=\bar{T}} \left. \frac{\partial \pi^y_\ell(s')}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \right) ds'.$$

This expression relies entirely on first order terms which we have derived in Section A.1.4. Using (A.7), we get

$$\sum_{\ell} \frac{\partial \pi_{\ell}^{y}(s)}{\partial \log T_l}\bigg|_{\boldsymbol{T}=\bar{T}} \frac{\partial \pi_{\ell}^{y}(s')}{\partial \log T_{l'}}\bigg|_{\boldsymbol{T}=\bar{T}} = \theta^2 I_{ll'} \left(\eta(s) - \mathbb{E}[\eta(S^y)]\right) \left(\eta(s') - \mathbb{E}[\eta(S^y)]\right) n^y(s') n^y(s),$$

which follows from $\sum_{\ell} I_{\ell l} I_{\ell l'} = I_{ll'}$. But $I$ is symmetric function, $I_{ll'} = I_{l'l}$. Hence, $\sum_{\ell} \partial_l \pi_{\ell}^{y}(s) \partial_{l'} \pi_{\ell}^{y}(s') = \sum_{\ell} \partial_{l'} \pi_{\ell}^{y}(s) \partial_l \pi_{\ell}^{y}(s')$, and we end up with

$$\frac{\partial^2 \mathbb{E}[S^o \mid s]}{\partial \log T_l \partial \log T_{l'}}\bigg|_{\boldsymbol{T}=1} = \frac{2}{L} \theta^2 I_{ll'} \Omega(s)$$

for

$$\Omega(s) \equiv \left(\eta(s) - \mathbb{E}[\eta(S^y)]\right) \int \gamma(s, s') \left(\eta(s') - \mathbb{E}[\eta(S^y)]\right) dN^y(s'). \tag{A.12}$$

We can therefore conclude that, to a second-order around $\boldsymbol{T} = \bar{T}$, the expected old skill workers with young skill $s$ is

$$\mathbb{E}\left[S^o \mid s\right] \approx \mathbb{E}\left[\mathrm{S}^o \mid s\right] + \vartheta^2 \mathbb{V}\mathrm{ar}[\log T_{\ell}] \Omega(s).$$

Aggregating across the skill distribution yields (1.12).

**Between-city wage inequality** Let $\bar{\omega}_{\ell}^{y} \equiv \mathbb{E}[\log W_{\ell}^{y}]$ be the average log wage in city $\ell$. Between-city wage inequality is then defined as

$$\mathbb{V}\mathrm{ar}[\bar{\omega}_{\ell}^{y}] = \sum_{\ell} \left(\bar{\omega}_{\ell}^{y} - \bar{\omega}^{y}\right)^2 N_{\ell}^{y},$$

for $\bar{\omega}^y \equiv \mathbb{E}[\log W^y]$ is the aggregate average log wage of young workers. As expected, there is no first order effect on the variance. The partial derivative of $\mathbb{V}\text{ar}[\bar{\omega}_\ell^y]$ w.r.t. $T_l$ reads

$$\frac{\partial \mathbb{V}\text{ar}[\bar{\omega}_\ell^y]}{\partial \log T_l} = 2\sum_\ell \left(\bar{\omega}_\ell^y - \bar{\omega}^y\right)\left(\frac{\partial \bar{\omega}_\ell^y}{\partial \log T_l} - \frac{\partial \bar{\omega}^y}{\partial \log T_l}\right) N_\ell^y + \sum_\ell \left(\bar{\omega}_\ell^y - \bar{\omega}^y\right)^2 \frac{\partial N_\ell^y}{\partial \log T_l},$$

which, when evaluated at $\boldsymbol{T} = \bar{T}$, simplifies to $\partial_l \mathbb{V}\text{ar}[\bar{W}_\ell^y] = 0$. Hence, turning to the second order effect, we have that the second order partial derivative of $\mathbb{V}\text{ar}[\bar{W}_\ell^y]$ w.r.t. $(T_l, T_{l'})$ evaluated at $\boldsymbol{T} = \bar{T}$ is

$$\partial^2_{ll'} \mathbb{V}\text{ar}[\bar{\omega}_\ell^y] = \frac{2}{L}\sum_\ell \left(\partial_{l'}\bar{\omega}_\ell^y - \partial_{l'}\bar{\omega}^y\right)\left(\partial_l \bar{\omega}_\ell^y - \partial_l \bar{\omega}^y\right). \tag{A.13}$$

All that is required is therefore the first order effect of cities' TFP on wages. Using (A.7), the first marginal effect reads

$$\partial_l \bar{\omega}_\ell^y = \mathbb{1}\{\ell = l\} + \theta I_{l\ell} \mathbb{C}\text{ov}[\log S^y, \eta(S^y)].$$

Using (1.2), the second marginal effect is

$$\partial_l \bar{\omega}^y = \frac{1}{L}.$$

Combined, (A.13) rewrites

$$\partial^2_{ll'} \mathbb{V}\text{ar}[\bar{\omega}_\ell^y] = \frac{2}{L} I_{ll'} \left(1 + \theta \mathbb{C}\text{ov}[\log S^y, \eta(S^y)]\right)\left(1 + \theta \mathbb{C}\text{ov}[\log S^y, \eta(S^y)]\right)^2.$$

where I have used $\sum_\ell I_{l'\ell} I_{l\ell} = I_{ll'}$. I therefore conclude that, to a second order, the between-city variance of wages is

$$\mathbb{V}\text{ar}[\bar{\omega}_\ell^y] \approx \mathbb{V}\text{ar}[\log T_\ell]\left(1 + \theta \mathbb{C}\text{ov}[\log S^y, S^y] + \theta^2 \beta \mathbb{C}\text{ov}[\log S^y, \Theta(S^y)]\right)^2. \tag{A.14}$$

### A.1.7 Proof of Lemma 1.1

Suppose that the learning technology is either supermodular or submodular, $\gamma_{12}(s, s_p) > 0$ or $\gamma_{12}(s, s_p) < 0$ a.e. I then guess and verify that $\mathbb{C}\text{ov}[\gamma_1(s, S^y), S^y] \geq -1$ implies $\eta'(s) \geq 0$. So guess that $\eta'(s) > 0$. I first sign $\Theta'$ under this guess. Note from (1.8) that $\Theta$ can be written $\Theta(s) = \mathbb{C}\text{ov}[\gamma(s, S^y), \eta(S^y)]$. Differentiate $\Theta$ to obtain $\Theta'(s) = \mathbb{C}\text{ov}[\gamma_1'(s, S), \eta(S)]$. The covariance of two increasing function is positive while the covariance of an increasing and a decreasing function is negative. Hence, since $\eta' > 0$, we have that $\gamma_{12} > 0$ implies $\Theta'(s) > 0$ while $\gamma_{12} < 0$ implies $\Theta'(s) < 0$. We can now verify the guess. Differentiate $\eta$ and plug in the expression for $\eta$ in $\Theta$ to find

$$\eta'(s) = 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(s, S^y), S^y] + (\theta\beta)^2 \mathbb{C}\text{ov}[\gamma_1(s, S^y), \Theta(S^y)].$$

I distinguish between two cases. First, suppose that $\gamma_{12} > 0$. Then, $\mathbb{C}\text{ov}[\gamma_1(s, S^y), S^y] > 0$. Furthermore, since $\Theta' > 0$, we also have $\mathbb{C}\text{ov}[\gamma_1(s, S^y), \Theta(S^y)] > 0$. Hence, $\eta' > 0$. Second, suppose that $\sigma_{12} < 0$. Then, $\mathbb{C}\text{ov}[\gamma_1(s, S^y), S^y] < 0$. Second, since $\Theta' < 0$ and the covariance of two decreasing functions is positive, we have $\mathbb{C}\text{ov}[\gamma_1(s, S), \Theta(S)] > 0$. Hence, $\eta'(s) > 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(s, S), S] \geq 0$, where the second inequality is from the assumption on $\gamma$. This verifies our guess.

### A.1.8 Proof of Proposition 1.2

**Proposition 1.2.1** To a first order, (A.4) and (A.9) imply that the size of city $\ell$ is

$$N_\ell \approx \mathcal{B}_\ell^y + \mathcal{B}_\ell^o + \theta \left( \mathcal{B}_\ell^y \mathcal{T}_\ell^y \mathbb{E}[S^y] + \mathcal{B}_\ell^o \mathcal{T}_\ell^o \mathbb{E}[S^o] \right) + \theta^2 \beta \mathcal{B}_\ell^y \mathcal{T}_\ell^y \mathbb{E}[\Theta(S^y)].$$

Meanwhile, if interactions are not spatially segmented, then

$$\tilde{N}_\ell \approx \mathcal{B}_\ell^y + \mathcal{B}_\ell^o + \theta \left( \mathcal{B}_\ell^y \mathcal{T}_\ell^y \mathbb{E}[S^y] + \mathcal{B}_\ell^o \mathcal{T}_\ell^o \mathbb{E}[S^o] \right).$$

When $\boldsymbol{T} \approx \bar{T}$, we thus have that for any $\ell$ such that $\mathcal{T}_\ell^y > 0$, then $N_\ell > \tilde{N}_\ell \iff \mathbb{E}[\Theta(S^y)] > 0$. Under Lemma 1.1, $\eta$ is an increasing function. Hence, $\gamma(s,\cdot)$ increasing implies $\Theta(s) = \mathbb{C}\text{ov}[\gamma(s,S^y),\eta(S^y)] > 0$. Conversely, $\gamma(s,\cdot)$ decreasing implies $\Theta(s) < 0$. When amenities are the same everywhere, $\mathcal{T}_\ell^y > 0 \iff T_\ell > \bar{T}$, which yields Proposition 1.2.1. □

**Proposition 1.2.2** From (1.9), the difference between the average skill in city $\ell$ when interactions are spatially segmented vs. when they are not is given by

$$\mathbb{E}[S_\ell^y] - \mathbb{E}[\tilde{S}_\ell^y] \approx \theta^2\beta\mathcal{T}_\ell^y\mathbb{C}\text{ov}[S^y,\Theta(S^y)].$$

Similarly, from (A.14), the difference between the between-city wage inequality when interactions are segmented vs. when they are not is

$$\mathbb{S}\text{d}[\bar{\omega}_\ell^y] - \mathbb{S}\text{d}[\tilde{\bar{\omega}}_\ell^y] \approx \mathbb{S}\text{d}[\log T_\ell]\theta^2\beta\mathbb{C}\text{ov}[\log S^y,\Theta(S^y)].$$

Hence, when $\boldsymbol{T} \approx \bar{T}$, we have: (i) for any $\ell$ such that $\mathcal{T}_\ell^y > 0$, then $\mathbb{E}[S_\ell^y] > \mathbb{E}[\tilde{S}_\ell^y] \iff \mathbb{C}\text{ov}[S^y,\Theta(S^y)] > 0$, and (ii) $\mathbb{S}\text{d}[\bar{\omega}_\ell^y] > \mathbb{S}\text{d}[\tilde{\bar{\omega}}_\ell^y] \iff \mathbb{C}\text{ov}[\log S^y,\Theta(S^y)] > 0$. If $\gamma$ is supermodular, then $\gamma_1(s,\cdot)$ is increasing. Hence $\Theta'(s) = \mathbb{C}\text{ov}[\gamma_1(s,S^y),\eta(S^y)] > 0$ (Lemma 1.1), and $\mathbb{C}\text{ov}[S^y,\Theta(S^y)] > 0$. Conversely, $\gamma_1(s,\cdot)$ is decreasing under $\gamma$, and $\mathbb{C}\text{ov}[S^y,\Theta(S^y)] < 0$. □

### A.1.9 Proof of Proposition 1.3

The second order approximation of $\mathbb{E}[S^o]$ is derived in Section A.1.6. Define $\chi(s) \equiv \eta(s) - \mathbb{E}[\eta(S^y)]$. From the fundamental theorem of calculus,

$$\gamma(s,s') = \int_{\underline{s}}^{s}\int_{\underline{s}}^{s'} \gamma_{12}(x,y)dydx + \gamma(s,\underline{s}) - \gamma(\underline{s},\underline{s}) + \gamma(\underline{s},s').$$

From the definition of $\chi$, $\int \gamma(s,\underline{s})\chi(s)ds = \int \gamma(\underline{s},s')\chi(s')ds' = \int \gamma(\underline{s},\underline{s})\chi(s)ds = 0$. Therefore, $\Omega$ rewrites

$$\Omega = \int\int\int_{\underline{s}}^{s}\int_{\underline{s}}^{s'} \gamma_{12}(x,y)\chi(s)\chi(s')dydxdN^y(s')dN^y(s) = \int\int \gamma_{12}(x,y)\Psi(x)\Psi(y)dxdy,$$

where $\Psi(x) \equiv \int_x \chi(s)dN^y(s)$ and the second equality follows from Fubini's. Note that $\Psi$ rewrites $\Psi(x) = (1-N(x))\left(\mathbb{E}[\eta(S^y) \mid S^y \geq x] - \mathbb{E}[\eta(S^y)]\right)$. Hence, $\Psi(x) > 0$ iff $\mathbb{E}[\eta(S^y) \mid S^y \geq x] > \mathbb{E}[\eta(S^y)]$. But Lemma 1.1 guarantees that $\eta$ is an increasing function. It follows that $s \to \mathbb{E}[\eta(S^y) \mid S^y \geq s]$ is increasing, and $\mathbb{E}[\eta(S^y) \mid S^y \geq s] \geq \mathbb{E}[\eta(S^y) \mid S^y \geq \underline{s}] = \mathbb{E}[\eta(S^y)]$. Hence, $\Psi(x)\Psi(y) > 0$ and $\Omega$ preserves the sign of $\gamma_{12}$.

### A.1.10 The Learning Consequences of Local Interactions across Skills

**Proposition A.4** (Learning inequality)**.** *Define $s^\star$ as the skill with the average willingness to sort, $\eta(s^\star) \equiv \mathbb{E}[\eta(S^y)]$.*

1. *If $\gamma(s,\cdot)$ is increasing for all $s$, then $\mathbb{E}[S^o \mid s] > \mathbb{E}[\bar{S}^o \mid s]$ if and only if $s > s^\star$;*
2. *If $\gamma(s,\cdot)$ is decreasing for all $s$, then $\mathbb{E}[S^o \mid s] > \mathbb{E}[\bar{S}^o \mid s]$ if and only if $s < s^\star$.*

*Proof.* The second order approximation for $\mathbb{E}[S^o \mid s]$ is derived in Section A.1.6. From the fundamental theorem of calculus, $\Omega(s)$ can be rewritten

$$\gamma(s,s') = \int_{\underline{s}}^{s'} \gamma_2(s,y)dydx + \gamma(s,\underline{s}).$$

From the definition of $\chi$, we know that $\int \gamma(s,\underline{s})\chi(s)ds' = 0$, and therefore $\Omega(s)$ reads

$$\Omega(s) = \chi(s)\int\int_{\underline{s}}^{s'} \gamma_2(s,y)dy\chi(s')dN^y(s') = \chi(s)\int \gamma_2(s,y)\Psi(y)dy.$$

The proof of Proposition 1.3 has shown that $\Psi(y) > 0$. The sign of $\Omega(s)$ therefore depends on the product between the sign of $\chi(s)$ and that of $\gamma_2$. Let $s^\star$ be such that $\eta(s^\star) = \mathbb{E}[\eta(S^y)]$

and $\eta(s) > \mathbb{E}[\eta(S^y)]$ iff $s > s^\star$. Lemma 1.1 guarantees that such $s^\star$ exists. For $s > s^\star$, if $\gamma_2 > 0$ for all $y$, then $\Omega(s) > 0$; if $\gamma_2 < 0$ for all $y$, then $\Omega(s) < 0$. Opposite equality exists for $s < s^\star$. □

## A.2 Efficiency

### A.2.1 Planner's problem

Suppose that the planner is utilitarian. The planner places unitary weights on skill and cities and discounts the future of future generations at rate $\beta_S$. Let $\mathbb{E}_\ell[V_t^{y\star}(s, \boldsymbol{\varepsilon}) \mid s]$ denote the expected utility of young workers with skill $s$ working in city $\ell$ at time $t$, where the expectation is taken across the unobserved idiosyncratic preferences $\boldsymbol{\varepsilon}$. Define $\mathbb{E}_\ell[V_t^{o\star}(s, \boldsymbol{\varepsilon}) \mid s]$ for old workers. The planner's social welfare function,

$$\int \sum_\ell \mathbb{E}_\ell[V_0^{o\star}(s, \boldsymbol{\varepsilon}) \mid s] n_{0\ell}^{o\star}(s) ds + \sum_{t>0} \beta_S^t \int \sum_\ell \mathbb{E}_\ell[V_t^{y\star}(s, \boldsymbol{\varepsilon}) \mid s] n_{t\ell}^{y\star}(s) ds.$$

The expected utility of workers must be consistent with workers' location choices. By the property of extreme-value type one distribution, the expected utility of old workers working in city $\ell$ is identical to the *ex-ante* expected utility:

$$\mathbb{E}_\ell[V_t^{o\star}(s, \boldsymbol{\varepsilon}) \mid s] = \mathbb{E}[V_t^{o\star}(s, \boldsymbol{\varepsilon}) \mid s] = \frac{1}{\vartheta} \log \left( \sum_{\ell'} \mathrm{e}^{\vartheta c_{t\ell'}^{o\star}(s)} \right) \equiv \mathcal{V}_t^{o\star}(s).$$

Similarly for the young,

$$\mathcal{V}_t^{y\star}(s) = \frac{1}{\vartheta} \log \left( \sum_\ell \mathrm{e}^{\vartheta \left( c_\ell^{y\star}(s) + \beta O_\ell^\star(s) \right)} \right),$$

where

$$O_\ell^\star(s) = \int \int \mathcal{V}^{o\star}[e\gamma(s, s_p)] dF(e) \pi_{t\ell}^{y\star}(s_p) ds_p.$$

Hence, the social welfare function simplifies to

$$\int \mathcal{V}_0^{o\star}(s) \sum_\ell n_{0\ell}^{o\star}(s) ds + \sum_{t>0} \beta_S^t \int \mathcal{V}_t^{y\star}(s) \sum_\ell n_{t\ell}^{y\star}(s) ds.$$

The problem of the planner is to maximize this objective function by choosing a sequence $\{\boldsymbol{c}_t^{y\star}, \boldsymbol{c}_t^{o\star}, \boldsymbol{n}_t^{y\star}, \boldsymbol{n}_t^{o\star}, n_t^\star\}_{t=0}^\infty$.

Next, I note that between-skill transfers are not pined down. Intuitively, workers' utility is linear in income. The idiosyncratic location preferences generate some concavity across cities but not across skills. Hence, any between-skill transfer can be sustained to the extent that these transfers are feasible. To see this, suppose that the planner considers two consumption plans at time $t$ for young workers: $\boldsymbol{c}_t^y$ on the one hand, and $\tilde{\boldsymbol{c}}_t^y$ on the other hand. The two plans differ in that $\tilde{c}_{t\ell}^{y\star}(s) = c_{t\ell}^{y\star}(s) + q(s)$ for some function $q$. Since $q$ is not city-specific, it does not affect the spatial allocation, and the learning value of cities is the same in the two plans. Similarly, total output is constant. Hence, these two plans are jointly feasible if $\int q(s) dN^y(s) ds = 0$. Meanwhile, the difference in the aggregate utility of cohort $t$ between the two consumption plans is

$$\frac{1}{\vartheta} \int \left( \log \left( \sum_\ell \mathrm{e}^{\vartheta \left( c_\ell^{y\star}(s) + \beta O_\ell^\star(s) \right)} \right) - \log \left( \sum_\ell \mathrm{e}^{\vartheta \left( \tilde{c}_\ell^{y\star}(s) + \beta O_\ell^\star(s) \right)} \right) \right) n^y(s) ds = \int q(s) n^y(s) ds = 0.$$

That is, any function $q$ yields the same aggregate welfare, and between-skill transfers are not determinate. I therefore restrict my attention to allocation without between-skill transfers, wlog. I also suppose that between-age transfers are not allowed to focus on the learning externalities.

The planner thus faces the following constraints, where the variables in the parenthesis refer to the associated Karush–Kuhn–Tucker multiplier:

1. Young and old consumption feasibility ($\kappa_t^y(s)$ and $\kappa_t^o(s)$):

$$\sum_\ell c_{t\ell}^{y\star}(s) n_{t\ell}^{y\star}(s) \leq \sum_\ell T_\ell s n_{t\ell}^{y\star}(s), \quad \text{and} \quad \sum_\ell c_{t\ell}^{o\star}(s) n_{t\ell}^{o\star}(s) \leq \sum_\ell T_\ell s n_{t\ell}^{o\star}(s), \quad \forall (s,t)$$

2. Young labor supply ($\lambda_{t\ell}^y(s)$):

$$n_{t\ell}^{y\star}(s) \leq n^y(s) \left( \frac{\mathrm{e}^{\vartheta\left(c_{t\ell}^{y\star}(s) + \beta O_{t\ell}^\star(s)\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(c_{t\ell'}^{y\star}(s) + \beta O_{t\ell'}^\star(s)\right)}} \right), \quad \forall (s,t,\ell).$$

3. Old labor supply $\lambda_{t\ell}^o(s)$:

$$n_{t\ell}^{o\star}(s) \leq n_t^{o\star}(s) \left( \frac{\mathrm{e}^{\vartheta c_{t\ell}^{o\star}(s)}}{\sum_{\ell'} \mathrm{e}^{\vartheta c_{t\ell'}^{o\star}(s)}} \right), \quad \forall (s,t,\ell).$$

4. The law of motion for the skill distribution of old workers ($\nu_t(s)$):

$$n_{t+1}^{o\star}(s_o) = \sum_\ell \int\int \frac{1}{\gamma(s_y, s_p)} f\left( \frac{s_o}{\gamma(s_y, s_p)} \right) n_{t\ell}^{y\star}(s_y) \pi_{t\ell}^{y\star}(s_p) ds_p ds_y, \quad \forall (s,t).$$

The Lagrangian associated to the planner's problem is

$$\begin{aligned}
\mathcal{L} = {} & \frac{1}{\vartheta} \int \log\left( \sum_\ell \mathrm{e}^{\vartheta c_{0\ell}^{o\star}(s)} \right) \sum_\ell n_{0\ell}^{o\star}(s) ds + \frac{1}{\vartheta} \sum_{t>0} \beta_S^t \int \log\left( \sum_\ell \mathrm{e}^{\vartheta\left(c_{t\ell}^{y\star}(s) + \beta O_{t\ell}^\star(s)\right)} \right) \sum_\ell n_{t\ell}^{y\star}(s) ds - \\
& \sum_t \sum_\ell \int \lambda_{t\ell}^y(s) \left( n_{t\ell}^{y\star}(s) - n^y(s) \left( \frac{\mathrm{e}^{\vartheta\left(c_{t\ell}^{y\star}(s) + \beta O_{t\ell}^\star(s)\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(c_{t\ell'}^{y\star}(s) + \beta O_{t\ell'}^\star(s)\right)}} \right) \right) ds - \\
& \sum_t \sum_\ell \int \lambda_{t\ell}^o(s) \left( n_{t\ell}^{o\star}(s) - n_t^{o\star}(s) \left( \frac{\mathrm{e}^{\vartheta c_{t\ell}^{o\star}(s)}}{\sum_{\ell'} \mathrm{e}^{\vartheta c_{t\ell'}^{o\star}(s)}} \right) \right) ds - \\
& \sum_t \int \nu_t(s_o) \left( n_{t+1}^{o\star}(s_o) - \sum_\ell \int\int \frac{1}{\gamma(s_y, s_p)} f\left( \frac{s_o}{\gamma(s_y, s_p)} \right) n_{t\ell}^{y\star}(s_y) \pi_{t\ell}^{y\star}(s_p) ds_p ds_y \right) ds_o - \\
& \sum_t \int \kappa_t^y(s) \sum_\ell \left( c_{t\ell}^{y\star}(s) - T_\ell s \right) n_{t\ell}^{y\star}(s) ds + \sum_t \int \kappa_t^o(s) \sum_\ell \left( c_{t\ell}^{o\star}(s) - T_\ell s \right) n_{t\ell}^{o\star}(s) ds,
\end{aligned}$$

where I have kept implicit that

$$O^{\star}_{t\ell}(s) = \frac{1}{\vartheta}\int\int \log\left(\sum_{\ell'} \mathrm{e}^{\vartheta c^{o\star}_{t+1\ell'}[e\gamma(s,s_p)]}\right) dF(e)\pi^{y\star}_{t\ell}(s_p)ds_p.$$

### A.2.2 Optimal allocation

Solving this maximization problem consists of finding the function $s \to c^{y\star}_{t\ell}(s)$, $s \to c^{o\star}_{t\ell}(s)$, $s \to n^{y\star}_{t\ell}(s)$, $s \to n^{o\star}_{t\ell}(s)$, and $s \to n^{o\star}_{t}(s)$, for each $\ell \in \{1, 2, \dots, L\}$ and $t \in \{0, 1, 2, \dots\}$. This is an infinite-dimensional maximization problem that is solved using functional derivatives. I lay down the complete argument for the optimality condition regarding the initial old allocation, $c^{o\star}_{0\ell}$ and $n^{o\star}_{0\ell}(s)$. I then present more succinct derivations for the remaining optimality conditions. For the remaining of the derivation, I drop the star superscript for notation simplicity.

**Initial old** Suppose the planner contemplates an alternative consumption plan given by $\tilde{c}^{o}_{0l}(s) = c^{o}_{0l}(s) + \varepsilon\eta(s)$ for some $l$ and some arbitrary function $\eta(s)$, while $\tilde{c}^{o}_{0\ell}(s) = c^{o}_{0\ell}(s)$ for $\ell \neq l$. Optimality requires that $d\mathcal{L}/d\varepsilon = 0$ at $\varepsilon = 0$, or

$$\int \eta(s)n^{o}_{0l}(s)ds + \vartheta\int \eta(s)n^{o}_{0l}(s)\left(\lambda^{o}_{0l}(s) - \sum_{\ell}\lambda^{o}_{0\ell}(s)\frac{n^{o}_{0\ell}(s)}{n^{o}_{0}(s)}\right)ds - \int \kappa^{o}_{0}(s)\eta(s)n^{o}_{0l}(s)ds = 0,$$

where I have already imposed that $\sum_{\ell} n^{o}_{0\ell}(s) = n^{o}_{0}(s)$. Optimality requires that this hold for any function $\eta(s)$. In particular, letting $\eta(s) = \delta_x(s) = \delta(s - x)$, where $\delta$ is the Dirac delta function, the above optimality condition simplifies to

$$1 + \vartheta\left(\lambda^{o}_{0l}(x) - \sum_{\ell}\lambda^{o}_{0\ell}(x)\frac{n^{o}_{0\ell}(x)}{n^{o}_{0}(x)}\right) = \kappa^{o}_{0}(x),$$

where I have also used that the idiosyncratic location preferences guarantee an interior allocation to simplify the $n^{o\star}_{0l}(x)$. In the above expression, the only $l$ specific term is the KKT multiplier $\lambda^{o}_{0l}(x)$. Hence, $\lambda^{o}_{0l}(x) = \lambda^{o}_{0}(x)$ for all $l$, and we can conclude that $\kappa^{o}_{0}(x) = 1$.

Proceeding in a similar fashion for the optimality condition regarding $n^o_{0l}(s)$ and using $\kappa^o_0(x) = 1$, we have

$$\mathcal{V}^o_0(x) - (c^o_{0l}(x) - T_l x) = \lambda^o_0(x).$$

Multiplying by $n^o_{0l}(x)$, summing across $l$ and using the age-skill specific feasibility condition yields $\lambda^o_0(x) = \mathcal{V}^o_0(x)$ and therefore $c^o_{0l}(x) = T_l x$.

**Old workers** I now derive the optimality conditions for $(\boldsymbol{n}^o_t, \boldsymbol{n}^o_{\ell t}, \boldsymbol{c}^o_{\ell t})$ for $t \geq 1$. Regarding $\boldsymbol{c}^o_{lt}$, we have

$$\begin{aligned}\int \kappa^o_t(s) n^o_{tl}(s)\eta(s)ds = \beta_S^{t-1}\beta \int \sum_\ell n^y_{t-1\ell}(s)\delta_{c^o_{tl}} O_{t-1\ell}(s)ds + \\ \vartheta\beta \sum_\ell \int \lambda^y_{t-1\ell}(s) n^y_{t\ell}(s) \left( \delta_{c^o_{tl}} O_{t-1\ell}(s) - \sum_{\ell'} \frac{n^y_{t-1\ell'}(s)}{n^y(s)} \delta_{c^o_{tl}} O_{t-1\ell'}(s) \right) ds + \\ \vartheta \int n^o_{tl}(s)\eta(s) \left( \lambda^o_{tl}(s) - \sum_\ell \lambda^o_{t\ell}(s) \frac{n^o_{t\ell}(s)}{n^o_t(s)} \right) ds,\end{aligned}$$

for any $\eta$ function, where $\delta_x F$ denote the functional derivative of $F$ w.r.t. $x$. I anticipate that, at the optimum, it must be that $\lambda^y_{t-1\ell}(s) \equiv \lambda^y_{t-1}(s)$ for all $\ell$ and $t$. Hence, the second line is equal to zero. From the expression for $O_{t\ell}$, we further know that

$$\delta_{c^o_{tl}} O_{t-1\ell}(s) = \int\int \eta[e\gamma(s,s_p)] \frac{n^o_{tl}[e\gamma(s,s_p)]}{n^o_t[e\gamma(s,s_p)]} dF(e)\pi^y_{t-1\ell}(s_p)ds_p.$$

Hence, the optimality condition turns into

$$\begin{aligned}\int \kappa^o_t(s) n^o_{tl}(s)\eta(s)ds = \beta_S^{t-1}\beta \int \eta(q)\frac{n^o_{tl}(q)}{n^o_t(q)} \sum_\ell \int\int \frac{1}{\gamma(s,s_p)} f\left(\frac{q}{\gamma(s,s_p)}\right) n^y_{t-1\ell}(s)\pi^y_{t-1\ell}(s_p)ds_p ds dq + \\ \vartheta \int n^o_{tl}(s)\eta(s) \left( \lambda^o_{tl}(s) - \sum_\ell \lambda^o_{t\ell}(s) \frac{n^o_{t\ell}(s)}{n^o_t(s)} \right) ds,\end{aligned}$$

where I have used the change of variable $e\gamma(s, s_p) \to q$ to rewrite the integral on the first line. Using the law of motion for the aggregate old skill distribution, the two inner most integrals on the first line cancel out with $n_t^o(q)$. Setting $\eta(s) = \delta_x(s)$, we then obtain

$$\kappa_t^o(x) = \beta_S^{t-1}\beta + \vartheta\left(\lambda_{tl}^o(x) - \sum_\ell \lambda_{t\ell}^o(x)\frac{n_{t\ell}^o(x)}{n_t^o(x)}\right),$$

where both sides have been divided by $n_{tl}^o(x)$. As for the initial old consumption allocation, there is no $l$-specific term in the above expression, and $\lambda_{tl}^o(x) = \lambda_t^o(x)$. Hence, $\kappa_t^o(x) = \beta_S^{t-1}\beta \equiv \kappa_t^o$ for $t \geq 1$. Setting $\beta_S = \beta$, then $\kappa_t^o = \beta^t$ for $t \geq 0$.

Turning to the optimality condition w.r.t. $n_{lt}^o$, note that $O_{t\ell}(s)$ is independent of $n_{tl}^o$. Hence, after using $\lambda_{tl}^o(x) = \lambda_t^o(x)$ and $\kappa_t^o = \beta_S^{t-1}\beta$, the optimality condition reads

$$\lambda_t^o(x) = \beta_S^{t-1}\beta\left(T_l s - c_{tl}^o(x)\right).$$

Multiplying by $n_{tl}^o(x)$, summing across $l$ and using the feasibility condition, this implies $\lambda_t^o(x) = 0$ for all $t \geq 1$, and therefore $c_{tl}^o(x) = T_l s$.[5]

Finally, there remains the optimality condition w.r.t. $n_t^o$. There, we have

$$\sum_\ell \int \lambda_{\ell t}^o(s)\eta(s)\frac{n_{\ell t}^o(s)}{n_t^o(s)}ds - \int \nu_{t-1}(s)\eta(s)ds = \int \lambda_t^o(s)\eta(s)ds - \int \nu_{t-1}(s)\eta(s)ds = 0,$$

for $t \geq 1$, where the first equality follows from $\lambda_{\ell t}^o(x) = \lambda_t^o(x)$. Setting $\eta = \delta_x(s)$, we obtain the optimality condition $\nu_{t-1}(x) = \lambda_t^o(x) = 0$.

**Young workers** I finally derive the optimality conditions for $(\boldsymbol{n}_t^y, \boldsymbol{n}_{\ell t}^y, \boldsymbol{c}_{\ell t}^y)$ for $t \geq 0$. For consumption, note that the learning value of cities is independent of $c_{tl}^y$. Hence, the optimality

[5]The same would hold if we were to allow for transfers within age across skills. Then, feasibility would require $\int \lambda_t^o(x)n_t^o(x)dx = \beta_S^{t-1}\beta\sum_l \int n_{tl}^o(x)\left(T_l s - c_{tl}^o(x)\right)dx = 0$. But $\lambda_t^o(x) \geq 0$ since it is a KKT multiplier, and therefore $\lambda_t^o(x) = 0$ for all $x$. Hence, on the contrary of the young allocation, the old allocation is uniquely pin down even with across-skill transfers. Why? Between-skill transfers for old workers affect the incentives to learn, and thus the spatial allocation of workers.

condition w.r.t. $c_{tl}^y$ is

$$\beta_S^t + \vartheta \left( \lambda_{tl}^y(x) - \sum_\ell \lambda_{t\ell}^y(x) \frac{n_{t\ell}^y(x)}{n^y(x)} \right) = \kappa_t^y(x).$$

Hence, here as well, this implies $\lambda_{lt}^y(x) \equiv \lambda_t^y(x)$ and $\kappa_t^y(x) = \beta_S^t \equiv \kappa_t^y$. When $\beta_S = \beta$, then $\kappa_t^y = \kappa_t^o$.

I conclude with the optimality condition for $n_{tl}^y$. There, and crucially, $n_{tl}^y$ affects $O_{tl}$. The optimality condition is

$$\begin{aligned}
0 = {} & \beta_S^t \beta \int \sum_\ell n_{t\ell}^y(s) \delta_{n_{tl}^y} O_{t\ell}(s) ds + \beta_S^t \int \mathcal{V}_t^y(s) \eta(s) ds - \\
& \int \left( \lambda_{tl}^y(s) \eta(s) - \vartheta \beta \sum_\ell \lambda_{t\ell}^y(s) n_{t\ell}^y(s) \left[ \delta_{n_{tl}^y} O_{t\ell}(s) - \sum_{\ell'} \frac{n_{t\ell'}^y(s)}{n^y(s)} \delta_{n_{tl}^y} O_{t\ell'}(s) \right] \right) ds + \\
& \int \nu_t(s_o) \int \int \frac{1}{\gamma(s_y, s_p)} f\left( \frac{s_o}{\gamma(s_y, s_p)} \right) \left( \eta(s_y) \pi_{tl}^y(s_p) + \pi_{tl}^y(s_y) \left[ \eta(s_p) - \pi_{tl}^y(s_p) \int \eta(\sigma) d\sigma \right] \right) ds_p ds_y ds_o - \\
& \int \kappa_t^y(s) \left[ c_{tl}^y(s) - s T_l \right] \eta(s) ds.
\end{aligned}$$

Using the spatial equalization of $\lambda_{lt}^y$, the second line boils down to $\int \lambda_t^y(s) \eta(s) ds$. Using $\nu_t^o(s) = 0$, the third line vanishes. Finally, from the expression for $O_{tl}(s)$, we have

$$\delta_{n_{tl}^y} O_{t\ell}(s) = \frac{1}{N_{tl}^y} \mathbb{1}\{\ell = l\} \left( \int \int \mathcal{V}^o[e\gamma(s, s_p)] dF(e) \eta(s_p) ds_p - O_{tl}(s) \int \eta(\sigma) d\sigma \right).$$

Using $\eta(s) = \delta_x(s)$ and $\int \delta_x(s) ds = 1$, this simplifies to

$$\partial_{n_{tl}(x)^y} O_{t\ell}(s) = \frac{1}{N_{tl}^y} \mathbb{1}\{\ell = l\} \left( \int \mathcal{V}^o[e\gamma(s, x)] dF(e) - O_{tl}(s) \right).$$

When the planner increases the measure of workers $x$ in $\ell$, it has two effects on the opportunities of the city. On the one hand, it increases the likelihood of interactions with $x$. The gains from these extra interactions are captured by the first term. On the other hand, it decreases the likelihood of interactions with every other workers in the city. When there are a continuum

of skills, this congestion cost is captured by the average value of interactions in $l$, which is the second term. Combining this expression in the optimality condition, and setting $\kappa_t^y = \beta_S^t$, we obtain

$$c_{tl}^y(x) = xT_l + \beta \int \left( \int \mathcal{V}^o[e\gamma(s,x)]dF(e) - O_{tl}(s) \right) \pi_{tl}^y(s)ds + \mathcal{V}_t^y(x) - \lambda_t^y(x)/\beta_S^t.$$

In steady state, it must be that $\lambda_t^y(s)/\beta_S^t \equiv \lambda^y(s)$ is constant over time. Then, the optimal allocation is given by

$$c_l^y(x) = xT_l + \tau_l(x) + \mathcal{V}^y(x) - \lambda^y(x),$$

for

$$\tau_l(x) \equiv \beta \int \int \left( \mathcal{V}^o[e\gamma(s,x)] - \mathbb{E}[\mathcal{V}^o[e\gamma(s,S_\ell^y)]] \right) dF(e)\pi_l^y(s)ds \tag{A.15}$$

after substituting the expression for $O_l(s)$. This expression corresponds to (1.15) in the main text. Using the feasibility condition, it must be that $\mathcal{V}^y(x) - \lambda^y(x) = -\sum_l n_l^y(x)/n^y(x)\tau_l(x)$, and we conclude that[6]

$$c_l^y(x) = xT_l + \tau_l(x) - \sum_\ell \left( \frac{n_\ell^y(x)}{n^y(x)} \right) \tau_\ell(x). \tag{A.16}$$

In the competitive equilibrium, workers' consumption equate their wage, i.e. $c_l^y(x) = xT_l$. (A.16) thus yields (1.14) in the main text. Alternatively, (A.16) can be rewritten

$$c_l^y(x) = w_l(x) + t_l(x),$$

where $t_l(x) \equiv \tau_l(x) - \sum_\ell \left( \frac{n_\ell^y(x)}{n^y(x)} \right) \tau_\ell(x)$. Hence, the optimal allocation can be decentralized

[6]If we were to allow for transfers across skill within the young, then the optimal allocation would be $c_l^y(x) = xT_l + \tau_l(x) + t(x)$, for any function $t(x)$ that satisfies $\int t(x)n^y(x)dx = -\int \sum_l n_l^y(x)\tau_l(x)dx$.

through the transfers $t_l(x)$.

The optimal allocation is given by (A.15), (A.16), and the local labor supply,

$$n_\ell^{y\star}(s) = n^y(s) \left( \frac{\mathrm{e}^{\vartheta\left(c_\ell^{y\star}(s)+\beta O_\ell^\star(s)\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(c_{\ell'}^{y\star}(s)+\beta O_{\ell'}^\star(s)\right)}} \right), \tag{A.17}$$

which together form a fixed point. This fixed point can be recasted as a function of the local labor supplies only. Hence, existence of the optimal allocation can be shown using similar techniques as in the proof of the first part of Proposition 1.1. Regarding uniqueness, note that if the local labor supplies are unique, then so are the transfers and consumption. Hence, using similar techniques as in the second part of the proof of Proposition 1.1, it is possible to show that, if $\vartheta$ is small enough, there exists a unique optimal allocation. Finally, consider the case where $T_\ell = \bar{T}$ for all $\ell$. It is straighforward to guess and verify that $\tau_l(x) = \tau(x)$ and $c_l^y(x) = x$ is a solution. Hence, when $\boldsymbol{T} = \bar{T}$ and $\vartheta$ is small, the unique optimal allocation is the symmetric allocation – as in the laissez-faire equilibrium.

### A.2.3 Linearizing the social optimum

I now approximate the optimal allocation around $\boldsymbol{T} \approx \bar{T}$ under the assumption of $\vartheta$ small so that the symmetric allocation is unique when $\boldsymbol{T} = \bar{T}$. Solving for the perturbed equilibrium requires to solve jointly for (A.15), (A.16) and (A.17). I drop the star superscript for notational simplicity.

From (A.17), we have

$$\left.\frac{\partial n_\ell^y(s)}{\partial \log T_l}\right|_{\boldsymbol{T}=\bar{T}} = \vartheta \left(\frac{n^y(s)}{L}\right) \left\{ \frac{\partial c_\ell^y(s)}{\partial \log T_l} + \beta \frac{\partial O_\ell(s)}{\partial \log T_l} - \frac{1}{L} \sum_{\ell'} \left( \frac{\partial c_{\ell'}^y(s)}{\partial \log T_l} + \beta \frac{\partial O_{\ell'}(s)}{\partial \log T_l} \right) \right\} \Bigg|_{\boldsymbol{T}=\bar{T}}. \tag{A.18}$$

The net change in the learning opportunities of city $\ell$ is derived as in Section A.1.4. To

continue, we need to derive the net change in consumption. From (A.16),

$$\partial_l \left(\tau_\ell(x) - t(x)\right) = \beta \int \partial_l \pi_\ell^y(s) \left(\gamma(s,x) - \mathbb{E}[\gamma(s,S^y)]\right) ds - \beta \int \mathbb{E}[\gamma(S^y,s_p)] \partial_l \pi_\ell^y(s_p) ds_p.$$

where $\partial_l x = \partial x / \partial \log T_l|_{\boldsymbol{T}=\bar{T}}$, and I have used $\tau_\ell(s) = 0$ under $\boldsymbol{T} = \bar{T}$ and $\sum_\ell \partial_l \pi_\ell(s) = 0$ from (1.2). It follows that the perturbed consumption allocation is

$$\partial_l c_\ell^y(x) = x \mathbb{1}\{l = \ell\} + \beta \int \partial_l \pi_\ell^y(s) \left(\gamma(s,x) - \mathbb{E}[\gamma(s,S^y)]\right) ds - \beta \int \mathbb{E}[\gamma(S^y,s_p)] \partial_l \pi_\ell^y(s_p) ds_p. \tag{A.19}$$

Using $\sum_\ell \partial_l c_\ell^y(x) = x$ and the expression for $\partial_l \pi_\ell^y(s)$ returns

$$\begin{aligned}\partial_l c_\ell^y(x) - \frac{1}{L}\sum_\ell \partial_l c_\ell^y(x) = x I_{\ell l} + \beta L \int \left(\partial_l n_\ell^y(s) - n(s) \int \partial_l n_\ell^y(q) dq\right) \left(\gamma(s,x) - \mathbb{E}[\gamma(s,S^y)]\right) ds - \\ \beta L \int \left(\partial_l n_\ell^y(s_p) - n(s_p) \int \partial_l n_\ell^y(q) dq\right) \mathbb{E}[\gamma(S^y,s_p)] ds_p.\end{aligned}$$

Combining the expression for $O_\ell(s)$ and $c_\ell(s)$ in (A.18) yields

$$\begin{aligned}\partial_l n_\ell^y(x) = \theta n^y(x) \Bigg\{ & \frac{x I_{\ell l}}{L} + \\ & \beta \int \left(\partial_l n_\ell^y(s) - n(s) \int \partial_l n_\ell^y(q) dq\right) \left(\gamma(s,x) - \mathbb{E}[\gamma(s,S^y)]\right) ds - \\ & \beta \int \left(\partial_l n_\ell^y(s_p) - n(s_p) \int \partial_l n_\ell^y(q) dq\right) \mathbb{E}[\gamma(S^y,s_p)] ds_p + \\ & \beta \int \partial_l n_\ell^y(s_p) \left(\gamma(x,s_p) - \mathbb{E}[\gamma(x,S^y)]\right) ds_p \Bigg\}.\end{aligned}$$

Guessing and verifying the form of the solution,

$$\partial_l n^y(s) = \theta n^y(s) \left(\frac{I_{\ell l}}{L}\right) \eta^\star(s),$$

we end up with the optimal integral equation

$$\eta^\star(x) = x + \theta\beta \int \left(\{\gamma(x,s) - \mathbb{E}[\gamma(x,S^y)]\} - \{\mathbb{E}[\gamma(S^y,s)] - \mathbb{E}[\gamma(S^y,S^y)]\}\right)\eta^\star(s)dN^y(s) + \\ \theta\beta \int \left(\{\gamma(s,x) - \mathbb{E}[\gamma(S^y,x)]\} - \{\mathbb{E}[\gamma(s,S^y)] - \mathbb{E}[\gamma(S^y,S^y)]\}\right)\eta^\star(s)dN^y(s). \tag{A.20}$$

Finally, using (A.19) and the fact that $\partial_l \pi^y_\ell(s) = \vartheta n^y(s) I_{l\ell}\left(\eta^\star(s) - \mathbb{E}[\eta^\star(S^y)]\right)$, the skill-by-city transfers $t_l(x)$ can be decomposed into a city-specific tax, $T$, and additional skill-by-city subsidies, $d_\ell(x)$. Specifically,

$$t^y_l(x) \approx T_l + d^y_l(x), \tag{A.21}$$

where

$$T_\ell = -\theta\beta \log\left(\frac{T_\ell}{\bar{T}}\right) \int \mathbb{E}[\gamma(S^y,s)]\left(\eta^\star(s) - \mathbb{E}[\eta^\star(S^y)]\right) dN^y(s)$$

and

$$d_\ell(x) = \theta\beta \log\left(\frac{T_\ell}{\bar{T}}\right) \int \left(\gamma(s,x) - \mathbb{E}[\gamma(s,S^y)]\right)\left(\eta^\star(s) - \mathbb{E}[\eta^\star(S^y)]\right) dN^y(s).$$

### A.2.4 Proof of Proposition 1.5

Using covariances, (A.20) rewrites

$$\eta^\star(x) = x + \theta\beta\mathbb{C}\text{ov}[\gamma(x,S^y),\eta^\star(S^y)] + \theta\beta\mathbb{C}\text{ov}[\gamma(S^y,x),\eta^\star(S^y)] - \\ \theta\beta\mathbb{C}\text{ov}[\mathbb{E}[\gamma(s,S^y) \mid s],\eta^\star(s)] - \theta\beta\mathbb{C}\text{ov}[\mathbb{E}[\gamma(S^y,s) \mid s],\eta^\star(s)].$$

The next lemma provides condition to ensure positive SPAM in the social optimum.

**Lemma A.2** (Optimal SPAM)**.** *Suppose that $\gamma$ is either (weakly) supermodular or submodular.*

*If*

$$\theta\beta\left(\mathbb{C}ov[\gamma_1(x,S^y),S^y]+\mathbb{C}ov[\gamma_2(S^y,x),S^y]\right)>-1,$$

*then* $\eta'^{\star}(x)>0$.

*Proof.* For notational simplicity, define $\tilde{\Theta}^{\star}(s)\equiv\mathbb{C}\text{ov}[\gamma(s,S^y),\eta^{\star}(S^y)]$ and $\tilde{\Upsilon}^{\star}(s)\equiv\mathbb{C}\text{ov}[\gamma(S^y,x),\eta^{\star}(S^y)]$, so that the expression for $\eta^{\star}$ reads

$$\eta^{\star}(x)=x+\theta\beta\tilde{\Theta}^{\star}(x)+\theta\beta\tilde{\Upsilon}^{\star}(x)-\theta\beta\mathbb{C}\text{ov}[\mathbb{E}[\gamma(s,S^y)\mid s],\eta^{\star}(s)]-\theta\beta\mathbb{C}\text{ov}[\mathbb{E}[\gamma(S^y,s)\mid s],\eta^{\star}(s)]$$

Differentiate w.r.t. $x$ and use the expression for $\tilde{\Theta}$ and $\tilde{\Upsilon}$ to obtain

$$\eta'^{\star}(x)=1+\theta\beta\mathbb{C}\text{ov}[\gamma_1(x,S^y),\eta^{\star}(S^y)]+\theta\beta\mathbb{C}\text{ov}[\gamma_2(S^y,x),\eta^{\star}(S^y)].$$

Plug the expressions for $\eta^{\star}(x)$ back in the above,

$$\begin{aligned}\eta'^{\star}(x)=1+\theta\beta\mathbb{C}\text{ov}[\gamma_1(x,S^y),S^y]+(\theta\beta)^2\mathbb{C}\text{ov}[\gamma_1(x,S^y),\tilde{\Theta}^{\star}(S^y)]+(\theta\beta)^2\mathbb{C}\text{ov}[\gamma_1(x,S^y),\tilde{\Upsilon}^{\star}(S^y)]+\\ \theta\beta\mathbb{C}\text{ov}[\gamma_2(S^y,x),S^y]+(\theta\beta)^2\mathbb{C}\text{ov}[\gamma_2(S^y,x),\tilde{\Theta}^{\star}(S^y)]+(\theta\beta)^2\mathbb{C}\text{ov}[\gamma_2(S^y,x),\tilde{\Upsilon}^{\star}(S^y)].\end{aligned}$$

Now, guess that $\eta$ is increasing. From the expression for $\tilde{\Theta}$ and $\tilde{\Upsilon}$, if $\gamma$ is supermodular (submodular), then $\tilde{\Theta}$ and $\tilde{\Upsilon}$ are increasing. Hence, if $\gamma$ is supermodular, then all the covariances in the expression above are positive, and $\eta'^{\star}(x>1>0$, which verifies the guess. Suppose now that $\gamma$ is submodular. Then, the covariances between $\gamma_1$, $\gamma_2$, $\tilde{\Theta}$ and $\tilde{\Upsilon}$ are positive since the covariance between two decreasing function is positive. Hence,

$$\eta'^{\star}(x)>1+\theta\beta\mathbb{C}\text{ov}[\gamma_1(x,S^y),S^y]+\theta\beta\mathbb{C}\text{ov}[\gamma_2(S^y,x),S^y].$$

It follows that $\theta\beta\left(\mathbb{C}\text{ov}[\gamma_1(x,S^y),S^y]+\mathbb{C}\text{ov}[\gamma_2(S^y,x),S^y]\right)>-1$ implies $\eta'^{\star}(x)>0$. □

This lemma is sufficient to prove the two elements of Proposition 1.5.

**Proposition 1.5.1** To a first order, the optimal spatial allocation is given by

$$n_\ell^{y\star}(s) \approx \frac{n^y(s)}{L} + \theta \left( \frac{n^y(s)}{L} \right) \log \left( \frac{T_\ell}{\bar{T}} \right) \eta^\star(s).$$

Note that $\mathbb{E}[\eta^\star(S^y)] = \int \eta^\star(x) n^y(x) dx = \mathbb{E}[S^y]$. Hence, the optimal number of young workers in city $\ell$ is

$$N_\ell^{y\star} \approx \frac{1}{L} + \frac{1}{L} \theta \log \left( \frac{T_\ell}{\bar{T}} \right) \mathbb{E}[S^y].$$

In the laissez-faire equilibrium, the number of young workers in city $\ell$ is

$$N_\ell \approx \frac{1}{L} + \theta \frac{1}{L} \log \left( \frac{T_\ell}{\bar{T}} \right) \mathbb{E}[S^y] + \theta^2 \beta \log \left( \frac{T_\ell}{\bar{T}} \right) \mathbb{E}[\Theta(S^y)].$$

Hence, it is immediate to conclude that, for productive cities, $N_\ell > N_\ell^\star \iff \mathbb{E}[\Theta(S^y)] > 0$ (and the converse holds for unproductive cities). But, we also know from Proposition 1.2 that $s_p \to \gamma(s, s_p)$ increasing implies $\Theta(s) > 0$. Hence, if $s_p \to \gamma(s, s_p)$, there is too much agglomeration in productive cities. Conversely, if $s_p \to \gamma(s, s_p)$ is decreasing, there is too little agglomeration in productive cities.

**Proposition 1.5.2** To a first order, the optimal skill density in city $\ell$ is

$$\pi_\ell^{y\star}(s) \approx n^y(s) + \theta n^y(s) \log \left( \frac{T_\ell}{\bar{T}} \right) \left( \eta^\star(s) - \mathbb{E}[S^y] \right).$$

In particular, the optimal average skill in city $\ell$ is

$$\mathbb{E}_\ell[S^{y\star}] \approx \mathbb{E}[S^y] + \theta \log \left( \frac{T_\ell}{\bar{T}} \right) \mathbb{C}\text{ov}[\eta^\star(S^y), S^y]$$

In the laissez-faire equilibrium, we had

$$\mathbb{E}_\ell[S^y] \approx \mathbb{E}[S^y] + \theta \log\left(\frac{T_\ell}{\overline{T}}\right) \mathbb{C}\text{ov}[\eta(S^y), S^y].$$

Hence, for productive cities, $\mathbb{E}_\ell[S^y] > \mathbb{E}_\ell[S^{y\star}] \iff \mathbb{C}\text{ov}[\eta(S^y), S^y] > \mathbb{C}\text{ov}[\eta^\star(S^y), S^y]$. A sufficient condition for $\mathbb{C}\text{ov}[\eta(S^y), S^y] > \mathbb{C}\text{ov}[\eta^\star(S^y), S^y]$ is $\eta' > \eta'^\star$ a.e.[7] I now show that $\gamma$ supermodular (submodular) implies $\eta'^\star > \eta'$ ($\eta'^\star < \eta'$). Differentiate the expression for $\eta^\star$ w.r.t. $x$ and use covariances to simplify the integrals,

$$\eta'^\star(x) = 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta^\star(S^y)] + \theta\beta\mathbb{C}\text{ov}[\gamma_2(S^y, x), \eta^\star(S^y)].$$

Taking a similar derivative in the decentralized equilbrium returns

$$\eta'(x) = 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta(S^y)].$$

The two functions $\eta$ and $\eta^\star$ are the solution to integrals equations. To prove the propositions, I therefore guess and verify the statement. First, suppose that $\gamma$ is supermodular, and guess that $\eta'^\star > \eta'$ a.e. Since $\gamma$ is supermodular, $s \to \gamma_2(s, x)$ is an increasing function, and $\mathbb{C}\text{ov}[\gamma_2(S^y, x), \eta^\star(S^y)] > 0$ from Lemma A.2. Hence,

$$\eta'^\star(x) > 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta^\star(S^y)] > 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta(S^y)] = \eta'(x),$$

where the second inequality follows from the guess that $\eta'^\star(x) > \eta'(x)$. Therefore, $\eta'^\star(x) > \eta'(x)$ for all $x$, which verifies the guess. Suppose now that $\gamma$ is submodular and guess that

[7] Take three increasing function, $f$, $g$ and $h$. Suppose that $g' > h'$. We want to show that $\mathbb{C}\text{ov}[f(S), g(S)] > \mathbb{C}\text{ov}[f(S), h(S)]$. Since the covariance is a linear operator, we have $\mathbb{C}\text{ov}[f(S), g(S)] - \mathbb{C}\text{ov}[f(S), h(S)] = \mathbb{C}\text{ov}[f(S), g(S) - h(S)] > 0$, which follows from the covariance of two increasing functions being positive and $g - h$ increasing from $g' > h'$.

$\eta'^{\star}(x) < \eta'(x)$. Then, by a similar argument, $\mathbb{C}\text{ov}[\gamma_2(S^y, x), \eta^{\star}(S^y)] < 0$, and

$$\eta'^{\star}(x) < 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta^{\star}(S^y)] < 1 + \theta\beta\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta(S^y)] = \eta'(x),$$

where $\mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta^{\star}(S^y)] < \mathbb{C}\text{ov}[\gamma_1(x, S^y), \eta(S^y)]$ follows from the guess. This verifies the guess, and proves Proposition 1.5.2.

**Conclusion of Proposition 1.5** I now conclude by showing that the optimal allocation implies a larger aggregate stock of human capital. Following exactly the same derivation as in Section A.1.6, the optimal average skill of old workers read

$$\mathbb{E}[S^{o\star}] \approx \mathbb{E}[S^o] + \theta\mathbb{V}\text{ar}[\log T_\ell]\Omega^{\star} \quad \text{where} \quad \Omega^{\star} = \int\int \gamma_{12}(s, s_p)\Psi^{\star}(s)\Psi^{\star}(s_p)dsds_p, \quad \text{(A.22)}$$

for $\Psi^{\star}(x) = (1 - N(x))\left(\mathbb{E}[\eta^{\star}(S^y) \mid S^y \geq x] - \mathbb{E}[\eta^{\star}(S^y)]\right)$ defined analogously to Section A.1.9. Hence, comparing (A.22) with (1.12), we have that

$$\mathbb{E}[S^{o\star}] - \mathbb{E}[S^o] \approx \theta\mathbb{V}\text{ar}[\log T_\ell] \int\int \gamma_{12}(s, s_p)\left[\Psi^{\star}(s)\Psi^{\star}(s_p) - \Psi(s)\Psi(s_p)\right] dsds_p.$$

Suppose for an instant that $\gamma$ supermodular implies $\Psi^{\star}(s) > \Psi(s)$, and inversely, $\gamma$ submodular implies $\Psi^{\star}(s) < \Psi(s)$. Then, it immediately follows that $\mathbb{E}[S^{o\star}] > \mathbb{E}[S^o]$.

# A.3 Extensions

## A.3.1 Production complementarities

In this Appendix, I show that Proposition 1.2 and 1.3 hold allowing for production complementarities.

**Model**

The framework is similar to that of Section 1.2.1 with one modification. Young and old workers differ in their employment opportunities. To earn an income, young workers need to be employed by the competitive local firms. These firms produce the freely-trade good with a CES production function,

$$R(T_\ell, \boldsymbol{n}) = T_\ell \left( \int a(s) n(s)^\sigma ds \right)^{\frac{1}{\sigma}},$$

where $\sigma \leq 1$ is the elasticity of substitution. Let $R(\boldsymbol{n})$ denote the CES aggregator, $R(\boldsymbol{n}) = R(T_\ell, \boldsymbol{n})/T_\ell$. In contrast to young workers, old workers have access to a backyard technology. Specifically, an old worker living in city $\ell$ with skill $s$ produces $sT_\ell$ of the freely traded good.[8] This framework nests the model of Section 1.2.1 under $\sigma \to 1^-$.

**Equilibrium**

The problem of young and old workers is unchanged up to the local wages. Local wages for young workers clear the local labor markets by equating the local labor supply and demand. They are given by

$$w_\ell(s) = T_\ell a(s) \left( \frac{R[\boldsymbol{n}^y_\ell(s)]}{n^y_\ell(s)} \right)^{1-\sigma}. \tag{A.23}$$

The equilibrium is given by (1.3), (1.4), (1.5), (1.6), and (A.23). As in the original problem, old workers are passive, and a unique equilibrium exists when $\vartheta\beta$ is small.

[8] I differentiate young and old work opportunities to simplify the analysis by decoupling the yound and old problem.

**Symmetry** Suppose that $T_\ell = \bar{T}$ for all $\ell$. Then, there always exists a symmetric equilibrium characterized by

$$n_\ell^y(s) = \frac{n^y(s)}{L} \quad \text{and} \quad w_\ell(s) = \bar{T} a(s) \left( \frac{R(\bar{\boldsymbol{n}})}{\bar{n}(s)} \right)^{1-\sigma}.$$

Define $w(s) = w_\ell(s)/\bar{T}$ in the symmetric equilibrium. This symmetric equilibrium is guaranteed to be unique when $\vartheta\beta$ is small enough. For the rest of the analysis, I suppose that wages are increasing in skills in the symmetric equilibrium. Since I do not impose any conditions on $\gamma$, this assumption is wlog.

**Assumption A.2.** *The functions $a$ and $n^y$ are such that wages are increasing in $s$ in the symmetric equilibrium, $w'(s) > 0$.*[9]

**Proposition 1.2 with production complementarities**

From (1.5), the partial derivative of the young allocation w.r.t. $T_l$ is

$$\begin{aligned}
\Delta_l n_\ell^y(s) \equiv & \left. \frac{\partial \log n_\ell^y(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \\
= & \vartheta \left( \left. \frac{\partial w_\ell(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} - \sum_{\ell'} \frac{n_{\ell'}^y(s)}{n^y(s)} \left. \frac{\partial w_{\ell'}(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \right) + \\
& \vartheta\beta \left( \left. \frac{\partial O_\ell(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} - \sum_{\ell'} \frac{n_{\ell'}^y(s)}{n^y(s)} \left. \frac{\partial O_{\ell'}(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} \right).
\end{aligned}$$

The second term is similar to the baseline economy of Section 1.2.1. From (A.23), the partial of wages w.r.t. $\log T_l$ is

$$\left. \frac{\partial w_\ell(s)}{\partial \log T_l} \right|_{\boldsymbol{T}=\bar{T}} = \mathbb{1}\{\ell = l\} \bar{T} w(s) + (1-\sigma) \bar{T} w(s) \left( \int \frac{w(s') n^y(s')}{R(\boldsymbol{n}^y)} \Delta_l n_\ell^y(s') ds' - \Delta_l n_\ell^y(s) \right).$$

[9] This is clearly satisfied if $a$ is increasing and $n^y$ is (weakly) decreasing. As such, Assumption A.2 necessarily holds for $a$ an iso-elastic function of $s$ and $n^y$ a Pareto distribution.

Combining the static and dynamic response, and solving for $\Delta_l n_\ell^y(s)$, we obtain

$$\Delta_l n_\ell^y(s) = \frac{\theta w(s)}{1+\theta(1-\sigma)w(s)} \left( I_{l\ell} + (1-\sigma) \int \frac{w(s')n^y(s')}{R(\boldsymbol{n}^y)} \Delta_l n_\ell^y(s') ds' \right) + \frac{\theta\beta}{1+\theta(1-\sigma)w(s)} \int \left(\gamma(s,s_p) - \mathbb{E}[\gamma(s,S^y)]\right) \Delta_l n_\ell^y(s_p) dN^y(s_p).$$

We can now guess a particular form for $\Delta_l n_\ell^y(s)$. Specifically, $\Delta_l n_\ell^y(s) = \theta I_{l\ell} \eta(s)$, for some function $\eta(s)$. Plugging in the original expression verifies the guess and yields the implicit expression for $\eta(s)$:

$$\eta(s) = \frac{w(s)\Sigma}{1+\theta(1-\sigma)w(s)} + \frac{\theta\beta\Theta(s)}{1+\theta(1-\sigma)w(s)}, \tag{A.24}$$

for

$$\Sigma = 1 + \theta(1-\sigma) \int \left( \frac{w(s')}{R(\boldsymbol{n}^y)} \right) \eta(s') dN^y(s'), \tag{A.25}$$

an implicit constant, and

$$\Theta(s) = \int \left(\gamma(s,s') - \mathbb{E}[\gamma(s,S^y)]\right) \eta(s') dN^y(s') = \mathbb{C}\text{ov}[\gamma(s,S^y), \eta(S^y)]. \tag{A.26}$$

an implicit function. The function $\Theta(s)$ is the same as in the main text, up to the value of the implicit function $\eta$. However, there are two novelties in (A.24). First, the presence of $\Sigma$, which captures the static production complementarities. If $\Sigma > 1$, production complementarities generate further agglomeration. Second, the three effects are normalized by a term which reflects the effect of other workers with similar skills on workers' wages. When $\sigma \to 1^-$, $\Sigma \to 1$, and $\eta(s)$ solves (1.8).

Despite these differences, I now show that Lemma 1.1 and Proposition 1.2 extends to $\sigma < 1$ when $\theta$ is not too large.

**Lemma A.3.** *Suppose that $\theta$ is small such that $\theta^2 \approx 0$. Then, if*

$$\theta\beta\mathbb{C}ov\left[\gamma_1(s, S^y), \frac{w(S^y)}{1+\theta(1-\sigma)w(S^y)}\right] > -w'(s),$$

*for all $s$, SPAM prevails: $\eta'(s) > 0$ for all $s$.*

*Proof.* Differentiate (A.24) w.r.t. $s$ to get

$$\eta'(s) \propto w'(s)\Sigma + \theta\beta\mathbb{C}\text{ov}[\gamma_1(s, S^y), \eta(S^y)] + \theta^2\beta(1-\sigma)\Theta'(s)w(s) - \theta^2\beta(1-\sigma)\Theta(s)w'(s).$$

When $\theta^2 \approx 0$, this expression simplifies to

$$\eta'(s) \propto w'(s)\Sigma + \theta\beta\Sigma\mathbb{C}\text{ov}\left[\gamma_1(s, S^y), \frac{w(S^y)}{1+\theta(1-\sigma)w(S^y)}\right].$$

after plugging in the expression for $\eta(S^y)$. Furthermore, plugging the expression for $\eta$ in $\Sigma$ and using $\theta^2 \approx 0$ yields[10]

$$\Sigma = \frac{1}{1-e} \quad \text{for } e \equiv \int \frac{w(s)n^y(s)}{R(\boldsymbol{n}^y)} \frac{\theta(1-\sigma)w(s)}{1+\theta(1-\sigma)w(s)} ds \in (0,1). \tag{A.27}$$

It is then automatic to see that the condition stated in Lemma A.3 implies $\eta'(s) > 0$. □

The condition stated in Lemma A.3 nests that of Lemma 1.1 under $a(s) = s$ and $\sigma \to 1^-$, such that $w'(s) = 1$. When $\sigma < 1$, it necessarily holds if the learning technology is supermodular. When the technology is submodular, Lemma A.3 imposes a bound on how submodular the function can be.

With Lemma A.3, it is now possible to generalize Proposition 1.2 to the case with $\sigma < 1$.

**Proposition A.5.** *Suppose that $\theta$ is small such that $\theta^2 \approx 0$ and Lemma A.3 holds. Then,*

1. *If $\gamma(s, \cdot)$ is increasing for all $s$, (un)productive cities are (smaller) larger when interactions are spatially segmented; the converse holds when $\gamma(s, \cdot)$ is decreasing;*

[10]The constant $e \in (0,1)$ since constant returns to scale imply $\int w(s)n(s)ds = R(\boldsymbol{n})$.

2. *If $\gamma$ is supermodular, the average skill of young workers in (un)productive cities is (smaller) larger when interactions are spatially segmented, which leads to greater between-city wage variance amongst young workers; the converse holds when $\gamma$ is submodular.*

*Proof.* Let $\tilde{x}$ denote the variable $x$ when interaction are not spatially segmented. Then, the equilibrium solves

$$\tilde{\eta}(s) = \frac{w(s)\tilde{\Sigma}}{1 + \theta(1-\sigma)w(s)},$$

where $\tilde{\Sigma}$ is defined analogously to (A.25) w.r.t. $\tilde{\eta}(s)$. Solving for $\tilde{\Sigma}$ implies $\tilde{\Sigma} = 1/(1-e) \approx \Sigma$, where the approximation is w.r.t. $\theta^2 \approx 0$. Accordingly, the difference between the equilibrium with and without spatially segmented interaction is

$$\eta(s) - \tilde{\eta}(s) = \frac{w(s)\left(\Sigma - \tilde{\Sigma}\right)}{1 + \theta(1-\sigma)w(s)} + \frac{\theta\beta\Theta(s)}{1 + \theta(1-\sigma)w(s)} \approx \frac{\theta\beta\Theta(s)}{1 + \theta(1-\sigma)w(s)}.$$

Proposition A.5.1 immediately follows: $\gamma(s,\cdot)$ increasing implies $\Theta(s) > 0$ (Lemma A.3), and therefore $\eta(s) > \tilde{\eta}(s)$. Meanwhile, the segmentation of interactions amplifies spatial inequality to the extent that $\eta(s) - \tilde{\eta}(s)$ is increasing. Differentiating w.r.t. $s$, we get

$$\eta'(s) - \tilde{\eta}'(s) \propto w'(s)\left(\Sigma - \tilde{\Sigma}\right) + \beta\Theta'(s)\left(\theta + \theta^2(1-\sigma)w(s)\right) - \theta^2\beta(1-\sigma)\Theta(s)w'(s) \approx \theta\beta\Theta'(s).$$

Proposition A.5.2 then follows: $\gamma$ supermodular (submodular) implies $\Theta' > 0$ ($\Theta' < 0$), and therefore $\eta'(s) > \tilde{\eta}'(s)$ for all $s$. □

**Proposition 1.3 with production complementarities**

Proposition 3 automatically generalizes to the presence of static complementarities – conditional on Lemma A.3 holding. To see that, three observations are useful. First, the expression for the average skill is the same regardless of $\sigma$. Second, the derivation of the second-order approximation is independent to the definition of $\eta$. Third, the proof of Proposition 3 only

relies on $\eta$ increasing.

## A.4 Quantitative Model

### A.4.1 Workers' problem

**Consumption** Young and old workers solve a standard consumption choice problem that maximize their static utility. A worker with income $y$ thus consumes

$$c_\ell(y) = \alpha y \quad ; \quad h_\ell(y) = \left(\frac{1-\alpha}{p_\ell}\right) y$$

on the numeraire and the housing good respectively. Accordingly, the local price index is $P_\ell = p_\ell^{1-\alpha}$ and indirect utility in city $\ell$ for a worker with income $y$ is $y/P_\ell$.

**Old workers** Worker' income are made of three sources: their wage, the subsidy they might receive, and the flat tax. Hence, the income of an old worker with skill $s$ living in city $l$ when young and choosing to live in city $\ell$ when old is $y^o_{l\ell}(s) = w_\ell(s) + \tau^o_{l\ell}(s) - t$. Given this income, old workers solve a static location choice problem,

$$V^o_l(s, \boldsymbol{\varepsilon}) = \max_\ell \ \frac{y_{l\ell}(s)}{P_\ell} + \varepsilon_\ell - \kappa^o_{l\ell}.$$

Letting $m^o_l(s)$ denote the spatial distribution of old workers before their new location choice, the spatial allocation of workers is given by

$$n^o_{l\ell}(s) = m^o_l(s) \left( \frac{\mathrm{e}^{\vartheta\left(y^o_{l\ell}(s)/P_\ell + B^o_\ell - \kappa^o_{l\ell}\right)}}{\sum_{\ell'} \mathrm{e}^{\vartheta\left(y^o_{l\ell'}(s)/P_{\ell'} + B^o_{\ell'} - \kappa^o_{l\ell'}\right)}} \right), \tag{A.28}$$

where $n^o_{l\ell}(s)$ is the measure of workers moving from $l$ to $\ell$ with skill $s$ when old. Consistently, the (unconditional) measure of old workers in city $\ell$ is

$$n^o_\ell(s) = \sum_l n^o_{l\ell}(s),$$

and the mass of old workers in $\ell$ is $N^o_\ell = \int n^o_\ell(s)ds$. Finally, let $\mathcal{V}^o_l(s) \equiv \mathbb{E}[V^o_l(s,\boldsymbol{\varepsilon}) \mid s,l]$ be the expected utility before the realization of the idiosyncratic preferences $\boldsymbol{\varepsilon}$, given by

$$\mathcal{V}^o_l(s) = \frac{1}{\vartheta}\log\left(\sum_\ell \mathrm{e}^{\vartheta\left(y^o_{l\ell}(s)/P_\ell + B^o_\ell - \kappa^o_{l\ell}\right)}\right).$$

**Young workers** Young workers solve a dynamic location choice problem which reads

$$V^y_l(s,\boldsymbol{\varepsilon}) = \max_\ell \frac{y^y_{l\ell}(s)}{P_\ell} + \varepsilon_\ell - \kappa^y_{l\ell} + \beta\int\int \mathcal{V}^o_\ell\left[e\gamma(s,s_p)\right]dF(e)\pi_\ell(s_p)ds_p,$$

where $y^y_{l\ell}(s) = w_\ell(s) + \tau^y_{l\ell}(s) - t$. In a similar fashion as for the old workers, the spatial allocation of young workers is given by

$$n^y_{l\ell}(s) = m^y_l(s)\left(\frac{\mathrm{e}^{\vartheta\left(y^y_{l\ell}(s)/P_\ell + B^y_\ell - \kappa^y_{l\ell} + \beta O_\ell(s)\right)}}{\sum_{\ell'}\mathrm{e}^{\vartheta\left(y^y_{l\ell'}(s)/P_{\ell'} + B^y_{\ell'} - \kappa^y_{l\ell'} + \beta O_{\ell'}(s)\right)}}\right). \tag{A.29}$$

I define $n^y_\ell(s)$ and $N^y_\ell$ in the same manner as $n^o_\ell(s)$ and $N^o_\ell$.

### A.4.2 General equilibrium

In equilibrium, the markets must clear, the expected spatial distribution of workers has to be consistent with workers' location choices, and the aggregate skill distribution must be consistent with workers' learning.

**Housing prices** Housing prices must clear the housing market in each location, and therefore they must satisfy[11]

$$p_\ell = \left( \frac{(1-\alpha)\, Y_\ell}{\mathcal{H}} \right)^{\frac{1}{1+\delta}}, \tag{A.30}$$

where $Y_\ell = T_\ell \int s \sum_a n_\ell^a(s) ds = \mathbb{E}_\ell[W] N_\ell$ is the output produced in city $\ell$. Taking logs give (1.21).

**Skill density** The within-city skill density of location $\ell$ must be consistent with workers' location choices, given by

$$\pi_\ell(s) = \frac{\sum_a n_\ell^a(s)}{\sum_a N_\ell^a}. \tag{A.31}$$

**Skill distributions** There are two distributions to be solved for. First, the location-specific distribution of old workers. Define $M_\ell^o(s)$ as the mass of old workers that lived in city $\ell$ when young and that obtained a skill less than $s$. Given the learning process, this is given by

$$M_\ell^o(s) = \int \int n_\ell^y(x) \pi_\ell(y) F\left( \frac{s}{\gamma(x,y)} \right) dy dx. \tag{A.32}$$

From here, one can define the location-specific measure as $m_\ell^o(s) = \partial M_\ell^o(s)/\partial s$ and the aggregate old skill density as $n^o(s) = \sum_\ell m_\ell^o(s)$. Finally, given the birth process, the location-specific distribution of young workers is $m_\ell^y(s) = N_\ell^y n^y(s)$.

**Definition A.1** (Steady state equilibrium)**.** *A steady state equilibrium is a collection of young and old spatial allocation, $s \to n_{l\ell}^y(s)$ and $s \to n_{l\ell}^o(s)$ for each $l, \ell \in \{1, 2, \ldots, L\}$, city-specific skill distribution, $s \to m_\ell^y(s)$ and $s \to m_\ell^o(s)$ for each $\ell \in \{1, 2, \ldots, L\}$, and housing prices $\{p_\ell\}_{\ell=1}^L$, such that*

1. *Taking as given the location decisions of other workers and housing prices, the young and old allocation satisfy* (A.29) *and* (A.28) *respectively;*

[11] Note that this implies $\Pi_\ell = p_\ell^{1+\delta} = (1-\alpha) Y_\ell$, where $\Pi_\ell$ are the revenues earned by the representative land owner.

2. *The local skill densities are consistent with workers' location decisions, given by* (A.31)*;*

3. *Given young workers' location decisions, the city-specific young and old skill distribution are given by* $m_\ell^y(s) = N_\ell^y n^y(s)$ *and* (A.32) *respectively;*

4. *Given workers' location decisions, housing prices solve* (A.30)*.*

### A.4.3 Algorithm

I use two different algorithms to solve for the steady state equilibrium depending on whether I am estimating the model or computing counterfactuals. Both algorithms are constituted of an inner loop and an outer loop. The inner loops are identical. Given city fundamentals, $\boldsymbol{T}$ and $\boldsymbol{B}$, housing prices $\boldsymbol{p}$, and skill distributions $\boldsymbol{m}^y$ and $\boldsymbol{m}^o$, they iterate on (A.29) and (A.28) to solve for the local skill densities, $\boldsymbol{\pi}$.

**Estimation** To estimate the model, I need to solve for the steady state equilibrium and recover the city characteristics, $\boldsymbol{T}$ and $\boldsymbol{B}$. The targeted moments for $\boldsymbol{T}$ and $\boldsymbol{B}$ imply that, when TFP and amenities are estimated, I match the city average wage and city size, and therefore cities' output, $\mathbb{E}_\ell[W]N_\ell$. Given estimates for $(\alpha, \delta, \mathcal{H})$, I can therefore read off (A.30) the housing prices that must hold in the baseline steady state equilibrium. To solve for the baseline steady state equilibrium, the outer loop therefore takes $\boldsymbol{p}$ as given and iterate on the skill distributions, $\boldsymbol{m}^y$ and $\boldsymbol{m}^o$, as well as on the city TFP and amenities through (1.23) and (A.39).

**Counterfactual** In the counterfactuals, city TFP and amenities are taken as given. The outer loop therefore iterates on (A.30) to solve for the housing prices, as well as on $m_\ell^y(s) = N_\ell^y n^y(s)$ and (A.31) to solve for $\boldsymbol{m}^y$ and $\boldsymbol{m}^o$.

**Simulation** I use the model to simulate a two-period panel datasets with $1,000,000$ workers. The simulation follows the structure of the model. All the variables, including the local skill

densities, are consistent with the model's general equilibrium. As in the data, I truncate the wage distribution at the 5% and 99.9% percentile.

## A.5 Descriptive Evidence

### A.5.1 Data description

As described in the main text, the French matched employer-employee dataset comes in two format. The first format is a long-panel that tracks the entire labor market history of 4% of the French workforce. The second format is a repeated short-panel (two years) that provides information on the job providing the main source of labor income within each year for the universe of the French workforce.[12] I apply a first set of similar restrictions on both datasets:

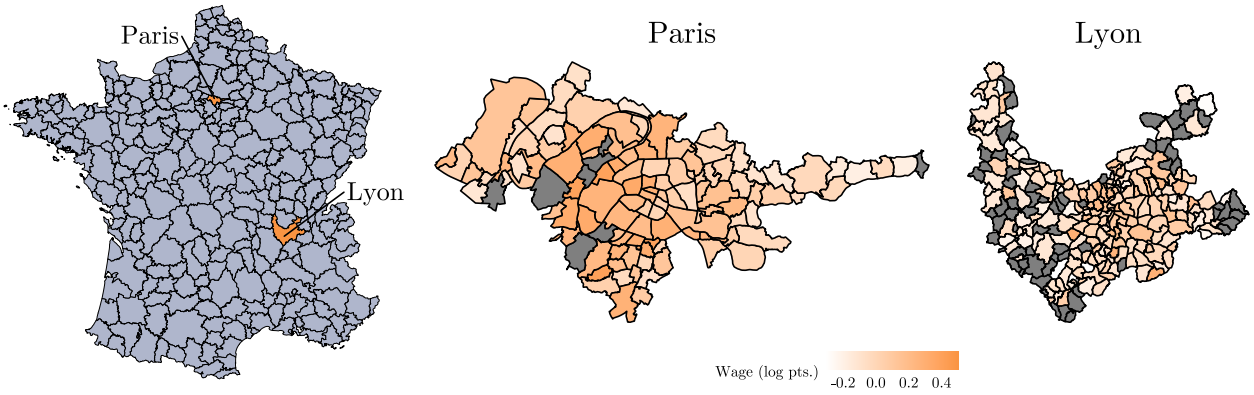
- Exclude workers younger than 25 and older than 55 year old;
- Exclude workers employed in the private sector;
- Exclude the agriculture, education and health industries;
- Keep only workers employed full-time;[13]
- Truncate the yearly wage distribution at the 5% and 99.9%.

The long-panel comes at the job-spell level, where a job-spell is defined as a year-establishment-occupation. Workers are observed multiple times within the same year if they switch jobs. Since the focus of this chapter is on middle- and long-term wage growth, I aggregate the long-panel at the yearly level. To do so, I keep for each worker the job that provides the highest earnings within the year. I exclude from the sample jobs that last less than 30 days, and I also exclude workers that have been unemployed for more than three consecutive years.

[12] In the short-panel, the worker ID are shuffled every year so that it is not possible to link workers for more than two periods.

[13] Full-time jobs are identified first by their contracts, and second by the fact that the individuals work at the job more than three hours per worked day.

Figure A.1: The geography of commuting zones and municipalities




### A.5.2 Neighborhood

As explained in Section 1.3.1, I define a neighborhood as a municipality, or commune in French. Commune are the most granular administrative division in France. They are also the most granular geographic unit I observe in the French matched employer-employee dataset.

Commune were introduced with the French revolution in 1789 to unify parishes (the houses and land around a church) and chartered cities (collection of parishes) under a common legal definition. By the end of the 18$^{th}$ century, 40,200 municipalities were created. Two reforms at the beginning of the 19$^{th}$ century (1831 and 1837) increased their administrative powers. Since then, both the geographic and administrative boundaries of municipalities have been stable over time. For instance, around 90% of commune are have similar geographic boundaries as when they were created during the French revolution.

In 2009, France had 36 783 communes. Their legislative power are analogous to civil townships and incorporated municipalities in the United States. In terms of geography, they are closer to ZIP codes. The three largest French communes, Paris, Lyon and Marseille, are themselves sub-divided into 20, 9, and 16 arrondissements. I observe those in the French matched employer-employee dataset, and therefore refer to these arrondissements also as neighborhoods.

Figure A.1 helps visualize the granularity of communes. The left map is a map of the

French commuting zones, where I have highlighted in orange the two largest French cities, Paris and Lyon. The two maps on the right are the commune included in the Paris (112 communes) and Lyon (191 communes) commuting zones. The orange shade represents the average wage of the neighborhood relative to the French average. Grey communes are communes excluded from the sample due to their small sample sizes. In total, 112 and 191 communes are kept in the sample in Paris and Lyon respectively. Throughout France, the average number of commune by commuting zones is 40. The city with the smallest number of commune is Chatillon (0.02% of the French workforce) with 4 communes. The city with the largest number of commune is Roissy (2.7% of the French workforce) with 212 communes.

### A.5.3 Identification

In this section, I lay down how (1.19) identifies the learning technology. I do under the assumption of $g_{12} = 0$ to reduce the length of the derivation. With a slight abuse of notation, let $s_{it}$ denote $\log s_{it}$. Then, the old skill of a worker $i$ who starts with skill $s_i^y$ and meets with a partner with skill $s_i^p$ is

$$s_i^o = g_0 + g_1 s_i^y + g_2 s_i^p + e_i. \tag{A.33}$$

Equation (A.33) always rewrite

$$s_i^o = g_0 + g_1 s_i^y + g_2 \mathbb{E}\left[s_j \mid \ell_i^y\right] + e_i + \nu_i, \tag{A.34}$$

where $\ell_i^y$ is the city where $i$ works when young, and $\nu_i \equiv g_2 \left(s_i^p - \mathbb{E}[s_j \mid \ell_i^y]\right)$. Equation (A.34) has two error terms. First, $e_i$ captures the learning sources other than interactions. Second, $\nu_i$ measures the difference between the particular interaction experienced by $i$ and the average interaction in city $\ell_i^y$. If the two error terms are uncorrelated with $s_i^y$ and $\mathbb{E}[s_j \mid \ell_i^y]$, then this justifies estimating the local projection (A.34) by OLS.

Assumption 1.6 guarantees the two error terms are orthogonal to the regressors. First,

Assumption 1.6.2 implies that $\mathbb{E}[s_i e_i] = \mathbb{E}[\mathbb{E}[s_j \mid \ell_i^y] e_i] = 0$. Second, Assumption 1.6.1 implies that $\nu_i$ is uncorrelated with the regressors. To see why, note that

$$\mathbb{E}[\nu_i] = \mathbb{E}[\mathbb{E}[\nu_i \mid s_i^y, \ell_i^y]] = \mathbb{E}[g_2 \mathbb{E}[s_i^y - \mathbb{E}[s_j \mid \ell_i^y] \mid s_i^y, \ell_i^y]] = 0,$$

where the first equality follows from the L.I.E., the second from the definition of $\nu_i$, and the third from $\mathbb{E}[s_j^p \mid s_i^y, \ell_i 6y] = \mathbb{E}[s_j \mid \ell_i^y]$ when interactions are random within cities. A similar argument implies $\mathbb{E}[\nu_i s_i^y] = \mathbb{E}[\nu_i \mathbb{E}[s_j \mid \ell_i^y]] = 0$.

These arguments justify estimating the learning technology through the local projection

$$s_i^o = \alpha + \beta s_i^y + \gamma \mathbb{E}[s_j \mid \ell_i^y] + u_i. \tag{A.35}$$

Is (A.35) exposed to the standard reflection problem (Manski, 1993)? In particular, when estimating the static framework $s_i = \alpha + \beta \mathbb{E}[s_j \mid \ell_i^y] + u_i$ by OLS, then $\hat{\beta} = 1$ regardless of the presence of peer effects. I now show that (A.35) does not suffer from this problem, and in particular, that $\hat{\beta} = g_2$.

**Proposition A.6.** *If $s_i$ and $\mathbb{E}_{\ell_i^y}[s_j]$ are not colinear,* (A.35) *yields $\hat{\gamma} = g_2$.*

*Proof.* By Frisch-Waugh–Lovell, we know that estimating

$$\epsilon_i = a + b\nu_i + \zeta_i \tag{A.36}$$

by OLS generates $\hat{b} = \hat{\gamma}$, where $\epsilon_i$ and $\nu_i$ are defined by

$$s_i^o = \mathtt{a} + \mathtt{b} s_i^y + \epsilon_i, \tag{A.37}$$

$$\mathbb{E}_{\ell_i^y}[s_j] = \mathrm{a} + \mathrm{b} s_i^y + \nu_i. \tag{A.38}$$

Using standard OLS algebra, the coefficient estimated by (A.37) are

$$\hat{\mathtt{a}} = \mathbb{E}[s_i^o] - \hat{\mathtt{b}}\mathbb{E}[s_i^y] \quad ; \quad \hat{\mathtt{b}} = \frac{\mathbb{C}\text{ov}\left(s_i^y, s_i^o\right)}{\mathbb{V}\text{ar}[s_i^y]}.$$

Using the learning process (A.33) yields

$$\hat{\mathtt{b}} = g_1 + \tilde{b}g_2 \qquad ; \qquad \epsilon_i = \tilde{a} + g_2\left(s_i^p - \tilde{b}s_i^y\right) + e_i$$

for $\tilde{a}$ some constant and

$$\tilde{b} \equiv \frac{\mathbb{C}\text{ov}\left(s_i^y, s_i^p\right)}{\mathbb{V}\text{ar}[s_i^y]} = \frac{\mathbb{E}\left[\mathbb{C}\text{ov}\left(s_i^y, s_i^p \mid \ell_i^y\right)\right]}{\mathbb{V}\text{ar}[s_i^y]} + \frac{\mathbb{C}\text{ov}\left(\mathbb{E}[s_j^y \mid \ell_i^y], \mathbb{E}[s_j \mid \ell_i^y]\right)}{\mathbb{V}\text{ar}[s_i^y]} = \frac{\mathbb{C}\text{ov}\left(\mathbb{E}[s_j^y \mid \ell_i^y], \mathbb{E}[s_j \mid \ell_i^y]\right)}{\mathbb{V}\text{ar}[s_i^y]},$$

where the second equality follows from the law of total covariance and the last from the random matching assumption within cities. Likewise, the slope estimated by (A.38) is

$$\hat{\mathrm{b}} = \frac{\mathbb{C}\text{ov}\left(s_i^y, \mathbb{E}\left[s_j \mid \ell_i^y\right]\right)}{\mathbb{V}\text{ar}\left[s_i^y\right]} = \frac{\mathbb{C}\text{ov}\left(\mathbb{E}\left[s_j^y \mid \ell_i^y\right], \mathbb{E}\left[s_j \mid \ell_i^y\right]\right)}{\mathbb{V}\text{ar}\left[s_i^y\right]} = \tilde{b} \qquad ; \qquad \nu_i = \mathbb{E}[s_j \mid \ell_i^y] - \hat{\mathrm{a}} - \tilde{b}s_i^y$$

Hence, altogether, the estimated slope of (A.36) is

$$\hat{b} = \frac{\mathbb{C}\text{ov}[\epsilon_i, \nu_i]}{\mathbb{V}\text{ar}[\nu_i]} = g_2\left(\frac{\mathbb{C}\text{ov}\left[s_i^p - \tilde{b}s_i^y, \nu_i\right]}{\mathbb{V}\text{ar}[\nu_i]}\right).$$

Expanding the numerator, using the definition of $\tilde{b}$ and $\nu_i$, and the law of total covariance, it is easy to conclude that $\mathbb{C}\text{ov}(s_i^p - \tilde{b}s_i^y, \nu_i) = \mathbb{V}\text{ar}(\nu_i)$. Hence, $\hat{b} = g_2$ if $\mathbb{V}\text{ar}[\nu_i] \neq 0$, which necessarily holds if $s_i^y$ and $\mathbb{E}[s_j \mid \ell_i^y]$ are not colinear. □

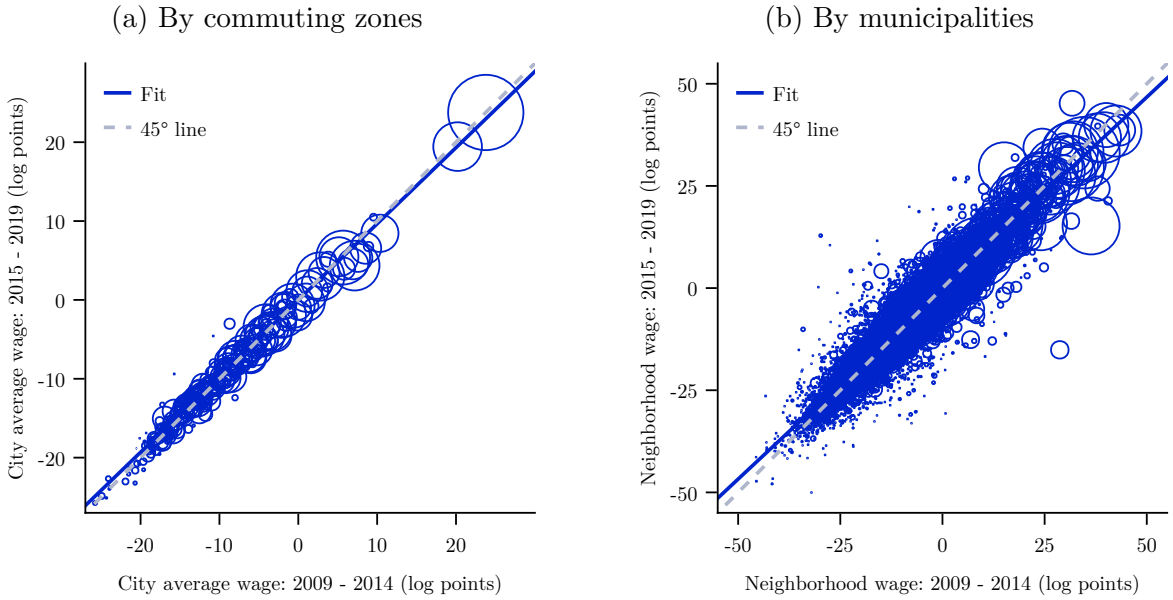
## A.5.4 Additional Tables and Figures

Table A.1: The returns to local interactions (robustness)

| Dep. variable | 5-year wage | | | | 3-year wage | Δ 3-year wage | |
|---|---|---|---|---|---|---|---|
| | (1) OLS | (2) 2SLS | (3) OLS | (4) OLS | (5) OLS | (6) OLS | (7) 2SLS |
| Skill ($g_1$) | 1.003 | 1.023 | 0.940 | 0.975 | 0.978 | | |
| | (0.002) | (0.003) | (0.004) | (0.002) | (0.002) | | |
| City skill ($g_2$) | 0.119 | 0.110 | | | 0.053 | | |
| | (0.010) | (0.005) | | | (0.003) | | |
| Skill × city skill ($g_{12}$) | 0.065 | 0.091 | | | 0.053 | | |
| | (0.019) | (0.007) | | | (0.006) | | |
| Neighborhood skill | | | 0.063 | 0.037 | | | |
| | | | (0.005) | (0.008) | | | |
| Wage × neighborhood skill | | | 0.083 | | | | |
| | | | (0.019) | | | | |
| Establishment wage | | | | 0.072 | | | |
| | | | | (0.006) | | | |
| Firm wage | | | | 0.011 | | | |
| | | | | (0.006) | | | |
| Δ Skill ($g_1$) | | | | | | -0.080 | 0.040 |
| | | | | | | (0.009) | (0.098) |
| Δ Local skill ($g_2$) | | | | | | 0.042 | 0.040 |
| | | | | | | (0.015) | (0.023) |
| Δ Skill × local skill ($g_{12}$) | | | | | | 0.280 | 0.161 |
| | | | | | | (0.017) | (0.078) |
| Controls | | | | | | | |
| Worker-level | · | ✓ | ✓ | ✓ | · | · | · |
| Distance work-residence | · | · | ✓ | ✓ | · | · | · |
| Instrument | | | | | | | |
| Skill ∼ past skill | · | ✓ | · | · | · | · | · |
| Δ Skill ∼ past Δ skill | · | · | · | · | · | · | ✓ |
| Fixed effects | | | | | | | |
| Year | ✓ | ✓ | · | · | ✓ | ✓ | ✓ |
| Year × residence × city | · | · | ✓ | ✓ | · | · | · |
| Year × occupation × industry × city | · | · | ✓ | · | · | · | · |
| Obs. | 971,401 | 971,401 | 971,401 | 508,236 | 520,938 | 520,938 | 520,938 |
| Weights | City size$^{-1}$ | Unit | Unit | Unit | Unit | Unit | Unit |
| Period | 2009-2019 | 2009-2019 | 2009-2019 | 2009-2019 | 2002-2019 | 2002-2019 | 2002-2019 |
| F-stat. | · | 103,412 | · | · | · | · | 20,308 |

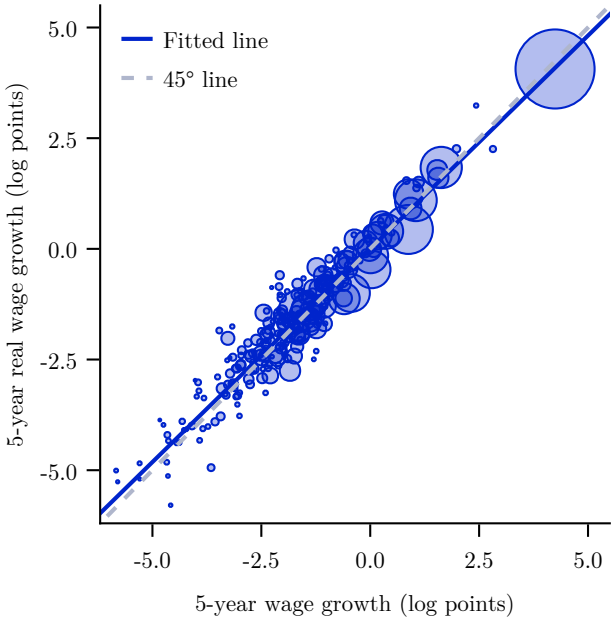
Standard errors clustered at the city level for column 1 and 2, at the neighborhood of work for 3 and 4, and at the worker level for 5 to 7. Sample includes all workers between 25 and 40 year old. Worker-level controls include age, job, occupation, and industry tenure fixed effects, and a dummy for employer switching. Occupations and industry measured at the one-digit level. First column weights observation by the inverse of the size of the city. Second column instrument present skills by the skills in the previous year. To introduce worker F.E., the last three columns extend the sample from 2002 to 2019 and looks at 3-year skill growth. Column 6 estimates the local projection in first difference to remove the worker F.E., and column 7 instruments skill change by past skill change to take into account the autocorrelation in the error term.

Figure A.2: Average wage at the beginning and the end of the 2010s




Note: left panel shows the average wage by commuting zone between 2009-2014 on the $x$-axis against the average wage in the same commuting zone between 2015-2019 on the $y$-axis. The marker size is proportional to the size of the commuting zone. The right-panel shows the same relationship but at the municipality level. The marker size is proportional to the size of the municipality. The aggregate average wage is normalized to zero in both periods for both panels. The blue solid line is the (unweighted) best fitted line between the 2009-2014 and 2015-2019 average wages. The grey dotted line is the 45 degree line.

Figure A.3: Nominal and real wage growth




Note: figure shows the relationship between real wage growth and nominal wage growth by commuting zones. Real wage growth are computed as $\mathbb{E}_\ell[\log w_{it+1}/w_{it}] - \alpha \log(p_{t+1\ell}/p_{t\ell})$, where the second term is the growth in rental prices. The housing expenditure share is set to 0.2 (see Table A.2). The marker size is proportional to the size of the commuting zone.

Figure A.4: The returns to local interactions by time horizon

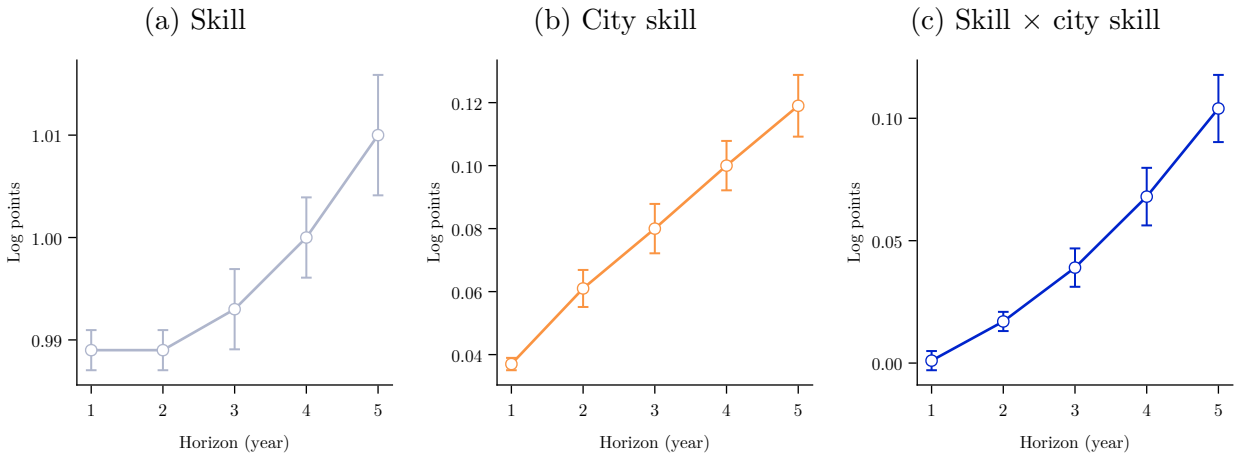



Figure A.5: The returns to local interactions by age

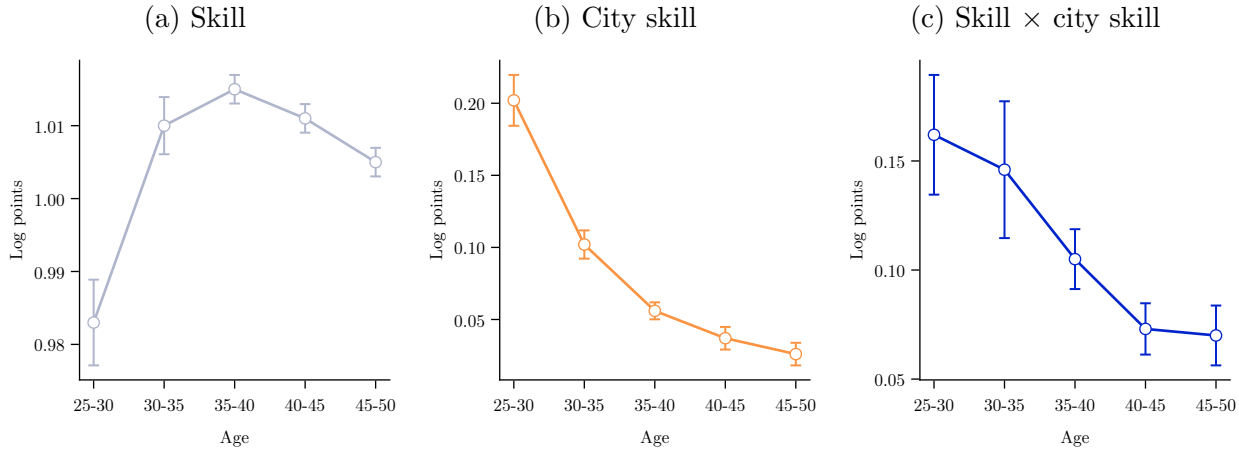



Figure A.6: The returns to local interactions by city size

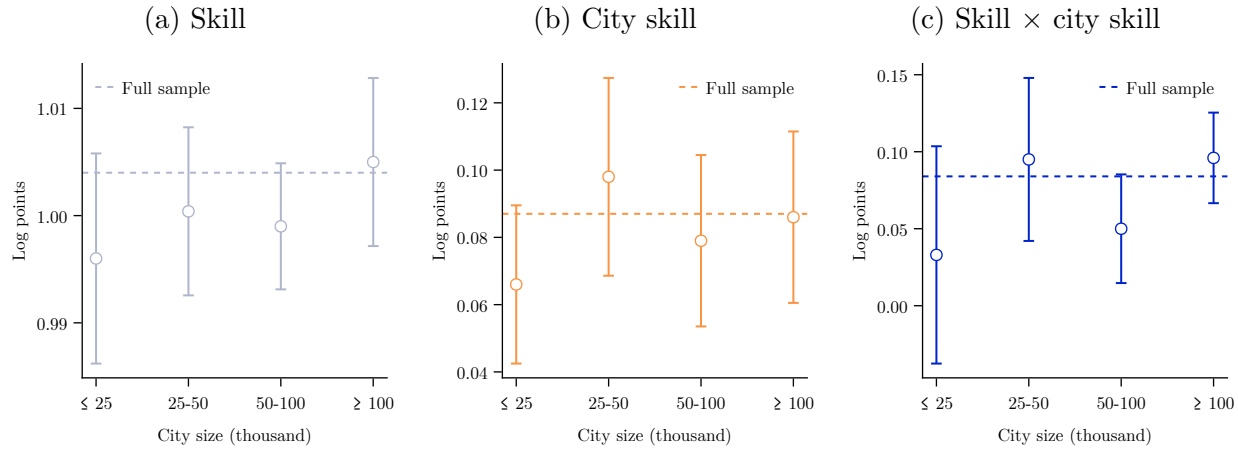


Figure A.7: The returns to local interactions by city inequality

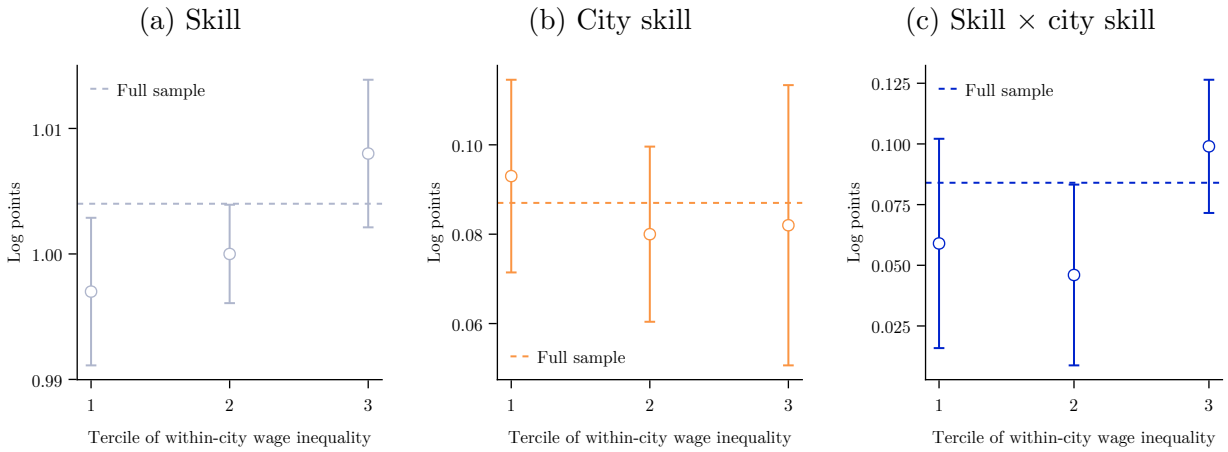
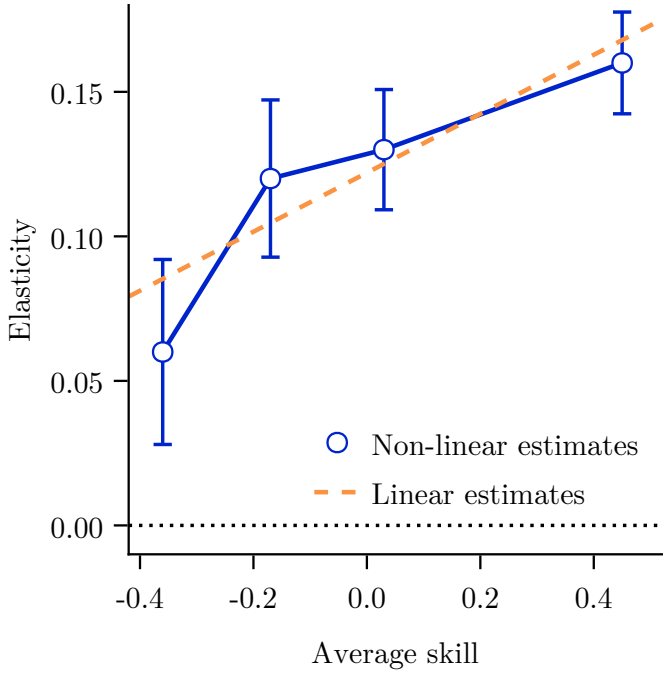
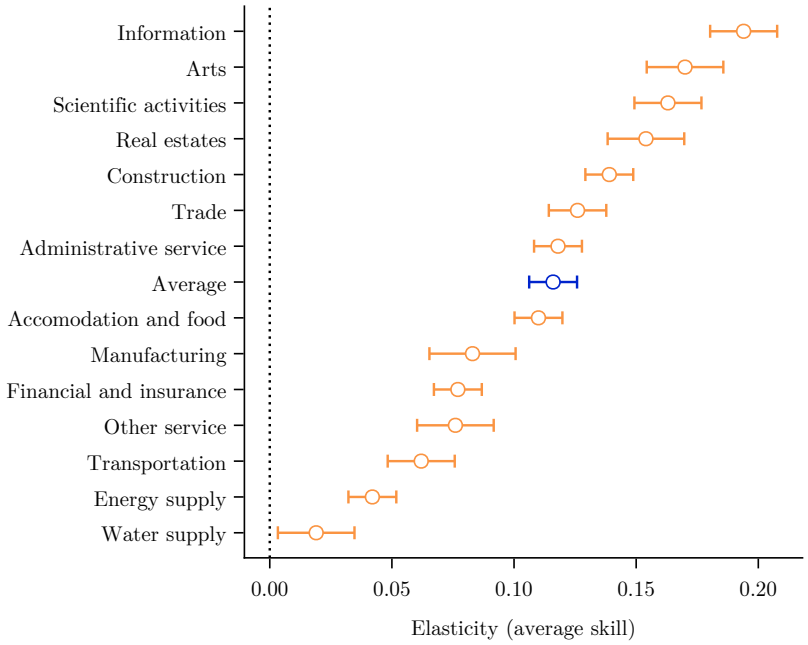



Figure A.8: Non-parametric estimation of the returns to local interactions




Note: estimates produced from a non-parametric version of (1.19), $\log s_{it+1} = \alpha_t + \beta \log s_{it} + \gamma_q \overline{\log s}_{\ell_{it}} + u_{it}$, for $q$ the quartile of workers' current skills. The figure displays $\{\hat{\gamma}_q\}_q$ (blue circles) along with 10% confidence intervals, and the parametric fit produced by (1.19) (orange line).

Figure A.9: The returns to local interactions by industry




Note: estimates produced from an industry-specific version of (1.19), $\log s_{it+1} = \alpha_{t\iota} + \beta_\iota \log s_{it} + \beta_\iota \overline{\log s}_{\ell_{it}} + u_{it}$, for $\iota$ the industry of worker $i$. The figure displays $\{\hat{\gamma}_\iota\}_\iota$ (orange circles) and the average effect (blue circle) along with 5% confidence intervals.

# A.6 Model Estimation

## A.6.1 Clustering

Cities are clustered into 30 city types. The ten first types are the ten largest French cities. The twenty remaining types are constructed by a k-mean algorithm so that there are five types within each French district (North-West, North-East, South-West, and South-East). I include in the clustering algorithm variables that proxy for the geographic primitives of the model: the coordinates of the city center to account for the migration costs, its average wage to account for city's TFP, and its population to account for its amenity.

Figure A.10 shows the mapping between cities to city types within each district. The grey areas are the ten largest French cities. Each other color represents a city type within the district. While k-mean is not a spatial clustering algorithm, including the latitude and longitude of the cities in the clustering variables is sufficient to generate city types that are

spatially contiguous. Table A.3 lists the number of cities by city type, the largest city in each type, the average size, as well as the average wage.

How good is the spatial approximation generate by the k-mean algorithm? Figure A.11 plots the within city-type sum of square residuals (SSR) as a fraction of the total SSR. With only 5 city types, the within city type is less than a fifth of the total SSR. A parsimonious geographic representation of France is therefore able to capture most of the variation of interest for the model.

### A.6.2 Estimating equations

I derive here the equations that I use for the estimation of the model. First, in each city, the housing market must clears. The housing supply is by assumption $H_\ell = \mathcal{H} p_\ell^\delta$, where the housing demand is $H_\ell^D = \alpha Y_\ell / p_\ell$. Equating the two, solving for $p_\ell$ and taking logs yield (1.21).

I then turn to the migration cost equation (1.22). I derive the expression for young workers, but a similar expression exists for the old. From (A.29), we know that

$$\frac{n_{l\ell}^y(s)}{n_{ll}^y(s)} = \frac{\mathrm{e}^{\vartheta\left(sT_\ell/P_\ell + B_\ell^y - \kappa_{l\ell}^y + \beta O_\ell(s)\right)}}{\mathrm{e}^{\vartheta\left(sT_l/P_l + B_l^y + \beta O_l(s)\right)}}.$$

after using $\kappa_{ll}^y = 0$. Similarly,

$$\frac{n_{\ell\ell}^y(s)}{n_{\ell l}^y(s)} = \frac{\mathrm{e}^{\vartheta\left(sT_\ell/P_\ell + B_\ell^y + \beta O_\ell(s)\right)}}{\mathrm{e}^{\vartheta\left(sT_l/P_l + B_l^y - \kappa_{\ell l}^y + \beta O_l(s)\right)}}.$$

Dividing the first expression by the second, using the symmetry $\kappa_{l\ell}^y = \kappa_{\ell l}^y$, and integrating across young workers w.r.t. $dN^y(s)$ yields (1.22).

Third and last, I derive the expression that I iterate on to obtain cities' amenities. Here as well, I derive the expression for young workers, but a similar expression exists for the old.

From (A.29), we know that

$$n^y_{l\ell}(s) = \mathrm{e}^{\vartheta\left(B^y_\ell - B^y_P\right)} \mathrm{e}^{\vartheta\left(\frac{sT_\ell}{P_\ell} - \frac{sT_P}{P_P} + \beta O_\ell(s) - \beta O_P(s)\right)} \mathrm{e}^{-\vartheta(\kappa^y_{l\ell} - \kappa^y_{lP})} n^y_{lP}(s),$$

for some reference city $P$. Using $n^y_\ell(s) = \sum_l n^y_{l\ell}(s)$, and then integrating across workers, we obtain

$$N^y_\ell = \mathrm{e}^{\vartheta\left(B^y_\ell - B^y_P\right)} \int \mathrm{e}^{\vartheta\left(\frac{sT_\ell}{P_\ell} - \frac{sT_P}{P_P} + \beta[O_\ell(s) - O_P(s)]\right)} \sum_l \mathrm{e}^{-\vartheta(\kappa^y_{l\ell} - \kappa^y_{lP})} n^a_{lP}(s) ds. \tag{A.39}$$

The left-hand side is the city $\ell$ employment share of young workers. The first-term is city $\ell$'s relative amenities. Finally, the integral on the right is a term that can be computed within the model. Normalizing $B^y_P = B^o_P$, cities' amenities are recovered by iterating on this expression.

### A.6.3 Migration heterogeneity

The model predicts substantial heterogeneity in migration probabilities across young workers. On average, an increase in young skills by one standard deviation is associated with a migration probability 1.1 percentage points larger. However, these elasticities differ across space. To see this, I estimate the reduced-form equation in the model

$$\mathrm{mig}_\ell(s) = \alpha_\ell + \beta_\ell \log s + u_\ell(s), \tag{A.40}$$

where $\mathrm{mig}_\ell(s)$ is the probability that a worker born in $\ell$ with skill $s$ moves out of their birthplace, $\alpha_\ell$ is a city fixed effect, and $\beta_\ell$ is the city-specific migration elasticity. Figure A.16a and Figure A.16b displays the migration elasticities for young and old workers. I find that young skilled workers born in productive cities are relatively more likely to stay in their birthplaces, whereas young skilled workers born in low-TFP locations are more likely to move out. The same pattern holds for old workers, but the migration elasticities are one order of

magnitude smaller, suggesting that most of the observed sorting patterns are determined at the onset of workers' career.

### A.6.4 Sensitivity

Figure A.17 and A.18 reports information to help assess the sources of identification in the indirect inference procedure. Specifically, Figure A.17 plots the value of the targeted moments in the indirect inference procedure as a function of the parameters. Each subplot is a given moment. Each line represents a particular parameter. The blue lines highlight the parameters that are meant to be identified by the moment.

Figure A.17 confirms that all parameters are well identified locally. Each targeted moment is indeed a steep function of the parameter they are meant to identify. For instance, the flattest relationship is between the wage variance for old workers and the learning shocks dispersion. Even then, a 1% increase in $\sigma_\nu$ leads to a 0.7% increase in the targeted moment. Figure A.17 also shows that some parameters affect mainly one moment whereas others drive several moments.

Figure A.18 complements the analysis of Figure A.17 by reporting the sensitivity measure of Andrews et al. (2017). Intuitively, this metric is the inverse of the moment elasticities reported in Figure A.17. It quantifies how variation in targeted moments would influence estimated parameter values. Theoretically, let $\boldsymbol{h}(\boldsymbol{\theta})$ denote the correspondence that maps the vector of parameters $\boldsymbol{\theta}$ into the vector of targeted moments. Let $\boldsymbol{J}_h$ denote the Jacobian matrix of this function evaluated at the estimated parameters. Then, the sensitivity matrix is computed as $\boldsymbol{\Lambda} = (\boldsymbol{J}_h'\boldsymbol{J}_h)^{-1}\boldsymbol{J}_h'$. Figure A.18 displays the absolute value of $\boldsymbol{\Lambda}$, and the sign of $\boldsymbol{\Lambda}$ is reported in parenthesis. As in Figure A.17, the blue bar highlights which moment is supposed to identify the specific parameter.

I draw two takeaways from Figure A.18. First, each moment is very informative for the parameters they are meant to identify. Second, some moments are informative for several parameters. For instance, the wage variance of old workers matters for the calibration of

$\sigma_s$, $\sigma_\nu$ and $\vartheta$. While this sensitivity measure sheds further light on the importance of the targeted moments for the calibration of the model, the magnitude of the sensitivity cannot however be interpreted.

### A.6.5 Over-identification exercises: sorting in the data and in the model

#### TFP estimates

In the theory, the average wage of city $\ell$ may be high because the city is relatively productive, or because the workers living there are particularly skilled. Specifically, the between-city wage gaps can be decomposed into a TFP gap and a skill gap,

$$\underbrace{\mathbb{E}[\log W_\ell] - \mathbb{E}[\log W]}_{\text{Between-city wage gap}} = \underbrace{\log T_\ell - \mathbb{E}[\log T]}_{\text{TFP gap}} + \underbrace{\mathbb{E}[\log S_\ell]}_{\text{Skill gap}}, \tag{A.41}$$

where $\mathbb{E}[\log S] = 0$ by normalization. Local TFPs are calibrated to match the local average wages. However, neither the TFP gaps nor the skill gaps are targeted in the estimation. In particular, for a given spatial distribution of average wages, greater sorting implies smaller TFP gaps. Comparing the spatial TFP and skill gaps in the data and in the model thus allows to quantify the extent to which the model captures sorting correctly, and with it, whether $\vartheta$ is correctly calibrated.

Computing the wage decomposition (A.41) empirically requires to directly estimate local TFPs. Through the lens of the model, the wage of worker $i$ when employed in city $\ell_{it}$ is $\log w_{it} = \log T_{\ell_{it}} + \log s_{it} + u_{it}$, for $u_{it}$ some measurement error. Hence, city TFP can be estimated as a city fixed effect upon observing workers' skill $s_{it}$.

I use workers' occupation at the four-digit level, interacted with their tenure at the occupation, to proxy for their skill. Proxying skills with occupations has the advantage that skills can evolve over time, much like in my model. In particular, worker fixed effects, as in

Card et al. (2023), cannot be used to estimate the productivity of a location in the presence of spatial heterogeneity in learning. Measuring skill through worker fixed effect would indeed confound the TFP estimates with the local learning opportunities since those fixed effects are constant over time.[14]

Figure A.19a and A.19b plots the TFP and skill gaps in the model against the between-city wage gaps. Cities with a relatively high wage are more productive and attract a higher density of skilled workers. Combined, these two panels show that skilled workers locate disproportionately in productive cities. Quantitatively, the spatial TFP differentials explain 43% of the spatial wage gaps in the model (blue dotted line in Figure A.19a), while the between-city skill gaps explain the remaining 57% of the variation (orange dotted line in Figure A.19b). In the data, I find that the TFP and skill gaps explain respectively 42% and 58% of the spatial wage variation (grey lines in both panels).

**Spatial sorting across the lifecycle**

In the model, workers' willingness to sort varies by skill and by age: young workers consider both the learning potential of cities and the local wage, whereas old workers only sort according to the earnings differential. When migration costs are small, the model predicts that the between-age difference in the number of workers in city $\ell$ with skill $s$ is

$$\log\left(\frac{n_\ell^y(s)}{n_\ell^o(s)}\right) = \vartheta\left(B_\ell^y - B_\ell^o\right) - \log\left(\frac{\mathcal{N}^y(s)}{\mathcal{N}^o(s)}\right) + \vartheta\beta O_\ell(s), \tag{A.42}$$

where $\mathcal{N}^a(s) \equiv \sum_\ell \mathrm{e}^{\vartheta[w_\ell(s)+B_\ell^a+\beta O_\ell^a(s)]}$ is independent from $\ell$. The last term captures the learning motive that is present only for young workers. The shape of the learning technology affects whether young skilled workers place a relatively higher learning value on productive cities than low skill workers. The dispersion in idiosyncratic preferences modulates the

[14]Suppose $s_{it} = i$ so that the empirical wage model reads $\log w_{it} = \alpha_{\ell_{it}} + \beta_i + u_{it}$. City TFPs are then estimated off within-worker time variation in wages for workers who switch locations: $\log w_{it+1} - \log w_{it} = \alpha_{\ell_{it+1}} - \alpha_{\ell_{it}} + u_{it+1} - u_{it}$. That is, city TFPs are estimated off wage growth variation for movers. Hence, these estimates conflate the proper city productivity with estimates of the learning technology.

between-age difference in willingness to sort. The spatial variations in young to old ratio can therefore be used to validate jointly the estimation of the learning technology and the dispersion in idiosyncratic preferences.

Motivated by (A.42), I measure in the data the between-age difference in willingness to sort through the reduced-form specification

$$\log\left(\frac{N_{\ell q}^{y}}{N_{\ell q}^{o}}\right) = \alpha_{\ell} + \beta_{q} + \gamma_{q}\mathbb{E}[\log W_{\ell}] + e_{\ell q}, \tag{A.43}$$

where $N_{\ell q}^{a}$ is the number of workers living in city $\ell$ with age $a$ in wage quintile $q$, and $e_{\ell q}$ is some residual. The parameters $\alpha_{\ell}$ and $\beta_{q}$ are a city and wage quintile fixed effects respectively. I proxy the quality of interactions in city $\ell$ by the city log wage, and let workers in different wage quintile value those interactions differently, as captured by $\gamma_{q}$. If $\gamma_{q} > 0$, young workers in quintile $q$ are more sensitive to spatial variations in wages than old workers in the same quintile. If $\gamma_{q'} > \gamma_{q}$, this between-age difference in willingness to sort is greater for workers in quintile $q'$ than $q$. The parameters $\{\gamma_{q}\}$ are identified up to a constant, and I normalize $\gamma_{1} = 0$ to focus on the difference in between-age willingness to sort across skills. I estimate (A.43) in the data and in the model by OLS.

Figure A.19c presents the point estimates. In the data, the extra incentives of young workers to sort to high-wage places is greater for skilled workers. The same pattern old in the model. Figure A.20 plots the model-implied spatial differences in option values for low, middle and high-skill. The future opportunities offered by productive cities are more attractive to skilled workers, which generates further sorting from them.[15]

[15]When migration costs are positive, the option value of productive cities reflect both the learning opportunities and the future migration costs. Figure A.20 plots the spatial differences in option values when interactions are not segmented by cities (Section 1.5). In this case, skilled workers continue to place a higher option value on productive cities to front-load the future migration costs, but the between-city differences are four times smaller.

### A.6.6 Additional Tables and Figures

Table A.2: Estimated parameters

| | Parameter | Value | Moment | Data | Model |
|---|---|---|---|---|---|
| | | A. Externally calibrated | | | |
| $\beta$ | 15-year discount factor | 0.684 | | · | · |
| | | B. Model inversion | | | |
| $1-\alpha$ | Housing exp. share | 0.200 | Housing exp. share | 0.200 | 0.200 |
| $\{\kappa^a_{\ell\ell'}\}_{\ell,\ell'}$ | Migration cost | · | Migration flows (1.22) | · | · |
| $\delta$ | Housing price elasticity | 7.473 | Housing price $\sim$ exp.: constant (1.21) | · | · |
| $\mathcal{H}$ | Supply stock | $7.2 \times 10^{-9}$ | Housing price $\sim$ exp.: slope (1.21) | · | · |
| $g_1$ | Learning technology | 1.057 | Wage growth $\sim$ wage (1.19) | · | · |
| $g_2$ | Learning technology | 0.327 | Wage growth $\sim$ city wage (1.19) | · | · |
| $g_{12}$ | Learning technology | 0.393 | Wage growth $\sim$ wage $\times$ city wage (1.19) | · | · |
| | | C. Indirect inference | | | |
| $\{T_\ell\}_\ell$ | City TFP | · | Average wage | · | · |
| $\{B_{a\ell}\}_{a,\ell}$ | City amenity | · | City size | · | · |
| $g_0$ | Learning technology | 0.400 | Old / young wage | 1.250 | 1.248 |
| $\vartheta$ | Taste shock dispersion | 0.009 | Local inequality $\sim$ local wage | 0.29 | 0.28 |
| $\mu^s_1$ | Mean young skill | $-0.200$ | Average skill | 0.000 | 0.003 |
| $\sigma^s_1$ | Variance young skill | 0.309 | Variance young wage | 0.139 | 0.141 |
| $\sigma_\nu$ | Var. learning shock | 0.310 | Variance old wage | 0.234 | 0.240 |

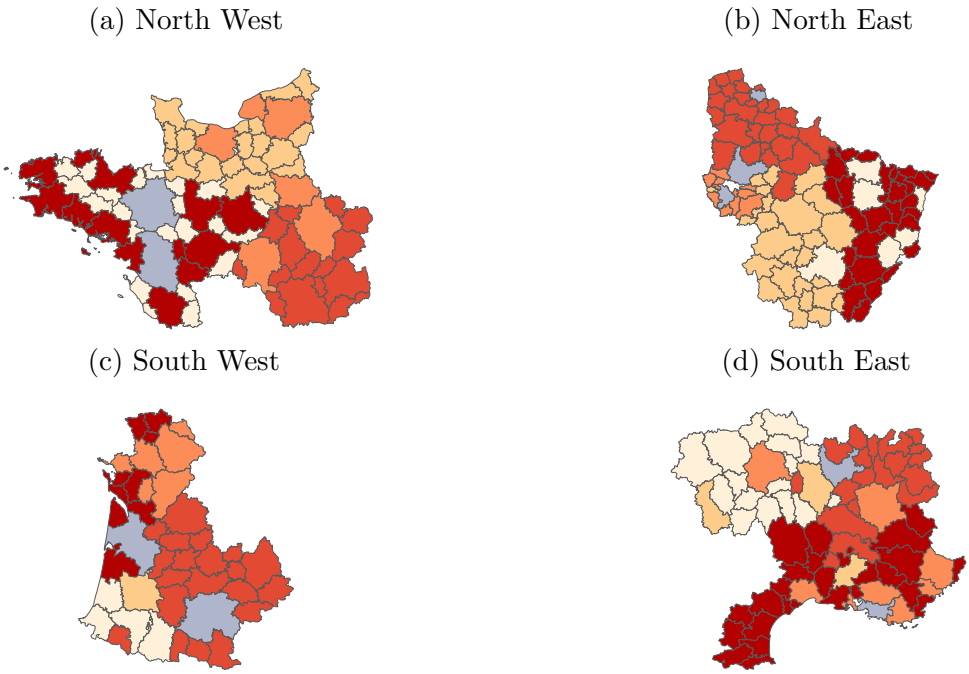
Parameter used in the quantitative model. The first two columns describe the parameter and the third column presents the value of the parameter. The fourth column describes the moment used to estimate or calibrate the parameter. The fifth and sixth columns show the value of that moment in the data and in the model. The migration costs are plotted in Figure A.12, the city TFP in Figure A.13a, and city amenities in Figure A.13b.

Table A.3: City types characteristics

| Cluster | District | # cities | Largest city | Average size (thousands) | Average wage |
|---|---|---|---|---|---|
| 1 | Paris | 1 | Paris | 2800.6 | 30.03 |
| 2 | SE | 1 | Lyon | 609.8 | 23.38 |
| 3 | NE | 1 | Roissy | 443.0 | 22.50 |
| 4 | NE | 1 | Saclay | 414.1 | 27.32 |
| 5 | SW | 1 | Toulouse | 403.8 | 22.42 |
| 6 | SW | 1 | Bordeaux | 332.6 | 21.07 |
| 7 | SE | 1 | Marseille | 316.8 | 22.14 |
| 8 | NW | 1 | Nantes | 305.8 | 20.94 |
| 9 | NE | 1 | Lille | 258.4 | 22.06 |
| 10 | NW | 1 | Rennes | 218.4 | 20.54 |
| 11 | NE | 6 | Strasbourg | 131.9 | 20.67 |
| 12 | NE | 26 | Troyes | 32.64 | 18.61 |
| 13 | NE | 11 | Orly | 109.7 | 22.88 |
| 14 | NE | 27 | Roubaix | 58.53 | 19.46 |
| 15 | NE | 28 | Besançon | 32.47 | 19.57 |
| 16 | NW | 22 | Challans | 16.70 | 17.65 |
| 17 | NW | 21 | Évreux | 26.13 | 18.80 |
| 18 | NW | 6 | Rouen | 141.9 | 20.33 |
| 19 | NW | 15 | Blois | 28.05 | 18.86 |
| 20 | NW | 13 | Angers | 74.63 | 18.94 |
| 21 | SW | 4 | Pau | 65.64 | 19.31 |
| 22 | SW | 1 | Mont-de-Marsan | 29.40 | 17.29 |
| 23 | SW | 6 | Poitiers | 54.48 | 19.42 |
| 24 | SW | 20 | Périgueux | 25.87 | 18.22 |
| 25 | SW | 9 | La Teste-de-Buch | 17.74 | 17.26 |
| 26 | SE | 16 | Limoges | 34.67 | 18.08 |
| 27 | SE | 3 | Saint-Étienne | 127.5 | 19.56 |
| 28 | SE | 8 | Grenoble | 146.2 | 21.67 |
| 29 | SE | 19 | Valence | 52.71 | 20.31 |
| 30 | SE | 26 | Nîmes | 34.06 | 18.29 |

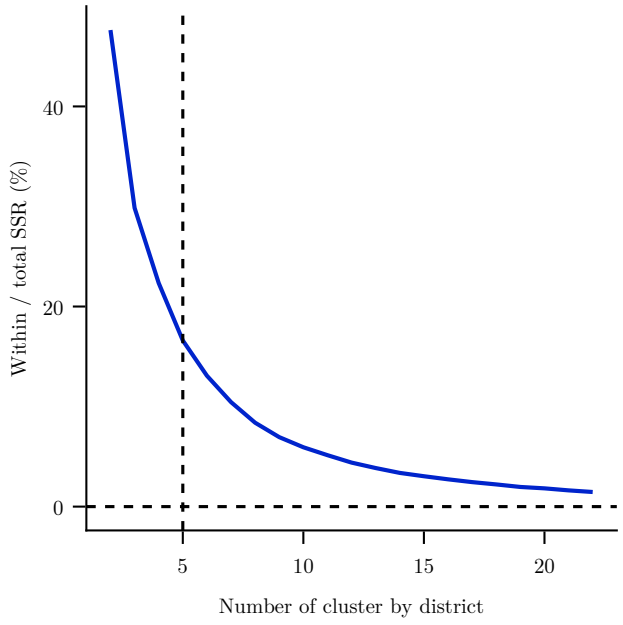
The tables report statistics on the city types used for the estimation of the quantitative model. The second column is the district in which the city type belongs (see Figure A.10). The third column is the number of cities included in the city city type. For the first ten types, there is a single city by definition. For the twenty remaining types, the number of cities per city type is determined by the k-mean algorithm. The fourth column reports the largest city in the city type, the fifth column the average number of employed workers (in thousands), and the sixth column the average wage.

Figure A.10: City clustering



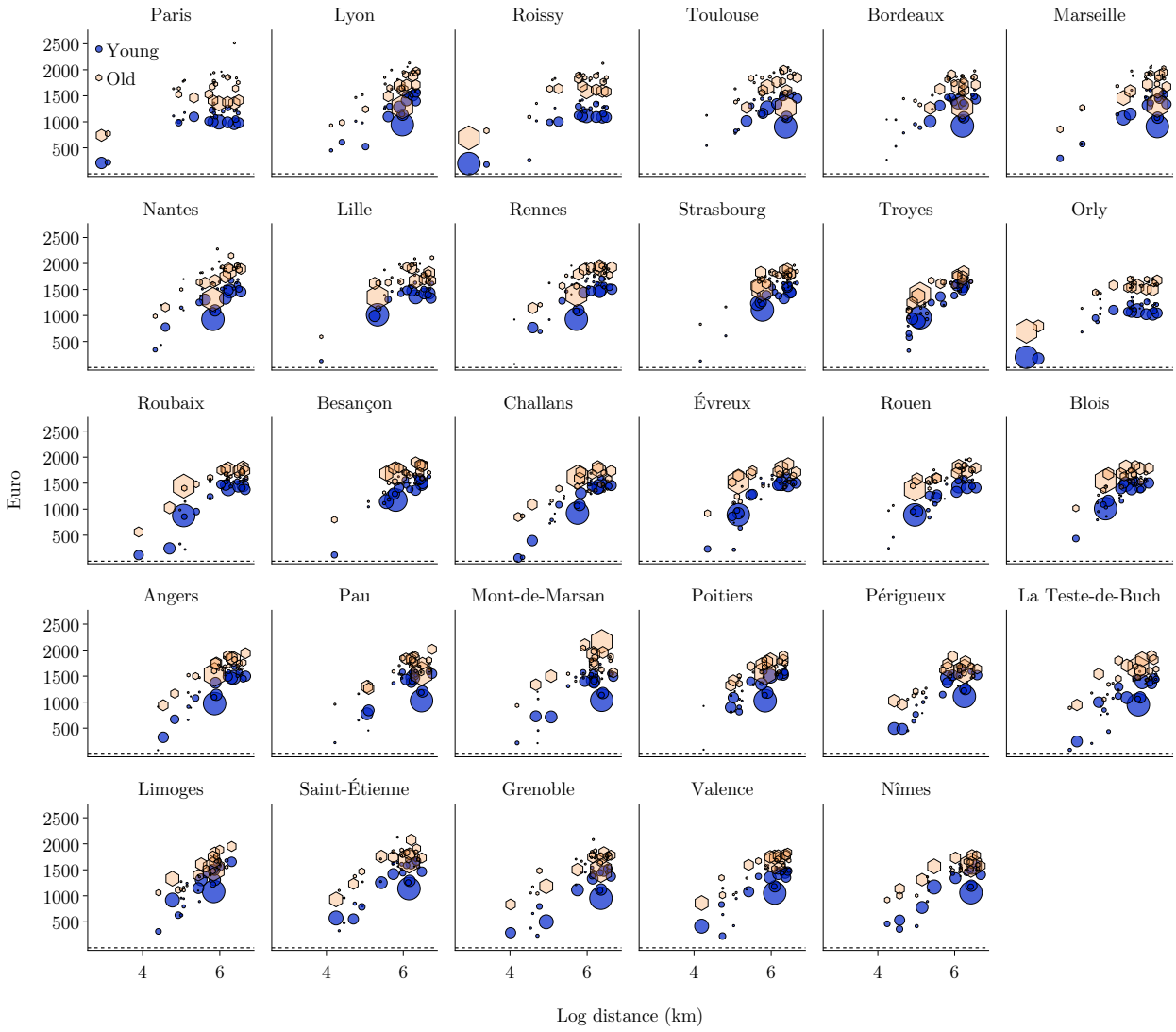

Note: each subplot is a French district. The grey areas are the ten largest French cities. The remaining commuting zones are subdivided into five groups within each district. Each city cluster is represented by a different color on the map. See Table A.3 for a list of the clusters.

Figure A.11: City clustering residual



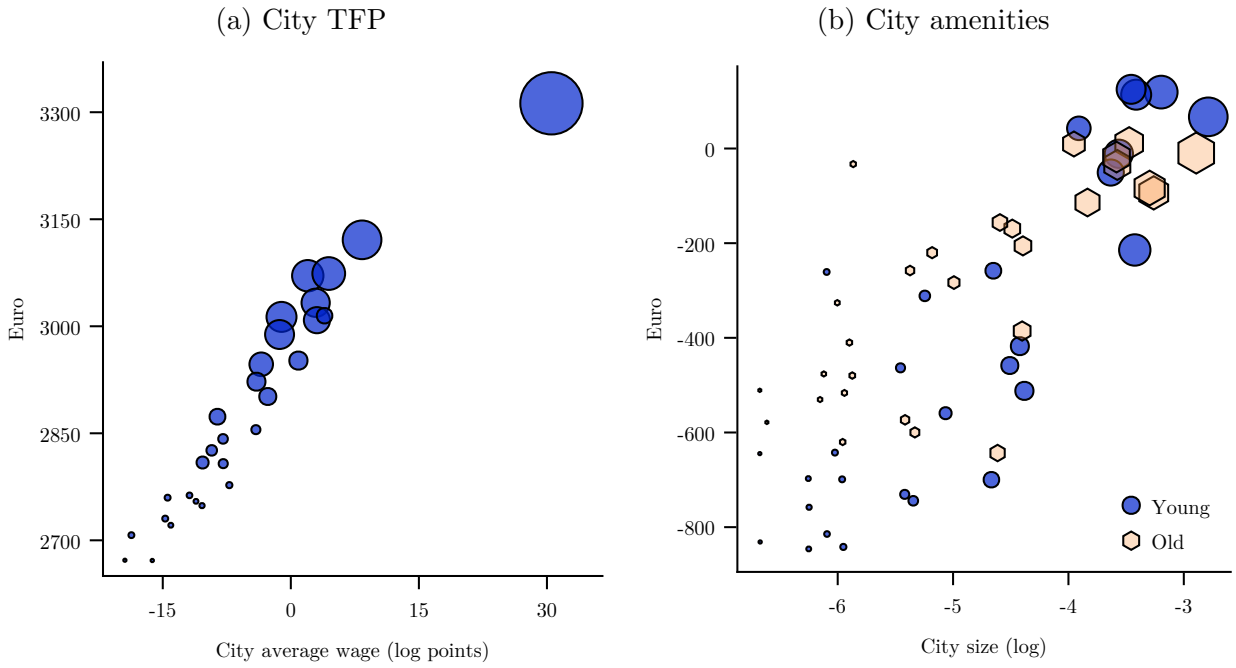

Note: average within cluster sum of square residuals (SSR) relative to the total SSR against the number of cluster (per district) used in the k-mean algorithm. The quantitative model features five city-types per district, as represented by the vertical dashed black line.

Figure A.12: Estimated age-specific migration costs by city of origin



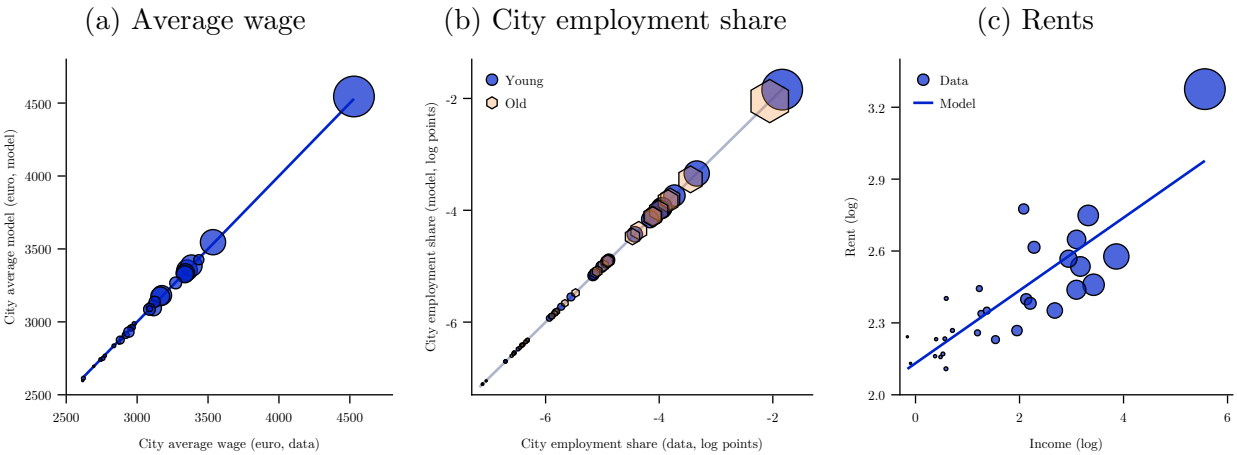

Note: estimated age-specific migration costs by city of origin as a function of the distance between the city of origin and the destination. Each subplot corresponds to a city of origin. The migration costs are estimated according to (1.22). The estimated costs are in utils, and they are reported here in euros through $\kappa^a_{\ell\ell} P_\ell$. The blue circles and orange hexagons are the migration costs for young and old workers. Distance between two cities computed as the Haversine distance. For city types that comprise more than one city, I define the coordinates of the city type as the coordinates of the most populous city in that cluster. For reference, the average wage in the economy is 3,278€. The size of the circles is proportional to the size of the city of destination.

Figure A.13: Estimated city characteristics



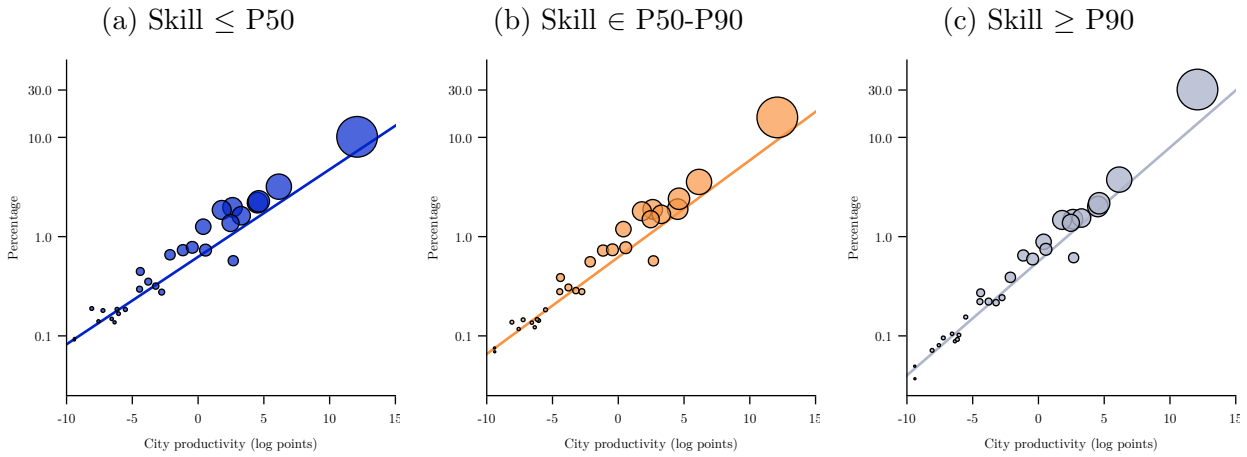

Note: panel (a) plots the estimated city TFP against the city average log wage. City TFP calibrated from (1.23). Panel (b) plots the estimated age-specific amenity against the age-specific city employment share in log. The blue circles and orange hexagons represent the young and old amenity. Amenities calibrated according to (A.39). The amenity value of Paris is normalized to zero for young and old and is omitted from panel (b) for improved visual clarity. Amenities are converted into monetary value through $B_\ell^a P_\ell$. The size of the circles is proportional to city size.

Figure A.14: Model fit



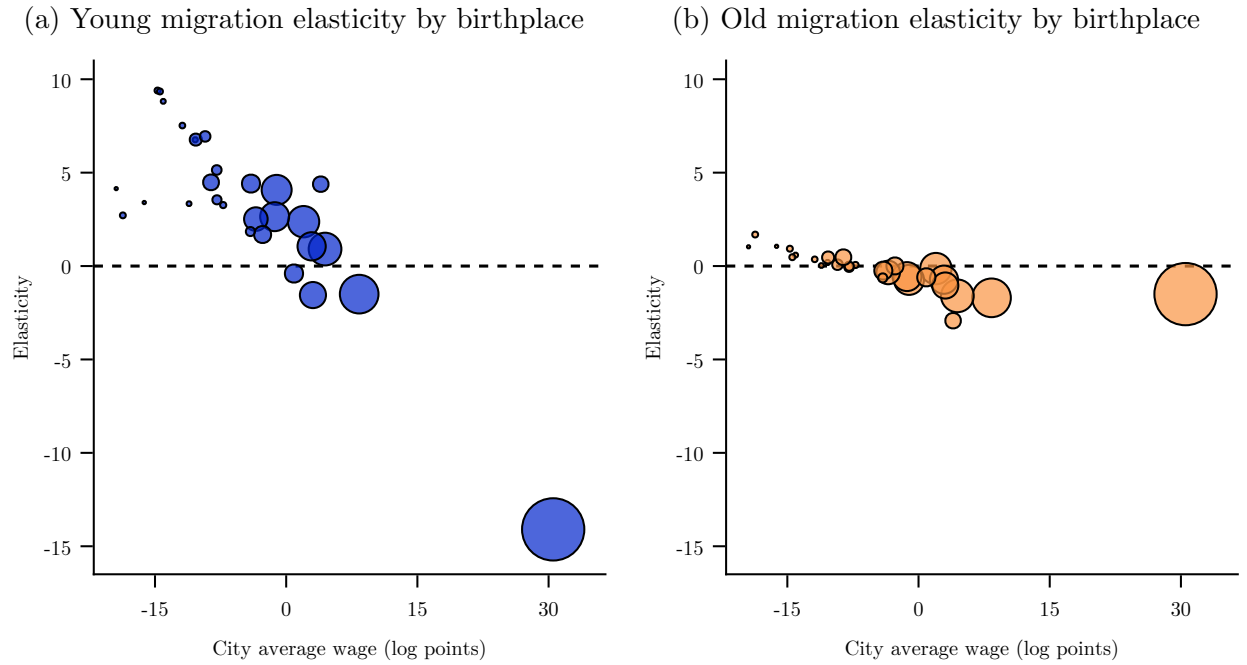

Note: panel (a) plots the average wage in the data (x-axis) against the average wage in the model (y-axis). Panel (b) plots the age-city city employment share for young (blue circles) and old workers (orange hexagons) in the data (x-axis) and in the model (y-axis). Panel (c) plots the rent in the data (blue circle) and the rent in the model (blue line) against city income. The size of the circles are proportional to the size of the cities.

Figure A.15: Spatial allocation of workers across the skill distribution



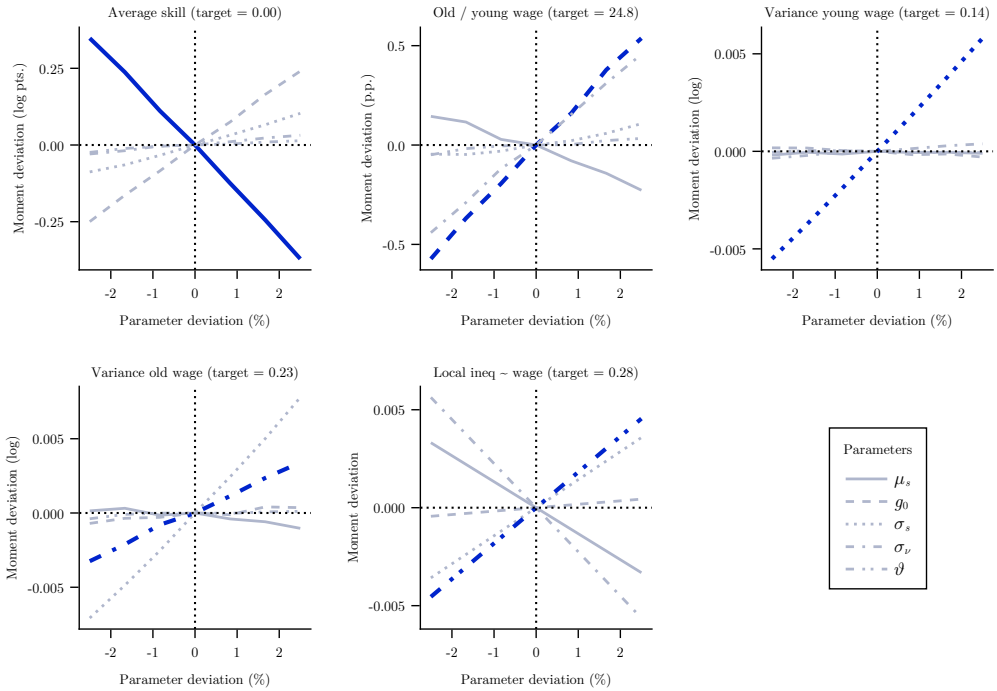

Note: three panels display the probability to work in a particular city if your skill is below the 50th percentile (a), between the 50th and 90th percentile (b), or above the 90th percentile (c). The $x$-axis is cities' relative log TFP, and the $y$-axis in log-scale. The size of the circles are proportional to the size of the cities.

Figure A.16: Migration elasticities across space



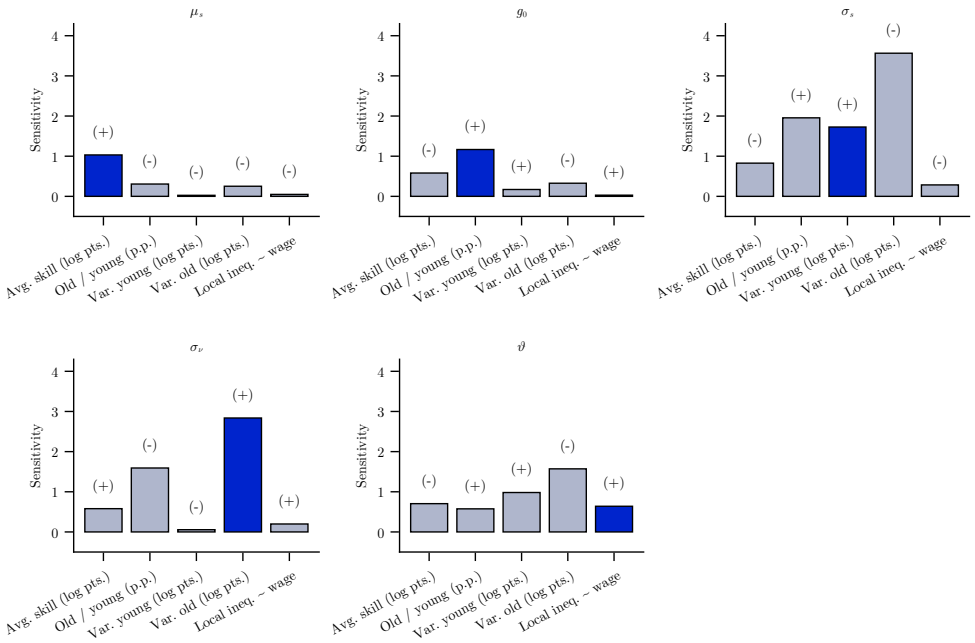

Note: panel (a) and (b) display the migration elasticities for young and old workers as estimated by (A.40). Skills are normalized in the regression, so that the elasticities correspond to the percentage point change in migration probability in response to an increase in skill by one standard deviation. The size of the circles are proportional to the size of the cities.

## Figure A.17: Sensitivity of targeted moments to parameters



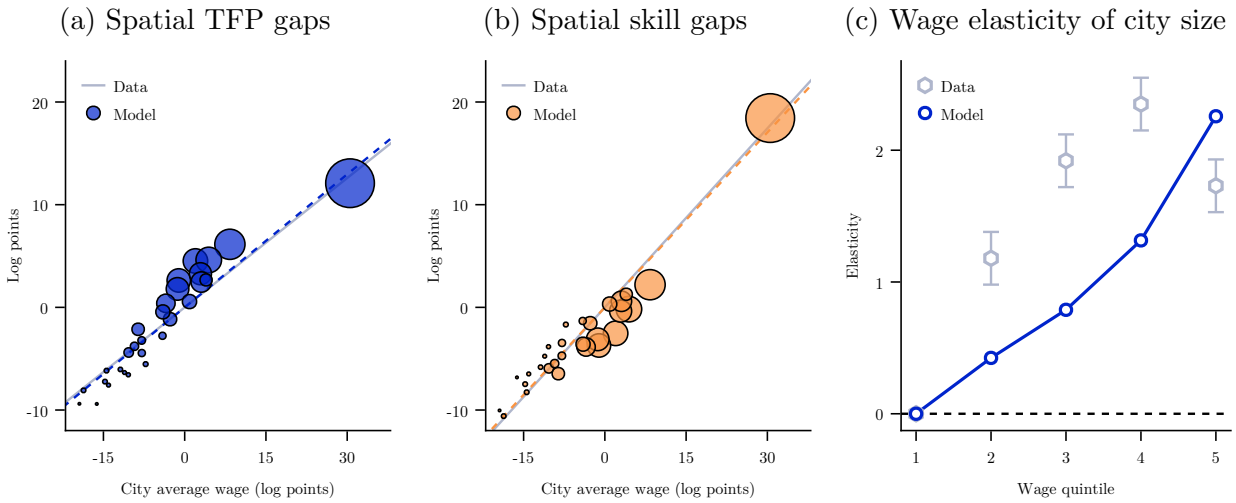

Note: each panel is a targeted moment in the indirect inference procedure. Each line is a parameter calibrated by the indirect inference procedure. The blue lines highlight the parameter that are heuristically identified by the particular moment of interest. I consider local deviations around the estimated parameters of 2.5% in each direction. The deviations are represented in percentages on the x-axis.

## Figure A.18: Sensitivity of parameters to targeted moments



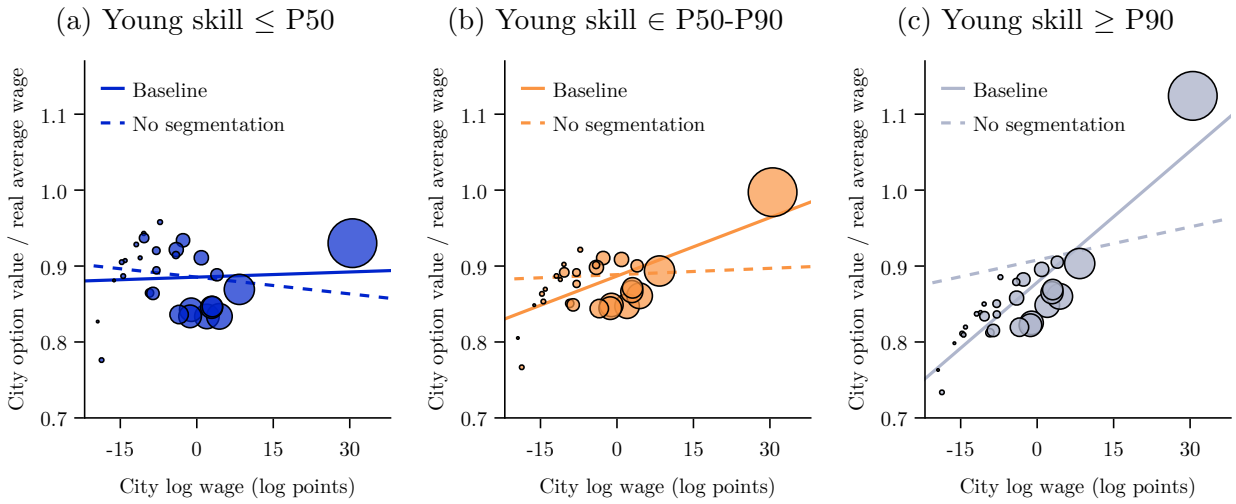

Note: figure reports the sensitivity measure of Andrews et al. (2017) in absolute value. The sign of the sensitivity is reported in parenthesis. Each panel is a parameter calibrated in the indirect inference procedure. Each column is a targeted moment. The blue bar highlights which moment is supposed to identify the parameter of interest.

Figure A.19: Worker sorting in the model and in the data.




Note: panel (a) displays the estimated city TFP. The grey solid line is computed as $\log \hat{T}_\ell = \alpha + \beta \log w_\ell$, where $\log \hat{T}_\ell$ are the location fixed effects estimated from a projection of log wages on city and skill fixed effects. Panel (b) displays the average skill in city $\ell$, $\mathbb{E}[\log S_\ell]$. The grey solid line is computed as $\mathbb{E}[\hat{\beta}_s \mid \ell] = \alpha + \beta \log w_\ell$, where $\hat{\beta}_s$ are the skill fixed effects from the TWFE regression. Panel (c) plots the point estimates of $\{\gamma_q\}$ in (A.43) obtained from the model (blue circles) and from the data (grey hexagons).

Figure A.20: The option value of cities across the skill distribution



## A.7 The Tradeoff Between Human Capital and Inequality

Figure A.21: Skill growth by birthplace and initial skill

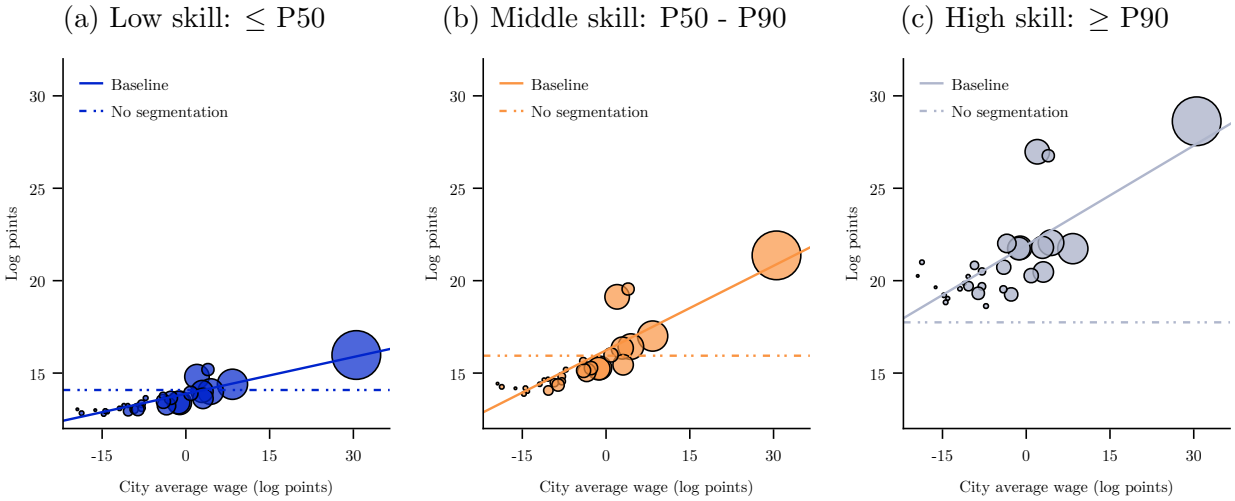



Figure A.22: Relative skil growth by birthplace

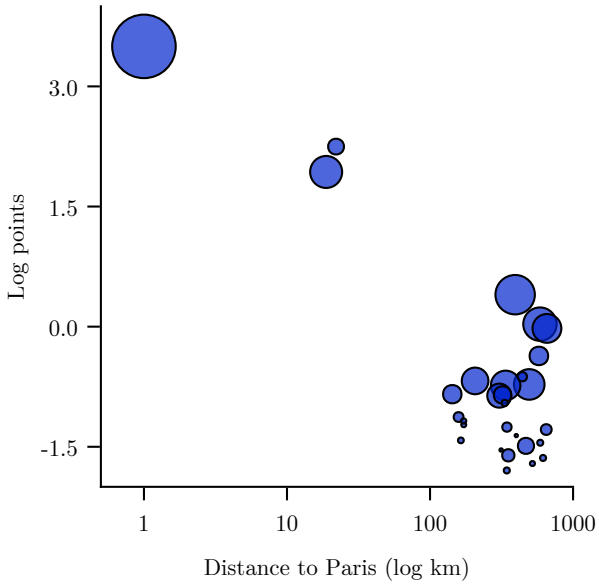
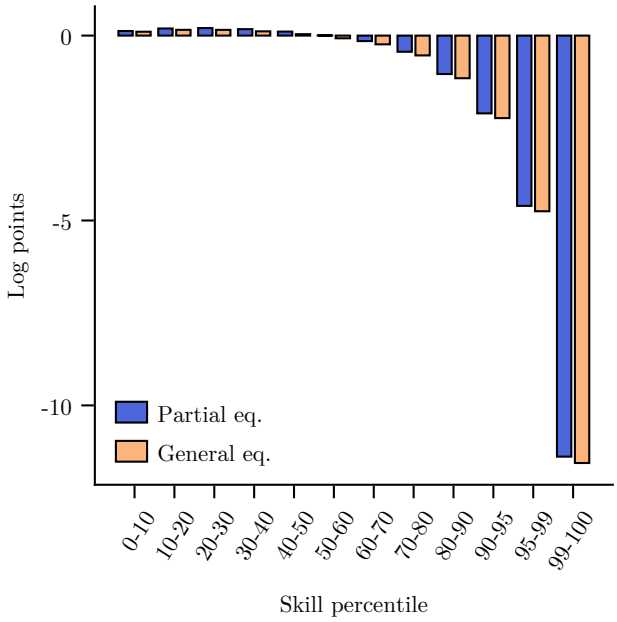
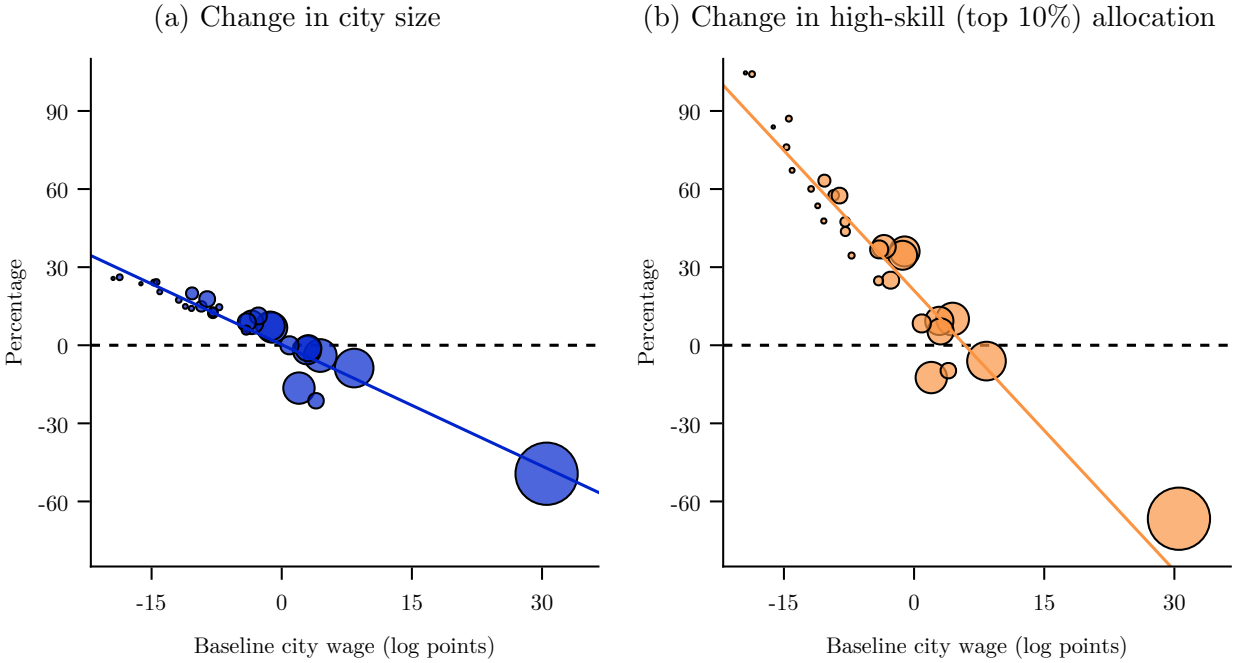


Figure A.23: The partial and general equilibrium effect of local interactions on skill growth



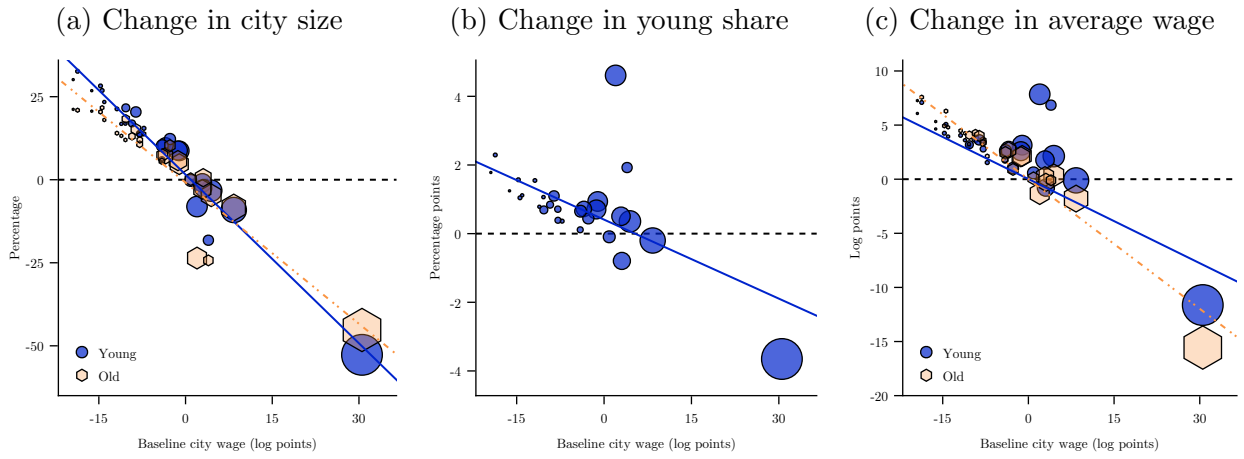

Note: figure plots the change in average skill growth between the baseline equilibrium and the no segmentation equilibrium across the skill distribution of young workers. The blue bars are the change in skill growth when interactions are no longer spatially segmented but the aggregate skill distribution is held constant (partial equilibrium). The orange bars are in general equilibrium letting the aggregate skill distribution adjusts.

Figure A.24: Local interactions as a source of agglomeration




Note: panel (a) displays the percentage change in city size between the baseline equilibrium and the equilibrium without segmented interaction against the baseline city average wage. The blue line is the fitted line. Panel (b) plots the percentage change in the propensity of high-skill workers (top 10% of the skill distribution) to work in a particular city. The orange line is the fitted line. The size of the markers is proportional to the baseline city size in both panels.

Figure A.25: The agglomeration effect of local interactions by age




# A.8 The Consequences of Spatial Policies

## A.8.1 Welfare analysis

**Consumption equivalence** I first present how I derive the consumption-equivalent welfare metric of Section 1.6.3. Let $\mathcal{V}_l^y(s,\zeta,\boldsymbol{\tau})$ denote the lifetime utility of a young worker born in $l$ with skill $s$ when their lifetime consumption is multiplied by $\zeta$ and they face a transfer function $\boldsymbol{\tau}$,

$$\mathcal{V}_l^y(s,\zeta,\boldsymbol{\tau}) = \frac{1}{\vartheta}\log\left(\sum_\ell \mathrm{e}^{\vartheta\left(\frac{\left[sT_\ell+\tau_{l\ell}^y(s)\right]\zeta}{P_\ell}+B_\ell^y-\kappa_{l\ell}^y+\beta O_\ell(s,\zeta,\boldsymbol{\tau})\right)}\right),$$

where[16]

$$O_\ell(s,\zeta,\boldsymbol{\tau}) = \frac{1}{\vartheta}\int\int\log\left(\sum_{\ell'}\mathrm{e}^{\vartheta\left(\frac{\left[s^oT_{\ell'}+\tau_{\ell\ell'}^o(s^o)\right]\zeta}{P_{\ell'}}+B_{\ell'}^o-\kappa_{\ell\ell'}^o\right)}\right)dF\left(\frac{s^o}{\gamma(s,s_p)}\right)\pi_\ell(s_p).$$

Let $\zeta_l(s)$ denote the consumption-equivalent change in welfare induced by the policy $\boldsymbol{\tau}$ for workers born in $l$ with skill $\ell$. This consumption-equivalent measure is defined implicitly by $\mathcal{V}_l^y[s,\zeta_\ell(s),0] = \mathcal{V}_l^y(s,0,\boldsymbol{\tau})$. Intuitively, $\zeta_l(s)$ measure by how much does the lifetime

[16] I use the change of variable $e\gamma(s,s_p)\to s^o$ to derive the option value of city $\ell$.

consumption of workers born in $l$ with skill $s$ needs to be multiplied in the baseline equilbrium to make these workers indifferent between the policy $\boldsymbol{\tau}$ and the baseline economy.

This metric can only be used to quantify the welfare impact of the policy for workers with identical skill born in the same location. In particular, it cannot be aggregated across skills or places. I therefore complement the worker-level metric with a welfare measure for "representative" workers. Specifically, I measure the welfare impact of the policy on group $g$ as the change in lifetime consumption required to make the average worker in that group indifferent between the two equilibrium,

$$\int \mathcal{V}_{\ell_i}^y(s_i, \zeta_g, 0) dF_i^g = \int \mathcal{V}_{\ell_i}^y(s_i, 0, \boldsymbol{\tau}) d\tilde{F}_i^g,$$

where $\zeta_g$ is the welfare measure, and $dF_i^g$ denote the distribution of workers in group $g$ for simplicity.[17] This second welfare metric is consistent with a utilitarian social welfare function that places equal weights on workers within group $g$.[18]

**Welfare decomposition** The total welfare impact of the policy can be decomposed into several margins. To see this, note that the expected lifetime utility of a young worker be written as

$$\mathcal{V}_l^y(s) = \mathbb{E}\left[\max_\ell \left\{ \frac{sT_\ell + \tau_{l\ell}}{P_\ell} - \kappa_{l\ell}^y + \beta O_\ell(s) + \varepsilon_\ell \right\}\right] = \sum_\ell \left( \frac{sT_\ell + \tau_{l\ell}}{P_\ell} - \kappa_{l\ell}^y + \beta O_\ell(s) \right) \frac{n_{l\ell}^y(s)}{m_\ell^y(s)} + \chi_l^y(s).$$

where $\chi_l^y(s) \equiv \mathbb{E}\left[\varepsilon_\ell^y \mid \ell \succ \ell' \, \forall \ell' \neq \ell\right]$ and I have subsumed the amenity value of cities into $\chi_l^y(s)$.[19] The first equality follows from the definition of lifetime utility, and the second line from $n_{l\ell}^y(s)/m_l^y(s) = \Pr[\varepsilon_\ell \geq \max_{\ell' \neq \ell} \varepsilon_{\ell'} + U_{l\ell'}(s) - U_{l\ell}(s)]$. Similarly, the expected option

[17] Note that $dF_i^g$ and $d\tilde{F}_i^g$ need not be the same as the policy is reallocating workers across space.

[18] Importantly however, no restrictions are placed on the social welfare function across groups. By making the groups more granular, one therefore recovers the worker-level welfare metric.

[19] No closed-form solution exists for $\chi_l^y(s)$ but it can be obtained as utility residual.

value of city $\ell$ can be rewritten

$$\begin{aligned} O_\ell(s) &= \int\int \mathbb{E}\left[\max_{\ell'}\left\{\frac{e\gamma(s,s_p)T_{\ell'}}{P_{\ell'}} - \kappa^o_{\ell\ell'} + \varepsilon_{\ell'}\right\}\right]\pi_\ell(s_p)dF(e)ds_p \\ &= \int\left(\sum_{\ell'}\frac{n^o_{\ell\ell'}(s^o)}{m^o_\ell(s^o)}\left(\frac{s^oT_{\ell'}}{P_{\ell'}} - \kappa^o_{\ell\ell'}\right) + \chi^o_\ell(s^o)\right)\omega_\ell(s^o,s)ds^o. \end{aligned}$$

where the second line follows from the change of variable $e\gamma(s,s_p) \to s^o$ and the definition

$$\omega_\ell(s^o,s) \equiv \int \frac{\pi_\ell(s_p)}{\gamma(s,s_p)} f\left(\frac{s^o}{\gamma(s,s_p)}\right) ds_p.$$

Combined with the expression for young workers, we can therefore write lifetime utility as

$$\begin{aligned} \mathcal{V}^y_l(s) = &\underbrace{\sum_\ell\left(\frac{\tau_{l\ell} - \kappa^y_{l\ell}P_\ell}{P_\ell}\right)\frac{n^y_{l\ell}(s)}{m^y_\ell(s)} - \beta\sum_\ell\frac{n^y_{l\ell}(s)}{m^y_\ell(s)}\int\omega_\ell(s^o,s)\left(\sum_{\ell'}\frac{n^o_{\ell\ell'}(s^o)}{m^o_\ell(s^o)}\kappa^o_{\ell\ell'}\right)ds^o}_{\text{Transfers net of lifetime migration costs } \equiv V^T_l(s)} \\ &\underbrace{\sum_\ell\left(\frac{sT_\ell}{P_\ell}\right)\frac{n^y_{l\ell}(s)}{m^y_\ell(s)}}_{\text{Present real income} \equiv V^I_l(s)} + \underbrace{\beta\sum_\ell\frac{n^y_{l\ell}(s)}{m^y_\ell(s)}\int\omega_\ell(s^o,s)\left(\sum_{\ell'}\frac{n^o_{\ell\ell'}(s^o)}{m^o_\ell(s^o)}\frac{s^oT_{\ell'}}{P_{\ell'}}\right)ds^o}_{\text{Future real income} \equiv V^{EI}_l(s)} + \\ &\underbrace{\chi_l(s) + \beta\sum_\ell\frac{n^y_{l\ell}(s)}{m_\ell y(s)}\int\chi_\ell(s^o)\omega_\ell(s^o,s)ds^o}_{\text{Preferences} \equiv V^A_l(s)}, \end{aligned}$$

Lifetime utility can therefore be decomposed into four terms. First, the value of the transfers net of the lifetime migration costs. Second, the present real income these workers have access to. Third, their expected future real income. And third, their preferences for the location they live in. Hence, $\mathcal{V}^y_l(s) = V^A_l(s) + V^T_l(s) + V^I_l(s) + V^{EI}_l(s)$.

This utility decomposition can be used to decompose the welfare impact of the policy.

The consumption-equivalent welfare change can indeed be rewritten

$$\begin{aligned}0 = \mathcal{V}_l^y[s, \zeta_l(s)] - \mathcal{V}_l^{y\star}(s) &= \mathcal{V}_l^y[s, \zeta_l(s)] - \mathcal{V}_l^y(s) + \mathcal{V}_l^y(s) - \mathcal{V}_l^{y\star}(s) \\ &= \mathcal{V}_l^y[s, \zeta_l(s)] - \mathcal{V}_l^y(s) + \left(V_l^A(s) - V_l^{A\star}(s)\right) + \left(V_l^T(s) - V_l^{T\star}(s)\right) + \\ &\quad \left(V_l^I(s) - V_l^{I\star}(s)\right) + \left(V_l^{EI}(s) - V_l^{EI\star}(s)\right),\end{aligned}$$

where the first equality is the definition of $\zeta_l(s)$, the second is an identity, and the third follows from the decomposition I just derived. From there, we can define the four consumption-equivalent welfare metric:

$$\begin{aligned}\mathcal{V}_l^y[s, \zeta_l^A(s)] &= \left(V_l^{A\star}(s) - V_l^A(s)\right) + \mathcal{V}_\ell^y(s), \\ \mathcal{V}_l^y[s, \zeta_l^{A+T}(s)] &= \left(V_l^{A\star}(s) - V_l^A(s)\right) + \left(V_l^{T\star}(s) - V_l^T(s)\right) + \mathcal{V}_\ell^y(s), \\ \mathcal{V}_l^y[s, \zeta_l^{A+T+I}(s)] &= \left(V_l^{A\star}(s) - V_l^A(s)\right) + \left(V_l^{T\star}(s) - V_l^T(s)\right) + \left(V_l^{I\star}(s) - V_l^I(s)\right) + \mathcal{V}_\ell^y(s), \\ \mathcal{V}_l^y[s, \zeta_l(s)] &= \mathcal{V}_l^{y\star}(s).\end{aligned}$$

The term $\zeta_l^A(s)$ measure the (consumption-equivalent) change in welfare coming solely from the idiosyncratic preferences for the locations. The second term $\zeta_l^{A+T}(s)$ adds to the first term the net value of the transfer. The third term $\zeta_l^{A+T+I}(s)$ brings in the value of the present real income opportunities. Finally, $\zeta_l(s)$ adds the expected future lifetime opportunities and corresponds to the total change in welfare. These four metrics are *not* additive but allow to measure the relative importance of each term.

Figure A.30 presents this welfare decomposition separately for treated workers, non-treated workers born outside of Paris, Lyon and Toulouse, and non-treated workers born in those three cities. The welfare impact of the policy on the treated is mainly explained by two offsetting forces. On the one hand, workers born in the treated locations that were choosing to stay there in the baseline equilibrium had on average strong preferences for these cities. On average, the policy is therefore reallocating treated workers to cities they dislike, which

reduces their lifetime utility. On the other hand, the policy improves their future real income opportunities by increasing their lifetime human capital. In net, the human capital channel dominates.[20]

To understand the welfare impact of the policy on the non-treated, recall from Proposition 1.5 that local interactions engender two externalities. First, a pulling externality: the concentration of skilled workers in a few cities requires many workers to forego their idiosyncratic spatial preferences. Second, a teaching externality: in the presence of strong within-skill complementarities, the economy features too few interactions between skilled workers. The voucher policy decreases the density of skilled workers in productive cities, and in doing so, mitigates the pulling externality while amplifying the teaching externality. Holding constant the human capital of non-treated workers, these workers would therefore also gain from the policy. However, in general equilibrium, the policy depresses their future income, which, together with the tax they have to pay to finance the policy, triggers welfare losses.

### A.8.2 Additional Figures

[20]Treated workers also experience welfare gains from the direct income effect of the subsidy.

Figure A.26: Average take-up rate of moving vouchers by birthplace

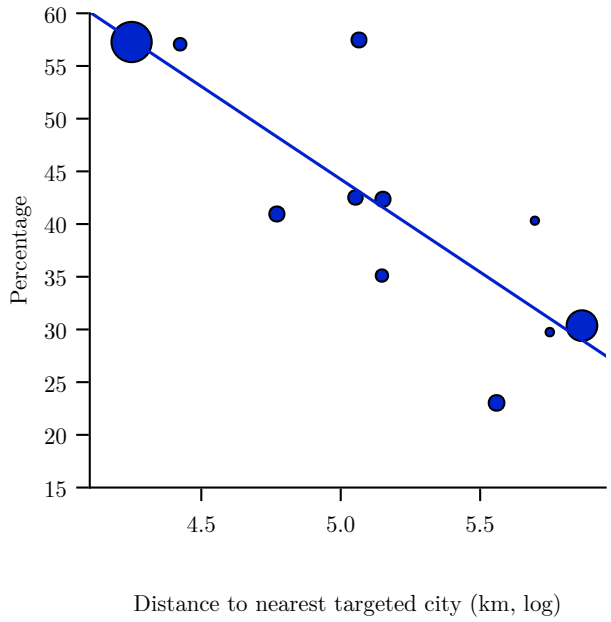



Figure A.27: The role of complementarities in shaping the learning costs of the voucher policy

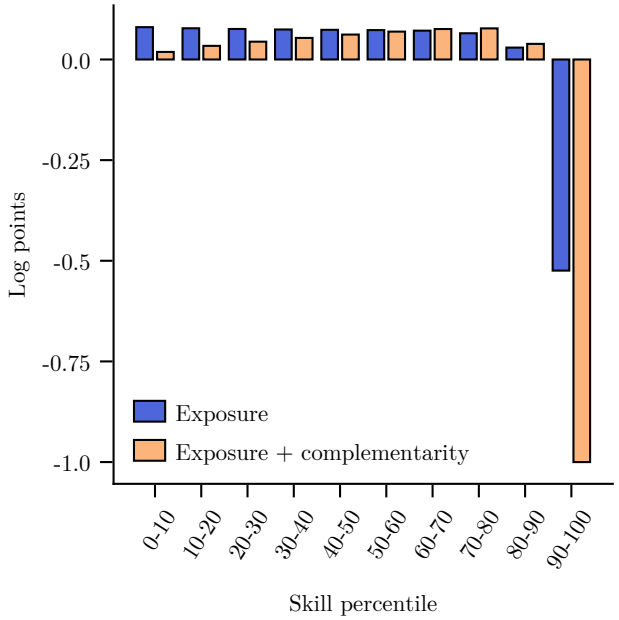


Figure A.28: The consequences of moving vouchers by skill and birthplace

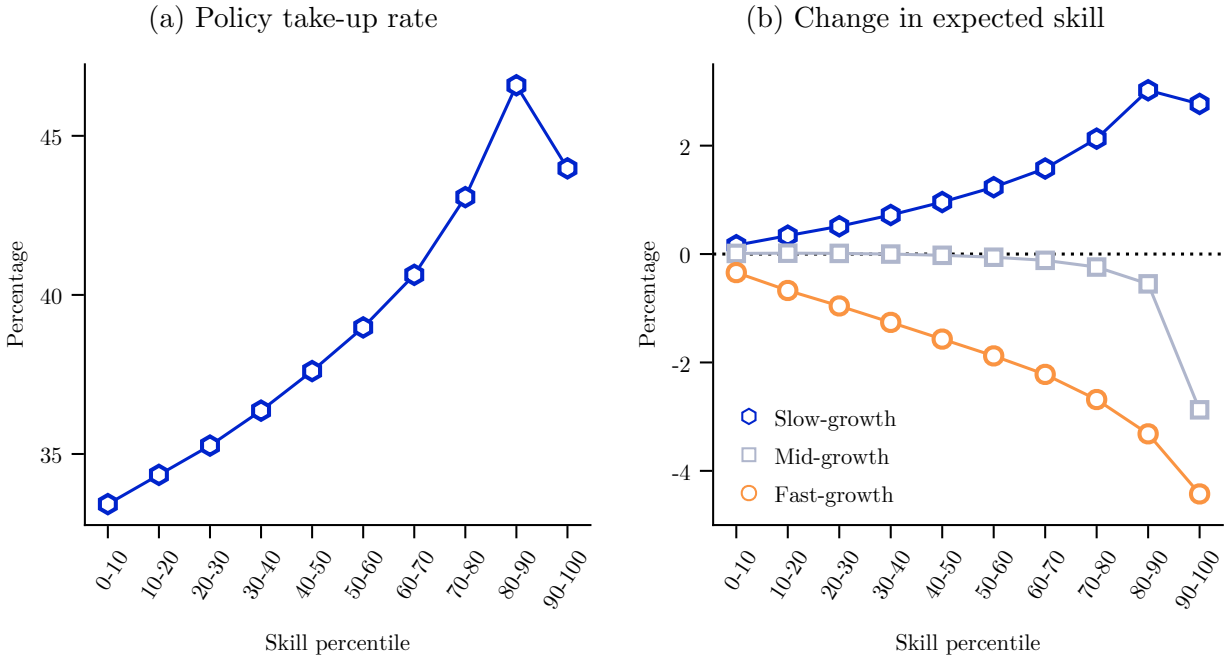



Figure A.29: The consequences of the voucher policy on local interactions

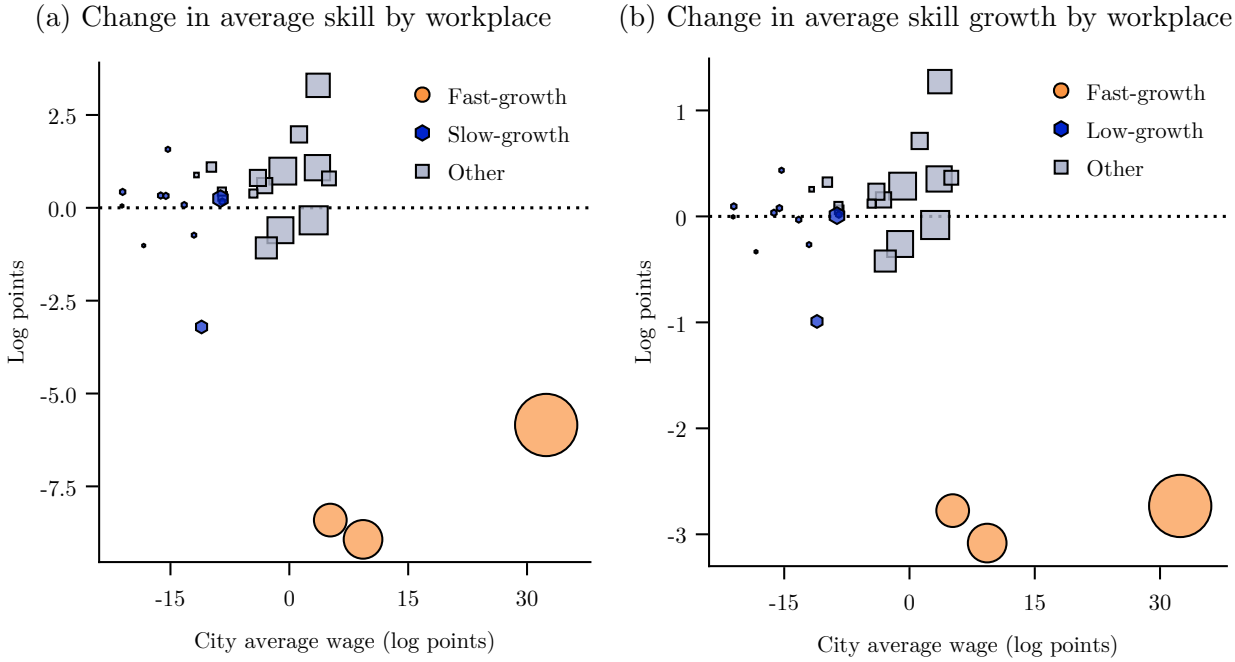


Figure A.30: Welfare decomposition of the moving voucher policy by birthplace

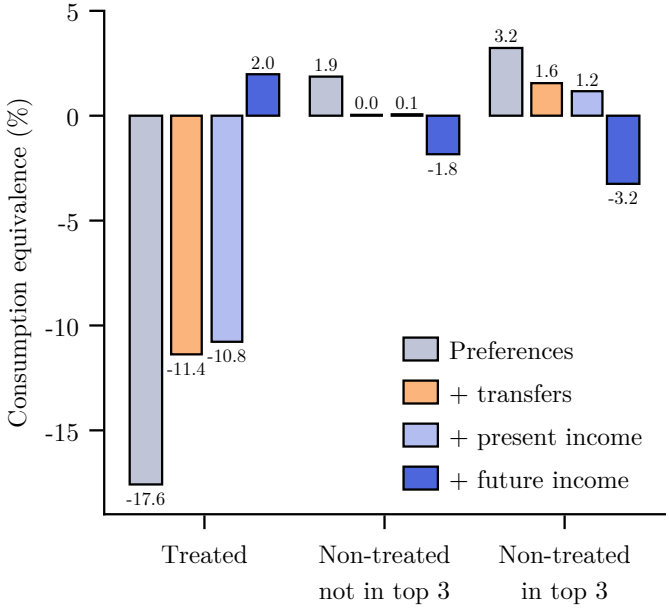
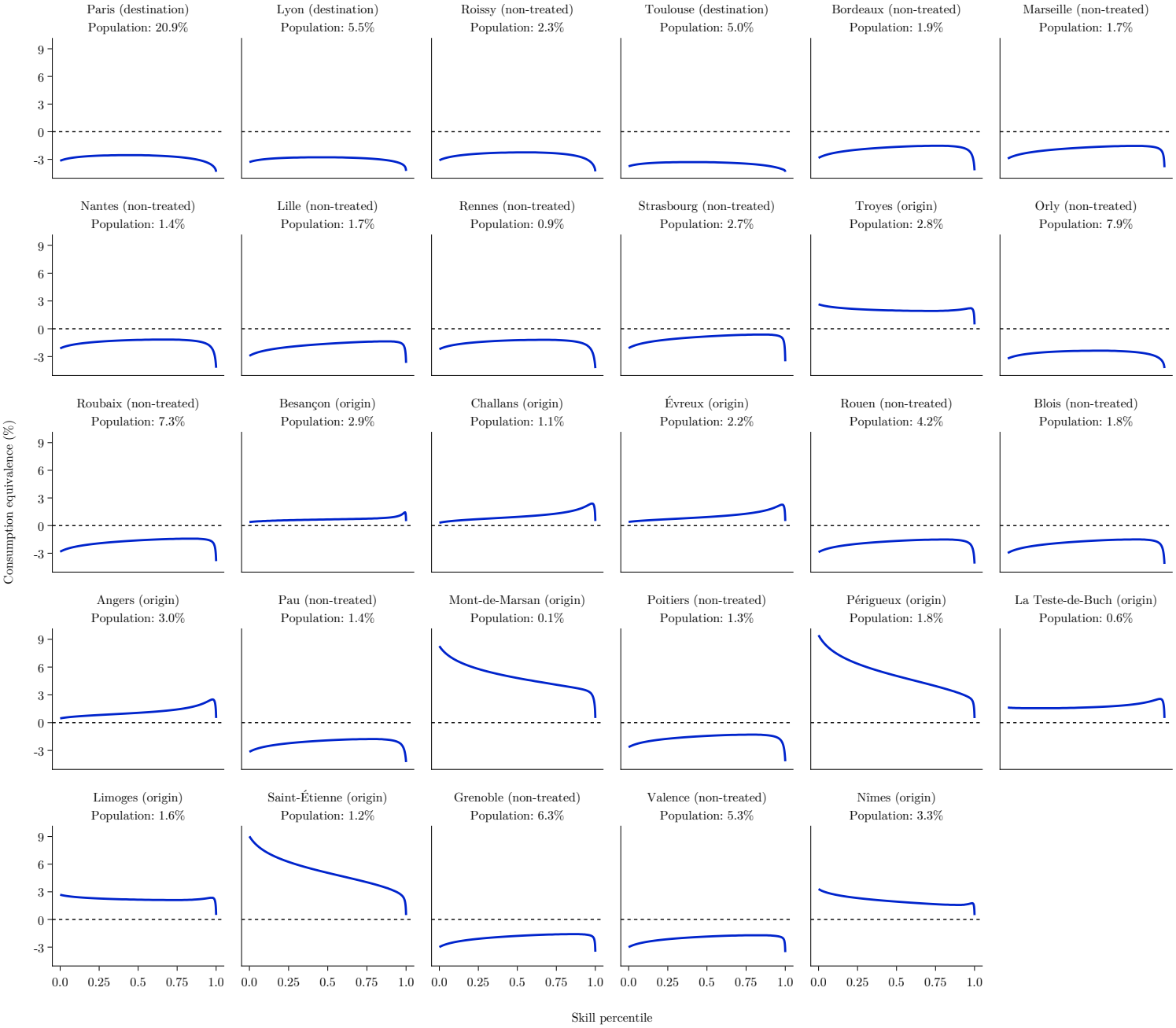


Figure A.31: The consequences of the moving voucher policy across skills and birthplaces



# Appendix B

# Outsourcing, Inequality and Aggregate Output

## B.1 Proofs

### B.1.1 Workers

We relate the wage offer distribution $F_s(w)$ to the wage distribution of employed workers $G_s(w)$ using worker flows. The flow of workers out of any wage interval $[\underline{w}_s, w)$ equals the flow of workers into that wage interval: $\lambda_s^U F_s(w) u_s = \big(\delta_s + \lambda_s^E (1 - F_s(w))\big)(m_s - u_s) G_s(w)$, where $u_s$ denotes the skill-specific unemployment rate. The left-hand-side is the flow of workers out of unemployment into the wage interval $[\underline{w}_s, w)$, while the right-hand-side is the flow of workers out of that wage interval. It consists of workers who exogenously lose their job, and those who transition into higher wages. A similar argument guarantees that $u_s = \frac{m_s \delta_s}{\delta_s + \lambda_s^U}$. Re-arranging delivers (2.1).

$N_s(w)$ is equal to the limit of the ratio $\frac{G_s(w) - G_s(w-\varepsilon)}{F_s(w) - F_s(w-\varepsilon)}$ when $\varepsilon \to 0$, times the number of employed workers $m_s - u_s$. Straightforward differentiation delivers (2.2).

### B.1.2 Reservation wage

Omit $s$ indices. Suppose without loss of generality that $F$ admits a density $f$. Then $\Big[r+\delta+\lambda^E(1-F(w))\Big]V(w) = w+\delta U+\lambda^E\int_w^\infty V(x)f(x)dx$. Differentiate w.r.t. $w$ to obtain $\Big[r+\delta+\lambda^E(1-F(w))\Big]V'(w)=1$. Integrate back to $V(w)=U+\int_{\underline{w}}^{w}\frac{dx}{r+\delta+\lambda^E(1-F(x))}$. Substituting into the value of unemployment, $rU = b+\lambda^U\int_{\underline{w}}^{\infty}\frac{(1-F(x))dx}{r+\delta+\lambda^E(1-F(x))}$. Since $V(\underline{w}) = U$, $(r+\lambda^U)U = b+\lambda^U\int_{\underline{w}}^{\infty}V(x)f(x)dx$ and $(r+\lambda^E)U = \underline{w}+\lambda^E\int_{\underline{w}}^{\infty}V(x)f(x)dx$. Thus, $rU=\frac{\lambda^U\underline{w}-\lambda^E b}{\lambda^U-\lambda^E}$. Therefore,

$$\lambda^U\underline{w} = \lambda^E b+(\lambda^U-\lambda^E)\left[b+\lambda^U\int_{\underline{w}}^{\infty}\frac{(1-F(x))dx}{r+\delta+\lambda^E(1-F(x))}.\right] \tag{B.1}$$

### B.1.3 Proof of Proposition 2.1

Impose Assumption B.1. Then $n_s(w)=\frac{n_{0s}}{[1+k_s(1-F_s(w))]^2}$, and $n_s'(w)=\frac{2kn_{0s}F_s'(w)}{[1+k_s(1-F_s(w))]^3}$. (2.3) becomes

$$\pi(z) = \max_{v_s\in[0,1]^S,w_s} R(z,\{n_s(w)v_s\}_s)-\int w_s n_s(w_s)v_s ds. \tag{B.2}$$

Start from the FOC for wages in (2.3). We obtain $R_{n_s}n_s'-n_s-w_sn_s'=0$.[1] Differentiating the objective in (B.2) w.r.t. $v_s$ and using the FOC for wages, we obtain $\frac{\partial\left(R(z,\{n_s(w)v_s\}_s)-\int w_sn_s(w_s)v_sds\right)}{\partial v_s} = (R_{n_s}-w_s)n_s>0$. Thus, firms are always at the corner $v_s=1$. Hence, (2.3) coincides with

$$\pi(z)=\max_{w_s}\Pi[z,\{w_s\}_s]\equiv R(z,\{n_s(w)\}_s)-\int w_sn_s(w_s)ds. \tag{B.3}$$

Since $n_s$ is increasing in $w$, $\Pi$ is continuously differentiable and strictly supermodular in any pair $(z,w_s)$. In addition, the profit function is supermodular in $\{w_s\}_s$, and exhibits increasing differences in $(z,w_s)$ for all $s$. In addition, the set of $\{w_s\}_s$ forms a lattice with the element-wise order. Therefore, we can apply Theorem 2.8.5. p. 79 in Topkis (1998). Thus,

[1] This equality implies $(R_{n_s}-w_s)n_s'=n_s>0$. Thus, $R_{n_s}>w_s$ and $n_s'>0$.

the set of maximizers $\{w_s(z)\}_s$ are strictly increasing in $z$ for each $s$. Given the ordering of the ordering of wages, $F(w_s(z)) = \Gamma(z)$

### B.1.4 Proof of Proposition 2.2

Given Proposition 2.1, it is immediate to verify that $n_s(z) = \frac{(1+k_s)e_s}{M[1+k_s(1-\Gamma(z))]}$.

### B.1.5 Proof of Proposition 2.3

Because wages are strictly increasing in $z$, they are continuous almost everywhere and we may take first-order conditions for almost every productivity $z$. Hence:

$$\frac{d\big(n_s(w)w\big)}{dw}\Big|_{w=w_s(z)} = \frac{dR(z, \boldsymbol{n}_{-s}(w_s(z)), n_s(w))}{dw}\Big|_{w=w_s(z)} = \frac{\partial R}{\partial n_s}(z, \boldsymbol{n}(z)) \cdot n_s'(w_s(z))$$

where $\boldsymbol{n}_{-s}$ denotes the vector $\boldsymbol{n}$ without its entry $s$. Multiplying both sides by $w_s'(z)$ and changing variables to $n_s(w_s(z)) \equiv n_s(z)$ delivers

$$n_s(z)w_s'(z) = n_s'(z)\big(R_{n_s}(z, \{n_t(z)\}_t) - w_s(z)\big). \tag{B.4}$$

Integrating over $z$ subject to the boundary condition $w_s(\underline{z}) = \underline{w}_s$ delivers the formula in Proposition 2.3.

### B.1.6 Proof of Proposition 2.4

**Existence and uniqueness among equilibria with continuous $F$.** Together, Propositions 2.1, 2.2 and 2.3 suffice to complete a guess and verify strategy to exhibit an equilibrium with a continuous wage offer distribution. The last condition to verify is whether a reservation wage compatible with those results exists. Omitting $s$ subscripts, re-write (B.1) as

$$\lambda^U \underline{w} = \lambda^E b + (\lambda^U - \lambda^E)\left[b + \lambda^U \int_{\underline{w}}^{\infty} \frac{(1-\Gamma(x))w'(x)dx}{r+\delta+\lambda^E(1-\Gamma(x))}\right]. \tag{B.5}$$

$w'(x)$ is a function of the reservation wage $\underline{w}$ through the ODE (B.4). To explicit its dependence, denote $d(z) = \frac{\partial w(z)}{\partial \underline{w}}$ where the partial derivative is understood as a derivative w.r.t. the initial condition of the ODE (B.4). Differentiating (B.4), we obtain $n(z)d'(z) = -n'(z)d(z)$. Solving this ODE explicitly and using $d(\underline{z}) = 1$ by definition, we obtain $d(z) = \frac{n(\underline{z})}{n(z)}$. Hence, $d'(z) = -\frac{n(\underline{z})n'(z)}{n(z)^2} < 0$. Thus, $w'(x)$ is a decreasing function of $\underline{w}$.

Hence, the right-hand-side of (B.5) is a decreasing function of $\underline{w}$ that goes to $\lambda^E b \leq \lambda^U b$ as $\underline{w}$ goes to infinity. Its left-hand-side is an increasing function of $\underline{w}$ that spans $\lambda^U b$ to $+\infty$. Therefore, there exists a unique reservation wage $\underline{w}$.

**Existence and uniqueness among all possible equilibria.** For expositional simplicity and without loss of generality, we present our trembling-hand refinement with a single skill. The maximization problem (2.3) becomes:

$$w(z) = \underset{w,n}{\operatorname{argmax}}\, R(z,n) - wn, \quad n \leq n(w) \equiv \frac{n_0}{[1+k(1-F(w))][1+k(1-F(w^-)]}.$$

Suppose that firms make mistakes $\varepsilon$ after choosing their target wage: firms post $w(z)+\varepsilon$ while having chosen $w(z)$ for an i.i.d. shock $\varepsilon$ across firms. Firm $z$ does not expect to make a mistake, but takes into account the equilibrium wage offer distribution inclusive of other firms' mistakes.

The distribution $F$ that enters the constraint is the vacancy-weighted distribution of posted wages $w+\varepsilon$ in the economy. We impose the following assumptions on the distribution of mistakes $\varepsilon$, $H_\sigma$. $H_\sigma$ has a $\mathcal{C}^\infty$ density with compact support. The variance of $H_\sigma$ (or any relevant measure of dispersion such as the size of its support) is given by $\sigma$. Convergence of $H_\sigma$ is uniform: $|H_\sigma(\varepsilon) - \mathbb{1}\{\varepsilon \geq 0\}| \leq h_0\sigma$, where $h_0$ is a constant independent from $\sigma$. $H_\sigma$ is strictly increasing. Standard results on convolution kernels imply that there exists such a distribution. Our trembling-hand refinement is to consider the limiting economy when $\sigma \downarrow 0$.

**Assumption B.1.** *When $\sigma = 0$, we restrict attention to decentralized equilibria that are the limit of a sequence of decentralized equilibria when $\sigma \downarrow 0$.*

The remainder of this section is structured as follows. First, we show that the wage distribution is smooth when $\sigma > 0$. Second, we show that wages are strictly increasing in $z$ when $\sigma > 0$. Third, we show that the wage rank converges to the productivity rank as $\sigma \to 0$. Fourth, we show that wages converge to our candidate equilibrium when $\sigma \to 0$.

**1. Smooth wage distribution when $\sigma > 0$.** $F$ is a convolution between the distribution of chosen wages $w(z)$ and an i.i.d. shock $\varepsilon$. Therefore, standard results on regularizing convolutions ensure that $F$ admits a $\mathcal{C}^\infty$ density when $\sigma > 0$. This conclusion follows from $F(w) = \int H_\sigma(w-\omega)d\Omega(\omega)$ together with dominated convergence, where $\omega = w(z)$ is a random variable that denotes chosen wages, and $\Omega$ is its c.d.f. In addition, $F$ is strictly increasing: $F'(w) > 0$. Since $F$ is smooth, for any $\sigma > 0$, $n(w) = \frac{n_0}{[1+k(1-F(w))]^2}$ and $n'(w) = \frac{2kn_0F'(w)}{[1+k(1-F(w))]^3}$.

**2. Binding constraint and increasing wages when $\sigma > 0$.** Conditional on $F$ being smooth, the argument is identical to Section B.1.3. Crucially, this conclusion would not be valid in general if there were a mass point in the distribution $F$.

**3. Wage rank and productivity rank when $\sigma \downarrow 0$.** Denote by $w_\sigma(z)$ the wage function for a given $\sigma$. Write $F(w(z_0)) = \mathbb{P}[\varepsilon \leq w(z_0) - w(z)] = \int H_\sigma\big(w(z_0) - w(z)\big)d\Gamma(z)$. Then

$$\int H_\sigma\Big(w_\sigma(z_0) - w_\sigma(z)\Big)d\Gamma(z) = \underbrace{\int \mathbb{1}\Big\{z \leq z_0\Big\}d\Gamma(z)}_{\text{By wage ranking}} + \underbrace{\int \Big[H_\sigma\Big(w_\sigma(z_0) - w_\sigma(z)\Big) - \mathbb{1}\Big\{w_\sigma(z_0) - w_\sigma(z) \geq 0\Big\}\Big]}_{\leq h_0\sigma \text{ by assumption}} d\Gamma(z)$$

Therefore, for all $z$, $F(w(z)) \to \Gamma(z)$ uniformly, and $n(w(z)) \to n(z) \equiv \frac{n_0}{[1+k(1-\Gamma(z))]^2}$ uniformly.

**4. Wages when $\sigma \downarrow 0$.** We go back to the maximization problem (B.2) and use an argument that resembles Berge's maximum theorem that we cannot apply directly. Re-write (B.2) as choosing the wage $w_\sigma(Z)$ of a firm with productivity $Z$. The wage function $w_\sigma$ must hence satisfy $z = \text{argmax}_Z\, R(z, n(w_\sigma(Z))) - w_\sigma(Z)n(w_\sigma(Z))\big]$. In particular, $Z^*(z) = z$ for all $\sigma$. Suppose for a contradiction that $w_\sigma$ was discontinuous in $\sigma$ at $\sigma = 0$ for some $z_0$. Since $n(w_\sigma(Z)) \to n(Z)$, it must be that $Z^*(z)$ jumps down at $\sigma = 0$ since firms downscale due to higher costs of labor. This contradicts $Z^*(z_0) = z_0$. Therefore, $w_\sigma$ is continuous in

$\sigma$ at $\sigma = 0$. At $\sigma = 0$, $w_0$ satisfies $z = \text{argmax}_Z R(z, n(Z)) - w_0(Z)n(Z)$. $w_0$ thus solves $(R_n(z, n(z)) - w_0(z))n'(z) = w_0'(z)n(z)$, which coincides with the wage ODE in our candidate equilibrium. Thus, the limit of any equilibrium under Assumption B.1 as $\sigma \downarrow 0$ converges to the candidate equilibrium.

### B.1.7 Micro-foundations for the cost of outsourcing

**Iceberg trade cost or productivity wedge.** To sell one unit of labor services to a goods producer, contractor firms must hire $1/\tau_s$ units of labor.

**Capital.** Assume that contractor firms for skill $s$ combine capital, in exogenous supply $K_s$, and labor to produce one unit of efficiency unit of labor services of a given skill $s$. The decision problem of the contractor firm is

$$\pi^C(w) = \max_k p_s k^{1-\beta} n_s(w)^\beta - r_s k - w n_s(w). \tag{B.6}$$

The optimality condition for capital is then $k = \left(\frac{(1-\beta)p_s}{r_s}\right)^{\frac{1}{\beta}} \cdot n_s(w)$. Market clearing for capital leads to $\frac{r_s}{1-\beta} = p_s(Q_s^{\text{Out}}/K_s)^\beta$ where $Q_s^{\text{Out}}$ is aggregate employment in contractor firms. Substituting back into (B.6), we obtain $\pi^C(w) = p_s \left(\frac{K_s}{Q_s^{\text{Out}}}\right)^{1-\beta} n_s(w) - w n_s(w)$. Assume further that $K_s = \tau_s^{\frac{1}{1-\beta}}$, and take $\beta \to 1$. Then, (B.6) becomes $\pi^C(w) = (\tau_s p_s - w)n_s(w)$.

### B.1.8 Proof of Proposition 2.5

We start by defining the cost function

$$C_s(n) = \min\left\{w_s(n)n, p_s n\right\} = n \min\left\{w_s(n), p_s\right\},$$

where $w_s(n)$ is the inverse function of $n_s(w)$. Then rewrite the profit-maximization problem (2.5) as

$$\pi(z) = \max_{\{n_s\}_s} R\big(z, \{n_s\}_s\big) - \int C_s(n_s)ds. \tag{B.7}$$

As in Proposition 2.1, the profit function in (B.7) is supermodular in $(z, \{n_s\}_s)$. We again use Theorem 2.8.1. p. 76 in Topkis (1998) to obtain that size is rising in productivity: $n_s^*(z)$ is increasing in $z$ for every $s$. Given that $w_s$ is increasing in $n$ and $n_s^*(z)$ is increasing in $z$, there must exist a threshold $\hat{z}_s$ such that the minimum of the cost function is attained in-house for $z \leq \hat{z}_s$, and attained outsourced for $z > \hat{z}_s$.

### B.1.9 Proof of Proposition 2.6

For clarity of exposition, we first present our proof when there is a single worker type, before moving to the multiple worker types case.

#### Single worker type

Focus on the case with a single worker type in the remainder of this proof and drop the $s$ index. We denote effective employment by $n$, and sometimes write $n_{\text{in}}$ or $n_{\text{out}}$ to denote in-house vs. outsourced employment.

**Proof that $n_{\text{in}}(\hat{z}) < n_{\text{out}}(\hat{z})$.** The theorem of the maximum implies that profits are continuous at the outsourcing cutoff $\hat{z}$. The indifference condition at $\hat{z}$ writes

$$R(\hat{z}, n_{\text{in}}(\hat{z})) - w(n_{\text{in}}(\hat{z}))n_{\text{in}}(\hat{z}) = R(\hat{z}, n_{\text{out}}(\hat{z})) - pn_{\text{out}}(\hat{z}) \tag{B.8}$$

where $w(n)$ is the (increasing) inverse function of the labor supply curve. The first-order condition for in-house employment is $R_n(\hat{z}, n_{\text{in}}(\hat{z})) = w(\hat{z}) + n_{\text{in}}(\hat{z})w'(n_{\text{in}}(\hat{z}))$. For outsourcing

it is $R_n(\hat{z}, n_{\text{out}}(\hat{z})) = p$. Substituting both into the indifference condition (B.8):

$$R(\hat{z}, n_{\text{in}}(\hat{z})) - R_n(\hat{z}, n_{\text{in}}(\hat{z}))n_{\text{in}}(\hat{z}) + n_{\text{in}}(\hat{z})^2 w'(n_{\text{in}}(\hat{z})) = R(\hat{z}, n_{\text{out}}(\hat{z})) - n_{\text{out}}(\hat{z})R_n(\hat{z}, n_{\text{out}}(\hat{z}))$$

$R$ being strictly concave in $n$, the function $R - nR_n$ is strictly increasing in $n$. Since $w'(n(\hat{z})) > 0$, we immediately obtain that $n_{\text{in}}(\hat{z}) > n_{\text{out}}(\hat{z})$.

**Proof that $R(\hat{z}, \boldsymbol{n_{\text{in}}(\hat{z})}) < \boldsymbol{R(\hat{z}, n_{\text{out}}(\hat{z}))}$.** Since effective employment jumps up at the outsourcing cutoff, so does revenue.

### Multiple worker types

We now return to the case with multiple worker types. We consider what happens to employment $n_s$ of a particular skill $s$ when a firm outsources. The overall structure of the proof follows closely that with a single worker type. The only difference is that we must control the difference in labor costs for other worker types $t \neq s$ in the indifference condition.

For notational simplicy, we omit dependence on $\hat{z}$ in this subsection since all function are evaluated at this value. We also denote for instance $R_{\text{in}} \equiv R(\hat{z}, n_{s,\text{in}}(\hat{z}), n_{-s,\text{in}})$ revenues at the outsourcing threshold when skill $s$ is in-house, and employment at all skills $t \neq s$ are chosen optimally given the in-house choice for $s$.

**Proof that $\boldsymbol{n_{s,\text{in}}(\hat{z}) < n_{s,\text{out}}(\hat{z})}$.** The indifference condition between in-house and outsourcing becomes

$$R_{\text{in}} - n_{s,\text{in}}R_{n_s,\text{in}} + (n_s^{\text{in}})^2 w_s'(n_{s,\text{in}}) - C_{-s}^{\text{in}} = R_{\text{out}} - n_{s,\text{out}}R_{n_s,\text{out}} - C_{-s}^{\text{out}} \tag{B.9}$$

where $C_{-s}$ denotes the cost function for skills other than $s$. Our goal is to show that $n_s^{\text{out}} > n_s^{\text{in}}$. We now guess and verify that $n_s^{\text{out}} > n_s^{\text{in}}$.

We seek to show that, under the guess, $C_{-s,\text{out}} - C_{-s,\text{in}} > 0$. Fix a particular $t \neq s$. Write

the F.O.C.s for $n_t$ depending on whether skill $s$ is in-house or outsourced as

$$R_{n_t}(n_{s,\text{in}}, n_{-s,\text{in}}) = C_{t,n_t}(n_{t,\text{in}}) \quad , \quad R_{n_t}(n_{s,\text{out}}, n_{-s,\text{out}}) = C_{t,n_t}(n_{t,\text{out}})$$

Thus, we only need to study how $n_t(x)$ varies with $x$ in the equation indexed by $x$:

$$R_{n_t}(x, n_{-s}(x)) \quad = \quad C_{n_t}(n_t(x)). \tag{B.10}$$

Differentiating (B.10) w.r.t. $x$, we obtain for all $t \neq s$:

$$R_{n_s n_t} + \sum_{k \neq s} R_{n_t n_k} n_k'(x) = C_{n_t n_t} n_t'(x) \quad \Longrightarrow \quad R_{n_s n_t} + \sum_{k \neq s,t} R_{n_t n_k} n_k'(x) = \Big( C_{n_t n_t} - R_{n_t n_t} \Big) n_t'(x).$$

Define the matrix

$$A_{tk} = \begin{cases} C_{n_t n_t} - R_{n_t n_t} > 0 & \text{if} \quad t = k \\ -R_{n_t n_k} < 0 & \text{if} \quad t \neq k \end{cases}$$

where the inequality in the first row follows from the S.O.C. and the inequality in the second row from supermodularity. We thus re-write our problem in vector form

$$A \cdot \{n_\ell'\}_{\ell \neq s} = \{R_{n_s n_\ell}\}_{\ell \neq s} > 0$$

Importantly, $A_{tk}$ is an M-matrix. As a result $A^{-1}$ has positive elements (see e.g. Berman and Plemmons, 1979). We conclude that $n_\ell' > 0$ for $\ell \neq s$, and so

$$n_{-s,\text{out}} > n_{-s,\text{in}} \tag{B.11}$$

Equation (B.11) ensures that size of other skills increase when firms outsource skill $s$. Because

costs rise with size, we obtain that

$$C_{-s}^{\text{out}} - C_{-s}^{\text{in}} > 0$$

We can now go back to the indifference condition (B.9). We thus have guaranteed that

$$(n_s^{\text{in}})^2 w_s'(n_s^{\text{in}}) + C_{-s}^{\text{out}} - C_{-s}^{\text{in}} > 0$$

As in the one-skill case, $R - n_s R_{n_s}$ is increasing because of concavity, and so we verify our guess that $n_{s,\text{out}} > n_{s,\text{in}}$.

**Proof that $\boldsymbol{R(\hat{z}, n_{s,\text{in}}(\hat{z}), n_{-s,\text{in}}(\hat{z})) < R(\hat{z}, n_{s,\text{out}}(\hat{z}), n_{-s,\text{out}}(\hat{z}))}$.** This inequality immediately follows from $n_{s,\text{out}} > n_{s,\text{in}}$ together with $n_{-s,\text{out}} > n_{-s,\text{in}}$ and $R$ being increasing in each argument.

### B.1.10 Proof of Proposition 2.7

We start our proof with the case in which contractors have neutral or low advantage $\tau_s \leq 1$. We then turn to the case of comparative advantage $\tau_s \gg 1$.

**Neutral or comparative disadvantage case $\tau_s \leq 1$**

**Proof that $\boldsymbol{w_s^{\text{cont.}} < w_s(\hat{z}_s)}$.** The F.O.C. for the marginal outsourcer at $\hat{z}_s$ writes

$$p_s \quad = \quad R_{n_s}(\hat{z}_s, n_{s,\text{out}}(\hat{z}_s), \boldsymbol{n}_{-s,\text{out}}(\hat{z}_s)) \tag{B.12}$$

Suppose for a contradiction that $w_{s,\text{in}}(\hat{z})$ is in the support of the wage distribution of contractors. Then the wage F.O.C. holds for goods producer $\hat{z}$ at $w = w_{s,\text{in}}(\hat{z})$. Then, the marginal benefit from a wage change $dw$ for a contractor at $w = w_{s,\text{in}}(\hat{z})$ is, up to $n_s'(w)$,

propotional to

$$\begin{aligned}\left(p_s\tau_s - w - \frac{n_s(w)}{n_s'(w)}\right)\Big|_{w_s=w_{s,\text{in}}(\hat{z})}dw &\leq \left(p_s - w - \tfrac{n_s(w)}{n_s'(w)}\right)\Big|_{w=w_{\text{in}}(\hat{z})}dw \qquad \text{(B.13)}\\ &= \left(R_{n_s}(\hat{z}_s, n_{s,\text{out}}(\hat{z}), \boldsymbol{n}_{-s,\text{out}}(\hat{z})) - w - \tfrac{n_s(w)}{n_s'(w)}\right)\Big|_{w=w_{s,\text{in}}(\hat{z})}dw\\ &< \left(R_{n_s}(\hat{z}_s, n_{s,\text{in}}(\hat{z}), \boldsymbol{n}_{-s,\text{out}}(\hat{z})) - w - \tfrac{n_s(w)}{n_s'(w)}\right)\Big|_{w=w_{s,\text{in}}(\hat{z})}dw\end{aligned}$$

where the first inequality follows from $\tau_s \leq 1$, the second uses (B.12), and the third relies on $n_{s,\text{in}}(\hat{z}) < n_{s,\text{out}}(\hat{z})$.

Our last step is to bound the marginal product $R_{n_s}(\hat{z}_s, n_{s,\text{in}}(\hat{z}), \boldsymbol{n}_{-s,\text{out}}(\hat{z}))$ of a firm that chooses to hire skill $s$ in-house but holding its employment fixed at the optimal size when skill $s$ is outsourced. To do so, we define a modified profit function

$$P(n; m_{-s}) = R(\hat{z}_s, n, m_{-s}) - w_s(n)n - C_{-s}(m_{-s})$$

where we hold productivity fixed at $\hat{z}$ and $m_{-s}$ fixed too. When evaluated at $m_{-s} = n_{-s,\text{out}}$, this modified profit function corresponds to the firm considering achieving the size $n_{s,\text{out}}$ and modified outsourced profits, but by hiring skill $s$ in-house. When evaluated at $m_{-s} = n_{s,\text{in}}$, this profit function is simply the fully in-house profit function.

Supermodularity implies that $\operatorname{argmax}_n P(n, m_{-s})$ is increasing in $m_{-s}$. Since Proposition 2.6 implies $n_{-s,\text{out}} > n_{-s,\text{in}}$, we obtain

$$\operatorname*{argmax}_n P(n, n_{-s,\text{out}}) > \operatorname*{argmax}_n P(n, n_{-s,\text{in}}) = n_{s,\text{in}}$$

As a result, $n_{s,\text{in}}$ lies to the right of the maximizer of $n \mapsto P(n, n_{-s,\text{out}})$, and so $n \mapsto P(n, n_{-s,\text{out}})$ must be decreasing when evaluated at $n = n_{s,\text{in}}$:

$$P_n(n_{s,\text{in}}, n_{-s,\text{out}}) < 0$$

This last condition rewrites

$$R_n(n_{s,\text{in}}, n_{-s,\text{out}}) < \frac{\partial}{\partial n}\left(w_s(n)n\right)\Big|_{n=n_{s,\text{in}}} \tag{B.14}$$

Substituting (B.14) into the last line of (B.13), we finally obtain

$$\left(p_s\tau_s - w - \frac{n_s(w)}{n_s'(w)}\right)\Big|_{w_s=w_{s,\text{in}}(\hat{z})} dw < 0 \tag{B.15}$$

Thus, contractors who consider posting wage $w_{s,\text{in}}(\hat{z})$ prefer to lower their wage offer. This observation implies that contractors post wages $w_s^{\text{cont.}}$ strictly below $w_{s,\text{in}}(\hat{z})$.

**Proof that $w_s^{\text{cont.}} \equiv \underline{w}_s$ under free-entry.** Impose free-entry. If contractors posted any wage $w > \underline{w}_s$, they could deviate to $w - \varepsilon$ for a small $\varepsilon$ and make positive profits, a contradiction. Thus, they all post the reservation wage.

**Comparative advantage case $\tau_s \gg 1$**

We start our proof under the assumption of a a single worker type and an isoelastic revenue function. Then we relax the assumption of isoelastic revenues.

**Isoelastic revenue function.** Suppose that $R(z,n) = zn^\rho$ with $0 < \rho < 1$. Market clearing for labor services writes

$$M\int_{\hat{z}}^{\overline{z}} \left(\frac{\rho z}{p}\right)^{\frac{1}{1-\rho}} d\Gamma(z) = \tau \times S \int n_{\text{out}}(w) dF(w)$$

Since in-house employment is always bounded below and above, $S\int n_{\text{out}}(w)dF(w) \equiv N_{\text{out}}$ remains bounded. Hence, to a leading order when $\tau \to \infty$, market clearing implies

$$\int_{\hat{z}}^{\overline{z}} (\rho z)^{\frac{1}{1-\rho}}\, d\Gamma(z) \sim \tau p^{\frac{1}{1-\rho}}$$

$\hat{z}$ is defined by the indifference condition $R - nR_n + n^2 w'(n)\big|_{\text{in}} = R - nR_n\big|_{\text{out}}$. In the isoelastic case it implies $c_1 \hat{z}^{\frac{1}{1-\rho}} p^{-\frac{\rho}{1-\rho}} \gtrsim c_2 \hat{z} + c_3$ for bounded and non-vanishing functions of $\tau$, $c_i(\tau)$. So if $p \to 0$, this identity implies $\hat{z} \to \underline{z}$, all firms outsource, and the indifference condition ceases to hold.

Now we are ready for a guess and verify. We guess that, as $\tau \to +\infty$, $p \to 0$. Then $\hat{z}$ hits $\underline{z}$. The integral in market clearing then becomes constant, and market clearing implies

$$1 \sim \tau p^{\frac{1}{1-\rho}} \quad \Longrightarrow \quad p \sim \tau^{\rho-1} \to 0$$

and the guess is verified. This then proves that $\tau p \sim \tau^{\rho} \to +\infty$: the marginal product of labor of contractors becomes infinite. A similar wage deviation argument to the one in Section B.1.10 then ensures that goods producers post wages below contractors.

**Relaxing isoelastic revenue function.** As long as $0 < \underline{\rho} \leq \frac{nR_n}{R} \leq \overline{\rho} < 1$, the previous arguments continue to apply since they are all estimates that only rely on $0 < \rho < 1$.

### B.1.11 Proof of Proposition 2.8

The hire rate from employment is increasing in $w$, while churn is decreasing in $w$. The comparative statics in Proposition 2.8 for these outcomes immediately follow from Proposition 2.7.

To show that net poaching is increasing in $w$ as well, combine equations (2.1) and (2.2) to obtain $\frac{G_s(w)}{n_s(w)} = \frac{\widehat{M}_s}{(1+k_s)e_s} F_s(w)(1 + k_s(1 - F_s(w)))$. From Online Appendix B.6.1, $q_s = \frac{e_s \delta_s (1+k_s)}{\widehat{M}_s}$. Hence $\text{NP}_s(w) = (1-\phi_s)F_s(w)(\delta_s + \lambda_s^E(1 - F_s(w))) - \lambda_s^E(1 - F_s(w)) = (1-\phi_s)\delta_s F_s(w) - \lambda_s^E(1 - F_s(w))\big[1 - (1-\phi_s)F_s(w)\big]$. Viewed as a function of $F_s(w)$, this quantity is weakly increasing in $F_s(w)$ if the second component $F \mapsto (1-F)\big[1-(1-\phi)F\big]$ is decreasing. This component is a second-order polynomial with roots equal to 1 and $1/(1-\phi) > 1$, with a positive coefficient on the quadratic term. Hence, it is decreasing on $[0, 1]$. Hence, net poaching is increasing in $w$. We conclude the proof once more with

Proposition 2.7.

### B.1.12 Outsourcing equilibrium

With some outsourcing in equilibrium and in the weakly neutral case $\tau_s \leq 1$, the wage distribution for a given skill $s$ has three regions. In the first, low wage region, only unproductive goods producers operate. In an intermediate wage region, contractors operate together with mid-productivity goods producers. In a high-wage region, only highly productive goods producers operate. Depending on parameter values, either the low-wage or the high-wage region may be empty. For the sake of brevity, we omit a lengthy distinction of all cases and describe the economy with all regions populated in the weakly neutral case $\tau_s \leq 1$. We denote by $\widehat{M}_s = M^C + M^G\Gamma(\hat{z}_s)$ the total measure of firms that hire skill $s$ in-house.

Denote by $z_{1s}$ the threshold productivity at which the low wage region ends, and $z_{2s}$ the threshold productivity at which the high wage region starts. Goods producers $z \in [\underline{z}, z_{1s}]$ behave similarly to the no-outsourcing economy. These goods producers are now poached by both contractors and high-productivity in-house firms. Their equilibrium rank in the job ladder and equilbrium size are

$$\Upsilon_{1s}(z) = \frac{M^G\Gamma(z)}{\widehat{M}_s}, \quad n_{1s}(z) = \frac{n_{0s}}{\widehat{M}_s[1 + k_s(1 - \Upsilon_{1s}(z))]^2}, \quad z \in [\underline{z}, z_{1s}]. \tag{B.16}$$

where $n_{0s} = (1 + k_s)e_s$. For goods producers in this low wage region, the only change to their wages relative to Proposition 2.3 stems from the number of workers they attract and retain with a wage offer given in equations (B.16). At the upper end of this low wage region, the marginal product of labor of goods producers equals that of the first contractor firm which determines the threshold $z_{1s}$: $R_{n_s}(z_{1s}, \boldsymbol{n}(z_{1s})) = p_s$.

In the intermediate wage region, contractors compete with in-house goods producers. Contractors being homogeneous, there are indifferent between paying any wage in this intermediate region. There, rank in the job ladder and size at any wage are directly

determined by contractors' indifference condition:

$$F_{2s}(w) = 1 + \frac{1}{k_s}\left[1 - \left(\frac{n_{0s}}{\widehat{M}_s n_{2s}(w)}\right)^{\frac{1}{2}}\right], \quad n_{2s}(w) = \frac{p_s - w(z_{1s})}{p - w} n_{1s}(z_{1s}), \tag{B.17}$$

for $z \in [z_{1s}, z_{2s}]$. Throughout this intermediate region, wages of goods producers keep rising with productivity so that the marginal product of labor of goods producers equals that of contractors: $R_{n_s}(z, n_{2s}(w_s(z)), n^*_{-s}(z)) = p_s$. The threshold productivity $z_{2s}$ is reached when there are no more contractors left $F_{2s}(w_s(z_{2s})) = M^C + M^G\Gamma(z_{2s})$.

The economy then enters the third, high wage region with only highly productive goods producers. This region resembles the low wage region in that size and rank are given by

$$\Upsilon_{3s}(z) = \frac{M^C + M^G\Gamma(z)}{\widehat{M}_s}, \quad n_{3s}(z) = \frac{n_{0s}}{\widehat{M}_s[1 + k_s(1 - \Upsilon_{3s}(z))]^2}, \quad z \in [z_{2s}, \hat{z}_s]. \tag{B.18}$$

Starting from $w_s(z_{2s})$, wages once again follow Proposition 2.3 but with an equilibrium size given in equations (B.18). Above the threshold productivity $\hat{z}_s$, no firm operates in-house. There, firms outsource their employment.

### B.1.13 Proof of Corollary 2.9

Write the problem of the firm as

$$\pi(z, \{\varepsilon_s\}_s) = \max_{n_s} R(z, \{n_s\}) - \sum_s C_s(n_s, \varepsilon_s) \quad , \quad C_s(n, \varepsilon) = n \min\{w_s(n), p_s\varepsilon\}$$

The profit function has the same supermodularity properties as before, but is also supermodular in $(n_s, 1/\varepsilon_s)$. Hence, optimal size $n^*_s(z, \varepsilon_s, \{\varepsilon_{s'}\}_{s' \neq s})$ is increasing in $z$ and weakly decreasing in $\varepsilon_s$.

Evaluating the cost function $C_s$ at optimal size $n^*_s$, one immediatley obtains that there is a threshold productivity $\hat{z}_s(\{\varepsilon_{s'}\}_{s'})$ at which the firm switches between both parts of the cost function, when $p_s \leq w_s(n^*_s(\hat{z}_s(\{\varepsilon_{s'}\}_{s'}), \{\varepsilon_{s'}\}_{s'}))/\varepsilon_s$. The right-hand-side is decreasing in $\varepsilon_s$,

and so the threshold $\hat{z}_s(\{\varepsilon_{s'}\}_{s'})$ is increasing in $\varepsilon_s$.

### B.1.14 Proof of Corollary 2.10

The proof follows exactly that of Proposition 2.6 once we condition on the idiosyncratic shock $\varepsilon_s$.

### B.1.15 Proof of Corollary 2.11

The proof follows exactly that of Proposition 2.7 once we condition on the shock $\varepsilon_s$ and use $\varepsilon_s \leq 1$ in the series of inequalities (B.13). The proof for the comparative advantage case also remains unchanged because market clearing for outsourcing becomes

$$M^G \int d\Gamma(\boldsymbol{\varepsilon}) \int_{\hat{z}_s(\boldsymbol{\varepsilon})}^{\overline{z}} n_s^*(z, \boldsymbol{\varepsilon}) d\Gamma(z|\boldsymbol{\varepsilon}) = \tau_s M^C \int n_s^{\text{cont.}}(w) dF_s(w)$$

and thus leaves the proof in Appendix B.1.10 unchanged.

### B.1.16 Proof that the outsourcing share rises with $1/\varepsilon_s$

Consider the setup from Section 2.5, and in particular the revenue function (2.18). The outsourcing share out of revenues conditional on outsourcing is $S(z, \boldsymbol{\varepsilon}) = \rho a_1$. The outsourcing share conditional on not outsourcing is 0. $\hat{z}_s(\boldsymbol{\varepsilon})$ is increasing in $\varepsilon_1$ (Corollary 2.9), so that the outsourcing share is weakly increasing in $1/\varepsilon_1$, and strictly increasing when $\hat{z}_s(\boldsymbol{\varepsilon})$ crosses $z$. Hence, $\mathbb{E}\left[\frac{\partial S(z,\boldsymbol{\varepsilon})}{\partial(1/\varepsilon_1)}\right] > 0$.

### B.1.17 Welfare and expected earnings

**Welfare.** The value function of a worker with state $x \in \{b, w\}$, where $b$ denotes unemployment, satisfies $V(x) = r\mathbb{E}_0 \int_0^\infty e^{-rt} x_t dt$. We rescale values by the discount rate $r$ as we require values to remain finite in the limit $r \to 0$. We show that, when $r \to 0$, the value of any worker, regardless of their state, converges to steady-state expected earnings $\mathbb{E}[x]$

when the process $x_t$ has a unique invariant distribution. Denote by $h(t,x)$ the solution to the time-dependent Kolmogorov Forward equation satisfied by the density of $x_t$. Then $V(x) = \int xr \int_0^\infty e^{-rt} h(t,x) d\mu(x)$ where $\mu$ is a base measure that has a Dirac mass point at $b$ and is the Lebesgue measure for all $x \geq \underline{w}$. If we can show that $r \int_0^\infty e^{-rt} u(t) dt \to \lim_{t\to\infty} u(t)$ for any smooth and bounded function $u$, we can apply this last result $x$ by $x$ and obtain $V(x) \to \int xh(\infty, x)dx \equiv \mathbb{E}[x]$. To show $r \int_0^\infty e^{-rt} u(t) dt \to \lim_{t\to\infty} u(t)$ for any smooth function $u$, change variables $\tau = rt$: $r \int_0^\infty e^{-rt} u(t) dt = \int_0^\infty e^{-\tau} u(\tau/r) d\tau$. $u(\tau/r) \to u(\infty)$ for all $\tau > 0$. We conclude the proof by dominated convergence.

**Expected earnings decomposition.** Let $I$ denote expected earnings of a given skill group. Omit skill subscripts for simplicity. Standard accounting ensures that

$$I = e^G w^G + e^C w^C + ub$$

where $e^G, e^C$ denote the employment rates of goods producers and contractors. $w^G, w^C$ denote the employment-weighted average wages paid by goods producers and contractors. Denote $\overline{w} = \frac{e^G w^G + e^C w^C}{e^G + e^C}$ the average wage in the economy, and $\overline{e} = e^G + e^C = 1 - u$ the employment rate. For any outcome $X$, we denote by $X$ its value in a baseline equilibrium, and $X'$ its value in the counterfactual equilibrium. Denote $\Delta X = X' - X$. Then the change in earnings between two equilibria of the model is

$$\Delta I = \underbrace{(\overline{w} - b)\Delta\overline{e}}_{\text{Employment}} + \underbrace{\left(\frac{(e^G)'}{\overline{e}'}\Delta w^G + \frac{(e^C)'}{\overline{e}'}\Delta w^C\right)}_{\text{Wage}} + \underbrace{\left(w^C - w^G\right)\Delta\left(\frac{e^C}{\overline{e}}\right)}_{\text{Wage penalty}}. \tag{B.19}$$

## B.2 Reduced-form results

### B.2.1 Selection into outsourcing

Table B.1: Summary statistics

*Panel A: Outsourcers vs. non-outsourcers*

| | Non-outsourcers | | Outsourcers | |
|---|---|---|---|---|
| | Mean | Median | Mean | Median |
| Employment (full-time eq.) | 65 | 32 | 180 | 50 |
| Sales (k) | 11,133 | 3,543 | 42,149 | 8,189 |
| Value added (k€) | 2,979 | 1,270 | 11,696 | 2,454 |
| Observations | | 91,608 | | 81,939 |

*Panel B: Outsourcing by industry*

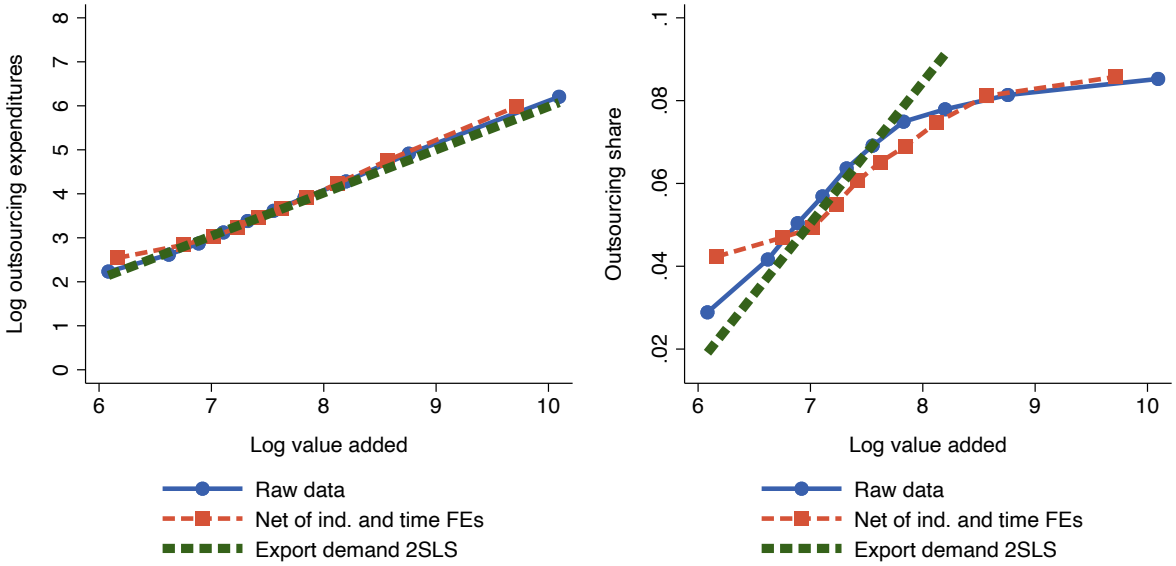
| Industry | Rank | Outsourcing share |
|---|---|---|
| Business supplies & equipment trade | 1 | 0.88 |
| Telecommunications | 2 | 0.49 |
| Terracotta manufacturing | 607 | 0.00 |
| Transport into space | 608 | 0.00 |

Data aggregated to three periods 1997-1999, 2000-2002, 2003-2007. Sample restricted to firms with at least 10 employees. Panel A: an outsourcer in a period is a firm that has positive expenditures on external workers in all years within the period.

Figure B.1: Outsourcing by value added.

(a) Outsourcing expenditures

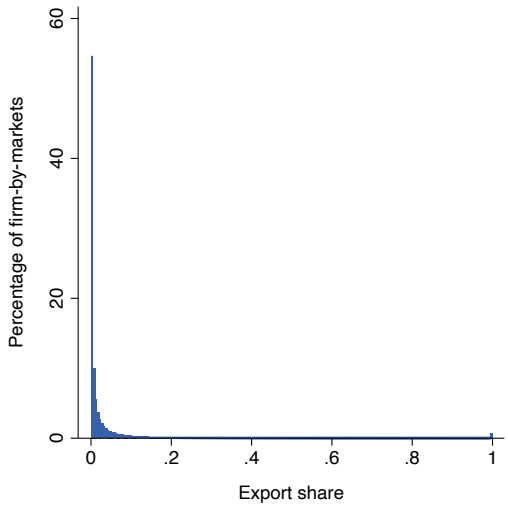
(b) Outsourcing share



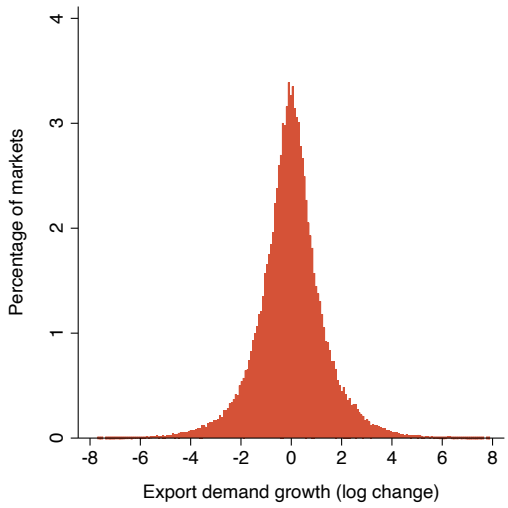

Note: Solid blue line : raw data. Dashed orange line: after removing 3-digit industry and time period fixed effects from the outsourcing share and log value added. Green line: 2SLS estimate using the export demand shift-share instrument in equation (2.14). Panel (a): log outsourcing expenditures. Panel (b): outsourcing share.

Figure B.2: Components of export demand instrumental variable.

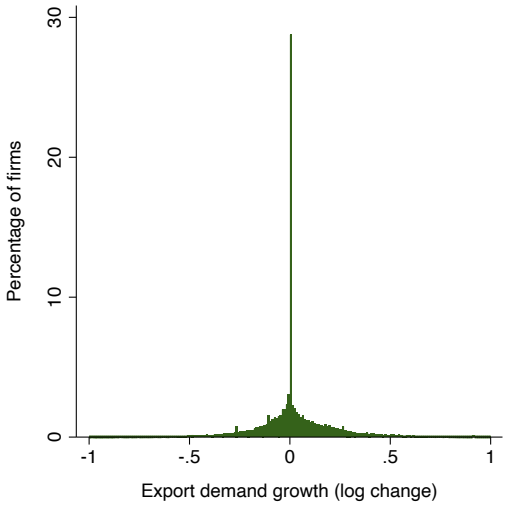
(a) Distribution of export shares by firm-market.



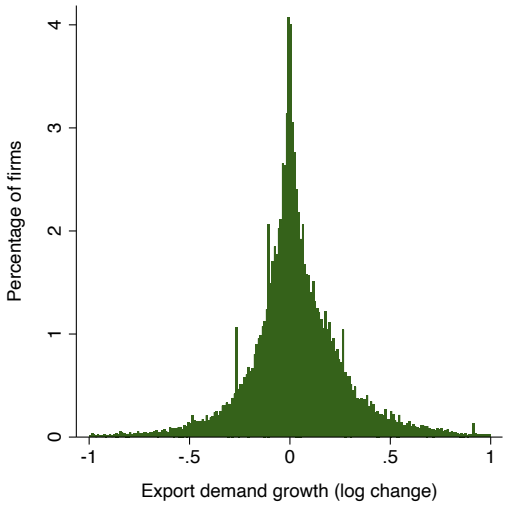

(b) Distribution of export demand by market.




(c) Distribution of IV




(d) Distribution of IV without zeros




Note: Distributions of components of the instrumental variable $Z_{ft}$ defined in equation (2.14). Panel (a) shows the distribution of the firm-by-market export shares $\pi_{f,t_0,j}$. Panel (b) shows that distribution of changes in export demand $\Delta \log X_{j,t,-f}$. Panel (c) shows the distribution of the IV $Z_{ft}$. Panel (d) shows the same distribution after removing zeros.

Table B.2: Dependent variable: log spending on external workers.

| | All | | | Exporters | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) OLS | (2) OLS | (3) OLS | (4) OLS | (5) 2SLS | (6) 2SLS | (7) 2SLS | (8) 2SLS |
| Log V.A. + Out. | 1.07*** | 1.11*** | 0.83*** | 0.81*** | 0.99*** | | | |
| | (0.01) | (0.01) | (0.02) | (0.02) | (0.21) | | | |
| Log V.A. | | | | | | 0.98*** | | |
| | | | | | | (0.22) | | |
| Log Size | | | | | | | 1.66*** | |
| | | | | | | | (0.34) | |
| Log Labor Prod. | | | | | | | | 2.41*** |
| | | | | | | | | (0.67) |
| *Fixed Effects* | | | | | | | | |
| Year | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Industry | | ✓ | | | | | | |
| Firm | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Obs. | 131734 | 131727 | 125356 | 38968 | 38968 | 38935 | 39272 | 38935 |
| Stand. coef. | 0.13 | 0.13 | 0.10 | 0.10 | 0.12 | 0.12 | 0.20 | 0.29 |
| $1^{st}$-stage F-stat. | . | . | . | . | 283.81 | 281.97 | 176.08 | 88.65 |

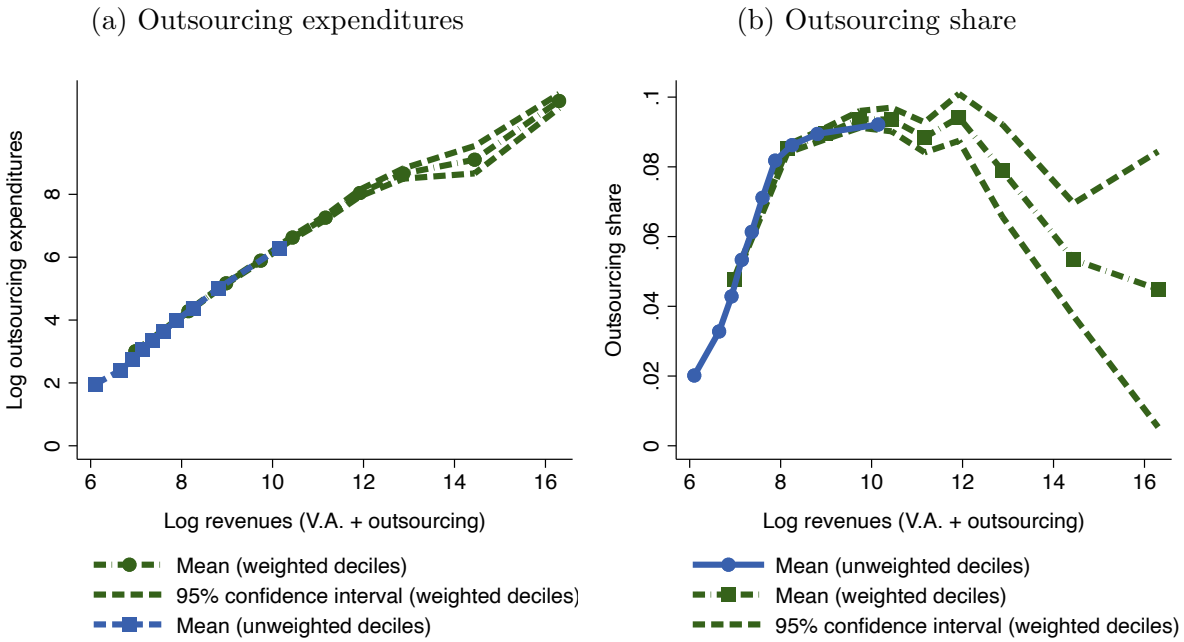
Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. Dependent variable: log spending on external workers. First independent variable: log of the sum of value added and expenditures on external workers. Instrument: shift-share of export demand growth by 4-digit industry, projected by firm using firm-level export shares in first period. All regressions at firm-period level and unweighted.

Table B.3: Dependent variable: spending on external workers as a fraction of labor costs, in p.p.

| | All | | | Exporters | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) OLS | (2) OLS | (3) OLS | (4) OLS | (5) 2SLS | (6) 2SLS | (7) 2SLS | (8) 2SLS |
| Log V.A. + Out. | 1.73*** | 1.82*** | 2.63*** | 1.85*** | 3.34** | | | |
| | (0.03) | (0.03) | (0.08) | (0.12) | (1.07) | | | |
| Log V.A. | | | | | | 3.41** | | |
| | | | | | | (1.09) | | |
| Log Size | | | | | | | 6.04*** | |
| | | | | | | | (1.82) | |
| Log Labor Prod. | | | | | | | | 8.48** |
| | | | | | | | | (2.98) |
| *Fixed Effects* | | | | | | | | |
| Year | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Industry | | ✓ | | | | | | |
| Firm | | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Obs. | 172490 | 172483 | 172350 | 45798 | 45798 | 45766 | 46152 | 45766 |
| Stand. coef. | 0.21 | 0.22 | 0.32 | 0.22 | 0.40 | 0.41 | 0.73 | 1.03 |
| $1^{st}$-stage F-stat. | . | . | . | . | 289.56 | 287.65 | 185.67 | 83.57 |

Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. Dependent variable: spending on external workers as a fraction of labor costs, in p.p. First independent variable: log of the sum of value added and expenditures on external workers. Instrument: shift-share of export demand growth by 4-digit industry, projected by firm using firm-level export shares in first period. All regressions at firm-period level and unweighted.

Figure B.3: Outsourcing by revenues.




Note: Solid blue line : deciles based on number of firms. Dotted green line: deciles based on revenue-weighted quantiles. Dashed green lines: 95% confidence interval for revenue-weighted decile means. Panel (a): log outsourcing expenditures. Panel (b): outsourcing share.

## B.2.2 The productivity effect

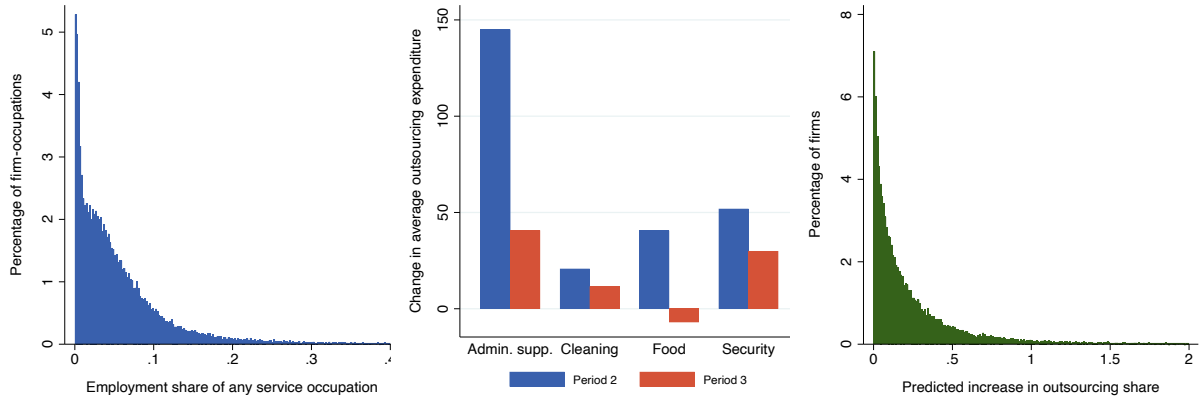
Table B.4: The productivity effect of outsourcing.

| | First stage | Reduced form | 2SLS |
|---|---|---|---|
| | $\Delta$ out. share | $\Delta$ log VA | $\Delta$ log VA |
| Change in out. IV | 0.245*** | 0.019*** | |
| | (0.050) | (0.003) | |
| Change in out. share | | | 0.079*** |
| | | | (0.020) |
| Obs. | 46677 | 46677 | 46677 |
| $1^{st}$-stage F-stat. | | | 23.757 |

Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. Variables winsorized at 5% level. Changes between only two periods. Instrument: shift-share of outsourcing expenditures growth by occupation, projected by firm using firm-level occupation shares in first period. Occupation codes are different in the first period and so only the second and third period can be used. Regression run in changes, leading to the lower number of observations relative to Table B.3.

Figure B.4: Components of outsourcing share instrumental variable.

(a) Payroll shares. (b) Change in expenditures. (c) IV




Note: Distributions of components of the instrumental variable $Z'_{ft}$ for the firm-level outsourcing share. Panel (a) shows the distribution of the firm-by-occupation wage shares shares $\omega_{f,t_0,o}$ for service occupations. Panel (b) shows changes in average outsourcing expenditures by occupation $\Delta\Omega_{o,t,-f}$. Panel (c) shows the distribution of the IV $Z'_{ft}$. Support for Panel (c) is restricted to positive values for graphical purposes, the fraction with negative values being negligible.

### B.2.3 The distributional effect

Table B.5: Firm size wage premium in France.

| | | | | |
|---|---|---|---|---|
| Log Firm In-house Employment | 0.032*** | 0.021*** | | |
| | (0.004) | (0.005) | | |
| Log Firm Value Added | | | 0.023*** | |
| | | | (0.004) | |
| Log Firm Mean Wage | | | | 0.036* |
| | | | | (0.018) |
| Year & 3-digit Industry Fixed Effects | ✓ | ✓ | ✓ | ✓ |
| Worker Fixed Effects | | ✓ | ✓ | ✓ |
| Obs. | 96697 | 94316 | 94316 | 94316 |

Note: Dependent variable: log worker daily wage. Standard errors in parenthesis, clustered by 3-digit industry. $^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. Regression for service workers only, defined as in Section 2.4.2. In-house firm employment, value added and mean wage computed from firm-level data. Regression equation: $\log w_{i,t} = \varphi_i + \psi_{I(i,t)} + \beta X_{J(i,t)} + \eta_{i,t}$. $i$ indexes workers, $t$ indexes year-quarters. $\varphi_i$ is a worker fixed effect. $\psi_{I(i,t)}$ is a fixed effect for the workers' employer's 3-digit industry $I(i,t)$. $J(i,t)$ denotes the worker's employer. $X$ denotes either log employment, log value added or log mean wage.

### B.2.4 Alternative explanations

Figure B.5: Outsourcing by industry volatility.

(a) Volatility by value added and industry. (b) Outsourcing by value added and industry.

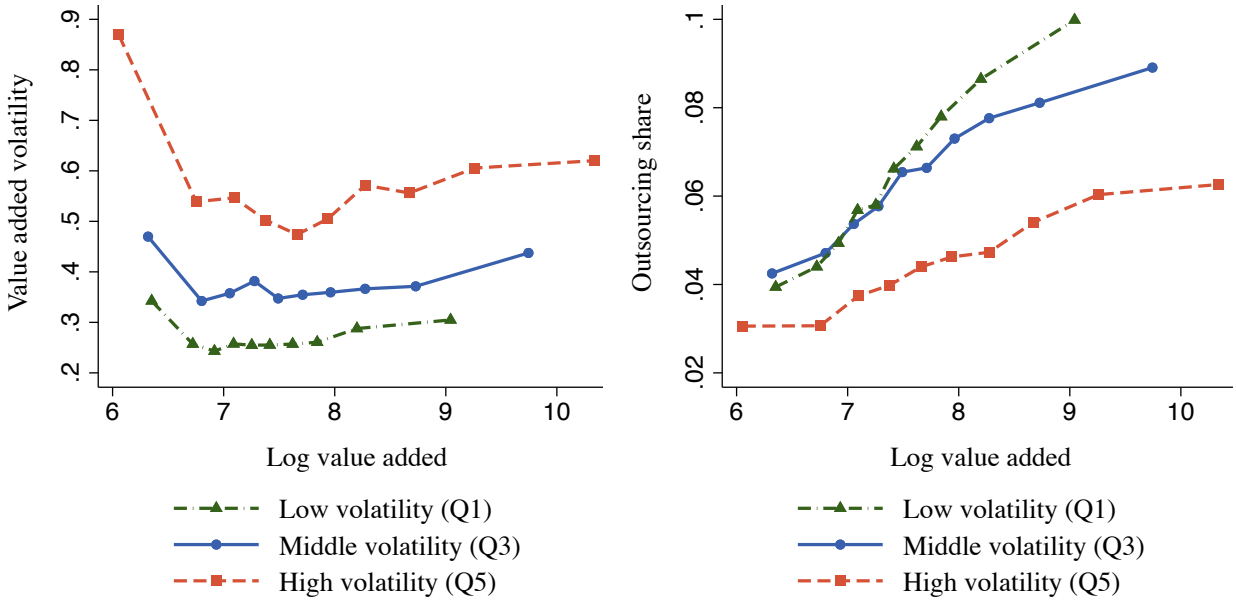



Figure B.6: Size-dependent policies and outsourcing.

(a) Firm size distribution (b) Outsourcing by size

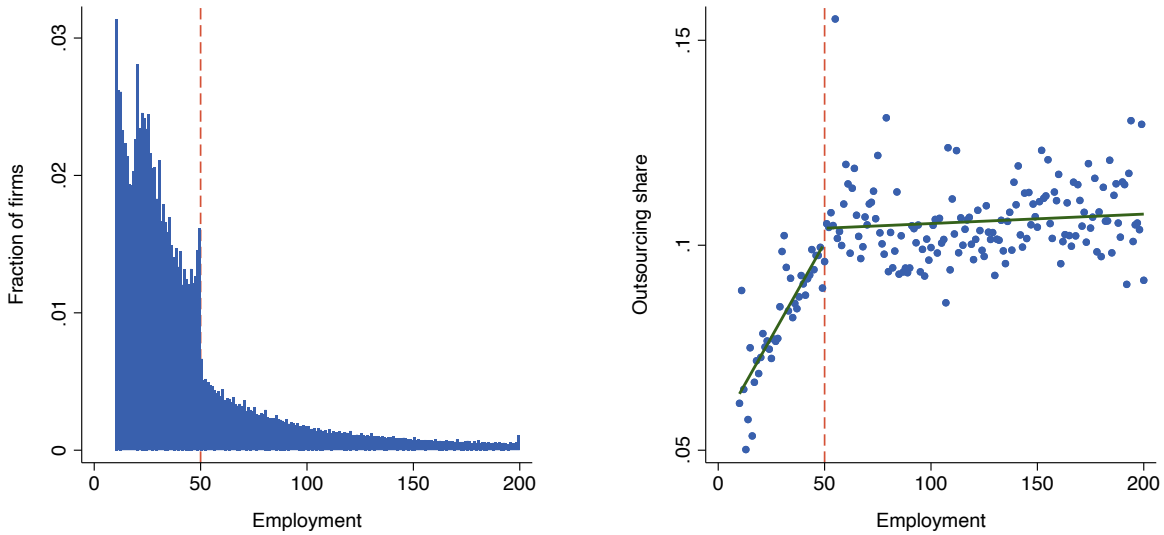


Table B.6: Dependent variable: spending on external workers as a fraction of labor costs, in p.p.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Log V.A. | 1.991*** | | 1.748*** | | 1.858*** | 1.719*** |
| | (0.017) | | (0.111) | | (0.023) | (0.105) |
| Volatility | | 0.605*** | | 0.634*** | 0.624*** | 0.707*** |
| | | (0.022) | | (0.036) | (0.022) | (0.037) |
| *Fixed Effects* | | | | | | |
| Constant | ✓ | ✓ | | | ✓ | |
| 4-digit industry | | | ✓ | ✓ | | ✓ |
| Obs. | 289243 | 183147 | 289243 | 183147 | 183147 | 183147 |
| $R^2$ | 0.043 | 0.004 | 0.168 | 0.184 | 0.040 | 0.207 |

Note: Standard errors in parenthesis, clustered at the industry level. $^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. Independent variables standardized to unit standard deviation. One observation is a firm. Firm fixed effects excluded because firm-level volatility computed using all sample periods.

Table B.7: Dependent variable: spending on external workers as a fraction of labor costs, in p.p.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Wage st. dev. | -0.06*** | -0.06*** | -0.01** | | | |
| | (0.00) | (0.00) | (0.00) | | | |
| Wage P90-P10 | | | | -0.02*** | -0.02*** | -0.00* |
| | | | | (0.00) | (0.00) | (0.00) |
| Log Revenues | | 0.02*** | 0.03*** | | 0.02*** | 0.03*** |
| | | (0.00) | (0.00) | | (0.00) | (0.00) |
| *Fixed Effects* | | | | | | |
| Year | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Firm | | | ✓ | | | ✓ |
| Obs. | 114656 | 113942 | 112716 | 114678 | 113962 | 112750 |

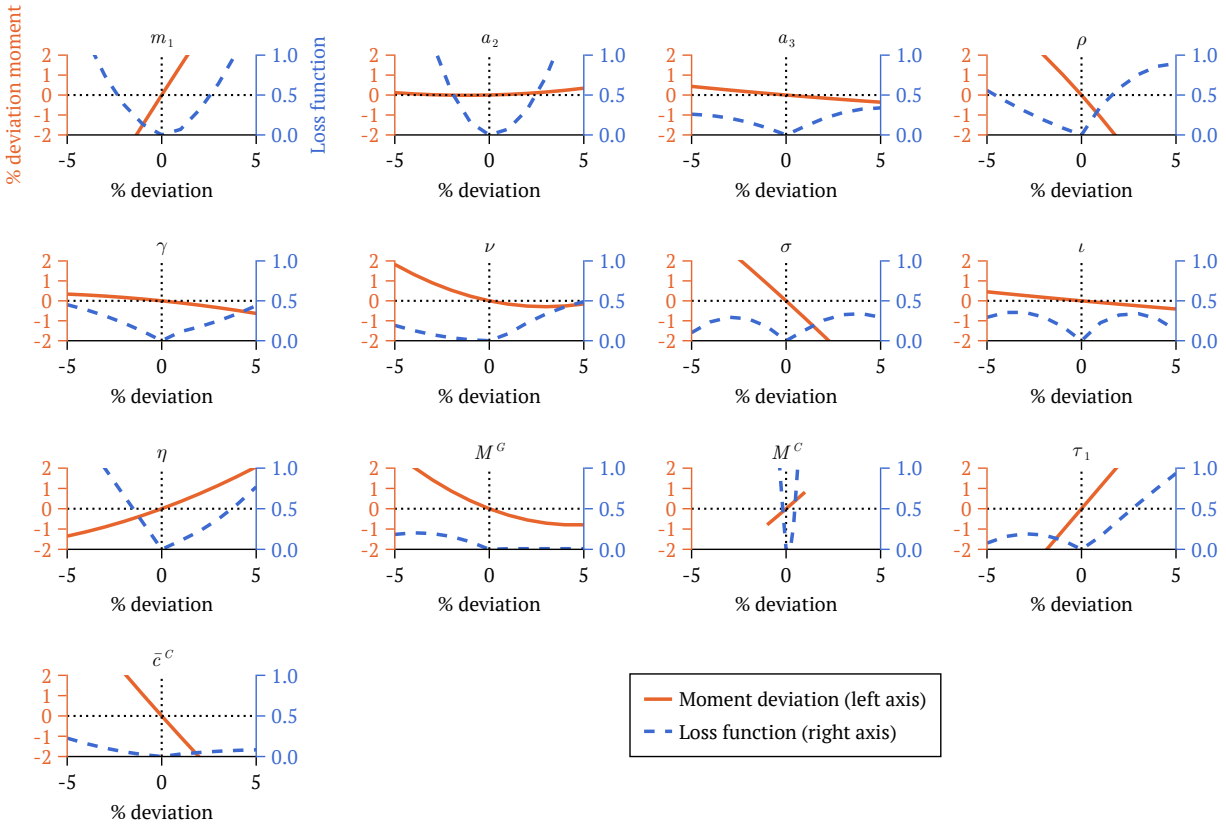
Standard errors in parenthesis, clustered by firm. $^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. Dependent variable: outsourcing share, defined as expenditures on external workers divided by total expenditures on labor (gross wage bill plus expenditures on external workers). First independent variable: within-firm standard deviation of log daily wages. Second independent variable: within-firm difference between 90th percentile of log daily wages and 10th percentile of log daily wages. Revenues defined as value added plus outsourcing expenditures. All regressions at firm-period level and unweighted.

## B.3 Estimation and identification

Table B.8: Parameter estimates and empirical targets.

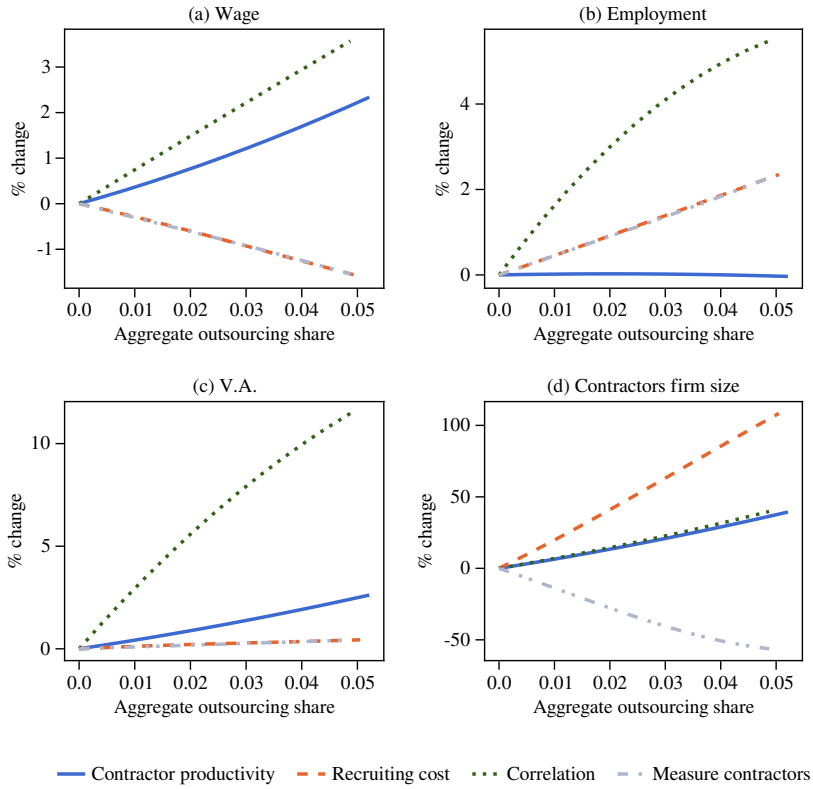
| Parameter | Interpretation | Target | Empirical Moment | Simulated Moment | Parameter Estimate |
|---|---|---|---|---|---|
| | | A. Parameters from direct inversion. | | | |
| $\delta_1$, $\delta_2$ | Job loss rate low-skill | EN rate low-skill | 0.04 | | 0.04 |
| $\delta_3$ | Job loss rate high-skill | EN rate high-skill | 0.03 | | 0.03 |
| $\zeta_1$, $\zeta_2$ | Rel. search. emp. low-skill | EE rate low-skill | 0.04 | | 0.75 |
| $\zeta_3$ | Rel. search. emp. high-skill | EE rate high-skill | 0.03 | | 0.68 |
| $b_1, b_2, b_3$ | Unemp. benefits | Replacement rate | 0.70 | | {0.53, 0.52, 0.94} |
| $\mu_1$ | Matching eff. low-skill 1 | NE rate low-skill | 0.17 | | 0.79 |
| $\mu_2$ | Matching eff. low-skill 2 | NE rate low-skill | 0.17 | | 0.56 |
| $\mu_1$ | Matching eff. high-skill | NE rate low-skill | 0.17 | | 1.13 |
| | | B. Parameters from MSM estimator. | | | |
| $m_1$ | Mass of low skill 1 | Out. share (employment) | 0.26 | 0.30 | 0.37 |
| $a_2$ | Rel. prod. low-skill 2 | Relative low-skill wage | 1.00 | 1.02 | 0.26 |
| $a_3$ | Rel. prod. high-skill | Skill premium | 1.74 | 1.78 | 2.62 |
| $\rho$ | Curvature in revenue | Labor share | 0.70 | 0.70 | 0.92 |
| $\gamma$ | Curvature vac. cost | St dev. log firm size | 0.98 | 1.03 | 10.1 |
| $\nu$ | Standard dev. prod. $z$ | St. dev. log VA | 1.17 | 1.26 | 0.63 |
| $\sigma$ | Standard dev. out. costs $\varepsilon$ | Frac. of firms OS $\leq 10\%$ | 0.89 | 0.96 | 0.19 |
| $\iota$ | Correlation btw. $z$ and $\varepsilon$ | VA elasticity of out. share | 1.83 | 1.63 | 0.15 |
| $\eta$ | Fixed cost | Frac. of firms $\leq 1$ employee | 0.30 | 0.29 | 0.00 |
| $M^G$ | Mass of producers | Average firm size | 60.0 | 45.4 | 0.06 |
| $M^C$ | Mass of contractors | Relative contractore firm size | 2.35 | 2.22 | 0.00 |
| $\tau_1$ | Outsourcing cost | Out. share (spending) | 0.08 | 0.08 | 0.65 |
| $\bar{c}^C$ | Providers' rel. vacancy cost | Outsourcing wage penaly | 0.14 | 0.13 | 0.55 |

Figure B.7: Simulated moments and loss function across parameter values.




Note: Numerical local identification of parameters. Solid orange line: percentage deviation of targeted moment relative to moment at estimated parameters, as a function of percentage deviation of parameter. Mapping as per Table B.8. Dashed blue line: percentage deviation of loss function relative to loss function at estimated parameters, as a function of percentage deviation of parameter.

Figure B.8: Effect of demand and supply shocks on equilibrium outcomes in the model.



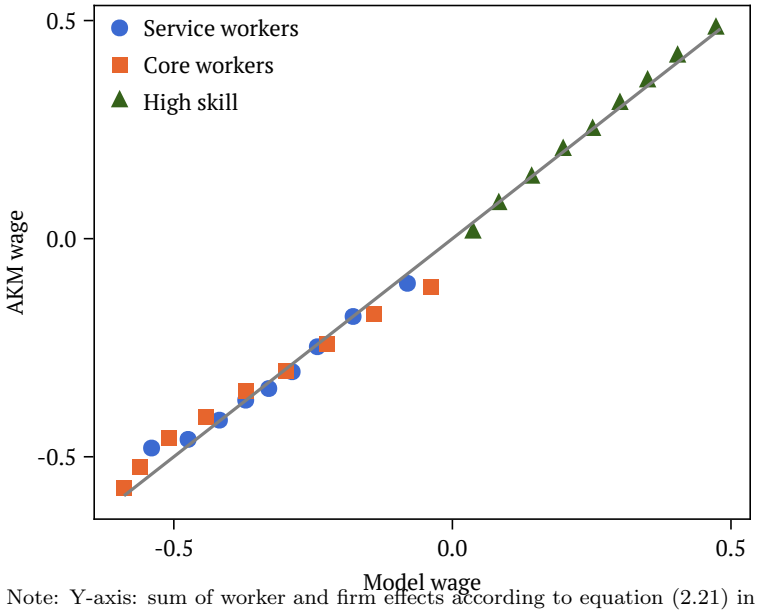

Note: Changes in wages, employment, value added and contractor size as a function of shocks to contractor productivity $\tau_1$, contractor recruiting efficiency $\bar{c}^C$, the productivity-outsourcing cost correlation $\iota$ and the measure of contractors $M^C$. All changes reported as a function of the resulting increase in aggregate outsourcing share.

Table B.9: Estimates of outsourcing shocks

| Interpretation | Parameter | 1997 | 2007 |
|---|---|---|---|
| Contractor rel. productivity | $\tau_1$ | 0.780 | 0.540 |
| Correlation $(z, \varepsilon)$ | $\iota$ | 0.170 | 0.113 |
| Measure of contractors | $M^C$ | 0.002 | 0.005 |
| Contractor rel. vacancy cost | $\bar{c}^C$ | 1.030 | 0.310 |

## B.4 Additional results

Figure B.9: Fit between estimated worker and firm effects and wages in model.



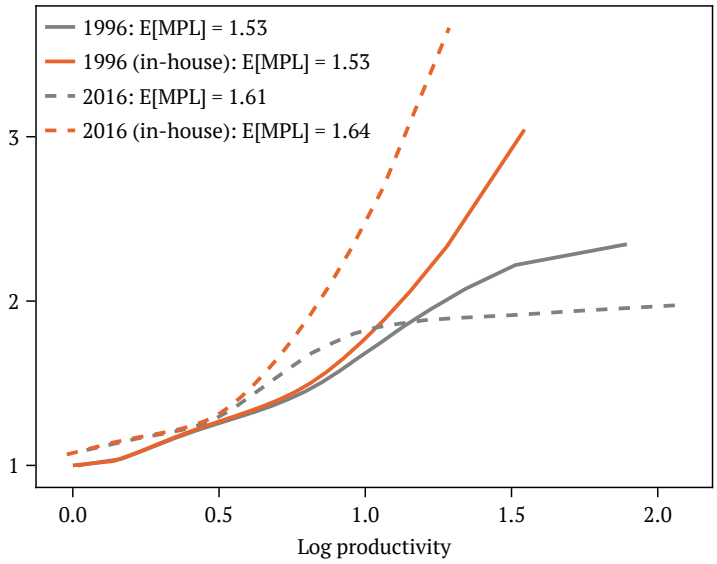

Note: Y-axis: sum of worker and firm effects according to equation (2.21) in model. X-axis: wages in model. Gray line: 45 degree line.

Figure B.10: Rent-sharing in the labor market for all firms.



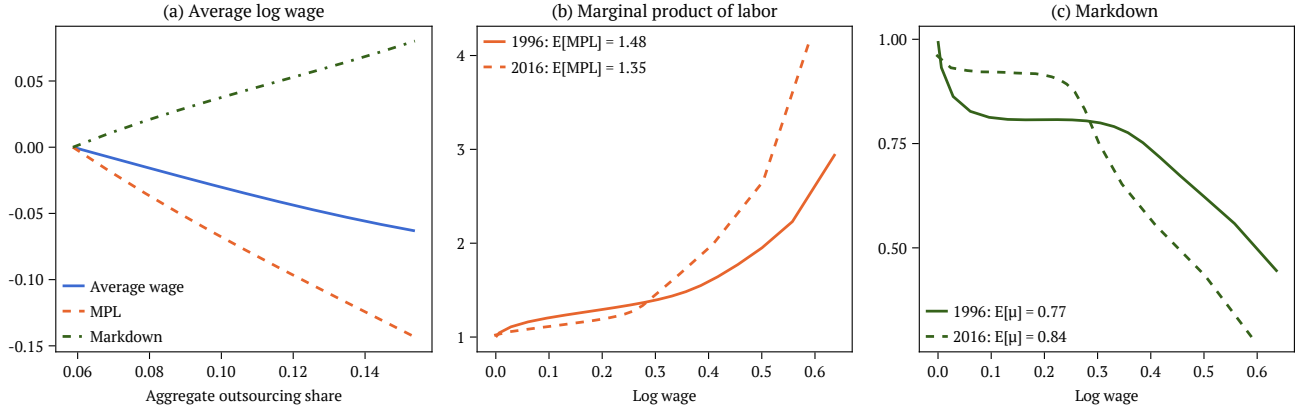

Note: Panel (a): value added-weighted distribution of labor shares. Panel (b): Labor share by average wage. Panel (c): Labor share by value added. All firms. Contractors contribute the additional mass at about Labor share = 0.9.

Figure B.11: The impact of outsourcing on the marginal product of labor with all firms.




Note: Marginal product of labor for service workers. Orange lines: goods producers who hire in-house only. Gray lines: all firms, including contractors.

Figure B.12: The impact of outsourcing on rent-sharing for service workers for all firms.




Note: Panel (a): decomposition of mean log wage for service workers for all firms according to the identity (2.24). Panel (b): distribution of the marginal product of labor for service workers for all firms. Panel (c): distribution of markdowns for service workers for all firms.

## B.5 Data description

**Firm-level balance sheet data.** We use the FICUS data (*"Fichier Complet Unifié de Suse"*) which covers the near universe of nonfarm French businesses. The unit of observation is a firm-year, and firms are identified by their tax identifier (*"siren"*). It details balance

sheet information. We construct value added by substracting purchases of intermediate goods and other intermediate purchases from firm sales.

**Firm-level survey data.** We use the EAE data (*"Enquête Annuelle d'Entreprise*). It covers a random sample of firms and tracks them across years. We link it to other sources using the common tax identifier (*"siren"*). The unit of observation is a firm-year. Among others, the dataset breaks down intermediate purchases of goods and services. In particular, we use expenditures on external workers (*"Dépenses de personnel extérieur"*) as our main measure of outsourcing expenditures.

**DADS panel.** We use the 4% sample of the DADS panel, between 1996 and 2007. Once a worker enters the dataset in any year after 1976, all her subsequent employment spells are recorded. Individuals' employment history is recorded in the dataset if (a) they have at least one employment spell, and (b) they are born in October in even years. The dataset provides start and end days of each employment spell, the job's wage, four-digit occupation and industry, as well as establishment and firm tax identifiers that can be linked to other datasets. We follow Bilal (2021) to set sample restrictions and define unemployment.

**DADS cross-section.** The DADS *Postes*, are used by the French statistical institute to construct the DADS *Panel*. They cover the universe of French workers, but in the version available to researchers, worker identifiers are reshuffled every two years. The DADS Postes allow to compute employment, wages, occupational mix for the near universe of French establishments.

**Firm-level customs data.** We use customs data for the universe of French importers and exporters. The unit of observation is at the firm-product-year-country-export/import level. We aggregate French exports for every firm, year and destination country at the 4-digit industry level to construct our firm-level instrument.

## B.6 Additional proofs

### B.6.1 Dynamic firm problem

We first show that the size constraint in (2.3) is consistent with the firm-level decision. Omit $s$ indices whenever unambiguous. Denote by $q$ the vacancy contact rate. Without loss of generality, we use a continuous offer distribution $F(w)$ to lighten notation. Start from the firm-level Kolmogorov Forward Equation:

$$\frac{dn(w,t)}{dt} = q[\phi + (1-\phi)G(w)] - [\delta + \lambda^E(1-F(w))]n,$$

where $\phi = \frac{u}{u+\frac{\lambda^E}{\lambda^U}(1-u)} = \frac{1}{1+k}$ is the probability of meeting an unemployed worker. In steady-state $dn/dt = 0$. Hence, from (2.1), $\phi + (1-\phi)G(w) = \frac{1}{1+k(1-F(w))}$, and so $n(w) = \frac{q}{\delta}\frac{1}{[1+k(1-F(w))]^2}$. Then, from a constant returns matching function, $\lambda^U = \theta q(\theta) = \frac{M}{m[u+(1-u)\lambda^E/\lambda^U]}q(\theta)$ where $\theta$ is labor market tightness. Re-arranging leads to $q = \frac{e\delta(1+k)}{M}$. Therefore,

$$n(w) = \frac{1}{M}\frac{(1+k)e}{[1+k(1-F(w))]^2}.$$

We now turn to showing that the decisions from the dynamic profit-maximization problem of the firm coincides with those from the static firm profit maximization problem (2.3) when the discount rate is low enough.

Consider the dynamic problem of a firm which may be out of its long-run size, while the rest of the economy is in steady-state. Assume that firms may freely adjust their wage each instant, but face an equal-pay constraint within worker type. Without loss of generality, we consider a single worker type to make notation lighter. Firms solve

$$rJ(z,n) = \max_w R(z,n) - wn + [q(\phi + (1-\phi)G(w)) - n(\delta + \lambda^E(1-F(w)))J_n(z,n).$$

Using $\phi = \frac{1}{1+k}$,

$$rJ(z,n) = \max_w R(z,n) - wn + \delta(1 + k(1 - F(w))(n(w) - n)J_n(z,n).$$

The first-order condition implies $-n + \delta(1 + k(1 - F))n'(w)J_n + kF'(n(w) - n)J_n = 0$. Evaluated at long-run size $n = n(w)$,

$$n(w) = \delta(1 + k(1 - F))n'(w)J_n(z, n(w)).$$

The envelope condition then yields $rJ_n = R_n - w + \delta(1 + k(1 - F))[-J_n + (n(w) - n)J_{nn}]$ which again evaluated at long-run size $n = n(w)$ leads to

$$rJ_n(z, n(w)) = R_n(z, n(w)) - w - \delta(1 + k(1 - F(w)))J_n(z, n(w)).$$

When the discount rate goes to zero $r \to 0$,

$$J_n(z, n(w)) = \frac{R_n(z, n(w)) - w}{\delta(1 + k(1 - F(w)))}.$$

Substituting into the first-order condition, we obtain

$$n(w) = n'(w)(R_n - w),$$

which coincides with the static first-order condition.

### B.6.2 Welfare and rent-sharing implications of compensating differentials models of wage inequality

Models of wage inequality based on compensating differentials have become popular due to their tractability (see e.g. Card et al., 2018, Sorkin, 2018, Berger et al., 2022, Lamadon et al.,

2022). These models are well-suited to study the effect of inequality on welfare at the market level. By contrast, they require strategic interactions to generate markdown variation across firms within markets. They also struggle to translate observed wage differences into welfare difference. We make that point by considering the simplest of such models.

There are $M$ firms $j$. Each firm has some productivity $z_j$. Each firm choose the wage $w_j$ it posts. For simplicity, suppose that each firm is small enough that it does not consider its impact on any aggregate. Workers choose from the continuum of wage offers. Worker draw random non-wage amenities $a_j$ specific to each firm. Workers value wages and non-work amenities $a_j$ to maximize $V = \max_j w_j a_j$.

Tractability is achieved by assuming that $a_j$ are independent across firms $j$ and have a Frechet distribution with shifter $T_j$ and shape parameter $\varepsilon$: $F_j(a) = e^{-T_j a^{-\varepsilon}}$.[2] Classical results in extreme value theory ensure several results (see e.g. Eaton and Kortum, 2002) that we derive here for completeness.

**Welfare implications.** First, the probability of choosing firm $i$ is $\pi_i = \frac{T_i w_i^{\varepsilon}}{\sum_j T_j w_j^{\varepsilon}}$. To see this, consider the calculation

$$\begin{aligned}
\mathbb{P}[w_i a_i \geq w_j a_j \ \forall j \neq i] &= \int_0^{\infty} \prod_{j \neq i} F_j(w_i a_i / w_j) dF_i(a_i) \\
&= \int_0^{\infty} e^{-\Phi_{-i} w_i^{-\varepsilon} a_i^{-\varepsilon}} dF_i(a_i) \\
&= \int_0^{\infty} \left(F_i(a_i)\right)^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF_i(a_i) \\
&= \int_0^{1} F^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF \\
&= \frac{1}{1 + \Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} \\
&= \frac{T_i w_i^{\varepsilon}}{\sum_j T_j w_j^{\varepsilon}}
\end{aligned}$$

[2] More involved correlation structures can be introduced with a nested Frechet distribution, but this additional complexity is not relevant for our purposes.

where we denote $\Phi_{-i} = \sum_{j \neq i} T_j w_j^{\varepsilon}$, and changed variables $F = F_i(a_i)$ from the third to fourth row.

This property has made this class of models particularly attractive to model an upward-sloping labor supply curve. When every firm is small enough that it negelects its effect on the denominator, the labor supply curve faced by every firm $j$ is of the form $n_j(w) = n_0 T_j w^{\varepsilon}$.

This tractability comes, however, at a cost. The cost is that welfare is equalized across firms, regardless of the wage they decide to post. To see this second result (also well-established, but less well-known), compute as above

$$\begin{aligned}
\mathbb{P}[w_i a_i \leq \omega \ \& \ w_i a_i > w_j a_i \ \forall j \neq i] &= \int_0^{\omega/w_i} \prod_{j \neq i} F_j(w_i a_i / w_j) dF_i(a_i) \\
&= \int_0^{\omega/w_i} e^{-\Phi_{-i} w_i^{-\varepsilon} a_i^{-\varepsilon}} dF_i(a_i) \\
&= \int_0^{\omega/w_i} \big(F_i(a_i)\big)^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF_i(a_i) \\
&= \int_0^{F_i(\omega/w_i)} F^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF \\
&= \frac{F_i(\omega/w_i)^{1+\Phi_{-i} w_i^{-\varepsilon}}}{1 + \Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} \\
&= \frac{T_i w_i^{\varepsilon}}{\sum_j T_j w_j^{\varepsilon}} e^{-\Phi \omega^{-\varepsilon}}
\end{aligned}$$

where $\Phi = \sum_j T_j w_j^{\varepsilon}$. Therefore, the conditional distribution of values within every chosen firm $i$ is

$$\begin{aligned}
\mathbb{P}[w_i a_i \leq \omega \mid w_i a_i > w_j a_i \ \forall j \neq i] &= \frac{\mathbb{P}[w_i a_i \leq \omega \ \& \ w_i a_i > w_j a_i \ \forall j \neq i]}{\mathbb{P}[w_i a_i \geq w_j a_j \ \forall j \neq i]} \\
&= e^{-\Phi \omega^{-\varepsilon}}
\end{aligned}$$

The distribution of indirect utility of employed workers at any firm is Frechet with shifter $\Phi$ and shape parameter $\varepsilon$. It is independent of the wage offered by the firm. Thus, welfare is equalized across firms. In particular, expected utility of employed workers at any firm is

$\Gamma(1-1/\varepsilon)\Phi^{1/\varepsilon}$, where $\Gamma$ denotes Euler's Gamma function. Non-work amenities introduce mixing, but they do not change the core implications relative to a standard free-mobility condition without amenities, $\max_j w_j$.

**Markdowns and labor shares.** To study rent-sharing in this economy, we specify a revenue function $R(z,n)$ that may depend on productivity $z$ and the number of workers that a firm hires $n$. Any given firm $j$ then solves

$$\max_{w,n} R(z_j,n) - wn \text{ s.t } n \leq \pi_j(w) \equiv \frac{T_j w^{\varepsilon}}{\sum_{i\neq j} T_i w_i^{\varepsilon}}$$

Assuming a large enough number of firms and thus no strategic interactions, $\sum_{i\neq j} T_i w_i^{\varepsilon} \equiv W_{0j}^{\varepsilon} T_j$ may be assumed to be constant from the perspective of firm $j$. The labor supply curve becomes $\pi_j(w) = W_{0j}^{-\varepsilon} w^{\varepsilon}$. The wage it pays is $w_j(n) = W_{0j} n^{1/\varepsilon}$. The problem of the firm becomes

$$\max_{n} R(z_j,n) - W_{0j} n^{1+1/\varepsilon}.$$

Thus, the firm chooses $n_j$ such that $R_n(z_j,n) = (1+1/\varepsilon) W_{0j} n_j^{1/\varepsilon}$. Under isoelastic revenue $R(z,n) = z n^{\rho}$, $R_n(z_j,n_j) = \rho R(z_j,n_j)/n_j = \rho z_j n_j^{\rho-1} = (1+1/\varepsilon) W_{0j} n_j^{1/\varepsilon} = (1+1/\varepsilon) w_j(n_j)$.

Hence, the markdown is constant across firms and given by

$$\text{markdown} = \frac{\rho\varepsilon}{1+\varepsilon}.$$

The labor share is also constant across firms and given by

$$\text{LS} = \frac{\varepsilon}{1+\varepsilon}$$

**Taking stock.** Thus, it is difficult to connect models of wage inequality based on compensating differentials to welfare inequality across firms.[3] These results are specific to the Frechet distribution however. Potentially, more complex distribution of compensating differentials could lead to meaningful welfare effects. Yet, more complex distributions would also break the tractability provided by the Frechet distribution. Similarly, models of wage inequality based on compensating differentials deliver constant markdowns and labor shares under standard isoelastic revenue functions absent market power.

## B.7 Simulation and estimation

### B.7.1 Model

**Good producers** Good producers solve the following problem

$$\pi(z,\varepsilon) = \max_{\boldsymbol{n},\boldsymbol{w},\boldsymbol{v},o} R(z,\boldsymbol{n}) - ((1-o)w_1 - op\varepsilon)\, n_1 - (1-o)c(v_1) - \sum_{s>1}(w_s n_s + c(v_s)) - \eta, \tag{B.20}$$

subject to

$$n_s \leq \mathrm{n}_s(w)\left(\frac{v_s}{V_s}\right) \quad \text{if } o_s = 0,$$

where $\boldsymbol{x} \equiv \{x_s\}_{s=1}^S$ and $\mathrm{n}_s$ is the firm-specific labor supply curve,

$$\mathrm{n}_s(w) \equiv \frac{(1+k_s)e_s}{(1+k_s(1-F_s(w_s)))^2}.$$

In the formulation of the problem above, we have already assumed that only the workers of skill type one (service workers) can be outsourced. We use the following parametric

[3]Of course, the shifter $\Phi$ depends on the distribution of wages inside the market and so cross-market welfare inequality is still meaningful.

assumptions. The revenue function is Cobb-Douglas nested in a decreasing returns upper tier,

$$R(z, \boldsymbol{n}) = \left( z \prod_{s=1}^{S} n_s^{a_s} \right)^{\rho}$$

with $\sum_s a_s = 1$. The vacancy cost function is isoelastic with elasticity $\gamma$,

$$c(v) = \frac{v^{1+\gamma}}{1+\gamma},$$

where we have normalized the vacancy cost of a vacancy for good producers to one, $c_0 = 1$. Finally, the joint distribution of $(z, \varepsilon)$, denoted by $\Gamma$, is log-normal with zero means and variance-covariance matrix $\mathbf{\Sigma}$.

**Contractors** To solve numerically the contractors' problem in a similar fashion as the problem of good producers, we introduce some minimal productivity heterogeneity across contractors. Specifically, let $\Psi$ denote the distribution of contractors' productivity. We parametrize it to be log-normal with zero mean and variance $\sigma^C$ and we set $\sigma^C \approx 0$. Then, contractors solve the problem

$$\pi^C(z) = \max_{n,w,v} \; p\tau z n - wn - \frac{(\bar{c}^C)^{\gamma} v^{1+\gamma}}{1+\gamma} \quad \text{s.t.} \quad n \leq \mathrm{n}_1(w) \left( \frac{v}{V_1} \right), \tag{B.21}$$

where $z$ denote the productivity of the contractor. The effective amount of labor that a contractor with productivity $z$ provides to the outsourcing market is $\tau z n$. Let $\underline{z}^C$ be the least productive contractor firms. In equilibrium, this firm offers the reservation wage and $\underline{z}^C = \underline{w}_1/(p\tau)$.[4]

[4]When contractor firms post their optimal number of vacancy, their profits are given by

$$\pi^C(z) \propto \max_{w} \; \left[ (p\tau z - w) \mathrm{n}_1(w) \right]^{\frac{1+\gamma}{\gamma}}.$$

Hence, $\underline{z}^C = w(\underline{z}^C)/(p\tau)$. Furthermore, the least productive active contractor firms must offer the reservation wage for otherwise contractor firms with lower productivity would be able to make a profit by posting a wage

### B.7.2 A tractable reformulation of the model

Due to its large amount of heterogeneity, this model is *a priori* numerically non-tractable. In particular, two objects are complicated to compute. First, the wage offered to in-house workers by good producers that outsource their service workers. These wages depend indeed on $(z, \varepsilon)$ rather than on a uni-dimensional variable. Second, the wage offer distributions is non-trivial to compute since different types of firms are now competing on the same skill-specific job ladder.[5] In this section, we derive a reformulation of the problem that simplifies these two problems. This derivation is feasible under three assumptions: a Cobb-Douglas revenue function, a single outsourceable worker type, and a log-normal distribution for $(z, \varepsilon)$.

**Outsourcing good producers** Index good producers that outsource their service workers by the superscript $o$. Similarly, index good producers that hire their service workers in-house by $i$. The problem of a good producer of type $o$ reads

$$\pi^o(z,\varepsilon) = \max_{\boldsymbol{n},\{w_s\}_{s>1},\{v_s\}_{s>1}} R(z,\boldsymbol{n}) - n_1 p\varepsilon - \sum_{s>1}\left(w_s n_s + c(v_s)\right) - \eta. \tag{B.22}$$

The optimal number of service workers hired from the contractor sector is obtained by taking the first order condition of this problem with respect to $n_1$,

$$n_1^o(z,\varepsilon) = \left(\frac{\rho a_1}{p\varepsilon}\right)^{\frac{1}{1-a_1\rho}} \left(z\prod_{s>1} n_s^{a_s}\right)^{\frac{\rho}{1-a_1\rho}}.$$

Plugged back into (B.22), the profit of the firm rewrites

$$\pi^o(z,\varepsilon) = \mathcal{G}\left(z\varepsilon^{-a_1}\prod_{s>1} n_s^{a_s}\right)^{\kappa} - \sum_{s>1} w_s n_s - c(v_s) - \eta \equiv \pi^o(\hat{z}),$$

in $[\underline{w}, w(\underline{z}^C))$.

[5] In a standard Burdett-Mortensen model, the wage offer distribution is directly recovered from two differential equations obtained from the wage and vacancy first-order conditions. This is not the case here as two firms with a similar revenue TFP $z$ may offer different wages depending on their outsourcing choice.

for $\kappa$ a parametric constant and $\mathcal{G}$ a general equilibrium constant.[6] In the above expression, payroll and vacancy costs are independent from $z$ and $\varepsilon$. Meanwhile, revenues only depend on the TFP aggregator $\hat{z} \equiv z\varepsilon^{a_1}$. As a result, the policy functions of type-$o$ good producers are only a function of $\hat{z}$ and it is not needed to keep track of $z$ and $\varepsilon$ separately. Since $(z, \varepsilon)$ is jointly log-normally distributed, so is $(z, \hat{z})$ and a closed-form expression exists for its variance-covariance matrix. Let $\Phi$ denote the log-normal distribution under the change of variable $(z, \varepsilon) \to (z, \hat{z})$. Finally, profits of in-house and outsourcing good producers are increasing in $z$ and $\hat{z}$ respectively, and there exists two productivity lower bounds, $\underline{z}$ and $\underline{\hat{z}}$, so that $\pi^i(z) > 0$ iff $z > \underline{z}$ and $\pi^o(\hat{z}) > 0$ iff $\hat{z} > \underline{\hat{z}}$. With a slight abuse of notation, we refer to $\Phi$ as the truncated log-normal distribution, $\Phi_z$ as the $z$-marginal, and $\Phi_{\hat{z}|z}$ as the distribution of $\hat{z}$ condition on $z$.

**Wage distributions** To derive the wage offer distributions, it is required to know which firms outsource their service workers and which do not. Let $\varphi(z)$ denote the productivity level that renders an outsourcing firm indifferent between the two outsourcing choices, $\pi^i(z) = \pi^o[\varphi(z)]$ and $\pi^i(z) < \pi^o(\hat{z})$ for all $\hat{z} > \varphi(z)$. With this notation, the wage offer distribution of skill $s > 1$ is given by

$$F_s(w) = \frac{M^G}{V_s}\left(\int \mathbb{1}\{w_s^i(z) \leq w\} v_s^i(z)\Omega^i(z)\mathrm{d}\Phi_z(z) + \int \mathbb{1}\{w_s^o(\hat{z}) \leq w\} v_s^o(\hat{z})\Omega^o(\hat{z})\mathrm{d}\Phi_{\hat{z}}(\hat{z})\right), \tag{B.23}$$

where

$$V_s = M^G\left(\int v_s^i(z)\Omega^i(z)\mathrm{d}\Phi_z(z) + \int v_s^o(\hat{z})\Omega^o(\hat{z})\mathrm{d}\Phi_{\hat{z}}(\hat{z})\right) \tag{B.24}$$

[6] Specifically, we have

$$\kappa = \frac{\rho}{1-\rho a_1} \quad \text{and} \quad \mathcal{G} = \frac{\rho}{\kappa}\left(\frac{\rho a_1}{p}\right)^{a_1\kappa}.$$

is the mass of vacancy posted for skill $s$ and $\Omega^i(x) \equiv \Phi_{\hat{z}|z}[\varphi(x) \mid x]$ is the probability that good producer $z$ hires its service workers in-house. Similarly, $\Omega^o(x) \equiv \Phi_{z|\hat{z}}[\varphi^{-1}(x) \mid x]$. The first integral in (B.23) is the relative mass of vacancy attached to wages lower than $w$ and offered by good producers hiring their service workers in-house. The second integral is the relative mass of vacancy attached to wages lower than $w$ and offered by good producers outsourcing their service workers. Similarly, the wage offer distribution for service workers is

$$F_1(w) = \frac{M^G}{V_1} \int \mathbb{1}\{w_1^i(z) \leq w\} v_1^i(z) \Omega^i(z) \mathrm{d}\Phi_z(z) + \frac{M^C}{V_1} \int \mathbb{1}\{w_1^C(z) \leq z\} v_1^C(z) \mathrm{d}\Psi(z), \tag{B.25}$$

where

$$V_1 = M^G \int v_1^i(z) \Omega^i(z) \mathrm{d}\Phi_z(z) + M^C \int v_1^C(z) \mathrm{d}\Psi(z). \tag{B.26}$$

In equation (B.25), the first integral is the relative mass of vacancy attached to wages lower than $w$ and offered by good producers hiring their service workers in-house. The second integral is the mass of vacancy attached to wages lower than $w$ and offered by contractors.

Finally, two further conditions close the equilibrium. First, the reservation wages which constitute the lower bound of the wage offer distributions,

$$\underline{w}_s = b_s + \left(k_s^U - k_s\right) \left( \int_{\underline{w}_s} \frac{1 - F_s(w)}{1 + k_s(1 - F_s(w))} \mathrm{d}w \right). \tag{B.27}$$

Second, the market clearing condition of the outsourcing market is

$$\begin{aligned} M^C \int \tau z n^c(z) \mathrm{d}\Psi(z) &= M^G \int \varepsilon n_1^o(z, \varepsilon) \mathbb{1}\{\pi^o(z, \varepsilon) > \pi^i(z)\} \mathrm{d}\Phi(z, \varepsilon) \\ &= M^G \left( \frac{a_1 \rho}{p} \right)^{\frac{\kappa}{\rho}} \int \left( \hat{z} \prod_{s>1} n_s^o(\hat{z})^{a_s} \right)^{\kappa} \Omega^o(\hat{z}) \mathrm{d}\Phi_{\hat{z}}(\hat{z}), \end{aligned} \tag{B.28}$$

where, as explained in the main text, $\varepsilon$ is interpreted as an idiosyncratic iceberg shock and as such appears in the aggregate demand for outsourced service workers. The second line is obtained from plugging the expression for $n^o$ and performing the change of variable $z\varepsilon^{-a_1} \to \hat{z}$.

**Definition 1** (Equilibrium)**.** *An equilibrium is a collection of wage and vacancy functions for good producers,* $\{w_s^\theta, v_s^\theta\}_{s\in\{1,\dots S\},\theta\in\{i,o\}}$*, wage and vacancy functions for contractor firms,* $w_1^C$ *and* $v_1^C$*, an indifference function,* $\varphi$*, wage distributions* $\{F_s\}_{s=1}^S$*, productivity cutoffs* $(\underline{z}, \hat{\underline{z}}, \underline{z}^C)$*, and aggregate quantities,* $\{\underline{w}_s\}_{s=1}^S$ *and* $p$*, such that*

1. *Given* $\{F_s\}_{s=1}^S$*,* $p$*, and an outsourcing decisions* $\theta \in \{i, o\}$*, the functions* $\{w_s^\theta, v_s^\theta\}_{s=1}^S$ *solve* (B.20)*;*
2. *Given* $F_1$ *and* $p$*, the functions* $w_1^C$ *and* $v_1^C$ *solve* (B.21)*;*
3. *Given the policy functions* $\{w_s^\theta, v_s^\theta\}_{s=1}^S$*, the indifference function is such that* $\pi^i(z) = \pi^o[\varphi(z)]$*;*
4. *Given the policy functions, the wage distributions satisfy* (B.23) *and* (B.25)*;*
5. *Given* $\{F_s\}_{s=1}^S$*, the reservation wages are given by* (B.27)*;*
6. *Given the firms' profits, the productivity cutoffs solve* $\underline{z} = \inf\{z : \pi^i(z) > 0\}$*,* $\hat{\underline{z}} = \inf\{\hat{z} : \pi^o(\hat{z}) > 0\}$ *and* $\underline{z}^C = \underline{w}_1/(p\tau)$*;*
7. *Given the policy functions and* $\varphi$*, the price* $p$ *solves the market clearing condition* (B.28)*.*

### B.7.3 Expressing the model as a system of differential equations

To compute numerically the equilibrium defined in Definition 1, we take the first order conditions of the firms' problem (B.20) and (B.21). In doing so, we show that it is never required to solve for the solution of the contractors' problem as long as their wage overlap with the wages offered by good producers. We then rewrite the wage offer distributions (B.23) and (B.25) in differentials so as to obtain them jointly with the wage and vacancy policy functions as the solution to a system of differential equations.

**Optimality conditions** Let $\Upsilon_s^\theta(z) \equiv F_s[w_s^\theta(z)]$ denote the rank on the skill-$s$ job ladder of a good producer with productivity $z$ and outsourcing choice $\theta \in \{i, o\}$. For instance, for $s > 1$ and $\theta = i$, we have

$$\Upsilon_s^i(z) = \frac{M^G}{V_s}\left(\int^z v_s^i(x)\Omega^i(z)\mathrm{d}\Phi_z(x) + \int \mathbb{1}\{w_s^o(\hat{z}) \leq w_s^i(z)\}v_s^o(\hat{z})\Omega^o(\hat{z})\mathrm{d}\Phi_{\hat{z}}(\hat{z})\right). \quad \text{(B.29)}$$

Taking the first-order condition of (B.20) with respect to $w$ and using the function $\Upsilon_s^\theta$, the wage optimality condition can be expressed as the differential equation

$$\frac{\partial w_s^\theta(z)}{\partial z} = \left(\frac{2k_s}{1 + k_s(1 - \Upsilon_s^\theta(z))}\right)\left(\frac{\partial R(z, \boldsymbol{n}^\theta(z))}{\partial n_s} - w_s^\theta(z)\right)\frac{\partial \Upsilon_s^\theta(z)}{\partial z}, \quad \text{(B.30)}$$

while the vacancy optimality condition is

$$v_s^\theta(z)^\gamma = \mathrm{n}_s^\theta(z)\left(\frac{\partial R(z, \boldsymbol{n}^\theta(z))}{\partial n_s} - w_s^\theta(z)\right). \quad \text{(B.31)}$$

Similarly, the wage optimal condition for a contractor firm is

$$\frac{\partial w_1^C(z)}{\partial z} = \left(\frac{2k_1}{1 + k_1(1 - \Upsilon_1^C(z))}\right)\left(p\tau z - w_1^C(z)\right)\frac{\partial \Upsilon_1^C(z)}{\partial z}, \quad \text{(B.32)}$$

while the vacancy optimality condition is

$$\left(\bar{c}^C v_1^C(z)\right)^\gamma = \mathrm{n}_1^C(z)(p\tau z - w_1^C(z)). \quad \text{(B.33)}$$

Solving for the wage in (B.30) and (B.32) requires to know $\mathrm{d}\Upsilon_s^\theta$. The goal of the algorithm developed in Section B.7.4 is to solve jointly for $w_s^\theta$ and $\Upsilon_s^\theta$. This requires to know how the second integral in (B.29) moves with $z$. To deal with this problem, we define two wage equivalence functions. First, let $\zeta^{G\to C}$ be the TFP of contractors firms posting wage $w_1^i(z)$; that is, $w_1^i(z) = w_1^C[\zeta^{G\to C}(z)]$. Similarly, let $\zeta_s^{i\to o}$ be such that $w_s^i(z) = w_s^o[\zeta_s^{i\to o}(z)]$. We now derive expressions for these two functions.

**Service workers' wages** Given that wages are striclty increasing in productivity, the function $\zeta^{G\to C}$ is well-defined on the joint support of the wages offered by good producers and contractor firms. Since $z$ is unbounded above for both good producers and contractor firms, $w_1^i(z) \to \bar{w}_1$ and $w_1^C(z) \to \bar{w}_1$ as $z \to \infty$. In addition, we have already argued that the least productive contractor firm offers the reservation wage. It follows that the function $\zeta^{i\to o}$ is well-defined for $z \geq \underline{z}$. Furthermore, a closed-form expression exists for $\zeta^{i\to o}$. For all $z \geq \underline{z}$, $w_1^i(z) = w_1^C[\zeta^{G\to C}(z)]$ implies by definition $\Upsilon_1^i(z) = \Upsilon_1^C[\zeta^{G\to C}(z)]$ and therefore $\mathrm{n}_1^i(z) = \mathrm{n}_1^C[\zeta^{G\to C}(z)]$. Furthermore, differentiating the first two equations implies $\partial_z w_1^i(z)/\partial_z \Upsilon_1^i(z) = \partial_z w_1^C[\zeta^{G\to C}(z)]/\partial_z \Upsilon_1^C[\zeta^{G\to C}(z)]$, where $\partial_z x(z) \equiv \partial x/\partial z$. Comparing equations (B.30) and (B.32), it must then be that the marginal product of labor of good producer $z$ and contractor firms $\zeta^{G\to C}(z)$ are equal. But since contractor firms are facing constant returns to scale, their MPL are independent from their size, and we can therefore invert this MPL equality condition and solve for $\zeta^{G\to C}(z)$ to obtain

$$\zeta^{G\to C}(z) = \left(\frac{\rho a_1}{p\tau}\right)\left(\frac{R^i(z)}{n_1^i(z)}\right). \tag{B.34}$$

Finally, since the right hand side of (B.31) and (B.33) are equal, it must also be that

$$v_1^i(z) = \bar{c}^C v_1^C[\zeta^{G\to C}(z)].$$

With the help of these results, we can differentiate $\Upsilon_1^i(z)$ for $z \geq \underline{z}$ to obtain

$$\frac{\partial \Upsilon_1^i(z)}{\partial z} = \frac{v_1^i(z)}{V_1}\left[M^G \Omega^i(z)\frac{\partial \Phi_z(z)}{\partial z} + \left(\frac{M^C}{\bar{c}^C}\right)\frac{\mathrm{d}\Psi[\zeta^{G\to C}(z)]}{\mathrm{d}z}\right]. \tag{B.35}$$

Ignoring for now the complementarity across skills, equations (B.30) and (B.35) constitute a system of differential equations, which, together with (B.31) and (B.34), allows us to solve for $(w_1^i, v_1^i, \Upsilon_1^i)$. These differential equations are subject to two boundary conditions, $\underline{w}_1^i \equiv w_1^i(\underline{z})$ and $\underline{\Upsilon}_1^i \equiv \Upsilon_1^i(\underline{z})$. If the least productive good producers decide to offer the reservation wage,

then $\underline{w}_1^i = \underline{w}_1$ and $\Upsilon_1^i(\underline{z}) = 0$. However, since good producers compete on the job ladder of service workers with the contractor firms, the least productive good producers may decide optimally *not* to offer the reservation wage.[7] If that is the case, then all wages between $[\underline{w}_1, \underline{w}_1^i)$ are offered by contractor firms. These wages have to satisfy (B.32) together with

$$\frac{\partial \Upsilon_1^C(z)}{\partial z} = \left(\frac{M^C}{V_1}\right) v_1^C(z) \frac{\partial \Psi(z)}{\partial z}.$$

Combined with (B.33), this differential equation becomes

$$\frac{\partial \Upsilon_1^C(z)}{\partial z} = \frac{1}{\bar{c}^C} \left((p\tau z - w_1^C(z))\mathrm{n}_1^C(z)\right)^{1/\gamma} \left(\frac{N}{V_1}\right) \frac{\partial \Psi(z)}{\partial z}. \tag{B.36}$$

This differential equation, together with (B.32) and the boundary conditions $w_1^C(\underline{z}^C) = \underline{w}_1$ and $\Upsilon_1^C(\underline{z}^C) = 0$, pin down the wages and the wage offer distribution for contractors with TFP in $[\underline{z}^C, \tilde{z}^C)$, where $\tilde{z}^C$ is such that $w_1^C(\tilde{z}^C) = w_1^i(\underline{z})$. By a similar argument as before, this wage equality also implies that the MPL of the two firms have to be equal, or

$$\tilde{z}^C = \left(\frac{\rho a_1}{p\tau}\right) \left(\frac{R^i(\underline{z})}{n_1^i(\underline{z})}\right). \tag{B.37}$$

Hence, on $[\underline{z}^C, \underline{\tilde{z}}^C)$, the policy functions of contractor firms are obtained from solving a standard uni-skill constant returns to scale Burdett-Mortensen model. For $z \geq \underline{\tilde{z}}^C$, it is not needed to solve for the problem of contractor firms since we know that $w_1^C[\zeta^{G\to C}(z)] = w_1^i(z)$ and $v_1^C[\zeta^{G\to C}(z)] = v_1^i(z)/\bar{c}^C$.

**Other skills** While only good producers are hiring workers with skills $s > 1$, there is effectively two type of goods producers in this model: those that hire service workers in-house, and those that outsource their service workers. As such, a similar derivation as in the previous

[7] Precisely, the least productive good producers may not want to offer the reservation wage due to two reasons. First, due to the presence of the contractor firms on the job ladder. Second, due to the existence of decreasing returns to scale in the revenue function. If either feature were absent, then $\underline{w}_1^i = \underline{w}_1$ as in the standard Burdett-Mortensen model.

paragraph is needed to solve efficiently for the wage distribution of skills $s > 1$. To avoid complicated combinatorial issues, we assume that the least productive good producers post the reservation wage regardless of their outsourcing decision; that is, $w_s^i(\underline{z}) = w_s^o(\underline{\hat{z}}) = \underline{w}_s$ for $s > 1$.[8] With this assumption, the wage equivalence function $\zeta^{i\to o}$ is globally well-defined. Then, using a similar argument as in the previous paragraph, $w_s^i(z) = w_s^o[\zeta^{i\to o}(z)]$ implies $v_s^i(z) = v_s^o[\zeta_s^{i\to o}(z)]$ as well as the equalization of the MPL of these two firms. Together, we therefore know that

$$\frac{R^i(z)}{n_s^i(z)} = \left(\frac{\kappa}{\rho}\right)\left(\frac{R^o[\zeta_s^{i\to o}(z)]}{n_s^o[\zeta_s^{i\to o}(z)]}\right) = \left(\frac{\kappa}{\rho}\right)\left(\frac{R^o[\zeta_s^{i\to o}(z)]}{n_s^i(z)}\right), \tag{B.38}$$

where the second equality follows from $\mathrm{n}_s^i(z) = \mathrm{n}_s^o[\zeta_s^{i\to o}(z)]$ and $v_s^i(z) = v_s^o[\zeta_s^{i\to o}(z)]$ so that $n_s^i(z) = n_s^o[\zeta_s^{i\to o}(z)]$. Hence, it must be that $R^o[\zeta_s^{i\to o}(z)] = \rho R^i(z)/\kappa$, which holds for each $s > 1$. This in turn implies $R^o[\zeta_s^{i\to o}(z)] = R^o[\zeta_{s'}^{i\to o}(z)]$ for any pair of skill $(s, s')$. But the function $R^o$ is strictly increasing in $z$, and therefore it must be that $\zeta_s^{i\to o}(z) = \zeta_{s'}^{i\to o}(z) \equiv \zeta^{i\to o}(z)$. The skill independence of the function $\zeta^{i\to s}$ allows us to simplify the equation (B.38) to

$$\zeta^{i\to o}(z) = \left(\frac{\rho}{\mathcal{G}\kappa}\right)^{\frac{1}{\kappa}}\left(z n_1^i(z)^{a_1}\prod_{s>1} n_s^i(z)^{a_s\left(1-\frac{\kappa}{\rho}\right)}\right)^{\frac{\rho}{\kappa}}. \tag{B.39}$$

Using the function $\zeta^{i\to o}$ in the expression for the wage offer distribution and differentiating the later, we obtain

$$\frac{\partial \Upsilon_s^i(z)}{\partial z} = M^G\left(\frac{v_s^i(z)}{V_s}\right)\left(\Omega^i(z)\frac{\partial \Phi_z(z)}{\partial z} + \Omega^o[\zeta^{i\to o}(z)]\frac{\mathrm{d}\Phi_{\hat{z}}[\zeta^{i\to o}(z)]}{\mathrm{d}z}\right). \tag{B.40}$$

Here as well, ignoring for now the skill complementarity, (B.30) and (B.40) together with (B.31) form a system of differential equations which, subject to the boundary conditions $\underline{w}_s^i(\underline{z}) = \underline{w}_s$ and $\Upsilon_s^i(\underline{z}) = 0$, returns the policy functions $(w_s^i, v_s^i)$ and the job ladder ranks $\Upsilon_s^\theta$.

[8]While we cannot guarantee that this assumption is satisfied for all calibrations of the model, we always check numerically whether there exists profitable deviations for $\underline{z}$ and $\underline{\hat{z}}$ and find that these do not exist.

### B.7.4 Algorithm

The algorithm has four levels of iteration. The most inner level solves for the policy functions and wage offer distributions using the system of differential equations obtained in Section B.7.3 while taking into account the complementarity across skills. The second most inner levels iterate on the aggregate number of vacancy, $\{V_s\}_s$, and the production cutoffs, $\underline{z}$ and $\hat{\underline{z}}$. The intermediate levels iterate on the indifference function $\varphi$, the reservation wages $\{\underline{w}_s\}_s$, and the productivity cutoff of contractor firms, $\underline{z}^C$. Finally, the outer level iterates on the marker clearing condition to solve for the price of outsourcing $p$.

**Differential equations** Given an outsourcing price $o$, the function $\varphi$, the reservation wages $\{\underline{w}_s\}_s$, the aggregate vacancies $\{V_s\}_s$ and the productivity cutoffs $\underline{z}$, $\hat{\underline{z}}$ and $\underline{z}^C$, the most-inner loop iterates forward on the differential equations to solve for the policy functions and the wage offer distributions. In particular, we iterate twice on the grid of productivity: one time to solve for the policy functions of good producers that hire their service workers in-house, and a second time to solve for the policy functions of good producers that outsource their service workers.

For the first iteration, we need to first find the initial conditions $\underline{w}_1^i$ and $\Upsilon_1^i$ depending on whether the least productive good producers offer the reservation wage to service workers. To find these boundary conditions, we proceed as follows. For each $z \geq \underline{z}^C$ starting from $\underline{z}^C$ for which we know that $w_1^C(\underline{z}^C) = \underline{w}_1$ and $\Upsilon_1^C(\underline{z}^C) = 0$:

1. Compute the policy functions of the least productive good producer with productivity $\underline{z}$ as if this firm was offering wages $w_1^i(\underline{z}) = w_1^C(z)$ and $w_s^i(\underline{z}) = \underline{w}_s$ for $s > 1$:

   (a) Compute $\mathrm{n}_s^i(\underline{z})$ for each $s$. For $s > 1$, this is $\mathrm{n}_s^i(\underline{z}) = \mathrm{n}_s(\underline{w}_s)$. For $s = 1$, this is $\mathrm{n}_1^i(\underline{z}) = \mathrm{n}_s[w_1^C(z)]$.

   (b) Compute $\{v_s^i(\underline{z})\}_s$ jointly by solving (B.31) using a non-linear solver.

   (c) Compute firm output and marginal products of labor.

2. Check if condition (B.37) holds at $z$.

   (a) If it does, set $w_1^i(\underline{z}) = w_1^C(z)$ and start the following algorithm.

   (b) If not, continue.

3. Compute $\partial_z w_1^C(z)$ and $\partial_z \Upsilon_1^C(z)$ from (B.32) and (B.36) respectively and go back to step 1.

Once the initial conditions $\underline{w}_1^i = w_1^C(\tilde{z}^C)$ and $\Upsilon_1^i = \Upsilon_1^C(\tilde{z}^C)$ have been found, we can iterate once on the differential equations for the good producers that hire their service workers in-house. The starting point for the wage equivalence functions are $\zeta^{i\to o}(\underline{z}) = \hat{\underline{z}}$ and $\zeta^{G\to C}(\underline{z}) = \tilde{z}^C$. Then, for any $z \geq \underline{z}$, given that we know $\{w_s^i(z)\}_s$, $\{\Upsilon_s^i(z)\}_s$ and the wage equivalence functions:

1. Compute $\{\mathrm{n}_s^i(z)\}_s$ from the labor supplies.

2. Compute $\{v_s^{Fi}(z)\}_s$ jointly by solving (B.31) using a non-linear solver.

3. Compute $\zeta^{G\to C}(z')$ from (B.34), where $z' = z + \Delta_z$ is the next point on the grid of $z$, and $R^i(z')$ and $n_1^i(z')$ are obtained from linear extrapolation of $\{n_s^i(z - \Delta_z), n_s^i(z)\}_s$. Similarly, compute $\zeta^{i\to o}(z')$ from (B.39).

4. Use (B.35) to compute $\Upsilon_1^i(z')$. Similarly, for $s > 1$, use (B.40) to compute $\Upsilon_s^i(z')$

5. Compute wages $\{w_s^i(z')\}_s$ from the wage ODEs (B.30).

Once this iteration over the $z$'s is finished, proceed to iterate over the $\hat{z}$ to compute the policy functions for good producers that outsource their service workers. Given that these firms do not hire service workers, the first step to find the lower bounds of $w_1^o(\hat{\underline{z}})$ and $\Upsilon_1^o(\hat{\underline{z}})$ is not necessary. The remaining steps are identical except for step 4: in this iteration, it is not required to compute $\zeta^{G\to C}$ but it is needed to find the numerical inverse of $\zeta^{i\to o}$. Once this iteration is finished, we have recovered all the policy functions for the good producers, $\{w_s^\theta(z), v_s^\theta(z)\}_{s\in\{1,\ldots,S\},\theta\in\{i,o\}}$, and the firms' rank on the job ladders, $\{\Upsilon_s^\theta(z)\}_{s\in\{1,\ldots,S\},\theta\in\{i,o\}}$.

The fact that we only need two iterations allows for a fast computation of the equilibrium despite its complexity.

**Inner iteration** Given an outsourcing price $o$, the function $\varphi$, the reservation wages $\{\underline{w}_s\}_s$ and the productivity cutoff of contractor firms $\underline{z}^C$, the inner iteration solves for the aggregate vacancies $\{V_s\}_s$ and the productivity cutoffs. In particular, $\{V_s\}_s$ need to be consistent with equations (B.24) and (B.26), while $\underline{z}$ and $\hat{\underline{z}}$ are given in condition 6 of Definition 1.

**Intermediate iteration** Given an outsourcing price $o$, the intermediate iteration solves for the indifference function, $\varphi$, the reservation wages, $\{\underline{w}_s\}_s$ and the productivity cutoffs of contractor firms $\underline{z}^C$. Specifically, given the profit functions $\pi^i$ and $\pi^o$, the function $\varphi$ is found through numerical inversion of the condition $\pi^i(z) = \pi^o[\varphi(z)]$. Then, from the policy functions $\{w_s^\theta(z), v_s^\theta(z)\}_{s\in\{1,\dots,S\},\theta\in\{i,o\}}$ and the updated indifference function $\varphi$, we compute the wage offer distributions $\{F_s\}_s$ that we then use to update the reservation wages according to (B.27). Finally, the productivity cutoff of contractor firms is computed as $\underline{z}^C = \underline{w}_1/(p\tau)$.

**Outer iteration** The outer iteration solves for the price of outsourcing through the market clearing condition (B.28). The supply of outsourcing services is computed directly from the contractors' policy functions on $[\underline{z}^C, \tilde{z}^C)$ and from the good producers' policy functions through $\zeta^{G\to C}$ for $z \geq \tilde{z}^C$.

### B.7.5 Estimation of parameters

As described in the main text, the estimation of the parameters is broken down into three steps. The first step inverts some equations of the model to directly estimates $\{\delta_s\}_{s=1}^3$ and $\{\zeta_s\}_{s=1}^3$ from the data (see Section B.7.6 to derive a closed form expression for the EE rate). The second step sets $\xi = 0.5$. Finally, the third step estimates together the remaining parameters via a minimum distance estimator. The remaining parameters can be divided into two sets. The first set of parameters can be estimated by model inversion given the

other parameters. This set consists of the matching efficiency, $\{\mu_s\}_s$, and the unemployment benefits, $\{b_s\}_s$. To recover the matching efficiency, note that, in equilibrium, unemployed workers always accept the job offers that they receive, so that $\lambda_s^U = \text{NE}_s$. Furthermore, in the model, $\lambda_s^U = \mathcal{M}(m_s[u_s + \zeta_s(1-u_s)], V_s)/m_s[u_s + \zeta_s(1-u_s)]$. Together, we obtain

$$\mu_s = \text{NE}_s \left( \frac{m_s[u_s + \zeta_s(1-u_s)]}{V_s} \right)^{1-\xi}.$$

Given $V_s$, the above equation identifies $\mu_s$. To recover the unemployment benefits, rewrite the expression for the reservation wage (B.27) as

$$\underline{w}_s = \text{RE}_s \mathbb{E}[w_s] + \left(k_s^U - k_s\right) \int_{\underline{w}_s} \frac{1 - F_s(w)}{1 + k_s(1 - F_s(w))} \mathrm{d}w,$$

where $\text{RE}_s \equiv b_s/\mathbb{E}[w_s]$ is the replacement rate and is targeted in the estimation. Importantly, the above expression is the only equation in which $b_s$ appears and $\mathbb{E}[w_s]$ is independent of $b_s$ up to $\underline{w}_s$. Hence, by directly setting $\text{RE}_s$, it is possible to compute the reservation wage without knowing $b_s$ and to recover $b_s$ as a residual,

$$b_s = \underline{w}_s - (k_s^U - k_s) \int_{\underline{w}_s} \frac{1 - F_s(w)}{1 + k_s(1 - F_s(w))} \mathrm{d}w.$$

To estimate the second set of parameters, we define the loss function

$$\mathcal{L}(\boldsymbol{\theta}) = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left[ h_n(\boldsymbol{\theta}) - \hat{h}_n \right]^2},$$

where $\boldsymbol{\theta}$ is the vector of parameter to be estimated, $\{\hat{h}_n\}_{n=1}^N$ is the set of empirical moments we are targeting, and $h : \mathbb{R}^N \mapsto \mathbb{R}^N$ maps parameters into simulated moments from our model. The simulated moments are computed as exact analogs of the empirical moments. To compute the simulated moments, we simulate a dataset in the $(z, \varepsilon)$ space, projecting onto this space the policy functions found in the $(z, \hat{z})$ space using linear interpolations.

To find the minimum of $\mathcal{L}$, we use a gradient descent algorithm. That is, starting from $\boldsymbol{\theta}^0$, we obtain a sequence of parameters $\{\boldsymbol{\theta}^j\}_j$ by iterating on $\boldsymbol{\theta}^{j+1} = \boldsymbol{\theta} - \gamma_j \nabla\mathcal{L}(\boldsymbol{\theta})$, where the endogenous step size follows the Barzilai–Borwein method. Namely, for $j > 1$,

$$\gamma_j = \frac{\max\left\{|\boldsymbol{\theta}^j - \boldsymbol{\theta}^{j+1}|^T \cdot |\nabla\mathcal{L}(\boldsymbol{\theta}^j) - \nabla\mathcal{L}(\boldsymbol{\theta}^{j-1})|, 10^{-3}\right\}}{\|\nabla\mathcal{L}(\boldsymbol{\theta}^j) - \nabla\mathcal{L}(\boldsymbol{\theta}^{j-1})\|^2}.$$

We impose a maximal step size as in Burdakov et al. (2019) to stabilize the descent. The gradient of the loss function is approximated with central finite difference to maximize accuracy. Given that we use $N = 13$ parameters, the loss function $\mathcal{L}(\boldsymbol{\theta})$ is high-dimensional and we cannot check for the existence of local minima. To avoid those, we first search manually to start the algorithm from a $\boldsymbol{\theta}^0$ with a relatively low loss function, in practice $\mathcal{L}(\boldsymbol{\theta}^0) \in [0.3, 0.5]$. The gradient descent attains its minimum at $\mathcal{L}(\boldsymbol{\theta}^*) = .085$. The gradient descent is implemented in Julia and parallelized over 6 CPUs. The descent is run on a standard laptop and takes about one hour to converge.

### B.7.6 Estimation: expression for the employment-employment transition rate

Omit $s$ indices for simplicity. Our argument requires only that the economy be stationary. Index firms by their wage offer $w$ and thie vacancy decision $v$. Denote $H(v|w)$ the conditional c.d.f. of vacancies given the wage offer. Then

$$EE = \frac{\lambda^E \iint n(w,v)(1 - F(w))dF(w)H(dv|w)}{\iint n(w,v)dF(w)H(dv|w)}.$$

The integral over $H(dv|w)$ produces the vacancy share of goods producers in the numerator and denominator, and hence drops out. Hence,

$$EE = \frac{\lambda^E \int \frac{(1+k)e}{(1+k(1-F(w)))^2}(1 - F(w))dF(w)}{\int \frac{(1+k)e}{(1+k(1-F(w)))^2}dF(w)} = \frac{\lambda^E \int_0^1 \frac{(1-F)dF}{(1+k(1-F))^2}}{\int_0^1 \frac{dF}{(1+k(1-F))^2}},$$

after changing variables to $F = F(w)$. Both integrals admit closed-form expressions, and thus:

$$EE = \lambda^E \frac{\big((1+k)\log(1+k) - k\big)\big/\big(k^2(1+k)\big)}{1\big/(1+k)} = \delta \frac{(1+k)\log(1+k) - k}{k}.$$

### B.7.7 Estimation of counterfactual

The baseline parameters of our model are estimated as if the economy was in 2002. To quantify the effects of the rise of outsourcing between 1996 and 2007, we therefore estimate two outsourcing shocks: a negative outsourcing shock that pushes back the economy to 1996, and a positive outsourcing shock that brings the economy to 2007. To compute both shocks, we estimate the mixture of parameter changes that matches the (rescaled) elasticities presented in Table 2.2. Specifically, let $o(\boldsymbol{\theta})$ be the outsourcing share and $\boldsymbol{X}(\boldsymbol{\theta})$ the variables of interest (e.g. log value added, log employment, etc.) under the parameter vector $\boldsymbol{\theta}$. For each shock, the new vector of parameters $\boldsymbol{\theta}'$ as to be such that

$$\begin{aligned} \frac{\boldsymbol{X}(\boldsymbol{\theta}') - \boldsymbol{X}(\boldsymbol{\theta})}{o(\boldsymbol{\theta}') - o(\boldsymbol{\theta})} &= \boldsymbol{\gamma}, \\ o(\boldsymbol{\theta}') - o(\boldsymbol{\theta}) &= \Delta_o, \end{aligned}$$

where $\boldsymbol{\gamma}$ are the cross-industry scaled elasticity and $\Delta_o$ is the size of the outsourcing shock considered. To implement a fast estimation of the shocks, we solve locally for $\boldsymbol{\theta}'$. Taking a first order approximation around $\boldsymbol{\theta}$, the system above becomes

$$\begin{aligned} \left[\boldsymbol{D}\boldsymbol{X}(\boldsymbol{\theta}) - \boldsymbol{\gamma}\boldsymbol{D}o(\boldsymbol{\theta})\right](\boldsymbol{\theta}' - \boldsymbol{\theta}) &= \mathbf{0}, \\ \boldsymbol{D}o(\boldsymbol{\theta})(\boldsymbol{\theta}' - \boldsymbol{\theta}) &= \Delta_o, \end{aligned} \tag{B.41}$$

where $\boldsymbol{D}$ is the differential operator with respect to $\boldsymbol{\theta}$ and $\boldsymbol{D}\boldsymbol{X}$ and $\boldsymbol{D}o$ can be computed by perturbating the economy around its 2002 steady state. We can then invert this linear system to recover $\{\boldsymbol{\theta}_{1996}, \boldsymbol{\theta}_{2007}\}$. To construct the counterfactual reported in Section 2.6, we fit cubic

polynomials on $\{\boldsymbol{\theta}_{1996}, \boldsymbol{\theta}_{2002}, \boldsymbol{\theta}_{2007}\}$ for each dimension of $\boldsymbol{\theta}$. We then use these polynomials to infer a vector of shock for each year in the 1996-2007 time frame. We also use the fitted cubic polynomials to extrapolate the consequences of outsourcing till 2016.

### B.7.8 Accounting

This section details how the main micro and macro variables are computed in the model. For that, suppose that we have simulated a cross-sectional data set at the firm level from our model. Let $i$ and $j$ describes the identity of a firm in this data set.

**Goods producers profits and value added.** Profits of goods producer $i$ are

$$\text{Profits}_i^G = R_i^G - \sum_s w_{is}^G n_{is}^G - p\varepsilon_i n_i^G - \sum_s c(v_{is}^G) - \eta,$$

where the notation follows closely that of Section B.7.1. The value added of goods producer $i$ is revenues net of spending on intermediaries, or

$$\text{VA}_i^G = R_i^G - p\varepsilon_i n_i^G.$$

The variable $\varepsilon_i$ indeed represents iceberg costs faced by a given goods producer $i$. Goods producer thus $i$ needs to purchase $\varepsilon_i n_i^G$ units of labor in the labor service market to obtain $n_i^G$ units of effective labor in production.

**Contractor profits and value added.** Contractors $j$ make profits

$$\text{Profits}_j^C = R_j^C - w_j^C n_j^C - c(v_j^C) = p\tau z_j n_j^C - w_j^C n_j^C - c(v_j^C),$$

and have value added

$$\text{VA}_j^C = R_j^C = p\tau z_j n_j^C.$$

**Aggregate output.** Aggregate output is the sum of value added of all sectors of the economy. Aggregate output coincides with the amount of goods available for consumption for workers, who receive wage payments, and capital owners, who receive vacancy costs and fixed costs. Thus,

$$\begin{aligned}
\text{Ag. output} &= \sum_i \text{VA}_i^G + \sum_j \text{VA}_j^C \\
&= \sum_i \left( R_i^G - p\varepsilon_i n_i^G \right) + p \sum_j \tau z_j n_j^C \\
&= \sum_i \left( R_i^G - p\varepsilon_i n_i^G \right) + p \sum_i \varepsilon_i n_i^G \\
&= \sum_i R_i^G
\end{aligned} \tag{B.42}$$

where the first equality uses the definitions of value added, and the second equality uses labor services market clearing (B.28).

**Aggregate TFP** We define TFP as

$$\text{TFP} = \frac{\text{Ag. output}}{\overline{N}}, \tag{B.43}$$

where the labor aggregator $\overline{N}$ is defined as

$$\overline{N} = \left( \prod_s \overline{N}_s^{a_s} \right)^{\rho},$$

and $\overline{N}_s = m_s(1 - u_s)$ is employment of skill $s$. To capture reallocation towards less productive contractors, we define the adjusted aggregator

$$\widetilde{N} = \widetilde{N}_1^{\rho a_1} \left( \prod_{s=2}^{3} \overline{N}_s^{a_s} \right)^{\rho} \quad , \quad \widetilde{N}_1 = \overline{N}_1^G + \tau_1 \overline{N}_1^C,$$

where $\overline{N}_1^G$ denotes aggregate employment of skill 1 by goods producers, and $\overline{N}_1^C$ aggregate employment by contractors. The effective measure $\widetilde{N}_1^G$ encodes the effective amount of labor used for task 1 in the economy. The ratio

$$\frac{\widetilde{N}}{\overline{\overline{N}}} = \left(\frac{\widetilde{N}_1}{\overline{\overline{N}}_1}\right)^{\rho a_1} = \left(\tau_1 x_1^C + (1 - x_1^C)\right)^{\rho a_1},$$

where $x_1^C = \overline{N}_1^C / \overline{N}_1$ is the employment share of contractors among low skill service workers, captures the TFP effect of reallocation towards more or less productive contractors. When $\tau_1 < 1$ and $x_1^C$ rises as outsourcing increases, $\widetilde{N}/\overline{N}$ decreases: workers are reallocated towards less productive jobs as far as production of labor services is concerned. Aggregate TFP then writes

$$\text{TFP} = \frac{\text{Ag. output}}{\widetilde{N}} \times \frac{\widetilde{N}}{\overline{\overline{N}}},$$

and so changes in aggregate TFP are

$$\Delta \log \text{TFP} = \underbrace{\Delta \log \frac{\text{Ag. output}}{\widetilde{N}}}_{\substack{\text{Allocative efficiency} \\ \text{given effective labor} \\ \text{in the economy}}} + \underbrace{\Delta \log \frac{\widetilde{N}}{\overline{\overline{N}}}}_{\substack{\text{Productivity gains/losses} \\ \text{from contractor comparative} \\ \text{advantage/disadvantage:} \\ \text{change in effective labor}}}.$$

# Appendix C

# Supply Chain Resilience: Should Policy Promote International Diversification or Reshoring?

This appendix is organized according to sections in the body of the chapter in order to make it easy for the reader to find these items.

## C.1 A Simple Model of Risky Supply Chains

### C.1.1 Supply Chain Management

We begin by deriving expected profits $\Pi_j := \Pi_j(\boldsymbol{\mu})$, $j \in \{f, h, b\}$, where $\boldsymbol{\mu} := (\mu_h, \mu_f, \mu_b)$. To this end, recall the profit function (3.7),

$$\pi^{J,K}(\omega) = \frac{s[p(\omega)/A]}{\sigma[p(\omega)/A]}(P^J)^{1-\varepsilon},$$

for $J \in \{H, J, B\}$ the state of the world and $K \in \{H, F\}$ the country from which the input is supplied from. Given a state $J$, all firms sourcing from the same location choose the same

prices. In states $H$ and $F$, only sourcing from one country is feasible, and we use $\pi^J := \pi^{J,J}$ to denote the realized profits in state $J$ for firms that have an active supply chain in country $J$.

First consider a state $J \in \{H, F\}$ in which supply chains from one country are disrupted but not so in the other country. In such a state, only firms that adopted a strategy of investing only in country $J$ and those that invested in both countries might be able to produce, provided that their bilateral relations do not suffer an idiosyncratic shock. Each such firm pays $q_J$ for its input. In this case, the market clearing condition (3.4) becomes

$$1 \equiv n^J(\boldsymbol{\mu})\, s[z^J(\boldsymbol{\mu})], \qquad J \in \{H, F\}\ , \tag{C.1}$$

where

$$n^J(\boldsymbol{\mu}) = (\mu_j + \mu_b)\,\rho$$

and $z^J = p^J/A^J$. These equations yield relative prices $z^J$ in state $J \in \{H, F\}$ as functions of $\boldsymbol{\mu}$, denoted $z^J(\boldsymbol{\mu})$.

Next note that, in state $J \in \{H, F\}$, the price index (3.5) can be expressed as

$$\log P^J = C_P + \log \frac{p^J}{z^J} - n^J \int_{z^J}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta,$$

where from (3.6),

$$p^J = \frac{\sigma\left(z^J\right)}{\sigma\left(z^J\right) - 1} q_J.$$

Using the function $z^J(\boldsymbol{\mu})$ and (C.1), we can express the price index $P^J$ as a function of $z^J(\boldsymbol{\mu})$,

$$\log P^J\left[z^J(\boldsymbol{\mu})\right] := C_P + \log \frac{\sigma\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right] - 1} + \log \frac{q_J}{z^J(\boldsymbol{\mu})} - \frac{1}{s\left[z^J(\boldsymbol{\mu})\right]} \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta,\ J \in \{H, F\}. \tag{C.2}$$

This function, together with (3.7) and $z^J(\boldsymbol{\mu})$, can be used to compute the profits of an active

firm in state $J$, which are

$$\pi^J\left[z^J(\boldsymbol{\mu})\right] := \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon}, \quad J \in \{H, F\}. \tag{C.3}$$

The functions $P^J(z)$ and $\pi^J(z)$, defined in (C.2) and (C.3), are decreasing in $z$. To see this, differentiate $\log P^J(z)$ with respect to $z$, which yields[1]

$$\frac{1}{P^J(z)}\frac{dP^J(z)}{dz} = -\frac{\sigma'(z)}{\sigma(z)\left[\sigma(z)-1\right]} + \frac{s'(z)}{s(z)^2}\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta < 0,\ J \in \{H, F\}. \tag{C.4}$$

Next, differentiate $\log \pi^J(z)$ with respect to $z$, which gives

$$\frac{1}{\pi^J(z)}\frac{d\pi^J(z)}{dz} = -\frac{\sigma'(z)}{\sigma(z)}\frac{\sigma(z)-\varepsilon}{\sigma(z)-1} + \frac{s'(z)}{s(z)}\left[1 - \frac{\varepsilon-1}{s(z)}\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right],\ J \in \{H, F\}. \tag{C.5}$$

Equation (3.3) in the main text implies (see Matsuyama and Ushchev (2020)):

$$\frac{s(\zeta)}{\zeta} = \frac{s'(\zeta)}{1-\sigma(\zeta)}.$$

Therefore

$$\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \int_{z^J}^{\bar{z}} \frac{-s'(\zeta)}{\sigma(\zeta)-1} d\zeta < \int_{z^J}^{\bar{z}} \frac{-s'(\zeta)}{\sigma(z^J)-1} d\zeta = \frac{s(z)-s(\bar{z})}{\sigma(z)-1} = \frac{s(z)}{\sigma(z)-1}. \tag{C.6}$$

Using this inequality, we obtain

$$\frac{1}{\pi^J(z)}\frac{d\pi^J(z)}{dz} < \left[\frac{s'(z)}{s(z)} - \frac{\sigma'(z)}{\sigma(z)}\right]\frac{\sigma(z)-\varepsilon}{\sigma(z)-1} < 0,\ J \in \{H, F\},$$

which we summarize in the following

**Lemma C.1.** *The functions $P^J(z)$ and $\pi^J(z)$ are declining in $z$ for $J \in \{H, F\}$.*

In state $B$, in which supply chains from both countries are viable, diversified firms prefer to

[1]Recall that $\sigma(z) > \varepsilon$ at our equilibrium points while $\sigma'(z) \geq 0$ and $s'(z) < 0$.

source from the cheaper country $F$ (recall that $q_F < q_H$), if they can. In this case, the number of firms that source from $F$ and pay $q_F$ for their inputs is $n^{B,F}(\boldsymbol{\mu}) = (\mu_f + \mu_b)\,\rho$. The number of firms that source from country $H$ and pay $q_H$ for inputs is $n^{B,H}(\boldsymbol{\mu}) = \mu_h\rho + \mu_b\rho\,(1-\rho)$. The market clearing condition (3.4) implies

$$1 \equiv n^{B,H}(\boldsymbol{\mu})\, s[z^{B,H}(\boldsymbol{\mu})] + n^{B,F}(\boldsymbol{\mu}) s[z^{B,F}(\boldsymbol{\mu})]\ , \tag{C.7}$$

which is equation (3.14) in the main text. The pricing equation (3.6) implies

$$\frac{z^{B,H}(\boldsymbol{\mu})}{z^{B,F}(\boldsymbol{\mu})} \equiv \left\{\frac{\sigma[z^{B,H}(\boldsymbol{\mu})]}{\sigma[z^{B,H}(\boldsymbol{\mu})]-1}\right\} \Big/ \left\{\frac{\sigma[z^{B,F}(\boldsymbol{\mu})]}{\sigma[z^{B,F}(\boldsymbol{\mu})]-1}\right\} \frac{q_H}{q_F}\ . \tag{C.8}$$

From here, we obtain solutions to the relative prices $z^{B,i}$, $i = H, F$, as functions of the vector $\boldsymbol{\mu}$, $z^{B,i}(\boldsymbol{\mu})$, $i \in \{H, F\}$. Furthermore, equation (C.8) implies that prices of the goods produced with inputs from country $F$ are strictly cheaper than goods produced with inputs from country $H$ in state $B$. To see this, suppose not, such that $p^{B,H} \leq p^{B,F}$ and therefore $z^{B,H} \leq z^{B,F}$. Equation (C.8) then returns

$$p^{B,H} = \left(\frac{\sigma(z^{B,H})}{\sigma(z^{B,H})-1}\right) q_H \leq \left(\frac{\sigma(z^{B,F})}{\sigma(z^{B,F})-1}\right) q_F = p^{B,F} < \left(\frac{\sigma(z^{B,F})}{\sigma(z^{B,F})-1}\right) q_H.$$

However, the mark-up function $z \to \sigma(z)/(\sigma(z)-1)$ is (weakly) increasing in $p$ under Assumption 3.2, thus contradicting the strict inequality above. It follows that, for any vector $\boldsymbol{\mu}$, we have $z^{B,H}(\boldsymbol{\mu}) > z^{B,F}(\boldsymbol{\mu})$.

To derive the price index (3.5) for state $B$, first note that the pricing equation (3.6) implies

$$\frac{1}{A^B(\boldsymbol{\mu})} = \frac{z^{B,i}(\boldsymbol{\mu})\left\{\sigma[z^{B,i}(\boldsymbol{\mu})]-1\right\}}{q_i \sigma[z^{B,i}(\boldsymbol{\mu})]}, \quad i \in \{H, F\}.$$

Using (3.14), we can write

$$\log A^B(\boldsymbol{\mu}) = \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) s\left[z^{B,i}(\boldsymbol{\mu})\right] \log\left\{\frac{q_i}{z^{B,i}(\boldsymbol{\mu})} \frac{\sigma\left[z^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,i}(\boldsymbol{\mu})\right]-1}\right\}.$$

Now, the price index (3.5) can be expressed as

$$\log P^B(\boldsymbol{\mu}) := \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu})\, s\left[z^{B,i}(\boldsymbol{\mu})\right] \log P^B\left[z^{B,i}(\boldsymbol{\mu})\right], \tag{C.9}$$

where the function $\log P^J(z)$ is defined in (C.2). Using this result for the price index, profits of a firm that sources from country $J$ in state $B$ amount to

$$\pi^{B,i}(\boldsymbol{\mu}) := \frac{s\left[z^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,i}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu})^{1-\varepsilon}, \quad i \in \{H,F\}. \tag{C.10}$$

Now consider expected profits from strategy $j$, $\Pi_j$, $j \in \{h,f,b\}$. For a firm that invests in a single supply chain, expected profits are

$$\Pi_h = \delta^H \pi^H \rho + \delta^B \pi^{B,H} \rho - k,$$

$$\Pi_f = \delta^F \pi^F \rho + \delta^B \pi^{B,F} \rho - k,$$

where $\delta^J$ is the probability that only supply chains from country $J$ will be available, $J \in \{H,F\}$, and $\delta^B$ is the probability that supply chains from both countries will be available. These probabilities are $\delta^H = \gamma_H(1-\gamma_F)$, $\delta^F = \gamma_F(1-\gamma_H)$, and $\delta^B = \gamma_F\gamma_H$. Using the profit functions (C.3) and (C.10), this yields

$$\Pi_h = \Pi_h(\boldsymbol{\mu}) := \delta^H \frac{s\left[z^H(\boldsymbol{\mu})\right]}{\sigma\left[z^H(\boldsymbol{\mu})\right]} P^H\left[z^H(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho + \delta^B \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu})^{1-\varepsilon} \rho - k, \tag{C.11}$$

$$\Pi_f = \Pi_f(\boldsymbol{\mu}) := \delta^F \frac{s\left[z^F(\boldsymbol{\mu})\right]}{\sigma\left[z^F(\boldsymbol{\mu})\right]} P^F\left[z^F(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho + \delta^B \frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu},\boldsymbol{q})^{1-\varepsilon} \rho - k. \tag{C.12}$$

For a firm that invests in supply chains in both countries, expected profits are

$$\Pi_b = \sum_{J=H,F} \delta^J \pi^J \rho + \delta^B \left[ \pi^{B,F} \rho + \pi^{B,H} (1-\rho) \rho \right] - 2k.$$

A firm that adopts this strategy expects profits $\pi^F$ if the supply chains survive only in country $F$, provided it does not suffer an idiosyncratic disruption there. Similarly, it expects profits $\pi^H$ if the supply chains survive only in country $H$, provided it does not suffer an idiosyncratic disruption there. In case supply chains in both countries are viable, the firm expects profits $\pi^{B,F}$ if its bilateral relation survives in country $F$ and profits $\pi^{B,H}$ if its bilateral relation in $F$ does not survive but that in $H$ does survive. Using (C.3) and (C.10), this yields

$$\begin{aligned} \Pi_b = \Pi_b(\boldsymbol{\mu}) := & \sum_{J=H,F} \delta^J \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J \left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho \\ & + \delta^B \left\{ \frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} + \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]} (1-\rho) \right\} P^B(\boldsymbol{\mu})^{1-\varepsilon} \rho - 2k. \end{aligned} \tag{C.13}$$

Using these functions and $P^J(\boldsymbol{\mu}) := P^J\left[z^J(\boldsymbol{\mu})\right], \ J \in \{H, F\}$, we obtain the welfare function (3.11) in the main text.

### C.1.2 The *Laissez-Faire* Equilibrium

Armed with these expressions, we now prove Figure 3.1. Suppose that $q_F \nearrow q_H$ and $\gamma_F < \gamma_H$. In this partially asymmetric world, given an aggregate state $J$, the country from which the input is supplied is irrelevant. In particular, in state $B$, equation (C.8) dictates that $z^{B,F} \to z^{B,H}$. As in state $H$ and $F$, we can then use the notation $z^B(\boldsymbol{\mu}) := z^{B,H}(\boldsymbol{\mu}) = z^{B,F}(\boldsymbol{\mu})$. Consequently, only the total number of products available in state $B$ matters, $n^B(\boldsymbol{\mu}) := n^{B,F}(\boldsymbol{\mu}) + n^{B,H}(\boldsymbol{\mu}) = \rho(1+(1-\rho)(1-\mu_h-\mu_f))$, and the market clearing condition (C.7) rewrites $1 = n^B(\boldsymbol{\mu}) s[z^B(\boldsymbol{\mu})]$.

Inasmuch as country $H$ is safer than country $F$, relatively more firms want to settle a single supply chain in $H$ than in $F$. To see this, suppose the contrary: firms invest relatively more in the risky country, $\mu_f > 0$ and $\mu_f \geq \mu_h$. From the market clearing conditions, we then have

$$s[z^F(\boldsymbol{\mu})] = \frac{1}{\rho(\mu_f + \mu_b)} \leq \frac{1}{\rho(\mu_h + \mu_b)} = s[z^H(\boldsymbol{\mu})].$$

Since $s'(z) < 0$, it must be that $z^H(\boldsymbol{\mu}) \leq z^F(\boldsymbol{\mu})$. But $\pi'(z) < 0$, so that $\delta^F \pi[z^F(\boldsymbol{\mu})) \leq \delta^F \pi[z^H(\boldsymbol{\mu})] < \delta^H \pi[z^H(\boldsymbol{\mu})]$. However, this in turn implies that the expected profits of the foreign strategy are lower, $\Pi_f < \Pi_h$, and therefore $\mu_f = 0$, a contradiction. Hence, in equilibrium, it must either be that no firms invest in the risky country, $\mu_f = 0$, or if some firms do, relatively more firms need to invest in the safe country, $\mu_f > 0$ and $\mu_h > \mu_f$. Accordingly, it must also be that the expected profits of the safer strategy are (weakly) higher than the expected profits of the less safe strategy, $\Pi_h \geq \Pi_f$.

Given that the expected profits of the home supply chain are weakly higher than those with a supplier in the foreign country, firms' investments are dictated by two comparisons: home versus foreign supply chains, $\Pi_h(\mu_h, \mu_f, \mu_b) - \Pi_f(\mu_h, \mu_f, \mu_b)$, and single supply chain at home versus diversification, $\Pi_h(\mu_h, \mu_f, \mu_b) - \Pi_b(\mu_h, \mu_f, \mu_b)$.[2] Using the expressions for expected profits (C.11:C.13), these two optimality conditions respectively read

$$\Pi_h(\mu_h, \mu_f, \mu_b) \geq \Pi_f(\mu_h, \mu_f, \mu_b) \quad \Longleftrightarrow \quad \delta^H \pi[z^H(\mu_h, \mu_f, \mu_b)] \geq \delta^F \pi[z^F(\mu_h, \mu_f, \mu_b)],$$

and

$$\Pi_b(\mu_h, \mu_f, \mu_b) \geq \Pi_h(\mu_h, \mu_f, \mu_b) \quad \Longleftrightarrow \quad \delta^F \pi[z^F(\mu_h, \mu_f, \mu_b)] + \delta^B \rho(1-\rho)\pi[z^B(\mu_h, \mu_f, \mu_b)] \geq k.$$

In addition, profits must be positive, $\Pi_j(\mu_h, \mu_f, \mu_b) \geq 0$ for $j \in \{h, f, b\}$. These three

[2]Without $\Pi_h \geq \Pi_f$, we would also need to compare $\Pi_f - \Pi_b$.

conditions together dictate the features of the equilibrium.

Figure 3.1 depicts the fraction of firms choosing each strategy as a function of $k$ when profits are unbounded; that is when $\lim_{z\to 0^+} \pi(z) = \infty$. We make this assumption throughout the proof, and come back to the case of bounded profit at the end of the section.

**Existence of $k_1$** For $k \to 0^+$, investing in resilience is clearly the most profitable option, $\Pi_b > \Pi_h > \Pi_f$, where the second inequality follows from $\delta^H \pi[z^H(0,0,1)] > \delta^F \pi[z^F(0,0,1)]$. Hence, for low $k$, $\mu_h = 0 = \mu_f$ and $\mu_b = 1$. As the fixed cost increases, the gap between $\Pi_h(0,0,1) - \Pi_b(0,0,1)$ shrinks to the point where the two strategies yield the same expected profits. This occurs at $k_1$, defined by

$$k_1 := \delta^F \rho \pi[z^F(0,0,1)] + \delta^B \rho(1-\rho)\pi[z^B(0,0,1)].$$

Furthermore, at $k_1$, excepted profits of strategy $h$ reads

$$\Pi_h = \delta^B \rho^2 \pi^B[z^B(0,0,1)] + \rho\left(\delta^H \pi[z^H(0,0,1)] - \delta^F \pi[z^F(0,0,1)]\right) > 0,$$

where the inequality follows from $z^H(0,0,1) = z^F(0,0,1)$ and $\delta^H > \delta^F$. Hence, $\Pi_b > 0$ for all $k \in [0, k_1]$.

**Existence of $k_2$** At $k = k_1$, we thus have $\Pi_b(0,0,1) = \Pi_h(0,0,1) > \Pi_f(0,0,1)$. For $k \in B^+(k_1)$, $\Pi_b(0,0,1) < \Pi_h(0,0,1)$, such that $(0,0,1)$ cannot be an equilibrium.[3] Instead, it must be that $\mu_h > 0$ and $\mu_b = 1 - \mu_h > 0$. When that is the case, firms must be indifferent

[3]The correspondence $B^+ : \mathbb{R} \mapsto 2^{\mathbb{R}}$ is defined as $B^+(x) = [x, x + \vartheta)$ for $\vartheta > 0$ small.

between the two strategies and must prefer them to the offshoring strategy,

$$\begin{aligned} k &= \delta^F \rho \pi[z^F(\mu_h, 0, 1-\mu_h)] + \delta^B \rho(1-\rho)\pi[z^B(\mu_h, 0, 1-\mu_h)], \\ 0 &> \delta^F \pi[z^F(\mu_h, 0, 1-\mu_h)] - \delta^H \pi[z^H(\mu_h, 0, 1-\mu_b)], \\ \mu_f &= 0,\ \mu_b = 1-\mu_h > 0. \end{aligned}$$

First, note that the first and second conditions imply that expected profits are positive. For instance, the expected profits of strategy $h$ is given by

$$\Pi_h = \delta^B \rho^2 \pi[z^B(\mu_h, 0, 1-\mu_h)] + \rho\left\{\delta^H \pi[z^H(\mu_h, 0, 1-\mu_h)] - \delta^F \rho \pi[z^F(\mu_h, 0, 1-\mu_h)]\right\} > 0,$$

where the first equality follows from the first condition and the inequality follows from the second condition. Second, when $\mu_h > 0$ and $\mu_b = 1-\mu_h > 0$, the market clearing conditions in states $F$, $H$ and $B$ read $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$, $s[z^H(\boldsymbol{\mu})]\rho = 1$ and $s[z^B(\boldsymbol{\mu})]\rho[1+(1-\mu_h)(1-\rho)] = 1$ respectively. Hence, $\pi[z^F(\boldsymbol{\mu})]$ and $\pi[z^B(\boldsymbol{\mu})]$ are increasing in $\mu_h$, while $\pi[z^H(\boldsymbol{\mu})]$ is independent of $\boldsymbol{\mu}$. This implies that $\mu_h$ is increasing in $k$ for $k \in B^+(k_1)$. Finally, unbounded profits imply that there exists a $\bar{\mu}_h^2$ such that the second condition cannot hold for any $\mu_h > \bar{\mu}_h^2$.[4] This limiting $\mu_h$ is defined by

$$\delta^H \pi(\bar{z}) = \delta^F \pi[z^F(\bar{\mu}_h^2, 0, 1-\bar{\mu}_h^2)].$$

Accordingly, we define the second fixed cost threshold $k_2$ as

$$k_2 = \delta^F \rho \pi[z^F(\bar{\mu}_h^2, 0, 1-\bar{\mu}_h^2)] + \delta^B \rho(1-\rho)\pi[z^B(\bar{\mu}_h^2, 0, 1-\bar{\mu}_h^2)].$$

For all $k \in B^+(k_2)$, we would have $\mu_h > \bar{\mu}_h^2$, which in turn would imply $\Pi_f > \Pi_h$ so that $(\mu_h, 0, 1-\mu_h)$ cannot be an equilibrium allocation.

[4] Otherwise, the equality would hold for $\mu_h \nearrow 1 \iff z^F \searrow 0$, at which point $\pi(z^F) \to \infty > \pi[z^H(1,0,0)]$, a contradiction.

**Existence of $k_3$** For $k \in B^+(k_2)$, the equilibrium must be of the type $\mu_h > 0$, $\mu_f > 0$ and $\mu_b = 1 - \mu_h - \mu_f > 0$. When that is the case, it must be that $\Pi_b = \Pi_h = \Pi_f$, or

$$\begin{aligned} k &= \delta^F \rho \pi[z^F(\boldsymbol{\mu})] + \delta^B \rho(1-\rho)\pi[z^B(\boldsymbol{\mu})], \\ 0 &= \delta^H[z^H(\boldsymbol{\mu})] - \delta^F \pi[z^F(\boldsymbol{\mu})], \\ 1 &\geq \mu_h + \mu_f(\mu_h). \end{aligned}$$

As before, note that the first and second conditions together imply that the expected profits of the three strategies are positive. The market clearing condition in each state are $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$, $s[z^H(\boldsymbol{\mu})]\rho(1-\mu_f) = 1$, and $s[z^B(\boldsymbol{\mu})]\rho[1+(1-\rho)(1-\mu_f-\mu_h)] = 1$. In particular, $z^F$, $z^H$ and $z^B$ are only functions of $\mu_h$, $\mu_f$ and $\mu_h+\mu_f$ respectively. Accordingly, let $\tilde{z}(\mu)$ and $\tilde{z}^B(\mu)$ be defined respectively by $s[\tilde{z}(\mu)]\rho(1-\mu) = 1$ and $s[\tilde{z}^B(\mu)]\rho(1+(1-\rho)(1-\mu)) = 1$. The first two equilibrium conditions then rewrite

$$\begin{aligned} k &= \delta^F \rho \pi[\tilde{z}(\mu_h)] + \delta^B \rho(1-\rho)\pi[\tilde{z}^B(\mu_h+\mu_f)], \\ 0 &= \delta^H \pi[\tilde{z}(\mu_f)] - \delta^F \pi[\tilde{z}(\mu_h)]. \end{aligned}$$

Both $\tilde{z}$ and $\tilde{z}^B$ are decreasing functions, such that $d\pi[\tilde{z}(\mu)]/d\mu > 0$. The second condition thus implies that $\mu_f$ is increasing in $\mu_h$, and the first condition implies that $\mu_h$ is increasing in $k$ for $k \in B^+(k_2)$. Finally, unbounded profits imply that there always exists an upper bound $\bar{\mu}_h^3$ such that

$$\delta^F \pi[\tilde{z}(\bar{\mu}_h^3)] = \delta^H \pi[\tilde{z}(1-\bar{\mu}_h^3)].$$

The first and third condition together then imply that there exists a fixed cost threshold $k_3$ such that $(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ is the equilibrium allocation, and $\Pi_b(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) < \Pi_h(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) =$

$\Pi_f(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ for $k \in B^+(k_3)$. This cutoff is defined by

$$k_3 = \delta^F \rho\pi[\tilde{z}(\bar{\mu}_h^3)] + \delta^B \rho(1-\rho)\pi[\tilde{z}^B(1)].$$

**Existence of $k_4$** At $k_3$, we have already argued that $\Pi_h(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) = \Pi_f(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) > 0$. Since $\Pi_h(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ is monotonically decreasing in $k$, there exists a $k_4 > k_3$ such that profits of the $h$ and $f$ strategy are nil,

$$k_4 = \delta^F \rho\pi[\tilde{z}(\bar{\mu}_h^3)] + \delta^B \rho\pi[\tilde{z}^B(1)].$$

**Beyond $k_4$** For $k \in B^+(k_4)$, it clearly cannot be that $(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ is an equilibrium. We first show that, locally, it must be that $\mu_h + \mu_f < 1$ and $\mu_b = 0$. For this to be an equilibrium, it clearly cannot be that $\Pi_h > 0$, for otherwise other firms would enter till either $\Pi_h = 0$ or $\mu_h + \mu_f = 1$. A symmetric argument exists for $\Pi_f$. Hence, for $k \in B^+(k_4)$, it must be that $\Pi_h = \Pi_f = 0 > \Pi_b$. Furthermore, when $\mu_h + \mu_f < 1$ and $\mu_b = 0$, the market clearing conditions in state $H$, $F$ and $B$ respectively read

$$s[z^H(\boldsymbol{\mu})]\rho\mu_h = 1, \quad s[z^F(\boldsymbol{\mu})]\rho\mu_f = 1, \quad s[z^B(\boldsymbol{\mu})](\mu_f + \mu_h)\rho = 1.$$

Let $\zeta(\mu)$ denote the (increasing) function that solves $s[\zeta(\mu)]\rho\mu = 1$. The equilibrium conditions $\Pi_h = \Pi_f = 0$ can then be written as

$$\begin{aligned} \delta^H \rho\pi[\zeta(\mu_h)] + \delta^B \rho\pi[\zeta(\mu_f + \mu_h)] &= k, \\ \delta^F \pi[\zeta(\mu_f)] - \delta^H \pi[\zeta(\mu_h)] &= 0, \\ \mu_f + \mu_h &< 1. \end{aligned}$$

From the second condition, $\mu_f$ is increasing in $\mu_h$. From the first condition, the right hand side is increasing in $k$ and the left-hand is decreasing in $\mu_h$. Hence, $\mu_h$ is decreasing in $k$ for

$k \in B^+(k_4)$. Finally, when profits are unbounded, as $k$ becomes infinitely large, the left-hand side of the first condition has to be large as well, which requires $\mu_h \searrow 0$. From the second condition, $\mu_h \searrow 0$ implies $\mu_f \searrow 0$. Hence, the two conditions above hold jointly for any $k > k_4$, which is depicted in Figure 3.1.

The proof of Figure 3.1 holds when profits are unbounded, $\lim_{z \searrow 0} \pi(z) = \infty$. Yet, with HSA preferences and $\varepsilon > 1$, profits may be bounded even as prices tend to zero. When that is the case, there may exist two further thresholds $k_5$ and $k_6$ such that, for $k$ between $k_5$ and $k_6$, we have $\mu_h > 0$ and $\mu_f = 0$, and for $k > k_6$, no firms enter, $\mu_f = \mu_h = \mu_b = 0$. Furthermore, when profits are bounded, the interval $(k_2, k_3)$ may be empty. However, $k_2 < k_3$ is guaranteed if the difference between $\gamma_F$ and $\gamma_H$ is not too large, or alternatively, if $\varepsilon - 1$ is small – in which case profits are necessarily unbounded. Numerically, in Section 3.5, we do find that $k_2 < k_3$ for relatively large risk premium and elasticity of substitution, namely $\gamma_F/\gamma_H = 0.7$ and $\varepsilon = 1.7$.

## C.2 The Unconstrained Social Optimum

We begin by deriving the social welfare function in the presence of consumption subsidies that equate consumer prices to marginal costs according to where inputs are sourced.

First, consider the pricing problem facing a producer that pays $q$ per unit for its inputs that faces an aggregator $A$ and that recognizes that consumers will pay only a fraction $\nu$ of the sticker price in view of the consumption subsidy at rate $1 - \nu$. Then the consumer price of the final product is $\nu p$, where $p$ is the producer price. As noted, the government sets the subsidy so that $\nu p = q$, and firms take the subsidy rate as given. They choose the sticker price as

$$p = \arg\max_{\breve{p}} P^{1-\varepsilon} s \left( \frac{\nu \breve{p}}{A} \right) (\nu \breve{p})^{-1} (\breve{p} - q) .$$

The solution to this problem yields

$$p = \frac{\sigma\left(p/A\right)}{\sigma\left(p/A\right) - 1} q.$$

Therefore, the optimal subsidy rates are

$$v\left(z^J\right) = \frac{\sigma\left(z^J\right) - 1}{\sigma\left(z^J\right)},\ J \in \{H, F\},$$

$$v\left(z^{B,i}\right) = \frac{\sigma\left(z^{B,i}\right) - 1}{\sigma\left(z^{B,i}\right)},\ i \in \{H, F\}.$$

These optimal subsidies vary across states of the world if the elasticity of substitution is not constant, and they vary in state $B$ according to the source of the inputs embodied in the final good.

We consider outcomes with $\boldsymbol{\mu} \gg 0$. Now, the market clearing conditions (3.12) to (3.14) must still be satisfied, but (C.8) is replaced with

$$\frac{z^{B,H}}{z^{B,F}} = \lambda := \frac{q_H}{q_F}. \tag{C.14}$$

It follows that the functions $z^J(\boldsymbol{\mu})$, $J \in \{H, F\}$ are the same as before, but the functions $z^{B,H}(\boldsymbol{\mu})$ and $z^{B,F}(\boldsymbol{\mu})$ are replaced by $\tilde{z}^{B,F}(\boldsymbol{\mu})$ and $\tilde{z}^{B,H}(\boldsymbol{\mu}) \equiv \lambda \tilde{z}^{B,F}(\boldsymbol{\mu})$, where the latter functions are obtained as solutions to (C.14) and (3.14). In what follows, we denote with a tilde any function that arise when the consumption subsidies are in place, except for those functions—like $z^H(\boldsymbol{\mu})$ and $z^F(\boldsymbol{\mu})$—that do not change as a result of the subsidies.

With the consumption subsidies in place, firms' operating profits in the various states are

$$\tilde{\pi}^J(\boldsymbol{\mu}) := \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right] - 1} \tilde{P}^J \left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon},\ J \in \{H, F\}, \tag{C.15}$$

$$\tilde{\pi}^{B,i}(\boldsymbol{\mu}) := \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right] - 1} \tilde{P}^B(\boldsymbol{\mu})^{1-\varepsilon},\ i \in \{H, F\}, \tag{C.16}$$

where, using (3.5), the price indexes are

$$\log \tilde{P}^J(\boldsymbol{\mu}) := \log \tilde{P}^J \left[ z^J (\boldsymbol{\mu}) \right] = C_P + \log \frac{q_J}{z^J (\boldsymbol{\mu})} - n^J (\boldsymbol{\mu}) \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \quad J \in \{H, F\}, \tag{C.17}$$

$$\log \tilde{P}^B(\boldsymbol{\mu}) = C_P + \log \tilde{A}^B(\boldsymbol{\mu}) - \sum_{i=H,F} n^{B,i} (\boldsymbol{\mu}) \int_{z^{B,i}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \tag{C.18}$$

and $\tilde{A}^B(\boldsymbol{\mu})$ is obtained from

$$1 \equiv n^{B,H} (\boldsymbol{\mu}) \, s \left[ \frac{q_H}{\tilde{A}^B(\boldsymbol{\mu})} \right] + n^{B,F} (\boldsymbol{\mu}) \, s \left[ \frac{q_F}{\tilde{A}^B(\boldsymbol{\mu})} \right].$$

Therefore,

$$\tilde{A}^B(\boldsymbol{\mu}) \equiv \frac{q_F}{\tilde{z}^{B,F} (\boldsymbol{\mu})} \equiv \frac{q_F}{\tilde{z}^{B,F} (\boldsymbol{\mu})}. \tag{C.19}$$

Lump-sum taxes are levied in state $J$ to finance the consumption subsidies paid in that state. Using the subsidy rates $v(z) = [\sigma(z) - 1] / \sigma(z)$, the required taxes are

$$\begin{aligned}
\tilde{T}^H(\boldsymbol{\mu}) &= -(\mu_h + \mu_b) \, \tilde{\pi}^H(\boldsymbol{\mu}) \rho, \\
\tilde{T}^F(\boldsymbol{\mu}) &= -(\mu_f + \mu_b) \, \tilde{\pi}^F(\boldsymbol{\mu}) \rho, \\
\tilde{T}^B(\boldsymbol{\mu}) &= -(\mu_f + \mu_b) \, \tilde{\pi}^{B,F}(\boldsymbol{\mu}) \rho - [\mu_h + \mu_b (1 - \rho)] \, \tilde{\pi}^{B,H}(\boldsymbol{\mu}) \rho.
\end{aligned}$$

It follows that

$$\sum_{J=H,F,B} \delta^J \tilde{T}^J(\boldsymbol{\mu}) + \sum_{j=h,f,b} \mu_j \tilde{\Pi}_j(\boldsymbol{\mu}) = -(\mu_h + \mu_f + 2\mu_b) \, k.$$

The welfare function (3.11) therefore becomes

$$\tilde{W}(\boldsymbol{\mu}) = \bar{Y} + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \tilde{P}^J (\boldsymbol{\mu})^{1-\varepsilon} - (\mu_h + \mu_f + 2\mu_b)\, k. \tag{C.20}$$

We next characterize the wedges that determine optimal supply chain policies. To this end, we first derive the first-order conditions for the optimal allocation $\boldsymbol{\mu}^o \gg 0$, which are characterized by $\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = 0$, $j = h, f$, where, for a general function $G(\boldsymbol{\mu})$, $dG(\boldsymbol{\mu})/d\mu_j$ is the change in $G(\cdot)$ from the variation $d\mu_j = -d\mu_b > 0$. Using the price indexes (C.17) and (C.18), together with (3.12), (3.13) and (C.19), we obtain

$$\begin{aligned}
\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = & - \sum_{J=H,F} \delta^J \tilde{P}^J (\boldsymbol{\mu}^o)^{1-\varepsilon} \left[ \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right] \frac{dn^J(\boldsymbol{\mu}^o)}{d\mu_j} \\
& - \delta^B \tilde{P}^B (\boldsymbol{\mu}^o)^{1-\varepsilon} \left[ \frac{d \log \tilde{A}^B (\boldsymbol{\mu}^o)}{d\mu_j} + \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}^o) \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu}^o)\right]}{\tilde{z}^{B,i}(\boldsymbol{\mu}^o)} \frac{d\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} \right] \\
& + \delta^B \tilde{P}^B (\boldsymbol{\mu}^o)^{1-\varepsilon} \sum_{i=H,F} \left[ \int_{z^{B,i}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} + k = 0,
\end{aligned}$$

for $j \in \{h, f\}$. Note, however, that $d \log \tilde{z}^{B,F}(\boldsymbol{\mu}^o)/d\mu_j = d \log \tilde{z}^{B,H}(\boldsymbol{\mu}^o)/d\mu_j$. Then, using (3.14),

$$\begin{aligned}
& \frac{d \log \tilde{A}^B (\boldsymbol{\mu}^o)}{d\mu_j} + \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]}{\tilde{z}^{B,i}(\boldsymbol{\mu})} \frac{d\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} \\
= & - \frac{d \log \tilde{z}^{B,F}(\boldsymbol{\mu})}{d\mu_j} \left[ 1 - \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu})\, s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right] \right] = 0.
\end{aligned}$$

In other words,

$$\frac{d \log \tilde{P}^B (\boldsymbol{\mu})}{d\mu_j} = - \sum_{i=H,F} \left[ \int_{\tilde{z}^{B,i}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j}. \tag{C.21}$$

The first-order conditions for the first-best allocation can therefore be written as

$$\frac{d\tilde{W}\left(\boldsymbol{\mu}^{o}\right)}{d\mu_{j}}=-\sum_{J=H,F}\delta^{J}\tilde{P}^{J}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{z^{J}(\boldsymbol{\mu})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\frac{dn^{J}\left(\boldsymbol{\mu}^{o}\right)}{d\mu_{j}}$$
$$+\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\sum_{i=H,F}\left[\int_{\tilde{z}^{B,i}(\boldsymbol{\mu}^{o})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\frac{dn^{B,i}\left(\boldsymbol{\mu}^{o}\right)}{d\mu_{j}}+k=0,$$

for $j=h,f$.

Next use $n^{F}\left(\boldsymbol{\mu}\right)=\left(1-\mu_{h}\right)\rho$, $n^{H}\left(\boldsymbol{\mu}\right)=\left(1-\mu_{f}\right)\rho$, $n^{B,F}\left(\boldsymbol{\mu}\right)=\left(\mu_{f}+\mu_{b}\right)\rho$, and $n^{B,H}\left(\boldsymbol{\mu}\right)=\left[\mu_{h}+\mu_{b}\left(1-\rho\right)\right]\rho$ to obtain $dn^{F}\left(\boldsymbol{\mu}\right)/d\mu_{f}=0$, $dn^{F}\left(\boldsymbol{\mu}\right)/d\mu_{h}=-\rho$, $dn^{H}\left(\boldsymbol{\mu}\right)/d\mu_{h}=0$, $dn^{H}\left(\boldsymbol{\mu}\right)/d\mu_{f}=-\rho$, $dn^{B,F}\left(\boldsymbol{\mu}\right)/d\mu_{f}=0$, $dn^{B,F}\left(\boldsymbol{\mu}\right)/d\mu_{h}=-\rho$, $dn^{B,H}\left(\boldsymbol{\mu}\right)/d\mu_{f}=-\left(1-\rho\right)\rho$, $dn^{B,H}\left(\boldsymbol{\mu}\right)/d\mu_{h}=\rho^{2}$ for $\boldsymbol{\mu}\gg0$. These expressions allow us to represent $d\tilde{W}\left(\boldsymbol{\mu}^{o}\right)/d\mu_{j}=0$ for $j\in\{h,f\}$ as

$$k=\delta^{H}\tilde{P}^{H}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{z^{H}(\boldsymbol{\mu}^{o})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\rho+\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{\tilde{z}^{B,H}(\boldsymbol{\mu}^{o})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\left(1-\rho\right)\rho \quad \text{(C.22)}$$

and

$$k=\delta^{F}\tilde{P}^{F}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{z^{F}(\boldsymbol{\mu}^{o})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\rho$$
$$+\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{\tilde{z}^{B,F}(\boldsymbol{\mu}^{o})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta-\rho\int_{\tilde{z}^{B,H}(\boldsymbol{\mu}^{o})}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\rho. \quad \text{(C.23)}$$

By definition,

$$w_{j}^{o}:=\tilde{\Pi}_{j}\left(\boldsymbol{\mu}^{o}\right)-\tilde{\Pi}_{b}\left(\boldsymbol{\mu}^{o}\right)-\frac{d\tilde{W}\left(\boldsymbol{\mu}^{o}\right)}{d\mu_{j}},\quad j\in\{h,f\}.$$

We therefore obtain

$$w_{f}^{o}=k-\delta^{H}\tilde{\pi}^{H}\left(\boldsymbol{\mu}^{o}\right)\rho-\delta^{B}\tilde{\pi}^{B,H}\left(\boldsymbol{\mu}^{o}\right)\left(1-\rho\right)\rho, \quad \text{(C.24)}$$

and

$$w_h^o = k - \delta^F \tilde{\pi}^F \left(\boldsymbol{\mu}^o\right) \rho - \delta^B \left[\tilde{\pi}^{B,F} \left(\boldsymbol{\mu}^o\right) - \rho \tilde{\pi}^{B,H} \left(\boldsymbol{\mu}^o\right)\right] \rho. \tag{C.25}$$

Next we use (C.15), (C.16) and (C.22) to derive

$$w_f^o = \delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \Phi \left[\tilde{z}^H \left(\boldsymbol{\mu}^o\right)\right] \rho + \delta^B \tilde{P}^B \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \Phi \left[\tilde{z}^{B,H} \left(\boldsymbol{\mu}^o\right)\right] (1-\rho) \rho, \tag{C.26}$$

where

$$\Phi(z) := \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{\sigma(z) - 1},$$

which is equation (3.17) in the main text. Moreover, using (C.15), (C.16) and (C.23), we obtain

$$\begin{aligned} w_h^o &= \delta^F \tilde{P}^F \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \Phi \left[\tilde{z}^F \left(\boldsymbol{\mu}^o\right)\right] \rho + \delta^B \tilde{P}^B \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \Phi \left[\tilde{z}^{B,H} \left(\boldsymbol{\mu}^o\right)\right] \rho (1-\rho) \\ &\quad + \delta^B \tilde{P}^B \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \left\{\Phi \left[\tilde{z}^{B,F} \left(\boldsymbol{\mu}^o\right)\right] - \Phi \left[\tilde{z}^{B,H} \left(\boldsymbol{\mu}^o\right)\right]\right\} \rho, \end{aligned} \tag{C.27}$$

which is equation (3.18) in the main text.

We now want to characterize the absolute and relative sign of these wedges. First, note that (C.6) implies $\Phi(z) < 0$ under Marshall's Second Law of Demand. Therefore, $w_f^o < 0$. Second,

$$\Phi'(z) = -\frac{s(z)}{z} - \frac{s'(z)}{\sigma(z) - 1} + \frac{s(z)}{[\sigma(z) - 1]^2} \sigma'(z) = \frac{s(z)}{[\sigma(z) - 1]^2} \sigma'(z) > 0.$$

Since $\tilde{z}^{B,H} \left(\boldsymbol{\mu}^o\right) = \lambda \tilde{z}^{B,F} \left(\boldsymbol{\mu}^o\right) > \tilde{z}^{B,F} \left(\boldsymbol{\mu}^o\right)$, this implies $\Phi \left[\tilde{z}^{B,F} \left(\boldsymbol{\mu}^o\right)\right] - \Phi \left[\lambda \tilde{z}^{B,F} \left(\boldsymbol{\mu}^o\right)\right] < 0$ and, therefore, $w_h^o < 0$. These findings are summarized in

**Lemma C.2.** *Let $\sigma'(z) > 0$ for $z \in (0, \bar{z})$. Then $w_j^o < 0$ for $j \in \{h, f\}$.*

Now consider two special cases. In the limiting case of symmetric CES preferences, $\sigma$

is constant and $s(z) := \alpha z^{1-\sigma}$, where $\alpha > 0$ is a constant. In this case $\Phi(z) = 0$ for all $z$ and thus $w_h^o = w_f^o = 0$. That is, the optimal allocation is achieved with no government intervention in the formation of supply chains; i.e., $\varphi_j = 0$ for $j \in \{h, f, b\}$.

In the case of symmetric translog preferences, $s(z) := -\theta \log z$, where $\theta > 0$ is a constant and $z \in (0, 1)$. These preferences imply

$$\int_z^1 \frac{s(\zeta)}{\zeta} d\zeta = \frac{1}{2} \frac{s(z)}{\sigma(z) - 1}.$$

The first-order conditions (C.22) and (C.23) become

$$2k = \delta^H \tilde{\pi}^H(\boldsymbol{\mu}^o) \rho + \delta^B \tilde{\pi}^{B,H}(\boldsymbol{\mu}^o)(1 - \rho)\rho,$$

$$2k = \delta^F \tilde{\pi}^F(\boldsymbol{\mu}^o) \rho + \delta^B \left[\tilde{\pi}^{B,F}(\boldsymbol{\mu}^o) - \rho \tilde{\pi}^{B,H}(\boldsymbol{\mu}^o)\right] \rho.$$

Combining these with (C.24) and (C.25) yields

$$w_f^o = w_h^o = -k. \tag{C.28}$$

That is, in the translog case, the optimal allocation is achieved by a policy that subsidizes fully the cost of all investments in single-country supply chains, i.e., $\varphi_b = 0$ and $\varphi_h = \varphi_f = k$.[5] We summarize these findings in

**Lemma C.3.** *(a) In the case of symmetric CES preferences, $w_j^o = 0$ for $j \in \{h, f\}$, which implies that $\varphi_j = 0$ for $j \in \{h, f, b\}$ induces the optimal allocation. (b) In the case of symmetric translog preferences $w_f^o = w_h^o = -k$, which implies that $\varphi_b = 0$ and $\varphi_h = \varphi_f = k$ induces the optimal allocation.*

Finally, consider the difference in the absolute sizes of the wedges. Using (C.26) and

[5] Alternatively, the planner can tax diversification with $\varphi_b = -k$, while leaving $\varphi_h = \varphi_f = 0$.

(C.27), we have

$$\begin{aligned} |w_h^o| - \left|w_f^o\right| &= \delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \Phi\left[\tilde{z}^H\left(\boldsymbol{\mu}^o\right)\right] \rho - \delta^F \tilde{P}^F \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \Phi\left[\tilde{z}^F\left(\boldsymbol{\mu}^o\right)\right] \rho \\ &\quad + \delta^B \tilde{P}^B \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \left\{\Phi\left[\tilde{z}^{B,H}\left(\boldsymbol{\mu}^o\right)\right] - \Phi\left[\tilde{z}^{B,F}\left(\boldsymbol{\mu}^o\right)\right]\right\} \rho. \end{aligned} \tag{C.29}$$

In the limit case $q_H \searrow q_F =: q$, the last term on the right-hand side of this equation equals zero. Moreover, the first-order conditions (C.22) and (C.23) imply

$$\delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \int_{z^H\left(\boldsymbol{\mu}^o\right)}^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta = \delta^F \tilde{P}^F \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \int_{z^F\left(\boldsymbol{\mu}^o\right)}^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta. \tag{C.30}$$

Therefore,

$$|w_h^o| - \left|w_f^o\right| = \delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \frac{s\left[z^H\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^H\left(\boldsymbol{\mu}^o\right)\right] - 1} \rho - \delta^F \tilde{P}^F \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \frac{s\left[z^F\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^F\left(\boldsymbol{\mu}^o\right)\right] - 1} \rho.$$

Using (C.30), this difference can be expressed as

$$|w_h^o| - \left|w_f^o\right| = \rho \delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \frac{\left\{\frac{s\left[z^H\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^H\left(\boldsymbol{\mu}^o\right)\right]-1}\right\} \left\{\frac{s\left[z^F\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^F\left(\boldsymbol{\mu}^o\right)\right]-1}\right\}}{\int_{z^F\left(\boldsymbol{\mu}^o\right)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta} \left\{\Psi\left[z^F\left(\boldsymbol{\mu}^o\right)\right] - \Psi\left[z^H\left(\boldsymbol{\mu}^o\right)\right]\right\}, \tag{C.31}$$

where

$$\Psi\left(z\right) := \int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta \Big/ \frac{s\left(z\right)}{\sigma\left(z\right) - 1}.$$

We have established

**Lemma C.4.** *Let $q_H \searrow q_F$. Then $|w_h^o| > |w_f^o|$ if and only if $\Psi\left[z^F\left(\boldsymbol{\mu}^o\right)\right] > \Psi\left[z^H\left(\boldsymbol{\mu}^o\right)\right]$.*

Next note from (C.17) that with equal costs in both countries, and $n^J\left(\boldsymbol{\mu}\right) s\left[z^J\left(\boldsymbol{\mu}\right)\right] = 1$,

$$\log \tilde{P}^J\left(\boldsymbol{\mu}\right) = \log \check{P}\left[z^J\left(\boldsymbol{\mu}\right)\right],$$

where

$$\log \check{P}(z) := C_P + \log \frac{q}{z} - \frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta.$$

It follows that $\check{P}(z) \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$ is a declining function of $z$. To see this, consider

$$\frac{d}{dz} \log \left\{ \check{P}(z)^{1-\varepsilon} \left[ \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right] \right\} = -(\varepsilon - 1) \frac{s'(z)}{s(z)^2} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{z \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta}.$$

We use

$$\frac{s'(z)}{s(z)} = -\frac{\sigma(z) - 1}{z}$$

to obtain

$$\frac{d}{dz} \log \left\{ \check{P}(z)^{1-\varepsilon} \left[ \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right] \right\} = (\varepsilon - 1) \frac{\sigma(z) - 1}{z s(z)} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{z \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta}.$$

Finally, from (C.6), we have

$$\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta < \frac{s(z)}{\sigma(z) - 1},$$

which implies

$$\begin{aligned} \frac{d}{dz} \log \left\{ P(z)^{1-\varepsilon} \left[ \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right] \right\} &= (\varepsilon - 1) \frac{\sigma(z) - 1}{z s(z)} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{z \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta} \\ &< -\frac{\sigma(z) - \varepsilon}{z} < 0. \end{aligned}$$

Applied to (C.30), this result implies

$$z^H(\boldsymbol{\mu}^o) > z^F(\boldsymbol{\mu}^o). \tag{C.32}$$

Therefore $|w_h^o| > |w_f^o|$ when $\Psi(z)$ is a decreasing function and $|w_h^o| < |w_f^o|$ when $\Psi(z)$ is an increasing function. We summarize this finding in

**Lemma C.5.** *Let $q_H \searrow q_F$ and $\sigma'(z) > 0$ for $z \in (0, \bar{z})$. If $\Psi'(z) < 0$ for all $z \in (0, \bar{z})$, then $|w_h^o| > |w_f^o|$ and if $\Psi'(z) > 0$ for all $z \in (0, \bar{z})$, then $|w_h^o| < |w_f^o|$.*

## C.3 The Constrained Social Optimum

We divide this section in two parts. First, we prove the theoretical foundation of Figure 3.2. Then, we derive the second-best supply chain policy.

Let $q_F \approx q_H$ and $\gamma_F \approx \gamma_H$ so that both the home and the foreign country are symmetric. When $\boldsymbol{\mu} \gg 0$, all strategies must yield equal expected profits, such that the following conditions must hold jointly

$$\begin{aligned}
\Pi_h(\mu_h, \mu_f, 1-\mu_h-\mu_f) &= \Pi_f(\mu_h, \mu_f, 1-\mu_h-\mu_f), \\
\Pi_h(\mu_h, \mu_f, 1-\mu_h-\mu_f) &= \Pi_b(\mu_h, \mu_f, 1-\mu_h-\mu_f), \\
\Pi_f(\mu_h, \mu_f, 1-\mu_h-\mu_f) &= \Pi_b(\mu_h, \mu_f, 1-\mu_h-\mu_f).
\end{aligned}$$

Using the expressions for expected profits (C.11) and (C.12), the first condition rewrites $\pi[z^H(\mu_h, \mu_f, 1-\mu_h-\mu_f))] = \pi[z^F(\mu_h, \mu_f, 1-\mu_h-\mu_f)]$, where the functiosn $z^H$ and $z^F$ solve, respectively, $s[z^H(\boldsymbol{\mu})]\rho(1-\mu_f) = 1$ and $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$. That is, the functions $z^H$ and $z^F$ are identical. Together with the monotonicity of $z \to \pi(z)$, this implies that $\Pi_h(\boldsymbol{\mu}) = \Pi_f(\boldsymbol{\mu})$ if and only if $\mu_h = \mu_f$.

Using (C.11) and (C.13), the second condition is

$$\delta\pi[z^F(\mu_h, \mu_f, 1-\mu_h-\mu_f)]\rho + \delta^B\pi[z^B(\mu_h, \mu_f, 1-\mu_h-\mu_f)](1-\rho)\rho = k.$$

The function $z^F$ and $z^B$ are given respectively by $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$ and $s[z^B(\boldsymbol{\mu})]\rho[1 + (1-\rho)(1-\mu_h-\mu_f)] = 1$, so that $z^F$ is solely a function of $\mu_h$ and $z^B$ is solely a function of $\mu_h + \mu_f$. Totally differentiating the equality above thus yields

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_h=\Pi_b} = -\frac{\delta^B d_{\mu_f}\pi[z^B(\mu_h, \mu_f, 1-\mu_h-\mu_f)](1-\rho)\rho}{\delta d_{\mu_h}\pi[z^F(\mu_h, \mu_f, 1-\mu_h-\mu_f)]\rho + \delta^B d_{\mu_h}\pi[z^B(\mu_h, \mu_f, 1-\mu_h-\mu_f)](1-\rho)\rho}.$$

In the above expression, the notation $d_x f[g(x)]$ refers to the total derivative of $f$ with

respect to $x$, $d_x f[g(x)] = f'[g(x)]g'(x)$. Since $z^B$ only depends on $\mu_h + \mu_f$, we have that $d\mu_f \pi(z^B) = d\mu_h \pi(z^B)$. Furthermore, $d\mu_f \pi(z^B) = d\mu_h \pi(z^B) > 0$ and $d\mu_h \pi(z^F) > 0$. Hence, it follows that the curve $\Pi_h = \Pi_b$ slopes downward with a slope in $(-1, 0)$. Finally, proceeding similarly with the third condition returns

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_f=\Pi_b} = -\frac{\delta d_{\mu_f}\pi[z^H(\mu_h,\mu_f,1-\mu_h-\mu_f)]\rho + \delta^B d_{\mu_f}\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho}{\delta^B d_{\mu_h}\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho} < -1.$$

These results explain the properties of Figure 3.2.

We now turn to deriving the general expressions for the wedges $w_f$ and $w_h$ in the constrained optimum, when consumption subsidies are not feasible. We use (3.11) to calculate $dW(\boldsymbol{\mu})/d\mu_j$. Evaluated at the constrained optimum $\boldsymbol{\mu}^*$, where $dW(\boldsymbol{\mu}^*)/d\mu_j = 0$, we obtain

$$\frac{dW(\boldsymbol{\mu}^*)}{d\mu_j} = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) + \sum_{i=h,f,b}\mu_i\frac{d\Pi_i(\boldsymbol{\mu}^*)}{d\mu_j} - \sum_{J=H,F,B}\delta^J P^J(\boldsymbol{\mu}^*)^{1-\varepsilon}\frac{d\log P^J(\boldsymbol{\mu}^*)}{d\mu_j} = 0, \;\; j \in \{h,f\}.$$

Rearranging terms, and using the definition of the wedges in the constrained optimum, i.e., $w_j^* = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*)$, yields (3.19) in the main text. Next, from the expressions for expected profits, (C.11)-(C.13), we have

$$\begin{aligned}
\frac{d\Pi_h(\boldsymbol{\mu}^*)}{d\mu_h} &= \delta^B\rho\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_h},\\
\frac{d\Pi_h(\boldsymbol{\mu}^*)}{d\mu_f} &= \delta^H\rho\frac{\partial\pi\left[z^H(\boldsymbol{\mu}^*)\right]}{\partial z}\frac{\partial z^H(\boldsymbol{\mu}^*)}{\partial\mu_f} + \delta^B\rho\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_f},\\
\frac{d\Pi_f(\boldsymbol{\mu}^*)}{d\mu_f} &= \delta^B\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_f},\\
\frac{d\Pi_f(\boldsymbol{\mu}^*)}{d\mu_h} &= \delta^F\rho\frac{\partial\pi\left[z^F(\boldsymbol{\mu}^*)\right]}{\partial z}\frac{dz^F(\boldsymbol{\mu}^*)}{d\mu_f} + \delta^B\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_h},\\
\frac{d\Pi_b(\boldsymbol{\mu}^*)}{d\mu_h} &= \delta^F\rho\frac{\partial\pi\left[z^F(\boldsymbol{\mu}^*),q_F\right]}{\partial z}\frac{dz^F(\boldsymbol{\mu}^*)}{d\mu_h} + \delta^B\left[\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_h} + \rho(1-\rho)\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_h}\right],\\
\frac{d\Pi_b(\boldsymbol{\mu}^*)}{d\mu_f} &= \delta^H\rho\frac{\partial\pi\left[z^H(\boldsymbol{\mu}^*),q_H\right]}{\partial z}\frac{dz^H(\boldsymbol{\mu}^*)}{d\mu_f} + \delta^B\left[\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_f} + \rho(1-\rho)\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_f}\right].
\end{aligned}$$

Substituting these derivatives into the expression for the wedges (3.19), we obtain

$$w_j^* = -\delta^K \left\{ \frac{1}{\sigma[z^K(\boldsymbol{\mu}^*)]} \frac{\partial \log \pi^K[z^K(\boldsymbol{\mu}^*)]}{\partial z} - \frac{\partial \log P^K[z^K(\boldsymbol{\mu}^*)]}{\partial z} \right\} P^K[z^K(\boldsymbol{\mu}^*)]^{1-\varepsilon} \frac{dz^K(\boldsymbol{\mu}^*)}{d\mu_j}$$
$$- \delta^B \left\{ \sum_K \frac{n^{B,K}(\boldsymbol{\mu}^*) s[z^{B,K}(\boldsymbol{\mu}^*)]}{\sigma[z^{B,K}(\boldsymbol{\mu}^*)]} \frac{d \log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} - \frac{d \log P^B(\boldsymbol{\mu}^*)}{d\mu_j} \right\} P^B(\boldsymbol{\mu}^*)^{1-\varepsilon}, \quad \text{(C.33)}$$

where $K = F$ if $j = h$ and $K = H$ if $j = f$. The first term on the right-hand side of (C.33) represents the net externality in state $K$, i.e., the business-stealing externality combined with the consumer-surplus externality. The second term represents the net externality in state $B$.

To compute these wedges, we need explicit expressions for the partial derivatives in (C.33). First note that the expressions for the semi-elasticities of the price index and profits in state $K \in \{H, F\}$ are given by (C.4) and (C.5), respectively. For state $B$, differentiate the expression for relative prices (C.8) to obtain

$$\frac{d \log z^{B,H}(\boldsymbol{\mu})}{d\mu_j} \Big/ \frac{d \log z^{B,F}(\boldsymbol{\mu})}{d\mu_j} = \left\{ 1 - z^{B,F}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,F}(\boldsymbol{\mu})]}{\partial z} \right\} \Big/ \left\{ 1 - z^{B,H}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,H}(\boldsymbol{\mu})]}{\partial z} \right\},$$

where $\eta(z) := \sigma(z)/(\sigma(z) - 1)$ is the markup factor. Together with condition (3.14), we obtain

$$\frac{d \log z^{B,K}(\boldsymbol{\mu})}{d\mu_h} = -\frac{\rho s[z^{B,F}(\boldsymbol{\mu})] - \rho^2 s[z^{B,H}(\boldsymbol{\mu})]}{\phi(\boldsymbol{\mu}) \left\{ 1 - z^{B,K}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,K}(\boldsymbol{\mu})]}{\partial z} \right\}}, \ K \in \{H, F\} \quad \text{(C.34)}$$

and

$$\frac{d \log z^{B,K}(\boldsymbol{\mu})}{d\mu_f} = -\frac{\rho(1-\rho) s[z^{B,H}(\boldsymbol{\mu})]}{\phi(\boldsymbol{\mu}) \left\{ 1 - z^{B,K}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,K}(\boldsymbol{\mu})]}{\partial z} \right\}}, \ K \in \{H, F\}, \quad \text{(C.35)}$$

where

$$\phi(\boldsymbol{\mu}) := \sum_{L=H,F} n^{B,L}(\boldsymbol{\mu}) s\left[z^{B,L}(\boldsymbol{\mu})\right] \left\{ \frac{\sigma[z^{B,L}(\boldsymbol{\mu})] - 1}{1 - z^{B,L}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,L}(\boldsymbol{\mu})]}{\partial z}} \right\}.$$

Differentiating the price index in state $B$, (C.9), we obtain

$$\begin{aligned}\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_h} = &\frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_h}\frac{\partial\log\eta[z^{B,H}(\boldsymbol{\mu})]}{\partial\log z}\\ &+ n^{B,F}(\boldsymbol{\mu})s[z^{B,F}(\boldsymbol{\mu})]\left[\frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_h} - \frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_h}\right]\\ &+ \rho\left[\int_{z^{B,F}(\boldsymbol{\mu})}^{z^{B,H}(\boldsymbol{\mu})}\frac{s(\zeta)}{\zeta}d\zeta + (1-\rho)\int_{z^{B,H}(\boldsymbol{\mu})}^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta\right],\end{aligned} \tag{C.36}$$

and

$$\begin{aligned}\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_f} = &\frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_f}\frac{\partial\log\eta[z^{B,F}(\boldsymbol{\mu})]}{\partial\log z}\\ &+ n^{B,H}(\boldsymbol{\mu})s[z^{B,H}(\boldsymbol{\mu})]\left[\frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_f} - \frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_f}\right]\\ &+ \rho(1-\rho)\int_{z^{B,H}(\boldsymbol{\mu})}^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta.\end{aligned} \tag{C.37}$$

Finally, the change in profits is given by

$$\begin{aligned}\frac{d\log\pi^{B,K}(\boldsymbol{\mu})}{d\mu_j} = &-\left(\sigma[z^{B,K}(\boldsymbol{\mu})] - 1 + \frac{\partial\log\sigma[z^{B,K}(\boldsymbol{\mu})]}{\partial\log z}\right)\frac{d\log z^{B,K}(\boldsymbol{\mu})}{d\mu_j}\\ &-(\varepsilon-1)\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_j},\ j\in\{h,f\},\ K\in\{H,F\}.\end{aligned} \tag{C.38}$$

To better understand these expressions, we consider the symmetric limiting case where $q_H \approx q_F = q$ and $\gamma_H \approx \gamma_F = \gamma$. In this setting, $\delta^F \approx \delta^H = \delta$, and $z^{B,F}(\boldsymbol{\mu}) \approx z^{B,H}(\boldsymbol{\mu}) =: z^B(\boldsymbol{\mu})$. As a result, the expression for the wedge (C.33) becomes

$$\begin{aligned}w_j^* = &-\delta\left\{\frac{\frac{\partial\log\pi[z^K(\boldsymbol{\mu}^*)]}{\partial z}}{\sigma\left[z\left(\boldsymbol{\mu}^*\right)\right]} - \frac{\partial\log P[z^K(\boldsymbol{\mu}^*)]}{\partial z}\right\}P[z^K(\boldsymbol{\mu}^*)]^{1-\varepsilon}\frac{dz^K(\boldsymbol{\mu}^*)}{d\mu_j}\\ &-\delta^B\left\{\frac{\frac{\partial\log\pi[z^B(\boldsymbol{\mu}^*),q]}{\partial z}}{\sigma\left[z^B\left(\boldsymbol{\mu}^*\right)\right]} - \frac{\partial\log P[z^B(\boldsymbol{\mu}^*)]}{\partial z}\right\}P[z^B(\boldsymbol{\mu}^*)]^{1-\varepsilon}\frac{dz^B(\boldsymbol{\mu}^*)}{d\mu_j}.\end{aligned} \tag{C.39}$$

The term $\partial\log P/\partial z$ represents the consumer-surplus externality, whereas the term $(\partial\log\pi/\partial z)/\sigma$ represents the business-stealing externality.

Before considering the signs of the wedges, we need to show that $\mu_h^* = \mu_f^*$. Recall that the necessary first-order conditions for an interior allocation are

$$W_j(\boldsymbol{\mu}^*) = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) - w_j^* = 0, \quad j = h, f.$$

For these two necessary conditions to hold jointly, it must be that

$$\Pi_h(\boldsymbol{\mu}^*) - w_h^* = \Pi_f(\boldsymbol{\mu}^*) - w_f^*.$$

Using (C.39) and the expressions for expected profits, we find that this equality indeed holds when $\mu_h^* = \mu_f^*$. This allocation corresponds to the unique optimal constrained allocation if $W$ is globally concave. Proving the global concavity of $W$ for general HSA preferences turns out to be a tricky task. Instead, we now show that $\mu_h^* = \mu_f^*$ is an optimum when preferences are symmetric translog. Additionally, we prove at the end of this Section that $W$ is indeed globally concave when preferences are CES.

To prove that $\mu_h^* = \mu_f^*$ is an optimum when preferences are symmetric translog, we show that increasing $\mu_f$ is welfare-improving if and only if $\mu_f < \mu_h$. Specifically, we consider the variation $d\mu = (d\mu_h, d\mu_f, 0)$ with $d\mu_h = -d\mu_f$. Totally differentiating the welfare function (3.11) and imposing $d\mu_h - d\mu_f$ returns

$$\frac{dW(\boldsymbol{\mu})}{d\mu_f} \propto \frac{\partial \Omega(\mu_h)}{\partial \mu_h} - \frac{\partial \Omega(\mu_f)}{\partial \mu_f},$$

where

$$\Omega(\mu) := -\left( n(\mu)\pi[z(\mu)] + \frac{1}{\varepsilon - 1} P[z(\mu)]^{1-\varepsilon} \right),$$

and $n(\mu) = \rho(1-\mu)$, the function $z$ solves $s[z(\mu)]n(\mu) = 1$, $P$ is given by (C.2) and $\pi(z)$ by (C.3). When preferences are symmetric translog, $s(z) = -\theta \log(z)$, and the function $\Omega$

becomes[6]

$$\Omega(\mu) \propto -\left(\frac{1}{1+\theta n(\mu)}+\frac{1}{\varepsilon-1}\right)\left(\frac{1+\theta n(\mu)}{\theta n(\mu)}\right)^{1-\varepsilon}\exp\left(\frac{1-\varepsilon}{2\theta n(\mu)}\right).$$

The function $\Omega$ is convex as long as $\sigma(\mu)=1+\theta n(\mu)>\varepsilon$, which holds through Assumption 3.2. Hence, $\partial_\mu\Omega(\mu)$ is increasing, and $d_{\mu_f}W(\boldsymbol{\mu})>0 \iff \partial_{\mu_h}\Omega(\mu_h)>\partial_{\mu_f}\Omega(\mu_f) \iff \mu_h>\mu_f$.

With these results in mind, we turn to signing the wedges. Since $\mu_h^*=\mu_f^*=:\mu^*$, the wedges for the two sole-sourcing strategies are equal, i.e., $w_h^*=w_f^*=:w^*$. Furthermore, from (C.39), we have

$$w^*>0 \quad \text{if} \quad \frac{\partial\log\pi(z)}{\partial z}>\sigma(z)\frac{\partial\log P(z)}{\partial z} \quad \text{for } z\in\{z^K(\mu^*),z^B(\mu^*)\},$$

and

$$w^\star<0 \quad \text{if} \quad \frac{\partial\log\pi(z)}{\partial z}<\sigma(z)\frac{\partial\log P(z)}{\partial z} \quad \text{for } z\in\{z^K(\mu^*),z^B(\mu^*)\},$$

which follows from the fact that $z^K$ and $z^B$ are decreasing in $\mu_j$. General HSA preferences do not yield simple parametric conditions that satisfy these inequalities. But we can gain further insight by considering the special cases of CES preferences and translog preferences.

First, with symmetric CES preferences, $s(z)=\alpha z^{1-\sigma}$ and $\sigma(z)=\sigma$ is a constant. Using (C.4) with this market-share function, the consumer-surplus externality becomes

$$\frac{\partial\log P(z)}{\partial z}=\frac{s'(z)}{s(z)}\frac{1}{\sigma-1}<0.$$

Next, using (C.5), the business-stealing externality simplifies to

$$\frac{\partial\log\pi(z)}{\partial z}=\frac{s'(z)}{s(z)}\frac{\sigma-\varepsilon}{\sigma-1}<0.$$

[6]Recall that under symmetric translog preferences, the elasticity of substitution is $\sigma(z)=1-1/\log(z)$, the market clearing condition implies $\log z(\mu)=-1/[\theta\rho(1-\mu)]$, and finally $[1/s(z)]\int_z^1 s(\zeta)/\zeta d\zeta=-\log(z)/2$.

Together they imply

$$\frac{\partial \log \pi(z)}{\partial z} - \sigma \frac{\partial \log P(z)}{\partial z} = -\frac{s'(z)}{s(z)} \frac{\varepsilon}{\sigma - 1} > 0 \text{ for all } z.$$

We have established

**Lemma C.6.** *Let $q_H \searrow q_F$, $\gamma_H \searrow \gamma_F$, and let consumers hold symmetric CES preferences. Then, $w_h^* = w_f^* > 0$.*

Turning to symmetric translog preferences, let $s(z) = -\theta \log(z)$ for $z \in (0, 1)$. Now (C.4) implies

$$\frac{\partial \log P(z)}{\partial \log z} = \frac{1}{\log z - 1} - \frac{1}{2}$$

while (C.5) implies

$$\frac{\partial \log \pi(z)}{\partial \log z} = \left(1 - \frac{1}{\log z} - \varepsilon\right)\left(\frac{1}{\log z - 1} - \frac{1}{2}\right) + \frac{1}{2 \log z}.$$

Together, these two expressions imply

$$\frac{\partial \log \pi(z)}{\partial \log z} - \sigma(z) \frac{\partial \log P(z)}{\partial \log z} = \varepsilon + \frac{1}{\log z} \frac{\log z - 1}{\log z - 3}.$$

Under symmetric translog preferences, the adding up constraints of market shares generate relative prices $\log z^J(\mu) = -1/[\theta n(\mu)]$ for $n^J(\mu) = n(\mu) := \rho(1 - \mu)$, $J \in \{H, F\}$, and $\log z^B(\mu) = -1/[\theta n^B(\mu)]$ for $n^B(\mu) := \rho[1 + (1 - \rho)(1 - 2\mu)]$. It follows that

$$\frac{\partial \log \pi[z(\mu^*)]}{\partial z} > \sigma[z(\mu^*)] \frac{\partial \log P[z(\mu^*)]}{\partial z} \quad \Longleftrightarrow \quad \varepsilon > \theta n^K(\mu^*) \frac{1 + \theta n^K(\mu^*)}{1 + 3\theta n^K(\mu^*)}, \ K \in \{H, F, B\}.$$

Finally, we note that $n^B(\mu) > n(\mu)$ for $\mu < 1/2$, and that the product $x(1 + x)/(1 + 3x)$ is increasing in $x$. We conclude that

$$\varepsilon < \theta n(\mu^*) \frac{1 + \theta n(\mu^*)}{1 + 3\theta n(\mu^*)} \Longrightarrow w^* < 0,$$

and

$$\varepsilon > \theta n^B(\mu^*)\frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)} \Longrightarrow w^* > 0.$$

Although the values of $n(\mu^*)$ and $n^B(\mu^*)$ are endogenous, it is possible to derive parametric restrictions that guarantee that one or the other of these inequalities holds. Specifically, if $\varepsilon < \theta n(\mu^*)\frac{1+\theta n(\mu^*)}{1+3\theta n(\mu^*)}$ holds for the smallest possible value of $n$, then it must hold for all $n$. Therefore $w^* < 0$ if $\varepsilon < \theta\rho(2+\theta\rho)/2(2+3\theta\rho)$. Similarly, if $\varepsilon > \theta n^B(\mu^*)\frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)}$ holds for the largest possible value of $n^B$, then it must hold for all $n$. Therefore $w^* > 0$ if $\varepsilon > \theta\rho(2-\rho)[1+\theta\rho(2-\rho))/(1+3\theta\rho(2-\rho)]$.[7]

**Lemma C.7.** *Let $q_H \searrow q_F$, $\gamma_H \searrow \gamma_F$ and suppose that consumers have symmetric translog preferences. Then*

$$\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)} \Longrightarrow w^* < 0,$$

*and*

$$\varepsilon > \frac{\theta\rho(2-\rho)\left[1+\theta\rho(2-\rho)\right]}{1+3\theta\rho(2-\rho)} \Longrightarrow w^* > 0.$$

To conclude this section, we return to the special case of CES preferences to show that Lemma C.6 generalizes to settings with asymmetric costs and risks. Returning to (C.33), we have already shown that the first term in parenthesis is negative. In state $B$, constant mark-ups simplify equations (C.34) and (C.35) to

$$\frac{d\log z^{B,K}}{d\mu_h} = -\frac{\rho\left\{s[z^{B,F}(\boldsymbol{\mu}^*)] - \rho s[z^{B,H}(\boldsymbol{\mu}^*)]\right\}}{\sigma-1} < 0,\ K \in \{H,F\},$$

and

$$\frac{d\log z^{B,K}}{d\mu_f} = -\frac{\rho(1-\rho)s[z^{B,H}(\boldsymbol{\mu}^*)]}{\sigma-1} < 0,\ K \in \{H,F\}.$$

Furthermore, the semi-elasticity of the price index (C.36) and (C.37) becomes

$$\frac{d\log P^B(\boldsymbol{\mu}^*)}{d\mu_j} = -\frac{d\log z^{B,H}(\boldsymbol{\mu}^*)}{d\mu_j} = -\frac{d\log z^{B,F}(\boldsymbol{\mu}^*)}{d\mu_j} > 0,\ j \in \{h,f\}.$$

[7]Technically, we also need to ensure that $\min\{\sigma[z(\mu^*)],\sigma[z^B(\mu^*)]\} = \sigma[z(\mu^*)] = 1+\theta n(\mu^*) > \varepsilon$. This is not a concern for the sufficient condition $\varepsilon < \theta n(\mu^*)\frac{1+\theta n(\mu^*)}{1+3\theta n(\mu^*)}$. Regarding $\varepsilon > \theta n^B(\mu^*)\frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)}$, a sufficient condition for $\sigma[z(\mu^*)] > \varepsilon$ for all $\mu^\star$ is $1+\theta n(1/2) = 1+\theta\rho/2 > \varepsilon$ since $n$ is decreasing in $\mu$.

Similarly, the semi-elasticity of profits (C.38) becomes

$$\frac{d\log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} = -(\sigma-\varepsilon)\frac{d\log z^{B,F}(\boldsymbol{\mu}^*)}{d\mu_j} = -(\sigma-\varepsilon)\frac{d\log z^{B,H}(\boldsymbol{\mu}^*)}{d\mu_j} > 0,\ j \in \{h,f\}\,.$$

Combining these expressions, the second term in (C.33) becomes

$$\begin{aligned}&\sum_{K=H,f} \frac{n^{B,K}(\boldsymbol{\mu}^*)s[z^{B,K}(\boldsymbol{\mu}^*)]}{\sigma}\frac{d\log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} - \frac{d\log P^B(\boldsymbol{\mu}^*)}{d\mu_j}\\ =\ & \frac{\varepsilon}{\sigma}\frac{d\log z^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} < 0,\ j \in \{h,f\} \text{ and } K \in \{H,F\}\,.\end{aligned}$$

Then (C.33) implies

$$\begin{aligned}w_h^* &= \rho\left(\frac{\varepsilon}{\sigma-1}\right)\left(\delta^F\pi[z^F(\boldsymbol{\mu}^*)] + \delta^B\left\{\pi[z^{B,F}(\boldsymbol{\mu}^*)] - \rho\pi[z^{B,H}(\boldsymbol{\mu}^*)]\right\}\right),\\ w_f^* &= \rho\left(\frac{\varepsilon}{\sigma-1}\right)\left\{\delta^H\pi[z^H(\boldsymbol{\mu}^*)] + \delta^B(1-\rho)\pi[z^{B,H}(\boldsymbol{\mu}^*)]\right\}.\end{aligned}$$

Together with the planner's first-order conditions, these expressions yield

**Lemma C.8.** *Suppose consumers have symmetric CES preferences. Then*

$$w_h^* = w_f^* = \left(\frac{\varepsilon}{\sigma+\varepsilon-1}\right)k > 0.$$

Evidently, in the CES case, the two wedges are positive and equal to one another, which implies that the constrained optimum can be achieved with a subsidy for diversification, i.e., $\varphi_b > 0$, with $\varphi_h = \varphi_f = 0$.

Finally, we conclude this section by showing that the first order conditions are necessary and sufficient when preferences are CES – that is, that the welfare function $W$ is globally concave. When preferences are symmetric CES, the price index (C.2) in state $J \in \{H,F\}$

simplifies to

$$P[z^J(\boldsymbol{\mu}), q_J] = \frac{q_J}{z^J(\boldsymbol{\mu})} = n^J(\boldsymbol{\mu})^{\frac{1}{1-\sigma}} \cdot q_J, \tag{C.40}$$

while the price index in state $B$ becomes

$$P^B(\boldsymbol{\mu}) = \left( \sum_{J=H,F} n^{B,J}(\boldsymbol{\mu}) q_J^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{C.41}$$

Additionally, the profit of an active firm in state $J \in \{H, F\}$ is

$$\pi[z^J(\boldsymbol{\mu}), q_J] = \left( \frac{q_J^{1-\varepsilon}}{\sigma} \right) n^J(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}}.$$

and the profit of an active firm in state $B$ purchasing an input from country $J \in \{H, F\}$ is

$$\pi^{B,J}(\boldsymbol{\mu}) = \left( \frac{q_J^{1-\sigma}}{\sigma} \right) \left( \sum_{\ell=H,F} n^{B,\ell}(\boldsymbol{\mu}) q_\ell^{1-\sigma} \right)^{\frac{\varepsilon-\sigma}{\sigma-1}}.$$

Under this special functional form, when the allocation is interior, $\mu_f > 0$, $\mu_h > 0$ and $\mu_b = 1 - \mu_f - \mu_h > 0$, the welfare function then simplifies to

$$W(\mu_h, \mu_f) = c \sum_{J=H,F,B} \delta^J P^J(\boldsymbol{\mu})^{1-\varepsilon} - k(2 - \mu_h - \mu_f),$$

where $c := 1/\sigma + 1/(\varepsilon - 1)$. Plugging in the expression for the price indices, (C.40) and (C.41), we have

$$W(\mu_h, \mu_f) = c \left[ \sum_{J=H,F} \delta^J n^J(\boldsymbol{\mu})^{\frac{1-\varepsilon}{1-\sigma}} q_J^{1-\varepsilon} + \delta^B \left( \sum_{\ell=H,F} n^{B,\ell}(\boldsymbol{\mu}) q_\ell^{1-\sigma} \right)^{\frac{1-\varepsilon}{1-\sigma}} \right] - k(2 - \mu_h - \mu_f).$$

Double differentiating $W$, we obtain that the elements of the Hessian matrix are

$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial(\mu_f)^2} \propto -\left(\delta^H n^H(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}-1} q_H^{1-\varepsilon} + \delta^B(1-\rho)^2 q_H^{2(1-\sigma)} P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon}\right) < 0,$$
$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial(\mu_h)^2} \propto -\left(\delta^F n^F(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}-1} q_F^{1-\varepsilon} + \delta^B\left[q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right]^2 P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon}\right) < 0,$$
$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial\mu_f\partial\mu_h} \propto -\delta^B(1-\rho) q_H^{1-\sigma}\left[\left(q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right] P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon} < 0,$$

where the constant of proportionality is the same. Inspection of the Hessian matrix shows that $W$ is globally concave.

## C.4 Numerical Exploration of the Constrained Optimum

In this section, we extend the numerical results of Section 6 in the main text to include simulations with both asymmetric risks *and* costs. Figure C.1 extends the comparative statics of panels (c) and (d) in Figure 3.4 by comparing the effect of cross-country differences in risk on the optimal supply-chain policies under two different cost discounts.[8] Panels (a) and (b) in Figure C.1 depict, respectively, the fraction of firms that adopt a particular supply chain strategy and the optimal policy under the symmetric cost simulation of Figure 3.4. Panels (c) and (d) plot the same variables for a positive cost discount of 5%.

When the cost discount is large but the risk premium is minimal, offshoring is *ceteris paribus* more profitable than onshoring, and firms locate their supply chains disproportionately in the foreign country, both in the equilibrium and in the constrained optimum. The wedges remain positive for both strategies, although they are no longer equal. Indeed, as discussed in Section 3.5, when risks are identical across countries but the input cost is lower in the foreign country, the social planner wants to tax relatively more the exclusive offshore relationships

[8]The effect of a positive cost differential on the comparative statics for the risk premium is qualitatively similar for the cases of $\varepsilon = 1.2$ and $\varepsilon = 1.7$. To conserve space, we present only the latter.

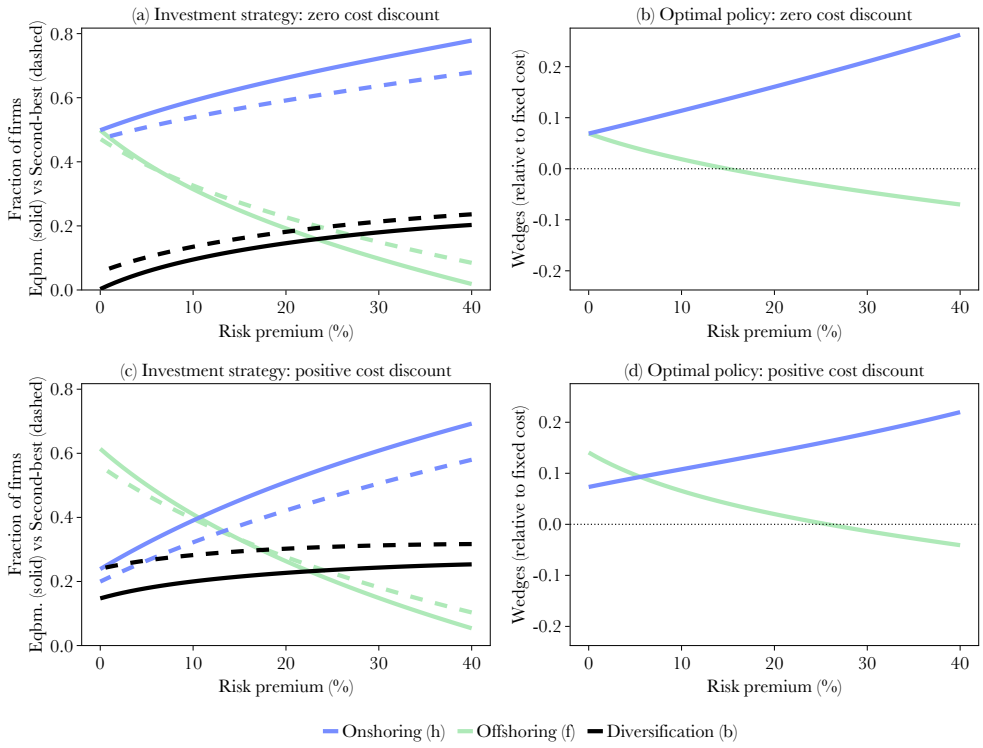



Figure C.1: Second-Best Policies: Risk Differences Across Locations with Two Cost Scenarios

Note: Baseline simulation is $\varepsilon = 1.7$, $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.37$. The risk premium is computed as $-(\gamma_F - \gamma_H)/\gamma_H$, where we keep $\gamma_H$ constant at its baseline value. The cost discount in panels (b) and (d) is 5%.

as the price index is lower in state $F$.

As in the case with symmetric costs depicted in Figure 3.4, when the cost differential is positive, an increase in the risk premium is associated with a greater fraction of diversified firms, a greater fraction of firms that form relationships only onshore, and a smaller fraction of firms that form relationships only abroad. However, in this case, firms face a tension between safe-but-expensive and riskier-but-cheaper suppliers. In panel (c), we see that for risk differentials greater than 10%, a cost discount of 5% is no longer enough to favor offshore investments, and firms locate disproportionately their supply chains in the safe-but-expensive home country.

Qualitatively, the effect of an increase in the risk premium on the optimal policies when the cost differential is positive also mimics what we have seen for symmetric costs. As the

foreign risk increases, relatively more firms locate their supply chains exclusively in the home country, which triggers a relative increase in the price index in state $F$ compared to state $H$, and with it an increase in $w_h^*$ but a decrease in $w_f^*$. Compared with the symmetric cost simulation, the difference is now that the price index was initially lower in state $F$ relative to state $H$ due to the lower input cost in the foreign country. Thus, an increase in foreign risk initially shrinks the market's misallocation between home sourcing and foreign sourcing, and the wedges converge for a risk premium of 5%. Then, as the risk premium continues to grow, the price index in state $F$ continues to increase, and the planner wants to tax relatively more the exclusive onshore relationships. Finally, for a sufficiently large risk premium, the planner's desire to shift the location of exclusive-sourcing from the home country to the foreign country implies again a tax on onshore relationships but a subsidy for investing in a single relationship abroad.

Figure C.2 extends the comparative statics of panels (c) and (d) in Figure 3.5 by allowing for a positive risk premium. Panel (a) and (b) reproduce the results illustrated in panels (c) and (d) of the earlier figure, where $\gamma_H = \gamma_F$, while panels (c) and (d) in Figure C.2 depict outcomes and policies with a positive risk premium of 15%.[9] When the cost discount is small relative to the risk premium, onshore sourcing relationships are relatively more attractive and a larger fraction of firms opt for strategy $h$. As the cost discount grows, the relative advantage of the foreign country increases, and a larger fraction of firms decide to form their exclusive relationship with foreign suppliers. This intuitive pattern mimics the findings for the case where risks are symmetric.

Regarding the optimal policies, the wedge for strategy $f$ is relatively smaller than that for strategy $h$ when the cost discount is relatively small. This echoes the discussion of Figure 3.4; when the risk premium is large but the cost discount is small, the monopoly distortion is more severe in state $F$ when the price index is higher, and the planner wishes to combat the higher prices in this state with a policy that tilts sourcing towards the foreign country.

[9] Once again the qualitative properties of the figure are similar for $\varepsilon = 1.2$ and $\varepsilon = 1.7$, so we present only the latter.

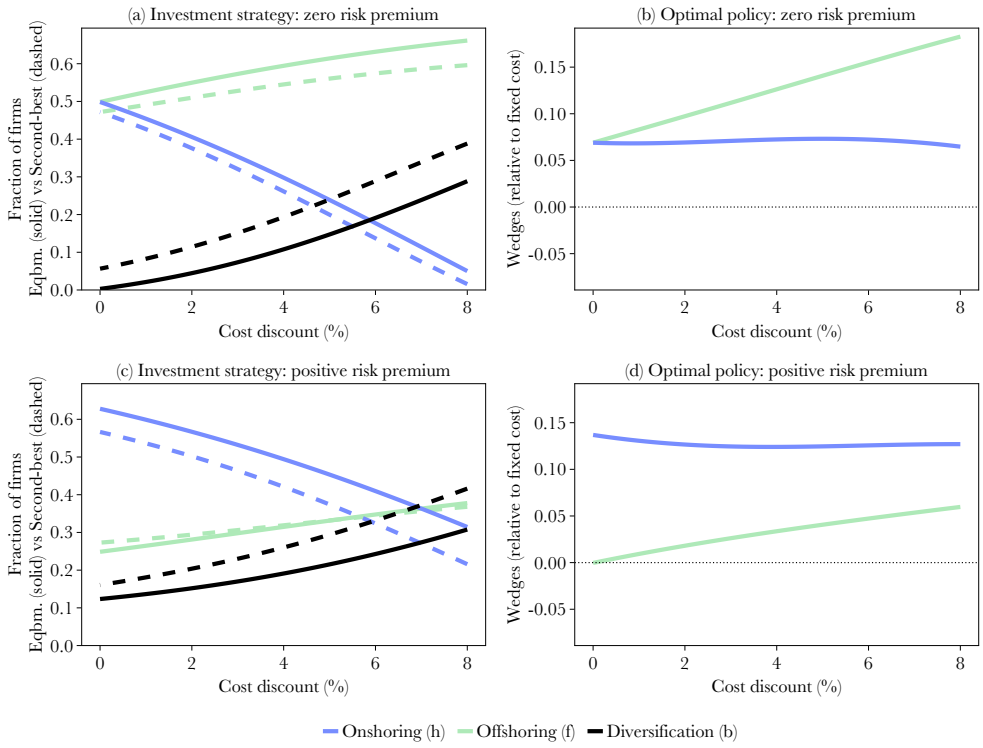



Figure C.2: Second-Best Policies: Cost Differences Across Locations with Two Risk Scenarios

Note: Baseline simulation is $\varepsilon = 1.7$, $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.37$. The cost discount is computed as $-(q_F - q_H)/q_H$, where we keep $q_H$ constant at its baseline value. The risk premium in panels (b) and (d) is 15%.

As the cost discount grows further, the fraction of firms that form exclusive relationships with foreign supplier rises, which, as in the scenario with symmetric risks, reduces the social benefit from promoting consumption in state $F$, and thus the gap between $w_f^*$ and $w_h^*$.

We have explored a large variety of parameters besides those illustrated here. In general, the optimal policies hinge on which country is more attractive for exclusive sourcing based on the tradeoff between risk and cost and the implications of these asymmetric investments on the sizes of the monopoly distortions in the various states of the world.

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