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Essays in Financial Economics

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ESSAYS IN FINANCIAL ECONOMICS

DONG BEOM CHOI

A DISSERTATION

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OF PRINCETON UNIVERSITY
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Abstract

This dissertation contains three essays that study liquidity crises and financial system stability.

The first chapter studies a model of systemic panic among heterogeneously leveraged financial institutions. Concerns about potential spillovers from each other generate strategic interactions among institutions and bring self-fulfilling panic. I show that systemic risk critically depends on the financial health of stronger institutions (less leveraged) in the contagion chain, although financial contagion originates in weaker institutions. My analysis highlights the striking contrast between macroprudential and non-systemic regulatory approaches yielding novel policy implications. Systemic stability can be enhanced by making the institutions more heterogeneous, and bolstering the strong institutions in the contagion chain, rather than the weak, more effectively contains systemic panic.

The second chapter studies a model of a credit crunch (an interbank market freeze) in which risk sharing among banks exacerbates financial fragility. Banks that wish to borrow with liquidity shortages may have to pay extra cost of credit if lenders have a better investment opportunity; collecting fire-sale assets at cheap price from the distressed banks. They thus have to compensate the lender for this outside option in order to borrow. With risk sharing among the banks, this option value can become more sensitive to aggregate uncertainty fluctuations since joint distress arises and the lender anticipates large price discounts. Credit costs and aggregate output can become more volatile, and credit rationing more likely with risk sharing.

The third chapter studies feedback between asset market distress and money market distress. The market clearing asset price can act as a public signal from which agents can extract information about the asset fundamental. As the asset price

drops, the creditors in the money market become concerned and less willing to lend. This distress in the money market forces financial institutions to liquidate their assets in the asset market, and the asset price becomes even lower, generating vicious cycle between the two markets. I combine noisy rational expectation equilibrium setup and global game setup to characterize this feedback. The asset price volatility becomes larger as the economic fundamental deteriorates, and the asset price distribution becomes negatively skewed.

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Chapter 1

Heterogeneity and Stability: Bolster the Strong not the Weak

1.1 Introduction

Panics in financial markets are contagious by nature. Financial spillovers, which spread distress among institutions, can arise through a variety of channels.¹ When market participants worry about this domino effect, the fear itself sometimes leads to self-fulfilling panics: If some become concerned and try to exit the market before the mess falls on, others follow and run for the exit in a panic to avoid being left behind (Pedersen (2009)). In such circumstances, coordination problems among market participants, which stem from concerns about spillovers, become critical.

¹For example, fire-sale externalities (Caballero and Krishnamurthy (2001), Diamond and Rajan (2005), and Brunnermeier and Pedersen (2009)), informational spillovers (Lee (1998), Aghion, Bolton, and Dewatripont (2000), and Kurlat (2010)) and direct/indirect exposures (Rochet and Tirole (1996), Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), and Dasgupta (2004)). See Brunnermeier (2009) for a description of the 2007-09 crisis.

This paper seeks to tackle a new question: How does systemic panic occur when financial institutions are heterogeneous (some are financially stronger than others)? Understanding this mechanism is important since it enables us to address the following policy question: What should we do to contain systemic crises in the future? Our model, by incorporating systemic concerns among heterogeneous institutions, presents answers to these questions with novel implications.

First thoughts suggest that financial distress simply spreads from financially weaker institutions to stronger ones (as in Diamond and Rajan (2005)). However, the underlying causal relationship is the opposite when strategic interactions from concerns about spillovers are taken into account. Indeed, measures taken by the stronger institutions to protect themselves from spillovers can exacerbate uneasiness among the weaker ones and ultimately destabilize the whole financial system. Hence systemic stability critically depends on the health of the stronger, rather than the weaker institutions.

In our model, strategic considerations about coordination concerns are present not only among the ex-ante identical institutions but also across those with differing financial health levels.² This implies that the stronger institutions are not passive. They do not simply sit and wait, worrying if the financial distress at weaker institutions will spill over to them. Rather, they try to run preemptively for the exit so as to avoid being dragged down. This, however, prompts the weaker ones to act the same, and run even faster. Even though not running is collectively better, such “pre-emption game”, in which one tries to exit the market before others,

²Panic models in the literature have mainly focused on coordination concerns among homogeneous participants (e.g., Diamond and Dybvig (1983), Rochet and Vives (2004), and Goldstein and Pauzner (2005)). Goldstein (2005) studies a coordination problem among investors in different markets, and Corsetti, Dasgupta, and Morris and Shin (2004) consider the problem among small and large traders. Sákovics and Steiner (2011) study policy effectiveness facing a coordination problem with heterogeneous agents.

may induce a coordination failure among heterogeneous institutions and undermine systemic stability.

What is remarkable here is that as economic fundamentals deteriorate, a systemic crisis materializes when the *stronger* institutions in the contagion chain lose their confidence and consider dropping out of the chain (exiting the market) to avoid the spillovers. This concern of the stronger eventually prompts the weaker to run preemptively, which self-fulfills the stronger's initial concern and leads them to run as well. Thus, what is critical in the systemic context is not the contagion trigger event itself (distress at the weaker institutions), but the level of the stronger's confidence facing the spillovers following that event. Therefore, systemic risk, or the probability of a systemic crisis, is related to the financial health of the *stronger* institutions which directly affects their levels of confidence.

This argument highlights a striking contrast between our systemic macroprudential approach and the benchmark non-systemic approach in which coordination problems among institutions are absent. In the benchmark case, the best way to contain contagious distress is to focus on the weakest link in the contagion chain: bolstering the weakest. The following quotation from *Lombard Street* (Bagehot (1873)) represents this view, which is consistent with conventional wisdom:

...In wild periods of alarm, one failure makes many, and the best way to prevent the derivative failures is to arrest the primary failure which causes them.

Our analysis suggests that incorporating strategic considerations can reverse this conventional wisdom. Unlike financial distress, which propagates from the weaker to the stronger institutions, the loss of confidence propagates from the stronger to the weaker eventually resulting in a self-fulfilling crisis. Thus, the best way to

reduce systemic risks is to bolster the strongest institutions in the contagion chain to prevent a destabilizing loss of confidence.

Our approach hence yields novel policy implications for financial system stability. We first argue that systemic risk is lower in more heterogeneous financial systems. This is because coordination problems are less severe in more heterogeneous systems and thus externalities from coordination failure are weaker. Since systemic soundness critically depends on the health of stronger institutions, it can be enhanced by separating the strong from the weak. This property indicates that bank stress tests can have a stabilizing effect on the financial system by differentiating and identifying strong and weak institutions. It also indicates that bank mergers (i.e., acquisitions of weak institutions by stronger ones, which are considered to be one of the handiest measures to protect fragile institutions) can undermine systemic stability by making the system more homogeneous.

Our analysis also brings novel implications for financial institution recapitalization. Recapitalization is commonly considered to be a robust measure to enhance financial soundness by increasing the size of loss-absorbing capital cushions, which is the rationale for tighter (ex-ante) capital requirements (Rochet and Vives (2004)). We demonstrate that although this argument is correct from microprudential perspective that focuses on individual institutions in isolation, it may not be correct when the strategic interactions among heterogeneous institutions are incorporated. Simply making the most highly leveraged (financially weakest) institutions stronger may fail to reduce the systemic risks because systemic panic materializes when the less leveraged (financially stronger) institutions lose their confidence, irrelevant of the weaker's relative financial health on the margin. Thus, we turn conventional

wisdom upside down: Recapitalize the strongest in the contagion chain, not the weakest, to enhance systemic stability effectively.

While our mechanism can be applied more generally when coordination concerns exist among heterogeneous agents with differing degrees of exposure to strategic uncertainty (i.e. coordination concerns), this paper specifically considers heterogeneously leveraged institutions holding an illiquid asset subject to a collateral constraint. Our explicit focus is on their optimal market-exit timing facing the following tradeoff:³

- Institutions prefer to keep this high-yield but illiquid asset (“stay” in the market) rather than to liquidate immediately (“exit” the market) at a discounted price.
- At the same time, they wish to avoid financial distress (forced liquidation of their assets) that occurs when their collateral constraint is violated.

This implies that the first best strategy is to delay immediate liquidation and exit the market right before the collateral constraint is binding, or funding liquidity evaporation.

What complicates the choice of this exit timing is coordination problems (strategic uncertainty) among the institutions holding the illiquid assets, whose liquidation value (i.e. collateral value) becomes lower as more of them are liquidated.⁴ Thus, collateral value becomes depressed as other institutions exit, which may result in a collateral constraint violation that could have been avoided if the institutions had

³The trader’s tradeoff in Morris and Shin (2004b) is the closest to that of our model. Investors of Bernardo and Welch (2004, 2010) also face similar tradeoff but fundamental uncertainty exists in their case.

⁴Theoretical models of this price impact are provided by Grossman and Stiglitz (1980), Kyle (1985), Grossman and Miller (1988), Shleifer and Vishny (1992), Allen and Gale (1994), and Brunnermeier and Pedersen (2009).

coordinated. In other words, funding liquidity is provided unless market liquidity is depleted, but the amount of remaining market liquidity depends on collective action of other institutions.

This dependence becomes the source of multiple self-fulfilling equilibria in the manner of Diamond and Dybvig (1983). We adopt the global game technique (Morris and Shin (2003) for the overview, and Toxvaerd (2008) for the exit game setup) to derive a unique equilibrium of our model. In equilibrium, systemic crises are triggered when deteriorating fundamentals cause institutions to run for limited market liquidity. The runs are self-fulfilling because they depress the market value of the assets to the point that collateral constraints are violated and funding liquidity evaporates.

A new source of externality arises in our model that results from the coordination failure among heterogeneously leveraged institutions. This externality precipitates the trigger of systemic panics and increases the systemic risk *ex ante*, and it becomes stronger if the institutions are more homogeneous. Using a continuous time approximation for our dynamic model, we apply a structural debt pricing framework (see Merton (1974), and Leland (1994)) to calculate credit spreads and systemic risks in a closed form. We explicitly examine the relationship between heterogeneity and credit spread dynamics, and how a microprudential analysis underestimates both credit spreads and true systemic risks by ignoring the externality from concerns about the spillovers in the system. The errors are negligible during normal times, but surge rapidly in a market downturn.

This paper is related to several strands of literature. We study liquidity crises in financial markets as in Holmström and Tirole (1998), and Allen and Gale (2004) where financial spillovers arise as in Allen and Gale (2000), Diamond and Rajan

(2005), and Brunnermeier and Pedersen (2009). Our approach is also related to the literature on coordination problems among market participants. Diamond and Dybvig (1983) provide a classic model of coordination problems that generate self-fulfilling multiple equilibria. Financial panic models with a unique equilibrium are developed using the global game technique by Rochet and Vives (2004), Goldstein and Pauzner (2005), and Morris and Shin (2004b). Goldstein (2004), and Corsetti, Dasgupta, Morris, and Shin (2004) extend them to the asymmetric global game setup. Dynamic extensions of a global game with an exit option are considered by Toxvaerd (2008), and Chassang (2010). Without adopting the global game setup, He and Xiong (2011a) study a coordination problem among creditors across different maturity dates. Coordination concerns about the liquidation timing in this paper are also studied by Brunnermeier and Pedersen (2005), Carlin, Lobo, and Vishwanathan (2007), and Oehmke (2010) in different contexts. This paper also employs the structural debt pricing approach originally proposed by Merton (1974). Leland (1994), and Leland and Toft (1996) provide solutions for debt valuation with endogenous default thresholds chosen by the equity holder whereas in our case debt contracts can also be terminated when the collateral constraint is violated. Our focus on the effect of coordination failure (thus higher rollover risk, whose effect is also considered in He and Xiong (2011b)) on credit costs is in a similar spirit to Morris and Shin (2001, 2004a). Bruche (2010) also studies this problem by employing a global game with a continuous time approximation, as in this paper.

The chapter is organized as follows. Section 1.2 describes the model setup. Section 1.3 analyzes the equilibrium of the model and Section 1.4 discusses its policy implications. Section 1.5 analyzes how coordination failure and heterogeneity affect asset pricing dynamics. Section 1.6 concludes.

1.2 Model Setup

We focus on how strategic interactions among heterogeneous institutions affect the systemic risk of a panic run for limited market liquidity (panic run for the exit), while taking their balance sheet structures and financial constraints as given.⁵ Consider an infinite horizon economy where time is discrete and advances by increments of Δ , indexed by $0, \Delta, \dots, t - \Delta, t, t + \Delta, \dots$. There are two groups of differently leveraged financial institutions (referred to as “institutions” hereafter). In each group, there is a continuum $[0, 1]$ of ex-ante identical institutions. The number of groups as well as that of institutions within a group is without loss of generality.

Each institution is endowed with one unit of an asset simultaneously used as collateral for the exogenous initial debt position (financed and purchased at the ex-ante period $t = 0$), with the debt principal value P .⁶ Since P reflects the scale of the initial debt on the liability side for one unit of asset holding on the asset side, we interpret P as a measure of the initial leverage where higher P implies a higher leverage level. Initial leverage levels are different between the two (H and L) groups. H -group institutions are more highly leveraged with $P = P_H$, than L -group (low-leverage) institutions with $P = P_L$, where $P_H > P_L$.

We focus on the interim market-exit decision of the institutions. At the beginning of each period, the institutions choose either to “stay” in the market (keep the

⁵Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Brunnermeier and Pedersen (2009) focus on the balance sheet fluctuations along the fundamental fluctuations. We consider a situation in which institutions are having difficulty in raising new capital, and focusing on the deleverage decision.

⁶We can consider that the asset position is financed ex ante partly by some debt using this particular asset as collateral and partly by its own capital, as with repo or ABCP.

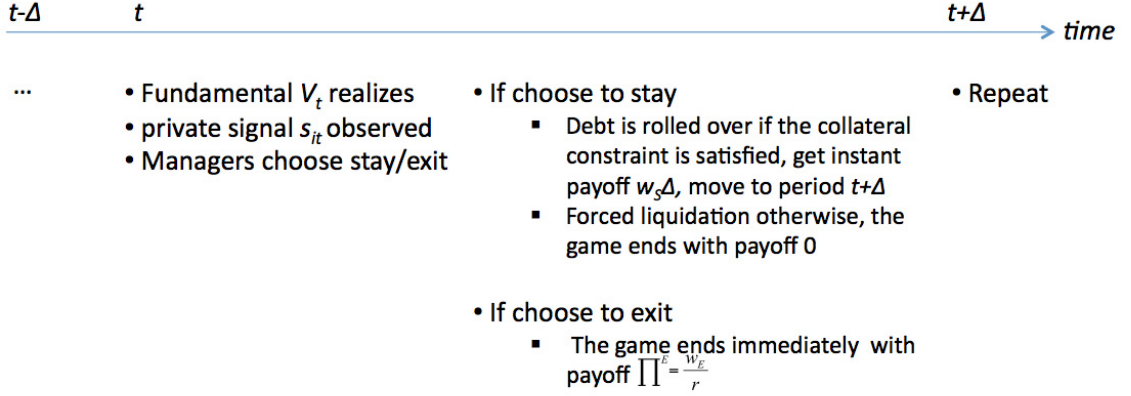


Figure 1.1: Timeline of the dynamic model

The institutions repeatedly choose either to stay in the market or exit the market after observing the private signal each period. When choosing to stay, the debt needs to satisfy a collateral constraint to be rolled over into the next period. The game ends if the institution chooses to exit or its debt rollover is refused.

asset to the next period) or to “exit” the market immediately⁷ (close their position and pay back the debt principal) facing the following tradeoff; they basically prefer keeping the asset over immediate liquidation, but also wish to avoid financial distress following the collateral constraint violation. When choosing to keep their leveraged position, the institutions have to roll over their debt to move on to the next period. Specifically, an institution’s debt will not be rolled over at some period if certain collateral constraint (Equation (1.2) to be specified later) is not satisfied, which brings financial distress to that institution. Given these balance sheet structures, the institutions choose an optimal timing of the market exit. This exit game is detailed in Section 1.2.3.

⁷Partial liquidation can be incorporated within our setup but will not be observed in equilibrium since coordination concerns among the institutions (desire for preemptive run) drive the equilibrium results.

1.2.1 Asset

The unlevered value of one unit of the asset V , which is non-verifiable, follows the stochastic process

$$V_{t+\Delta} = V_t + (r - \delta)V_t\Delta + \sigma V_t z_{t+\Delta}$$

where the innovations $\{z_{t+\Delta}\}$ are i.i.d. $N(0, \Delta)$. Here, r is the risk-free rate and δ is the cash payout ratio.⁸ V_t is referred to as the “fundamental” value of the asset at period t .

The assets are illiquid in a sense that their interim liquidation price deviates from the fundamental value as more of them are liquidated in the market. Denote the liquidation price of one unit of the asset at the end of period t as L_t , and let f_t be the amount of the assets that have been liquidated previously. Thus, $f_t \in [0, 2]$ is simply the mass of the institutions that have chosen to exit the asset market up to the beginning of period t . Given the fundamental V_t , the liquidation price L_t at the end of period t follows

$$L_t = V_t - \lambda f_t \tag{1.1}$$

where λ is the measure of (il)liquidity for this asset.⁹ Since the asset is also used as collateral for the debt, we will use the terms “liquidation price” and “collateral value” interchangeably, both indicating L_t . Equation (1.1) implies that the collateral value becomes lower as more institutions liquidate their asset holdings to exit the

⁸As $\Delta \rightarrow 0$, $\{V_t\}$ converges to a geometric Brownian motion $dV = (r - \delta)Vdt + \sigma VdW$, commonly used in the structural debt pricing literature. We assume that probabilities are measured under the risk-neutral measure.

⁹ λ reflects market depth in Kyle (1985), market liquidity in Brunnermeier and Pedersen (2009), liquidity premium in Allen and Gale (1994), risk premium in Grossman and Stiglitz (1980), and dislocation cost in Shleifer and Vishny (1992).

market, thereby reducing market liquidity. The decrease is larger when the asset is more illiquid.

1.2.2 Debt contract and collateral constraints

The institution uses its one unit of the initial asset holding as collateral for its initial debt position. Since the fundamental V is non-verifiable, the debt (with the principal $P = P_H, P_L$) is subject to a collateral constraint (Hart and Moore (1998), Kiyotaki and Moore (1997)) that requires the perceived value of the collateral asset (marked to market asset value) L_t to exceed certain threshold proportional to the debt size P . For simplicity, we assume that the debt needs to be fully collateralized. At the end of each period t , a j -group institution ($j = H, L$) can roll over its debt if and only if the collateral value L_t exceeds its debt principal P_j :

$$L_t > P_j \tag{1.2}$$

If the collateral value is below this threshold, the institution is under “financial distress” facing rollover refusal, which leads to forced liquidation of its asset holding.¹⁰

The debt pays a constant coupon payment $C_j\Delta$ each period until its termination, which is triggered by either the rollover refusal (puttable option) or the institution’s voluntary exit (callable option), paying the principal P_j at that point.¹¹

¹⁰We assume that the institutions can’t raise new capital once in financial distress. We also rule out debt renegotiation. Diamond and Rajan (2001) provides a theoretical model in which a coordination problem among lenders prevents debt renegotiation. See Choi (2011) for the lenders’ coordination problem in this setup.

¹¹The full-collateral requirement (1.2) guarantees the payment of principal P_j . This setup is not critical.

1.2.3 Market-exit game

We solve for the optimal timing of the institutions' market-exit as the fundamentals deteriorate. At the beginning of each period, each institution (indexed by $i \in [0, 2]$) decides either to stay in the market ($a_{it} = 0$) or to exit immediately ($a_{it} = 1$). We consider the following economic tradeoff microfounded in Appendix B: (i) the institutions prefer to stay in the market rather than to liquidate the asset immediately for a discounted price, but (ii) the institutions at the same time wish to avoid financial distress (following rollover refusal) since voluntary liquidation with early exiting is preferred over forced liquidation.¹² Facing the risk of potential rollover refusal, each institution chooses an optimal action of the two given the state variables. No re-entry is allowed once exiting the market.

Since the collateral value (thus, the debt rollover) depends on collective action of the other institutions (characterized by f_t), the optimal exit decision also depends on what others do. We adopt a dynamic global game setup (as in Toxvaerd (2008)) by introducing a noisy private signal about the fundamental, such that a unique equilibrium can be pinned down. At the beginning of the typical period t , the fundamental V_t is realized but is not common knowledge to the institutions. Instead, a typical institution i receives an idiosyncratic signal s_{it} that follows $s_{it} = V_t + \epsilon_{it}$, where ϵ_{it} is independently uniform over $[-\epsilon, \epsilon]$ with $\epsilon = o(\Delta^{\frac{1}{2}})$.¹³ Let $h_t = \{V_s, f_s | s < t\}$ be the past history up to period t , which is common knowledge.

¹²While this assumption is exogenous in our setup, it could result from (i) reputational reasons (stigma effect), (ii) higher liquidation price with preemptive liquidation (as in Morris and Shin (2004b) and Bernardo and Welch (2005)), or (iii) the fear of facing predatory trading (Brunnermeier and Pedersen (2005)).

¹³Using uniform distribution is for simplicity and without loss of generality with $\epsilon \rightarrow 0$ as $\Delta \rightarrow 0$. If the order of convergence is larger, multiple equilibria exist with too informative public signals (here, past fundamentals). See Angeletos and Werning (2006).

In each period, the institutions make an optimal decision (stay/exit) repeatedly based on these state variables. We suppose a reduced-form setup (as in Rochet and Vives (2004)) in which the optimal decision is delegated to a manager with a simple payoff structure while preserving the tradeoff described previously. This simplifies our analysis to focus explicitly on the strategic interactions without affecting the model's implications.¹⁴

Managers are risk neutral and discount with the risk-free rate r . At the beginning of period t , a typical manager (indexed by $i \in [0, 2]$, irrespective of the groups they belong to) chooses either to stay in the illiquid market or to exit after observing the private signal s_i and the past history h_t . At any interim period, a manager basically prefers keeping the illiquid asset to immediate liquidation; he obtains a high wage of $w_S \Delta$ at the end of each period as long as he keeps the asset and the debt gets rolled over, but only obtains a low wage of $w_E \Delta$ each period (with $0 < w_E < w_S$) once liquidating his position and exiting the market.¹⁵ Therefore, “staying” is strictly better than “exiting” at any given period, as long as it is certain that the debt will be rolled over in that period (funding liquidity is secured).

The downside of staying in the market is that the institution may fall into financial distress at the end of that period if the collateral constraint (1.2) is violated. The managers of distressed institutions will then be penalized for making the “wrong” decision of remaining in the market, receiving 0 afterwards in that case.

¹⁴This delegation assumption is for simplicity. We can get essentially the same results by directly analyzing the equity holder's payoff who collects a dividend payment $(\delta V_t - C_j) \Delta$ every period, imposing an extra penalty for going under financial distress. See Appendix B.

¹⁵Although we arbitrarily pick w_S and w_E further assuming that these values are identical across the managers belonging to different groups, the specific parametrization of the salary schedule are irrelevant in the limit. Our results require only the minimum assumptions that voluntary preemptive liquidation is preferred over forced liquidation for any institution. See Appendix B.

This tradeoff between (i) the higher wage for staying over exiting and (ii) the heavy penalty for staying mistakenly when funding liquidity dries up, is the driving force behind the interim exit decision. The managers (institutions) wish to stay in the illiquid market to enjoy high salaries (high yields) while funding liquidity is provided, but do not wish to stay too long so as to avoid the financial mess following funding liquidity evaporation.

When ignoring other institutions, the institutions thus try to delay their market exit until the collateral constraint is surely violated. However, they fail to achieve this outcome since coordination concerns arise among them; one's collateral value (thereby one's debt rollover) depends on collective action of other institutions, thus one may also have to exit when sufficiently many other institutions are exiting. The global game technique enables us to solve for the unique equilibrium exit strategy that takes this strategic uncertainty into account.

We focus on the Markov threshold strategies characterized by respective “(panic) exit thresholds” $s_H^*(h_t)$ and $s_L^*(h_t)$ for the two groups; given the history h_t , the manager i of group $j(= H, L)$ chooses to stay in that period if his signal about the fundamentals is high enough exceeding j -group's exit threshold ($a_{it}(s_{it}, h_t) = 0$ if $s_{it} > s_j^*(h_t)$), but exit otherwise ($a_{it}(s_{it}, h_t) = 1$ if $s_{it} \leq s_j^*(h_t)$). In the next section, we define and derive a unique Markovian Perfect Bayesian equilibrium in threshold strategies.

The timeline of the model is summarized in Figure 1.1. At the beginning of a typical period t , the fundamental V_t is realized and the managers who remain in the market receive their private signals $\{s_{it}\}_{i \in [0, 2]}$. Each manager then chooses either to stay or to exit according to their strategy profiles. At the end of period t , the debt rollover is allowed for the institution staying in the market if and only if its

collateral constraint (1.2) is satisfied, which from (1.1) depends on the aggregate size of the past asset liquidation f_t and the current fundamental V_t . The wages are then paid according to the salary schedule. If the manager chooses to stay and the debt rollover is allowed, it moves on to the next period $t + \Delta$ and the same game is repeated. The game ends otherwise, either by exiting voluntarily or by a rollover refusal.

1.3 Bayesian Equilibrium

A single-group global game with a unique equilibrium (Morris and Shin (2003) for the overview) can be easily extended to a multiple-group setup as shown in Frankel, Morris, and Pauzner (2003) or Goldstein (2005). For now, we assume that the difference in initial leverages (i.e., debt principal values P_j) between the two groups is not very large while the assets they hold are illiquid, satisfying $P_H - P_L < \lambda$. As will subsequently be discussed, this condition implies a domino effect between the two groups when one of them becomes distressed.

Given the past history h_t , two exit thresholds characterize an equilibrium of the model, $s_H^*(h_t)$ for H -group institutions and $s_L^*(h_t)$ for L -group, with a signal below which a manager loses confidence and chooses to exit the market. A Bayesian equilibrium is defined such that one's strategy in the profile maximizes his conditional expected payoff when all others are following the equilibrium strategy profiles. We take a continuous time approximation of our discrete time model with $\Delta \rightarrow 0$. There are three advantages of the continuous time setup: (i) closed-form solutions can be derived, (ii) we can employ the structural debt pricing framework to define and calculate the dynamics of credit spreads, and (iii) the model's results become

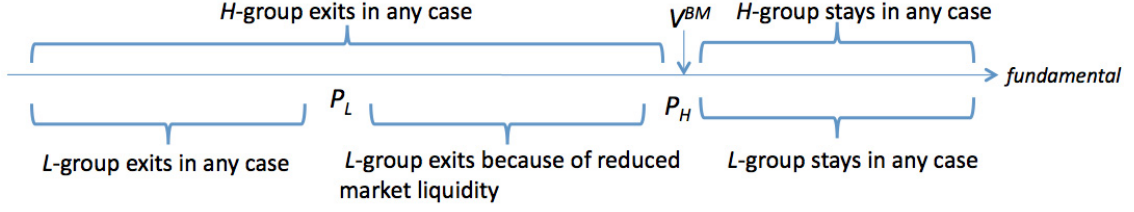


Figure 1.2: Crisis threshold V^{BM} in the benchmark case

When the institutions can perfectly coordinate, all (both H and L group) institutions exit the market when the fundamental V hits $V^{BM} = P_H$ from above. Here, the crisis is simply triggered when H -group institutions are distressed.

robust to the specific parametrization of the delegated manager's payoffs (i.e., w_S and w_E).

1.3.1 Benchmark case with perfect coordination

Prior to the equilibrium analysis with strategic interactions, consider the first-best benchmark case in which the institutions can perfectly coordinate. As depicted in Figure 1.2, we verify that all (both H and L group) institutions delay their market exit until the fundamental V eventually hits P_H .

In the benchmark case with perfect coordination, the institutions delay their liquidation as much as possible, and liquidate their position right before their collateral constraints (1.2) are surely violated. H -group institutions thus exit the market when the fundamental V hits P_H , at which point depressing the collateral value by $\lambda \times 1$ to $P_H - \lambda$, through mass 1 of H -group's liquidation. This generates financial spillovers (fire-sale externality) to L -group, since L -group's collateral constraint will then be violated with reduced market liquidity; the collateral value L_t is now lower than L -group's debt principal value P_L under our assumption of $P_H - P_L < \lambda$. In

anticipation of this domino effect (H -group's liquidation drags down L -group into financial distress contagiously), L -group institutions also choose to exit immediately at this point, setting their exit thresholds at P_H . Therefore, all institutions in both groups choose to exit simultaneously when the fundamentals V eventually deteriorate to $V^{BM} = P_H$, where V^{BM} is referred to as the crisis threshold of the fundamental under the benchmark setup. Here, a “crisis” refers to a systemic event in which all institutions in the system choose to liquidate their asset simultaneously to exit the market, and systemic risk refers to the risk of this systemic event. Note that the crisis simply happens when *weaker* H -group get distressed at $V = P_H$ in this case, and the crisis threshold (thus the systemic risk) depends on H -group's financial health (leverage level) P_H .

1.3.2 Equilibrium with coordination concerns

When strategic considerations about coordination concerns are incorporated, the institutions don't wait passively until others exit, but instead, worry about what others will do. The fear itself can generate a self-fulfilling panic in this case, which increases the systemic risk *ex ante*. Our contribution is to demonstrate novel implications on systemic stability arising from strategic interactions among heterogeneous institutions. A “pre-emption game” starts between the two groups, and on the margin, it is in effect *stronger* L -group that is critical in initiating the systemic crisis as opposed to the benchmark case.

We begin the equilibrium analysis by defining the critical level of liquidation pressure for each group given V_t , denoted as $f_H^*(V_t)$ and $f_L^*(V_t)$. Let the critical

pressure $f_j^*(V_t)$ for group j be such that $V_t - \lambda f_j^*(V_t) = P_j$, then we get

$$f_j^*(V_t) = \frac{V_t - P_j}{\lambda}. \quad (1.3)$$

This condition along with (1.1) and (1.2) implies that given the fundamental V_t , the debt rollover for the remaining j -group institutions is allowed at that period if the total mass of exited institutions (liquidation pressure) turns out to be lower than this threshold ($f_t < f_j^*(V_t)$) but is refused otherwise ($f_t \geq f_j^*(V_t)$). This characterizes the source of coordination concerns among the institutions; the rollover of one's debt depends on collective action of others which may deplete limited market liquidity for the asset. Note that $f_H^*(V_t) < f_L^*(V_t)$ with $P_H > P_L$, implying highly leveraged institutions are more vulnerable to liquidation pressure than less leveraged institutions. Thus, strategic uncertainty (concerns about the spillovers) is more critical for H -group institutions. H -group institutions can become distressed even when L -group institutions are not, yet the opposite cannot happen. H -group is thus financially “weaker” and L -group is “stronger”.

We derive a unique equilibrium using the global game technique. As shown in Toxvaerd (2008), this dynamic global game with an exit option can be solved as a sequence of one-shot games, with appropriately defined value functions for the two actions. We focus on deriving the closed-form solutions for our exit game under the continuous time approximation ($\Delta \rightarrow 0$).

We now define the value functions for the respective actions (stay or exit) as of the beginning of period t given the signal s_{it} and the history h_t . When choosing to exit, the manager receives $w_E \Delta$ constantly afterwards and under the continuous

time approximation, the value function Π^E is simply defined by

$$\Pi^E = \int_0^\infty e^{-rt} w_E dt = \frac{w_E}{r} \quad (1.4)$$

for both groups, independent of the private signal. This represents the option value of an immediate exit, or the outside option value for the manager.

When choosing to stay, the value function Π_j^S of j -group given information about the state variables can be defined as

$$\begin{aligned} \Pi_j^S(s_{it}, h_t) = E \Big[& \left(w_S \Delta + e^{-r\Delta} \max\{\Pi_j^S(s_{it+\Delta}, h_{t+\Delta}), \Pi^E\} \right) \times \mathbf{1}_{[f_t < f_j^*(V_t)]} \\ & + 0 \times \mathbf{1}_{[f_t \geq f_j^*(V_t)]} \Big| s_{it}, h_t \Big]. \end{aligned} \quad (1.5)$$

The righthand side of (1.5) consists of two parts. If the debt contract is rolled over in that period (with $f_t < f_j^*(V_t)$), the manager receives an instant high wage of $w_S \Delta$ and in the next period (at $t + \Delta$) again gets to choose either to stay or to exit, captured by the continuation value $e^{-r\Delta} \max\{\Pi_j^S(s_{it+\Delta}, h_{t+\Delta}), \Pi^E\}$. If the debt is not rolled over (with $f_t \geq f_j^*(V_t)$), he gets fired (getting 0) and the game ends immediately.

Decomposing the total mass of the exited institutions up to period t as $f_t = f_{H,t} + f_{L,t}$, where $f_{j,t} \in [0, 1]$ is the mass of the exited institutions in group j , it is straightforward that strategic complementarities exist not only within one group but also across the different groups (Π_j^S is decreasing both in $f_{j,t}$ and in $f_{-j,t}$); one has to care not only about its own group institutions' run for limited market liquidity but also about the other group's run. Note that $\Pi_j^S(s_{it}, h_t)$ is increasing in s_{it} —"staying" is more attractive with higher signals about the fundamentals since rollover becomes more likely—which enables us to pin down the indifference

thresholds $s_j^*(h_t)$ on which switching of the actions occurs ($\Pi_j^S(s_{it}, h_t)$ is greater (less) than Π^E with s_{it} right (left) to that threshold).

As shown in the appendix (Lemma A1), we can interpret our dynamic exit game as a sequence of the identical history-independent one-shot games. $s_j^*(h_t)$ and $\Pi_j^S(s_{it}, h_t)$ can thus be denoted as s_j^* and $\Pi_j^S(s_{it})$ which are history independent, where

$$\begin{aligned} \Pi_j^S(s_{it}) = E \left[\left(w_S \Delta + e^{-r\Delta} \max\{\Pi_j^S(s_{it+\Delta}), \Pi^E\} \right) \times \mathbf{1}_{[f_t < f_j^*(V_t)]} \right. \\ \left. + 0 \times \mathbf{1}_{[f_t \geq f_j^*(V_t)]} \middle| s_{it} \right]. \quad (1.6) \end{aligned}$$

We take three steps in solving for the equilibrium threshold (s_H^*, s_L^*) . We first derive the optimal exit threshold of each group ignoring the other group, which becomes the lower bound of the equilibrium threshold. We next derive a best response of one group given the other group's exit threshold. We then derive the equilibrium thresholds incorporating full strategic interactions, which are best responses of one another.

Optimal exit threshold ignoring the other group

As a first step, we focus on the coordination problem within one group ignoring the existence of the other group. Let $\underline{s}_j^*$ be the optimal exit threshold of j -group when $-j$ -group does not exit. Thus $f_{-j,t} = 0$ in this case, and given the signal s_{it} , the option value of staying can be defined by

$$\begin{aligned} \Pi_j^S(s_{it}) = E \left[\left(w_S \Delta + e^{-r\Delta} \max\{\Pi_j^S(s_{it+\Delta}), \Pi^E\} \right) \times \mathbf{1}_{[f_{j,t} < f_j^*(V_t)]} \right. \\ \left. + 0 \times \mathbf{1}_{[f_{j,t} \geq f_j^*(V_t)]} \middle| s_{it} \right]. \quad (1.7) \end{aligned}$$

in which only within j -group strategic uncertainty is taken into consideration. The switching threshold $\underline{s}_j^*$ can then be derived from the indifference condition $\Pi_j^S(\underline{s}_j^*) = \Pi^E$ as shown in the appendix.

Lemma 1. (Exit threshold ignoring the other group)

When $-j$ -group is ignored, a j -group institution chooses to exit if and only if its signal about the fundamentals is below

$$\underline{s}_j^* = P_j + \lambda + \epsilon. \quad (1.8)$$

Before adding the between-group strategic interaction, we also define the upper bound of s_j^* , denoted as $\bar{s}_j^*$. It is easy to verify that $\bar{s}_j^* = P_j + 2\lambda + \epsilon$ since staying in the market is the dominant action for a j -group institution if $s_{it} > P_j + 2\lambda + \epsilon$, where its debt will be surely rolled over regardless of the others' collective action f_t . It is obvious that the equilibrium exit threshold (with full strategic interactions) s_j^* should be bounded by these two extreme thresholds, that

$$\underline{s}_j^* \leq s_j^* \leq \bar{s}_j^* \quad (1.9)$$

holds for both $j = H, L$.

Equilibrium exit thresholds with full strategic interactions

We now derive the equilibrium thresholds (s_H^*, s_L^*) when institutions of different groups are acting strategically, anticipating the effect of one's action on the others and vice versa. Since strategic complementarities exist both within one group and between different groups, one's incentive to exit increases not only in the number

of exiting institutions in its own group, but also in the number of those in the other group. These additional coordination concerns generate a novel externality through a spiral of growing concerns between the two groups; the concern about the other group's panic makes my group more concerned, which in turn makes the other group's concern grow, generating a feedback loop. The spiral is described as a “pre-emption game” between the two groups in which one group raises its exit threshold in response to the other's raise in a vicious cycle, so as to run for the exit faster than the other. In the end, the spiral only stops when stronger L -group drops out, and the equilibrium exit thresholds get pushed up to L -group's upper bound $\bar{s}_L^*$ via this new source of the externality (Figure 1.3).

The mechanism of this pre-emption game can be best described using the following best response functions, the optimal exit threshold of one group given the other group's exit threshold. Let $s_H^{BR}(s_L^*)$ refer to H -group institution's best response given s_L^* , and define $s_L^{BR}(s_H^*)$ analogously. Note that in equilibrium these two optimal thresholds have to be the best responses of one another, that the system of equations $s_H^* = s_H^{BR}(s_L^*)$ and $s_L^* = s_L^{BR}(s_H^*)$ have to hold. We first derive the following lemma shown in the appendix.

Lemma 2. (Pre-emption game between the two groups)

- For L -group, if $s_H^* < \bar{s}_L^*$, then $s_L^{BR}(s_H^*) > s_H^*$.
- For H -group, if $s_L^* < \bar{s}_H^*$, then $s_H^{BR}(s_L^*) > s_L^*$.

Lemma 2 characterize the process of pre-emption game between the two groups. It would be mutually beneficial for all institutions to delay their exit and keep their exit thresholds as low as possible, but coordination failure prevents them from achieving this. The institutions in one group have an incentive to avoid the

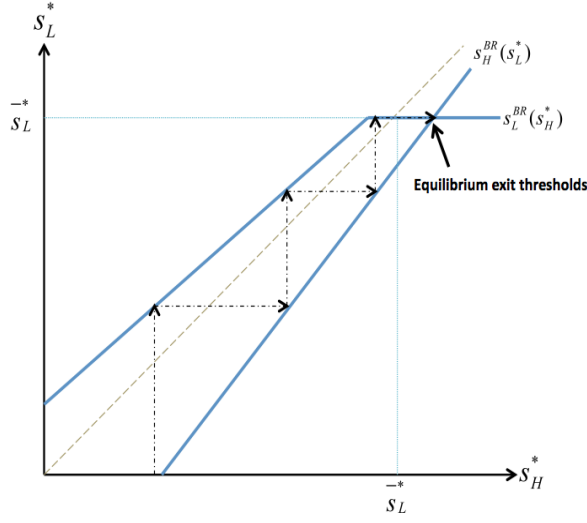


Figure 1.3: Pre-emption game between the two groups

One group raises its exit threshold in response to the other group's raise to run faster. The spiral only stops when L -group drops out at $\bar{s}_L^*$ beyond which no concern about coordination failure exists. The difference between the two groups' equilibrium exit thresholds becomes negligible in the limit case and both exit at the same threshold $\bar{s}_L^*$.

spillovers from the other group by acting preemptively, and try to raise their exit threshold slightly higher than that of the other group so that they can run for the exit faster before market liquidity evaporates.

Combining Lemma 2 and (1.9), we get $s_L^* = \bar{s}_L^*$ (Figure 1.3); the pre-emption game continues until L -group institutions drop out of the spiral at their upper bound $\bar{s}_L^*$, beyond which they are confident enough to stay in the market with strong enough fundamentals irrespective of what H -group institutions do. We can subsequently derive $s_H^* = s_H^{BR}(s_L^*)$ from their indifference condition given $s_L^* = \bar{s}_L^*$, which can be shown to be converging to s_L^* as the noise becomes small. Intuitively, H -group have no reason to exit “too early” when they know the timing of L -

group's exit. They try to exit “right before” L -group do, and the difference becomes negligible in the limit.

In equilibrium, panic runs of H -group simultaneously lead to contagious runs of L -group. Consequently, the institutions all exit together when the fundamental V eventually hits a “crisis threshold” $V^* = P_L + 2\lambda$, while all stay in when V is higher than V^* , as summarized in the following Proposition proved in the appendix.

Proposition 1. (Systemic panic run)

Systemic panic run for market liquidity is triggered when the fundamental V hits the crisis threshold $V^* = P_L + 2\lambda$ from above, at which point all institutions exit the asset market simultaneously.

Two points should be remarked on. First, ex-ante systemic risk increases with the coordination failure. The crisis threshold of the fundamental V^* is higher than that under the benchmark case $V^{BM} = P_H$ —concerns about the spillovers lead to a self-fulfilling crisis that would not take place if coordination failure were absent. This implies a discrepancy between systemic risk (from macroprudential perspective) and individual institution-wise risk (from microprudential perspective) that will further be analyzed in Section 1.5.

Second, more importantly, a novel implication on systemic stability arises; the crisis materialization (thus the systemic risk) depends critically on *stronger* L -group on the margin. Notice that the pre-emption game stops eventually (equivalently, systemic panic materializes) at L -group's upper bound $\bar{s}_L^*$ that is independent of P_H as highlighted in Figure 1.3. Contrast this with the crisis threshold under the benchmark approach $V^{BM} = P_H$. When coordination concerns are absent, what is critical in initiating the crisis is the materialization of the triggering event— H -

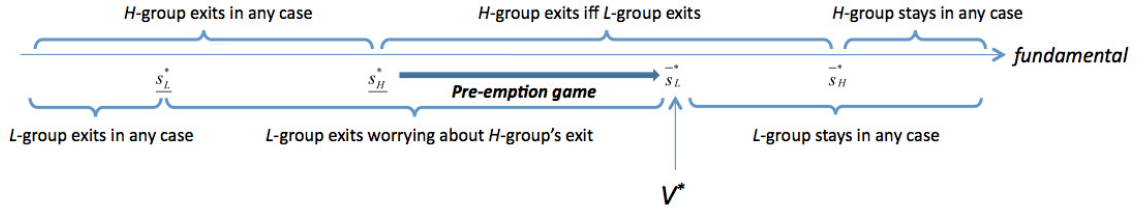


Figure 1.4: Crisis threshold V^* with between-group coordination failure

H -group exits at $\underline{s}_H^*$ and L -group at $\underline{s}_L^*$ if between-group coordination failure is absent. The crisis threshold gets pushed up through the pre-emption game to $\bar{s}_L^*$ and systemic panic is triggered (both H and L group institutions exit all together) when the fundamental hits $V^* = P_L + 2\lambda$.

group's distress—itself which directly depends on the financial health (debt level) of the weaker group P_H . As the fundamental deteriorates, H -group eventually liquidates when V hits P_H , which consequently prompts the contagious liquidation of L -group as described in Section 1.3.1.

When strategic interactions are involved, the crisis initiates in a self-fulfilling way from the strategic concerns about the spillovers at $V^* = P_L + 2\lambda$. What essentially causes the crisis on the margin is not the triggering event itself, but the loss of confidence among the institutions about staying in the market. Here, an asymmetry between weaker H -group and stronger L -group exists in terms of whose loss of confidence matters more (Figure 1.4).

Weaker H -group institutions act as a second mover when choosing their optimal exit timing. What is critical for them is to conjecture when stronger L -group lose confidence and consider exiting, such that they can exit right before that happens. H -group's confidence level is thus subject to L -group's confidence level. Their own financial health (characterized by the debt level P_H) is of secondary importance on the margin, since H -group have to exit any way if L -group exit to avoid spillovers.

Stronger L -group institutions, on the other hand, anticipate that weaker H -group will always try to exit preemptively, generating financial spillovers to them in equilibrium. Thus, when choosing their own exit threshold (i.e., when to exit the market), L -group take the spillovers from H -group *as given* following the conservative presumption.¹⁶ Hence, on the margin, they only care about whether other L -group institutions will endure the anticipated spillovers from H -group without panicking, which depends directly on L -group's financial health P_L . This L -group's choice is independent of the H -group's health P_H on the margin, since spillovers from H -group will arise to L -group in any case and the scale of the spillovers is irrelevant with the H -group's health. It is when this stronger L -group lose confidence at $\bar{s}_L^*$ that eventually prompts the preemptive run of H -group, which self-fulfills the concern of L -group and leads them to join the run contagiously. Note that in this region, H -group institutions have no reason to panic unless L -group do since the fundamental is higher than $\underline{s}_H^*$ as in Figure 1.4.

In sum, the crisis materialization on the margin depends critically not on the weaker but on the stronger institutions in the contagion chain, and the crisis threshold $V^* = P_L + 2\lambda$ depends on P_L but not on P_H . This novel externality from coordination concerns among heterogeneous institutions generates important policy implications on the financial system stability, which will be examined in the next section.

¹⁶Strictly speaking, an L -group institution anticipates that all H -group institutions will exit faster than herself if she turns out to be the one with the smallest signal within L -group.

1.4 Policy implications

A growing number of recent studies discusses macroprudential policies incorporating systemic considerations. These studies argue that controlling the individual institution's risk, analyzed in isolation, is insufficient in enhancing systemic stability (e.g., Acharya and Richardson (2009), Hanson, Kashyap, and Stein (2010), French et al. (2010)). Our model provides a novel framework for evaluating the stability of the financial system with institutions of heterogeneous financial health. We compare different systems (or effects of certain policy measures) based on their crisis thresholds of the fundamental V^* , where lower V^* implies higher systemic stability (or lower systemic risks).¹⁷

Compared to the benchmark approach without the coordination problem, policy implications change drastically when the strategic interactions in the systemic context are incorporated. We begin by discussing the relationship between financial institution heterogeneity and financial system stability. We then suggest several policy measures to enhance systemic stability.

1.4.1 Heterogeneity and systemic stability

First, we examine how heterogeneity of financial institutions affects financial system stability. Concretely, we consider whether the system in which institutions are different (in terms of degrees of their individual financial health) is more sound.

Surprisingly, the answers are quite the contrary depending on whether the strategic interactions are taken into account. Financial systems with more homogeneous institutions are more sound when the externality from coordination failure

¹⁷Although not explicitly modeled, we implicitly assume that there's welfare loss when the assets are liquidated in the secondary market (i.e., dislocation cost).

is ignored, but the opposite is true from our macroprudential perspective. That is, systemic stability can be enhanced by making the system more heterogeneous.

To illustrate the mechanism, consider a system with two groups (H and L) as in the previous sections and let the respective endowed debt levels be $P_L = P - u$ and $P_H = P + u$. The “heterogeneity parameter” u is positive and not too large ($u < \frac{\lambda}{2}$) such that a domino effect is anticipated as before. Here, higher u implies a more heterogeneous system, while fixing the average degrees of financial strength for the entire system (total debt outstanding in our case, $\frac{P_H + P_L}{2} = P$ for all u).¹⁸ We now compare the crisis thresholds of different systems $V^*(u)$ as the dispersion u is varied.

Recall that in the benchmark case without coordination concerns, the crisis threshold of the fundamentals is $V^{BM} = P_H \equiv P + u$ as discussed in Section 1.3.1, which is increasing in u . The crisis initiation in this case is critically related to financial distress of weaker H -group institutions, thus depending on their financial health (debt level P_H). Here, systemic stability can be enhanced by making the system more homogeneous with smaller u since it directly suppresses the liquidation triggering event (distress at the weaker institutions) by making the weaker less fragile.

This recommendation is reversed in our macroprudential approach with the strategic interactions. As discussed at the end of Section 1.3.2, a self-fulfilling panic is triggered when stronger L -group lose confidence and consider exiting, which directly depends on the financial health of L -group institutions. From Proposition 1, the crisis threshold is now given by $V^*(u) = P_L + 2\lambda \equiv P - u + 2\lambda$ which is decreasing in u . From macroprudential perspective, therefore, making the system

¹⁸It can also be interpreted as controlling the aggregate bank capital in the system since the aggregate asset size is also fixed (each institution holds one unit of the asset). We can also consider it as a separation of good banks and bad banks.

more heterogeneous reduces the systemic risk since it further bolsters stronger L -group institutions to induce them to stay in when exposed to the spillovers, such that a self-fulfilling panic can be contained.

Corollary 1. (Financial institution heterogeneity and systemic stability)

When the average degrees of financial strength are fixed across the systems,

- *Heterogeneous system is more robust from systemic perspective.*
- *Homogeneous system is more robust when the externality from coordination failure is ignored.*

With the above argument, we analyze stabilizing effects of several intervention measures.

Stabilizing effects of bank stress tests Bank stress tests diagnose the financial health of individual institutions with the aim of reducing uncertainty in the financial system. Although in principal an individual institution does not become financially stronger simply by knowing its condition, the tests can have a stabilizing effect on the system as a whole by practically increasing the heterogeneity of institutions.

Consider that institutions *ex ante* do not have a clear idea of how strong or weak they individually are when uncertainty is high;¹⁹ they presume that they are around the “average” robustness level. That is, the distribution of the institutions is perceived to be very condensed (small u) prior to the tests.

¹⁹In Choi (2011), banks have some prior beliefs about their robustness levels before the tests.

Stress tests notify institutions who are strong and weak, thus making the system more heterogeneous by separating the strong from the weak (larger u). We know from Corollary 1 that a more heterogeneous system is more sound, so the stress tests enhance systemic stability in this case by mitigating the coordination concerns among the institutions.

Corollary 2. (Bank stress tests and systemic stability)

Financial system can become more sound after bank stress tests even without accompanying further interventions such as recapitalizations or liquidity injections.

Merging banks We next consider the effects of bank mergers on systemic stability. Weak institutions are often acquired by stronger institutions as one handy way to protect them. It is obvious that the mergers can prevent financial distress of weak institutions from microprudential perspective, but it is not clear if they will enhance systemic stability as a whole. Our analysis suggests that bank mergers can be *destabilizing* since institutions become more homogeneous and the system become more condensed.

Suppose, as an extreme example, that all the L -group institutions are acquired by H -group institutions in our setup. Now, the distribution of institutions becomes very tight with $u \rightarrow 0$ and the crisis threshold $V^*(u)$ becomes higher after the mergers. Note that the opposite is true from benchmark perspective (straightforward from Corollary 1). Thus, we derive the following implication.

Corollary 3. (Bank mergers and systemic stability)

Bank mergers may increase systemic risks (whereas they reduce systemic risks if concerns about spillovers are absent).

1.4.2 Recapitalization

We next discuss the stabilizing effect of recapitalization. From microprudential perspective, recapitalization (either through tighter ex-ante requirements or public funds injections) should increase the financial stability of individual institutions by increasing their loss-absorbing capital cushions (Rochet and Vives (2004)). Our novel result is to demonstrate that it may fail to reduce systemic risks when the strategic considerations are incorporated; recapitalizing the weaker institutions in the contagion chain may not have a stabilizing effect.

Capital requirements We first analyze the stabilizing effect of tighter capital requirements. We argue that requiring a higher level of a minimum loss-absorbing capital cushion may fail to enhance systemic stability. Tighter rules primarily focus on bolstering the weakest institutions in the system, but on the margin it is the stronger in the chain that matters, not the weak.

Note that requiring a higher level of a minimum capital cushion ex-ante is equivalent to allowing a lower level of maximum initial debt in our setup, which is captured by the debt principal value P . We therefore consider a policy that places a cap on the maximum value of debt P allowed in the system.

When analyzing a single institution in isolation, it is obvious that tighter requirements are universally effective; lowering P enhances the stability of an individual institution, consistent with the objective of a thicker capital cushion. This is also true in the benchmark approach without coordination concerns. Going back to our

example, consider a policy that restricts the maximum value of P allowed in the system with the new upper bound denoted by $\overline{P}$. When this tighter requirement is implemented, P_H has to be adjusted so as to stay below $\overline{P}$ if initial P_H exceeds this limit (suppose $P_L < \overline{P} < P_H$). Without coordination failure, this policy should lower the crisis threshold $V^{FB} = P_H$ (from Section 1.3.1) by reducing P_H .

This tighter rule, however, fails to lower systemic risks when coordination concerns arise since the crisis threshold $V^* = P_L + 2\lambda$ of Proposition 1 is independent of P_H . As discussed in the second remark after Proposition 1, what is critical with self-fulfilling crises is the stronger institutions' level of confidence, which is related to the level of P_L but not to P_H on the margin. Tighter capital requirements focus on recapitalizing the weaker (reducing their debt P_H), but systemic risks remain the same unless the stronger group are also recapitalized and the concerns among the institutions are eased. This is because H -group's confidence level is subject to L -group's confidence level, and what L -group care about is whether they can endure the spillovers from H -group without panicking, which critically depends on their own financial health. The new requirements should be strict enough to affect both groups ($\overline{P}$ should be lower than P_L) to enhance systemic stability.

This argument implies that enhancing the weakest institutions' individual soundness does not guarantee a more sound financial system. The aggregate distribution of capital in the entire system should be taken into account, as is also suggested by Greenlaw, Kashyap, Schoenholtz, and Shin (2011). Studies on capital requirements with systemic concerns have primarily been focusing on the bank size, interconnectedness, liquidity, or the balance sheet mismatch structure (e.g. French et al. (2010), Kashyap, Rajan, and Stein (2008)), while our model suggests a new source of systemic risk which is related to the heterogeneity of the system.

Corollary 4. (Capital requirements and systemic stability)

- *When concerns about the spillovers exist, tighter capital requirements may fail to enhance systemic stability.*
- *When coordination concerns are absent, however, tighter capital requirements always enhance systemic stability.*

Capital injections The above argument also provides a novel implication on the issue of capital injections. Unlike Bagehot's suggestion to focus on the weakest which is correct without the coordination problems, we suggest recapitalizing the strongest incorporating systemic concerns.

Since a capital injection lowers the debt level (or leverage level) of an institution, it lowers the debt size P in our model's context. Following the same argument, systemic risks cannot be reduced on the margin if capital is injected into weaker H -group institutions (lowering P_H). The panic run still arises at the same crisis threshold when coordination concerns exist (V^* is independent of P_H), unless H -group become even stronger than L -group. The system becomes more sound if capital is injected to stronger L -group institutions (lower P_L implies lower V^*), since it makes them more confident and induces them to stay in such that self-fulfilling panics can be contained. Thus, we should bolster the stronger in the contagion chain, not the weaker. We can generalize this argument to the systems with more groups in the contagion chain and obtain the same implication—bolster the strongest in the chain.

This is counter-intuitive but to contain self-fulfilling crises, what is necessary is to prevent the institutions from dropping out of the chain (exiting the market). Our argument suggests that on the margin, it is more effective to inject funds into the

stronger, rather than the weaker, to induce them to stay in. As argued previously, the opposite holds if the coordination problem is absent.

Corollary 5. (Capital injections and systemic stability)

- *Based on the macroprudential approach with coordination concerns, stronger L -group institutions should be recapitalized in order to enhance systemic stability effectively.*
- *Based on the benchmark approach without coordination concerns, recapitalizing weaker H -group institutions is more effective.*

This argument implies that it is not enough to regulate only the most troubled institutions to enhance systemic stability; even stronger institutions should be subject to regulatory constraints. Some relatively “strong” institutions may be reluctant to raise additional capital while the fundamentals are still weak because they conceive the extra capital to be unnecessary. Those claims may be correct from an individual institution’s perspective, but not when systemic considerations are taken into account. In the next section, we explicitly examine how stronger institutions’ degree of financial health can affect weaker institutions’ risks.

1.5 Credit spreads, rollover risks, and coordination failure

In this section, we explicitly analyze how heterogeneity and coordination failure within the system affect the asset pricing dynamics using the structural debt pricing approach. We specifically focus on one institution (belonging to H -group) and

examine its funding costs (credit spreads) and funding risks (rollover risks) as the heterogeneity in the system varies.

We first begin with analyzing a system in which the two groups are heterogeneous enough such that no between-group coordination problem arises. We then make the two groups less heterogeneous such that between-group coordination problems arise, explicitly examining the effect of heterogeneity on the funding dynamics.

1.5.1 Very heterogeneous system

As a baseline setup, we assume a high enough between-group heterogeneity, $P_H - P_L > \lambda$. This condition rules out strategic interactions between the groups, such that only within-group coordination problems exist. Financial health of L -group has no effect on H -group's risk in this case.

Since a domino effect between the groups is absent with high enough heterogeneity in this case, it is sufficient to only consider within-group coordination problems ignoring strategic interactions between the groups. This is the case because H -group's $\underline{s}_H^*$ is higher than L -group's $\bar{s}_L^*$; L -group institutions will always choose to stay when H -group institutions exit at $\underline{s}_H^*$, and knowing this, H -group can ignore L -group institutions when solving for their own optimal exit timing. We thus solve H -group's problem in isolation ignoring L -group, which corresponds to the result of Lemma 1. There, all institutions in H -group run for the exit when the fundamental hits $V^{**} = P_H + \lambda$ from above, where V^{**} is H -group's crisis threshold of the fundamental in this very heterogeneous system.

Note that panics are self-fulfilling in this case; concerns about the potential rollover refusal trigger the panic run for limited market liquidity when the fundamentals hit the crisis threshold, followed by the actual violation of the collateral

constraint (2) at that point. The crisis threshold V^{**} can thus alternatively be interpreted as H -group's rollover threshold of the debt on which the debt rollover is anticipated to be refused. Following corollary summarizes the argument above.

Corollary 6. (Crisis/rollover threshold of H -group under high between-group heterogeneity)

- *All H -group institutions stay in the market as long as the fundamental V is beyond their crisis threshold $V^{**} = P_H + \lambda$, but run for the exit materializes as soon as V hits V^{**} .*
- *The debt rollover is allowed for H -group as long as V is beyond the rollover threshold $V^{**} = P_H + \lambda$, but is refused as soon as V hits V^{**} resulting in funding liquidity evaporation.*

Compare these results with those of the benchmark case in which all institutions can perfectly coordinate (alternatively, an individual institution is analyzed in isolation from microprudential perspective). All H -group institutions act as a single entity and it is easy to verify that they delay their market-exit until the fundamental hits $V^{BM} = P_H$, right before the collateral constraint (1.2) binds. Externality from coordination concerns thus precipitates the market exit and funding liquidity evaporation (triggered at the higher fundamental $V^* = P_H + \lambda$) compared to the benchmark microprudential case, implying a discrepancy between macroprudential systemic risk and microprudential risk. We now examine the asset pricing implications of this externality by analyzing the dynamics of the discrepancy between the two differently measured risks.

1.5.2 Credit spreads and rollover risk dynamics

Using the above Corollary, we can apply the structural debt pricing framework to our setup to examine how the externality from coordination concerns affects credit spreads and rollover risks in times of crises, where rollover risk refers to the risk of the debt rollover refusal.

In terms of asset pricing, the debt contract under consideration can be interpreted as a perpetual coupon debt with both callable and puttable options. Notice that there exists an endogenous threshold of the fundamental (rollover threshold) V^R determined in the model, and the pre-determined coupon $C_H \Delta$ is paid each period until the fundamentals V hits that threshold, at which point the contract is terminated paying the principal. One distinction from the standard debt contract is that the creditor can also terminate this contract (puttable, by refusing to roll over) upon the violation of the collateral constraint (1.2), not passively waiting for the borrower's default decision (as in Leland (1994), and Leland and Toft (1996)).

Note that coordination failure leads the debt to be terminated at a higher fundamental threshold, at $V^{**} = P_H + \lambda$ with coordination concerns rather than $V^{BM} = P_H$ without coordination concerns. We now calculate the debt value with these endogenous thresholds and the payoffs on those thresholds. Under a continuous time approximation, the debt contract pays the coupon C_H continuously until the fundamentals V hit the rollover threshold $V^R (= V^{**} \text{ or } V^{BM})$ where the rollover is refused and the principal P_H is paid.^{20 21}

²⁰To be precise, the institutions exit from the contract voluntarily (exercise the callable option) in anticipation of this rollover refusal, but still paying P_H at $V_{\tau(V^R)} = V^R$.

²¹With this reduced-form setup, we try to capture the qualitative effect of the coordination failure on credit spreads, rather than the quantitative effect which is the focus of the credit spread puzzle literature (Huang and Huang (2003) for the overview). We can consider cases with partial recovery upon termination but the qualitative implications remain the same since what drives our results is the

The fundamentals follow $dV = (r - \delta)Vdt + \sigma VdW$ in the limit and the debt termination paying the principal P_H is triggered at $V_{\tau(V^R)} = V^R$, where the stopping time is defined by $\tau(V^R) = \inf\{t | V_t \leq V^R\}$. The debt value given the current state V_t , denoted as $D(V_t; V^R)$, can be derived from the Bellman equation

$$D(V_t; V^R) = E\left[\int_t^{\tau(V^R)-t} e^{-r(s-t)} C_H ds + e^{-r(\tau(V^R)-t)} P_H | V_t\right]. \quad (1.10)$$

Using Ito's formula, we get an ODE

$$C_H + \frac{1}{2}\sigma^2 V^2 D_{VV} + (r - \delta)V D_V - rD = 0,$$

with the boundary conditions

$$\lim_{V \rightarrow \infty} D(V) = \frac{C_H}{r},$$

$$D(V^R; V^R) = P_H.$$

Solving this, the debt value can be calculated as

$$D(V_t; V^R) = \frac{C_H}{r} + [P_H - \frac{C_H}{r}] \times \left[\frac{V_t}{V^R}\right]^X \quad (1.11)$$

where X is the negative root of $X(X - 1)\frac{\sigma^2}{2} + X(r - \delta) = r$.

We define the credit spread as the difference between the yield and riskfree rate following the standard definition:

$$CS(V_t; V^R) = \frac{C_H}{D(V_t; V^R)} - r. \quad (1.12)$$

changes in the endogenous termination (rollover) threshold when coordination concerns arise.

In addition, consider the following measure of the rollover risk $RR(V_t; V^R)$ ranging from 0 to 1:

$$RR(V_t; V^R) \equiv E[e^{-r(\tau(V^R)-t)} | V_t], \quad (1.13)$$

which is a normalized distance to the rollover threshold V^R given the current fundamental V_t , reflecting how likely the rollover refusal will occur in the near future. It is decreasing in the fundamentals V_t , converging to 1 as V_t approaches to the rollover (crisis) threshold V^R . We can calculate this rollover risk in a closed form, such that $RR(V_t; V^R) = [\frac{V_t}{V^R}]^X$.

Different asset pricing dynamics result from different levels of the rollover thresholds with or without coordination concerns ($V^R = V^{**}$ or V^{FB}). Since higher V^R implies lower debt value, thereby higher credit spreads and rollover risks, coordination failure results in additional spreads along with additional risks in the systemic context compared to microprudential perspective. Figure 1.5 compares how credit spreads and rollover risks under our systemic (one-group coordination failure) approach vary differently from those under the benchmark microprudential approach. The difference in credit spreads can be interpreted as the “coordination failure premium”, and that in rollover risks as “additional systemic risk”.

This result implies that we underestimate both credit spreads and true systemic risks if we follow the microprudential analysis. Notice that credit spreads as well as rollover risks from the two approaches stay close at low levels while the fundamentals are robust, but the discrepancies widen as the fundamentals deteriorate. Thus, the errors are small during normal times but become larger quickly in a downturn.

1.5.3 Flight to quality

We next examine the effects of asset illiquidity. Note that the rollover threshold V^R is given by $V^{**} = P_H + \lambda$ from Corollary 6, which is increasing in asset illiquidity λ . Coordination concerns about limited market liquidity are more severe with more illiquid assets, thus the panic exit accompanying funding evaporation is triggered at a higher threshold. Reflecting this additional risk, credit spreads are higher for more illiquid collateral assets.

Figure 6 describes flight to quality resulting from the different severity of coordination failure problems. Credit spreads rise faster in a downturn for a more illiquid collateral asset, while the differences are very small when the fundamentals are robust.²² This channel of flight to quality is absent if coordination concerns do not exist because then the rollover thresholds are constant $V^{BM} = P_H$ irrespective of the illiquidity measure λ .

1.5.4 More homogeneous system with between-group interactions

We now make the two groups more homogeneous such that strategic interactions between heterogeneous groups arise. Given fixed P_H , consider that stronger L -group now become less robust than before (higher P_L satisfying $P_H - P_L < \lambda$). Coordination concerns now arise not only within a single group as in the baseline setup, but also across the different groups as seen in Section 1.3. H -group's risk is now affected by stronger L -group's degree of financial health, and we get the following Corollary from Proposition 1.

²²Gorton and Metrick (2011), and Krishnamurthy, Nagel, and Orlov (2011) provide empirical evidences for this prediction.

Corollary 7. (Crisis/rollover threshold of H -group with reduced heterogeneity)

- *All H -group institutions stay in the market as long as the fundamental V is beyond their crisis threshold given by $V^* = P_L + 2\lambda$, but exit as soon as V hits V^* .*
- *The debt rollover is allowed for H -group as long as V is beyond their rollover threshold $V^* = P_L + 2\lambda$, but is refused as soon as V hits V^* resulting in funding liquidity evaporation.*

This indicates the emergence of additional systemic risks with reduced heterogeneity (see the right panel of Figure 1.5). Now, the panic is triggered at the higher fundamental threshold (at $V^* = P_L + 2\lambda$, rather than $V^{**} = P_H + \lambda$) because the stronger L -group has become less robust and the coordination problems have become more severe. We observe in Figure 1.5 that although its financial health is unchanged, credit spreads and rollover risks for an H -group institution become higher with reduced heterogeneity between the two groups (two-group coordination failure), compared to when L -group is robust enough that no domino effect is anticipated (one-group coordination failure). Besides, V^* is now increasing in P_L , which implies that H -group's credit spreads and rollover risks become even higher simply when the stronger L -group institutions' financial health deteriorates. This additional premium and additional risks result from the strategic concern among heterogeneous institutions, which is unique in this paper.

1.6 Conclusion

This paper presents a novel framework for studying systemic panic in financial markets with heterogeneous participants. When we anticipate a contagious chain reaction, conventional wisdom dictates that we ought to focus on the weakest link: Bolster the weakest since it all starts from the distress of the weakest. Our analysis, taking macroprudential perspective, suggests we turn this upside down: Bolster the strongest since it actually starts when the strongest loses confidence in the market. When the strongest begins evaluating the possibility of an exit, this prompts the weak to run before the mess materializes and, in turn, a systemic crisis occurs in a self-fulfilling manner.

The framework of this paper can apply more generally to situations in which coordination problems exist among agents with differing degrees of exposure to strategic uncertainty. A companion paper (Choi (2011)) studies contagious bank runs by creditors across multiple banks and generates analogous policy implications: The most effective way to address systemic concerns is, in fact, to bolster the strongest bank. The mechanism can further shed lights on the ongoing European crisis in which contagious distress is well suspected. Regardless, the key message of our macroprudential analysis stays constant: The critical factor is the confidence level of the strongest in the contagion chain. Bolster the strong, not the weak.

1.7 Appendix A: Proofs

Given the history h_t , let $V_j^*(h_s)$ be the threshold of the fundamental on which j -group institutions' collateral constraint binds and the exit game is terminated. We first claim that $V_H^*(h_s) \geq V_L^*(h_s)$. Suppose $V_H^*(h_s) < V_L^*(h_s)$. Note that at

$V_L^*(h_s)$, f_s satisfies $V_L^*(h_s) - \lambda f_s = P_L$, so $f_s = \frac{V_L^*(h_s) - P_L}{\lambda}$. Given this f_s and the fundamental $V_L^*(h_s)$, however, H -group's collateral constraint is violated since $L_s = V_L^*(h_s) - \lambda f_s = P_L < P_H$. This implies that $V_H^*(h_s) > V_L^*(h_s)$ which is contradiction. Thus, $V_H^*(h_s) \geq V_L^*(h_s)$, which implies that H -group institutions will always get distressed when L -group are distressed, but not vice versa. We first prove the following lemma.

Lemma A1. (History independence in the limit case)

- (i) If the exit game has not terminated at period t (i.e., $V_s > V_j^*(h_s)$ for all $s < t$), then $f_s = 0$ a.s. for all $s < t$.
- (ii) s_{it} is a sufficient statistic for V_t

Proof of Lemma A1

- (i) Suppose $V_s > V_H^*(h_s) \geq V_L^*(h_s)$. Note that $V_t - V_{t-1} = O_p(\Delta) \forall t$, thus $V_s - V_H^*(h_s) = O_p(\Delta)$ and $V_s - V_L^*(h_s) = O_p(\Delta)$. Also, $s_{is} - V_s = o_p(\Delta) \forall i$, thus $s_H^*(h_s) - V_H^*(h_s) = o_p(\Delta)$ and $s_L^*(h_s) - V_L^*(h_s) = o_p(\Delta)$. These imply that $s_{is} > s_j^*(h_s)$ a.s. $\forall i$ for both $j = H, L$, as $\Delta \rightarrow 0$. Thus $f_{j,s} = 0$ a.s. for both $j = H, L$. We get $f_s = 0$ a.s. for all $s < t$.

- (ii) This is straightforward from $s_{it} - V_t = o_p(\Delta^{\frac{1}{2}})$ and $V_t - E[V_t|V_s] = O_p(\Delta^{\frac{1}{2}}) \forall s < t$. ■

This Lemma implies that equilibrium strategy profile $s_j^*(h_t)$ (thus the exit game) is history-independent and V_t is the only state variable of the exit game. Since s_{it} is a sufficient statistic for V_t in the limit case, it contains all the relevant information for the optimal decision making and $V_j^*(h_t)$, $s_j^*(h_t)$, $\Pi_j^S(s_{it}, h_t)$ can hence be denoted

as V_j^* , s_j^* , and $\Pi_j^S(s_{it})$. Value function of staying is thus simplified as

$$\begin{aligned} \Pi_j^S(s_{it}) = E \left[\left(w_S \Delta + e^{-r\Delta} \max\{\Pi_j^S(s_{it+\Delta}), \Pi^E\} \right) \times \mathbf{1}_{[f_t < f_j^*(V_t)]} \right. \\ \left. + 0 \times \mathbf{1}_{[f_t \geq f_j^*(V_t)]} \middle| s_{it} \right] \quad (1.14) \end{aligned}$$

which is history independent. Thus we can interpret our dynamic exit game as a sequence of the identical one-shot games.

Proof of Lemma 1

When $f_{-j,t} = 0$ is given, the option value of staying for a j -group institution given the private signal s_{it} can be defined as

$$\Pi_j^S(s_{it}) = E \left[\left(w_S \Delta + e^{-r\Delta} \max\{\Pi_j^S(s_{it+\Delta}), \Pi^E\} \right) \times \mathbf{1}_{[f_{j,t} < f_j^*(V_t)]} + 0 \times \mathbf{1}_{[f_{j,t} \geq f_j^*(V_t)]} \middle| s_{it} \right] \quad (1.15)$$

We can solve for the optimal switching threshold $\underline{s}_j^*$ from the indifference condition $\Pi_j^S(\underline{s}_j^*) = \Pi^E$. Let $\underline{V}_j^*$ be the crisis threshold of the fundamental V on which the collateral constraint of j -group is breached in this case. We now solve for $\underline{V}_j^*$ and $\underline{s}_j^*$ from the following two equations. First, the actual mass of exit f_j has to be equal to the critical liquidation pressure given the fundamental $\underline{V}_j^*$, defined as $f_j^*(\underline{V}_j^*)$. Following the definition, $f_j^*(\underline{V}_j^*) = \frac{\underline{V}_j^* - P_j}{\lambda}$. Since j -group institutions with signals below $\underline{s}_j^*$ exit, from uniform distribution, $f_{j,t} = \Pr[s_{it} \leq \underline{s}_j^* | \underline{V}_j^*] = \frac{\underline{s}_j^* - (\underline{V}_j^* - \epsilon)}{2\epsilon}$. Equating the two, we get

$$\underline{s}_j^* = \frac{2\epsilon}{\lambda}(\underline{V}_j^* - P_j) + \underline{V}_j^* - \epsilon. \quad (1.16)$$

Next, the two actions have to be indifferent at the switching threshold $\underline{s}_j^*$, so $\Pi_j^S(\underline{s}_j^*) = \Pi^E$. Rewriting this condition using (1.15), we get

$$\begin{aligned} Pr(V_t \geq \underline{V}_j^* | s_{it} = \underline{s}_j^*) \times \left[w_S \Delta + e^{-r\Delta} E \left(\max \{ \Pi_j^S(s_{it+\Delta}), \Pi^E \} \middle| s_{it} = \underline{s}_j^* \right) \right] \\ = \Pi^E \end{aligned} \quad (1.17)$$

Note that optimal switching occurs at $\underline{s}_j^*$ with $\Pi_j^S(\underline{s}_j^*) = \Pi^E$, we get

$$E[\max \{ \Pi_j^S(s_{it+\Delta}), \Pi^E \} | s_{it} = \underline{s}_j^*] \rightarrow \Pi^E$$

as $\Delta \rightarrow 0$.

Plug this in (1.17), we get $Pr(V_t \geq \underline{V}_j^* | s_{it} = \underline{s}_j^*) = 1$ as $\Delta \rightarrow 0$, which can be rewritten as

$$\frac{\underline{s}_j^* + \epsilon - \underline{V}_j^*}{2\epsilon} = 1 \quad (1.18)$$

since $V_t | (s_{it} = \underline{s}_j^*)$ is uniformly distributed over $[\underline{s}_j^* - \epsilon, \underline{s}_j^* + \epsilon]$. From (16) and (18), we get $\underline{s}_j^* = P_j + \lambda + \epsilon$ and $\underline{V}_j^* = P_j + \lambda$ ■

Proof of Lemma 2

As in the proof of Lemma 1, we define two crisis thresholds of the fundamental V_j^* with $j = H, L$, on which j -group's collateral constraint is breached.

We first solve for H -group's optimal threshold s_H^* given s_L^* (best response $s_H^{BR}(s_L^*)$). On the fundamental threshold of V_H^* , note that

$$f_{H,t} = \Pr[s_{it} \leq s_H^* | V_H^*] = \frac{s_H^* - (V_H^* - \epsilon)}{2\epsilon}$$

and

$$f_{L,t} = \Pr[s_{it} \leq s_L^* | V_H^*] = \frac{s_L^* - (V_H^* - \epsilon)}{2\epsilon}$$

thus

$$f_t = f_{H,t} + f_{L,t} = \frac{s_H^* + s_L^* - 2(V_H^* - \epsilon)}{2\epsilon}.$$

Since this actually coincides with $f_H^*(V_H^*) = \frac{V_H^* - P_H}{\lambda}$, we get

$$s_H^* = \frac{2\epsilon}{\lambda}(V_H^* - P_H) + 2(V_H^* - \epsilon) - s_L^*. \quad (1.19)$$

The indifference condition at s_H^* implies $\Pi_H^S(s_H^*) = \Pi^E$, and with (1.6) we get

$$\begin{aligned} \Pr(V_t \geq V_H^* | s_{it} = s_H^*) \times \left[w_S \Delta + e^{-r\Delta} E \left(\max\{\Pi_H^S(s_{it+\Delta}), \Pi^E\} \middle| s_{it} = s_H^* \right) \right] \\ = \Pi^E. \end{aligned} \quad (1.20)$$

Note that optimal switching occurs at s_H^* with $\Pi_H^S(s_H^*) = \Pi^E$, we get

$$E[\max\{\Pi_H^S(s_{it+\Delta}), \Pi^E\} | s_{it} = s_H^*] \rightarrow \Pi^E$$

as $\Delta \rightarrow 0$.

Plug this in (1.20), we get $Pr(V_t \geq V_H^* | s_{it} = s_H^*) = 1$ as $\Delta \rightarrow 0$, which can be rewritten as

$$\frac{s_H^* + \epsilon - V_H^*}{2\epsilon} = 1 \quad (1.21)$$

since $V_t | (s_{it} = s_H^*)$ is uniformly distributed over $[s_H^* - \epsilon, s_H^* + \epsilon]$.

From (1.19) and (1.21), solving for s_H^* , we get

$$s_H^* = \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1} \right] \times [s_L^* + \frac{2\epsilon}{\lambda} P_H + 3\epsilon] + \epsilon. \quad (1.22)$$

We now show that $s_H^* > s_L^*$ (i.e., $s_H^{BR}(s_L^*) > s_L^*$) if $s_L^* < \bar{s}_H^* (= P_H + 2\lambda + \epsilon)$.

From the above,

$$\begin{aligned} s_H^* - s_L^* &= \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1} \right] \times [s_L^* + \frac{2\epsilon}{\lambda} P_H + 3\epsilon] + \epsilon - s_L^* \\ &= \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1} \right] \times [3\epsilon + \frac{2\epsilon}{\lambda} (P_H - s_L^*)] + \epsilon \\ &> \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1} \right] \times [3\epsilon - \frac{2\epsilon}{\lambda} (2\lambda + \epsilon)] + \epsilon \\ &= \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1} \right] \times [-\epsilon - \frac{2}{\lambda} \epsilon^2] + \epsilon = -\epsilon + \epsilon = 0 \end{aligned}$$

where the inequality comes from $s_L^* < P_H + 2\lambda + \epsilon$. We thus get $s_H^{BR}(s_L^*) > s_L^*$ if $s_L^* < \bar{s}_H^*$.

Repeating the same steps for the L -group, we get

$$s_L^* = \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1} \right] \times [s_H^* + \frac{2\epsilon}{\lambda} P_L + 3\epsilon] + \epsilon$$

given s_H^* , and $s_L^{BR}(s_H^*) > s_H^*$ if $s_H^* < \bar{s}_L^*$. ■

Proof of Proposition 1

By definition, $\bar{s}_L^* < \bar{s}_H^*$. This implies that $s_L^* < \bar{s}_H^*$ as $s_L^* \leq \bar{s}_L^*$. This implies that $s_H^* > s_L^*$ always has to hold from Lemma 2. Also, notice that $s_H^* \geq \bar{s}_L^*$ since $s_H^* < \bar{s}_L^*$ implies $s_L^* > s_H^*$ from Lemma 2, which contradicts with the above.

Now, suppose $s_L^* < \bar{s}_L^*$ as $\Delta \rightarrow 0$ (thus $\epsilon \rightarrow 0$). Note that from (1.22) $s_H^* \rightarrow s_L^*$ in this case, implying $s_H^* < \bar{s}_L^*$. But then this implies $s_L^* > s_H^*$ from Lemma 2, contradicting with $s_H^* > s_L^*$. Combining with $s_L^* \leq \bar{s}_L^*$, we thus get $s_L^* \rightarrow \bar{s}_L^*$ in the limit. $s_H^* \rightarrow s_L^*$ implies that $s_H^* \rightarrow \bar{s}_L^*$. Therefore, in the limit, $\bar{s}_L^* \rightarrow P_L + 2\lambda$, and both H and L group institutions exit the market altogether when the fundamental V hits $P_L + 2\lambda$. ■

1.8 Appendix B: Non reduced-form setup

We examine the non reduced-form setup without the delegation assumption by analyzing the equity holder's payoff directly. Following the same setup, we first focus only on one-group in isolation as in Section 3. We claim that the non reduced-form result is the same as Lemma 1 based on the reduced-form setup. Given this, deriving the same Proposition 1 is straightforward following the same steps of Section 1.3.2.2. We take the following two steps: (i) Show that the equity holder prefers keeping his position rather than liquidating immediately when the collateral constraint is satisfied, (ii) incorporating coordination concerns, show that the solutions are the same as in Lemma 1. We focus on an H -group institution without loss of generality.

We impose a restriction on the coupon rate such that $\frac{C_H}{P_H} < \frac{r}{1-\frac{1}{1-X}}$ where X is the negative root of $X(X-1)\frac{\sigma^2}{2} + X(r-\sigma) = r$. This condition holds for most of the

realistic parametric assumptions. We here rule out the extreme cases in which the coupon rate $\frac{C_H}{P_H}$ is so high that the collateral constraint never binds in equilibrium.

First, suppose that there exists a fundamental threshold V^D on which the institution (equity holder) wishes to close its position paying back its debt principal P_H , even though the debt can be surely rolled over (i.e., $f_{H,t} < f_H^*(V^D)$). Then, we can calculate the equity value at V_t , denoted by $\Pi_H^S(V_t) = V_t - D(V_t; V^D)$, as follows

$$\Pi_H^S(V_t) = V_t - \frac{C_H}{r} + \left[\frac{C_H}{r} - P_H \right] \times \left[\frac{V_t}{V^D} \right]^X.$$

However, we can show that $\left. \frac{\partial \Pi_H^S(V_t)}{\partial V_t} \right|_{V_t=V^D} > 0$ in this case which violates the smooth pasting condition. This implies that when rollover is certainly allowed, the exit decision at V^D will be “too early” since the option value of staying is higher than that of immediate liquidation $V^D - P_H$. Therefore, no such V^D exists and the institutions prefer to stay if funding liquidity is surely provided. This implies that the equity value should be higher than the payoff from immediate liquidation in that region, so $\Pi_H^S(V_t) > V_t - P_H$ if $V_t > P_H + \lambda$ (where $f_{H,t} < f_H^*(V_t)$ with probability 1).

Now we incorporate (coordination) concerns about the rollover refusal. We impose a non-zero “penalty” c to the equity holder in the case of forced liquidation. It can be any positive number, and we can interpret this as a loss in the franchise value or reputational costs. We use the history-independent property of Lemma A1.

When choosing to exit given the signal s_{it} , the equity holder simply expects to get

$$\Pi_H^E(s_{it}) = \max\{s_{it} - P_H, 0\}.$$

When choosing to stay, the equity value incorporating the potential rollover refusal is

$$\begin{aligned} \Pi_H^S(s_{it}) = E \Big[& \left((\delta V_t - C_H) \Delta + e^{-r\Delta} \max\{\Pi_H^S(s_{it+\Delta}), \Pi_H^E(s_{it+\Delta})\} \right) \\ & \times \mathbf{1}_{[f_{H,t} < f_H^*(V_t)]} - c \times \mathbf{1}_{[f_{H,t} \geq f_H^*(V_t)]} \Big| s_{it} \Big] \quad (1.23) \end{aligned}$$

similar to (1.6) in Section 1.3.2. Note that the equity value is higher than the outside option when $Pr(f_{H,t} < f_H^*(V_t)|s_{it}) = 1$, so $\Pi_H^S(s_{it}) > \Pi_H^E(s_{it})$ if $s_{it} > P_H + \lambda + \epsilon$. It is straightforward that upper/lower dominance regions exist, strategic complementarities and state monotonicity hold. Thus by Toxvaerd (2008), there exists a unique s^* such that $\Pi_H^E(s^*) = \Pi_H^S(s^*)$. Given $s_{it} = s^*$, as $\Delta \rightarrow 0$, $\Pi_H^S(s_{it+\Delta}) \rightarrow \Pi_H^S(s^*)$ a.s. and $\Pi_H^E(s_{it+\Delta}) \rightarrow \Pi_H^E(s^*)$ a.s., thus we get $Pr(f_{H,t} < f_H^*(V_t)|s^*) = 1$. The rest of the proof is the same as that of Lemma 1, and we can get $s^* = P_H + \lambda + \epsilon$ which is the same as in Lemma 1.

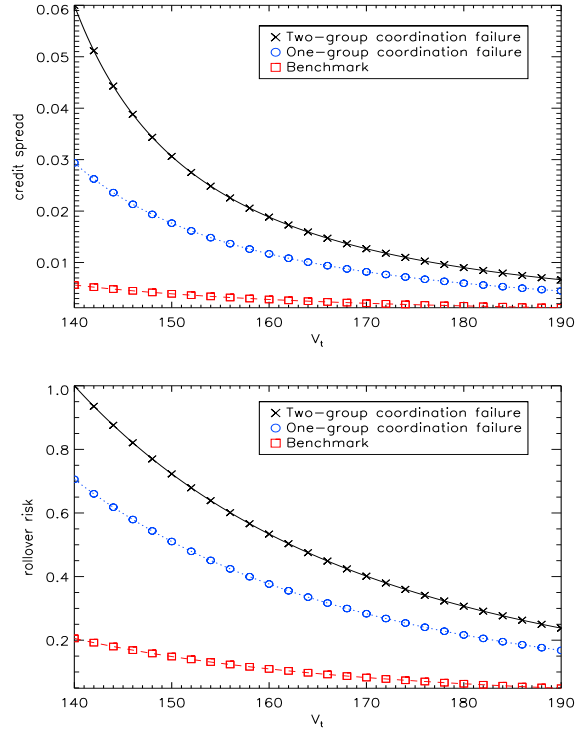


Figure 1.5: Credit spreads and rollover risks of an H -group institution with different degrees of coordination failure

Both credit spreads and rollover risks increase as the heterogeneity decreases and the coordination problems become more severe. The difference between credit spreads is “coordination failure premium” and that between rollover risks is “additional systemic risk”. The parameters in this example are $r = 0.04$, $\sigma = 0.1$, $\delta = 0.02$, $P_H = 100$, $P_L = 60$, $C_H = 10$, $\lambda = 30$. In the two-group coordination failure case of Section 1.5.4, the difference in two group’s initial leverage levels is smaller with $P_L = 80$.

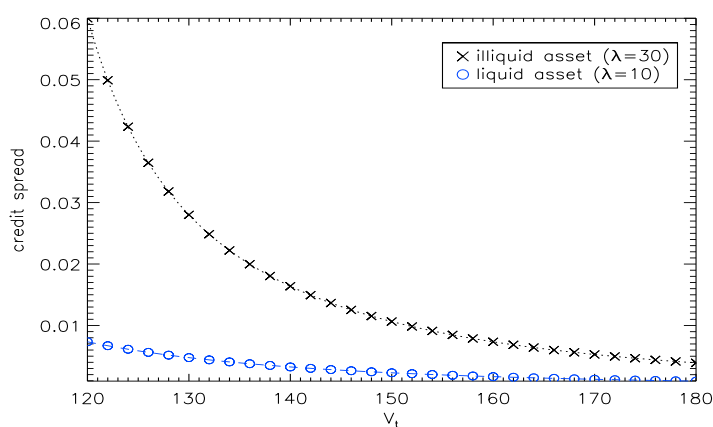


Figure 1.6: Flight to quality

Flight to quality emerges as the fundamentals deteriorate from more severe coordination concerns for illiquid collateral assets. Other parameters in this example are the same as in Figure 1.5.

Chapter 2

Risk Sharing, Credit Crunch, and Financial Fragility

2.1 Introduction

A credit crunch generally arises when lenders worry about creditworthiness of borrowers or their own liquidity needs in the future. The 2007-09 credit crisis, however, was unprecedented in both its scope and severity.¹ Even relatively unscathed banks were reluctant to lend, and some solvent but illiquid banks found it difficult to borrow. Interbank lending broke down with skyrocketed TED spread, leading to the credit freeze. Flight to liquidity and fire sale of assets prevailed. Natural questions included why the credit crunch was so universal, and so severe.

In this paper, I present a model with a novel link between market liquidity, funding liquidity, and risk sharing. Risk sharing among banks enables them to diversify their individual risks and smooth the potential interim liquidity shock.

¹Furfine (2002) provides an evidence that the interbank market was stable during the LTCM crisis in spite of increased uncertainty in the financial market.

Thus, diversification should enhance individual banks' stability in principal. When market liquidity is scarce, however, it can have the opposite effect as a whole; risk sharing can make the interbank market more fragile. This fragility may explain the universal credit crunch and flight to liquidity, corresponding output drops during the crisis.

There are three main ingredients in my model featuring the 2007-09 crisis. The first one is scarce liquidity in secondary asset markets; the asset price in the secondary market deviates from the fundamental value when a large number of assets are liquidated, and the wedge becomes larger as more are sold. The second ingredient is friction coming from a lending relationship in the interbank market.² The financial intermediaries (banks) can borrow from a relationship lender when necessary, but these lenders are also potential buyers in the secondary market. Thus, lenders may refuse to provide a loan and become "vulture" buyers instead, if that outside option is more profitable.³ Therefore, the borrowing banks have to compensate the lenders for this outside option (rent) in order to induce them to lend. The last ingredient is risk sharing among banks which diversifies individual risks but exposes them to each other's risk. This can help reduce the risk of an individual bank, but it also becomes a source of a systemic (joint) distress, since banks are now interconnected.

I first focus on the ex ante liquidity provision problem of banks facing interim liquidity risks. When experiencing an interim liquidity deficit, banks can meet this demand either by hoarding cash beforehand (inside liquidity) or borrowing from

²See Bech and Atalay (2010), Cocco, Gomes, and Martins (2009) on the evidences of a lending relationship in the interbank market.

³On Jan 27, 2009, Citi CEO Vikram Pandit said "It is cheaper to buy a loan in the secondary market than make a new one because of the liquidity premium in the secondary markets".

outside (outside liquidity), but their (long term) assets will be liquidated if they fail to secure sufficient liquidity. More cash hoarding implies less investment in the productive long run asset, thus banks *ex ante* choose an optimal mix of inside and outside liquidity given the cost of outside liquidity. In contrast to the literature that assumes that lenders are breaking even, the cost of this outside liquidity is nontrivial and endogenous in my model; it reflects the lenders' outside option value (rents).

The lender in the model is a relationship lender, since the borrower's asset is complex; only those who possess a proper monitoring technology or knowledge are willing to lend. The main feature is that this lender can alternatively become a buyer for the asset in the secondary market. Therefore, he may refuse to lend and store his liquidity and instead wait for the future lucrative opportunity to come if he anticipates higher profit by collecting the fire-sale assets in the secondary market. Predatory behavior can thus arise, as in Brunnermeier and Pedersen (2005), or in Carlin, Lobo, and Vishwanathan (2007). The borrowers in this case can then borrow only if they pay extra costs for outside liquidity (rents) to compensate the lender for this higher opportunity cost. This higher cost of outside liquidity may in turn affect the banks' *ex ante* portfolio choice of inside liquidity hoarding and long term investment. In other words, the lenders' *ex post* opportunism can affect the borrower's *ex ante* liquidity provision.

To be specific, the asset price can deviate from the fundamental value with limited potential buyers (cash) in the secondary market, hence a potential buyer may buy at a profit when there is a fire sale (cash in the market pricing as in Allen and Gale (1994)). Since the lender is also a potential buyer, his rents are now nonzero, and he will not lend unless he gets an extra cost of credit for a loan.

Borrowing in the spot market (after the shock realizes) in this case is less efficient, since the lender will always try to turn to a “vulture” buyer, extracting all of the rents ex post, whenever a bank is under distress. Facing this lender opportunism, the borrowing bank ex ante has to hoard an excessive level of cash in order to avoid financial distress. Alternatively, the bank can contract a line of credit⁴ ex ante so as to avoid the lender’s ex post opportunism, which enables him to reduce the wasteful cash hoarding. Nonetheless, the lender will still not agree to lend unless he is compensated for the rent (expected profit upon refusing to lend) with an extra “ex-ante” cost of outside liquidity. Here, both this cost of outside liquidity and the liquidity provision decision is endogenous.

I next focus on how risk sharing between banks affects this cost of outside liquidity. Risk sharing, such as interbank deposit, CDS, or mergers, enables an individual bank to smooth its interim cash demand through diversification. The main result is that risk sharing can become the source of financial fragility, making the credit cost more volatile in response to the changes in aggregate uncertainty, even though it may reduce the credit cost during normal times with low aggregate uncertainty.

First of all, risk sharing can in principle reduce the cost of outside liquidity by decreasing the likelihood of the distress event through diversification, depressing the outside option value for the lender. However, note that it can become the source of a joint (systemic) distress when a sufficiently large shock is realized as more banks become interconnected. Since the liquidation price is decreasing in the amount liquidated, the asset price is very low when this episodic joint distress ever happens and the buyers can potentially make a significant profit. On the contrary,

⁴This can also be interpreted as securing a relationship lending in the interbank market.

the profit margin is much lower with a single bank distress (which is mostly the case without risk sharing). However, if this systemic distress is anticipated as a rare event, with decreased likelihood *ex ante*, the overall outside option value can be decreased with risk sharing.

The main result is that the effect of increase in aggregate uncertainty (higher likelihood of a tail event, for instance) differs in the two cases (with and without risk sharing). If risks are not shared, the effect of this change on the lender's rent is relatively small. This is because, although the future fire-sale event becomes more likely, not many assets should be liquidated at the same time, implying low profit margin and thus little impact on the lender's expected profit. If risks are shared, however, the lender's rent increases rapidly; when the tail event (in this case, mostly joint distress) becomes more likely, the lender's expected profit rises faster because of the higher profit margin with larger fire-sale. This sensitivity of *ex-ante* rent is equivalent to the sensitivity of the cost of credit (outside liquidity) to changes in aggregate uncertainty, which indicates that risk sharing makes funding liquidity more vulnerable when aggregate uncertainty fluctuates. Since an increase in the cost of outside liquidity gives a borrower bank more incentives to hoard inside liquidity and cut down its long term investment, aggregate output may shrink as well.

This rapid increase in the cost of outside liquidity can also become a source of credit rationing. A borrowing bank cannot commit to pay too much *ex ante* due to the asset substitution problem as in Stiglitz and Weiss (1981). This suggests that, if the cost of outside liquidity is beyond some threshold level, no loan will be provided and outside liquidity evaporates. Then, the lender refuses to lend in expectation of higher profit in the secondary market. Without outside liquidity supply, the bank has to cut down its long term investment to hoard more inside

liquidity, and aggregate output drops. Credit crunch in the interbank market then causes a credit crunch in the debt market. Since the cost of outside liquidity can rise more rapidly with risk sharing, these credit rationing and aggregate output drops can become more likely when risks are shared.

In terms of welfare, the banks may *ex ante* tend to share their risks “excessively” in pursuit of cheap credit, which eventually brings fragility. From the social welfare perspective, aggregate welfare is maximized when resources are allocated in a most efficient way, and the cost of credit is not of concern itself unless it distorts resource allocation. The banks, however, do take the credit cost into account. If the benefit of cheap credit achieved by risk sharing is greater than the cost of potential credit rationing, they choose to share risks, although this is not desirable from a social perspective.

The credit crunch and financial fragility discussed in this paper can be avoided through proper policy interventions. Note that the fragility comes from fluctuations of the lender’s outside option value. Liquidity requirements, asset repurchase, and liquidity injection are some of the tools that stabilize the economy. Liquidity requirements force banks to hold enough inside liquidity when the lenders refuse to supply outside liquidity. Acting as a commitment device, this reduces the lenders’ rents and lowers the cost of credit. This measure thus reduces fragility *ex ante* without generating inefficiency in resource allocation, since it only affects the off-the-equilibrium path outcome. Asset repurchase and liquidity injection can directly affect market liquidity, with some potential caveats.

The remainder of the chapter is organized as follows. Section 2.2 reviews the related literature. Section 2.3 presents the model. Section 2.4 analyzes the equilibrium. Section 2.5 provides an argument of excessive risk sharing. Section 2.6 studies

several policy interventions. Section 2.7 discusses the robustness issues and some extensions of the model. Section 2.8 concludes.

2.2 Related Literature

This paper is mainly related to two strands of literature. The first is on bank liquidity provision and banking/interbanking crises. Diamond and Dybvig (1983) develop a seminal model of bank liquidity provision and a bank run. Allen and Gale (1994, 1998) introduce an aggregate uncertainty, with bank failures and fire sale as an equilibrium phenomenon. Shleifer and Vishny (1992), Gorton and Huang (2004) also develop a model of fire sale with lack of liquidity in the market. Diamond and Rajan (2005) formalize contagious bank failures through the shrinkage of the liquidity pool and increase in real interest rate. In this paper, market liquidity shrinks through fire sale, which leads to higher cost of outside liquidity affecting ex ante portfolio decisions of banks.

The banks in this paper also has an access to outside liquidity (borrowing in the interbank market) with potential credit rationing. Stiglitz and Weiss (1981) first examine the effects of private information on debt markets and the possibility of credit rationing. Battacharya and Gale (1986) study liquidity provision in the interbank market with private information, and Heider, Hoerova, and Holthausen (2009) provide a model of interbank market breakdown with severe asymmetric information problems. Holmström and Tirole (1998, 2008), and Bolton, Santos and Scheinkman (2010) study an optimal mix of different sources of liquidity. In Holmström and Tirole, cash hoarding (inside liquidity in this paper) and a line of credit (outside liquidity in this paper) are equivalent as a liquidity buffer since the lender has no outside option and always breaks even. Outside liquidity in their model

stands for exogenous liquidity supply by the government. In Bolton et al., banks can meet their liquidity demand with either cash hoarding (inside liquidity) or asset sales (outside liquidity). Their focus is on the effect of asymmetric information on this liquidity provision. Banks can borrow from outside instead of selling in this paper, and our focus is how risk sharing affects the volatility of the cost of outside liquidity and corresponding outside liquidity supply.

This paper also introduces a novel mechanism of liquidity evaporation. Brunnermeier and Pedersen (2009) develop a model of feedback effects between market liquidity and funding liquidity. Morris and Shin (2005), and Malherbe (2010) study a sudden market liquidity evaporation during the financial crisis. Other two papers that share a similar motivation with this paper are Acharya, Gromb, and Yorulmazer (2009), and Diamond and Rajan (2010). Their models contain a strategic behavior of a potential buyer with better investment opportunity in the future. There are also a growing number of empirical research on liquidity provision during the financial crisis (Ivashina and Scharfstein (2009), Nagel (2010)). The findings in Gara, Kovner, and Schoar (2010) is the closest to the predictions of this paper. They find that during the 2008 crisis, interbanking market (the Federal funds market) didn't completely freeze up (the average amount of lending did not contract), but the lending rates did increase and the increments varied across the banks depending on a borrowing bank's characteristic. My model predicts that when aggregate uncertainty increases, outside liquidity should still be provided but with higher lending rate. It also predicts that the sensitivity of lending rates depend on the lender's rent, which in turn depends on the market liquidity for the borrowing bank's assets and the risk sharing structure.

The second related strand of literature is on the aggregate effect of financial network, homogenization, or risk sharing. Some of the findings in the literature can be summarized as follows: (1) The network becomes more robust with more interconnections (Allen and Gale (2000), Zawadowski (2010)); (2) A systemic failure can happen with more interconnections while the likelihood of individual failure becomes smaller (Freixas, Parigi, and Rochet (2000), Leitner (2005), Brusco and Castiglionesi (2007), Castiglionesi, Feriozzi and Lorenzoni (2010), Wagner (2009), Ibragimov, Jaffee, and Walden (2010)). I argue that (1) only holds with abundant market liquidity, and risk sharing causes different type of fragility when market liquidity is scarce. (2) implies certain financial fragility, which is an optimal financial crisis coming incorporating a tradeoff between an individual safety and a systemic risk (Acharya (2001), Leitner (2005), Castiglionesi et al. (2010), Wagner (2009), and Ibragimov et al (2010)). In all of the models above, however, financial crisis happens only as a rare event. It is hard to conclude in that sense that risk sharing induces financial fragility since the likelihood of that episodic crisis is in fact very low.⁵ In my model, however, credit crunch is not a rare event; it actually becomes more likely with risk sharing when aggregate uncertainty increases. Risk sharing does induce financial fragility in this sense.

The motivations of risk sharing (or homogenization) are different across papers. Acharya and Yorulmazer (2007, 2008b) suggest that “too many to fail” argument leads investors to be homogenized so that they get bailed out when in trouble. In Acharya and Yorulmazer (2008a), possible information contagion induces the banks

⁵We may not have to worry about the systemic crisis since it is literally a rare event. Alan Greenspan’s testimony “We have to recognize that this is almost surely a once-in-a-century phenomenon, and in that regard, to realize the types of regulation that would prevent this from happening in the future are so onerous as to basically suppress the growth rate in the economy and ... the standards of living of the American people.”

to herd. The sources of negative externality of risk sharing are often exogenous in the literature. Acharya (2001), Wagner (2009), Ibragimov et al. (2010) assume some social costs of joint failure. In Castiglionesi, Feriozzi and Lorenzoni (2009), excessive risk sharing comes from pecuniary externality. In my model, banks share their risks not only to diversify their interim shocks but also in seeking cheap credit, and this can impose a negative externality by causing fragility and inefficient resource allocation.

2.3 The Model Setup

2.3.1 Agents, Technology, Interim Shock

Consider a three period ($t = 0, 1, 2$) economy with a single consumption good. There are three investors (banks) in this economy, two ex ante identical borrowers (A and B) and a lender.⁶ All of them are risk neutral, and only consume at $t = 2$.

The borrowers can initiate a productive long-term project, with 1 unit of initial endowment.⁷ At $t = 0$, they can invest in either of two assets (projects), a short term asset or a long term asset. The short term asset (liquid asset) is a storage technology and one unit of the good stored produces one unit in the next period, acting as liquidity cushion for potential interim liquidity needs. The long term asset takes two periods to mature. One unit of the good invested in the long term asset at $t = 0$ produces $R > 1$ at $t = 2$. It produces nothing at $t = 1$, but can be sold in the secondary market at endogenous liquidation price P per unit. The borrower chooses an optimal portfolio decision at $t = 0$, y in the short term asset and $1 - y$ in

⁶We can also think of them as three types of investors. See Section 2.7.2.

⁷For simplicity, I ignore the agency problem within the borrower institution so that capital structure is irrelevant. But I implicitly assume that a substantial part of the endowment is from debt financing. See Section 2.7.1.

the long term asset (plus how much to borrow from the lender), facing an interim liquidity shock described below. I assume that all investors take the asset price as given.

At $t = 1$, the borrower $i = A, B$ may experience an idiosyncratic interim shock $\tilde{\rho}_i$, which is an amount of required liquidity injection (see Figure 2.1). This can be understood as a drawdown on a committed loan, or liquidity deficits from a loss of asset-in-place. For simplicity, I assume that the shocks are exclusive, such that at most one of the two borrowers (either A or B) may receive a positive shock, and each borrower ex ante anticipates to receive non-zero shock with probability $p(< \frac{1}{2})$. When hit by a shock, the size of the potential shock follows $\tilde{\rho} \sim U[0, \bar{\rho}]$.⁸ Thus, ex ante distribution of an individual liquidity shock can be represented as

$$\tilde{\rho}_i = \begin{cases} 0 & \text{with prob } 1 - p \\ \tilde{\rho} & \text{with prob } p, \text{ where } \tilde{\rho} \sim U[0, \bar{\rho}] \end{cases} \quad (2.1)$$

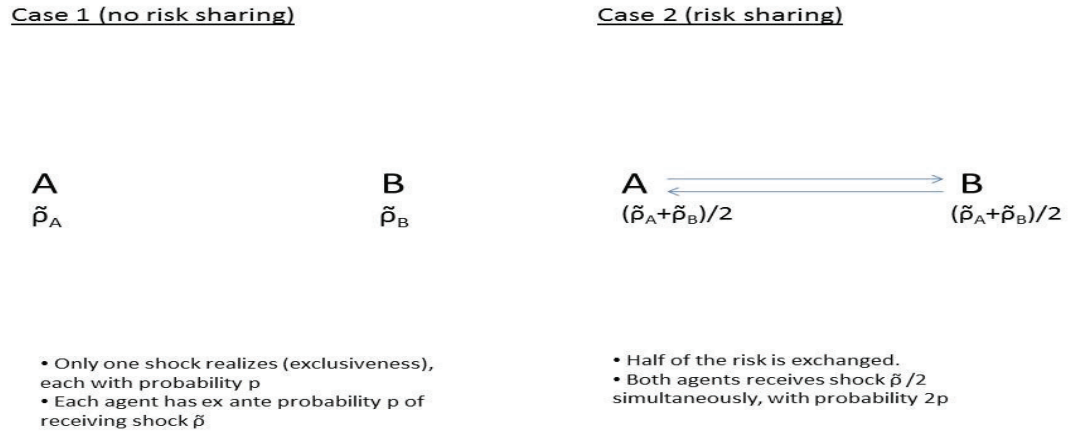
I interpret $\bar{\rho}$ as a measure of aggregate uncertainty where higher $\bar{\rho}$ implies higher aggregate uncertainty in this economy.⁹ I assume $\bar{\rho} < 1$ such that the size of the shock is less than the insider's initial endowment.

Let ρ be a realization of the liquidity shock $\tilde{\rho}$. The insiders can meet this liquidity demand from two sources of liquidity: inside liquidity and outside liquidity. Inside liquidity is liquid asset y hoarded at $t = 0$, and outside liquidity is a loan from the lender described below. If the borrower cannot secure enough liquidity to pay ρ , he is in financial distress and forced to liquidate (fire sale) his long term asset $1 - y$ in the secondary market at a price P per unit.¹⁰

⁸We can assume i.i.d. shocks instead of exclusive shocks, but the main implications of the model remain the same.

⁹This is for tractability. An alternative measure is mean-preserving spread of $\tilde{\rho}$.

¹⁰This no partial liquidation assumption is not crucial. See Section 2.7.1.

**Figure 1 : Risk sharing****Figure 2.1: Liquidity shock and risk sharing**

2.3.2 Loan from the Lender

Before the portfolio decision at $t = 0$, the borrowers can approach the lender for a credit line contract (or secure a lending relationship). A credit line contract is characterized by the limit amount and an (endogenous) interest rate r for the loan. If the lender accepts, the borrower can borrow up to the specified limit at $t = 1$ if necessary, and pays $r \cdot \rho$ at $t = 2$ upon borrowing ρ at $t = 1$.¹¹ I consider r as a measure of borrowing cost. Instead of contracting an ex ante credit line, the borrowers can alternatively borrow from the lender in the spot market at $t = 1$ after the shock realizes.¹²

¹¹I assume that this credit line contract is ex-post enforceable. See Boot, Greenbaum, and Thakor (1993) for the theoretical model based on reputation concerns.

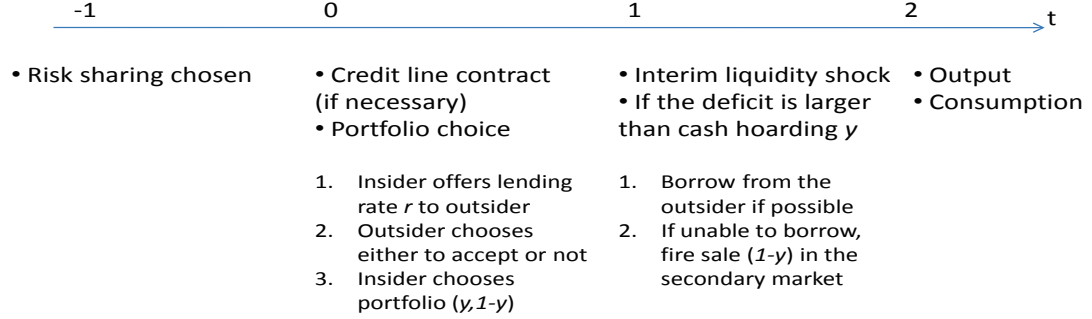
¹²This is strictly dominated by the credit line contract, thus don't arise in equilibrium. See Appendix.

2.3.3 Lender's Investment Alternatives, Time Line

The (relationship) lender is the only investor (bank) in this economy who is willing to lend to the borrowers with the proper monitoring technology. He is endowed with $M(> \bar{p})$ units of the good and has three investment alternatives: (i) Lending to the borrowers in the interbank market; (ii) Buying fire sale assets in the secondary market; (iii) Storage. I assume that the lender cannot initiate the long term project by himself without the proper screening knowledge. However, he can run the long term project upon an acquisition in the secondary market at $t = 1$ and produces R per unit at $t = 2$.

Note that the lender can rather choose to become a buyer than lending, which is one of the main assumptions. In the $t = 1$ spot market, he may refuse to lend to the borrowers if he can collect higher profits by letting the borrowers fall in distress. The borrower thus has to pay extra (cost of credit) to induce the lender to lend in this case. However, the borrower can avoid this ex-post opportunism and secure outside liquidity provision by contracting a (ex-ante) line of credit (a lending relationship) ex ante at $t = 0$. Again, the lender may refuse to accept this credit line offer if he has a better outside option: waiting for the borrower's distress and collecting the discounted fire sale assets in the secondary market. Thus, extra cost of credit needs to be paid ex ante at $t = 0$ if the lender's expected profit (rent) is nonzero, even for the credit line contract.

The time line is summarized as follows (Figure 2.2). At $t = 0$, the borrowers first approach the lender for a credit line contract if necessary. The lender then chooses either to accept the offer or not. Next, the borrowers choose their investment portfolio $(y, 1 - y)$ which is not contractible. At $t = 1$, a liquidity shock realizes and the borrowers experiencing the liquidity shocks try to borrow from the lender if

**Figure 2 : Time line of the model****Figure 2.2: Timeline**

necessary. The distressed borrower's long term asset is liquidated in the secondary market if he fails to secure enough liquidity. At $t = 2$, output is produced and agents consume.

2.3.4 Asset Substitution

At $t = 0$, the borrowers can secretly invest in other riskier, less efficient asset other than the long term asset. This asset substitution problem can bring credit rationing as in Stiglitz and Weiss (1981). For simplicity, I impose an exogenous threshold $\hat{\Pi}$ such that the substitution happens if the borrower's expected loan payment is larger than $\hat{\Pi}$. That is, the borrowers cannot commit to pay more than $\hat{\Pi}$ in expectation at $t = 0$. I assume that

$$\hat{\Pi} > E[\tilde{\rho}_i] = p \times E[\tilde{\rho}] \quad (A1)$$

which implies that asset substitution does not arise with no extra cost of credit ($r = 0$).

2.3.5 Risk Sharing

The borrowers may arrange (ex-ante) risk sharing between them.¹³ This risk sharing diversifies and smooths their individual interim shock, enabling them to insure each other. Here, I simply characterize risk sharing as an exchange of their idiosyncratic risks; after risk sharing, each borrower owns half of his own risk and half of the other's risk.¹⁴ With the exclusiveness assumption of the two shocks, both borrowers will now be hit by the same shock with probability $2p$, but the size of the shock one gets is half of that without risk sharing, which is distributed uniformly between 0 and $\frac{\bar{p}}{2}$. Thus we can denote the individual liquidity shock with risk sharing as

$$\tilde{\rho}_i = \begin{cases} 0 & \text{with prob } 1 - 2p \\ \frac{\tilde{p}}{2} & \text{with prob } 2p, \text{ where } \tilde{p} \sim U[0, \bar{p}] \end{cases} \quad (2.2)$$

for both borrower $i = A, B$. This risk sharing is described in Figure 2.1.

2.4 Liquidity Provision and Cost of Credit

I now solve for the cost of credit (outside liquidity) and ex-ante liquidity provision of the borrower I first fix the level of aggregate uncertainty ($\bar{p}$) and solve the optimal liquidity provision problem of the borrower and the corresponding outside liquidity supply for the case without risk sharing (Case 1). Then I repeat the same steps

¹³Risk sharing is contracted at $t = -1$ although we will take the risk sharing arrangement as given until Section 2.5.

¹⁴The merger between the two borrowers can be an alternative interpretation of this risk sharing.

for the case with the risk sharing arrangement (Case 2). Finally, I analyze how the changes in aggregate uncertainty $\bar{p}$ affect these. I discuss that financial fragility (more volatile credit cost and output) may emerge with risk sharing by comparing the two cases.

For simplicity, I assume that the lender is the only potential buyer in the secondary market and other investors don't have a knowledge about the complex underlying asset.¹⁵ For exactly the same reason, those investors are reluctant to lend to the borrowers. The lender is hence the only potential buyer (lender) to the borrowers in our setup.

The asset price in the secondary market can deviate from the fundamental value with limited market liquidity, and this wedge becomes larger as more assets are sold. Thus the lender may collect nonzero profit in the secondary market when there is fire sale, which gives him an incentive to become a “vulture” buyer instead of being a “friendly” relationship lender.

Note that the outsider endowment is M . When market liquidity is not enough to clear the asset market at a price equal to the fundamental value R , the asset is priced by cash-in-the-market pricing as in Allen and Gale (1994,1998) such that

$$P = \min\left\{R, \frac{M}{Q}\right\} \quad (2.3)$$

where $Q = N \times (1 - y)$ is the total amount of long term asset liquidated in the secondary market, and $N = 1, 2$ is the number of the borrowers under distress. If $P < R$, the lender gets a positive profit $(R - P)$ per unit as a buyer. Thus the

¹⁵This is for simplicity. We can still preserve the main results with a weaker assumption of downward sloping asset demand curve. See Section 2.7.3.

borrowers have to compensate the lender for this “rent” if they wish to borrow from the lender, and this rent affects the demand and supply of outside liquidity.

Recall that a line of credit contract is characterized by a credit limit and an accompanying interest rate r . In the appendix, I show the optimality of the credit line contract such that; (i) The borrowers prefer contracting a line of credit at $t = 0$, to not contracting it (borrowing in the $t = 1$ spot market or not borrowing at all); (ii) Credit limit on the optimal credit line is large enough to fully cover their interim liquidity deficit when credit is not rationed. Without loss of generality, I set the credit limit to be $\bar{p}$ and focus on deriving an optimal interest rate (cost of credit) r^* of a credit line contract which the borrower offers to the lender at $t = 0$. Here the optimality stands for maximum expected payoff for the borrower, while inducing the lender to supply outside liquidity.

I solve the borrower’s optimal problem (choice of an outside interest rate r and inside liquidity hoarding y) taking the following steps. First, I calculate the lender’s rent (off-the-equilibrium payoff) when he refuses to be a relationship lender and chooses to collect fire sale assets (if any). Notice that the borrower has to pay this rent to the lender as an additional cost in order to induce him to lend. Given this rent, I next solve for the optimal loan contract (r^*) and the optimal portfolio decision (y^*) for the borrower. I focus on the symmetric equilibrium and treat the borrowers A and B in an equal way.

2.4.1 Case 1: No Risk Sharing (No Hedging)

I begin with the case in which risk sharing is not provided. We first calculate the borrower’s $t = 0$ inside liquidity hoarding when the lender refuses to lend. We then calculate the lender’s expected profit when refusing to lend, given this

borrower's optimal response (outsider's rent). Given this lender's rent, we then solve the optimal portfolio problem of the borrowers, under the outside liquidity provision derived above.

Lender's Expected Profit (Rent) in the Secondary Market

Without a credit line contract, the borrower i 's payoff given his inside liquidity hoarding y is defined as $U_i^V(y)$ such that

$$U_i^V(y) = [(1 - y)R + y] \times \Pr(\tilde{\rho}_i \leq y) + [(1 - y)P + y] \times \Pr(\tilde{\rho}_i > y) - E(\tilde{\rho}_i) \quad (2.4)$$

where $\tilde{\rho}_i$ follows (2.1), and $P = \min\{R, \frac{M}{1-y}\}$ taken as given. The superscript V stands for the lender as a “vulture buyer”, and the borrower has to sell if the shock $\tilde{\rho}$ is larger than his liquidity cushion y without any loan provision. Since this is strictly concave in y , we can pin down unique $y^V \in [0, 1]$ such that

$$y^V = \operatorname{argmax}_y U_i^V(y) \quad (2.5)$$

where y^V is the optimal inside liquidity hoarding when there's no outside liquidity available.

Now, let $\Pi^V \equiv \Pi^V(y^V)$ be the lender's expected profit (rent) when he refuses to lend, given that the borrower hoards y^V as the inside liquidity (optimal response). We get

$$\begin{aligned} \Pi^V(y^V) &= (1 - y^V) \times (R - P) \times \Pr(\tilde{\rho} > y^V) \times 2p \\ &= (1 - y^V) \times (R - \min\{R, \frac{M}{1-y^V}\}) \times (1 - \frac{y^V}{\bar{\rho}}) \times 2p \quad (2.6) \end{aligned}$$

where the first term is the amount of asset sold, the second term is profit margin per unit of asset, and the rest is the likelihood of buying opportunity.

Optimal Loan Contract and Portfolio Decision

Given the lender's rent (2.6), we now solve for the borrower's optimal line of credit contract and optimal investment portfolio. To be specific, I derive an optimal interest rate $r = r^*$ (cost of credit) and inside liquidity hoarding $y = y^*$ maximizing the borrower's expected payoff.

Since the lender expects to get positive profit Π^V by refusing to lend, the borrower has to compensate at least Π^V in expectation as a cost of credit, in order to borrow. Thus, optimal investment y^* and interest rate r^* are such that: (i) The lender expects to receive no less than Π^V as an interest payment; (ii) The borrower maximizes his expected payoff.

Given some combination of r and y , the borrower i 's expected payoff is characterized as

$$U_i^L(y, r) = (1 - y)R + y - rE[\max(0, \tilde{\rho}_i - y)] - E(\tilde{\rho}_i) \quad (2.7)$$

the superscript L stands for the loan providing lender. As a first step, we can easily derive the maximizer y^* as a function of r .

Lemma 1. Optimal inside liquidity hoarding given r *For a given interest rate r , with $\bar{r} = \frac{R-1}{p}$*

- *if $r \leq \bar{r}$, then $y^*(r) = 0$. $U_i^L(y(r); r)$ is independent of $y(r)$, strictly decreasing in r .*

- if $r > \bar{r}$, then $y^*(r) = \bar{\rho}[1 - \frac{(R-1)}{rp}]$.

This implies that when outside liquidity is not very costly, the borrower will not rely on costly inside liquidity so as to invest more in the long term asset. Inside liquidity hoarding is increasing as the cost of outside liquidity becomes higher, beyond certain threshold $\bar{r}$.

Let $\Pi^L(y^*(r); r)$ be the lender's expected return when accepting a credit line contract, given an interest rate r . Without risk sharing, only one borrower will borrow, so

$$\Pi^L(y^*(r); r) = 2p \times r \times E[\max\{0, \tilde{\rho} - y^*(r)\}] \quad (2.8)$$

where the first term is the probability with which liquidity shock arises, the second term is the interest rate, and the last term is the amount of liquidity that the lender expects to provide. From (2.8) and Lemma 1, we get the following lemma.

Lemma 2.

$\Pi^L(y^*(r); r)$ is strictly increasing in r if $r \leq \bar{r}$, strictly decreasing in r if $r > \bar{r}$.

This implies that the borrower can promise higher return to the lender by offering higher interest rate only if $r \leq \bar{r}$. When $r > \bar{r}$, however, the borrower cannot promise higher payoff simply by raising the cost of outside liquidity since he should try to hoard more inside liquidity with too costly outside liquidity once the term is contracted. We denote $\Pi^L(\bar{r}) \equiv \bar{\Pi}^L$ as the maximum profit the lender can expect to collecting by lending, ignoring the asset substitution problem for now. Thus, the borrower cannot ex ante commit to pay more than this amount to the lender.

Since the lender's rent (outside option value) is Π^V , interest rate r has to satisfy $\Pi^L(y^*(r); r) \geq \Pi^V$ in order to induce him to lend which is a participation condition for the lender. Note that only the LHS is a function of r .

Next, we solve for the optimal contract offered by the borrowers given the lender's rent Π^V . Since y is a function of r , the optimal credit line contract boils down to choosing r maximizing the borrower payoff U_i^L which is a function of r , subject to the participation constraint:

$$\max_r U_i^L(y^*(r)) \quad (2.9)$$

$$s.t. \Pi^L(y^*(r)) \geq \Pi^V \quad (IR) \quad (2.10)$$

First, consider the case $\Pi^V > \bar{\Pi}^L \equiv \max\{\Pi^L(r)\}$. From Lemma 2, we can see that no r can satisfy IR condition (2.10). The borrower cannot borrow by simply offering higher r in this case since the lender's rent is beyond the level that the borrower can commit to pay. The lender will then choose to be a “buyer” as he can expect higher payoff by going to the secondary market. As we saw before, the optimal response (inside liquidity hoarding) with outsider “buyer” is $y^* = y^V$ from (2.5) and the borrower gets $U_i^V(y^V)$.

Now consider the case $\Pi^V \leq \bar{\Pi}^L$. From Lemma 2, $y^* = 0$ if $r < \bar{r}$, and it is obvious from (2.7) and (2.8) that $U^L(y^*(r))$ decreases in $\Pi^L(y^*(r))$ when $y^*(r) = 0$. Thus the minimum $\Pi^L(y^*(r))$ maximizes $U^L(y^*(r))$, and IR condition (2.10) has to bind. From Lemma 2, we can find a unique $r^*(< \bar{r})$ such that (2.10) binds.

Recall from Section 2.3.4. that the borrower cannot commit to pay more than $\hat{\Pi}$ because of the asset substitution problem. In this context, if the lender's rent Π^V is higher than $\hat{\Pi}$, a line of credit will not be provided since the lender expects to be better off by rejecting to lend. Without outside liquidity supply, again the

borrower hoards y^V as a (optimal response) liquidity cushion as we saw in (2.5). The following summarizes our findings.

Proposition 1 (Inside and Outside Liquidity Provision)

- If $\Pi^V < \min\{\hat{\Pi}, \bar{\Pi}^L\}$, there exists a unique optimal r^* such that $\Pi^L(y^*(r^*)) = \Pi^V$. No need for inside liquidity, $y^* = 0$.
- If $\Pi^V \geq \min\{\hat{\Pi}, \bar{\Pi}^L\}$, the insider cannot borrow, hoard inside liquidity $y^* = y^V$.

This implies that when cost of outside credit is relatively cheap, the borrowers mainly rely on outside liquidity than hoarding costly inside liquidity. As the lender's rents become larger, the insider pays higher interest rate to borrow but still does not hoard inside liquidity. But if the rents are beyond $\min\{\hat{\Pi}, \bar{\Pi}^L\}$, the borrower cannot borrow from the lender any more. The lender will rather choose to become a “vulture” buyer and the borrowers have to self-prepare by hoarding some inside liquidity. We should note that the lender's rent Π^V , optimal inside liquidity y^* , interest rate on the loan r^* are all functions of the level of aggregate uncertainty $\bar{\rho}$. This implies that changes in aggregate uncertainty affects the lender's outside option value and ex ante liquidity provision which is the focus of Section 2.4.3. where we will argue that the lender's rent becomes larger as aggregate uncertainty increases, affecting ex ante liquidity provision of the borrowers through the higher cost of outside liquidity. In that section, we denote (y^*, r^*, Π^V) of this Case 1 as (y_1^*, r_1^*, Π_1^V) to compare them with those in the next case where risks are shared among the borrowers.

2.4.2 Case 2: With Risk Sharing

We again derive the optimal inside liquidity hoarding y^* and cost of credit r^* , along with the lender's outside option value Π^V in the same way as in Case 1. The only difference is that now the borrowers are interconnected (risks are shared) such that distribution of the liquidity shocks they anticipate ($\tilde{\rho}_i$) will be different.

The borrower's expected payoff is again defined as (2.4) if the lender refuses to lend (U_i^V), and (2.7) if he agrees to lend (U_i^L). The difference is now $\tilde{\rho}_i$ follows (2.2) instead of (2.1), and $P = \min\{R, \frac{M}{2(1-y)}\}$ from (2.3) with $N = 2$. We can derive y^V , the optimal inside liquidity hoarding with no outside liquidity provision, in the same way and now the lender's rent $\Pi^V \equiv \Pi^V(y^V)$ is characterized by

$$\begin{aligned}\Pi^V(y^V) &= 2(1 - y^V) \times (R - P) \times \Pr\left(\frac{\tilde{\rho}}{2} > y^V\right) \times 2p \\ &= 2(1 - y^V) \times \left(R - \min\left\{R, \frac{M}{2(1 - y^V)}\right\}\right) \times \left(1 - \frac{2y^V}{\bar{\rho}}\right) \times 2p\end{aligned}\quad (2.11)$$

where the first term is the amount asset the lender expects to buy in the secondary market, the second term is profit per unit of the asset purchased, and the rest is the probability of that event.

If the lender chooses to lend, then the expected payoff at interest rate r is

$$\Pi^L(y^*(r); r) = 2p \times 2r \times E[\max\{0, \frac{\tilde{\rho}}{2} - y^*(r)\}] \quad (2.12)$$

which is similar to (2.8).

Given these, we can solve for the optimal contract and liquidity provision problem by maximizing (2.9) subject to (2.10) as before. We first get the similar results as Lemma 1 and 2 in the previous subsection of Case 1.

Lemma 3. *For a given interest rate r ,*

- *if $r \leq \bar{r}$, then $y^* = 0$. $U_i^L(y(r); r)$ is independent of y , strictly decreasing in r .*
- *if $r > \bar{r}$, then $y^* = \frac{\bar{L}}{2}[1 - \frac{(R-1)}{2rp}]$. where $\bar{r} = \frac{R-1}{2p}$.*

Lemma 4.

$\Pi^L(y^*(r); r)$ is strictly increasing in r if $r \leq \bar{r}$, strictly decreasing in r if $r > \bar{r}$.

With these, we can solve for the optimal inside liquidity hoarding y^* as well as the equilibrium cost of credit r^* as in Proposition 1 of Case 1.

Proposition 2.

- *If $\Pi^V < \min\{\hat{\Pi}, \bar{\Pi}^L\}$, there exists a unique optimal r^* such that $\Pi^L(y^*(r^*)) = \Pi^V$. No need for inside liquidity, $y^* = 0$.*
- *If $\Pi^V \geq \min\{\hat{\Pi}, \bar{\Pi}^L\}$, the insider cannot borrow, hoard $y^* = y^V$.*

The economic interpretations are the same as in the previous case. Cheap credit implies no need for inside liquidity, but credit should be rationed with high outsider's rents (expensive credit) and inside liquidity hoarded with vulture buyers. Note that Π^V , y^* , r^* are again functions of $\bar{\rho}$, and we denote them as Π_2^V , y_2^* , r_2^* for this risk sharing case. Our next focus is on the sensitivities of these variables with respect to changes in aggregate uncertainty $\bar{\rho}$.

2.4.3 Aggregate Uncertainty and Cost of Credit

So far, we took the measure of aggregate uncertainty $\bar{\rho}$ as given. In this section, I argue that risk sharing can become the source of financial fragility. I particularly

focus on the volatilities of the cost of credit (funding liquidity, credit crunch) characterized by Π^V (or r^*), total output y^* , and the emergence of credit rationing (liquidity evaporation) with respect to the changes in $\bar{\rho}$.

I first show that the lender's outside option value (equivalently, credit cost) can become more sensitive to aggregate uncertainty fluctuations when risks are shared and the cash in the market is scarce (limited market liquidity).

Proposition 3. (Sensitivity of the Lender's rent, or Cost of Credit, to Aggregate Uncertainty Fluctuations)

- When $\bar{\rho}$ is not very low and market liquidity is not abundant, the risk sharing make the credit cost more volatile to the changes in aggregate uncertainty $\bar{\rho}$.
- Formally, there exists $\underline{M}$ and $\overline{M}$ such that if $\underline{M} < M < \overline{M}$,

$$\frac{d\Pi_2^V}{d\bar{\rho}} > \frac{d\Pi_1^V}{d\bar{\rho}} \quad (2.13)$$

The assumption $\underline{M} < M < \overline{M}$ implies that cash in the market is large enough to absorb small amount of fire sale without price discount, but is not sufficient to absorb large block trade. The proposition implies that when this is the case, the value of the lender's outside option (wait and buy rather than lend) rises faster as aggregate uncertainty increases (higher $\bar{\rho}$) if risks are shared (Case 2) compared to the case without risk sharing (Case 1). Equivalently, the cost of credit (outside liquidity) rises more rapidly when risks are shared.

The intuition is straightforward from the definition of Π_1^V and Π_2^V in (2.6) and (2.11). Note that more number of the borrowers are distressed at the same time when risks are shared, so the fire sale price is lower with more asset liquidation

during the distress episodes if it ever happens. This is reflected on the higher profit margin on buying ($R - P$) for Case 2 than that for Case 1. Apparently, this distress event rarely happens with risk sharing since the financial network becomes more resilient to liquidity shock ($Pr(\tilde{\rho} > y_1^V) > Pr(\tilde{\rho}_2 > y_2^V)$); there is a trade off between severity and frequency of the distress as discussed in the literature. What's novel here is that small changes in the ex-ante likelihood of financial distress (or tail events) with increased uncertainty, characterized by higher $\bar{\rho}$, can have significantly different effects on the outcome variables in the two cases. In this ex-ante perspective, what matters when considering the effect of the marginal *changes* in aggregate uncertainty is not the size of that event's likelihood itself, but the difference in profit margins since those are the factors that are critical for the sensitivity of the lender's outside option value to fluctuating aggregate uncertainties. With risk sharing, the rent increases more rapidly when the ex-ante likelihood of buying event is increased (with fatter/longer tailed distribution) because of the higher margin on buying whereas it changes only slightly without risk sharing since margin on buying is small (that is, $2(1 - y_2^V) \times (R - P_2) \gg (1 - y_1^V) \times (R - P_1)$). This brings higher volatility in outside liquidity cost and financial fragility with risk sharing. This argument can be summarized as follows.

Corollary 1 (Financial Fragility with Risk Sharing 1)

With risk sharing, the cost of the outside liquidity rises more rapidly as aggregate uncertainty increases. Credit crunch becomes severe rapidly.

Recall that if the lender's outside option value Π^V is greater than $\min\{\hat{\Pi}, \bar{\Pi}^L\}$, outside liquidity evaporates since the lender refuses to lend. If this happens, credit

rationing arises, inside liquidity y^V is hoard, and the long term investment drops (Proposition 2, 4).

Now suppose that Proposition 3 holds. As $\bar{p}$ increases, Π^V rises faster with risk sharing than without risk sharing. If it eventually hits its upper bound ($\min\{\hat{\Pi}, \bar{\Pi}^L\}$) with smaller value of $\bar{p}$ with risk sharing, this implies that credit rationing and output drops emerges with lower level of aggregate uncertainty when risks are shared.

Corollary 2. (Financial Fragility with Risk Sharing 2)

The following fragility may arise with risk sharing:

1. *Credit is rationed with lower level of aggregate uncertainty.*
2. *Aggregate output suddenly drops with lower level of aggregate uncertainty.*
3. *Both lender and borrowers start to hoard liquid asset with lower aggregate uncertainty (flight to liquidity in both sectors).*

In sum, risk sharing can make the lender's outside option value more volatile because of joint failure and corresponding low asset price. This becomes the source of financial fragility. The following example displays these results.

Numerical Example

Consider the following parameters: $R = 1.4$, $M = 1.3$, $p = 0.2$, $\hat{\Pi} = 0.027$, and aggregate uncertainty ranging $\bar{p} \in [0.65, 0.76]$. The results are summarized in the figures below.

When aggregate uncertainty is low, the borrowers can have access to cheaper credit with risk sharing. Cost of credit (the lender's rent Π^V), however, rises faster with risk sharing as aggregate uncertainty rises (credit crunch in the interbank mar-

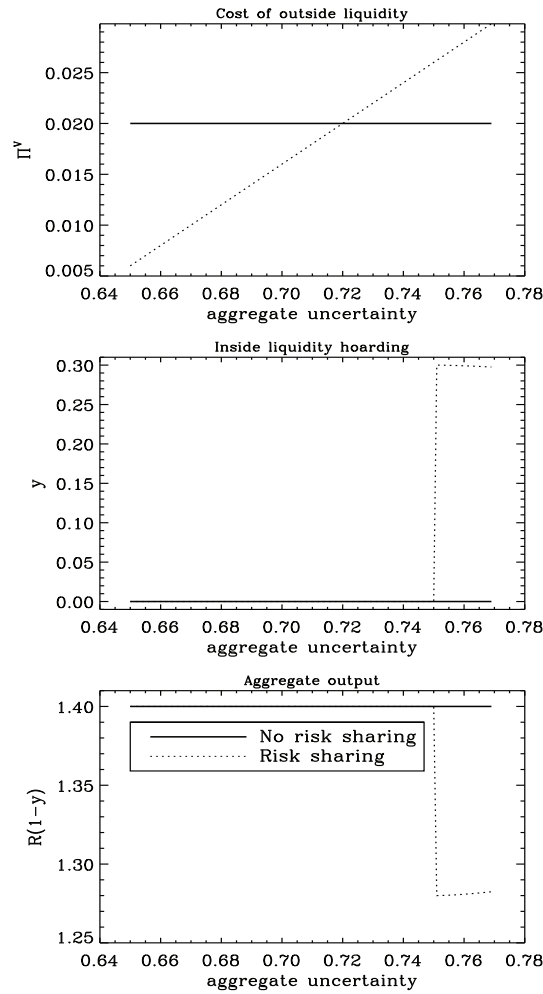


Figure 2.3: Financial fragility with risk sharing

ket), and hit $\hat{\Pi}$ when $\bar{p} = 0.75$. At this point, asset substitution becomes problematic and credit rationing arises. Outside liquidity then evaporates, and the borrowers cut down their investment level to hoard inside liquidity (positive y). Now the credit crunch in interbank market brings a credit crunch in the debt market (lower long term investment), with real effects on the economy through flight to liquidity. Note that these wouldn't have happened if risks had not been shared in our example. Financial fragility emerged with risk sharing.

2.5 Excessive Risk Sharing

In this section, I discuss that the borrowers may ex ante provide excessive level of risk sharing in seeking cheap credit. I present an example in which the borrowers choose to share their risks when no risk sharing is socially welfare enhancing.

Note that cost of credit is a mere transfer between the two sectors, and it doesn't directly affect aggregate welfare unless credit is rationed ($y^* = 0$ with outside liquidity provision). Thus, aggregate welfare is maximized when the expected total output is maximized and inside liquidity hoarding is minimized. The borrowers, however, try to reduce their cost of credit since it directly affects their payoff. Consider the borrower's ex ante (at $t = -1$) decision of risk sharing before the aggregate uncertainty $\bar{p}$ realizes at $t = 0$. If the benefit of cheap credit is greater than the loss from potential credit rationing, the borrowers make a socially suboptimal choice of excessive risk sharing provision. Consider the following 2-state case using the numbers from the previous example.

Suppose that there are two possible states realizing at $t = 0$, H with $\bar{p} = \bar{p}_H = 0.76$ and L with $\bar{p} = \bar{p}_L = 0.70$. Let the ex ante probability of state H be 0.05 and that of state L be 0.95 as of $t = -1$. Denote $\Pi_{1,H}^V$ and $\Pi_{1,L}^V$ as the lender's rents for the two state without risk sharing, $\Pi_{2,L}^V$ and $\Pi_{2,H}^V$ with risk sharing.¹⁶ From the figures of Example 1, we can observe that $\Pi_{2,L}^V < \Pi_{1,L}^V \leq \Pi_{1,H}^V < (\hat{\Pi} <) \Pi_{2,H}^V$. This implies that with risk sharing, the borrowers can borrow at a low interest rate in L state, but credit will be rationed in H state whereas no credit rationing arises without risk sharing. However, as H state is unlikely ex ante at $t = -1$, the borrowers choose to share risks in a pursuit of cheap credit in L state at $t = -1$. This is not socially optimal since total output is smaller than the first best level

¹⁶Note that lower rent implies lower cost of credit, thus higher expected payoff for the borrowers.

in H state with risk sharing, but no credit rationing happens without risk sharing and the total output will always be in its maximum level. Excessive risk sharing arises here bringing less expected total output and higher volatility.

2.6 Policy Intervention

As discussed previously, cost of credit is a mere transfer between the borrowers and the lender. Hence the policy maker's primary concern is to avoid credit rationing and maximize aggregate output of the economy, rather than reducing the loan cost itself within this setup. First best level of output is produced when $y^* = 0$ (no precautionary saving).

The traditional central bank intervention through open market operation doesn't directly ease the credit crunch as it doesn't tackle the roots of the high cost of credit: the lender's reluctance to lend comes from their better investment opportunity. In this section I analyze three policy interventions which can possibly relax credit rationing: liquidity requirements, asset repurchase, and liquidity injection.

2.6.1 Liquidity Requirements

The banking industry has argued that strict liquidity requirements are counter-productive since it reduces their long-term investment. In my model, however, liquidity requirements can actually benefit the borrowers by working as a commitment device resolving the time-inconsistency problem.

If the borrower could commit to hoard large amount of liquidity when the lender refuses the credit line offer at $t = 0$, the lender's anticipated profit in the secondary market would be much lower since abundant inside liquidity should be hoarded. However, this is not a credible threat. The lender knows that the borrowers will

not hoard such an excessive inside liquidity once he turns down the offer (and only hoard the optimal response y^V), thus positive profit as a buyer will do remain.

Now consider mandatory liquidity requirements.¹⁷ If the borrowers are forced to hoard high enough level of inside liquidity when outside liquidity dries up, this drives down the lender's rent. Lower rent implies lower cost of outside liquidity, and credit rationing can be avoided. In fact, we can show that

Proposition 4

For a given level of aggregate uncertainty $\bar{p}$, there exists a minimum level of liquidity $\bar{y}$ such that if the borrowers are required to secure at least $\bar{y}$ of liquidity (either inside liquidity or outside liquidity, or both), the first best level of output without credit rationing is achieved.

Here, social welfare is enhanced without inducing any inefficiency in resource allocation, by changing the outcome in the off-the-equilibrium path. Liquidity requirements act as a commitment device which ex ante rules out financial fragility.

2.6.2 Asset Repurchase

The government can act as a buyer in the secondary market to stabilize asset price. This directly increases market liquidity and the lender's rents go down, alleviating credit crunch.

This policy can be effective in principle, but in reality the government has the same problem as the investors (other than the lender) in the secondary market: a lack of special knowledge. The government may not be able to evaluate the com-

¹⁷Liquidity here includes both inside and outside liquidity.

plicated assets' value and potentially lose public money by investing in this asset,¹⁸ which imposes political pressure impairing the credibility of the policy actually being implemented. If this is the case, credit crunch won't be alleviated even with the proposed government buy-out program until it gets implemented, as we witnessed during the 2007-09 credit crunch.

2.6.3 Liquidity injection

Direct liquidity injection can also reduce the lender's rent and cost of credit. I distinguish liquidity injection to the borrower sector and the lender sector, and argue that both policies can be less effective in some cases.

The objective of injecting liquidity to the lender sector is to raise the secondary market asset price P by providing more market liquidity, which reduces the lender's rent and cost of credit. However, there's no guarantee that this policy will work as planned. The lender may use injected liquidity in other uses, though not modeled in this paper. If liquidity doesn't flow into the asset market, credit crunch will still remain.

Liquidity injection to troubled borrower can resolve credit crunch, but this causes a moral hazard issue accompanying with bail-out. Knowing that they will be bailed out by the government when in trouble, the borrower tends to take excessive risks. If this liquidity injection is to be implemented in an unanticipated manner, that wouldn't resolve our ex-ante credit crunch since the lender's ex ante expected rent will not be reduced, either.

¹⁸In other words, the government cannot distinguish between illiquid and insolvent banks.

2.7 Discussions and Extensions

2.7.1 No Partial Liquidation Assumption

The no partial liquidation assumption is not crucial for the main results for the model. Financial fragility with risk sharing still holds even when partial liquidation is allowed, as long as the one of the following two conditions is satisfied.

- (a) The borrower fails (liquidating all assets) when hit by sufficiently large liquidity shock
- (b) The lender expects to get more profit when there is a joint distress compared to a single-borrower distress, even the aggregate liquidity deficits in the two cases are the same.

Beginning with (a), the main driving force of financial fragility in my model is not the likelihood of failure itself, but the difference between profit margins with a single failure and a joint failure when this episodic event happens. If these two are significantly different, small changes in the likelihood of the episodic tail event (implied by changes in aggregate uncertainty, or fatter tail of the shock distribution) affect the lender's rent more in the case with risk sharing than without risk sharing. Thus as long as there exists an upper bound of liquidity deficit $\rho - y$ beyond which the borrower fails, the argument still holds.

When hit by a large shock, the bank is forced to liquidate all of its assets because of the creditors' run. Two possible rationales of this run are a principal-agent problem (the creditors' concern about moral hazard) and coordination failure among creditors.

As assets are (partially) liquidated, less assets will remain in the bank's balance sheet to back up its debt. The original creditors of the bank (who financed $t = 0$

endowment) start to worry about the manager's moral hazard, and withdraw all the deposits when the remaining assets are below certain level. This is a Calomiris and Kahn (1991) or Holmström and Tirole (1998) type failure through large interim shocks, with endogenous shock capacity.

As an alternative rationale, a solvent bank can fail from the fear of the other creditors' run. Rochet and Vives (2004) provide a model in which a illiquid but solvent bank fails from the creditors' coordination failure.

(b) directly implies that the profit margin is higher with risk sharing even when the size of the shock is the same. This is equivalent to lower liquidation prices with a joint distress. This lower price can arise from the competition between the distressed borrowers when they are facing certain price impact of their liquidation. If there's only one seller, he can liquidate his asset relatively in an orderly way or try to search for an alternative financing before liquidating. With more sellers, however, a competition among them becomes critical since one doesn't want to sell after the market liquidity is washed away by the other sellers. This increases selling pressure and pushes down the asset price. Oehmke (2010) builds a model of illiquid asset liquidation in which the sellers get lower payoff when more number of asset holders are selling at the same time even though aggregate liquidation volume is the same.

2.7.2 Coordination Problem

I have been ignoring the coordination problem among the lenders by assuming (i) the borrower makes an offer, (ii) a single lender. With multiple outsiders, their rent could be significantly reduced by a competition between them; a lender tries to steal the other lender's loan by offering a lower cost of credit.

This competition will not be so severe when a relationship lending between the lender and the borrower is critical.¹⁹ Consider a case in which there are more than one lender, and more than one borrowers in each region A and B . A borrower initially borrows from only one lender since lending relationships matter;²⁰ this lender has a better monitoring technology about the borrower than the other lenders. This is not an unrealistic assumption in my model since the asset the borrower is investing in is assumed to be very complicated. When this is the case, one lender is hesitant to steal another lender's customer since he doesn't have knowledge about the particular customer's asset, which requires additional costs. This reduces an incentive of competition between the lenders and their rent remains.

2.7.3 Exogenous Market Liquidity

I assume scarce market liquidity, which comes from limited cash in the market. The assumption of fixed lender's endowment M is for tractability, and the main results hold as long as the asset price in the secondary market is decreasing in the amount of the liquidated assets. This is a common assumption in the market microstructure literature where information asymmetry between the seller and the market maker exists (e.g. Kyle (1985)).

As an extension of the model, market liquidity can be endogenized. Suppose that if there is a single-borrower distress, uninformed market makers in the secondary market (other than the informed lender) infers that this is more likely from temporary liquidity shock rather than fundamental shock, thus market liquidity is

¹⁹Bech and Atalay (2010) finds that most banks have only a few counterparties in the federal funds market. Cocco, Gomes, and Martins (2009) use Portuguese dataset to find that relationships are an important determinant of banks' access to the interbank market.

²⁰We can alternatively assume a small number of outside lenders with limited capital for one borrower.

abundant. With joint distress, however, they infer that there must be some bad news that they are unaware of, even though it's actually caused by mere liquidity shock. Thus, price becomes lower with joint distress and the differences in profit margin lead to our previous result; financial fragility with risk sharing.

2.7.4 Repeated Setup

Although this model is a one-stage game, it can be easily extended to a repeated stage game setup of lending relationships as in Carlin et al. (2007).

At the beginning of each stage, a level of aggregate uncertainty $\bar{p}$ is revealed. The analysis in a single stage given certain $\bar{p}$ is the same as before. Cooperation (relationship lending) between the lender and the borrower implies a fixed cost of credit between the two within one stage. This cooperation breaks down and the lender becomes a “vulture” when the stakes in current stage are high, which means the lender’s (implicit) rent is large.

Note that the stakes are stable across stages without risk sharing, but volatile with risk sharing. Thus, without risk sharing, relatively high cost of credit should be paid in each stage but cooperation breakdown (credit crunch) rarely happens. On the contrary, cheap credit can be achieved with risk sharing but cooperation is more likely to break down. Again, financial fragility emerges with risk sharing.

2.8 Conclusion

This paper links market liquidity, funding liquidity, and the effect of risk sharing. It argues that when market liquidity in the secondary market is scarce, risk sharing can make the interbank market more fragile. This can explain why interbank lending

universally froze up, why the credit crunch was so severe and flight to liquidity prevailed during the 2007-09 crisis.

Financial innovation has provided investors novel ways to diversify their individual risks. This should in principle resolve some of the market frictions and enhance social welfare by making the market more complete. The financial market on the other hand has become more complicated, and generated a different kind of market friction. When these two are combined, an unanticipated type of fragility could emerge: financial innovation destabilizes the economy.

Financial fragility in this paper is not the sort of rare event. Risk sharing not only brings forth a systemic failure, but also affects an *ex-ante* provision of liquidity. Proper regulations are required to control this fragility, so that financial innovation actually brings benefits.

2.9 Appendix

2.9.1 Optimality of credit line contract

I solve for the case 1 (without risk sharing). The proofs for case 2 is similar and omitted.

Claim 1: *The insiders prefer contracting a line of credit (a lending relationship) at $t = 0$ to borrowing in the spot market at $t = 1$ or not borrowing at all.*

First, notice that the borrower's expected payoffs of no borrowing and spot market borrowing are equivalent. When borrowing in the spot market, the lender tries to extract all the rents which is equal to $(1 - y)(R - P)$. Thus, given inside

liquidity y , the borrower's payoff when he borrows from the outsider in the spot market at $t = 1$ is

$$(1 - y)R + y - ((1 - y)(R - P)) = ((1 - y)P + y)$$

Since the insider borrows when $\rho_i > y$, the insider's $t = 0$ objective function is equal to (2.6), the objective function with no borrowing. Thus, the borrower's choice of y at $t = 0$ is the same in the two cases with identical objective function, which implies they hoard y^V in both cases, and expected payoffs are $U_i^V(y^V)$.

Note that using the definition of Π^V , we can represent it as

$$U_i^V(y^V) = [(1 - y^V)R + y^V] - p\Pi^V - E[\tilde{\rho}_i] \quad (2.14)$$

Now, we verify that securing a line of credit generates higher expected payoff than (2.16).

Note that the insider payoff with credit line is from (14) above,

$$\begin{aligned} U_i^L(y^*, r) &= \max_y [(1 - y)R + y - rE[\max(0, \tilde{\rho}_i - y)] - E(\tilde{\rho}_i)] \\ &= \max_y [(1 - y)R + y - p\Pi^L(r) - E(\tilde{\rho}_i)] \\ &= \max_y [(1 - y)R + y - p\Pi^V - E(\tilde{\rho}_i)] \end{aligned}$$

where the last equality is from binding IR condition (2.12) such that $\Pi^L(r) = \Pi^V$.

By comparing this and (2.14), it is obvious that

$$U_i^L(y^*, r) \geq U_i^V(y^V)$$

Thus a credit line contract is weakly better than spot market (or no) borrowing.

Claim 2: *Credit limit of the optimal credit line is no less than $\bar{p}$ (full coverage) when credit is not rationed.*

Suppose that a credit line provides a full coverage. When credit is not rationed, $y^* = 0$ as seen in Proposition 2 (a). Therefore, the borrower's payoff in this case is

$$U_i^L = R - \Pi^V - E(\tilde{\rho}_i)$$

since IR condition (2.12) is binding.

Note that regardless of the amount of credit limit, in expectation Π^V has to be transferred to the lender in order to induce him to lend. It is obvious that U_i^L above is the maximum payoff that the borrower can achieve with any credit limit since no inside liquidity is hoarded. Suppose that full coverage is not provided (credit limit is low) such that the borrower cannot borrow if ρ_i is too large. The cost of outside liquidity is still Π^V , but if the borrower increases the inside liquidity hoarding y , then U_i^L is decreased. Therefore, full coverage with high enough credit limit provides (weakly) higher expected payoff than partial coverage.

2.9.2 Proofs

The proofs of Lemma 3, Lemma 4, and Proposition 2 are similar with those of Lemma 1, Lemma 2, and Proposition 1.

Proof of Lemma 1

From the first order condition,

$$(1 - R) + pr - r \frac{y}{\bar{\rho}} p = 0$$

Solving for y ,

$$y = \bar{\rho} \left[1 - \frac{(R - 1)}{rp} \right].$$

Since $0 \leq y \leq 1$, from the concavity of U_i^L ,

- $y^*(r) = 0$ if $r \leq \bar{r}$,
- $y^*(r) = \bar{\rho} \left[1 - \frac{(R-1)}{rp} \right]$ if $r > \bar{r}$

where $\bar{r} = (R - 1)/p$.

Proof of Lemma 2

From (2.8),

$$\Pi^L(y^*(r); r) = 2p \times r \times \int_y^{\bar{\rho}} (\rho - y^*(r)) \frac{1}{\bar{\rho}} d\rho$$

From Lemma 1, $y^*(r) = 0$ if $r \leq \bar{r}$, so

$$\Pi^L(y^*(r); r) = 2p \times r \times \int_0^{\bar{\rho}} \rho \frac{1}{\bar{\rho}} d\rho$$

which is increasing in r .

Also, $y^*(r) = \bar{\rho} \left[1 - \frac{(R-1)}{rp} \right]$ if $r > \bar{r}$, thus

$$\Pi^L(y^*(r); r) = 2p \times r \times \int_y^{\bar{\rho}} \left(\rho - \bar{\rho} \left[1 - \frac{(R-1)}{rp} \right] \right) \frac{1}{\bar{\rho}} d\rho = 2p \times \int_y^{\bar{\rho}} \left(r\rho - r\bar{\rho} + \frac{(R-1)}{p} \right) \frac{1}{\bar{\rho}} d\rho$$

Since $\rho < \bar{\rho}$, we get $\frac{\partial \Pi^L}{\partial r} < 0$ in this region.

Proof of Proposition 1

(a) When $\Pi^V < \min\{\hat{\Pi}, \bar{\Pi}^L\}$.

We maximize (2.7) with respect to r and y such that (2.10) is satisfied. From Lemma 1, y^* can be represented as a function of r , so our program can be represented by

$$\begin{aligned} U_i^L(r) &= [(1 - y(r))R + y(r)] - p\Pi^L(r) - E[\tilde{\rho}_i] \\ \text{s.t. } \Pi^L(r) &\geq \Pi^V \end{aligned}$$

First, suppose that $r < \bar{r}$. Then from Lemma 1, $y^* = 0$ and our program becomes

$$\begin{aligned} U_i^L(r) &= R - p\Pi^L(r) - E[\tilde{\rho}_i] \\ \text{s.t. } \Pi^L(r) &\geq \Pi^V \end{aligned}$$

It is obvious that $U_i^L(r)$ is maximized when $\Pi^L(r)$ is minimized, therefore $\Pi^L(r) = \Pi^V$. From Lemma 2, $\Pi^L(r)$ is strictly increasing in r , so there exists unique r^* such that $\Pi^L(r^*) = \Pi^V$. The insider payoff in this case is

$$U_i^L(r^*) = R - p\Pi^V - E[\tilde{\rho}_i] \tag{2.15}$$

Now suppose $r > \bar{r}$, then (2.7) becomes

$$U_i^L(r) = [(1 - y^*(r))R + y^*(r)] - p\Pi^L(r) - E[\tilde{\rho}_i]$$

Since $y^*(r) > 0$ and $\Pi^L(r) \geq \Pi^V$, this is strictly less than (2.15), thus $y^* = 0$ and r^* that we derived above are the optimal solution.

(b) When $\Pi^V \geq \min\{\hat{\Pi}, \bar{\Pi}^L\}$.

The outsider refuses to contract a line of credit since his expected payoff is higher when he chooses to buy in the secondary market. Thus, $y^* = y^V$ which given by (2.5).

Proof of Proposition 3

Note that from (2.6)

$$\Pi_1^V = p(1 - y_1^V) \times (R - P_1) \times (1 - \frac{y_1^V}{\bar{\rho}}) = p(1 - y_1^V) \times (R - \min\{R, \frac{M}{1 - y_1^V}\}) \times (1 - \frac{y_1^V}{\bar{\rho}})$$

and from (2.11)

$$\begin{aligned} \Pi_2^V &= 2p(1 - y_2^V) \times (R - P_2) \times (1 - \frac{2y_2^V}{\bar{\rho}}) \\ &= 2p(1 - y_2^V) \times (R - \min\{R, \frac{M}{2(1 - y_2^V)}\}) \times (1 - \frac{2y_2^V}{\bar{\rho}}) \end{aligned} \quad (2.16)$$

We can derive the exact condition by plug in y_1^V, y_2^V and differentiate with respect to $\bar{\rho}$, but the closed form solutions are very complicated. Not very small $\bar{\rho}$ is needed so that the insiders don't fully insure in case 2. Suppose that M is chosen such that $R - P_1 = R - \min\{R, \frac{M}{1 - y_2^V}\}$ is small but $R - P_2 = R - \min\{R, \frac{M}{2(1 - y_2^V)}\}$ is not very small. For this M , as long as $R - P_2 \gg R - P_1$ then $\frac{\partial \Pi_2^V}{\partial \bar{\rho}} > \frac{\partial \Pi_1^V}{\partial \bar{\rho}}$ holds from the previous equations. By continuity, there exist $\underline{M}$ and $\bar{M}$ such that the lender's rent becomes more sensitive with risk sharing.

Proof of Proposition 4

Note that from (2.6) and (2.11), Π^V is a function of $\bar{\rho}$ and y . Observe that Π_s^V is increasing in $\bar{\rho}$ and decreasing in y . Therefore for any $\bar{\rho}$, we can find $\bar{y}$ such that

$$\Pi^V(\bar{y}; \bar{\rho}) \leq \min\{\hat{\Pi}, \bar{\Pi}^L\}$$

With liquidity requirements of $\bar{y}$, the lender's expected profit in the secondary market is less than $\min\{\hat{\Pi}, \bar{\Pi}^L\}$, which means that cost of outside liquidity is less than the upper bound beyond which credit is rationed. The lender then provides outside liquidity since he wouldn't get higher payoff by rejecting the credit line offer.

Chapter 3

Asset Market Distress and Money Market Distress

3.1 Introduction

This paper studies a model of joint distress in the asset market and the money market. Two markets can interact with each other during the economic downturn, as presented in Figure 3.1. The first market is the asset market, where assets are liquidated and traded, and the second market is the money market, where the asset holders (financial institutions) finance their position using short term debt. My model formalizes a feedback between the distress of the two markets combining the noisy rational expectation equilibrium setup (Grossman and Stiglitz (1980)) and global game technique (Morris and Shin (1998, 2003)). As a result, there emerges an endogenous vicious cycle between market liquidity dry-up and funding liquidity dry-up (as in Brunnermeier and Pedersen (2009)) during the market downturn.

The asset price in our model plays a dual role; it works as a market clearing device in the asset market, while at the same time, it also acts as an informative public

signal in the money market (equivalently, in the banking sector). Concern about the creditors' panic run in the banking sector is one of the main problems during a financial crisis, and this problem is exacerbated as the asset price is depressed in my model; the creditors interpret the price drop as a bad signal about the economic fundamental and then become less willing to lend, which leads the asset-holding institutions to face more difficulty in maintaining their position (rolling over their debt). This distress at the funding stage brings fire-sale of the assets and price drops further, but this again makes the creditors even more concerned. Thus the vicious cycle begins, becoming the source of joint distress between the two markets.

When assets are liquidated in the asset market, it is purchased by risk averse market makers. Their demand for the assets is constructed under the rational expectation, which follows a downward sloping demand curve with the random supply from the noisy traders. The difference between this and the conventional setup of Grossman and Stiglitz is that in this case there can be additional supply of the assets that comes from the asset fire-sale by the financial institution raising its debt in the money market. As a result, the market clearing price, which is a function of the fundamental and the noise supply becomes non-linear as in Genotte and Leland (1990). However, agents can still extract certain information out of the asset price under the rational expectation.

This asset price aggregates individual agents' information about the asset's fundamental, thus acting as a public signal which agents' can use to update their beliefs about the fundamental in addition to their own private signals. Now moving to the money market, the creditors of certain financial institution holding this asset can also have access to this information when the asset is marked-to-market. As a result, the creditors in the money market can also learn from the asset price in the

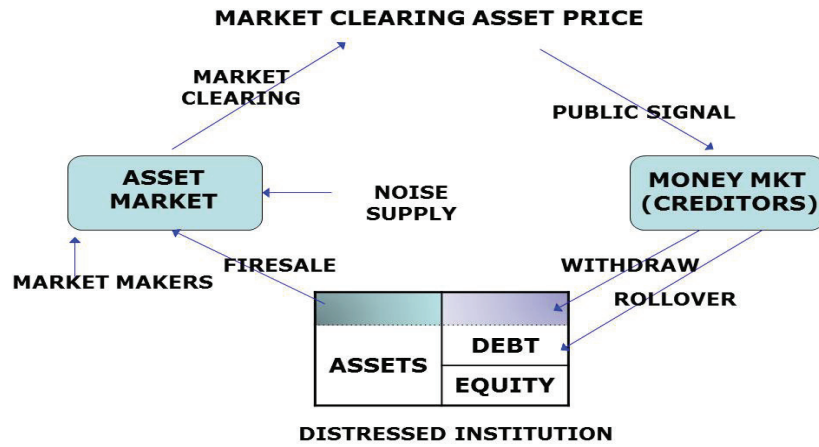


Figure 3.1: Overview of the Mechanism

asset market; they become more pessimistic about the institution's outlook when they observe a drop in the asset price.

The creditors of the financial institution are facing strategic complementarity which is common in the bank-run literature (Diamond and Dybvig (1983)). They choose whether they should roll over their lending to the institution or not, and the institution can fail if a sufficient number of creditors refuse to roll over their debt (run in the money market). I impose the global game setup to pin down a unique equilibrium, which enables us to calculate the exact number of creditors who choose to withdraw their lending in the money market. As in Angeletos and Werning (2006), the creditors have an access to the public signal, the asset price in the asset market, in addition to the private signal. Therefore, they can become less willing to lend when the asset price drops since they interpret it as a bad signal about the fundamental.

Thus, drops in asset price can lead to refusal to roll over among a greater number of creditors, and subsequently the distressed institution is forced to liquidate more

of its asset, since it faces difficulty in raising capital in the money market. This asset fire-sale is absorbed by the market makers in the asset market, but it pushes down the asset price further, which again concerns the creditors further. Now the feedback between the asset market and the money market emerges (Figure 3.2); distress in the asset market (market liquidity) brings distress in the money market (funding liquidity), and this again distresses the asset market. Eventually, asset prices can drop significantly and financial institutions can have more difficulty in the money market through this vicious cycle. Distress in one market causes distress in the other market, and the joint distress emerges.

This feedback process makes the asset price volatility heterogeneous, and the asset price distribution negatively skewed. When the economic fundamental is robust, there's no feedback between the two markets, since the creditors in the money market are confident enough to let their lending rolled over even when the asset price drops through the noise supply. As the fundamental deteriorates, however, they become concerned about the other creditor's action, which becomes the source of the negative spiral between the two markets. As a result, the asset price volatility becomes higher in the downturn, and the ex ante asset price distribution is skewed to the left while it is symmetric in the conventional rational expectation setup. Financial institution failures can be more likely with the learning from the asset price, which implies that marked-to-market accounting can undermine financial stability during the economic downturn.

Technically, I combine the noisy rational expectation equilibrium setup and the global game setup to derive the equilibrium. The asset market equilibrium is characterized by the market clearing price function given the supply (both from the noise supply and fire-sale supply) under the rational expectation, containing (pub-

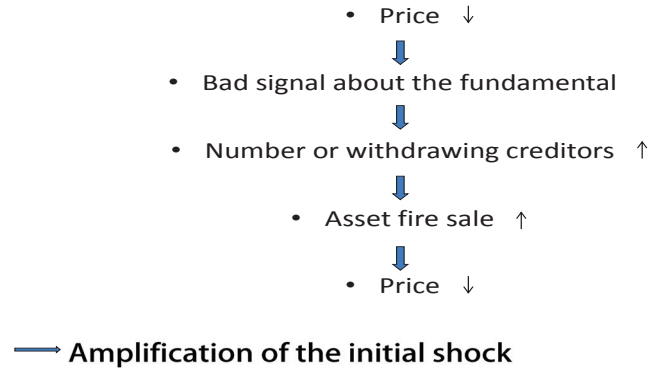


Figure 3.2: Feedback between the Two Markets

lic) information about the fundamental. The money market equilibrium is defined by the threshold of the private signal given the asset price, which enables us to pin down the amount of fire-sale (the percentage of creditors who refuse to roll over) given the asset price. The model's equilibrium is derived by solving the fixed point of the two functions incorporating the feedback between the two markets. This feedback process can increase the fire-sale assets and depress the asset price, making them more volatile, particularly when the fundamental is low.

Brunnermeier and Pedersen (2009) also characterizes the spiral effects between market liquidity and funding liquidity using the destabilizing margin constraint. In their paper, the price drops raise margin/haircut, which induces further asset liquidation and price drops, but the decision for the margin follows an exogenous rule. My model explicitly explains how funding liquidity can become scarce and generate the feedback by analyzing the creditors' problem in the money market. I use the noisy rational expectation equilibrium model to solve the equilibrium asset price as in Grossman and Stiglitz, but there is an additional source of asset

supply besides the noise trading, as in Gennotte and Leland. In Gennotte and Leland, hedging supply π increases automatically as the asset price drops, but in my model the fire-sale asset supply is brought to the market endogenously. The agents interpret the asset price drops as bad news about the fundamental, but the fire-sale is brought by the decisions of agents in a different (money) market; the amount of fire sale supply is pinned down through the endogenous decision of creditors, which comes out of fear of coordination failure. This learning process along with the market clearing process generates an endogenous feedback effect between the two markets. Ozdenoren and Yuan (2008) also solve the noisy REE model with feedback effect between asset price and firm cash flow, but they take the feedback process as given. The amount of the fire-sale asset, coming from the creditors' decisions over rolling over, is solved by using the global game technique under strategic complementarity which is also studied by Diamond and Dybvig, Rochet and Vives (2004), and Goldstein and Pauzner (2005). In addition to the private signal commonly used in the conventional global game setup, they can also use the asset price as an endogenous public signal as in Angeletos and Werning. This access to the public signal becomes the source of the feedback between the two markets while the feedback process is absent in Angeletos and Werning.

The chapter is organized as follows. Section 3.2 describes the model setup. Section 3.3 solves for the equilibrium of the model and Section 3.4 discusses its implications. Section 3.5 concludes.

3.2 Model Setup

The model is static with $t = 0, 1$, and we focus on agents' decision making at $t = 0$. Consider an asset that produces θ at $t = 1$, where θ is random variable. θ is realized

at $t = 0$, but is not common knowledge to the agents in the model. A priori, θ is uniformly distributed over $\mathbb{R}$. We denote θ as the “fundamental” value of the asset, and assume the risk free rate to be zero.

There are two markets which can affect each other, an asset market where this risky asset is traded, and a money market where agents (financial institutions) finance their debt backed by this asset. There are four types of agents: (i) market makers in the asset market, (ii) creditors in the money market, (iii) a financial institution (“institution” hereafter) which finances in the money market and trades in the asset market, (iv) noisy traders in the asset market.

3.2.1 Agents

Market makers provide liquidity in the asset market and absorb the asset liquidated by the (troubled) institutions at $t = 0$. There are continuum measure 1 of them indexed by $i \in [0, 1]$, each with constant absolute risk aversion γ . These market makers are “informed traders” in the sense that they can form their expectation about the fundamental based on available information, the market price of the asset (denoted as P) and a private signal x_i about the fundamental θ , where

$$x_i = \theta + \epsilon_i, \quad \epsilon_i \sim N(0, \tau_x^{-1}). \quad (3.1)$$

They also know how the market clearing asset price is determined in equilibrium with given parameters in the rational expectation sense. Using the Gaussian uncertainty and CARA utility, the demand for the asset by the market maker i can be represented as

$$d_i = \frac{E[\theta|x_i, P] - P}{\gamma \text{var}[\theta|x_i, P]}. \quad (3.2)$$

We denote the aggregate demand of the market making sector by $D = \int d_i di$.

Creditors in the money market hold short-term debt (CP, Repo) of the institution (which invests in this long run asset), and decide whether to roll over their debt or withdraw at $t = 0$. The creditors are risk neutral, and concern potential failure of the their borrower (institution) as in Diamond and Dybvig type maturity mismatch setup. There are continuum measure 1 of ex ante identical creditors lending to the institution, and creditor i 's choice at $t = 0$ is characterized by

$$y_i = \begin{cases} 1 & \text{if withdraw} \\ 0 & \text{if roll over} \end{cases}$$

Then aggregate fraction of creditors who refuse to roll over at $t = 0$ can be denoted by $Y \equiv \int y_i di$.

Creditors are concerned with the institution's potential failure caused by others' run on the institution given the maturity mismatch. Note that the long-term asset the institution owns delivers θ only at $t = 1$. As of $t = 0$, however, the bank has to liquidate fraction of its asset inefficiently if any creditors decide to withdraw early.¹ When the fundamental is weak (small θ) and too many of them withdraw in this early stage (large Y), not enough funds will be left and the institution fails in that case with little payoff to the remaining creditors. For simplicity, we assume that the bank fails when $\theta < L \times Y$ where L is a exogenous measure of leverage for this institution.² We use the following simplified payoff structure for the creditors which reflects the main strategic complementarity they are facing;

¹We assume that no new equity can be raised in the market with low fundamental, thus the institution has to liquidate some of its asset to repay to the creditors.

²See Rochet and Vives for the microfoundation of this setup.

	$\theta \geq LY$	$\theta < LY$
$y = 0$ (rollover)	1	0
$y = 1$ (withdraw)	c	c

where $0 < c < 1$. The institution thus survives if only small fraction of them withdraw early and the creditors who roll over get more compensation compared to the early withdrawal payoff. However, they end up with the least payoff if they choose to roll over and the institution fails.

As of $t = 0$, each creditor observes two kinds of signals about the fundamental of the asset the institution holds. One is a noisy private signal of θ which is common in the global game setup. The other is an asset price P which is observed in the asset market.³ The (function of) asset price P works as an endogenous public signal of fundamental θ . That is,

$$\begin{cases} \text{Private signal} & x_i = \theta + \epsilon_i \quad \epsilon_i \sim \text{uniform}(-\eta, \eta) \\ \text{Public signal} & z = \theta + \epsilon_z \quad \epsilon_z \sim N(0, \tau_z^{-1}) \end{cases}$$

Here z is a function of P which will be specified later. We use uniformly distributed private noises for tractability.

The institution only has a passive role in our model. It owns the long term asset and is financed with the short-term debt from the money market. It is forced to (partially) liquidate its asset holding when meeting with the creditors' rollover refusal, which is cleared in the asset market. The amount of this fire-sale supply of the asset by the institution depends on how many creditors refuse to roll over the debt, and we assume that the size of the fire-sale by the institution (denoted as f)

³Alternatively, the creditors can learn the asset price changes through the marked to market balance sheet.

is given by $f = \kappa \times Y$, where κ is some constant which is increasing in the initial leverage level L .⁴

The last agents are the noise traders in the asset market commonly used in the noisy rational expectation equilibrium setup. At $t = 0$, there is random supply of the asset from the noise trader which follows $S \sim N(0, \tau_s^{-1})$.

3.2.2 Two markets

The decision-making agents (market makers and creditors) receive private signal after the realization of θ , and choose their actions according to the equilibrium strategies. The asset is traded in the asset market at $t = 0$. Aggregate demand is represented by D , and the total supply is given by $S + f$ as defined before. Market clearing price in the equilibrium is given by P which equalizes the supply and the demand. Notice that both D and f are the functions of P .⁵

In the money market, each creditors choose either to roll over or withdraw, and this endogenously decides the size of the fire-sale asset that needs to be liquidated in the asset market (i.e. f). Note that f affects the market clearing price P , and this again affects the creditor's decision making since they try to learn some information out of the changing price. This again can change f , and the asset price P subsequently. This is the feedback process between the two markets described in Figure 3.1.

⁴If $\kappa = 0$, no fire-sale arises and the asset market equilibrium is the conventional noisy rational expectation equilibrium. The linear setup is for tractability.

⁵ Y is a function of P and f depends on Y .

3.3 Equilibrium

We now characterize the rational expectation equilibrium of the model. The equilibrium is defined by the two optimal strategies (for market makers and creditors) as well as the market clearing price P . It is under the rational expectation in the sense that the agents have the correct conjecture of the market clearing asset price function, construct their optimal strategies based on this conjecture, and their actions with the corresponding asset price in fact clear the asset market.

We solve for the equilibrium in two stages. In the first stage, we focus on the asset market and solve for the market maker's demand as well as the market clearing price using the guess and verified method, taking the creditor's strategy as given. This enables us to pin down what kind of public information about the fundamental the agents can learn from the asset price. Then we move to the money market in the second stage. Here, we solve for the creditor's optimal strategy given the public and private signals, using the global game technique as in Angeletos and Werning, while taking the price function as given. Finally, we combine the two stages and the equilibrium is given by the fixed point.

Note that the agents' have access to the public signal (asset price P) as well as the private signal x_i . The equilibrium of the model can be formally defined in the following way:

Definition (Equilibrium) *An equilibrium is a price function $P = P(\theta, S)$, the market maker's individual demand $d_i = d_i(x_i, P)$, and the creditor's rollover decision $y_i = y_i(x_i, P)$ such that individual strategies maximize their respective expected utilities and the corresponding asset price clears the asset market in the rational expectation sense.*

We focus on monotone equilibrium for the creditors. Now, the creditors' optimal strategies can be characterized by a threshold given the asset price; there exist a threshold $x^*(P)$ such that a creditor withdraws if and only if $x_i < x^*(P)$.

3.3.1 Analysis

Asset market (REE)

We first focus on the asset market while taking the creditor's optimal strategy $x^*(P)$ as given. Here, we derive the market clearing price in the rational expectation equilibrium as well as the public information that the asset price contains. We then solve for $x^*(P)$ by moving our focus to the money market as the next step.

We distinguish three regions of (θ, P) as follows:⁶ (i) $\theta > x^*(P) + \eta$; (ii) $x^*(P) - \eta < \theta < x^*(P) + \eta$; (iii) $\theta < x^*(P) - \eta$. We focus on the region (ii) which is the case with feedback between the markets that we are interested in. No creditor withdraws in region (i) and all withdraw in region (iii) and the analysis for those cases are trivial.

First of all, note that $Y(P; \theta)$ is the mass of the creditors who received signals lower than $x^*(P)$, given the fundamental θ . From the uniform distribution, we get

$$Y(P; \theta) = \frac{1}{2\eta} [x^*(P) - (\theta - \eta)] \quad (3.3)$$

⁶Note that P is also a function of θ as verified later, but we take $x^*(P)$ as given at this point.

Therefore, the number of anticipated fire-sale assets by the institution, given the fundamental θ and the market clearing price P should be

$$f(Y(P; \theta)) = \frac{\kappa}{2\eta} [x^*(P) - (\theta - \eta)]. \quad (3.4)$$

We focus on the asset market, and solve the noise REE by the guess and verify method. Suppose that the agents know the correct price function $P = g(\cdot)$ (and also $x^*(P)$) in the rational expectation sense. At this point, we conjecture that the asset price follows $P = g(\theta - \alpha S)$ and verify that this is the case in equilibrium where α is some constant. Denote $z(\equiv z(P)) \equiv g^{-1}(P) = \theta - \alpha S$. If this is the case, z is a function of known variables and $z \sim N(\theta, \tau_z^{-1})$ where $\tau_z^{-1} = \alpha^2 \sigma_s^2$. It has the same information about the fundamental θ as the asset price P , thus the agents use z to infer θ when observing P . In other words, $z(= z(P))$ works as a public signal of θ which any agent can observe in the asset market to update their beliefs.

Recall that the market maker's demand can be given by $d_i(x_i, P) = \frac{E[\theta|x_i, P] - P}{\gamma \text{var}[\theta|x_i, P]}$. Note that the agent use z to extract information out of P , from Bayesian updating, we get $E[\theta|x_i, P] = \frac{\tau_x x_i + \tau_z z}{\tau}$, $\text{var}[\theta|x_i, P] = \tau^{-1}$ where $\tau = \tau_x + \tau_z$. Using this, we can calculate d_i as well as the aggregate demand D given θ and P which can be shown to be

$$D = \frac{\frac{\tau_x \theta + \tau_z g^{-1}(P)}{\tau} - P}{\gamma \tau^{-1}} \quad (3.5)$$

Now the market clearing condition is $D = S + f(Y)$, plug (3.4) and (3.5) in this, we get

$$\frac{\frac{\tau_x \theta + \tau_z g^{-1}(P)}{(\tau_x + \frac{\tau_s}{\alpha^2})} - P}{\gamma(\tau_x + \frac{\tau_s}{\alpha^2})^{-1}} = S + \frac{\kappa}{2\eta} x^*(P) - \frac{\kappa}{2\eta} \theta + \frac{\kappa}{2} \quad (3.6)$$

Rearranging this, we can derive the following results as shown in the appendix.

Proposition 1. (Asset market equilibrium) *The market clearing price P in the asset market is defined implicitly by the following equation:*

$$\theta - \alpha S = \frac{\alpha^2 \gamma}{\alpha \gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right) \quad (3.7)$$

where $\alpha = (\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta})^{-1}$.

Note that $z(\equiv z(P)) \equiv g^{-1}(P) = \theta - \alpha S$, which is normally distributed given θ . From Proposition 1, we get the following Corollary.

Corollary 1. (Public signal) *By observing the market price P , the agent can get the public signal z about the fundamental θ , which follows $z \sim N(\theta, \tau_z^{-1})$ with $\tau_z = (\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta})^2 \tau_s$*

Given these results, now we solve for $x^*(P)$ using the global game. As in Angeletos and Werning (2006), there can be multiple $x^*(P)$ in equilibrium when the endogenous public signal becomes very precise. This itself is a source of volatility, but we focus on the unique $x^*(P)$ case with some parametric restrictions in order to characterize the endogenous feedback between the two markets. More details are explained in the appendix.

Money Market (Global Game)

We next move to the money market. Here, we solve for the creditor's optimal strategy using the global game technique. Note that each creditor has access to two

types of the signals; private signal x_i and public signal P (equivalently $z \equiv z(P)$). We denote $\theta^*(P)$ as the threshold of the fundamental below which the institution fails given the asset price P . We can derive $x^*(P)$ and $\theta^*(P)$ jointly, from the following two steps.

First of all, recall that $Y(P; \theta) = \frac{1}{2\eta}[x^*(P) - (\theta - \eta)]$ from (3.3). By the definition of $\theta^*(P)$, the institution just fails when $\theta = \theta^*(P)$ given P , and since the institution fails if and only if $\theta \leq LY$, we get

$$L \times Y(P; \theta^*(P)) = \theta^*(P) \Rightarrow x^*(P) = \left(\frac{2\eta}{L} + 1\right)\theta^*(P) - \eta \quad (3.8)$$

A creditor forms his posterior distribution of θ given x_i and P (i.e. $z(P)$). Notice that the posterior is the equivalent to the posterior of θ given x_i and $z(P)$, thus follows truncated normal distribution with mean $z(P)$ and precision τ_z bounded by $(x_i - \eta, x_i + \eta)$.

Since the creditor has to be indifferent between withdrawing and rolling over at the threshold $x_i = x^*(P)$, we get

$$c = 1 \times Pr(\theta > \theta^*(P) | x^*(P), z(P)) + 0 \times Pr(\theta \leq \theta^*(P) | x^*(P), z(P)) \quad (3.9)$$

and this can be written as

$$\begin{aligned} & \Phi(\sqrt{\tau_z}(\theta^*(P) - z(P))) - \Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - 2\eta - z(P))) \\ &= (1 - c)[\Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - z(P))) - \Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - 2\eta - z(P)))] \end{aligned} \quad (3.10)$$

using the truncated normal posterior.

Substituting (3.8) in $z(P) \equiv g^{-1}(P) = \frac{\alpha^2\gamma}{\alpha\gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right)$, we get

$$z(P) = \frac{\alpha^2\gamma}{\alpha\gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} \left(\frac{2\eta}{L} + 1 \right) \theta^*(P) \right), \quad (3.11)$$

and plug this in the above equation, we get the equation for $\theta^*(P)$;

$$\begin{aligned} & \Phi \left[\sqrt{\tau_z} \left(\left(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)} (2\eta/L + 1) \right) \theta^*(P) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s} P \right) \right] \\ & - \Phi \left[\sqrt{\tau_z} \left((2\eta/L + 1) \left(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)} \right) \theta^*(P) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s} P - 2\eta \right) \right] \\ & = (1 - c) \left[\Phi \left[\sqrt{\tau_z} \left((2\eta/L + 1) \left(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)} \right) \theta^*(p) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s} p \right) \right] \right. \\ & \quad \left. - \Phi \left[\sqrt{\tau_z} \left((2\eta/L + 1) \left(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)} \right) \theta^*(p) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s} p - 2\eta \right) \right] \right] \quad (3.12) \end{aligned}$$

From this equation, we can solve for $\theta^*(P)$. As discussed in the appendix, there exists unique $0 < \theta^*(P) < 1$ when η is not too small.⁷ We can subsequently solve for $x^*(P)$ from (3.8).

Proposition 2. (Money market equilibrium) *Given the asset price P , there exists unique $x^*(P)$ if η is not too small.*

We can now derive the equilibrium of the model solving the two stages (markets) jointly. We have solved the asset market equilibrium (market clearing price P) taking $x^*(P)$ as given, and have solved the money market equilibrium $x^*(P)$ taking P as given. Therefore, the equilibrium is derived as the fixed point of the

⁷Multiple equilibria exist if η is very small as is in Angeletos and Werning (2006). The endogenous public signal which aggregates private signals becomes too precise, which becomes the source of multiple equilibria. We focus on the unique equilibrium case with large η .

two. We hence get the following result.

Theorem 1. *There exists a unique equilibrium of the model if the noise in the private signal (η) is not too small.*

We can describe the feedback process between the two markets as follows. From Proposition 1, taking $x^*(P)$ as constant, notice that the market clearing price P is decreasing in $x^*(P)$ (equivalently $\theta^*(P)$, or $f(Y)$). Also, Proposition 2 implies that $x^*(P)$ is decreasing in P .⁸ This generates the amplification of the initial shock through the feedback, described in Figure 3.2. When there is large noisy supply S in the asset market, this will push down the asset price P in order to clear the market. However, in the downturn with not high θ , the creditors can interpret this as a bad signal about the fundamental. This updated belief then can cause the creditors less willing to lend (higher $x^*(P)$, and thus higher $Y(P; \theta)$ from (3.3)) and subsequently the institution has to liquidate more of its asset (fire-sale) in order to meet the rollover refusal (higher $f(Y)$ from (3.4)). This additional asset liquidation will further push down the market clearing price P , and the creditors become even more concerned with higher $x^*(P)$. The spiral stops when the fixed point is reached, or it can even drag down the institution to failure while it would have survived if there had been no feedback process.

⁸This can be shown by contradiction. If $x^*(P)$ is increasing in P , both z and x^* are increasing in P . However, this cannot satisfy the indifference condition, thus contradiction.

3.4 Implications

3.4.1 Heterogeneous Price Volatility

The feedback between the two markets becomes the source of heterogeneous asset price volatility. Note that the asset price is given by $P = \theta - \gamma\sigma_s^2 S$ if there's no feedback between the two markets as in the conventional noisy REE setup. This corresponds to the case when we analyze the asset market in separation. It is easy to verify that the asset price volatility conditional on a given fundamental $\text{var}(P|\theta)$ is constant, and the ex ante distribution of the asset price P is symmetric. The asset price volatility is homogeneous regardless of the economic states.

The asset price distribution becomes negatively skewed and the volatility becomes heterogeneous once the feedback process is incorporated. Notice that when the fundamental is robust (high θ), the creditors will let the debt to be rolled over in almost all of the realizations of S . Hence, the asset price volatility can be calculated in the same way as above in this case. When θ is low, however, the asset price becomes more sensitive to the realizations of S since this makes the creditors concerned and leads to the asset fire-sale caused by the funding liquidity dry-up. Since the volume of fire-sale is larger with lower θ for the same realizations of S , the asset price becomes more volatile as the fundamental deteriorates. Ex ante, this makes the asset price distribution negatively skewed.

Corollary 2. *The conditional asset price volatility $\text{var}(P|\theta)$ is heterogenous, and decreasing in the fundamental θ (higher volatility during the downturn compared to the normal times). Ex ante, the unconditional distribution of the asset price P is negatively skewed.*

3.4.2 Joint distress in the two markets

We next focus on the likelihood of bank failure. When there's no public information, the creditor's problem is reduced to the conventional bank run problem as in Rochet and Vives (2004); there exists a threshold of the fundamental θ^{**} such that the institution fails if and only if $\theta < \theta^{**}$. With the feedback process, however, this failure threshold becomes higher depending on the realizations of S as seen in the previous section. With large S , the price P drops and this increases the failure threshold $\theta^*(P)$ by making the creditor more concerned. Even the noise supply of the asset can bring the institution failure by making the creditors panicked, if $\theta^{**} < \theta < \theta^*(P)$.

Notice that $\theta^*(P)$ is the function of P , and P is the function of θ and S . As seen in the previous subsection, P becomes more sensitive to S as θ decreases, thus $\theta^*(P)$ tend to deviate more from θ^{**} as θ becomes lower, or during the economic downturns. This implies that given θ , the joint distress (asset and money market distressed) through the noisy supply S becomes more likely with lower θ . Asset liquidation by the liquidity traders may depress the asset market, but this can further bring the institutions financing from the money market into trouble. This joint distress becomes more likely during the downturn, as the fundamental deteriorates.

Corollary 3. *The institution that could have survived otherwise may fail because of the distress in the asset market. This joint distress becomes more likely in the downturn.*

3.4.3 The role of leverage levels

The institution's leverage level affects its robustness $\theta^*(P)$ through two channels. The first channel is conventional; if an institution is more highly leveraged, it fails more easily when certain fraction of the creditors try to run compared to lower leveraged ones.⁹ This happens only within the money market, and higher L implies a higher failure threshold for an institution.

There's additional channel which comes through the asset market, and feeds back into the decision making of the creditors in the money market. Note that asset (fire-sale) liquidation size by the institution, which is captured by κ , is increasing in the leverage level. For given fraction of creditors who refuse to roll over the debt, the institution has to liquidate more assets to meet the withdrawal pressure if it is more highly leveraged. Larger asset liquidation implies larger asset price drop in the asset market, and this makes the creditors of the institutions more concerned and the institution more fragile with higher $\theta^*(P)$. The two channels reinforce each other, and the increased leverage level makes the institution more fragile compared to the case when the institution's financing problem in the money market is analyzed in isolation ignoring the asset market.

3.5 Conclusion

This paper constructs a model that endogenously characterizes the feedback between market liquidity (asset market distress) and funding liquidity (money market distress). When the asset price drops during the economic downturn, the creditors in the money market can become more concerned, and the financial institutions

⁹It fails if $\theta < LY$.

can have a difficulty in securing their funding to maintain the position. The institutions are then forced to liquidate some of the asset holdings, which further drags down the asset price and exacerbates the stress in the money market subsequently. This becomes the source of the joint distress between the asset market and money market.

The technical contribution of this paper is to combine the noisy rational expectation equilibrium setup and the global game setup. Unlike the model of Angeletos and Werning that follows arbitrary two steps in which one simply precedes the other, there is a feedback effect between the steps. This feedback works as an amplification mechanism of an initial shock, which does not arise when the fundamental is sufficiently robust but only arises as the fundamental deteriorates. As a result, the asset price volatility becomes higher as the fundamental deteriorates and the asset price distribution becomes negatively skewed.

3.6 Appendix

Proof of Proposition 1

Suppose $P = g(\theta - \alpha S)$ and we later verify that this conjecture is right in equilibrium. Define $z(\equiv z(P)) \equiv g^{-1}(P) = \theta - \alpha S$, then $z \sim N(\theta, \alpha^2 \sigma_s^2)$. CARA-normal setup implies that

$$d_i(x_i, P) = \frac{E[\theta|x_i, P] - P}{\gamma \text{var}[\theta|x_i, P]} = \frac{E[\theta|x_i, z] - P}{\gamma \text{var}[\theta|x_i, z]}.$$

From the Bayesian updating, we get $E[\theta|x_i, z] = \frac{\tau_x x_i + \tau_z z}{\tau}$, $\text{var}[\theta|x_i, z] = \tau^{-1}$ where $\tau = \tau_x + \tau_z$. Plug these in $d_i(x_i, P)$ above, we get $D = \int d_i(x_i, P) di$ given

(θ, P) as

$$D = \frac{\frac{\tau_x \theta + \tau_z g^{-1}(P)}{\tau} - P}{\gamma \tau^{-1}}$$

From (3.4), $f(Y) = \frac{\kappa}{2\eta} x^*(P) - \frac{\kappa}{2\eta} \theta + \frac{\kappa}{2}$. Now let $\alpha = (\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta})^{-1}$, then the market clearing condition is $D = S + f(Y)$ can be written as

$$\frac{\frac{\tau_x \theta + \tau_z g^{-1}(P)}{(\tau_x + \frac{\tau_s}{\alpha^2})} - P}{\gamma(\tau_x + \frac{\tau_s}{\alpha^2})^{-1}} = S + \frac{\kappa}{2\eta} x^*(P) - \frac{\kappa}{2\eta} \theta + \frac{\kappa}{2}.$$

Rearranging this,

$$\theta - \frac{S}{\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta}} = \frac{1}{\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta}} \times \left(-\frac{\tau_s}{\alpha^2 \gamma} g^{-1}(P) + \frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right)$$

Denote the RHS of the above equation as $g^{-1}(P)$, we get

$$g^{-1}(P) = \frac{1}{\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta}} \times \left(-\frac{\tau_s}{\alpha^2 \gamma} g^{-1}(P) + \frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right)$$

Solving this for $g^{-1}(P)$, and get

$$g^{-1}(P) = \frac{\alpha^2 \gamma}{\alpha \gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right)$$

The asset price P implicitly defined by this g does clear the market, thus it is a rational expectation equilibrium. ■

Proof of Corollary 1.

Given P and $x^*(P)$, z can be uniquely defined as

$$z = \frac{\alpha^2 \gamma}{\alpha \gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right)$$

where $\alpha = (\frac{\tau_x}{\gamma} + \frac{\kappa}{2\eta})^{-1}$.

We show in the proof of Proposition 1 that $z \sim N(\theta, \alpha^2 \sigma_s^2)$. ■

Proof of Proposition 2.

Given θ , x_i is uniformly distributed between $[\theta - \eta, \theta + \eta]$. Since the creditor with private signal $x_i \leq x^*(P)$ withdraws, the fraction of the creditors who withdraw given the asset price P is equal to

$$Y(P; \theta) = \frac{1}{2\eta} [x^*(P) - (\theta - \eta)]$$

Now, by the definition of $\theta^*(P)$, the institution just fails when $\theta = \theta^*(P)$ given P , and using the above this implies that

$$L \times \frac{1}{2\eta} [x^*(P) - (\theta^*(P) - \eta)] = \theta^*(P) \Rightarrow x^*(P) = \left(\frac{2\eta}{L} + 1 \right) \theta^*(P) - \eta \quad (3.13)$$

We next consider the posterior of $\theta|x_i, P$. Note that this posterior is equivalent to $\theta|x_i, z$. Since $\theta|z$ is normally distributed with mean z (equivalently $z(P)$) and precision τ_z which is specified in Corollary 1, and $\theta|x_i$ is uniformly distributed between $[x_i - \eta, x_i + \eta]$, Bayesian updating implies that the posterior $\theta|x_i, z$ follows truncated normal distribution with mean z and precision τ_z bounded by $(x_i - \eta, x_i + \eta)$.

Since the creditor has to be indifferent between the two alternative actions at the switching threshold $x_i = x^*(P)$, it has to be

$$c = 1 \times Pr(\theta > \theta^*(P)|x^*, P) + 0 \times Pr(\theta \leq \theta^*(P)|x^*, P)$$

and this can be written as

$$1 - c = Pr(\theta < \theta^*(P) | x^*, P) \quad (3.14)$$

Using the posterior distribution of θ , this can be written as

$$\frac{\Phi(\sqrt{\tau_z}(\theta^*(P) - z(P))) - \Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - 2\eta - z(P)))}{\Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - z(P))) - \Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - 2\eta - z(P)))} = (1 - c),$$

thus

$$\begin{aligned} & \Phi(\sqrt{\tau_z}(\theta^*(P) - z(P))) - \Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - 2\eta - z(P))) \\ &= (1 - c)[\Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - z(P))) - \Phi(\sqrt{\tau_z}((2\eta/L + 1)\theta^*(P) - 2\eta - z(P)))] \end{aligned} \quad (3.15)$$

from the truncated normal posterior.

Substituting (3.13) in $z(P) \equiv g^{-1}(P) = \frac{\alpha^2\gamma}{\alpha\gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} x^*(P) + \frac{\kappa}{2} \right)$, we get

$$z(P) = \frac{\alpha^2\gamma}{\alpha\gamma + \tau_s} \left(\frac{\tau_x + \frac{\tau_s}{\alpha^2}}{\gamma} P + \frac{\kappa}{2\eta} \left(\frac{2\eta}{L} + 1 \right) \theta^*(P) \right).$$

Plug this in (3.15), we get the equation for $\theta^*(P)$;

$$\begin{aligned}
& \Phi[\sqrt{\tau_z}((1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)}(2\eta/L + 1))\theta^*(P) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s}P)] \\
& - \Phi[\sqrt{\tau_z}((2\eta/L + 1)(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)})\theta^*(P) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s}P - 2\eta)] \\
& = (1 - c)[\Phi[\sqrt{\tau_z}((2\eta/L + 1)(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)})\theta^*(p) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s}p)] \\
& - \Phi[\sqrt{\tau_z}((2\eta/L + 1)(1 - \frac{\kappa\alpha^2\gamma}{2\eta(\alpha\tau + \tau_s)})\theta^*(p) - \frac{\alpha^2\tau_x + \tau_s}{\alpha\gamma + \tau_s}p - 2\eta)]] \quad (3.16)
\end{aligned}$$

Now, we wish to find a unique $0 \leq \theta^*(P) \leq 1$ that satisfies (3.16). First of all, since $0 < c < 1$, we get LHS $>$ RHS for (3.16) when $\theta^*(P) = 0$. Also, LHS $<$ RHS when $\theta^*(P) = L$. From the continuity, there exists at least $\theta^*(P)$ between 0 and 1. We need some additional restrictions for the uniqueness since agents have access to a public signal as in Angeletos and Werning.

In order to have a unique equilibrium, the public signal should not be too precise compared to the private signal. For our case, we can achieve this by making η, σ_x^2 , or σ_s^2 not too small. ■

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