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ESSAYS ON POLITICAL ECONOMY

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Abstract

This collection of essays investigates issues related to elections and wars. In the first essay, we present an auction-type model of war and characterize its unique equilibrium. We offer three different explanations for the causes of wars. First, as an individual-level explanation of war, we show that wars occur due to information asymmetry between governments. Second, as a societal-level explanation of war, we show that countries have smaller armies, they fight less severe wars, and their citizens are better off in a democratic world than in an autocratic world. Finally, as a systemic-level explanation of war, we show that wars will completely disappear when one of the countries has absolute superiority in military technology.

In the second essay, we examine the impact of positive altruism, that is, voting for a candidate to please voters who prefer the same candidate, on policy choices of strategic candidates. We establish the existence of equilibrium consistent with Duverger's Law and polarization. We show that this equilibrium is unique. Moreover, our model offers a theoretical basis for the decrease in turnout and increase in polarization observed in the U.S. after the 1960s. We also consider negative altruism, that is, voting for a candidate to anger voters who dislike the candidate, and show that it is qualitatively similar to positive altruism in terms of its effects on candidates, that is, the equilibria are consistent with Duverger's Law and polarization.

In the final essay, we offer an explanation to 'policy convergence' in Downsian models of two-party systems with unidimensional policy space and to 'paradox of voting' in large elections. In our model, voters vote to improve the lives of their

fellow voters. Candidates are office motivated. They try to increase voter support and energize their base to win the election. The incentive to increase their voter support pushes candidates towards each other, while the incentive to energize their bases pulls them apart. We examine the trade-off between these two incentives and establish the existence of a unique equilibrium in which candidates diverge from each other and voters vote in high proportions.

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Chapter 1

A Theory of Democratic Peace and Power Shift

1.1 Introduction

Wars are destructive. The countries involved are generally rational and know the costs of war, but nonetheless wars recur. In the literature, there are different explanations for the causes of wars.¹ International relations theorists classify the causes of war in terms of their origins in the individual level, the societal level and the international-system level.² In terms of individual-level explanations of war, we see uncertainty, bounded rationality, and the differences in people's characteristics, that is, their personalities, decision-making processes, emotions, belief systems or biases.

¹The rational explanations of wars include: uncertainty, differences between the incentives of leaders and citizens, differences in countries' beliefs about the outcome of war, arms races, internal political competition, preventive or preemptive strategic advantage, and commitment problems. For a detailed survey of the motivations for war, see Jackson and Morelli [57] and Levy [67].

²Ever since Waltz [103] made this classification, international relations theorists have agreed on using it for the levels of analysis.

Among societal-level explanations of war, the most famous is based on the existence of undemocratic states. It is generally known as “the democratic peace theory”. According to this theory, democracies are hesitant to engage in wars with other democracies, and therefore the likelihood of war between two democracies is smaller than the likelihood of war between two countries of which at least one is not a democracy.³ A second consensus, which is also a societal-level explanation of war, has developed in parallel to democratic peace theory. This is the consensus that democracies are not different than other types of regimes in terms of frequency of war, but they are less warlike, that is, their wars are less severe.⁴ Among systemic-level explanations, we see two theories that have developed through studies on the relationship between power distributions and war. The *balance-of-power approach* insists on peaceful consequences of power equality, while the *power-shift approach* claims the opposite, that is, a shift from power equality towards power inequality produces a more peaceful world. There are a number of strong adherents to both approaches, but recent empirical analyses tend to grant greater validity to power-shift approach.⁵

On the other hand, auctions and wars are closely related, because both of them feature several players competing for a prize or set of prizes by expending resources.⁶ Hence, the tools of auction theory bring an insight to the discussions of the causes of war. In this chapter, we take a step in the direction of understanding the causes of war with the help of auction theory. We provide an auction-type model of war which offers a theoretical background in order to explain how countries’ regimes and power

³See Kant [59], Paine [80], de Tocqueville [25], Small and Singer [95], and Doyle [27, 28].

⁴See Rummel [89] for a discussion of this consensus.

⁵See Sabrosky [91] and Kugler and Lemke [63].

⁶See Krishna and Morgan [62] and Hodler and Yektas [52].

distributions affect wars. Moreover, our model explains the causes of war in terms of its origin in all three levels of analysis: governments' lack of information about each other in the individual level; discrepancies between autocratic governments and citizens, which lead to warlikeness of autocracies in terms of the severity of their wars, in the societal level; none of the countries having absolute superiority in military technology, i.e., power-shift approach, in the international system level.

First, we begin with the individual-level analysis. In our model, the individual-level explanation of war depends on governments' uncertainty about each other. We show that, when governments have complete information, they choose mixed strategies and create uncertainty that paves the way for wars, whereas when they have incomplete information, wars recur due to their exogenous uncertainty about each other.

Second, we have the societal-level analysis. In 1795, Immanuel Kant [59] wrote:

“...if the consent of the citizens is required in order to decide that war should be declared...nothing is more natural than that they would be very cautious in commencing such a poor game, decreeing for themselves all the calamities of war. Among the latter would be: having to fight, having to pay the costs of war from their own resources, having painfully to repair the devastation war leaves behind, and, to fill up the measure of evils, load themselves with a heavy national debt that would embitter peace itself and that can never be liquidated on account of constant wars in the future. But, on the other hand, in a constitution which is not

republican, and under which the subjects are not citizens, a declaration of war is the easiest thing in the world to decide upon, because war does not require of the ruler, who is the proprietor and not a member of the state, the least sacrifice of the pleasures of his table, the chase, his country houses, his court functions, and the like. He may, therefore, resolve on war as on a pleasure party for the most trivial reasons...”

Following Kant’s logic, we argue that democratic governments are accountable to the citizens, they are highly attentive to the costs of wars and bear all burdens of wars; however, the same cannot be said about autocratic governments, which enjoy the benefits of wars and avoid most of their costs. In other words, democratic governments have incentives similar to those of their “citizens”, but the incentives of autocratic governments are different from those of their “subjects”, and wars are more costly for democratic governments than for their autocratic counterparts. However, we deviate from Kant’s conclusion and argue that the discrepancy between autocratic governments and their “subjects” is the reason for autocracies fighting more severe wars than democracies. In other words, unlike Kant, we measure a regime’s warlikeness in terms of the severity of wars instead of the frequency count of wars, which gives the same count of one for a country that has suffered only a few casualties and for another country that has lost several million citizens. We show that wars between autocracies are expected to be more severe than wars between democracies. Hence, our model offers a theoretical background to the validity of the second consensus on democratic peace, that is, a world of democracies is expected to be more peaceful

than a world of autocracies - when peace is measured in terms of mildness of wars - and the existence of undemocratic regimes is one of the reasons for severe wars.

Finally, we have an international system-level analysis. In our model, the war game between two governments is like all-pay auctions. When we look at the relationship between power distributions and war, our model offers a theoretical explanation for the power-shift approach through the following observation on auctions. In all-pay auctions, an increase in the strong bidder's valuation or a decrease in the weak bidder's valuation causes a decrease in the weak bidder's probability of victory, and this leads to a decrease in his bid. The strong bidder also bids less aggressively due to the decrease in the weak bidder's bid. Therefore, bids will be higher when bidders' valuations are equal than when one bidder has a preponderance of valuation over the other.⁷ This observation works similarly in our model when bids are replaced with effective bids, where a government's effective bid is equal to its bid amplified by its military technology, and bidders' valuations are replaced with governments' levels of aggression, which is their victory value amplified by their military technology.

We offer a new interpretation of power-shift approach, and show that our model offers a theoretical background that supports both our and classical interpretations of this approach. Powerful countries do not intervene in the conflicts between other countries - or powers - as long as the conflict is not important to them. For example, France intervened in Libya during the Arab Spring in 2011, but chose not to intervene in Bosnia and Herzegovina two decades ago; the U.S. intervened in Korea, Vietnam, Iraq etc., but failed to intervene in Rwanda and tried to avoid intervention in Syria

⁷See Baye, Kovenock and de Vries [10] and Amann and Leininger [9].

for nearly three years. These show that countries do not intervene in conflicts just because they are powerful, but they choose to become a part of the conflict if they have a motive for intervening. Hence, countries' levels of aggression is a better measure of their motivation than their power levels, and we offer a different interpretation of power-shift approach which depends on countries' aggression levels. According to our interpretation, power-shift approach states that a shift towards higher aggression inequality, instead of power inequality, produces a more peaceful world. In line with our understanding of power-shift approach, we show that when governments have complete information, investment in warfare will decrease and losses from wars will decline with an advancement in the more aggressive government's military technology or a decline in the less aggressive government's military technology, that is, a step towards higher aggression inequality will also be a step towards a more peaceful world. Moreover, in line with the classical interpretation of power-shift approach, independent of whether governments have complete or incomplete information, we show that investments in warfare and wars will completely disappear with the absolute superiority of one of the countries in military technology.

1.2 Related Literature

There is a large body of literature on models analyzing international conflicts in which pre-play communication precedes a possible military conflict.⁸ However, only a few of them typically focus on matters related to strategic ambiguity about military

⁸See Hummel [54] for a small portion of this literature.

capacity, a possible individual-level explanation of war. Baliga and Sjöström [9] present a model in which one of the states possesses advanced weapons while the other does not possess such weapons but may attempt to acquire them, and strategic ambiguity arises since the deterrence of such an attempt by doubt can be a substitute for a costly weapons program. Meirowitz and Sartori [71]’s model has a similar flavor to ours in that there are two competing states that endogenously decide how much to invest in developing their military capacity before bargaining and then possibly going to war, because they, acting in their own self-interest, generally choose mixed strategies for military production in equilibria and create ambiguity about their true military capacity. However, Meirowitz and Sartori [71] are unable to derive the equilibrium strategies due to the generality of their model. Hummel [54] considers a model in which a nation must decide whether to reveal its military capacity when it faces two possible adversaries, one inclined to attack if it has a weak military capacity, and the other inclined to attack preemptively if it is developing a strong military capacity. He derives conditions based on hawkishness of the adversaries and the accuracy of the adversaries’ national intelligence, under which the nation chooses to be ambiguous about its military capacity. In his model, strategic ambiguity arises because countries have strategic interactions with multiple adversaries and ambiguity can decrease their probability of being attacked. Hodler and Yektas [52] is the closest to our model in this literature. They model wars as asymmetric auctions with incomplete information. In their model, the outcome of the war is uncertain since countries’ realized resource levels are private information. Each country allocates its

resources to production and warfare, and the country with greater military power wins and consumes all goods that have been produced in the two countries. They show that in equilibrium both comparative and absolute advantages matter. There are major differences between their model and ours. In our model, *(i)* countries' victory values, which play a similar role to the "resource levels" of Hodler and Yektas [52], can be both private information and common knowledge, whereas in Hodler and Yektas [52] resource levels are common knowledge *(ii)* the winner gains only its own value of the war, unlike Hodler and Yektas [52] in which the winner consumes both countries' production, and *(iii)* both governments pay their bids and the costs of war, unlike Hodler and Yektas [52] in which the loser pays nothing. Our results are also different in that our model has a say about power-shift approach and democratic peace.

Among societal-level explanations of war, the idea that democracies almost never go to war with each other has acquired a nearly law-like status. The foundations of this idea date back to Paine [80] and Kant [59], and it has been revitalized by Doyle [27, 28], which have been followed by an outbreak of studies on democracy. However, there are criticisms of democratic peace, as well. For instance, Russett [90] says that democracies were too scarce and far apart before the Second World War, hence the absence of wars between democracies was not surprising. Even though his core thesis is that democracies do not fight with one another, he lists several other factors such as alliances, wealth and political stability that have influenced the alleged peace between democracies, and restricts the influence of democracy on peace to the

very recent past. Small and Singer [95] find an absence of wars between democratic states with two "marginal exceptions", but argue that this pattern is statistically insignificant. Spiro [97] shows that the absence of wars between democracies is not statistically significant, except for a brief period during World War I. Gowa [47] shows that the democratic peace before 1945 is insignificant, and offers an alternate explanation for the following period such that peace is an artifact of the Cold War, and the threat from the communist states forced democracies to ally with one another. Schwartz and Skinner [93] take the criticism further, and argue that neither the historical record nor the theoretical arguments advanced for the purpose provide any support for democratic pacifism, and show that there have been as many wars between democracies as one would expect between any other pair of states. However, they include wars between young and dubious democracies as well as very small wars. Rummel [89] tries to reconcile the democratic peace theory with these criticisms, and argues that a regime's warlikeness should be measured in terms of the severity of wars instead of the frequency count of wars, and shows that democracies are less warlike in terms of the severity of their wars between 1900 and 1987. Rummel [89]'s results are the closest to ours in the literature on democratic peace, but his paper, unlike ours, is an empirical study.

In our model, we follow Kant [59] and argue that democratic governments have similar incentives to those of their citizens unlike autocratic governments, which do not suffer some of the costs of war. As a result of the discrepancy between autocratic governments and their citizens, autocracies are more warlike than democracies. The

differences between the incentives of governments and citizens have been studied in the literature. For instance, Lake [64] argues that in democracies the society's cost of controlling the state is relatively low compared that cost in autocracies, so democracies are less likely to fight wars with each other while they are more likely to fight wars with autocratic states. Schultz [92] presents an incomplete information model of crisis bargaining to explore the effect of domestic political competition on the escalation of international crises and comes up with a similar result: the inclusion of a strategic opposition party decreases the ex-ante probability of war by helping to reveal information about a state's preferences. Hence, he suggests a mechanism through which democratic states overcome the informational asymmetries between their governments and their citizens, a central obstacle to negotiation. Jackson and Morelli [55] examine how countries' incentives to engage in a war depend on the political bias of their leaders, where political bias refers to the discrepancy between the interests of decision makers and citizens. They show that when it is possible to avoid a war when there is no political bias, but when there is sufficiently strong bias on the part of one or both countries, war cannot be prevented via transfers, and the probability of winning the war depends on the countries' levels of wealth. They show that when transfers are possible, at least one country will choose a biased leader as that leads to a strong bargaining position and extraction of transfers. We differ from the literature on the discrepancy between autocratic governments and their citizens by modeling the war as an auction⁹ and by showing that this discrepancy makes

⁹It is an auction with heterogenous valuations where the winner is the bidder with the highest effective bid.

autocracies more warlike than democracies even though there exists no informational asymmetry between governments and their citizens, nor do there exist any possible transfers between countries.

In terms of systemic-level explanations of war, there are two opposite approaches: the balance-of-power approach and the power-shift approach. Balance-of-power approach is concerned with explaining national strategies, the formation of blocking coalitions, the avoidance of hegemony, and the stability of the system. Balance-of-power supporters agree that some form of equilibrium of military capabilities, bipolarity¹⁰ or multipolarity¹¹, increases the stability of the system, which is generally defined as the relative absence of major wars, and that movements toward unipolarity are destabilizing because they trigger a hegemonic war to restore equilibrium. Even though they rely heavily on polarity as a key explanatory variable, they do so with very little supporting evidence. For instance, Sabrosky [91] shows that bipolarity is no less war-prone than multipolarity, that wars occur under a variety of structural conditions, and that polarity is not a primary causal factor in the outbreak of war. As an alternative to balance-of-power approach, power-shift approach is a form of hegemonic theory that emphasizes the peaceful consequences of a hierarchical system.¹² Many of the theoretical analyses of power-shift approach focus on transitions between a hegemonic state and a challenger, and study the causes of the rise and fall of hegemons as well as the precise identity of hegemonic war such as who initiated the war, whether the declining hegemonic state did to block the rising chal-

¹⁰See Waltz [104] and Mearsheimer [70]

¹¹See Morgenthau [72] and Gulick [48]

¹²See Organski and Kugler [76], Gilpin [45], and Thompson [98].

lenger while the chance was still available or the challenger did so to bring its benefits from the system into line with its rising military power.¹³ Analyses of power-shift approach generally include a broader international system with a hierarchy among states. However, some studies of power-shift approach apply to any two states in the system, and require that countries are more warlike when there is an equality of power and less warlike when one state has a preponderance of power over the other. Kugler and Lemke [63] shows the widespread support for peaceful consequences of power preponderance of a state over another in the empirical literature. Hence, the empirical studies tend to favor power-shift approach over balance-of-power approach. In this chapter we present an auction type model and offer a theoretical background for power-shift approach.

As we model war as an auction, our study also relates to the literature on auction theory. Wars between autocracies share some features with all-pay auctions. Resources allocated to warfare cannot be used to produce consumption goods, hence, as in all-pay auctions, all bids need to be paid. However, there is a small difference. In our game, the winner is the government with the highest effective bid, whereas in an all-pay auction the winner is the bidder with the highest bid. Baye, Kovenock and de Vries [10] fully characterize equilibrium for all-pay auctions with complete information, and we adapt their characterization for wars between autocracies that have complete information. On the other hand, Amann and Leininger [9] prove the existence and uniqueness of Bayesian equilibrium for a class of generally asymmetric all-pay auctions with incomplete information, which resembles our model when

¹³For a detailed survey see Levy [66] and Vasquez [102].

the war is between autocracies that have incomplete information. When the war is between democracies, however, our model deviates from the literature on all-pay auctions in that the payoff structures are quite different, because governments' payoffs depend on each others' bids.

The remaining of this chapter is organized as follows. Section 1.3 lays out the basic model. We deal with the complete information case in Section 1.4 and with the incomplete information case in Section 1.5. Section 1.6 concludes. All the proofs are provided in Appendix A.

1.3 The Basic Model

Two countries, country A and country B , engage in a war to acquire the spoils of victory. Each country $i \in \{A, B\}$ is characterized by three parameters, its *initial resource level* $R_i \in \mathbb{R}^+$, its *military technology* $\lambda_i \in \mathbb{R}^+$, and its *victory value* $v_i \in \mathbb{R}^+$, that is, country $i \in \{A, B\}$ has R_i units of resources, it can produce λ_i units of military goods by using one unit of resources, and it gains a benefit of v_i units of resources by winning the war.¹⁴ Each country has a government. Governments simultaneously decide how much of their own resources they will allocate to military production. A country's initial resource level is not a binding constraint on their military production, because they can borrow from other countries. After the production of military goods, countries engage in war. The outcome of the war is deterministic and the

¹⁴Our model covers a wide range of possible conflict cases. For example, if the reason for war is to acquire a land or a mineral mine, v_i represents the value of that land or mine for country i . On the other hand, if the two countries are fighting due to political differences, such as separation versus unification, then v_i represents the extra benefit country i gets by implementing its own policy instead of its opponent's policy.

country with greater military power wins the war. More formally, let b_i denote the amount of resources allocated for military production by government $i \in \{A, B\}$. Then, country $i \in \{A, B\}$ will have a *military power* of $\lambda_i b_i$. We call b_i *government i 's bid* and $\lambda_i b_i$ *government i 's effective bid*. Effective bids determine the winner of the war, such that the country with the higher effective bid wins the war and in case of equality both countries win with equal probabilities.

War is costly, because both countries produce military goods before the war and also because war causes losses, such as wounded civilians and soldiers, destroyed or damaged buildings and other financial costs. We assume that countries spend one unit of their military power in order to destroy one unit of their opponent's military power, and they do not give up fighting as long as they have military power, that is, the war is over whenever the military power of one of the countries is completely destroyed. As a result of this, the size of the smaller army determines the countries' losses during the war. We measure all losses in terms of resources. Therefore, more formally, given b_i, b_j and $\lambda_j b_j \leq \lambda_i b_i$, country i and country j spend b_i and b_j units of resources to produce $\lambda_i b_i$ and $\lambda_j b_j$ units of military goods, respectively, and suffer losses of $\frac{\lambda_j}{\lambda_i} b_j$ and b_j units of resources during the war, respectively. We call the total war-related losses of the involved countries, which are equal to $b_A + b_B + \min\{\lambda_A b_A, \lambda_B b_B\} \left(\frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right)$, *the size of the war*.

In each country, there is a representative citizen; we call them *citizen A* and *citizen B*. These representative citizens are risk-neutral and they enjoy the resources in their countries after the war. If country i wins the war, i.e., if $\lambda_j b_j < \lambda_i b_i$, then

it uses b_i units of resources for military production, suffers a loss of $\frac{\lambda_j}{\lambda_i}b_j$ units of resources during the war,¹⁵ and acquires v_i units of resources by winning the war; hence, citizen i enjoys $R_i + v_i - b_i - \frac{\lambda_j}{\lambda_i}b_j$ units of resources. Following the same reasoning, if both countries win the war with equal probabilities, i.e., if $\lambda_j b_j = \lambda_i b_i$, then citizen i enjoys $R_i + v_i - 2b_i$ or $R_i - 2b_i$ units of resources with equal probabilities. Finally, if country i loses the war, i.e., if $\lambda_j b_j > \lambda_i b_i$, then citizen i enjoys only $R_i - 2b_i$ units of resources.

Governments are one of two types: autocratic or democratic. Autocratic governments suffer the costs of military production, however, unlike their representative citizens, they do not suffer any losses during the war. Hence, it follows from the above discussion that when country i is an autocracy, government i 's utility is:

$$u_{AR_i}(b_i, b_j) = \begin{cases} R_i - b_i, & \text{for } \lambda_i b_i < \lambda_j b_j \\ R_i + \frac{v_i}{2} - b_i, & \text{for } \lambda_i b_i = \lambda_j b_j \\ R_i + v_i - b_i, & \text{for } \lambda_i b_i > \lambda_j b_j \end{cases} \quad (1.1)$$

Democratic governments, unlike their autocratic counterparts, are accountable to their citizens and bear the burdens of wars just like their representative citizens.

Hence, a democratically elected government i 's utility is the same as citizen i 's utility,

¹⁵Country j suffers a loss of b_j units of resources. Hence, the country that is more advanced in military technology suffers less in terms of resources during the war.

and it is:

$$\begin{aligned}
u_{DR_i}(b_i, b_j) &= u_{AR_i}(b_i, b_j) - \frac{\min\{\lambda_i b_i, \lambda_j b_j\}}{\lambda_i} \\
&= \begin{cases} R_i - 2b_i, & \text{if } \lambda_i b_i < \lambda_j b_j \\ R_i + \frac{v_i}{2} - 2b_i, & \text{if } \lambda_i b_i = \lambda_j b_j \\ R_i + v_i - b_i - \frac{\lambda_j b_j}{\lambda_i}, & \text{if } \lambda_i b_i > \lambda_j b_j. \end{cases} \quad (1.2)
\end{aligned}$$

In this paper, we examine wars between two democracies and between two autocracies and compare *a world of democracies* with *a world of autocracies*.¹⁶ We make this comparison with two different information setups: complete and incomplete information. The parameters that characterize country A and country B are common knowledge in the complete information setup, whereas countries' victory values, i.e., v_A and v_B , are private information in the incomplete information setup.

1.4 Complete Information

Initially, we study the complete information setup, in which all parameters of country A and country B are common knowledge. Government $i \in \{A, B\}$ can produce $\lambda_i v_i$ units of military goods by using the spoils of victory.¹⁷ We call $\lambda_i v_i$ *government i 's level of aggression*. We say that government i *becomes more aggressive* when $\lambda_i v_i$ increases, and similarly *becomes less aggressive* when $\lambda_i v_i$ decreases. Without loss of

¹⁶We leave the wars between a democracy and an autocracy for future research.

¹⁷Government i has no incentive to produce more than $\lambda_i v_i$ units of military goods, since it is better off with no military production than with a military production of more than $\lambda_i v_i$ units, even though having no military production leads to a sure failure in the war. Hence, $\lambda_i v_i$ units of military goods is the maximal military power that government i will choose to produce.

generality, we assume that government A is weakly more aggressive than government B , i.e., $\lambda_A v_A \geq \lambda_B v_B$, and call government A and government B *the aggressive government* and *the unaggressive government*, respectively. We call the ratio of the aggressive government's level of aggression over the unaggressive government's level of aggression *aggression inequality*, and denote it with:

$$s = \frac{\lambda_A v_A}{\lambda_B v_B}. \quad (1.3)$$

Hence, governments' aggression levels are alike when aggression inequality is low, whereas government A is much more aggressive than its opponent when aggression inequality is high.

First, we look at wars between two autocratic governments. The war game between two autocracies is very similar to all-pay auctions with complete information and private valuation, and differs only in that the winner is the government with the highest effective bid.¹⁸ The equilibrium will require mixed strategies. We denote governments' bidding strategies by the distribution functions F_A and F_B , where $F_i(x)$, $i \in \{A, B\}$, is the probability that the amount of resources allocated for military production by government i is less than x units.

¹⁸Notice that an all-pay auction is a specific case of this war game when $\lambda_A = \lambda_B = 1$.

Proposition 1.4.1. *When both countries are autocracies, there is a unique Nash equilibrium. In this equilibrium, governments' bidding strategies are:*

$$F_A(x) = s \cdot \frac{x}{v_A} \text{ for } x \in \left[0, \frac{v_A}{s}\right], \quad (1.4)$$

$$F_B(x) = 1 - \frac{1}{s} + \frac{1}{s} \cdot \frac{x}{v_B} \text{ for } x \in [0, v_B]. \quad (1.5)$$

War does not produce anything of use to citizens other than its determination of who will acquire the spoils of victory, and is costly both in terms of military production and losses. Hence, war always causes inefficiency. Governments have the option of choosing pure strategies and avoiding uncertainty, which causes war. However, Proposition 1.4.1 shows us that autocratic governments, acting in their own self-interest, choose mixed strategies, create uncertainty about their military powers, and cause war. In the unique equilibrium, government A 's effective bid has first-order stochastic dominance over government B 's effective bid. As a result of this, government A , which values the victory more than its opponent, bids more aggressively, produces more military goods, and hence is more likely to win the war.¹⁹

¹⁹In the equilibrium, the probabilities of victory for the countries are:

$$\begin{aligned} P(\text{country A wins}) &= \int_0^{\frac{v_A}{s}} F_B\left(\frac{\lambda_A x}{\lambda_B}\right) f_A(x) dx \\ &= \int_0^{\frac{v_A}{s}} \left(1 + \frac{1}{s} \frac{\frac{\lambda_A x}{\lambda_B} - v_B}{v_B}\right) \frac{s}{v_A} dx \\ &= \frac{2s-1}{2s}, \text{ and} \end{aligned} \quad (1.6)$$

$$P(\text{country B wins}) = 1 - P(\text{country A wins}) = \frac{1}{2s}. \quad (1.7)$$

Since $s > \frac{1}{2}$, it follows that $P(\text{country A wins}) > P(\text{country B wins})$. On the other hand, we get that country A has greater military power by comparing (A.17) and (A.18).

Second, we look at wars between democracies, which, unlike their autocratic counterparts, bear all the burdens of war. As in the case of autocratic governments, the equilibrium requires mixed strategies, and we denote governments' bidding strategies by the distribution functions of the resources they allocate for military production, i.e., F_A and F_B .

Proposition 1.4.2. *When both countries are democracies, there is a unique Nash equilibrium. In this equilibrium, governments' bidding strategies are:*

$$F_A(x) = 2 - 2 \cdot e^{-s \cdot \frac{x}{v_A}} \text{ for } x \in \left[0, \frac{v_A \ln 2}{s}\right] \quad (1.8)$$

$$F_B(x) = 2 - 2^{\frac{1}{s}} \cdot e^{-\frac{1}{s} \cdot \frac{x}{v_B}} \text{ for } x \in [0, v_B \ln 2]. \quad (1.9)$$

Proposition 1.4.2 shows that when there is a conflict between two democracies which have complete information, both democratic governments, just like their autocratic counterparts, choose mixed strategies for military spending, create uncertainty about their military powers, and cause war.

From Proposition 1.4.1 and Proposition 1.4.2, we see that our model offers an individual-level explanation of wars in that the informational asymmetry created endogenously by governments causes war. Moreover, when both involved countries have the same regime, even the aggressive government chooses to lose the war with a positive probability²⁰, since its mixed bidding strategy results in a higher payoff due to its bearing a lower cost of military production and lower losses during the

²⁰The aggressive government bids less than the unaggressive government and loses the war with a positive probability.

war when compared with other bidding strategies that lead to a sure victory. On the other hand, in equilibrium, the highest effective bids of both governments are equal because their payoffs decrease in the resources allocated to warfare, and therefore they never bid more than the amount necessary for a sure victory. However, the aggressive government's effective bid has first-order stochastic dominance over the unaggressive government's effective bid, and it is more likely to win the war and to acquire the spoils of victory due to its aggressive bidding strategy.²¹

In the standard models, such as Garfinkel and Skaperdas [44], both countries win the war with equal probabilities regardless of their initial resource distribution. In our model, as in standard models, the initial resource distribution has no effect on the probabilities of victory. However, unlike standard models, in our model unequal probabilities of victory are plausible due to the following. In the standard models, the winner claims all of the resources of both countries, which have the same value for both governments, whereas in our model, the winner claims the spoils of victory, which may have different values for governments in terms of the military powers that can be produced using them. If both governments can produce the same amount of

²¹In the equilibrium, the probabilities of victory for the countries are: The probabilities of victory for the countries in the equilibrium are:

$$\begin{aligned} P(\text{country A wins}) &= \int_0^{\frac{v_A \ln 2}{s}} F_B \left(\frac{\lambda_A x}{\lambda_B} \right) f_A(x) dx \\ &= \int_0^{\frac{v_A \ln 2}{s}} \left(2 - e^{\frac{v_B \ln 2 - \frac{\lambda_A x}{\lambda_B}}{s v_B}} \right) \frac{2s}{v_A} e^{-\frac{s x}{v_A}} dx \\ &= 2 + \frac{s}{s+1} \left(1 - 2^{\frac{s+1}{s}} \right), \text{ and} \end{aligned} \tag{1.10}$$

$$P(\text{country B wins}) = \frac{s}{s+1} \left(2^{\frac{s+1}{s}} - 1 \right). \tag{1.11}$$

It can be easily shown that $P(\text{country A wins}) > \frac{1}{2} > P(\text{country B wins})$.

military goods using the spoils of victory, then our model will lead to the same result as the standard models, and both countries will win the war with equal probabilities.

Next, we compare the wars between two democracies with the wars between two autocracies by keeping all parameters fixed in Proposition 1.4.1 and Proposition 1.4.2. As a result of this comparison, we obtain the following corollary which shows that our model offers a societal-level explanation of wars.

Corollary 1.4.1. *In a world of democracies, countries have smaller military powers, wars are less severe, and both representative citizens are better off than they would be in a world of autocracies.*²²

Democratic governments, unlike autocratic governments, suffer all the burdens of the war just as their citizens do. As a result of this, they are expected to invest less in military production and build smaller armies than their autocratic counterparts. Wars between smaller armies will be less severe. Therefore, if all parameters other than regime types are the same, a war between democracies is expected to be less severe than a war between autocracies. Finally, all representative citizens are expected to be better off in a world of democracies than they would be in a world of autocracies because of the following. Consider a change in countries' regimes from autocracy to democracy. Then, the aggressive government's probability of victory decreases. However, the expected sizes of armies and wars also decrease as a result of this regime change. The benefit to representative citizen of the aggressive government from building smaller armies and fighting less severe wars exceeds the cost

²²We compare the expected values of the corresponding parameters, because governments choose mixed strategies.

of losing the spoils of victory with a higher probability, and he is better off.²³ The unaggressive government's representative citizen is also better off because after the regime change his country wins the war with a higher probability, which adds further to the benefit received from building smaller armies and fighting less severe wars.

Corollary 1.4.1 is about the effects of a regime change from democracy to autocracy. In the following, we keep countries' regimes fixed - democracy or autocracy - and determine what happens with an increase in aggression inequality due to a change in the military technology of one of the countries. By doing so, we show that our model also offers a systemic-level explanation of wars.

Corollary 1.4.2. *Assume both countries have the same regime. Then, if there is an advancement in country A 's military technology or a decline in country B 's military technology, both countries will have smaller armies, the war will be less severe, and both representative citizens will be better off. Moreover, wars will completely disappear when country A has absolute superiority in military technology, i.e., $\lambda_A/\lambda_B \rightarrow \infty$.*

When aggression inequality increases due to an advancement in country A 's military technology or a decline in country B 's military technology, government B realizes that it is unlikely to be victorious and invests less in warfare. In response to this, government A also decreases its investment in warfare. Therefore, both countries will have smaller military powers, and the war will be less severe. This result is in line with our interpretation of the power-shift approach, since a step towards aggression inequality is also a step towards a more peaceful world. Moreover, this change in

²³The democratic governments care about their citizens more than their autocratic counterparts and this sometimes requires them to have a higher probability of losing the war.

military technology is in favor of government A , and government A uses this advantage to increase its probability of victory. The representative citizen of the aggressive government benefits from this increase in aggression inequality, because it increases his country's probability of victory while decreasing the costs of military production and war. When we look at the representative citizen of the unaggressive government, he is hurt by the decrease in the winning probability of his country. However, his benefit, due to the decrease in the costs of military production and war, exceeds his loss and he is weakly better off.

On the other hand, if country A 's military technology is overwhelmingly better than its opponent's, government B will choose to produce almost no military goods since it will have almost no hope to win the war. As a result of this, government A will win almost surely by investing almost no resources in warfare and war will almost disappear. This result supports our interpretation of the power-shift approach, since aggression inequality leads to peace. It also offers an explanation for the validity of the classical interpretation of power-shift approach. Country A 's absolute superiority in military technology leads to absolute power inequality, where country i 's power is measured by the size of its army or by the magnitude of military power it can produce using its resources; and this power inequality induced by the absolute superiority of one of the countries in military technology paves the way for absolute peace.

The countries' victory values, v_A and v_B , have been common knowledge in this section. In the following section, we investigate the war game in which these parameters are private information.

1.5 Incomplete Information

The countries' victory values, v_A and v_B , are independently and randomly drawn from the intervals $[0, \bar{v}_A]$ and $[0, \bar{v}_B]$, respectively. The realizations of v_A and v_B are private information, whereas their distributions are common knowledge. Country $i \in \{A, B\}$ can produce up to $\lambda_i \bar{v}_i$ units of military goods by using the spoils of victory. As in the previous section, we call $\lambda_i \bar{v}_i$ *government i 's level of aggression*, and we say that government i *becomes more aggressive* when $\lambda_i \bar{v}_i$ increases, and it *becomes less aggressive* when $\lambda_i \bar{v}_i$ decreases. Without loss of generality, we assume government A is weakly more aggressive than government B . i.e., $\lambda_A \bar{v}_A \geq \lambda_B \bar{v}_B$. We call government A and government B *the aggressive government* and *the unaggressive government*, respectively. We denote the ratio of government A 's level of aggression over government B 's level of aggression with:

$$\bar{s} = \frac{\lambda_A \bar{v}_A}{\lambda_B \bar{v}_B} \quad (1.12)$$

and we call it *aggression inequality*.

First, we investigate the war game between two autocracies. Governments have incomplete information about each other's victory values, hence it is a Bayesian game. Following Hoder and Yektas [52], we look for Bayesian Nash equilibria in monotone continuous strategies that are differentiable with non-zero derivatives everywhere except at the boundary points.

Proposition 1.5.1. *When both countries are autocracies, there is a unique Bayesian Nash equilibrium. In this equilibrium, governments' bidding strategies are:*

$$b_A(v_A) = \frac{1}{\bar{s} + 1} \cdot \left(\frac{v_A}{\bar{v}_A} \right)^{1/\bar{s}} \cdot v_A \text{ for } v_A \in [0, \bar{v}_A], \text{ and} \quad (1.13)$$

$$b_B(v_B) = \frac{\bar{s}}{\bar{s} + 1} \cdot \left(\frac{v_B}{\bar{v}_B} \right)^{\bar{s}} \cdot v_B \text{ for } v_B \in [0, \bar{v}_B]. \quad (1.14)$$

Proposition 1.5.1 shows that when autocratic governments have incomplete information, unlike the complete information setup, they choose pure strategies in equilibria and do not create uncertainty. However, the exogenous uncertainty, that is, governments' uncertainty about each other's victory values, causes war. We get results parallel to those of Proposition 1.4.1, that is, government A , which values the victory more than its opponent, bids more aggressively, produces more military goods, and as a result of this it is more likely to win the war.²⁴

On the other hand, the size of the war does not necessarily decrease with an increase in aggression inequality. However, as stated in the following corollary, if

²⁴Country A wins the war if and only if:

$$\lambda_A \frac{1}{\bar{s} + 1} \left(\frac{v_A}{\bar{v}_A} \right)^{\frac{\bar{s}+1}{\bar{s}}} \bar{v}_A > \lambda_B \frac{\bar{s}}{\bar{s} + 1} \left(\frac{v_B}{\bar{v}_B} \right)^{\bar{s}+1} \bar{v}_B \Leftrightarrow \frac{v_B}{\bar{v}_B} < \left(\frac{v_A}{\bar{v}_A} \right)^{1/\bar{s}}.$$

Since v_A and v_B are independent and uniformly distributed on $[0, \bar{v}_A]$ and $[0, \bar{v}_B]$, the probabilities of victory for the countries in the equilibrium are:

$$P(\text{country A wins}) = \int_0^{\bar{v}_A} \left(\frac{v_A}{\bar{v}_A} \right)^{1/\bar{s}} \frac{1}{\bar{v}_A} dv_A = \frac{\bar{s}}{\bar{s} + 1}, \text{ and} \quad (1.15)$$

$$P(\text{country B wins}) = \frac{1}{\bar{s} + 1}. \quad (1.16)$$

It is obvious that $P(\text{country A wins}) > P(\text{country B wins})$. On the other hand, we get that country A has greater military power by comparing A.47 with A.46.

country A 's military technology is overwhelmingly better than country B 's military technology, wars will disappear.

Corollary 1.5.1. *Assume both countries are autocracies. Then, wars will completely disappear when country A has absolute superiority in military technology, i.e., $\lambda_A/\lambda_B \rightarrow \infty$.*

When country A 's military technology is overwhelmingly better than country B 's, government B chooses to spend almost no resources on military production since it will almost surely lose the war. In response to this, government A also chooses to invest almost no resources in warfare, however wins almost surely due to its superiority in military technology. As a result of the lack of investment in warfare, wars will disappear.²⁵ In other words, power inequality as well as aggression inequality, induced by one of the two countries' absolute superiority in military technology, paves the way for peace. Hence, our model offers a theoretical background to the validity of the power-shift approach independent of whether governments have complete or incomplete information.

Next, we look at the war game between two democracies. In this case, a general closed form solution cannot be found. Hence, we study a specific case in which both countries have the same victory value and the same military technology, i.e., $\bar{v}_A = \bar{v}_B$ and $\lambda_A = \lambda_B$. In this Bayesian game, as in the previous section, we look for the

²⁵On the other hand, the disappearance of wars due to country A 's absolute superiority in military technology makes citizen A best off, more so than any other possible state of military technologies, however the same cannot be said for citizen B , who is better off when both countries have the same levels of aggression - and will be even better off if his country has absolute superiority in military technology -.

Bayesian Nash equilibria in monotone continuous strategies that are differentiable with non-zero derivatives everywhere except at the boundary points.

Proposition 1.5.2. *Assume both countries are democracies, $\bar{v}_A = \bar{v}_B = \bar{v}$, and $\lambda_A = \lambda_B$. Then, there is a unique Bayesian Nash equilibrium. In this equilibrium, governments' bidding strategies are:*

$$b(v) = 2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v} - v} \right) - v \text{ for } v \in [0, \bar{v}]. \quad (1.17)$$

Proposition 1.5.2 shows that when democratic governments have incomplete information, they choose pure strategies and do not create uncertainty. Hence, the war is caused only by the exogenous uncertainty of governments about each other's victory values. Moreover, governments' bids increase in their victory values, i.e., $b'(v) > 0$, because they will be willing to invest more for a higher expected return.

Using Proposition 1.5.1 and Proposition 1.5.2, we compare the wars between two democracies with the wars between two autocracies under the incomplete information setup, and obtain the following corollary.

Corollary 1.5.2. *Assume $\bar{v}_A = \bar{v}_B$ and $\lambda_A = \lambda_B$. Then, the countries have smaller military powers, wars are less severe, and both representative citizens are better off in a world of democracies than in a world of autocracies.*

This corollary shows that our model's societal-level explanation of wars, which states that a regime change from democracy to autocracy aggravates wars, is valid for both complete and incomplete information setups. In line with the democratic

peace theory, democratic governments invest less in military production, build smaller armies and fight less severe wars than their autocratic counterparts. Moreover, both representative citizens are better off in a world of democracies than they would be in a world of autocracies, due to the decreases in the costs of armies and wars.

1.6 Conclusion

We have presented a game-theoretic model in which information asymmetry is the source of uncertain outcomes in wars. We have characterized the unique equilibria in the wars between two democracies and between two autocracies for the complete and incomplete information cases. We have obtained three significant results which offer individual-level, societal-level and systemic-level analyses for the causes of war, respectively. First, governments' informational asymmetries, which exist exogenously in the incomplete information case and are created endogenously by governments' mixed strategies in the complete information case, pave the way for wars. Second, in line with the democratic peace theory, a world of democracies is more peaceful than a world of autocracies in terms of countries having smaller military powers and fighting less severe wars, and all citizens are better off in the democratic world. Third, in line with the power-shift approach, if one of the countries' military technology is overwhelmingly better than its opponent's military technology, then investments in warfare and wars will almost disappear. Moreover, when countries have complete information, even a small shift from aggression equality towards aggression inequality will produce a more peaceful world.

Chapter 2

Polarization and Duverger's Law with Altruistic Voting

2.1 Introduction

In recent years, motivated by the various empirical and experimental findings that support the idea of other-regarding voters, several scholars have proposed models that incorporate altruistic voters who are concerned with the welfare of others.¹ These models have generally been used to explain voter behavior, especially the substantial turnout rates that occur even in large elections.² Altruistic voters vote solely for the purpose of affecting the election's outcome, as in the standard rational-choice models of turnout, or they vote as an act of expressive behavior or ethical behavior, as in the

¹For the empirical and experimental findings see Feddersen, Gailmard and Sandroni [36], Fowler [41], Fowler and Kam [42], Kinder and Kiewiet [61], and Markus [68].

²See Andreoni [5], Coate and Conlin [21], Evren [31], and Feddersen and Sandroni [34, 35].

expressive voting models.³ Evren’s [31] game-theoretic model of costly voting is of the first type. In his model, an altruistic agent compares his private voting cost with the expected contribution of his vote to the welfare of society when deciding whether to vote or not, and significant turnout rates are observed even when the electorate is arbitrarily large. Feddersen and Sandroni [34, 35] is of the second type. They predict significant turnout rates with a model in which only altruistic voters consider casting a vote and receive payoff for acting according to the strategy that is best for society. Both Evren [31] and Feddersen and Sandroni [34, 35] have two key features that jointly drive the results: *(i)* some voters are altruistic, and *(ii)* the fraction of altruistic voters among the voters who prefer any given candidate is uncertain, and this uncertainty fosters a higher turnout among altruistic voters.

To the best of our knowledge, Glazer [46] is the only paper on altruistic voting in which candidates are strategic actors. He investigates the influence of altruistic voters on strategic candidates’ behavior, and shows that altruism is consistent with high turnout and with candidates’ adoption of divergent positions when candidates are plurality-motivated and voters’ negative altruism exceeds their positive altruism.⁴ According to Glazer [46], voters identify themselves as supporters of the candidate, and their expressive benefit of voting varies with the fraction of the population who prefer the same candidate and the fraction of the population who dislike the candidate. The behavior of altruistic voters in Glazer [46] is in line with Rabin’s [86]

³For the first type see Edlin, Gelman and Kaplan [29], and Evren [31]; for the second type see Andreoni [5], Coate and Conlin [21], Feddersen and Sandroni [34, 35], and Glazer [46].

⁴Different factors, such as policy motivation (Osborne and Slivinski [77]; Besley and Coate [12]), uncertainty (Berger, Munger and Potthoff, [11]; Roemer [88]) and non-policy considerations (Adams [2]) have been considered in the literature to explain the policy divergence that occurs when candidates have office motivation.

idea that people like to help those who are helping them and to hurt those who are hurting them. In most economic models, agents are motivated solely by self-interest. In contrast, in some models people pursue simple altruistic behaviors, i.e., they care not only about their own well-being but also about the well-being of others. However, altruistic behavior is often complex, and people who are altruistic to other altruists may be spiteful towards those who hurt them.⁵ Rabin calls outcomes reflecting such motivations “fairness equilibria,” and incorporates fairness into economics instead of self-interest and simple altruism. Hence, voters in Glazer [46] are fair in Rabin’s sense. There are studies that find empirical support for this type of fair voting. For instance, Kan and Yang [58] show that people turn out to cast their votes simply because they want to “cheer” or “boo” their favored or unfavored candidates. Using data from the 1988 American National Election Study, they find that the “cheering” and “booing” effects are statistically significant and have substantial influence on both turnout and voter choice. They also obtained evidence against the proposition that people turn out to vote because they consider themselves potentially decisive with regard to the election’s outcome.⁶

We follow Glazer [46] and examine the effects on the behavior of plurality-motivated and strategic candidates of voting for a certain candidate in order to please voters who prefer the same candidate and voting for a certain candidate in order to anger voters who dislike the candidate. Our model brings the research on altruism one step further by showing that this type of altruism is consistent with

⁵See Rabin [86] for the evidence from the psychological literature regarding these facts.

⁶See Ashworth, Geys and Heyndels [7] as well. They show that when voters like winners, turnout may rise with the winner’s expected plurality, and supports this prediction with data from Belgian municipal elections.

Duverger’s Law, which asserts that the plurality rule favors a two-party system, and by offering an explanation for the inverse relationship between turnout and polarization. In our model, as in Evren [31] and Feddersen and Sandroni [34], only altruistic voters receive payoff for voting and the fraction of altruistic voters among those who support any given candidate is uncertain; and like Glazer [46], a voter’s incentive to vote depends on the number of people expected to vote for his favorite candidate and the number expected to vote for the others. We incorporate candidates’ policy competition and Glazer [46]’s type of altruism to the framework of Evren [31] and Feddersen and Sandroni [34], and offer an explanation for several phenomena related to the candidates’ behavior.⁷ In this literature, Glazer [46] is the closest model to ours, however, unlike Glazer [46], in our model positive altruism is qualitatively similar to negative altruism in its influence on candidates’ policy choices. Moreover, Glazer [46] does not address Duverger’s Law and the inverse relation between turnout and polarization.

In the literature, the formal models of Duverger’s Law have generally concentrated on strategic voting, also known as tactical voting.⁸ Their main insight is that strategic voters may support candidates other than their true favorites in order to prevent undesirable outcomes, and such voting behavior deters the entry of the candidates who are likely to lose the election. Some recent models argue that Duverger’s Law can be explained by the actions of policy-motivated candidates without relying on strategic voting.⁹ The general idea in these models is as follows. When a candidate

⁷These phenomena include policy divergence, Duverger’s Law and the inverse relation between turnout and polarization.

⁸See Feddersen [32], Fey [37], Myerson and Weber [75], and Palfrey [82].

⁹See Besley and Coate [12], Fey [38], and Osborne and Slivinski [77].

drops out of the race, her potential supporters will vote for the candidates whose policy choices are closest to hers. Therefore, policy-motivated candidates who are likely to lose the election will drop out of the race in order to increase the probability of victory for the candidates whose policy choices are closest to their own. The extent of strategic voting and the presence of policy-motivated candidates in actual elections have been questioned by empirical researchers. They have shown that there are cases in which voters do not necessarily vote strategically and candidates are not necessarily motivated by policies, calling into question the explanations of Duverger’s Law relying on these.¹⁰ Callander and Wilson [20] provide a formal model without strategic voting and policy-motivated candidacy. They show that voter abstention due to alienation, which they measure with a parameter called “voter tolerance,” yields consistency with Duverger’s Law and policy divergence.¹¹ They argue that there is a connection between Duverger’s Law, polarization and abstention by establishing the existence of an equilibrium in which only two candidates enter and choose noncentrist policy positions to deter subsequent entry.

In our model, as in that of Callander and Wilson [20], we incorporate the threat of the entry of a third candidate and identify a mechanism that relates altruism to Duverger’s Law without strategic voting and policy-motivated candidacy. Our results differ from those in Callander and Wilson [20]. First, we obtain the closed form of the equilibria. Second, the relationship between policy divergence and voter turnout, which is not monotonic in their model, is monotonic in ours. In Callander

¹⁰See Alvarez and Nagle [4], Blais [16], Blais, Young and Turcotte [17], and Fredriksson, Wang and Mamun [43].

¹¹Thus voter abstention plays a more central role in elections than it does in earlier literature.

and Wilson [20], policy divergence increases in voter tolerance at lower values and decreases at higher values, while voter turnout strictly increases in voter tolerance. However, in our model, policy divergence decreases and voter turnout increases in the expected number of altruistic voters. Hence, the decrease in turnout¹² and increase in polarization¹³ observed in the U.S. after the 1960s is a result of the decrease in altruism in our model, whereas it is the result of the decrease in voter tolerance in Callander and Wilson [20].¹⁴ Finally, unlike Callander and Wilson [20], our model enables us to calibrate divergent two-candidate equilibria that covers all turnout levels observed in the U.S. electoral system.¹⁵ Our model has some technical differences as well. In Callander and Wilson [20], if successful entry is possible then dominant candidates will lose the election to the entrant. In our model, the entrant may have a small probability of victory. Moreover, in our model, the cost of voting is random rather than fixed, and candidates maximize their votes rather than their probability of victory.

The remainder of the paper is organized as follows. Section 2.2 lays out the basic model. The model is solved for positive and negative altruism in sections 2.3 and 2.4, respectively. Section 2.5 concludes. All proofs are provided in Appendix B.

¹²See Fiorina [39].

¹³See McCarty, Poole and Rosenthal [69], and Poole and Rosenthal [84].

¹⁴In contrast to this observation, there are also some empirical studies, such as Moser [74] and Powell [85], which show that polarized electorates have high turnout.

¹⁵According to the American Presidency Project at the University of California Santa Barbara, the turnout rate in the U.S. has never exceeded 81.8%, the rate that has occurred in the 1876 presidential election. All turnout levels, other than full turnout, can be achieved in our model, whereas the maximum level of turnout that can be achieved in Callander and Wilson [20] is approximately 70.7%.

2.2 The Model

We develop a simple model of electoral competition. There are three potential candidates: two dominant candidates, D_1 and D_2 , and a potential entrant, E .¹⁶ The dominant candidates are vote-motivated, that is, they try to maximize the number of votes they get. The potential entrant is office-motivated. She chooses to compete when her probability of victory is strictly positive, and she tries to maximize her probability of victory when she chooses to compete.¹⁷

There is a continuum of voters, and each voter has an *optimal policy*. For each voter $i \in [0, 1]$, i represents this voter's optimal policy. Voters prefer the candidate whose policy choice is closest to their own optimal policy. If the policy choices of two dominant candidates are the same distance from the optimal policy of a voter, then this voter is indifferent between them, and supports them with equal probabilities. However, if the policy choices of a dominant candidate and the potential entrant are the same distance from the optimal policy of a voter, then he prefers the dominant candidate to the potential entrant.¹⁸

¹⁶The three-candidate model, which features two dominant candidates and a potential entrant, and the sequential structure of the candidates' movements are borrowed from Palfrey [81] and Callander and Wilson [20]. The three-candidate structure of the model helps us to examine how the entry of an office-motivated third party is deterred by the two dominant parties in a tractable manner. Palfrey [81] generates equilibrium with policy divergence, but he does not address Duverger's Law. His model has also some critical differences: there is full turnout, the third candidate enters the election regardless of her chances of victory, and at least one third of the voters resides between the two dominant candidates, unlike ours, in which policy divergence varies from almost nonexistent to significant levels.

¹⁷The motivation of the potential entrant has no effect on our results, as long as she chooses not to compete when her probability of victory is zero.

¹⁸Therefore, a potential entrant will never pick a policy chosen by a dominant candidate.

There are two types of voters, *altruists* and *abstainers*.¹⁹ Abstainers receive zero payoff for voting, whereas altruists receive payoff by pleasing citizens who prefer the same candidate or angering citizens who dislike the candidate.²⁰ The payoff that an altruistic voter who prefers candidate j receives by voting for candidate j is $y(k_j, k_{-j})$, where k_j is the number of people who prefer j to the other candidates and k_{-j} is the number of people who prefer one of the other candidates. Voting is costly, and the cost of voting is uniformly distributed over the interval $[0, c]$, where $c > 0$. Hence, abstainers always abstain, whereas altruistic voters vote for their preferred candidate if the payoff they receive by voting is more than their cost of voting. A voter who supports candidate $j \in \{D_1, D_2, E\}$ is altruistic with probability q_j . The random variables q_{D_1} , q_{D_2} and q_E are independent and uniformly distributed over the interval $[q, 1]$, where $q \in [0, 1)$ is *the minimum fraction of altruistic voters among the supporters of any candidate*. The uncertainty about the fraction of altruists among the supporters of each candidate is the only source of uncertainty in our model.

The order of strategic play in the election is as follows. The dominant candidates move first, and simultaneously select their policy positions from the policy space $[0, 1]$. The potential entrant responds to the dominant candidates, chooses whether to compete, and if so, chooses a policy position. Then, nature determines the fractions of altruistic voters among the supporters of each candidate, that is, for each $i \in \{D_1, D_2, E\}$ q_i is chosen randomly from $[q, 1]$, and q_i fraction of voters supporting candidate i are randomly chosen as altruistic voters. Finally, the election takes

¹⁹Evren [31] and Feddersen and Sandroni [34] have a similar voter structure. Evren [31] uses the same names for these two types of voters, whereas Feddersen and Sandroni [34] refer to them as “ethicals” and “abstainers”.

²⁰Hence, altruists also receive a payoff by pleasing or angering the abstainers.

place among the candidates who chose to compete, and the winner is determined by plurality rule.

The equilibrium concept is subgame perfect Nash. In the following, equilibrium refers to subgame perfect Nash equilibrium. Voters are not strategic actors, therefore we describe the equilibrium in terms of candidates' strategies. Candidates choose their policy platforms from the set $\{[0, 1], \emptyset\}$, where $\emptyset$ represents the choice not to compete; and we denote the policy platforms of candidates D_1 , D_2 and E with d_1 , d_2 and e , respectively. We say that an equilibrium is a *two-candidate equilibrium* if only the two dominant candidates enter the election and the potential entrant is deterred.²¹ Moreover, a two-candidate equilibrium is *divergent* if $d_1 \neq d_2$, and *symmetric* if $d_1 = 1 - d_2$. The candidates have no ideological motivation. Hence, without loss of generality, we assume $d_1 \leq d_2$, that is, D_1 is a left-wing candidate and D_2 is a right-wing candidate.

2.3 Positive Altruism

An altruistic voter is said to have *positive altruism* if he votes to please others who prefer the same candidate, and the payoff he receives by voting for his favorite candidate is given by:

$$y(k_j, k_{-j}) = Y k_j^n, \quad (2.1)$$

where $Y \in \mathbb{R}^+$ is *the maximum benefit a voter can get for voting* and $n \in \mathbb{R}$ is the *power of altruism*. In real world elections, there is always abstention among all types

²¹This definition is borrowed from Callander and Wilson [20].

of voters.²² Hence, in order to ensure abstention among altruistic voters in all cases, we assume $Y \leq c$.²³ On the other hand, an altruistic voter gains a higher utility by pleasing more citizens. Therefore, we assume that the power of altruism is positive, i.e., $n > 0$.

When the minimum fraction of altruistic voters, q , is very low, dominant candidates may get too few votes, and the potential entrant can win the election even with a small group of supporters. Therefore, for low values of q , no two-candidate equilibrium exists.²⁴ The following proposition shows that this observation determines the necessary and sufficient conditions for a stable two-candidate system under the plurality rule.

Proposition 2.3.1. *When voters have positive altruism, there exists $\underline{q} \in (0, 1)$ such that a two-candidate equilibrium exists if and only if $q \geq \underline{q}$.²⁵ Moreover, the equilibrium is unique and the policy platforms of the candidates are $(d_1, d_2, e) = \left(\frac{q^{\frac{1}{n+1}}}{2}, 1 - \frac{q^{\frac{1}{n+1}}}{2}, \emptyset\right)$.*

Proposition 2.3.1 shows the existence of a unique equilibrium simultaneously consistent with the dual empirical regularities of Duverger's Law and polarization. We see that, as long as the fraction of altruistic voters is high, the dominant candidates deter the entry of the potential entrant by choosing divergent policies that are neither

²²For instance, the leader of the opposition party in Turkey did not vote in the referendum on 12 September, 2010.

²³ $Y \leq 2^n c$ would have been enough for our results.

²⁴Moreover, when all voters are altruistic, i.e., $q = 1$, there exists no two-candidate equilibrium. Because, if $q = 1$, then the dominant candidates will tend to choose the median policy to maximize the number of votes they get, however, in that case the entrant will enter the election since she will win with a positive probability by locating on one of the flanks.

²⁵In the proof of Proposition 2.3.1, it is shown that $\underline{q} = \left(\frac{\sqrt{17}-3}{2}\right)^{n+1}$.

too extremist nor too centrist.²⁶ To understand how the equilibrium works, we have to revisit our two important assumptions, the threat of a third candidate's entry and the randomness of the fraction of altruists among the supporters of each candidate. Because altruism is random, the candidates do not know how many votes they will get. When q is high, it is possible for the dominant candidates to deter the potential entrant by choosing divergent policies that leave small policy intervals on the flanks and between themselves. Since q is high, in such a situation the potential entrant cannot win more votes than either of the dominant candidates.²⁷ However, when q is low, the dominant candidates cannot deter E 's entry with such policies, because it is possible that both of the dominant candidates will receive too few votes, and the potential entrant can win even with a small group of supporters.

In the equilibrium, the policy choices of the candidates are symmetric, because if the potential entrant can be deterred with a non-symmetric policy pair, then the dominant candidate who is further away from the median voter's optimal policy will still be deterring the potential entrant by locating in the position symmetrical to that of the other dominant candidate, and she will deviate because of her vote motivation.

Finally, the two-candidate equilibrium is unique when it exists, because vote-motivated candidates will choose policies as close as possible to each other - to increase the expected number of votes they get -, and this incentive makes them choose the

²⁶Notice that $d_1 \in \left[\frac{q^{\frac{1}{n+1}}}{2}, \frac{1}{2}\right)$ and $d_2 \in \left(\frac{1}{2}, 1 - \frac{q^{\frac{1}{n+1}}}{2}\right]$ where $\frac{q^{\frac{1}{n+1}}}{2} = \frac{\sqrt{17}-3}{4}$.

²⁷However, if one of the dominant candidates converges towards the median voter, the potential entrant can win more votes than either of the dominant candidates by choosing a policy on the larger flank. This is because when she locates on that flank, there will be lots of voters who will support her, and the altruists among the supporters of the dominant candidates may be fewer than the altruists among her supporters.

symmetric entry-deterring policy pair that has the minimum polarization. Hence, it turns out that the existence of a symmetric entry-deterring policy pair for dominant candidates is the necessary and sufficient condition for the existence of two-candidate equilibria, and $\underline{q}$ is the critical value of the minimum fraction of altruistic voters above which dominant candidates have a symmetric entry-deterring policy pair. However, our model's consistency with Duverger's Law and polarization is not strictly bound to the magnitude of the fraction of altruistic voters. We can calibrate divergent two-candidate equilibria for any $q \in (0, 1)$. Because, given any $q \in (0, 1)$, there exists a power of altruism, n , for which a divergent two-candidate equilibrium exists.²⁸

The expected voter turnout in the two-candidate equilibrium is:

$$E \left[q_{D_1} k_{D_1} \frac{y(k_{D_1}, k_{D_2})}{c} + q_{D_2} k_{D_2} \frac{y(k_{D_2}, k_{D_1})}{c} \right] = \frac{(q+1)Y}{2^{n+1}c} \quad (2.2)$$

which increases in q . The intuition is clear: if there are more altruistic voters, there will be more voters willing to cast a vote, and the turnout will be higher. The difference between candidates' policies is:

$$d_2 - d_1 = 1 - q^{\frac{1}{n+1}} \quad (2.3)$$

which increases with decreasing q , that is, when the voters are expected to be less altruistic, the candidates will choose policies further from the median voter and from each other. To understand this, let us consider a decrease in q . Then, the dominant

²⁸It follows from $\lim_{n \rightarrow \infty} \left(\frac{\sqrt{17}-3}{2} \right)^{n+1} = 0$.

candidates are likely to get fewer votes, and E becomes more likely to enter because of a possible victory with the support of a smaller voter group. Therefore, the dominant candidates will choose policies further from each other in order to leave smaller voter groups on the flanks and deter E 's entry. Combining these two observations, we conclude that expected voter turnout is strictly increasing in the expected number of altruistic voters, while polarization is strictly decreasing in it.

Corollary 2.3.1. *When the expected fraction of altruistic voters changes, expected voter turnout and polarization move in opposite directions.*

As this corollary shows, our model relates turnout to polarization through the expected number of altruistic voters, and allows us to establish the inverse relation between the two. According to our model, the reason for the decrease in turnout and the increase in polarization observed in the United States after the 1960s is the decrease in the number of altruistic voters.

Callander and Wilson [20] offer an explanation for the same observation using a model in which abstention is due to the voters' alienation from the candidates' policy choices. They use the range of tolerance for each voter, which represents how close a candidate must be to a voter to induce the voter to turn out, to measure alienation. They show that turnout and policy divergence move in the same direction for small values of voter tolerance and in opposite directions for large values of voter tolerance. According to their model, the decrease in turnout and the increase in polarization experienced in the U.S. is because of a decrease in voter tolerance, that is, nowadays candidates have to be closer to their supporters in order to induce them to turn out.

Our model differs from Callander and Wilson [20]’s in that it allows us to calibrate all turnout levels, other than full turnout, in the divergent two-candidate equilibria.²⁹

Finally, in our model, the total expected welfare of voters is:

$$E \left[\sum_{i \in \{1,2\}} q_{D_i} k_{D_i} (y(k_{D_i}, k_{D_j}) - \text{voting cost}) \right] = \frac{(q+1)Y}{2^{n+2}}, \quad (2.4)$$

from which we conclude that society is better off when the percentage of altruists increases.³⁰ We know that policy divergence decreases in q . Therefore, the total welfare of the society increases with policy platforms converging to the median voter’s optimal policy.³¹

Up to now, we have examined voters who have positive altruism, and have established the existence of a unique two-candidate equilibrium under the plurality rule in which the candidates choose divergent policy positions. Next, we investigate negative altruism.

2.4 Negative Altruism

An altruistic voter is said to have *negative altruism* if he votes to anger voters who prefer other candidates, and the payoff he receives by voting for the candidate he supports is given by:

$$y(k_i, k_{-i}) = Nk_{-i} \quad (2.5)$$

²⁹From (2.2), $\lim_{q \rightarrow 1, Y \rightarrow c, n \rightarrow 0} E[q_{D_1} k_{D_1} y(k_{D_1}, k_{D_2}) + q_{D_2} k_{D_2} y(k_{D_2}, k_{D_1})] = 1$ and $\lim_{c \rightarrow \infty} E[q_{D_1} k_{D_1} y(k_{D_1}, k_{D_2}) + q_{D_2} k_{D_2} y(k_{D_2}, k_{D_1})] = 0$.

³⁰Moreover, it can be easily shown that none of the voters get worse off when q increases.

³¹Conversely, in Van Weelden [105], total voter welfare is expected to be higher with non-converging platforms.

where N is *the maximum benefit a voter can get for voting*. In order to ensure abstention among altruistic voters, we assume that $N \leq c$.

As in the previous section, for low values of q , the dominant candidates may get too few votes, and the potential entrant may win the election even with a small group of supporters. Therefore, no two-candidate equilibrium exists for low values of q . As this observation indicates, large q turns out to be necessary and sufficient for the existence of divergent two-candidate equilibria.

Proposition 2.4.1. *When voters have negative altruism, there exists $\underline{q} \in (0, 1)$ such that a two-candidate equilibrium exists if and only if $q \geq \underline{q}$.³² Moreover, the policy platforms of candidates in the two-candidate equilibria are:*

$$\left\{ (d_1, d_2, e) : d_1 \in \left[\frac{4 - 3q}{8 - 2q}, \frac{1 - \sqrt{1 - q}}{2} \right], d_2 = 1 - d_1, e = \emptyset \right\}.$$

Proposition 2.4.1 shows that some of the effects of negative altruism are qualitatively the same as those of positive altruism. There exist two-candidate equilibria as long as the fraction of altruistic voters in the society is high,³³ and candidates choose symmetric and divergent policies; however, unlike the positive altruism case, there are infinitely many equilibria. The dynamics of each equilibrium are similar to those in the positive altruism case. The dominant candidates want to prevent the entry of the potential entrant on the flanks or between them, and they can do this by choosing divergent policies that are neither too extremist nor too centrist, for high

³²In the proof of proposition 2.4.1, it is shown that $\underline{q}$ is equal to the unique real root of the equation $q^3 - 5q^2 + 24q - 16 = 0$ which is approximately 0.772.

³³There exist two-candidate equilibria for $q = 1$ as well, and the policy platforms of candidates in these two-candidate equilibria are $\{(d_1, d_2, e) : d_1 \in (\frac{1}{6}, \frac{1}{2}), d_2 = 1 - d_1, e = \emptyset\}$.

values of q . However, for low values of q , they cannot deter the entry of the potential entrant, since she can win the election even with a small group of supporters due to the possibility of dominant candidates receiving too few votes. The policy choices of the dominant candidates are symmetric due to the following.³⁴ Vote-motivated candidates have two aims: they want voters to support them and to vote aggressively for them. The first aim makes them increase their own voter support, whereas the second aim makes them increase their opponent's voter support - so that their supporters will vote aggressively in order to anger all those voters who support the opponent -. The trade-off between these two aims makes dominant candidates choose symmetric policies. Moreover, there exist multiple equilibria because the dominant candidates are satisfied with any symmetric policy pair that prevents E 's entry, and unlike the positive altruism case, they do not want to increase their voter support to more than half of the potential voters. Finally, as in the previous section, $\underline{q}$ is the critical value of the minimum fraction of altruistic voters above which a symmetric entry-detering policy pair exists for the dominant candidates, because the existence of a symmetric entry-detering policy pair for the dominant candidates is necessary and sufficient for the existence of two-candidate equilibria.

The expected voter turnout in all two-candidate equilibria is the same, and is equal to:

$$E[q_{D_1}k_{D_1}y(k_{D_1}, k_{D_2}) + q_{D_2}k_{D_2}y(k_{D_2}, k_{D_1})] = \frac{(q+1)N}{4c} \quad (2.6)$$

³⁴The number of votes candidate i will get is $q_i k_i y(k_i, k_{-i}) = N q_i k_i k_{-i}$.

and, as in the positive altruism case, turnout increases in q because when there are more altruistic voters, there will be more voters casting their votes. The differences between candidates' policies in these two-candidate equilibria are found within the following interval:

$$d_2 - d_1 \in \left[\sqrt{1 - q}, \frac{4q}{8 - 2q} \right] \quad (2.7)$$

the lower bound of which decreases in q while the upper bound increases in q . To understand the dynamics, let us consider a decrease in q . The dominant candidates are happy with any symmetric policy-pair that prevents E 's entry. However, when the voters are expected to be less altruistic, they are likely to get fewer votes and the potential entrant becomes more likely to enter. Therefore, the dominant candidates have to leave smaller voter groups on the flanks and between each other. In order to leave smaller voter groups on the flanks, they tend to choose policies further from each other, as a result of which the lower bound of $d_2 - d_1$ increases. On the other hand, in order to leave a smaller voter group between each other, they tend to choose policies closer to each other, as a result of which the upper bound of $d_2 - d_1$ decreases. Thus, the interval of polarization shrinks with decreasing q , or inversely expands with increasing q . When the number of altruistic voters is expected to be high, candidates can choose policies closer to those preferred by the median voter but, unlike the positive altruism case, they do not have to. In other words, with a change in the expected fraction of altruistic voters, expected voter turnout and polarization do not necessarily move in opposite directions, but they can.

The total welfare of voters in all two-candidate equilibria are the same and are expected to be equal to:

$$E\left[\sum_{i \in \{1,2\}} q_{D_i} k_{D_i} (y(k_{D_i}, k_{D_j}) - \text{voting cost})\right] = \frac{(q+1)N}{8}. \quad (2.8)$$

Society is better off when the percentage of altruists increases. Hence, the increase in expected turnout increases the welfare of the society, even though it causes an increase in the total cost of voting.

Glazer [46] shows that candidates' policy choices diverge if and only if negative altruism exceeds positive altruism, hence negative altruism causes polarization. In our model, as shown in Proposition 2.3.1 and Proposition 2.4.1, both types of altruism cause polarization and have similar comparative statics. However, recalling (2.3) and (2.7), we see that negative altruism causes more polarization than positive altruism for all $n > 0$, $Y \leq c$ and $N \leq c$, which is in line with Glazer [46].³⁵ To put it differently, candidates tend towards moderate policies when voters want to please other voters supporting the same candidate, whereas they tend towards extreme policies when voters want to anger others who dislike the candidate they support. The intuition can be understood as follows. Let us consider a candidate with low voter support. Her negative altruist supporters will be more eager to vote relative to her positive altruist supporters, because they will anger more voters than they will please with

³⁵Given $q \in [\underline{q}, 1)$ and $n > 0$, the polarization in any equilibrium under negative altruism is higher than the polarization in the unique equilibrium under positive altruism, which can be seen from the following calculations: for $q < 1$, $q^{1/n+1} > q \Rightarrow 1 - q > 1 - q^{1/n+1} \Rightarrow \sqrt{1 - q} > \sqrt{1 - q^{1/n+1}}$; moreover, noticing that $0 < \sqrt{1 - q^{1/n+1}} < 1$ and multiplying the right hand side of the final inequality with $\sqrt{1 - q^{1/n+1}}$, we obtain $\sqrt{1 - q} > 1 - q^{1/n+1}$.

their votes, and she will be more likely to win with negative altruist supporters. For this reason, when voters have negative altruism, the dominant candidates have to leave smaller intervals on the flanks in order to deter the potential entrant.

Finally, a two-candidate equilibrium consistent with Duverger's Law and polarization exists for a larger set of parameters under positive altruism than under negative altruism, i.e., $\underline{q} > \underline{q}$ for all $n > 0$, $Y \leq c$ and $N \leq c$.³⁶ The intuition follows the same line of reasoning as above, which shows that it is harder to deter the potential entrant when voters have negative altruism. Thus, we conclude that negative altruism causes more polarization but for a smaller set of parameters, and unlike Glazer [46], negative altruism is not the main reason for polarization. Rather, it is mainly the threat of other potential candidates' entry that causes polarization.

2.5 Conclusion

In this paper, we focus on altruistic voting. Assuming that the number of altruistic voters among the supporters of each candidate is random and there exists a third candidate who threatens to compete in the election, we explain several important characteristics of elections, including Duverger's Law, polarization, turnout and the inverse relation between policy divergence and voter turnout. We consider two types of altruism, positive and negative altruism, and show that a unique equilibrium exists for positive altruism, whereas multiple equilibria exist for negative altruism. How-

³⁶It can be easily seen that the unique real root of the equation $q^3 - 5q^2 + 24q - 16 = 0$ is strictly greater than $\left(\frac{\sqrt{17}-3}{2}\right)^{n+1}$ for all $n > 0$.

ever, both types of altruism are qualitatively similar in explaining the phenomena stated above. Altruism plays a key role in turnout, that is, voters vote to fulfill their altruistic desires. The randomness of altruistic voters and the threat of third candidate's entry are the major driving forces for Duverger's Law and polarization such that the dominant candidates diverge from each other in order to deter the third candidate who will take the advantage of uncertainty in the number of altruistic voters and run in the election if the dominant candidates converge to each other. Finally, changing the expected number of altruistic voters enables us to relate polarization to voter turnout and offer an explanation for the inverse relation between the two. Our model shows that the decrease in turnout and increase in polarization observed in the U.S. after the 1960s could be a result of the decrease in the expected number of altruistic voters.

Chapter 3

Polarization and Participation with Group Rule Utilitarian Voters

3.1 Introduction

Small costs are expected to dissuade turnout in large elections, since a single vote is unlikely to be decisive.¹ However, we observe significant turnouts in large elections. The voting literature has worked around this inconsistency between strategic behavior and high voter turnout in costly voting models, known as “the paradox of voting,” by introducing different motivations for voters to vote other than the probability of being pivotal, such as signaling, the role of the leader, expressive voting and rule-utilitarianism.² In this literature, Feddersen and Sandroni [34] have provided a

¹See Downs [26].

²In the literature on rule-utilitarianism, voters derive utility from adhering to moral principles and receive payoffs for acting according the strategy that maximizes the total utility of the society (Harsanyi [49, 50], Coate and Conlin [21], Feddersen and Sandroni [34]). In leader-follower theories, the leaders play a role in motivating voters to participate in elections (Uhlaner [101], Morton [73], Shachar and Nalebuff [94], Herrera and Martinelli [51]). Expressive voting models take voting as a

canonical rational choice model in which voting is costly, voter turnout is high, and voters behave strategically.³ In their model, there are two office-seeker candidates, and two types of voters, ethicals and abstainers, of which only one type, ethicals, consider voting. These ethical voters are rule-utilitarian, that is, they receive payoffs for acting according the strategy that is best for the society, and there is high turnout among them as long as there is significant support for both candidates. Coate and Conlin [21], building on Feddersen and Sandroni [34], develop a group rule-utilitarian model, that is, voters receive payoffs for acting according the strategy that is best for their group. They show that group rule-utilitarian approach provides a good explanation for turnout in Texas liquor referenda and outperforms expressive voting approach. Rule-utilitarianism has been a crucial step in explaining turnout. However, the candidates are not strategic in the rule-utilitarian models proposed thus far. Therefore, they do not address any of the phenomena related to the candidate end. In this study, we extend the rule-utilitarian approach to develop a model that explains polarization of candidates together with high turnout.

In two-candidate election models, office seeker candidates, who are competing along a one-dimensional political spectrum, converge to the median voter's optimal

consumption good and voters vote to express themselves (Brennan and Hamlin [19], Engelen [30], Glazer [46]). In signaling models, voters vote in order to signal their unobserved characteristics to others (Aytimur, Boukouras and Schwager [8]).

³In the literature, there is support for the claim that voters are strategic agents. Cox [22] shows that the predictions of the models in which voters are strategic are consistent with voting behavior under the plurality rule. Moreover, there are studies that show voting costs (Riker and Ordeshook [87]), the closeness of elections (Blais [15]), the viability of candidates (Abramson, Aldrich, Paolino and Rohde [1]), education, income (Wolfinger and Rosenstone [106]), and information levels (Feddersen and Pesendorfer [33]) influence turnout, and all of them support that voting is a strategic decision.

policy as long as voter preferences are single-peaked.⁴ Yet, candidates have diverse political positions in all countries. Therefore, these models, known as Downsian models, are empirically untenable. Political scientists have worked around this inconsistency, known as “Downsian convergence,” by introducing policy motivation⁵, uncertainty⁶ and non-policy considerations⁷ to Downsian models. However, they have generally focused on the candidate side and have not addressed the voter side of the election. Glazer [46]’s model is one of the models addresses both candidate and voter sides of the election, and , to the best of our knowledge, it is the only model that offers an explanation the paradox of voting along with Downsian convergence.

According to Glazer [46], voters vote for a candidate to please citizens who prefer the same candidate, a phenomenon that he refers to as positive altruism, and to anger citizens who prefer the other candidate, a pattern he refers to as negative altruism. He shows that this expressive voting behavior is consistent with high turnout and with candidates adopting divergent positions. In his model, negative altruism turns out to be the driving force of policy divergence, and if voters exhibit more positive altruism than negative altruism, then there will be no divergence at all. Voters in Glazer [46]’s model are not strategic in the sense that their payoffs - and hence strategies - do not

⁴Black [13] shows that the median voter’s optimal policy is a Condorcet winner, when there is only one policy dimension and voter preferences are single-peaked. Downs [26] argues that median voter’s optimal policy is always an equilibrium and both parties have to adopt it, without recognizing the single-peaked preferences requirement.

⁵Osborne and Silvinski [77] and Besley and Coate [12] use citizen candidates, who have policy motivations, and show policy divergence when voters are sincere and voting is mandatory.

⁶Berger, Munger and Potthoff [11] assume that voters are uncertain about the candidates’ positions, and predict divergence in equilibrium. Roemer [88] introduces candidates’ uncertainty about the distribution of voter traits, and shows that electoral equilibrium, in which candidates propose different policies, generally exists.

⁷Adams [2] shows that an unpopular candidate moves away from other candidate’s position to increase his chances of victory.

depend on the effect of their votes on the outcome. When we compare Glazer [46] and Feddersen and Sandroni [34], we see that unlike those in Glazer [46], voters in Feddersen and Sandroni [34] are strategic, since their payoffs depend on the effect of their groups' votes on the outcome. Moreover, as opposed to Glazer [46], there is empirical support for the solid performance of Feddersen and Sandroni [34]'s group rule-utilitarian model in explaining turnout.

In this study, as in that of Feddersen and Sandroni [34], we incorporate the rule-utilitarian approach and identify a mechanism that relates group rule-utilitarianism to polarization by endogenizing “the importance of the election”, which is the aggregate benefit a group of voters get in case of their favorite candidate's victory.⁸ In our model, unlike Feddersen and Sandroni [34]'s model, candidates policy choices influence the importance of the election. Voters' payoff depends on the distance of the candidates' policy choices from voters' optimal policy, such that they are better off when their favorite candidate gets closer to them or the opponent moves away. In addition, candidates can brand themselves as underdogs in the eyes of their supporters through their policy choices and capitalize on the gains of the “underdog effect”. We define the underdog effect as follows: Public polls influence voting behavior, as voters' sympathy for a candidate depends on whether the candidate is leading or trailing in the polls.⁹ If voters are more likely to vote for a candidate when they expect him to lose than when they expect him to win, it is said that there is an

⁸In Feddersen and Sandroni [34] voters are rule-utilitarian, whereas they are group rule-utilitarian in our model and Coate and Conlin [21]'s. However, our results on polarization and turnout would still have hold, if voters were rule-utilitarians.

⁹This influence has been recognized through considerable research. (See Fleitas [40], Levine and Palfrey [65], Paharia, Keinan and Avery [78], Paharia, Keinan, Avery and Schor [79], and Keinan, Paharia and Avery [60].)

“underdog effect”.¹⁰ This effect is of two types: (i) voters may shift their preferences towards the minority candidate, or (ii) their tendency to cast a vote for their favorite candidate may increase in response to a decrease in their voter support. In our model, we only consider the second type of underdog effect. Voters look only at candidates’ policy choices to determine who they will support. That is, the percentage of candidates’ voter support does not influence voters’ decision whom to support. Yet, their tendency to cast a vote for their favorite candidate increases when she is more likely to lose.

The dynamics of our model is as follows: Candidates’ policy choices have an influence on ethical voters’ voting behavior, and hence the outcome of the election, through two factors: the number of voters supporting them and the importance of the election for ethical voters. When a candidate moves away from the other candidate, her supporters’ aggregate additional benefit increases due to the increase in difference between candidates’ optimal policies,¹¹ and the increase in these supporters’ sympathy for their favorite candidate whose voter support decreased. Hence, candidates have the tendency to move away from each other to increase the importance of the election for their ethical voters. On the other hand, they have the tendency to move towards each other to increase the number of voters supporting them. The trade-off between these tendencies determines the equilibrium in which candidates choose divergent policies.

¹⁰Both Barack Obama and John McCain in the 2008 U.S. presidential election Mitt Romney in the 2012 U.S. presidential election positioned themselves as underdogs to earn voter sympathy.

¹¹This is up to a point, when a candidate moves away from the other candidate, the additional benefit for some of her supporters whose optimal policies lay between the candidates’ policy choices will decrease, and some of these voters will start to support the other candidate.

The remainder of the paper is organized as follows. The model is stated in Section 3.2. The case of exogenous importance of the election for ethical voters is analyzed in Sections 3.3, and it is used as a benchmark in section 3.4, where these variables are endogenized. Section 3.5 concludes. All proofs are provided in Appendix C.

3.2 The Basic Model

Two strategic candidates, A and B , compete in an election. They choose policies from $[0, 1]$, x_A and x_B , which they are committed to implement if elected. They have a lexicographic objective. They first determine the policy choices that maximize their probability of victory and then among these policies choose the one that maximizes the aggregate utility of voters who support them.¹²

There is a continuum of voters whose *optimal policies* are uniformly distributed over $[0, 1]$. They support the candidate closer to their optimal policy.¹³ Voters supporting candidate $i \in \{A, B\}$ constitute *group i* , and k_i denotes the size of group i .¹⁴ Voting is costly and voting cost is uniformly distributed over $[0, c]$. Moreover, a single vote is never pivotal.¹⁵ Hence, voters are not motivated by the impact of their

¹²If there are multiple policies that maximize the aggregate utility of voters who support them, then they will randomly choose one of them.

¹³A voter supports each candidate equally when his optimal policy is the same distance from both candidates.

¹⁴

$$k_i = 1 - k_j = \begin{cases} \frac{1}{2} & \text{if } x_i = x_j, \\ \frac{x_i + x_j}{2} & \text{if } x_i < x_j, \text{ and} \\ 1 - \frac{x_i + x_j}{2} & \text{if } x_i > x_j. \end{cases}$$

¹⁵Our results would not be significantly altered, if there is a small, but positive, probability of a vote being pivotal.

vote on the outcome of the election. Instead, ethical considerations motivate them to vote.

As in Feddersen and Sandroni [34]’s model, all voters are of two kinds depending on their payoffs, *ethical*s and *abstainers*. Ethical voters are *group rule utilitarian*. They are motivated to vote by the impact of their group’s voting strategy on their aggregate expected utility. *The aggregate expected utility of voters in group $i \in \{A, B\}$ is given by:*

$$u_i = w_i p_i - \phi_i \tag{3.1}$$

where w_i is *the importance of the election for ethical voters in group i* , p_i is *the probability of candidate i ’s victory*, and ϕ_i is *the total cost of voting for voters in group i* . Each ethical voter determines the strategy that maximizes the aggregate expected utility of voters in his group if all ethical voters in his group act according to it, and he follows this strategy.¹⁶ According to an ethical voter, his voting behavior influences his fellow ethical voters’ voting behavior. Therefore, he can influence the outcome of the election and the aggregate expected utility of voters in his group. On the other hand, abstainers receive no payoff for voting. Therefore, they prefer to abstain.¹⁷ The probability of a voter in group $i \in \{A, B\}$ being ethical is q_i . The random variables q_A and q_B are uniformly and independently distributed over $[0, 1]$. Candidates and voters know only the distribution of q_A and q_B , but do not know their realized values.

¹⁶Like Feddersen and Sandroni [34], we can consider that ethical voters receive a payoff $D > c$ for voting according to the strategy that maximizes the aggregate utility of their group.

¹⁷According to an abstainer, his voting behavior has no impact on the voting behavior of other voters. Moreover, his vote is never pivotal since there is a continuum of voters. Therefore, his voting behavior does not affect the outcome of the election, and he receives no payoff for voting.

All ethical voters in a group act like a single player, because all of them try to maximize their favorite candidate's probability of victory while trying to minimize the total cost of voting. Hence, keeping the number of votes cast for a candidate fixed, it is better for the candidate's ethical supporters if the ones with lower voting costs cast a vote while the ones with higher costs abstain. Therefore, ethical voters' optimal strategies are cut-off strategies, $(\sigma_A, \sigma_B) \in [0, 1]^2$, which require ethical voters in group $i \in \{A, B\}$ with voting costs below the threshold, $\sigma_i c$, to vote for candidate i , and others to abstain.¹⁸

Like Feddersen and Sandroni [34], we call a strategy profile of ethical voters, (σ_A^*, σ_B^*) , *consistent*, if the following inequality holds:

$$w_i p_i(\sigma_i^*, \sigma_j^*) - \phi_i(\sigma_i^*) \geq w_i p_i(\sigma_i, \sigma_j^*) - \phi_i(\sigma_i),^{19} \quad (3.2)$$

for all $\sigma_i \in [0, 1]$ and $i \in \{A, B\}$.²⁰ In a consistent strategy profile, taking the behavior of abstainers and ethical voters in the other group as given, each ethical voter in group $i \in \{A, B\}$ independently considers what would occur if all ethical voters in his group follow the strategy determined by cut-off point σ_i , and reasons to follow σ_i^* that maximizes aggregate utility of group i . Hence, ethical voters, who are group rule utilitarian, will follow the consistent strategy profiles.

¹⁸See Feddersen and Sandroni [35].

¹⁹ $p_i(\hat{\sigma}_i, \hat{\sigma}_j)$ is the probability of candidate i 's victory, and $\phi_i(\hat{\sigma}_i)$ is the total cost of voting for voters in group i when the cut-off strategies of ethical voters in groups i and j are $\hat{\sigma}_i$ and $\hat{\sigma}_j$, respectively.

²⁰The consistency concept was first used in Feddersen and Sandroni [34]. It has the flavor of Nash equilibrium, because a consistent strategy profile of ethical voters is a Nash equilibrium of the game played between the two representative agents, each of whom represents the ethical voters in one of the groups.

The election proceeds through the following steps. (i) Candidates choose and announce the policies that they are committed to implement if elected. (ii) Nature determines the ethical voters in each group.²¹ (iii) Ethical voters determine a consistent strategy profile and act according to it in the election. The winner is determined by the plurality rule. In the following section, we take the importance of the election for ethical voters as exogenous variables and we use it as a benchmark in Section 3.4, where we endogenize these variables.

3.3 Exogenous Importance of the Election

When the importance of the election for the ethical voters is exogenous, candidates' policy choices have an effect on the consistent strategy profile, and thus on the outcome of the election, only through their effect on the sizes of groups A and B . Hence, each candidate tries to increase the size of their voter group - and to decrease the size of her opponent's voter group - in order to increase their probability of victory. As a result of this competition, both candidates converge to the optimal policy of median voter in the Nash equilibrium. Ethical voters, on the other hand, face a trade off while determining their optimal strategy that maximizes the aggregate expected utility of their group: a higher cut-off point implies a higher probability of victory for their favorite candidate, but also a higher social cost. This trade off determines the consistent strategy profile ethical voters follow, and as shown in the following

²¹ q_A and q_B are randomly drawn from the interval $[0, 1]$, and, for $i \in \{A, B\}$, q_i percent of voters in group i are chosen randomly as ethical voters.

proposition, when the election is important for ethical voters, there will be a high voter turnout in both groups.

Proposition 3.3.1. *Assume w_A and w_B are exogenous, and without loss of generality, let $0 < w_A \leq w_B$. Then, there exists a unique Nash equilibrium. In this equilibrium, the policy choices of candidates are:*

$$(x_A, x_B) = \left(\frac{1}{2}, \frac{1}{2} \right), \quad (3.3)$$

and the consistent strategy profile of ethical voters is:

$$(\sigma_A^*, \sigma_B^*) = \begin{cases} \left(\sqrt[4]{\frac{4w_A^3}{w_B c^2}}, \sqrt[4]{\frac{4w_A w_B}{c^2}} \right) & \text{if } \sqrt{4w_A w_B} < c, \\ \left(\frac{2w_A}{c}, 1 \right) & \text{if } 2w_A < c \leq \sqrt{4w_A w_B}, \\ (1, 1) & \text{if } c \leq 2w_A. \end{cases} \quad (3.4)$$

Using Proposition 3.3.1, we can do comparative statics on the equilibrium. When ethical voters follow the consistent strategy profile (σ_A^*, σ_B^*) , a fraction of $k_i q_i \sigma_i^*$ of the electorate will vote for candidate $i \in \{A, B\}$. Hence, expected voter turnout in the Nash equilibrium is:

$$E(k_A q_A \sigma_A^* + k_B q_B \sigma_B^*) = \frac{\sigma_A^* + \sigma_B^*}{4}. \quad (3.5)$$

From Proposition 3.3.1, it follows that expected voter turnout is strictly positive as long as w_A and w_B are greater than zero. In other words, voting cost does not

dissuade large-scale turnout as long as ethical voters get additional benefit for their favorite candidate's victory.²² However, the ethical voters in group A , who get weakly less benefit, are weakly less likely to vote than the ethical voters in group B . Finally, expected voter turnout in both groups weakly increase in the importance of the election for them and weakly decrease in their voting costs.

In the Nash equilibrium, candidates' probabilities of victory are:²³

$$(p_A, p_B) = \begin{cases} \left(\frac{1}{2} \sqrt{\frac{w_A}{w_B}}, 1 - \frac{1}{2} \sqrt{\frac{w_A}{w_B}} \right) & \text{if } \sqrt{4w_A w_B} < c, \\ \left(\frac{w_A}{c}, 1 - \frac{w_A}{c} \right) & \text{if } 2w_A < c \leq \sqrt{4w_A w_B}, \\ \left(\frac{1}{2}, \frac{1}{2} \right) & \text{if } c \leq 2w_A, \end{cases} \quad (3.6)$$

If the maximum voting cost is high, i.e., $\sqrt{4w_A w_B} < c$, the candidate for whose voters the election is more important has a higher probability of winning the election. The gap between candidates' probabilities of victory decreases with decreasing maximum voting cost;²⁴ and both candidates are equally likely to win when it is lower than a threshold, i.e., $c \leq 2w_A$. The intuition for this is the following: if voting costs are high with respect to the benefits of victory, ethical voters who are expected to get more benefit for their favorite candidate's victory will vote with higher probabilities when compared with other ethical voters; however the gap between voting probabilities will decrease with decreasing voting costs, and all ethical voters will vote when voting costs are very low.

²²This result is Property 2 in Feddersen and Sandroni [34].

²³We obtain these by plugging the consistent strategy profile from Proposition 3.3.1 into (C.3).

²⁴This gap strictly decreases in c for $2w_A < c \leq \sqrt{4w_A w_B}$.

The aggregate expected utilities of voters in the equilibrium are:²⁵

$$(u_A, u_B) = \begin{cases} \left(\frac{1}{4} \sqrt{\frac{w_A^3}{w_B}}, w_B - \frac{3}{4} \sqrt{w_A w_B} \right) & \text{if } \sqrt{4w_A w_B} < c, \\ \left(\frac{w_A^2}{2c}, w_B - \frac{w_A w_B}{c} - \frac{c}{8} \right) & \text{if } 2w_A < c \leq \sqrt{4w_A w_B}, \\ \left(\frac{w_A}{2} - \frac{c}{8}, \frac{w_B}{2} - \frac{c}{8} \right) & \text{if } c \leq 2w_A. \end{cases} \quad (3.7)$$

It can be easily seen that, the aggregate expected utilities of voters in both groups are always positive. Hence, ethical voters are better off by following the consistent strategy profile than by abstaining. However, ethical voters in group B are expected to get more benefit than ethical voters in group A , because the election is more important for them and their favorite is weakly more likely to win. Finally, all voters are weakly better off with decreasing voting costs.

In this benchmark case, there is large-scale turnout. However polarization does not simultaneously arise with it. In the following section, we endogenize the importance of the election for ethical voters, such that they depend on candidates' policy choices. Candidates try to make the election more important for their supporters and less important for their opponent's supporters in order to increase their probability of victory. We show that the dynamics of this two dimensional competition between candidates leads to polarization.²⁶

²⁵We obtain these by plugging the consistent strategy profile from Proposition 3.3.1 into (C.5).

²⁶The competition is two dimensional, because candidates try to increase the sizes of their voter groups together with the importance of the election for their ethical voters, while trying to decrease the sizes of each other's voter groups and the importance of the election for each other's ethical voters.

3.4 Endogenous Importance of the Election

A voter will get more benefit if his favorite candidate wins the election instead of the other candidate. This additional benefit will be higher if his favorite candidate's policy choice is closer to his optimal policy. This model of voting behavior was first proposed by Hotelling [53]²⁷ and has been used in numerous empirical²⁸ and formal²⁹ studies. We extend it by introducing that the payoff a voter receives from his favorite candidate's victory depends on the distances of all candidates' policy choice from his optimal policy. Moreover, we assume that the smaller the support for a voter's favorite candidate, the more this voter will enjoy his favorite candidate's victory.³⁰ Therefore, candidates try to brand themselves as underdogs in the eyes of their supporters through their policy choices. Unlike the previous models dealing with the underdog effect, voters support the candidate whose policy choice is closest to their optimal policy and the sizes of candidates' voter supports do not affect whom voters will support.³¹ However, the sizes of candidates' voter support affect the benefit a voter gets in case of his favorite candidate's victory. More formally, a voter who is in group $i \in \{A, B\}$ and for whom the distance between his optimal policy and the policy choice of the winning candidate is x , thinks that voters in his group will get a total benefit of $g(x)$, and that this benefit will be distributed evenly to the voters in his group such that each voter will get a benefit of $\frac{g(x)}{k_i}$. Hence, the

²⁷See Smithies [96] as well.

²⁸See Adams and Merrill [3], Plane and Gershtenson [83], Thurner and Eymann [99].

²⁹See Palfrey [81], and Callander and Wilson [20].

³⁰In other words, a voter is better off with the decrease in the number of people with whom he will share the benefits of victory.

³¹See Fleitas [40] and Levine and Palfrey [65] for models in which the sizes of candidates' voter supports have an influence on voters preference.

additional benefit this voter will get in case of his favorite candidate's victory is:

$$b(x, x_i, x_j) = \frac{g(|x - x_i|) - g(|x - x_j|)}{k_i(x_i, x_j)} \quad (3.8)$$

where x is his optimal policy, x_i is the policy choice of his favorite candidate, x_j is the policy choice of the opponent, and $k_i(x_i, x_j)$ is the size of his group. We take:

$$g(x) = \begin{cases} \alpha - x & \text{if } x \in [0, \alpha) \\ 0 & \text{if } x \geq \alpha. \end{cases} \quad (3.9)$$

where $\alpha \in (0, 1/2)$.³² Hence, a voter gets a positive benefit from the implemented policy as long as the distance between his optimal policy and winning candidate's policy choice is less than α . This benefit structure is similar to the abstention structure in Callander and Wilson [20]. In their model voting is compulsory for those voters whose distance from their favorite candidate is less than a threshold, which they call voter tolerance, and abstention is compulsory for other voters. In our model, these distances influence voting decision indirectly through their effect on voters' benefit, which exists as long as they are less than the threshold, α .

The importance of the election for an ethical voter is equal to the aggregate additional benefit voters in his group will get in case of his favorite candidate's victory. Moreover, he considers voters' own estimates of their additional benefits, which they will get in case of their favorite candidate's victory, while determining the importance

³²Voting cost and polarization are related for $\alpha \in (0, 1/2)$. However, for $\alpha \geq 1/2$, as shown in the proof of Proposition 3.4.1, both candidates converge to the median voter's optimal policy independent of voting costs.

of the election for himself. Therefore, the importance of the election for an ethical voter in group $i \in \{A, B\}$ is:

$$w_i(x_i, x_j) = \int_{S_i} b(x, x_i, x_j) dx \quad (3.10)$$

where S_i is the set of voters in group i .³³ Moreover, the benefit function, g , and the distributions of voters' optimal policies are common knowledge. As a result of this, all ethical voters can correctly predict w_A and w_B .

Candidates' policy choices determine the voter groups as well as the importance of the election for ethical voters. Hence, candidates tend to move towards each other in order to have more supporters, and tend to move towards the flanks in order to make the election more important for their supporters. The equilibrium will emerge as a result of the trade-off between these two tendencies.

Before stating the equilibrium, we present the following results that will help us to understand how equilibrium emerges:³⁴ (i) if one of the candidates locates on $[0, \alpha) \cup (1 - \alpha, 1]$, then her opponent will be more likely to win with a policy choice closer to the median voter's optimal policy; (ii) for $i \in \{A, B\}$, if candidate i has the support of the majority of the voters and the average additional benefit a voter in group j gets from candidate j 's victory is higher than the maximum voting cost, i.e., if $k_i(x_i, x_j) > \frac{1}{2}$ and $\frac{w_j(x_j, x_i)}{k_j(x_j, x_i)} > c$, then candidate i will be more likely to win than her opponent; and (iii) both candidates shall be equally likely to win in the equilibrium,

³³The importance of the election for ethical voters in the minority group would always be higher than the importance of the election for ethical voters in the majority group.

³⁴All the proofs are in the Appendix.

because they can guarantee to win with 50% probability by playing symmetrically. Combining these three results, we obtain the following two features of the equilibrium. First, candidates will locate themselves on $[\alpha, 1 - \alpha]$ in the equilibrium. Second, if any of the candidates deviates from her equilibrium policy choice to a policy choice that will make her the majority candidate, then the average additional benefit that her opponent's supporters get from her opponent's victory shall not be more than the maximum voting cost. More formally, let us define the function $\Phi : [0, 1]^2 \rightarrow [0, \infty)$ such that:³⁵

$$\Phi(x_A, x_B) = \max \left\{ \max_{y \in [1-x_B, x_B]} \frac{w_B(x_B, y)}{k_B(x_B, y)}, \max_{z \in [1-x_A, x_A]} \frac{w_A(x_A, z)}{k_A(x_A, z)} \right\}. \quad (3.11)$$

If (x_A, x_B) is an equilibrium, then:

$$x_A, x_B \in [\alpha, 1 - \alpha], \text{ and} \quad (3.12)$$

$$\Phi(x_A, x_B) \leq c. \quad (3.13)$$

Candidates want to increase the aggregate utility of their supporters as long as it does not decrease their probability of victory. We show that, as a result of this incentive, in the equilibrium candidates will diverge as much as possible without violating the constraints above, (3.12) and (3.13). If c is high, $\Phi(x_A, x_B) \leq c$ will hold for all $x_A, x_B \in [\alpha, 1 - \alpha]$. Therefore, for high values of c , (3.12) will be the only binding constraint, and candidates will choose the most extreme policy pair that

³⁵ $[1 - x_i, x_i]$ is well-defined even if $1 - x_i > x_i$, because we take $[1 - x_i, x_i] = [\min\{x_i, 1 - x_i\}, \max\{x_i, 1 - x_i\}]$.

fulfills it, i.e., $(x_A, x_B) \in \{(\alpha, 1 - \alpha), (1 - \alpha, \alpha)\}$. However, for lower values of c , (3.13) will become the binding constraint, and candidates will choose the most extreme symmetric policy pair that fulfills it, i.e., $(x_A, x_B) \in \{(x(c), 1 - x(c)), (1 - x(c), x(c))\}$ where $x(c) = \min\{x \in (\alpha, 1/2) : \Phi(x, 1 - x) = c\}$. The following proposition combines these results and states the equilibrium.

Proposition 3.4.1. *When the importance of the election for ethical voters are endogenous variables, there exists a unique Nash equilibrium. In this equilibrium, the policy choices of candidates are:*

$$(x_i, x_j) = \begin{cases} (\alpha, 1 - \alpha) & \text{if } c \geq \Phi(\alpha, 1 - \alpha), \\ (x(c), 1 - x(c)) & \text{if } c < \Phi(\alpha, 1 - \alpha), \end{cases} \quad (3.14)$$

and the consistent strategy profile of ethical voters is:

$$\sigma_i^* = \sigma_j^* = \sqrt{\frac{2w_i(x_i, x_j)}{c}}. \quad (3.15)$$

Moreover, $x(c)$ is decreasing in c , $\lim_{c \rightarrow 0} x(c) = 1/2$, and $\lim_{c \rightarrow 0} \sigma_i^* = \lim_{c \rightarrow 0} \sigma_j^* = 1$.

Proposition 3.4.1 shows that when the importance of the election for ethical voters depends endogenously on candidates' policy choices, candidates diverge from each other and voters vote in high proportion. The polarization is at its maximum, when the maximum voting cost is high, i.e., above the threshold $\Phi(\alpha, 1 - \alpha)$. Below this threshold, polarization decreases with decreasing voting costs, and disappears with the disappearance of voting costs. The intuition is as follows: when voting costs

are high, then increasing the importance of the election for ethical voters is more important than increasing voter support for candidates. As a result of this, candidates diverge to the extremes. However, as voting costs decrease, candidates do not have to make the election very important to make voters vote, and their incentive to increase voter support makes them move closer towards each other. On the other hand, even though candidates' policy choices will be very close to each other for low voting costs, the importance of the election caused by these policy choices will be enough to ensure that the ethical voters of both candidates vote. Hence, there will be high turnout even if the polarization is very low.

At equilibrium, candidates choose symmetric policies. As a result of this, their voter support and the importance of the election for their ethical voters are the same, and they are equally likely to win the election, i.e., $p_A = p_B = \frac{1}{2}$.³⁶ The aggregate expected utilities of voters are:³⁷

$$u_A = u_B = \frac{w_i(x_i, x_j)}{4} = \frac{\int_0^{1/2} [g(|x - x_i|) - g(|x - x_j|)] dx}{2}. \quad (3.16)$$

Hence, all voters are expected to have the same utility, because of the symmetricity of candidates' policy choices. This utility increases with increasing polarization, because voters' favorite candidates become much closer to their group than the opponent candidates. As shown in proposition 3.4.1, polarization increases with voting cost. The positive effect of the increase in polarization outweighs the negative effect of the

³⁶Follows from plugging the consistent strategy profile from Proposition 3.4.1 into (C.3).

³⁷Follows from plugging the consistent strategy profile from Proposition 3.4.1 into (C.5).

increase in voting costs and, as seen from (3.16), ethical voters become better off with increasing voting costs.

3.5 Conclusion

The main result of our model is that it provides an explanation for both the polarization and the paradox of voting. In our model, as in Feddersen and Sandroni [34], some voters, whom we call ethicals, have a sense of civic duty. These voters act as if they are a single player, and vote to maximize the aggregate expected utility of their group. As a result of this behavior, there is a high probability that they will vote even though voting is costly and the electorate is large. We have introduced three features to Feddersen and Sandroni [34]’s framework to address polarization. First, candidates are strategic actors with lexicographic objectives, that is, they choose the policy that maximizes the aggregate expected utility of their supporters among the policies which maximize their probability of victory. Second, the importance of the election for ethical voters is endogenous and depends on candidates’ policy choices. Third, voters have more sympathy for their favorite candidate when the candidate is more likely to lose. Thus, when candidates diverge from each other by moving towards the flanks, the election will become more important because they will be closer to their voters and they will be seen as more of an underdog. We have shown that in the equilibrium, candidates choose divergent policies to make their voters better off by making the election more important for them.

Appendix A

Proofs for Chapter 1

A.1 Proof of Proposition 1.4.1:

Let us define two new countries A' and B' such that given the bid of other government, $b_{j'}$, government i' tries to maximize:

$$u_{i'}(b_{i'}, b_{j'}) = \begin{cases} -b_{i'} & \text{for } b_{i'} < b_{j'} \\ \frac{v_{i'}}{2} - b_{i'} & \text{for } b_{i'} = b_{j'} \\ v_{i'} - b_{i'} & \text{for } b_{i'} > b_{j'} \end{cases} \quad (\text{A.1})$$

where $v_{i'} = \lambda_i v_i$. It can be easily seen $(b_{A'}, b_{B'})$ is an equilibrium of this game if and only if $(\frac{b_{A'}}{\lambda_A}, \frac{b_{B'}}{\lambda_B})$ is an equilibrium of our original war game. Hence, we will characterize the equilibria of this new game to complete the proof. Notice that this

new game is the original all-pay auction with complete information which has been solved by Baye, Kovenock and de Vries [10], and we repeat their solution.¹

First, we assume that $s > 1$, i.e. $v_{A'} > v_{B'}$. For $i' \in \{A, B\}$, let $\underline{s}_{i'}$ and $\bar{s}_{i'}$ be the lower and upper bounds of government i' 's equilibrium bid distribution function, i.e., $F_{i'}$. Let $\alpha_{i'}(x)$ denote the size of mass at point x in government i' 's distribution function,² and $u_{i'}^*$ be government i' 's equilibrium payoff. We first obtain the supports of the mixed strategies in the equilibrium.

Step 1: Let us show that $\underline{s}_{A'} = \underline{s}_{B'} = 0$. Government i' can guarantee at least a non-negative payoff by bidding $b_{i'} = 0$, so all bids greater than $v_{i'}$ are ruled out. Thus, $v_{i'} \geq \bar{s}_{i'} \geq \underline{s}_{i'} \geq 0$.

Given $\underline{s}_{A'} = \underline{s}_{B'}$, $\alpha_{A'}(\underline{s}_{A'}) > 0$ and $\alpha_{B'}(\underline{s}_{B'}) > 0$, we see that both government A' and government B' has an incentive to raise their lower bound by a small amount. Therefore, it follows that:

$$\underline{s}_{A'} = \underline{s}_{B'} \Rightarrow \text{there exists } h \in \{A, B\} \text{ s.t. } \alpha_h(\underline{s}_h) = 0. \quad (\text{A.2})$$

Let $u_{j'}(b_{j'}, F_{i'})$ denote government j' 's payoff. If $\underline{s}_{i'} \geq \underline{s}_{j'}$ and $\alpha_{i'}(\underline{s}_{j'}) = 0$, then $u_{j'}(\underline{s}_{j'}, F_{i'}) = -\underline{s}_{j'} < 0 = u_{j'}(0, F_{i'})$ for $s_{j'} > 0$. We will get the same result when we replace $s_{j'}$ with any $b_{j'} \in (0, \underline{s}_{i'})$. Thus, it follows that:

$$\underline{s}_{i'} \geq \underline{s}_{j'} \text{ and } \alpha_{i'}(\underline{s}_{j'}) = 0 \Rightarrow \underline{s}_{j'} = 0 \text{ and } F_{j'}(0) = \lim_{x \uparrow \underline{s}_{i'}} F_{j'}(x). \quad (\text{A.3})$$

¹The proof for all-pay auctions under complete information when the valuations are homogeneous or heterogenous can be seen in Baye, Kovenock and de Vries [10] .

² $\alpha_{i'}(x)$ is greater than zero if and only if x is a masspoint.

Moreover, it can be easily seen that:

$$\underline{s}_{i'} \geq \underline{s}_{j'} \text{ and } \alpha_{i'}(\underline{s}_{i'}) = 0 \Rightarrow \underline{s}_{j'} = 0 \text{ and } F_{j'}(0) = F_{j'}(\underline{s}_{i'}). \quad (\text{A.4})$$

Now, let us show that $\underline{s}_{A'} = \underline{s}_{B'}$. Assume not. Then, there exists i' s.t. $\underline{s}_{i'} > \underline{s}_{j'}$. If $\alpha_{i'}(\underline{s}_{i'}) = 0$, then from (A.4) we get $F_{j'}(0) = F_{j'}(\underline{s}_{i'})$, and as a result of this $u_{i'}(\underline{s}_{i'}, F_{j'}) < u_{i'}(x, F_{j'})$ for x very small but greater than 0, which is a contradiction. On the other hand, if $\alpha_{i'}(\underline{s}_{i'}) > 0$, then $\alpha_{j'}(\underline{s}_{i'}) = 0$, since otherwise government i' has an incentive to raise his lower bound by a small amount. In this case, bidding a positive amount lower than $\underline{s}_{i'}$ causes only loss for government j' , we get that $F_{j'}(0) = F_{j'}(\underline{s}_{i'})$. As a result of this, $u_{i'}(\underline{s}_{i'}, F_{j'}) < u_{i'}(x, F_{j'})$ for x very small but greater than 0, which is a contradiction. Thus, we conclude that $\underline{s}_{A'} = \underline{s}_{B'}$. Combining this result with (A.2) and (A.3), it follows that:

$$\underline{s}_{A'} = \underline{s}_{B'} = 0. \quad (\text{A.5})$$

Step 2: We will show that $\bar{s}_{A'} = \bar{s}_{B'} = v_{B'}$ and $u_{B'}^* = 0$.

Let us denote $\bar{s} = \max\{\bar{s}_{A'}, \bar{s}_{B'}\}$. Government B' will never bid above $v_{B'}$, since 0 strictly dominates any bid above $v_{B'}$. Therefore, government A' has no incentive to bid above $v_{B'}$, and it follows that $\bar{s} \leq v_{B'}$. Moreover, government A' 's equilibrium payoff, i.e., $u_{A'}^*$, cannot be less than $v_{A'} - v_{B'} > 0$, because $\bar{s}_{B'} \leq v_{B'}$ and government A' wins the election for sure with any bid greater than $\bar{s}_{B'}$.

Let us show that government A' does not have a masspoint at 0. Assume not. Then, government A' has a masspoint at 0. From (A.2) and (A.5), we know that at least one of the governments does not have a masspoint at 0, and if it is not government A' , then it is government B' . Given government B' does not have a masspoint at 0, it follows that government A' will get a payoff 0 - which is less than $v_{A'} - v_{B'}$ - by bidding $\underline{s}_{A'} = 0$, a contradiction.

Since $u_{A'}^* > 0$, government A' should outbid government B' with a probability bounded away from zero. This is possible only when government B' has a masspoint at 0, because $\underline{s}_{A'} = 0$. Thus, 0 is a masspoint for government B' but not for government A' , and we get that $u_{B'}^* = 0$.

If $\bar{s} < v_{B'}$, then government B can get more than 0 by bidding a little above $\bar{s}$, which is a contradiction. Hence, $\bar{s} = v_{B'}$. Since none of the governments have an incentive to bid more than the highest bid by the other government in the equilibrium (which follows from step 3, i.e., there exists no masspoints on $(0, v_{B'}]$), we conclude that $\bar{s}_{A'} = \bar{s}_{B'} = v_{B'}$.

Step 3: Let us show that there are no point masses on the interval $(0, v_{B'}]$. Suppose that $F_{i'}$ has a masspoint at $x_{i'} \in (0, v_{B'}]$. If $x_{i'} < v_{j'}$, then government j' has an incentive to transfer mass from $(x_{i'} - \epsilon, x_{i'}]$ to $x_{i'} + \delta$ for a small ϵ and a very small δ . If $x_{i'} = v_{j'}$, then government j' has an incentive to transfer mass from $(v_{j'} - \epsilon, v_{j'}]$ to 0 for a small ϵ . Thus, there would be a neighborhood below $x_{i'}$ in which government j' would put no mass. However, then it is not an equilibrium strategy for player i' to put mass at $x_{i'}$, which is a contradiction.

Step 4: Let us show that, for $x, y \in [0, v_{B'}]$ and $i' \in \{A, B\}$, if $x > y$, then $F_{i'}(x) - F_{i'}(y) > 0$.

Assume that there exists $x, y \in (0, v_{B'}]$ such that $x > y$ and $F_{i'}(x) - F_{i'}(y) = 0$. $x, y \in (0, v_{B'}]$ cannot be masspoints, which follows from step 3, and putting a mass on $(y, x]$ causes only loss for government j' . Therefore, $F_{j'}(x) - F_{j'}(y) = 0$. Denote $\bar{x} = \sup\{z \in [x, v_{B'}] : F_{A'}(z) - F_{A'}(y) = 0 \text{ or } F_{B'}(z) - F_{B'}(y) = 0\}$. Then $F_{A'}(\bar{x}) - F_{A'}(y) = 0$ and $F_{B'}(\bar{x}) - F_{B'}(y) = 0$. If $\bar{x} < v_{B'}$, government A (and government B) has an incentive to transfer mass from $(\bar{x}, \bar{x} + \epsilon)$ for ϵ small to y , which is a contradiction. If $\bar{x} = v_{B'}$, then $y \geq \bar{s}_{A'}$, which is a contradiction, because we already have $v_{B'} > y$ and $\bar{s}_{A'} = v_{B'}$. On the other hand, the case for $y = 0$ is straight forward, because there exists $y' \in (0, x)$ and it directly follows that $F_{i'}(x) > F_{i'}(y') \geq F_{i'}(0)$.

Step 5: For $i' \in \{A, B\}$, $F_{i'}$ is increasing at all points $\in [0, v_{B'}]$. Therefore, the payoff government i' gets by bidding any $x_{i'} \in [0, v_{B'}]$ is constant and equal to $u_{i'}^*$, where $u_{A'}^* = v_{A'} - v_{B'}$ and $u_{B'}^* = 0$. As a result of this, for all $x \in [0, v_{B'}]$, we have:

$$(v_{A'} - x)F_{B'}(x) - x(1 - F_{B'}(x)) = u_{A'}^* \Leftrightarrow F_{B'}(x) = \frac{v_{A'} - v_{B'} + x}{v_{A'}}, \text{ and} \quad (\text{A.6})$$

$$(v_{B'} - x)F_{A'}(x) - x(1 - F_{A'}(x)) = u_{B'}^* \Leftrightarrow F_{A'}(x) = \frac{x}{v_{B'}}. \quad (\text{A.7})$$

Rearranging these distribution functions, we obtain that the unique Bayesian Nash equilibrium of the war game is given by (1.4) and (1.5). The same proof can easily be adapted to the case $s = 1$.

A.2 Proof of Proposition 1.4.2:

Let us define two new governments A' and B' such that given the bid of other government, $b_{j'}$, government i' tries to maximize:

$$u_{i'}(b_{i'}, b_{j'}) = \begin{cases} -2b_{i'} & \text{for } b_{i'} < b_{j'} \\ \frac{v_{i'}}{2} - 2b_{i'} & \text{for } b_{i'} = b_{j'} \\ v_{i'} - b_{i'} - b_{j'} & \text{for } b_{i'} > b_{j'} \end{cases} \quad (\text{A.8})$$

where $v_{i'} = \lambda_i v_i$. It can be easily seen $(b_{A'}, b_{B'})$ is an equilibrium of this game if and only if $(\frac{b_{A'}}{\lambda_A}, \frac{b_{B'}}{\lambda_B})$ is an equilibrium of our original war game. Hence, we will characterize the equilibria of this new game to complete the proof.

First, we assume that $s > 1$, i.e. $v_{A'} > v_{B'}$. Let $\underline{s}_{i'}$, $\bar{s}_{i'}$, $F_{i'}$, $\alpha_{i'}$, and $u_{i'}^*$ be same as in the proof of Proposition 1.4.1. We begin with obtaining the supports of the mixed strategies in the equilibrium.

Step 1: $\underline{s}_{A'} = \underline{s}_{B'} = 0$. It follows from the proof of step 1 in the proof of proposition 1.4.1 with a small difference, which is that $u_{j'}(\underline{s}_{j'}, F_{i'}) = -2\underline{s}_{j'}$.

Step 2: Let us show that $\bar{s}_{A'} = \bar{s}_{B'} = \bar{s} \leq v_{B'}$ and there are no masspoints on the interval $(0, \bar{s}]$. There are no masspoints on the interval $(0, v_{B'}]$, which is shown same as step 3 in the proof of proposition 1.4.1. It is obvious that government B' has no incentive to bid more than $v_{B'}$, and since $v_{B'}$ is not a masspoint for government B' , government A' has no incentive to bid more than $v_{B'}$. On the other hand, since there exists no masspoints on $(0, v_{B'}]$, none of governments has an incentive to bid more

than the highest bid of the other government. Thus, $\bar{s}_{A'} = \bar{s}_{B'} = \bar{s} \leq v_{B'}$, and there exists no masspoints on $(0, \bar{s}]$.

Step 3: For $x, y \in [0, \bar{s}]$ and $i' \in \{A, B\}$, if $x > y$, then $F_{i'}(x) - F_{i'}(y) > 0$. The proof of this step follows from the proof of step 4 of the proof of proposition 1.4.1, when we replace $v_{B'}$ with $\bar{s}$.

Step 4: For $i' \in \{A, B\}$, $F_{i'}$ is increasing at all points in the interval $[0, \bar{s}]$. Therefore, the payoff government i' gets by bidding any $x_{i'} \in [0, \bar{s}]$ is constant and equal to $u_{i'}^*$. As a result of this, for all $x \in [0, \bar{s}]$, the payoff government A' gets by bidding x is:

$$\begin{aligned} u_{A'}^* &= \int_0^x (v_{A'} - x - y) f_{B'}(y) dy + \int_x^{\bar{s}} (-2x) f_{B'}(y) dy \\ &= v_{A'} F_{B'}(x) - x(2 - F_{B'}(x)) - \int_0^x y f_{B'}(y) dy \end{aligned} \quad (\text{A.9})$$

and the payoff government B' gets by bidding x is:

$$\begin{aligned} u_{B'}^* &= \int_0^x (v_{B'} - x - y) f_{A'}(y) dy + \int_x^{\bar{s}} (-2x) f_{A'}(y) dy \\ &= v_{B'} F_{A'}(x) - x(2 - F_{A'}(x)) - \int_0^x y f_{A'}(y) dy \end{aligned} \quad (\text{A.10})$$

Taking the derivative of (A.9) with respect to x we get:

$$\begin{aligned} 0 &= v_{A'} f_{B'}(x) - 2 + F_{B'}(x) + x f_{B'}(x) - x f_{B'}(x) \\ &\Leftrightarrow F_{B'}(x) = 2 - v_{A'} f_{B'}(x) \end{aligned} \quad (\text{A.11})$$

Since $F_{B'}$ is differentiable, it follows that $f_{B'}$ is also differentiable. Then, taking the derivative of (A.11) with respect to x , we get:

$$f_{B'}(x) = -v_{A'} f'_{B'}(x) \Rightarrow f_{B'}(x) = M e^{-x/v_{A'}} \text{ and } F_{B'}(x) = 2 - M v_{A'} e^{-x/v_{A'}}$$

where M is a constant. Since $F_{B'}(\bar{s}) = 1$, we obtain $M = \frac{e^{\bar{s}/v_{A'}}}{v_{A'}}$ and:

$$F_{B'}(x) = 2 - e^{\frac{\bar{s}-x}{v_{A'}}} \quad (\text{A.12})$$

Taking the derivative of (A.10) with respect to x we get:

$$0 = v_{B'} f_{A'}(x) - 2 + F_{A'}(x) + x f_{A'}(x) - x f_{A'}(x)$$

$$\Leftrightarrow F_{A'}(x) = 2 - v_{B'} f_{A'}(x) \quad (\text{A.13})$$

Since $F_{A'}$ is differentiable, it follows that $f_{A'}$ is differentiable. Taking the derivative of (A.13) with respect to x we get:

$$f_{A'}(x) = -v_{B'} f'_{A'}(x) \Rightarrow f_{A'}(x) = N e^{-x/v_{B'}} \text{ and } F_{A'}(x) = 2 - N v_{B'} e^{-x/v_{B'}}$$

where N is a constant. We know that $F_{A'}(\bar{s}) = 1$. Therefore, $N = \frac{e^{\bar{s}/v_{B'}}}{v_{B'}}$ and:

$$F_{A'}(x) = 2 - e^{\frac{\bar{s}-x}{v_{B'}}} \quad (\text{A.14})$$

From step 1 and (A.2), it follows that there exists i' such that $\alpha_{i'}(0) = 0$, i.e., $F_{i'}(0) = 0$. Since $v_{A'} > v_{B'}$, we have $F_{B'}(0) > F_{A'}(0)$. Hence, $F_{A'}(0) = 0$, and, from (A.14), it follows that:

$$\bar{s} = v_{B'} \ln 2. \quad (\text{A.15})$$

Plugging (A.15) in (A.12) and (A.14), we obtain:

$$F_{A'}(x) = 2 - 2e^{\frac{-x}{v_{B'}}} \text{ and } F_{B'}(x) = 2 - e^{\frac{v_{B'} \ln 2 - x}{v_{A'}}} \text{ on } x \in [0, v_{B'} \ln 2]. \quad (\text{A.16})$$

Rearranging these distribution functions, we conclude that governments' bidding strategies in the unique Bayesian Nash equilibrium of the original war game are given by (1.8) and (1.9). The same proof can easily be adapted to the case $s = 1$.³

A.3 Proof of Corollary 1.4.1:

In a world of autocracies, using governments' bidding strategies stated in proposition 1.4.1, we get that country A 's expected military power is:

$$MP_A = \int_0^{\frac{v_A}{s}} \lambda_A x f_A(x) dx = \frac{\lambda_B v_B}{2}, \quad (\text{A.17})$$

country B 's expected military power is:

$$MP_B = \int_0^{v_B} \lambda_B x f_B(x) dx = \frac{\lambda_B v_B}{2s}, \quad (\text{A.18})$$

³There is a small difference, in that at the end of step 4, we obtain $F_{B'}(0) = F_{A'}(0) = 0$ which follows from $v_{A'} = v_{B'}$. We use it to find upper bound of the governments' bids.

the size of the war is:

$$\begin{aligned}
WS &= \int_0^{\frac{v_A}{s}} x f_A(x) dx + \int_0^{v_B} x f_B(x) dx \\
&\quad + \left(\frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right) \int_0^{\frac{v_A}{s}} \int_0^{v_B} \min\{\lambda_A x, \lambda_B y\} f_B(y) f_A(x) dy dx \\
&= \frac{v_A}{2s} + \frac{v_B}{2s} + \left(\frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right) \int_0^{\frac{v_A}{s}} \left[\int_0^{\frac{\lambda_A x}{\lambda_B}} \lambda_B y dy + \int_{\frac{\lambda_A x}{\lambda_B}}^{v_B} \lambda_A x dy \right] \frac{1}{v_B v_A} dx \\
&= \left(\frac{1}{2s} + \frac{1}{3s^2} \right) v_A + \frac{5}{6s} v_B,
\end{aligned} \tag{A.19}$$

the expected utility of citizen A is:

$$\begin{aligned}
EU_A &= R_A + \int_0^{\frac{v_A}{s}} \left[\int_0^{v_B} u_{DR_A}(x, y) f_B(y) dy \right] f_A(x) dx \\
&= R_A + \int_0^{\frac{v_A}{s}} \left[\frac{1}{s v_B} \left(\int_0^{\frac{\lambda_A x}{\lambda_B}} \left(v_A - x - \frac{\lambda_B y}{\lambda_A} \right) dy - \int_{\frac{\lambda_A x}{\lambda_B}}^{v_B} 2x dy \right) + \frac{s-1}{s} (v_A - x) \right] \frac{s}{v_A} dx \\
&= R_A + v_A \left(1 - \frac{1}{s} - \frac{1}{3s^2} \right),
\end{aligned} \tag{A.20}$$

and the expected utility of citizen B is:

$$\begin{aligned}
EU_B &= R_B + \int_0^{v_B} \left[\int_0^{\frac{v_A}{s}} u_{DR_B}(y, x) f_A(x) dx \right] f_B(y) dy \\
&= R_B + \int_0^{v_B} \left[\int_0^{\frac{\lambda_B y}{\lambda_A}} \left(v_B - y - \frac{\lambda_A x}{\lambda_B} \right) dx - \int_{\frac{\lambda_B y}{\lambda_A}}^{\frac{v_A}{s}} 2y dx \right] \frac{1}{v_A v_B} dy \\
&= R_B - \frac{v_B}{3s}.
\end{aligned} \tag{A.21}$$

Similarly, in a world of democracies, using governments' bidding strategies stated in proposition 1.4.2, we get that country A 's expected military power is:

$$MP_A = \int_0^{\frac{v_A \ln 2}{s}} \lambda_A x f_A(x) dx = (1 - \ln 2) \lambda_B v_B, \quad (\text{A.22})$$

country B 's expected military power is:

$$MP_B = \int_0^{v_B \ln 2} \lambda_B x f_B(x) dx = [s(2^{\frac{1}{s}} - 1) - \ln 2] \lambda_B v_B, \quad (\text{A.23})$$

the size of the war is:

$$\begin{aligned} WS &= \int_0^{\frac{v_A \ln 2}{s}} x f_A(x) dx + \int_0^{v_B \ln 2} x f_B(x) dx \\ &\quad + \left(\frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right) \int_0^{\frac{v_A \ln 2}{s}} \int_0^{v_B \ln 2} \min\{\lambda_A x, \lambda_B y\} f_B(y) f_A(x) dy dx \\ &= (1 - \ln 2) \frac{v_A}{s} + [s(2^{\frac{1}{s}} - 1) - \ln 2] v_B \\ &\quad + \left(\frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right) \frac{2^{1+\frac{1}{s}}}{v_A v_B} \int_0^{\frac{v_A \ln 2}{s}} \left[\int_0^{\frac{\lambda_A x}{\lambda_B}} \lambda_B y e^{-\frac{y}{s v_B}} dy + \int_{\frac{\lambda_A x}{\lambda_B}}^{v_B \ln 2} \lambda_A x e^{-\frac{y}{s v_B}} dy \right] e^{-\frac{s x}{v_A}} dx \\ &= \left[\frac{s}{1+s} \left(1 - 2^{\frac{1}{s}} \right) + \frac{1}{1+s} 2^{\frac{1}{s}} \right] v_A + \left[\frac{s}{1+s} \left(2^{1+\frac{1}{s}} - 1 \right) - 1 \right] v_B, \quad (\text{A.24}) \end{aligned}$$

the expected utility of citizen A is:

$$\begin{aligned}
EU_A &= R_A + \int_0^{\frac{v_A \ln 2}{s}} \left[\int_0^{v_B \ln 2} u_{DR_A}(x, y) f_B(y) dy \right] f_A(x) dx \\
&= R_A + \int_0^{\frac{v_A \ln 2}{s}} \left[2^{\frac{1}{s}} \frac{1}{sv_B} \int_0^{\frac{\lambda_A x}{\lambda_B}} \left(v_A - x - \frac{\lambda_B y}{\lambda_A} \right) e^{-\frac{y}{sv_B}} dy - \right. \\
&\quad \left. 2^{\frac{1}{s}} \frac{1}{sv_B} \int_{\frac{\lambda_A x}{\lambda_B}}^{v_B \ln 2} 2x e^{-\frac{y}{sv_B}} dy + (v_A - x) \left(2 - 2^{\frac{1}{s}} \right) \right] 2 \frac{s}{v_A} e^{-\frac{sx}{v_A}} dx \\
&= R_A + v_A \left(2 - 2^{\frac{1}{s}} \right). \tag{A.25}
\end{aligned}$$

and the expected utility of citizen B is:

$$\begin{aligned}
EU_B &= R_B + \int_0^{v_B \ln 2} \left[\int_0^{\frac{v_A \ln 2}{s}} u_{DR_B}(y, x) f_A(x) dx \right] f_B(y) dy \\
&= R_B + \int_0^{v_B \ln 2} \left[\int_0^{\frac{\lambda_B y}{\lambda_A}} \left(v_B - y - \frac{\lambda_A x}{\lambda_B} \right) \frac{2s}{v_A} e^{-\frac{sx}{v_A}} dx - \right. \\
&\quad \left. \int_{\frac{\lambda_B y}{\lambda_A}}^{\frac{v_A \ln 2}{s}} 2y \frac{2s}{v_A} e^{-\frac{sx}{v_A}} dx \right] 2^{\frac{1}{s}} \frac{1}{sv_B} e^{-\frac{y}{sv_B}} dy = R_B \tag{A.26}
\end{aligned}$$

Comparing these, we get the desired results.

A.4 Proof of Corollary 1.4.2:

It follows from the calculations in the proof of Corollary 1.4.1.

A.5 Proof of Proposition 1.5.1:

For $i \in \{A, B\}$, let us define s_i as the percentage of country i 's maximal military power in the total maximal military power, that is:

$$s_i = \frac{\lambda_i \bar{v}_i}{\lambda_A \bar{v}_A + \lambda_B \bar{v}_B} \quad (\text{A.27})$$

We begin with three observations. First, $b_A(0) = b_B(0) = 0$. Second, since governments' bidding strategies are monotone with non-zero derivative, they are strictly increasing functions, and their inverse is also differentiable everywhere but boundary points. Third,

$$\lambda_A b_A(\bar{v}_A) = \lambda_B b_B(\bar{v}_B), \quad (\text{A.28})$$

because if $\lambda_i b_i(\bar{v}_i) > \lambda_j b_j(\bar{v}_j)$ for some $i \in \{A, B\}$, then government i is better off by deviating to lower values than $b_i(\bar{v}_i)$, which is a contradiction.

For $i \in \{A, B\}$, government i bids 0 if and only if $v_i = 0$, which follows from the first observation. Now we focus on the bids when victory values are positive.

Assume that $v_A > 0$ and $v_B > 0$. Then, we know that the bids are positive. We assume that government B 's bidding strategy, b_B , is differentiable with non-zero derivatives everywhere except at the boundary points, and solve for the bid of government A . From the second observation, it follows that b_B is a strictly increasing differentiable function whose inverse is also differentiable everywhere but boundary points. On the other hand, country A wins the war if and only if government A bids y which is greater than $\frac{\lambda_B b_B(v_B)}{\lambda_A}$, i.e., $v_B < b_B^{-1}(y/k)$ where $k = \frac{\lambda_B}{\lambda_A}$. Hence, given

country A 's victory value is $v_A \in [0, \bar{v}_A]$ and government A bids y , the expected payoff of government A is:

$$\begin{aligned}
EU_{AR_A}(y, v_A) &= \int_0^{b_B^{-1}(\frac{y}{k})} (R_A + v_A - y) \frac{1}{\bar{v}_B} dv_B + \int_{b_B^{-1}(\frac{y}{k})}^{\bar{v}_B} (R_A - y) \frac{1}{\bar{v}_B} dv_B \\
&= R_A + \frac{v_A}{\bar{v}_B} b_B^{-1}(y/k) - y \\
\Rightarrow \frac{dEU_{AR_A}(y, v_A)}{dy} \Big|_{v_A=b_A^{-1}(y)} &= \frac{b_A^{-1}(y)}{\bar{v}_B} \frac{db_B^{-1}(y/k)}{dy} - 1 = 0 \\
\Rightarrow b_A^{-1}(y) \frac{db_B^{-1}(y/k)}{dy} &= \bar{v}_B. \tag{A.29}
\end{aligned}$$

Similarly, given country B 's victory value is $v_B \in [0, \bar{v}_B]$ and government B bids z , the expected payoff of government B is:

$$\begin{aligned}
EU_{AR_B}(z, v_B) &= \int_0^{b_A^{-1}(kz)} (R_B + v_B - z) \frac{1}{\bar{v}_A} dv_A + \int_{b_A^{-1}(kz)}^{\bar{v}_A} (R_B - z) \frac{1}{\bar{v}_A} dv_A \\
&= R_B + \frac{v_B}{\bar{v}_A} b_A^{-1}(kz) - z \\
\Rightarrow \frac{dEU_{AR_B}(z, v_B)}{dz} \Big|_{v_B=b_B^{-1}(z)} &= \frac{b_B^{-1}(z)}{\bar{v}_A} \frac{db_A^{-1}(kz)}{dz} - 1 = 0 \\
\Rightarrow b_B^{-1}(z) \frac{db_A^{-1}(kz)}{dz} &= \bar{v}_A \Rightarrow b_B^{-1}(z) \frac{db_A^{-1}(kz)}{dkz} = \frac{\bar{v}_A}{k},
\end{aligned}$$

and when we replace z with y/k , we get:

$$b_B^{-1}(y/k) \frac{db_A^{-1}(y)}{dy} = \frac{\bar{v}_A}{k}. \tag{A.30}$$

Dividing (A.30) with (A.29) side by side we get:

$$\begin{aligned} \frac{b_B^{-1}(y/k) \frac{db_A^{-1}(y)}{dy}}{b_A^{-1}(y) \frac{db_B^{-1}(y/k)}{dy}} &= \frac{\bar{v}_A}{\bar{v}_B} \Rightarrow \frac{d \ln b_A^{-1}(y)}{dy} = \frac{\bar{v}_A}{k \bar{v}_B} \frac{d \ln b_B^{-1}(y/k)}{dy} \\ \Rightarrow b_B^{-1}(y/k) &= K_0 [b_A^{-1}(y)]^{\frac{k \bar{v}_B}{\bar{v}_A}} \end{aligned} \quad (\text{A.31})$$

where K_0 is a constant. Plugging (A.31) in (A.29), we get:

$$\begin{aligned} b_A^{-1}(y) K_0 \frac{k \bar{v}_B}{\bar{v}_A} [b_A^{-1}(y)]^{\frac{k \bar{v}_B}{\bar{v}_A} - 1} \frac{db_A^{-1}(y)}{dy} &= \bar{v}_B \\ \Rightarrow \frac{db_A^{-1}(y)}{dy} &= \frac{\bar{v}_A}{k K_0} [b_A^{-1}(y)]^{-\frac{k \bar{v}_B}{\bar{v}_A}} \\ \Rightarrow [b_A^{-1}(y)]^{\frac{k \bar{v}_B}{\bar{v}_A}} db_A^{-1}(y) &= \frac{\bar{v}_A}{k K_0} dy \\ \Rightarrow s_A [b_A^{-1}(y)]^{1/s_A} &= \frac{\bar{v}_A}{k K_0} y + \text{constant} \end{aligned} \quad (\text{A.32})$$

Since $b_A^{-1}(0) = 0$, we get $\text{constant} = 0$. Therefore, (A.32) becomes:

$$b_A^{-1}(y) = \left(\frac{\bar{v}_A}{k K_0 s_A} y \right)^{s_A} \quad (\text{A.33})$$

and plugging (A.33) in (A.31), we get:

$$b_B^{-1}(y) = K_0 \left(\frac{\bar{v}_A}{K_0 s_A} y \right)^{s_B} \quad (\text{A.34})$$

From (A.33) and (A.34), it follows that:

$$b_A(x) = \frac{kK_0s_A}{\bar{v}_A}x^{1/s_A} \text{ for } x \in [0, \bar{v}_A], \text{ and} \quad (\text{A.35})$$

$$b_B(x) = \frac{K_0s_A}{\bar{v}_A}\left(\frac{x}{K_0}\right)^{1/s_B} \text{ for } x \in [0, \bar{v}_B]. \quad (\text{A.36})$$

Plugging (A.35) and (A.36) in (A.28), and solving for K_0 we get $K_0 = \bar{v}_B\bar{v}_A^{-s_B/s_A}$.

Therefore, it follows that:

$$b_A(x) = s_B\left(\frac{x}{\bar{v}_A}\right)^{1/s_A}\bar{v}_A \text{ for } x \in [0, \bar{v}_A], \text{ and} \quad (\text{A.37})$$

$$b_B(x) = s_A\left(\frac{x}{\bar{v}_B}\right)^{1/s_B}\bar{v}_B \text{ for } x \in [0, \bar{v}_B]. \quad (\text{A.38})$$

Notice that governments are bidding less than their victory values, i.e., $b_i(x) < x$ for $i \in \{A, B\}$. We conclude the proof by plugging $s_A = \frac{\bar{s}}{1+\bar{s}}$ and $s_B = \frac{1}{1+\bar{s}}$ in (A.37) and (A.38).

A.6 Proof of Corollary 1.5.1:

In a world of autocracies, using governments' bidding strategies stated in proposition 1.5.1, we get that the size of the war is:

$$\begin{aligned}
WS &= \int_0^{\bar{v}_A} \frac{1}{\bar{s}+1} \left(\frac{x}{\bar{v}_A} \right)^{\frac{1+\bar{s}}{\bar{s}}} \bar{v}_A \frac{1}{\bar{v}_A} dx + \int_0^{\bar{v}_B} \frac{\bar{s}}{\bar{s}+1} \left(\frac{x}{\bar{v}_B} \right)^{1+\bar{s}} \bar{v}_B \frac{1}{\bar{v}_B} dx \\
&\quad + \left(\frac{1}{\lambda_A} + \frac{1}{\lambda_B} \right) \left(\int_0^{\bar{v}_A} \left[\int_0^{\bar{v}_B(\frac{x}{\bar{v}_A})^{\frac{1}{\bar{s}}}} \lambda_B \frac{\bar{s}}{1+\bar{s}} \left(\frac{y}{\bar{v}_B} \right)^{1+\bar{s}} \bar{v}_B dy + \right. \right. \\
&\quad \left. \left. \int_{\bar{v}_B(\frac{x}{\bar{v}_A})^{\frac{1}{\bar{s}}}}^{\bar{v}_B} \lambda_A \frac{1}{1+\bar{s}} \left(\frac{x}{\bar{v}_A} \right)^{\frac{1+\bar{s}}{\bar{s}}} \bar{v}_A dy \right] \frac{1}{\bar{v}_A \bar{v}_B} dx \right) \\
&= \frac{\bar{s}(2\bar{s}+7)}{2(\bar{s}+1)(\bar{s}+2)(2\bar{s}+1)} \bar{v}_A + \frac{\bar{s}(7\bar{s}+2)}{2(\bar{s}+1)(\bar{s}+2)(2\bar{s}+1)} \bar{v}_B \tag{A.39}
\end{aligned}$$

When country A has absolute superiority in military technology, it can be easily seen that $\bar{s} \rightarrow \infty$. The desired result follows immediately from $\lim_{\bar{s} \rightarrow \infty} WS = 0$.

A.7 Proof of Proposition 1.5.2:

The three observations in the proof of proposition 1.5.1 still hold. First, $b_A(0) = b_B(0) = 0$. Second, since governments' bidding strategies are monotone with non-zero derivative, they are strictly increasing functions, and their inverse is differentiable everywhere but boundary points. Third, $\lambda_A b_A(\bar{v}_A) = \lambda_B b_B(\bar{v}_B)$, and it becomes $b_A(\bar{v}) = b_B(\bar{v})$ because $\bar{v}_A = \bar{v}_B = \bar{v}$ and $\lambda_A = \lambda_B$. We denote $y^* = b_A(\bar{v}) = b_B(\bar{v})$.

Government i bids 0 if and only $v_i = 0$. Assume $v_A > 0$ and $v_B > 0$. Then, we know that the bids are positive. We assume government B 's bidding strategy, b_B , is differentiable with non-zero derivatives except at the boundary points, and solve for

the bid of government A . From the second observation, we get that b_B is a strictly increasing differentiable function and its inverse is also differentiable everywhere but boundary points. Country A wins the war if and only if government A bids y greater than $b_B(v_B)$, i.e., $v_B < b_B^{-1}(y)$. Hence, given country A 's victory value is $v_A \in [0, \bar{v}]$ and government A bids $y \in (0, y^*]$, the expected payoff of government A is:

$$\begin{aligned}
EU_{DR_A}(y, v_A) &= \int_0^{b_B^{-1}(y)} [R_A + v_A - y - b_B(v_B)] \frac{1}{\bar{v}} dv_B + \int_{b_B^{-1}(y)}^{\bar{v}} (R_A - 2y) \frac{1}{\bar{v}} dv_B \\
&= R_A + \frac{v_A}{\bar{v}} b_B^{-1}(y) - y \left(\frac{2\bar{v} - b_B^{-1}(y)}{\bar{v}} \right) - \int_0^{b_B^{-1}(y)} \frac{b_B(v_B)}{\bar{v}} dv_B \\
&\Rightarrow \frac{dEU_{DR_A}(y, v_A)}{dy} \Big|_{v_A=b_A^{-1}(y)} = \frac{b_A^{-1}(y)}{\bar{v}} \frac{db_B^{-1}(y)}{dy} - 2 + \frac{b_B^{-1}(y)}{\bar{v}} = 0 \\
&\Rightarrow b_A^{-1}(y) \frac{db_B^{-1}(y)}{dy} = 2\bar{v} - b_B^{-1}(y)
\end{aligned} \tag{A.40}$$

Similarly, from symmetry we get:

$$b_B^{-1}(y) \frac{db_A^{-1}(y)}{dy} = 2\bar{v} - b_A^{-1}(y). \tag{A.41}$$

for $y \in (0, y^*]$. First let us show that in equilibrium $b_A^{-1}(y) = b_B^{-1}(y)$ for $y \in (0, y^*)$. Assume that there exists $x \in (0, y^*)$ such that $b_A^{-1}(x) > b_B^{-1}(x)$. Then, dividing equation (A.40) and (A.41) side by side and rearranging we get:

$$\frac{\frac{db_B^{-1}(x)}{dx}}{\frac{db_A^{-1}(x)}{dx}} = \frac{2\bar{v} - b_B^{-1}(x)}{2\bar{v} - b_A^{-1}(x)} \cdot \frac{b_B^{-1}(x)}{b_A^{-1}(x)}$$

Since $b_B^{-1}(x) < b_A^{-1}(x) \leq \bar{v}$, it follows that $\frac{db_B^{-1}(x)}{dx} < \frac{db_A^{-1}(x)}{dx}$. Then, it can be easily seen that $b_B^{-1}(z)$ has to be lower than $b_A^{-1}(z)$ for all $z \geq x$. However, this is a contradiction, because $b_B^{-1}(y^*) = v$ is not lower than $b_A^{-1}(y^*) = v$. We get the same contradiction when there exists $x \in (0, y^*)$ such that $b_A^{-1}(x) < b_B^{-1}(x)$. Therefore, b_A^{-1} and b_B^{-1} are the same function, and we will denote it with g . Replacing b_A^{-1} and b_B^{-1} with g in (A.40), we get:

$$\begin{aligned} g \frac{dg}{dy} &= 2\bar{v} - g \Rightarrow \frac{g}{2\bar{v} - g} dg = dy \\ \Rightarrow \int_{g(0)}^{g(y)} \frac{g}{2\bar{v} - g} dg &= \int_0^y dy \text{ for } y \in [0, y^*] \end{aligned} \quad (\text{A.42})$$

For any $y \in (0, y^*]$, we have $0 < g(y) \leq \bar{v}$, and therefore we can find $\theta \in [0, \pi/2)$ such that $g(y) = 2\bar{v}(1 - \cos \theta)$ (and $dg = 2\bar{v} \sin \theta d\theta$). We pick $\hat{y} \in (0, y^*]$, and denote θ that solves $g(\hat{y}) = 2\bar{v}(1 - \cos \theta)$ with $\hat{\theta}$. Plugging these back into equation (A.42), we get:

$$\begin{aligned} \int_0^{\hat{\theta}} \frac{2\bar{v}(1 - \cos \theta)}{2\bar{v} \cos \theta} 2\bar{v} \sin \theta d\theta &= \int_0^{\hat{y}} dy \\ \Rightarrow 2\bar{v} \int_0^{\hat{\theta}} (\tan \theta - \sin \theta) d\theta &= \hat{y} \\ \Rightarrow 2\bar{v}(\cos \hat{\theta} - \ln \cos \hat{\theta}) - 2\bar{v}(\cos 0 - \ln \cos 0) &= \hat{y} \end{aligned} \quad (\text{A.43})$$

Since $g(\hat{y}) = 2\bar{v}(1 - \cos \hat{\theta})$, we get:

$$\cos \hat{\theta} = \frac{2\bar{v} - g(\hat{y})}{2\bar{v}}. \quad (\text{A.44})$$

Plugging (A.44) back into equation (A.43) and rearranging we get:

$$2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v} - g(\hat{y})} \right) - g(\hat{y}) = \hat{y} \text{ for all } \hat{y} \in (0, y^*] \quad (\text{A.45})$$

For any $t \in (0, \bar{v}]$, we have $b_A(t) = b_B(t) = g^{-1}(t) \in (0, y^*]$. Plugging $g^{-1}(t)$ instead of $\hat{y}$ in (A.45), and remembering $b_A(0) = b_B(0) = 0$, we conclude that:

$$b_A(t) = b_B(t) = g^{-1}(t) = 2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v} - t} \right) - t \text{ for all } t \in [0, \bar{v}].$$

A.8 Proof of Corollary 1.5.2:

In a world of autocracies, using governments' bidding strategies stated in proposition 1.5.1, we get that country A 's expected military power is:

$$MP_A = \lambda_A \int_0^{\bar{v}_A} \frac{1}{\bar{s} + 1} \left(\frac{x}{\bar{v}_A} \right)^{\frac{1+\bar{s}}{\bar{s}}} \bar{v}_A \frac{1}{\bar{v}_A} dx = \frac{\bar{s}^2}{(\bar{s} + 1)(2\bar{s} + 1)} \lambda_B \bar{v}_B \quad (\text{A.46})$$

country B 's expected military power is:

$$MP_B = \lambda_B \int_0^{\bar{v}_B} \frac{\bar{s}}{\bar{s} + 1} \left(\frac{x}{\bar{v}_B} \right)^{1+\bar{s}} \bar{v}_B \frac{1}{\bar{v}_B} dx = \frac{\bar{s}}{(\bar{s} + 1)(\bar{s} + 2)} \lambda_B \bar{v}_B \quad (\text{A.47})$$

the expected utility of citizen A is:

$$\begin{aligned}
EU_A &= R_A + \int_0^{\bar{v}_A} \left[\int_0^{\bar{v}_B \left(\frac{x}{\bar{v}_A} \right)^{\frac{1}{\bar{s}}}} \left(x - \frac{1}{\bar{s}+1} \bar{v}_A \left(\frac{x}{\bar{v}_A} \right)^{\frac{\bar{s}+1}{\bar{s}}} - \frac{\lambda_B \frac{\bar{s}}{\bar{s}+1} \left(\frac{y}{\bar{v}_B} \right)^{\bar{s}+1} \bar{v}_B}{\lambda_A} \right) dy - \right. \\
&\quad \left. \int_{\bar{v}_B \left(\frac{x}{\bar{v}_A} \right)^{\frac{1}{\bar{s}}}}^{\bar{v}_B} 2 \frac{1}{\bar{s}+1} \left(\frac{x}{\bar{v}_A} \right)^{\frac{\bar{s}+1}{\bar{s}}} \bar{v}_A dy \right] \frac{1}{\bar{v}_A \bar{v}_B} dx \\
&= R_A + \bar{v}_A \left[\frac{\bar{s}(\bar{s}-1)}{(2\bar{s}+1)(\bar{s}+1)} + \frac{\bar{s}}{(\bar{s}+1)(2\bar{s}+4)} \right]
\end{aligned} \tag{A.48}$$

and similarly the expected utility of citizen B is:

$$EU_B = R_B + \bar{v}_B \left[\frac{1-\bar{s}}{(\bar{s}+1)(\bar{s}+2)} + \frac{\bar{s}}{(\bar{s}+1)(4\bar{s}+2)} \right] \tag{A.49}$$

On the other hand, in a world of democracies, using governments' bidding strategies stated in proposition 1.4.2, and assuming $\bar{v}_A = \bar{v}_B = \bar{v}$ and $\lambda_A = \lambda_B = \lambda$, we get that country A 's expected military power is equal to country B 's expected military power, which is:

$$\begin{aligned}
MP_A &= MP_B = \lambda \int_0^{\bar{v}} \left[2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v}-x} \right) - x \right] \frac{1}{\bar{v}} dx \\
&= \lambda \left(2\bar{v} \ln(2\bar{v}) - \frac{\bar{v}}{2} - 2 \left[(x-2\bar{v}) \ln(2\bar{v}-x) - x \right]_{x=0}^{x=\bar{v}} \right) \\
&= \left(\frac{3}{2} - 2 \ln 2 \right) \lambda \bar{v},
\end{aligned} \tag{A.50}$$

the size of the war is:

$$\begin{aligned}
WS &= \int_0^{\bar{v}} \left[\int_0^x \left(2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v}-y} \right) - y \right) dy + \int_x^{\bar{v}} \left(2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v}-x} \right) - x \right) dy \right] \frac{1}{\bar{v}^2} dx \\
&= \left(2 \ln 2 - \frac{4}{3} \right) \bar{v},
\end{aligned} \tag{A.51}$$

the expected utility of citizen A is:

$$\begin{aligned}
EU_A &= R_A + \int_0^{\bar{v}} \left[\int_0^x \left(x - 2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v}-x} \right) + x - 2\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v}-y} \right) + y \right) dy + \right. \\
&\quad \left. \int_x^{\bar{v}} \left(2x - 4\bar{v} \ln \left(\frac{2\bar{v}}{2\bar{v}-x} \right) \right) dy \right] \frac{1}{\bar{v}^2} dx = R_A + \frac{\bar{v}}{6},
\end{aligned} \tag{A.52}$$

and similarly the expected utility of citizen B is:

$$EU_B = R_B + \frac{\bar{v}}{6},$$

The result follows from the comparison of these results and (A.39) for $\bar{v}_A = \bar{v}_B = \bar{v}$

and $\lambda_A = \lambda_B = \lambda$.

Appendix B

Proofs for Chapter 2

B.1 Proof of Proposition 2.3.1:

For $i \in \{D_1, D_2\}$, candidate i tries to maximize the number of votes she gets, which is $\frac{q_i Y(k_i)^{n+1}}{c}$. Hence, she tries to maximize the number of voters supporting her, i.e., k_i , and she never benefits from E 's entry. On the other hand, if E chooses to enter the election, hence if her probability of victory is strictly positive, she will choose the policy that maximizes her probability of victory. We complete the proof in three steps.

Step 1: Let us show that a two-candidate equilibrium has to be symmetric. Assume not. Then, there exists a two-candidate equilibrium (d_1, d_2, e) such that $d_1 \neq 1 - d_2$ and $e = \emptyset$. Without loss of generality, let us assume $d_1 > 1 - d_2$. If E decides to enter, she has three choices: locate on the left flank, locate between the dominant candidates, or locate on the right flank. We know that, in a two-candidate equilibrium, E chooses not to enter the election, i.e., her probability of victory is zero

for all policy choices. Therefore, E loses with a policy choice on the left flank, i.e., for all $e \in [0, d_1)$:

$$\left(\frac{e + d_1}{2}\right)^{n+1} \leq q \max \left\{ \left(1 - \frac{d_1 + d_2}{2}\right)^{n+1}, \left(\frac{d_2 - e}{2}\right)^{n+1} \right\} \quad (\text{B.1})$$

From (B.1), it follows that:

$$E \text{ loses on the left flank} \Leftrightarrow (d_1)^{n+1} \leq q \left(1 - \frac{d_1 + d_2}{2}\right)^{n+1}. \quad (\text{B.2})$$

Similarly, comparing the maximum number of votes E will get by locating on the other intervals with the minimum number of votes dominant candidates will get, we get:

$$E \text{ loses on the right flank} \Leftrightarrow (1 - d_2)^{n+1} \leq q \left(\frac{d_1 + d_2}{2}\right)^{n+1}, \quad (\text{B.3})$$

$$E \text{ loses between } D_1 \text{ and } D_2 \Leftrightarrow \text{for all } a \in \left(0, \frac{d_2 - d_1}{2}\right) \\ \left(\frac{d_2 - d_1}{2}\right)^{n+1} \leq q \max \left\{ (d_1 + a)^{n+1}, \left(1 - \frac{d_1 + d_2}{2} - a\right)^{n+1} \right\}. \quad (\text{B.4})$$

We will get a contradiction by showing that $\tilde{d}_2 = 1 - d_1$ is a profitable deviation for D_2 , whose objective is to maximize k_{D_2} . It is obvious that this deviation will increase the number of votes D_2 gets if E chooses not to compete. Hence, it will be enough to show that E 's probability of victory will be zero if he locates on any of the intervals after D_2 's deviation.

When D_2 deviates to $\tilde{d}_2$, E will lose on the left flank since:

$$1 - \frac{d_1 + d_2}{2} < 1 - \frac{d_1 + \tilde{d}_2}{2} \Rightarrow (d_1)^{n+1} \leq q \left(1 - \frac{d_1 + \tilde{d}_2}{2} \right)^{n+1} \quad (\text{B.5})$$

From symmetry, E will also lose on the right flank. Finally, since $\tilde{d}_2 < d_2$, we have $(\frac{\tilde{d}_2 - d_1}{2})^{n+1} < (\frac{d_2 - d_1}{2})^{n+1}$ and $q \cdot \max\{(d_1 + a)^{n+1}, (1 - \frac{d_1 + d_2}{2} - a)^{n+1}\} \leq q \cdot \max\{(d_1 + a)^{n+1}, (1 - \frac{d_1 + \tilde{d}_2}{2} - a)^{n+1}\}$ for all $a \in (0, \frac{\tilde{d}_2 - d_1}{2})$. Combining these with (B.4), we get that for all $a \in (0, \frac{\tilde{d}_2 - d_1}{2})$:

$$\left(\frac{\tilde{d}_2 - d_1}{2} \right)^{n+1} < q \max \left\{ (x + a)^{n+1}, \left(1 - \frac{d_1 + \tilde{d}_2}{2} - a \right)^{n+1} \right\}, \quad (\text{B.6})$$

and E will lose between the dominant candidates. Thus, $\tilde{d}_2$ is a profitable deviation for D_2 , which is a contradiction.

Step 2: Let us show that if a two-candidate equilibrium exists, then $q \in [\underline{q}, 1)$, where $\underline{q} = \left(\frac{\sqrt{17}-3}{2} \right)^{n+1}$.

Assume that (d_1, d_2, e) is a two-candidate equilibrium. Then, from step 1, $d_1 = 1 - d_2$ and:

$$E \text{ loses on the left and right flanks} \Leftrightarrow (d_1)^{n+1} \leq q \left(\frac{1}{2} \right)^{n+1}, \quad (\text{B.7})$$

$$\begin{aligned} E \text{ loses between the dominant candidates} &\Leftrightarrow \text{for all } a \in \left(0, \frac{1 - 2d_1}{2} \right) \\ \left(\frac{1 - 2d_1}{2} \right)^{n+1} &\leq q \max \left\{ (d_1 + a)^{n+1}, \left(\frac{1}{2} - a \right)^{n+1} \right\} \\ \Leftrightarrow \left(\frac{1 - 2d_1}{2} \right)^{n+1} &\leq q \left(\frac{1 + 2d_1}{4} \right)^{n+1}. \end{aligned} \quad (\text{B.8})$$

From (B.7), it follows that:

$$d_1 \leq \frac{q^{\frac{1}{n+1}}}{2}. \quad (\text{B.9})$$

Similarly, from (B.8), it follows that:

$$\frac{2 - 4d_1}{1 + 2d_1} \leq q^{\frac{1}{n+1}} \quad (\text{B.10})$$

Combining (B.9) and (B.10), we get:

$$\max \left\{ 2d_1, \frac{2 - 4d_1}{1 + 2d_1} \right\} \leq q^{\frac{1}{n+1}} \quad (\text{B.11})$$

Let us define a function $f : [0, 1] \rightarrow \mathbb{R}$ such that $f(x) = \max \left\{ 2x, \frac{2-4x}{1+2x} \right\}$. It can be easily shown that $\min \{ f(d_1) : d_1 \in [0, 0.5] \} = f\left(\frac{\sqrt{17}-3}{4}\right) = \frac{\sqrt{17}-3}{2}$. Therefore, if a two-candidate equilibrium exists, then $q \geq \underline{q} = \left(\frac{\sqrt{17}-3}{2}\right)^{n+1}$.

Let us also show that there exists no two-candidate equilibrium for $q = 1$. Assume that (d_1, d_2, e) is two-candidate equilibrium for $q = 1$. First, we show that $d_1 = d_2 = 1/2$. Assume not. Then, from step 1, it follows that $d_1 = 1 - d_2 \neq 1/2$. However, D_1 is better off by deviating to $\tilde{d}_1 = d_1 + \epsilon$ for a small ϵ , which is a contradiction. Hence, $d_1 = d_2 = 1/2$. Finally, if $d_1 = d_2 = 1/2$, E will win the election for sure with a policy choice close to $1/2$ on the left or right flank, which is a contradiction.

Step 3: We will complete the proof by showing that if $q \in [\underline{q}, 1)$, then a two-candidate equilibrium exists and the policy platforms of candidates are $(d_1, d_2, e) = \left(\frac{q^{\frac{1}{n+1}}}{2}, 1 - \frac{q^{\frac{1}{n+1}}}{2}, \emptyset\right)$.

Let (d_1, d_2, e) be a two-candidate equilibrium. From step 1 and (B.9), we get:

$$d_1 = 1 - d_2 \leq \frac{q^{\frac{1}{n+1}}}{2}. \quad (\text{B.12})$$

First, let us assume that $d_1 < \frac{q^{\frac{1}{n+1}}}{2}$. We will get a contradiction by showing that D_1 has a profitable deviation.

It can be easily seen that there exists $\epsilon_1 > 0$ such that

$$d_1 < q^{\frac{1}{n+1}}/2 \Rightarrow (d_1 + \epsilon)^{n+1} < q \left(\frac{1 - \epsilon}{2} \right)^{n+1} \text{ for all } \epsilon < \epsilon_1$$

and therefore if D_1 deviates to $\tilde{d}_1 = d_1 + \epsilon$ for any $\epsilon < \epsilon_1$, E will lose with any policy choice on the left flank. Similarly, if D_1 deviates to $\tilde{d}_1 = d_1 + \epsilon$ for any $\epsilon < \epsilon_1$, right flank will be still shorter than the left flank, and therefore E will lose with any policy choice on the right flank. Let us show that, there exists $\epsilon < \epsilon_1$ such that when D_1 deviates to $\tilde{d}_1 = d_1 + \epsilon$, E will lose with any policy choice between the dominant candidates.

From (B.11), we can easily obtain $\left(\frac{1-2d_1}{2} \right)^{n+1} \leq q \left(\frac{1+2d_1}{4} \right)^{n+1}$. Pick up $\epsilon < \min\{\epsilon_1, 1 - 2d_1\}$. Then, we have:

$$\left(\frac{1 - 2d_1 - \epsilon}{2} \right)^{n+1} \leq q \left(\frac{1 + 2d_1 + \epsilon}{4} \right)^{n+1}. \quad (\text{B.13})$$

Moreover, it can be easily seen that:

$$q \left(\frac{1 + 2d_1 + \epsilon}{4} \right)^{n+1} \leq q \max \left\{ (d_1 + \epsilon + a)^{n+1}, \left(\frac{1 - \epsilon}{2} - a \right)^{n+1} \right\} \quad (\text{B.14})$$

for all $a \in (0, \frac{1-2d_1-\epsilon}{2})$. Combining (B.13) and (B.14), we get:

$$\left(\frac{1 - 2d_1 - \epsilon}{2} \right)^{n+1} \leq q \max \left\{ (d_1 + \epsilon + a)^{n+1}, \left(\frac{1 - \epsilon}{2} - a \right)^{n+1} \right\}$$

for all $a \in (0, \frac{1-2d_1-\epsilon}{2})$. Hence, if D_1 deviates to $\tilde{d}_1 = d_1 + \epsilon$, E will lose with any policy choice between the dominant candidates. As a result of this, it follows that D_1 benefits from a deviation to $\tilde{d}_1 = d_1 + \epsilon$ for $\epsilon \in (0, \min\{\epsilon_1, 1 - 2d_1\})$, which is a contradiction. Hence, our initial assumption is wrong, and $d_1 = \frac{q^{\frac{1}{n+1}}}{2}$. Moreover, if there exists a two-candidate equilibrium, then it is $\left(d_1 = \frac{q^{\frac{1}{n+1}}}{2}, d_2 = 1 - \frac{q^{\frac{1}{n+1}}}{2}, e = \emptyset \right)$. Let us show that these policy platforms constitute a two-candidate equilibrium.

From (B.7) and (B.8), it follows that E 's probability of victory is zero for all policy choices, i.e., E has no profitable deviation. If D_1 deviates to a policy greater than d_1 , then E will choose to enter (because there will be a positive probability of victory for her on the left flank) and D_1 will get less votes (and will lose the election for sure). Therefore, D_1 does not deviate to policies greater than d_1 . D_1 does not deviate to a policy smaller than d_1 , since deviating towards the left flank will decrease the number of votes she gets. Similarly, D_2 does not have any profitable deviation. Thus, we conclude that $\left(d_1 = \frac{q^{\frac{1}{n+1}}}{2}, d_2 = 1 - \frac{q^{\frac{1}{n+1}}}{2}, e = \emptyset \right)$ is a two-

candidate equilibrium, and $q \in [\underline{q}, 1)$ is the unique necessary and sufficient condition for these policy platforms being a two-candidate equilibrium.

B.2 Proof of Proposition 2.4.1:

First, let us do an observation about the objective of dominant candidates. For $i \in \{D_1, D_2\}$, candidate i wants to maximize the expected number of votes he gets, i.e., $\frac{1+q}{2} N k_{-i} k_i$. Hence, dominant candidates try to increase the number of their supporters up to $1/2$ and does not care about E 's entry as long as it does not affect the number of their supporters.

Lemma B.2.1. *All two-candidate equilibria are symmetric.*

Proof of Lemma B.2.1: Assume not. Then, there exists a two-candidate equilibrium such that $d_2 \neq 1 - d_1$. Without loss of generality, let us assume $d_2 > 1 - d_1$.¹ It can be easily seen that $d_1 < 1/2 < d_2$.² We know that if E locates on the left flank, she will lose, i.e.:

$$\frac{e + d_1}{2} \left(1 - \frac{e + d_1}{2}\right) \leq q \max \left\{ \frac{d_2 - e}{2} \left(1 - \frac{d_2 - e}{2}\right), \left(1 - \frac{d_1 + d_2}{2}\right) \frac{d_1 + d_2}{2} \right\}$$

for all $e \in [0, d_1)$ and hence it follows that:

$$d_1(1 - d_1) \leq q \left(1 - \frac{d_1 + d_2}{2}\right) \frac{d_1 + d_2}{2}. \quad (\text{B.15})$$

¹Notice that if $(d_1, d_2, e = \emptyset)$ is an equilibrium, then $(1 - d_2, 1 - d_1, e = \emptyset)$ will constitute an equilibrium as well; and if $d_2 < 1 - d_1$, then $(1 - d_2) > 1 - (1 - d_1)$.

²Because, otherwise E can win the election by locating on the flanks, which is a contradiction.

We know that $1 - d_1 < d_2$, and the right hand side of (B.15) increases with decreasing $d_2 \in [1 - d_1, 1]$. Hence, we have:

$$q \left(1 - \frac{d_1 + d_2}{2} \right) \frac{d_1 + d_2}{2} \leq q \frac{1}{4}. \quad (\text{B.16})$$

Combining (B.15) and (B.16), we obtain:

$$d_1(1 - d_1) \leq q \frac{1}{4}. \quad (\text{B.17})$$

On the other hand, E loses between the dominant candidates, that is, her probability of victory is zero when she chooses $e \in (d_1, d_2)$. Hence, we have:

$$k_E(d_1, d_2, e)(1 - k_E(d_1, d_2, e)) \leq qg(d_1, d_2, e), \quad (\text{B.18})$$

for all $e \in (d_1, d_2)$, where $k_j(d_1, d_2, e)$ is the number of voters supporting candidate $j \in \{D_1, D_2, E\}$ and $g(d_1, d_2, e) = \max\{k_i(d_1, d_2, e)(1 - k_i(d_1, d_2, e)) : i \in \{D_1, D_2\}\}$.

Then, plugging $e = \frac{1}{2}$ in (B.18), we obtain:

$$\frac{d_2 - d_1}{2} \left(1 - \frac{d_2 - d_1}{2} \right) \leq q \frac{1 + 2d_1}{4} \frac{3 - 2d_1}{4} \quad (\text{B.19})$$

Since $1 - d_1 < d_2$ and the left hand side of (B.19) decreases with decreasing $d_2 \in [1 - d_1, 1]$, we have:

$$\frac{1 - 2d_1}{2} \left(\frac{1 + 2d_1}{2} \right) \leq \frac{d_2 - d_1}{2} \left(1 - \frac{d_2 - d_1}{2} \right). \quad (\text{B.20})$$

Combining (B.19) and (B.20) we obtain:

$$\frac{1-2d_1}{2} \left(\frac{1+2d_1}{2} \right) \leq q \frac{1+2d_1}{4} \frac{3-2d_1}{4} \quad (\text{B.21})$$

We will complete the proof by showing that D_2 is better off by deviating to $\tilde{d}_2 = 1-d_1$.

If E 's entry is still deterred after D_2 's deviation, then D_2 's voter support will increase to $1/2$, and she will be better off. Hence, it will be enough to show that E will lose if she locates on any of the intervals after D_2 's deviation.

From (B.17), it follows that:

$$d_1(1-d_1) \leq q \frac{1}{4} = q \left(1 - \frac{d_1 + \tilde{d}_2}{2} \right) \frac{d_1 + \tilde{d}_2}{2}. \quad (\text{B.22})$$

Therefore, E will lose if she locates on the flanks when dominant candidates' policy platforms are d_1 and $\tilde{d}_2$. Moreover, it can be easily seen that:

$$\min\{g(d_1, \tilde{d}_2, \hat{e}) : \hat{e} \in (d_1, \tilde{d}_2)\} = g\left(d_1, \tilde{d}_2, \frac{1}{2}\right) \quad (\text{B.23})$$

where $g\left(d_1, \tilde{d}_2, \frac{1}{2}\right) = \frac{1+2d_1}{4} \frac{3-2d_1}{4}$, and

$$k_E(d_1, \tilde{d}_2, \tilde{e}) = \frac{1-2d_1}{2} \text{ for all } \tilde{e} \in (d_1, \tilde{d}_2). \quad (\text{B.24})$$

From (B.23), (B.24) and (B.21), we get:

$$k_E(d_1, \tilde{d}_2, \tilde{e})(1 - k_E(d_1, \tilde{d}_2, \tilde{e})) \leq qg(d_1, \tilde{d}_2, \tilde{e}), \quad (\text{B.25})$$

for all $\tilde{e} \in (d_1, \tilde{d}_2)$. Therefore, E will lose if she locates between the dominant candidates when their policy platforms are d_1 and $\tilde{d}_2$. We conclude that D_2 is better off by deviating to $\tilde{d}_2 = 1 - d_1$, which is a contradiction. ■

Now let us find the necessary and sufficient conditions for the existence of a two-candidate equilibrium. Assume that $(d_1, d_2, e = \emptyset)$ is a two candidate equilibrium. Then, from lemma B.2.1, $d_1 = 1 - d_2$. Moreover, $d_1 = 1 - d_2 < 1/2$, because E will locate on the left or right flank and win with a positive probability for $d_1 = 1 - d_2 = 1/2$, which is a contradiction. Dominant candidates are satisfied with any symmetric policy pair that prevents E 's entry.³ Hence, there isn't any profitable deviation for the dominant candidates, and finding the conditions for the deterrence of E 's entry will complete the proof.

From (B.15), it follows that E 's probability of victory is zero for all policy choices on the left flank if and only if:

$$d_1(1 - d_1) \leq q \left(1 - \frac{d_1 + d_2}{2} \right) \frac{d_1 + d_2}{2} = \frac{q}{4}. \quad (\text{B.26})$$

Similarly, E 's probability of victory is zero for all policy choices on the right flank if and only if:

$$d_2(1 - d_2) \leq \frac{q}{4}. \quad (\text{B.27})$$

³The expected number of dominant candidates get is maximized in symmetric two-candidate equilibria.

Finally, her probability of victory is zero for all policy choices between the dominant candidates if and only if (B.18) holds for all $e \in (d_1, d_2)$. E 's voter support is:

$$k_E(d_1, d_2, e) = \frac{1 - 2d_1}{2} \text{ for all } e \in (d_1, d_2), \text{ and} \quad (\text{B.28})$$

$$\min\{g(d_1, d_2, \hat{e}) : \hat{e} \in (d_1, d_2)\} = g(d_1, d_2, 1/2). \quad (\text{B.29})$$

Hence, E 's probability of victory is zero for all $e \in (d_1, d_2)$ if and only if:

$$\frac{1 - 2d_1}{2} \frac{1 + 2d_1}{2} \leq q \frac{1 + 2d_1}{4} \left(1 - \frac{1 + 2d_1}{4}\right) \quad (\text{B.30})$$

From these observations, we infer that $(d_1, d_2 = 1 - d_1, e = \emptyset)$ constitutes an equilibrium for the values of d_1 that satisfy (B.26) and (B.30). Finding the roots of $d_1(1 - d_1) = \frac{q}{4}$ and $\frac{1 - 2d_1}{2} \frac{1 + 2d_1}{2} = q \frac{1 + 2d_1}{4} \left(1 - \frac{1 + 2d_1}{4}\right)$, and taking into account that $d_1 \leq \frac{1}{2}$, we get:

$$(\text{B.26}) \Leftrightarrow d_1 \leq \frac{1 - \sqrt{1 - q}}{2} \quad (\text{B.31})$$

$$(\text{B.30}) \Leftrightarrow d_1 \geq \frac{4 - 3q}{8 - 2q} \quad (\text{B.32})$$

Combining (B.31) and (B.32), it follows that $(d_1, d_2 = 1 - d_1, e = \emptyset)$ constitutes an equilibrium if and only if:

$$\frac{4 - 3q}{8 - 2q} \leq d_1 \leq \frac{1 - \sqrt{1 - q}}{2}, \quad (\text{B.33})$$

and a two-candidate equilibrium exists if and only if:

$$\frac{4-3q}{8-2q} \leq \frac{1-\sqrt{1-q}}{2}. \quad (\text{B.34})$$

Arranging the terms and taking the square we get:

$$(\text{B.34}) \Leftrightarrow q^3 - 5q^2 + 24q - 16 \geq 0. \quad (\text{B.35})$$

$q^3 - 5q^2 + 24q - 16 = 0$ has a unique real root, which we denote with $\underline{\underline{q}}$ and is approximately 0.772, and (B.35) holds if and only if $q \geq \underline{\underline{q}}$. Hence, a two-candidate equilibrium exists if and only if $q \geq \underline{\underline{q}}$, and the set of two-candidate equilibria is $\left\{ (d_1, d_2, e) : d_1 \in \left[\frac{4-3q}{8-2q}, \frac{1-\sqrt{1-q}}{2} \right], d_2 = 1 - d_1, e = \emptyset \right\}$.

Appendix C

Proofs for Chapter 3

C.1 Proof of Proposition 3.3.1:

We solve the model backwards. First, given candidates' policy choices, and hence given fixed voter groups and fixed importance of the election for ethical voters, we solve for ethical voters' strategies.

Lemma C.1.1. ¹ Assume k_A, k_B, w_A and w_B are fixed, and without loss of generality $k_i w_j \geq k_j w_i$. Let f be the density function of q_A/q_B .² Then, the consistent strategy profile followed by ethical voters is:

¹This lemma is an extended version of the consistent profiles in Feddersen and Sandroni [34]. The importance of the election for all ethical voters are equal in their model, whereas they are not necessarily equal in our model.

²The cumulative distribution and density function of $\frac{q_A}{q_B}$ and q_B/q_A , which we denote with F and f , respectively, are:

$$F(x) = \begin{cases} P\left(\frac{q_i}{q_i} \leq x\right) = \int_0^1 x q_i \partial q_i = \frac{x}{2} & \text{if } x \leq 1, \\ P\left(\frac{q_i}{q_i} \leq x\right) = \int_0^{1/x} x q_i \partial q_i + \int_{1/x}^1 \partial q_i = 1 - \frac{1}{2x} & \text{if } x > 1, \end{cases} \quad (\text{C.1})$$

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } x \leq 1, \\ \frac{1}{2x^2} & \text{if } x > 1. \end{cases} \quad (\text{C.2})$$

$$\begin{aligned}
(\sigma_i^*, \sigma_j^*) &= \left(\sqrt[4]{\frac{4f^2 \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) w_i^3}{k_i k_j w_j c^2}}, \sqrt[4]{\frac{4f^2 \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) k_i w_i w_j}{k_j^3 c^2}} \right) \text{ if } c > \frac{2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \sqrt{k_i w_i w_j}}{\sqrt{k_j^3}}, \\
(\sigma_i^*, \sigma_j^*) &= \left(\sqrt[3]{\frac{k_j w_i}{c k_i^2}}, 1 \right) \text{ if } \frac{k_j w_i}{k_i^2} < c \leq \min \left\{ 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}}, \frac{k_i w_i}{k_j^2} \right\}, \\
(\sigma_i^*, \sigma_j^*) &= \left(\frac{w_i}{c k_j}, 1 \right) \text{ if } \max \left\{ \frac{w_i}{k_j}, \frac{k_i w_i}{k_j^2} \right\} < c \leq 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}}, \text{ and} \\
(\sigma_i^*, \sigma_j^*) &= (1, 1) \text{ if } c \leq \min \left\{ \max \left\{ \frac{w_i}{k_j}, \frac{k_i w_i}{k_j^2} \right\}, \frac{k_j w_i}{k_i^2}, 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}} \right\}.
\end{aligned}$$

Proof of Lemma C.1.1: Assume that ethical agents follow the cut-off strategy (σ_A, σ_B) . Candidate $i \in \{A, B\}$ wins the election if he receives the majority of votes, i.e., if:

$$q_i k_i \sigma_i \geq q_j k_j \sigma_j \Leftrightarrow \frac{q_j}{q_i} \leq \frac{k_i \sigma_i}{k_j \sigma_j}.$$

Hence, the probability of candidate i 's victory is:

$$p_i = F \left(\frac{k_i \sigma_i}{k_j \sigma_j} \right). \quad (\text{C.3})$$

The total cost of voting for voters in group i is:³

$$\phi_i = c k_i E(q_i) \int_0^{\sigma_i} x \partial x = c k_i E(q_i) \frac{\sigma_i^2}{2}. \quad (\text{C.4})$$

Plugging (C.3) and (C.4) in (3.1), we get:

$$u_i = w_i F \left(\frac{k_i \sigma_i}{k_j \sigma_j} \right) - c k_i E(q_i) \frac{\sigma_i^2}{2}. \quad (\text{C.5})$$

³If the voters were rule utilitarian, the total cost of voting for voters in group i would have been $\phi_i(\sigma_i, \sigma_{-i}) = \frac{c}{2} (k_i E(q_i) \sigma_i^2 + k_{-i} E(q_{-i}) \sigma_{-i}^2)$.

The objective of ethical voters in group i is to maximize u_i . Hence, the first order condition is:

$$w_i f\left(\frac{k_i \sigma_i}{k_j \sigma_j}\right) \frac{k_i}{k_j \sigma_j} - c k_i E(q_i) \sigma_i \begin{cases} = 0 & \text{if } \sigma_i \in (0, 1), \\ \geq 0 & \text{if } \sigma_i = 1. \end{cases} \quad (\text{C.6})$$

Similarly, the first order condition of the maximization problem of ethical voters in group j is:

$$w_j f\left(\frac{k_i \sigma_i}{k_j \sigma_j}\right) \frac{k_i \sigma_i}{k_j \sigma_j^2} - c k_j E(q_j) \sigma_j \begin{cases} = 0 & \text{if } \sigma_j \in (0, 1), \\ \geq 0 & \text{if } \sigma_j = 1. \end{cases} \quad (\text{C.7})$$

Assume $\sigma_i, \sigma_j \in (0, 1)$. Then, plugging in $E(q_i) = E(q_j) = \frac{1}{2}$, and solving (C.6) and (C.7) together, we obtain:

$$\frac{w_i \sigma_j}{w_j \sigma_i} = \frac{k_i \sigma_i}{k_j \sigma_j} \Rightarrow k_i \sigma_i = k_j \sigma_j \sqrt{\frac{w_i k_i}{w_j k_j}}. \quad (\text{C.8})$$

Plugging (C.8) in (C.6) and rearranging, we get:

$$w_i f\left(\sqrt{\frac{w_i k_i}{w_j k_j}}\right) k_i = \frac{c}{2} k_j^2 \sigma_j^2 \sqrt{\frac{w_i k_i}{w_j k_j}} \Rightarrow \sigma_j = \frac{\sqrt{2f\left(\sqrt{\frac{w_i k_i}{w_j k_j}}\right)} \sqrt[4]{k_i w_i w_j}}{\sqrt[4]{k_j^3} \sqrt{c}} \quad (\text{C.9})$$

Plugging (C.9) in (C.8), we obtain:

$$\sigma_i = \frac{\sqrt{2f\left(\sqrt{\frac{w_i k_i}{w_j k_j}}\right)} \sqrt[4]{w_i^3}}{\sqrt[4]{k_i k_j w_j} \sqrt{c}}. \quad (\text{C.10})$$

From $k_i w_j \geq k_j w_i$, it follows that $\sigma_i = \sigma_j \sqrt{\frac{w_i k_j}{w_j k_i}} \leq \sigma_j$. Hence:

$$\sigma_i, \sigma_j \in (0, 1) \Leftrightarrow 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}} < c. \quad (\text{C.11})$$

Therefore, if (C.11) holds, the consistent rule profile followed by ethical voters will be given by (C.9) and (C.10).

On the other hand,

$$\sigma_j = 1 \Leftrightarrow c \leq 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}}. \quad (\text{C.12})$$

Given $\sigma_j = 1$, if $\sigma_i < 1$, then from (C.6), we will get:

$$w_i f \left(\frac{k_i \sigma_i}{k_j} \right) \frac{k_i}{k_j} = \frac{c k_i \sigma_i}{2} \Rightarrow \sigma_i = \begin{cases} \sqrt[3]{\frac{k_j w_i}{c k_i^2}} & \text{if } k_i \sigma_i \geq k_j, \\ \frac{w_i}{c k_j} & \text{if } k_i \sigma_i < k_j, \end{cases} \quad (\text{C.13})$$

Using (C.12) in (C.13), it follows that:

$$\sigma_j = 1 \text{ and } \sigma_i < 1 \Rightarrow \sigma_i = \begin{cases} \sqrt[3]{\frac{k_j w_i}{c k_i^2}} & \text{if } \frac{k_j w_i}{k_i^2} < c \leq \frac{k_i w_i}{k_j^2}, \\ \frac{w_i}{c k_j} & \text{if } \max\left\{\frac{w_i}{k_j}, \frac{k_i w_i}{k_j^2}\right\} < c. \end{cases} \quad (\text{C.14})$$

Therefore, we conclude:

$$\sigma_i = \begin{cases} \sqrt[3]{\frac{k_j w_i}{c k_i^2}} & \text{if } \frac{k_j w_i}{k_i^2} < c \leq \min \left\{ 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}}, \frac{k_i w_i}{k_j^2} \right\}, \\ \frac{w_i}{c k_j} & \text{if } \max \left\{ \frac{w_i}{k_j}, \frac{k_i w_i}{k_j^2} \right\} < c \leq 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}}, \\ 1 & \text{if } c \leq \min \left\{ \max \left\{ \frac{w_i}{k_j}, \frac{k_i w_i}{k_j^2} \right\}, \frac{k_j w_i}{k_i^2}, 2f \left(\sqrt{\frac{w_i k_i}{w_j k_j}} \right) \frac{\sqrt{k_i w_i w_j}}{\sqrt{k_j^3}} \right\}. \blacksquare \end{cases}$$

When we plug the consistent strategy profiles from Lemma C.1.1 in (C.3), it follows that candidates try to increase the size of their voter groups to fulfil their objective, that is, to maximize their probability of victory. As a result of this, candidates converge each other and choose the median voter's optimal policy. Therefore, we get $k_A = k_B = \frac{1}{2}$. Plugging $k_A = k_B = \frac{1}{2}$ in Lemma C.1.1 concludes the proof.

C.2 Proof of Proposition 3.4.1:

Lemma C.2.1. *Assume candidates' policy choices are fixed, the importance of the election for ethical voters are endogenous variables, and without loss of generality $k_i \geq k_j$. Then, the consistent strategy profile followed by ethical voters is:*

$$(\sigma_i^*, \sigma_j^*) = \begin{cases} \left(\frac{\sqrt[4]{k_i k_j}}{k_i \sqrt{c}} \sqrt[4]{w_i w_j}, \frac{1}{\sqrt[4]{k_i k_j} \sqrt{c}} \frac{w_j}{\sqrt[4]{w_i w_j}} \right) & \text{if } \sqrt{\frac{w_j^3}{k_i k_j w_i}} \leq c, \\ \left(\sqrt[3]{\frac{k_j w_i}{c k_i^2}}, 1 \right) & \text{if } \frac{k_j w_i}{k_i^2} < c < \sqrt{\frac{w_j^3}{k_i k_j w_i}}, \\ (1, 1) & \text{if } c \leq \frac{k_j w_i}{k_i^2}. \end{cases}$$

Proof of Lemma C.2.1: First, we will show that if $k_i \geq k_j$, then $k_i w_j \geq k_j w_i$.

Let us assume $k_i \geq k_j$. Then, either (i) $1 - x_j \leq x_i \leq x_j$, or (ii) $x_j \leq x_i \leq 1 - x_j$. The proof for both cases follow the same steps. Therefore, we assume $1 - x_j \leq x_i \leq x_j$.

If $\alpha \leq 1 - x_j$, then from (3.10), we get:⁴

$$\begin{aligned}
k_i w_i &= \int_{S_i} g(|x - x_i|) - g(|x - x_j|) dx \\
&= \begin{cases} 2 \int_0^{\frac{x_j - x_i}{2}} g(x) dx & \text{if } \frac{x_j - x_i}{2} < \alpha \\ 2 \int_0^\alpha g(x) dx & \text{if } \frac{x_j - x_i}{2} \geq \alpha \end{cases} \\
&= \int_{S_j} g(|x - x_j|) - g(|x - x_i|) dx = k_j w_j \tag{C.15}
\end{aligned}$$

Then, since $k_i \geq k_j$, we obtain $w_i \leq w_j$, and it follows that $k_i w_j \geq k_j w_i$.

Otherwise, if $1 - x_j \leq \alpha$, then, from the graph of $k_j w_j$, it will be easily seen that:

$$\begin{aligned}
k_j w_j &= \int_{S_j} g(|x - x_j|) - g(|x - x_i|) dx \geq k_j \frac{g(0) - g(|x_j - x_i|/2)}{2} \\
\Rightarrow w_j &\geq \frac{g(0) - g(|x_j - x_i|/2)}{2}. \tag{C.16}
\end{aligned}$$

Similarly, from the graph of $k_i w_i$, it can be easily seen that:

$$k_i w_i = \int_{S_i} g(|x - x_i|) - g(|x - x_j|) dx \leq k_j w_j + (k_i - k_j)[g(0) - g(|x_j - x_i|/2)]$$

⁴Throughout the proof of Lemma C.2.1 and Proposition 3.4.1, if the graph of the integration given by (3.10) is visualized, then it will be much more simple to understand the computations of w_i and w_j (or, similarly, $k_i w_i$ and $k_j w_j$).

Using (C.16) in this inequality, we get:

$$k_i w_i \leq k_j w_j + 2(k_i - k_j)w_j = (2k_i - k_j)w_j \quad (\text{C.17})$$

Multiplying both sides of (C.17) with k_j/k_i and noticing that $k_j(2k_i - k_j) \leq k_i^2$ (which follows from the inequality of arithmetic and geometric means), we obtain $k_j w_i \leq k_i w_j$.

Thus, we conclude that:

$$k_i \geq k_j \Rightarrow k_i w_i \geq k_j w_i \text{ for } 0 < \alpha \leq 1. \quad (\text{C.18})$$

Since $k_i w_i = \int_{S_i} g(|x - x_i|) - g(|x - x_j|) dx$, it follows that:

$$\begin{aligned} k_i \geq k_j &\Rightarrow k_i w_i = k_j w_j + \int_0^{x_i + x_j - 1} g(|x - x_i|) - g(|x - x_j|) dx \\ &\Rightarrow k_i w_i \geq k_j w_j \end{aligned} \quad (\text{C.19})$$

Plugging (C.18) and (C.19) in Lemma C.1.1, and rearranging, we get the desired result. ■

Let (x_i, x_j) be Nash equilibrium (NE), and let the short notations of functions stand for the value of the corresponding function at (x_i, x_j) , i.e., $k_i = k_i(x_i, x_j)$, $w_i = w_i(x_i, x_j)$, $p_i = p_i(x_i, x_j)$, and $u_i = u_i(x_i, x_j)$ for $i \in \{A, B\}$. Without loss of generality, let us assume that $k_i \geq k_j$, and $x_i \leq x_j$. The probability of candidate i 's

victory is given by (C.3), that is $p_i = F\left(\frac{k_i\sigma_i^*}{k_j\sigma_j^*}\right)$. From Lemma C.2.1, we get:

$$\frac{k_i\sigma_i^*}{k_j\sigma_j^*} = \begin{cases} \sqrt{\frac{k_i w_i}{k_j w_j}} & \text{if } \sqrt{\frac{w_j^3}{k_i k_j w_i}} \leq c, \\ \sqrt[3]{\frac{k_i w_i}{c k_j^2}} & \text{if } \frac{k_j w_i}{k_i^2} < c < \sqrt{\frac{w_j^3}{k_i k_j w_i}}, \\ \frac{k_i}{k_j} & \text{if } c \leq \frac{k_j w_i}{k_i^2}. \end{cases} \quad (\text{C.20})$$

Each candidate has the option to choose the same policy with the other candidate and to win the election with %50 probability. Therefore, both candidates are equally likely to win in the NE, and it follows that:

$$\begin{aligned} p_i = F\left(\frac{k_i\sigma_i^*}{k_j\sigma_j^*}\right) = \frac{1}{2} &\Rightarrow \frac{k_i\sigma_i^*}{k_j\sigma_j^*} = 1 \Rightarrow (I) \ k_i w_i = k_j w_j \text{ if } \sqrt{\frac{w_j^3}{k_i k_j w_i}} \leq c, \\ (II) \ c &= \frac{k_i w_i}{k_j^2} \text{ if } \frac{k_j w_i}{k_i^2} < c < \sqrt{\frac{w_j^3}{k_i k_j w_i}}, \\ (III) \ k_i &= k_j \text{ if } c \leq \frac{k_j w_i}{k_i^2}. \end{aligned}$$

From (C.19), it follows that:

$$k_i \geq k_j \Rightarrow k_i w_i \geq k_j w_j \Rightarrow (k_i w_i)^{3/2} \geq (k_j w_j)^{3/2} \Rightarrow \frac{k_i w_i}{k_j^2} \geq \sqrt{\frac{w_j^3}{k_i k_j w_i}}.$$

Therefore, (II) is not possible. Assume $c < \sqrt{\frac{w_j^3}{k_i k_j w_i}}$. Then, (III) will hold, and we will $x_i = 1 - x_j < \frac{1}{2}$.⁵ If candidate i deviates to $\tilde{x}_i = x_i + \epsilon$ for a very small ϵ , we will have $c < \sqrt{\frac{w_j^3(x_j, \tilde{x}_i)}{k_i(\tilde{x}_i, x_j)k_j(x_j, \tilde{x}_i)w_i(\tilde{x}_i, x_j)}}$, $k_i(\tilde{x}_i, x_j) > k_j(\tilde{x}_i, x_j)$ and

⁵The importance of the election is zero for all ethical voters when $x_i = 1 - x_j = \frac{1}{2}$, and it is not possible for $c < \sqrt{\frac{w_j^3}{k_i k_j w_i}}$.

$k_i(\tilde{x}_i, x_j)w_i(\tilde{x}_i, x_j) \geq k_j(\tilde{x}_i, x_j)w_j(\tilde{x}_i, x_j)$. Using these in (C.20) and (C.3), and rearranging we get $p_i(\tilde{x}_i, x_j) > \frac{1}{2}$, which is a contradiction. Hence, (I) holds in the NE, that is:

$$(x_i, x_j) \text{ is a NE} \Rightarrow c \geq \sqrt{\frac{w_j^3}{k_i k_j w_i}} \text{ and } k_i w_i = k_j w_j. \quad (\text{C.21})$$

Case 1: Assume $\alpha \geq \frac{1}{2}$. Then, from (C.19) and (C.21), it follows that:

$$k_i w_i = k_j w_j \Rightarrow k_i = k_j \Rightarrow x_i = 1 - x_j.$$

Assume $x_i = 1 - x_j < \frac{1}{2}$. If candidate i deviates to $x'_i = x_i + \epsilon < 1 - x_j$ for a small $\epsilon > 0$, then we get:

$$k_i(x'_i, x_j) > k_j(x_j, x'_i), \quad (\text{C.22})$$

Using (C.22) in (C.19), we obtain:

$$k_i(x'_i, x_j)w_i(x'_i, x_j) > k_j(x_j, x'_i)w_j(x_j, x'_i). \quad (\text{C.23})$$

If $\sqrt{\frac{w_j^3(x_j, x'_i)}{k_i(x'_i, x_j)k_j(x_j, x'_i)w_i(x'_i, x_j)}} \leq c$, from (C.20) and (C.23), we will get:

$$\frac{k_i(x'_i, x_j)\sigma_i^*(x'_i, x_j)}{k_j(x'_i, x_j)\sigma_j^*(x'_i, x_j)} = \sqrt{\frac{k_i(x'_i, x_j)w_i(x'_i, x_j)}{k_j(x_j, x'_i)w_j(x_j, x'_i)}} > 1. \quad (\text{C.24})$$

Otherwise, if $\frac{k_j(x_j, x'_i)w_i(x'_i, x_j)}{k_i^2(x'_i, x_j)} < c < \sqrt{\frac{w_j^3(x_j, x'_i)}{k_i(x'_i, x_j)k_j(x_j, x'_i)w_i(x'_i, x_j)}}$, then from (C.20) and (C.23), we will obtain:

$$\frac{k_i(x'_i, x_j)\sigma_i^*(x'_i, x_j)}{k_j(x'_i, x_j)\sigma_j^*(x'_i, x_j)} = \sqrt[3]{\frac{k_i(x'_i, x_j)w_i(x'_i, x_j)}{ck_j^2(x_j, x'_i)}} > \sqrt{\frac{k_i(x'_i, x_j)w_i(x'_i, x_j)}{k_j(x_j, x'_i)w_j(x_j, x'_i)}} > 1. \quad (\text{C.25})$$

Finally, if $c \leq \frac{k_j(x_j, x'_i)w_i(x'_i, x_j)}{k_i^2(x'_i, x_j)}$, then from (C.20) and (C.22), we will get:

$$\frac{k_i(x'_i, x_j)\sigma_i^*(x'_i, x_j)}{k_j(x'_i, x_j)\sigma_j^*(x'_i, x_j)} = \frac{k_i(x'_i, x_j)}{k_j(x_j, x'_i)} > 1. \quad (\text{C.26})$$

Combining (C.24), (C.25) and (C.26) we conclude that:

$$p_i(x'_i, x_j) = F\left(\frac{k_i(x'_i, x_j)\sigma_i^*(x'_i, x_j)}{k_j(x'_i, x_j)\sigma_j^*(x'_i, x_j)}\right) > F(1) = \frac{1}{2}.$$

Thus, x'_i is a profitable deviation for candidate i , a contradiction.

Now, let us show that $x_i = x_j = \frac{1}{2}$ is a NE. If candidate i deviates to $x''_i \neq \frac{1}{2}$, then:

$$\begin{aligned} k_i(x''_i, x_j) < k_j(x_j, x''_i) &\Rightarrow k_i(x''_i, x_j)w_i(x''_i, x_j) < k_j(x_j, x''_i)w_j(x_j, x''_i) \\ &\Rightarrow \frac{k_i(x''_i, x_j)\sigma_i^*(x''_i, x_j)}{k_j(x_j, x''_i)\sigma_j^*(x_j, x''_i)} < 1 \\ &\Rightarrow p_i(x''_i, x_j) < \frac{1}{2}. \end{aligned}$$

Therefore, there is no profitable deviation for any of the candidates, and $x_i = x_j = \frac{1}{2}$

is a unique NE for $\alpha \geq \frac{1}{2}$.

Case 2: Assume $\alpha < \frac{1}{2}$. If $x_i < \alpha$, then from (C.19) and (C.21), it follows that $x_i = 1 - x_j$.⁶ When candidate i deviates to $\hat{x}_i = x_i + \epsilon < x_j$, we will have:

$$\begin{aligned} k_i(\hat{x}_i, x_j) > k_j(\hat{x}_i, x_j) &\Rightarrow k_i(\hat{x}_i, x_j)w_i(\hat{x}_i, x_j) > k_j(x_j, \hat{x}_i)w_j(x_j, \hat{x}_i) \\ &\Rightarrow \frac{k_i(\hat{x}_i, x_j)\sigma_i^*(\hat{x}_i, x_j)}{k_j(x_j, \hat{x}_i)\sigma_j^*(x_j, \hat{x}_i)} > 1 \\ &\Rightarrow p_i(\hat{x}_i, x_j) > \frac{1}{2}. \end{aligned}$$

Hence, $\hat{x}_i$ is a profitable deviation for candidate i , which is a contradiction. Since candidates are symmetric, we conclude that:

$$\alpha \leq x_i \leq x_j \leq 1 - \alpha. \quad (\text{C.27})$$

Claim: $x_i = 1 - x_j$.

Proof: Assume not. Then, $x_i > 1 - x_j$, and $k_i(x_i, x_j) > k_j(x_j, x_i)$. Let us show that $\check{x}_i = x_i - \epsilon > 1 - x_j$, for a small $\epsilon > 0$, is a profitable deviation for candidate i .

If $\sqrt{\frac{w_j^3(x_j, \check{x}_i)}{k_i(\check{x}_i, x_j)k_j(x_j, \check{x}_i)w_i(\check{x}_i, x_j)}} \leq c$, then from Lemma C.2.1 we get:

$$\frac{k_i(\check{x}_i, x_j)\sigma_i^*(\check{x}_i, x_j)}{k_j(x_j, \check{x}_i)\sigma_j^*(x_j, \check{x}_i)} = 1. \quad (\text{C.28})$$

⁶Because given $x_i < \alpha$, we have $k_i w_i < k_j w_j$ for $x_i < 1 - x_j$ and $k_i w_i > k_j w_j$ for $x_i > 1 - x_j$.

Plugging (C.28) in (C.3) we obtain:

$$p_i(\check{x}_i, x_j) = p_i(x_i, x_j) = \frac{1}{2},$$

that is, candidate i 's deviation does not increase his probability of victory. However, from (C.5) and Lemma C.2.1 we get:

$$u_i(\check{x}_i, x_j) = \frac{w_i(\check{x}_i, x_j)}{2} - \frac{\sqrt{k_i(\check{x}_i, x_j)k_j(x_j, \check{x}_i)}}{4k_i(\check{x}_i, x_j)} \sqrt{w_i(\check{x}_i, x_j)w_j(x_j, \check{x}_i)}$$

Since $k_i(\check{x}_i, x_j)w_i(\check{x}_i, x_j) = k_j(x_j, \check{x}_i)w_j(x_j, \check{x}_i)$, it follows that:

$$u_i(\check{x}_i, x_j) = \frac{w_i(\check{x}_i, x_j)}{4}$$

Similarly, going through the same steps, we get:

$$u_i(x_i, x_j) = \frac{w_i(x_i, x_j)}{4}.$$

From $\alpha < \check{x}_i < x_i$, it can be easily seen that $k_i(\check{x}_i, x_j) < k_i(x_i, x_j)$ and $k_i(\check{x}_i, x_j)w_i(\check{x}_i, x_j) \geq k_i(x_i, x_j)w_i(x_i, x_j)$. Therefore, we get $w_i(\check{x}_i, x_j) > w_i(x_i, x_j)$ and $u_i(\check{x}_i, x_j) > u_i(x_i, x_j)$. Hence, $\check{x}_i$ is a profitable deviation for candidate i , which is a contradiction.

Otherwise, $c < \sqrt{\frac{w_j^3(x_j, \check{x}_i)}{k_i(\check{x}_i, x_j)k_j(x_j, \check{x}_i)w_i(\check{x}_i, x_j)}}$, and $k_i(\check{x}_i, x_j) > k_j(x_j, \check{x}_i)$. Using these in (C.20) and (C.3), we get:

$$p_i(\check{x}_i, x_j) > p_i(x_i, x_j) = \frac{1}{2}.$$

Hence, $\check{x}_i$ is a profitable deviation for candidate i , which is a contradiction. Therefore, $x_i = 1 - x_j$. ■

From the claim and (C.27), it follows that:

$$\alpha \leq x_i = 1 - x_j \leq x_j \leq 1 - \alpha.$$

For any $s, t \in (\alpha, 1 - \alpha)$, we have:

$$\begin{aligned} \sqrt{\frac{w_j^3(t, s)}{k_i(s, t)k_j(t, s)w_i(s, t)}} &= \sqrt{\frac{k_j^3(t, s)w_j^3(t, s)}{k_i(s, t)w_i(s, t)} \frac{1}{k_j^2(t, s)}} \\ &= k_j(t, s)w_j(t, s) \frac{1}{k_j^2(t, s)} = \frac{w_j(t, s)}{k_j(t, s)} \end{aligned} \quad (\text{C.29})$$

Let us define $M(x) = \Phi(x, 1 - x)$. It can be easily seen that M is a continuous and decreasing function on $[\alpha, \frac{1}{2}]$. Moreover, $M(\frac{1}{2}) = 0$.

Assume $M(\alpha) \leq c$. Then,

$$\frac{w_j(1 - \alpha, \alpha)}{k_j(1 - \alpha, \alpha)} \leq c. \quad (\text{C.30})$$

Moreover, it can be easily seen that:

$$k_i(\alpha, x_j)w_i(\alpha, x_j) = k_j(x_j, \alpha)w_j(x_j, \alpha), \quad (\text{C.31})$$

and

$$\frac{w_j(x_j, \alpha)}{k_j(x_j, \alpha)} \leq \frac{w_j(1 - \alpha, \alpha)}{k_j(1 - \alpha, \alpha)}. \quad (\text{C.32})$$

Combining (C.30), (C.31), and (C.32), we obtain:

$$\sqrt{\frac{w_j^3(x_j, \alpha)}{k_i(\alpha, x_j)k_j(x_j, \alpha)w_i(\alpha, x_j)}} \leq c. \quad (\text{C.33})$$

Using (C.31) and (C.33) in (C.20) and (C.3), we obtain:

$$p_i(\alpha, x_j) = \frac{1}{2}.$$

If $x_i > \alpha$, a deviation to α by candidate i will not decrease his probability of victory, however it will increase the aggregate expected utility of his voters, which is a contradiction. On the other hand, if $x_i = 1 - x_j = \alpha$, then a candidate's deviation away from her opponent will decrease her probability of victory, whereas a deviation towards her opponent will decrease her voters' aggregate expected utility without changing her probability of victory. Hence, given $M(\alpha) \leq c$, $(\alpha, 1 - \alpha)$ is a unique NE.

Finally, assume $M(\alpha) > c$. Then, there exists $x \in (\alpha, 0.5)$ such that $M(x) = c$. Let us denote $x(c) = \min\{x \in (\alpha, 1/2) : M(x) = c\}$. From (C.21) and (C.29) it

follows that:

$$c \geq \frac{w_j}{k_j}$$

and since M is a decreasing function, we get:

$$x_i = 1 - x_j \geq x(c).$$

If $x_i = 1 - x_j > x(c)$, then $M(1 - x_j) < c$, and candidate i 's a deviation to $x_i - \epsilon$, for a small $\epsilon > 0$, will increase her voters' aggregate expected utility without changing her probability of victory, a contradiction. If $x_i = 1 - x_j = x(c)$, then a candidate's deviation away from her opponent will decrease her probability of victory, whereas converging to her opponent will decrease her voters' aggregate expected utility without changing her probability of victory. Hence, given $M(\alpha) > c$, $(x(c), 1 - x(c))$ is a unique NE.

$M(x)$ is a decreasing function. Therefore, it follows that $x(c)$ is decreasing in c . Moreover, $\lim_{c \rightarrow 0} x(c) = \frac{1}{2}$, because $w_j(1 - x(c), x_i) \approx 0$ and $k_j(1 - x(c), x_i) \approx \frac{1}{2}$ for $x_i \in [x(c), 1 - x(c)]$ when $x(c) \approx \frac{1}{2}$. Finally, when c is very small, (using the graphs of w_A and w_B) it can be easily seen that:

$$x(c) = \arg \max_{y \in [x(c), 1 - x(c)]} \frac{w_j(1 - x(c), y)}{k_j(1 - x(c), y)}. \quad (\text{C.34})$$

From (C.34), it follows that:

$$\frac{w_j(1 - x(c), x(c))}{k_j(1 - x(c), x(c))} = c. \quad (\text{C.35})$$

Moreover, we have:

$$k_j(1 - x(c), x(c)) = k_i(x(c), 1 - x(c)) = \frac{1}{2}, \text{ and} \quad (\text{C.36})$$

$$w_j(1 - x(c), x(c)) = w_i(x(c), 1 - x(c)). \quad (\text{C.37})$$

Using (C.35), (C.36) and (C.37) in Lemma C.2.1, we obtain:

$$\lim_{c \rightarrow 0} \sigma_i^* = \lim_{c \rightarrow 0} \sigma_j^* = 1.$$

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