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Three essays in financial market price informativeness

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THREE ESSAYS IN FINANCIAL MARKET PRICE
INFORMATIVENESS

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A DISSERTATION
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Abstract

The thesis studies the interaction between financial market and the real investment and their impacts on price informativeness. The first chapter provides theoretical and empirical evidences that private information conveyed through trading activities alleviate information asymmetry and facilitate equity financing. The second chapter studies an environment where the central bank learns the information about the economy fundamental from the financial security price before making monetary policy. The feedback effect from the financial market to the policy maker can impair the price informativeness by reducing the incentive for the speculators to coordinate and reveal their correlated information. The third chapter explores the interaction between financial markets and real economy of two regions whose investment increases each others productivity. The capital provider in each region learns from its own and the other regions financial market in making their investment decisions. The coordination among capital providers amplifies the trading frenzies in each financial market.

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To my daughter Jiahan.

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Chapter 1

Order Flow Informativeness and Equity Financing

1.1 Introduction

This paper studies the interaction between two important roles of the financial market: information production and fund raising. On the one hand, information asymmetry between managers and the market prevents firms from obtaining external financing efficiently (Myers and Majluf (1984) [44]). On the other hand, trading process transmits private information from traders to the market (e.g., Grossman and Stiglitz (1980) [30], Glosten and Milgrom (1985) [27], and Kyle (1985) [38]). This study investigates the question whether private information conveyed through trading process alleviates information asymmetry and helps firms raise more capital.

In a stylized model based on Myers and Majluf (1984) [44] and Kyle (1985) [38], I develop the hypothesis that equity financing is negatively correlated with the probability of information asymmetry and positively correlated with the cross term between the probability of information asymmetry and the stock's order flow informativeness as measured by the relative amount of informed trade to noise trade. In other words,

the probability of information asymmetry impairs firm's equity financing, however, the private information conveyed through the trading activities alleviates this friction. I assume that the manager and informed traders know more about the firm value than the market. Then the order flow provides an additional signal for the market. The informativeness of the order flow signal, determined by the intensity of informed trade, reduces information asymmetry between the manager and the market, and therefore, facilitates equity financing.

One of the key issues in testing the hypothesis is to find the proxy for the information asymmetry and the order flow informativeness. While these two factors impact equity financing in opposite directions, they both increase the informed trade. High amount of private information-based order flows can occur for two reasons: first, the firm is less transparent and the informed traders have more chances to obtain information advantage; second, there are more informed traders relative to noise traders in the market and they explore and trade on their information more intensively. The first is related to firm's level of information asymmetry and should impair equity issuance; while the second determines the market's awareness of the private information and should facilitate fund raising. My order flow informativeness should be relevant to the second effect. To decouple these two effects, first, I use the *PIN* measure developed in Easley, Kiefer, and O'Hara (1996 [21] 1997a [19], b [20]) as the measure of informed trade; Then I decompose the measure of informed trade into two components: the probability of information asymmetry and the order flow informativeness conditional on and event of information asymmetry.

PIN has been tested and used in a series of articles and has been proved to be a valid measure for the level of informed trade. It has been used as proxies for both information asymmetry (e.g. Bharath, Pasquariello et al. (2009) [8]) and price informativeness (e.g. Chen, Goldstein et al. (2007) [13]). The measure captures the probability of informed trade from order flow data through a market micro structure

model. In the model, informed trade happens upon an information event in which information asymmetry occurs between different groups of traders. PIN is determined by both the probability of the information event and relative amount of informed trade to noise trade conditional on the information event. The first component represents the level of information asymmetry from its definition. Brown and Hillegeist (2007) [9] find evidence that the probability of information event links to lower disclosure quality. The second component is correspondent to the informativeness of the order flow when private information arrives. Here I use the first component as the proxy for the probability of information asymmetry and the second component, the relative amount of informed trade to noise trade conditional on the information event, as the proxy of order flow informativeness in the test (details are described in Section 3).

I find that firm's net equity financing is strongly negatively correlated with the probability of information asymmetry and is strongly positively correlated with the cross term between the probability of information asymmetry and order flow informativeness, consistent with the hypothesis that information conveyed through trading activities help alleviate the adverse selection problem in equity financing. There also exists an important conflicting effect from the order flow informativeness. Both theoretical and empirical studies have been carried out on the negative effect from informed trading on market liquidity (Manove (1989) [41], Diamond and Verrecchia (1991) [16], Easley and O'Hara (2004) [22], Easley, Hvidkjaer et al. (2002) [18], Elul and Pagano (2006) [25], Chan, Menkveld et al. (2008) [12], Choi, Jin and Yan (2013) [14], Hwang, Lee et al. (2013) [34]). In their theories, when there is information asymmetry, the existence of informed trade increases the cost of capital by draining up the market liquidity. Taking this into account, my result is even stronger in proving the positive effect from the information transmission of the trading process.

I perform more tests to assess the validity of this conclusion. First, the increase in net equity finance can be attributed to more equity financing needs. Equity financing

needs are determined by both investment opportunity and firm's equity dependence. To control for investment opportunity, I include Tobin's q into the basic test. It is also important to check that the result is not purely driven by equity dependence, as data shows financial constrained firms tend to have high order flow informativeness. I control for equity dependence by conducting analysis on five quintile subsamples which are sorted according to the KZ index. I show that both the equity dependence and the order flow informativeness play a role in determining the amount of net equity issuance. Another important concern is the effect of firm size. My order flow informativeness is negatively correlated with firm size. Firm size, in turn, may affect the equity financing for reasons unrelated to the amount of private information reflected in the order flow. Thus, to ensure that my results are not driven by firm size, I conduct our analysis on five quintile subsamples sorted by firm size. I also control for size directly in my main regressions by including it as an additional variable. In both these tests, I find that information asymmetry and the order flow informativeness remain important after size is controlled for.

As the hypothesis is about the private information revealed in the order flow, I control for the effect from public information by adding the analyst coverage into the analysis. The effect from the probability of information asymmetry and the order flow informativeness stays significant. The result shows that firm's net equity issuance decreases in the amount of analyst coverage. In addition, the analyst coverage is negatively related to the order flow informativeness. As argued by Easley, O'Hara and Paperman (1998) [23], the presence of analysts can attract more noise trading to the stock. This reduces the informativeness of the order flow and thus decreases net equity issuance when there is information asymmetry.

The interpretation of the results in the paper depends on my measure of the order flow informativeness. I also conduct some analysis to jointly test the validity of my measure of order flow informativeness and the robustness of the results. *PIN*

and its components are closely related to market illiquidity. Furthermore, Duarte and Young (2009) [17] pointed out that *PIN* could be driven by liquidity factors that are not related to information. To make sure that my result is not driven by non-information liquidity factors, I study the relation between the information variables and four liquidity measures and then add these liquidity factors to my basic test. The four measures I use here are: shares turnover, daily idiosyncratic return volatility, dollar volume weighted daily average absolute return, and return reversal as developed in Pastor and Stambaugh (2003) [46]. It is important to note that the first three measures can be driven by both information and non-information factors; while the last measure is constructed to capture illiquidity that is not related to information. My results show that both my information variables are correlated with shares turnover, idiosyncratic daily return volatility and dollar volume weighted daily average absolute return. This is consistent with previous study that information based trading links to market liquidity. However, it is interesting that return reversal does not have a significant effect on order flow informativeness. This provides additional evidence for using *PIN* and its components as information factors. Furthermore, my main results stays intact after adding these liquidity measures to the main regression.

I also analyze the effect from the insider and institutional trade on my main results, and their relation with the information variables. Both *PIN* and the order flow informativeness are positively related to the intensity of insider trade and the aggressiveness of the institutional position change, though at a mild level. This again indicates that my measure captures private information based trade, as both insider and institutional traders are widely assumed to possess information advantage. The mild correlation is reasonable, as insider trade only composes of a very small fraction of the trading activities, and a high amount of institutional trade are triggered by liquidity reasons. The effect from intensity of insider trade is not significant, which is expected, as insider trade only takes up very small part of the trading volume.

The aggressiveness of institutional position change is negatively related to net equity issuance. This is consistent with the fact that a lot of institutional trades arise from non-information incentive (Coval and Stafford (2007) [15], Brunnermeier and Pedersen (2009) [11], Adrian and Shin (2014) [1]). The effect from my order flow informativeness stays significant after adding these two variables to the regression.

This paper is related to the group of literature that studies the effect of informed trade on cost of capital. The theory papers mainly model the liquidity impact of the information based trade; however the sign of the predicted relationship differs both within and across models (Manove (1989) [41], Leland (1992) [40], Diamond and Verrecchia (1991) [16], Garleanu and Pedersen (2004) [26], Vayanos and Wang (2012) [51], Easley and O'Hara (2004) [22], Hughes, Liu and Liu (2007) [33]). The related empirical works are mainly on identifying the related market pricing factor. Easley, Hvidkjaer, and O'Hara (2002) [18] find that *PIN* affects asset return and interpret the result as the cost of information risk. Choi, Li and Yan (2013) [14] also document links between informed trade and higher expected return using the Chinese stock market data. These studies focus on the relation between post issuance market liquidity and the price of the equity in the secondary market. The strategy of the manager is not considered. This paper studies the interaction between the informed trade and manager's adverse selection problem in making the financing decisions. I directly examine the relation between the intensity of informed trade and the amount of equity financing, and find that the positive effect from order flow informativeness out-weights its negative impact on market liquidity.

A more closely related article is Alti and Sulaeman (2012) [3]. They find that firms tend to issue equity following high market return only when it coincides with high institutional demand. My paper is different from Alti and Sulaeman (2012) [3] in two important dimensions. First, I analyze effect from order flow informativeness on manager's financing decisions. This effect may be a channel through which their result

takes effect. Second, I decompose the level of the informed trade into an information asymmetry component and an order flow informativeness component. To the best of my knowledge, this paper is the first one to separate these two driving forces behind the informed trade.

The remainder of the paper is organized as follows. In Section 2 I develop the hypothesise in the context of a simple model. The model provides a framework for understanding the interaction between informed trade and firm's equity financing decision, and also gives some guidance as to how to measure the order flow informativeness. In Section 3 I describe the *PIN* measure and its decomposition, and in Section 4 I present the empirical results. Section 5 concludes.

1.2 A simple model and the hypothesis

Base on a simplified setting of Myers and Majluf (1984) [44], I develop the hypotheses about order flow informativeness and equity financing by adding the trading activities. There is an all equity funded firm with an existing asset and an investment opportunity. The existing asset V_a can take on two possible values V_H and V_L ($V_H > V_L$, H and L denote high and low state), with probability p and $1 - p$. The investment opportunity requires an upfront cost I , and the net present value (NPV) of the project is V_b , therefore the total final payoff from the new project will be $I + V_b$. The firm does not have financial slack and need to raise the upfront cost from the capital market in order to take the investment opportunity. The manager owns θ fraction of the firm's equity and maximizes his own share value when making finance and investment decisions.

There is information asymmetry about the value of the existing asset with probability α . The manager always observes the exact value of the existing asset, while the market can observe the value only when there is no information asymmetry (with

probability $1 - \alpha$). $V_H, V_L, V_b, I, p, \theta$ and α are all common knowledge. Denote $\mathbb{M}$ the manager's information about the existing asset value. After the firm announces its decision to issue equity and invest, the market updates its belief on the value of the existing asset and sets the share price such that the new shares take λ fraction of the firm and its total value satisfies the following break even condition:

$$\lambda [E(V_a | Issue, \mathbb{M}) + I + V_b] = I \quad (1.1)$$

The manager issues equity if and only if his share value after equity issuance is higher, which can be expressed by the following inequality:

$$V_a \leq (1 - \lambda) (V_a + I + V_b) \quad (1.2)$$

Without information asymmetry, it is obvious that the firm always issues equity and take on the investment opportunity. In the case with information asymmetry, the key friction is that the announcement of equity financing could be a signal of overvaluation to the market.

Proposition 1 describes the equilibrium without trading activity which illustrates the adverse selection problem in Myers and Majluf (1984) [44].

Proposition 1 *When there is no information asymmetry ($\mathbb{M} = \{V_H\}$ or $\{V_L\}$), the manager always issues equity. When there is information asymmetry ($\mathbb{M} = \{V_H, V_L\}$), if $p \geq \frac{V_H \frac{I}{I+V_b} - V_b - V_L}{V_H - V_L}$, there exists an equilibrium where the manager always issues equity; otherwise, there is unique equilibrium where the manager issues equity only in the low state.*

When there is information asymmetry, the manager has to issue equity at the price lower than its true value in the high state. There is trade off between the loss from selling the existing asset at a low price and the profit from taking on the

investment opportunity. Therefore the manager's decision depends on the market valuation of the existing asset which is driven by the market belief of the high state probability here. Let $p^* = \frac{V_H \frac{I}{I+V_b} - V_b - V_L}{V_H - V_L}$, the threshold market belief of the high state probability at which the manager is indifferent between issuing or not. Keeping other parameters fixed, this threshold probability is increasing in the upfront investment amount I . With higher upfront cost, more share of existing asset is required to be sold for the investment opportunity, therefore less under-valuation can be afforded. In the case where $p < p^*$, the manager bypasses the investment opportunity, as the value dilution cannot be justified by the profit from the new project. I focus on this case and introduce trading activities to analyze the effect from order flow informativeness on equity financing.

There are a continuum number of risk neutral informed traders with measure μ who also observe the existing asset value. Each informed trader can trade an amount within $[-1, 1]$ shares of the firm's stock. The demand from noise trader ε is normally distributed with mean 0 and variance σ_ε^2 . Denote the demand from informed trader as X_I . Then the aggregate order flow $X = X_I + \varepsilon$. The manager makes the financing decision after observing the aggregate order flow X . The market set the share price conditioning on X and the manager's decision.

It is obvious that the market valuation of the existing asset is always higher than or equal to that of the informed traders in the low state and lower than or equal to that of the informed traders in the high state. Assume that the informed traders always long one share in the high state and short one share in the low state, when he is indifferent. Then the demand from the informed traders is μ in the high state and $-\mu$ in the low state. Therefore, the order flow becomes another source of information to the market. The relative amount of informed traders to noise traders influences the precision of the order flow signal. I define the order flow informativeness as $\frac{\mu}{\sigma_\varepsilon}$, the ratio of the amount of informed trade over that of noise trade. Assume that manager

issues equity in both states, the probability of state H conditional on information asymmetry, equity issuance and aggregate order flow X can be expressed as follows:

$$\Pr(H|Issue, \mathbb{M} = \{V_H, V_L\}, X) = \frac{p \cdot \exp\left(-\frac{(X-\mu)^2}{2\sigma_\varepsilon^2}\right)}{p \cdot \exp\left(-\frac{(X-\mu)^2}{2\sigma_\varepsilon^2}\right) + (1-p) \cdot \exp\left(-\frac{(X+\mu)^2}{2\sigma_\varepsilon^2}\right)} \quad (1.3)$$

which is strictly increasing in the aggregate order flow X .

The market demand λ share of the new company according to the following break even condition:

$$\lambda [E(V_a|Issue, X, \mathbb{M}) + I + V_b] = I \quad (1.4)$$

Proposition 2 summarizes manager's behave in the equilibrium where the information contained in the order flow takes an effect on both the market's belief of the asset value and manager's financing decision.

Proposition 2 *Assume that $p < p^*$. When there is no information asymmetry ($\mathbb{M} = \{V_H\}$ or $\{V_L\}$), the manager always issues equity. When there is information asymmetry ($\mathbb{M} = \{V_H, V_L\}$), there exists an equilibrium with a threshold value X^* such that for $X \geq X^*$ the manager issues equity in both states; for $X < X^*$ the manager issues equity only in the low state.*

The intuition behind Proposition 2 is as follows. The market updates the probability of state H according to the order flow signal which can also be observed by the manager. In state H , when the signal from the order flow is high enough, the manager will know that the market valuation of the existing asset is close enough to its true value and choose to issue equity. In this way, the adverse selection problem is eliminated. Denote E the ex ante expected amount of equity issuance. Proposition 3 summaries the effect from both the probability of information asymmetry α and order flow informativeness $\frac{\mu}{\sigma_\varepsilon}$ on equity financing which generates the hypothesis to be empirically tested in the later sections.

Proposition 3 *For the equilibrium in Proposition 2, the ex ante expected amount of equity issuance E is decreasing in the probability of information asymmetry α , i.e. $\frac{\partial E}{\partial \alpha} < 0$; the negative effect from the probability of information asymmetry is decreasing in the order flow informativeness $\frac{\mu}{\sigma_\varepsilon}$, i.e. $\frac{\partial^2 E}{\partial \alpha \partial (\frac{\mu}{\sigma_\varepsilon})} > 0$.*

First, it is straightforward that the ex ante expected amount of equity issuance is decreasing in the probability of information asymmetry which is the key friction in the model. Then as the order flow informativeness increases, the market belief on the existing asset is more likely to be close to the true value, which reduces the information asymmetry between the manager and the market. This mitigates the negative effect from the probability of information asymmetry.

Therefore, Proposition 3 leads to the following testable hypothesis: the amount of equity issuance is negatively related to the probability of information asymmetry and positively related to the interaction between the probability of information asymmetry and the order flow informativeness.

1.3 The *PIN* measure and its decomposition

This section describes the measure of the probability of information asymmetry and the order flow informativeness conditional on information asymmetry from the market data. I resort to the *PIN* measure developed by Easley et al. (1996) [21] to detect the amount of informed trade from the order flow data in the security market. I then decompose the *PIN* measure into the probability of information event and the intensity of informed trade upon an information event. The first term correspondent to the probability of information asymmetry. I use the second term as the proxy for the order flow informativeness in the empirical test.

The *PIN* measure has solid theoretical foundations as a measure of informed trading in the security market. The measure was used in a series of articles by

Easley, Kiefer, and O'Hara (1996 [21], 1997a [19], b [20]), Easley, O'Hara and Paperman (1998) [23], Easley, O'Hara and Srinivas (1998) [24], and Easley, Hvidkjaer and O'Hara (2002) [18]. These articles also directly test the validity of the *PIN* measure by comparing the predictions of the information-based model with other alternative models. Furthermore, Vega (2005) [52] found further evidence supporting *PIN* as a measure of private information. She showed that stocks with higher *PIN* values have smaller post-earnings-announcement drifts, suggesting high *PIN* stocks adjust to fundamentals quicker. As she found that this quicker adjustment is not because of media coverage or other public news releases. The overall results from these tests strongly support *PIN* as capturing the private information in the trading activities.

PIN is estimated from a structural market microstructure model in which trades can come from noise traders or from informed traders. Let me briefly describe the basic idea behind the measure. Suppose the daily arrival rates of noise traders that submit buy and sell orders are ε_b and ε_s , respectively. The probability that an information event occurs is α , in which case the probability of bad news is δ and the probability of good news is $(1 - \delta)$. If an information event occurs, the arrival rate of informed traders is μ . Informed traders submit a sell order if they get bad news and a buy order if they get good news. Thus, on a day with no information event [which happens with probability $(1 - \alpha)$], the arrival rate of a buy order will be ε_b and the arrival rate of a sell order will be ε_s . On a day with a bad information event (which happens with probability $\alpha\delta$), the arrival rate of a buy order will be ε_b and the arrival rate of a sell order will be $\mu + \varepsilon_s$. On a day with a good information event [which happens with probability $\alpha(1 - \delta)$], the arrival rate of a buy order will be $\mu + \varepsilon_b$ and the arrival rate of a sell order will be ε_s . Let $\theta = \{\varepsilon_s, \varepsilon_b, \alpha, \delta, \mu\}$. The likelihood

function for a single trading day is given by:

$$\begin{aligned}
L(\theta|B, S) = & (1 - \alpha)e^{-\varepsilon_b} \frac{(\varepsilon_b)^B}{B!} e^{-\varepsilon_s} \frac{(\varepsilon_s)^S}{S!} + \alpha\delta e^{-\varepsilon_b} \frac{(\varepsilon_b)^B}{B!} e^{-(\varepsilon_s + \mu)} \frac{(\varepsilon_s + \mu)^S}{S!} \\
& + \alpha(1 - \delta)e^{-(\varepsilon_b + \mu)} \frac{(\varepsilon_b + \mu)^B}{B!} e^{-\varepsilon_s} \frac{(\varepsilon_s)^S}{S!}
\end{aligned} \tag{1.5}$$

Here, B is the number of buy orders and S is the number of sell orders in a single trading day. Using trading information over J days and assuming cross-trading-day independence, one can estimate the parameters of the model $\theta = \{\varepsilon_s, \varepsilon_b, \alpha, \delta, \mu\}$ by maximizing the following likelihood function:

$$V = \prod_1^J L(\theta|B_j, S_j) \tag{1.6}$$

Then, the probability of informed trading in a given stock for a given period, which determines the *PIN* measure, is estimated as:

$$PIN = \frac{\alpha\mu}{\alpha\mu + \varepsilon_s + \varepsilon_b} \tag{1.7}$$

The above expression can be rewritten as:

$$PIN = \frac{\frac{\mu}{\varepsilon_s + \varepsilon_b}}{\frac{\mu}{\varepsilon_s + \varepsilon_b} + \frac{1}{\alpha}} \tag{1.8}$$

By observing equation (1.8), the *PIN* measure is positively related to both the probability of information event and the relative amount of informed trade to noise trade $\frac{\mu}{\varepsilon_s + \varepsilon_b}$. These two components affect equity financing in opposite directions. The first measures the probability of the event that there is information asymmetry about the firm's value. Brown and Hillegeist (2007) [9] documents negative correlation between the probability of information event and the disclosure quality. Their study supports using α as the proxy for information asymmetry. Therefore α should be

negatively related to firm's equity finance according to Myers and Majluf (1984) [44]. The second term corresponds to the order flow informativeness defined in Section 2 which is proved to alleviate the negative effect from information asymmetry.

I use α , the probability of information event, as the proxy for the probability of information asymmetry and $\frac{\mu}{\varepsilon_s + \varepsilon_b}$ as the measure for order flow informativeness in the empirical test.

1.4 The empirical test

1.4.1 Data description and variable definition

The data is collected from six databases. I obtain firms' stock price and return information from Center for Research in Security Prices (CRSP), equity financing and other financial data from Compustat, intraday transaction data from Trade And Quote (TAQ), insider and institutional trading information and analysts' coverage data from the Thomson Reuter's IFDF, 13F and IBES databases. The sample consists of an unbalanced panel of firms from 1993 to 2012, excluding firms in the financial industries (SIC code 6000–6999) and utility industries (SIC code 4200). I exclude firm-year observations with less than \$10 million market capitalization or with less than 60 days of trading activities in a year. The final sample consists of 44,875 firm-year observations with 6457 firms.

The baseline equation for testing the hypothesis is as follows:

$$\begin{aligned} NET_ISSUE_{it} = & \beta_1 \cdot \alpha_{i,t-1} + \beta_2 \cdot \alpha_{i,t-1} \cdot ORDER_INFO_{i,t-1} \\ & + \theta_j \cdot Control_{i,j,t-1} + \eta_t + \xi_i + \varepsilon_{it} \end{aligned} \quad (1.9)$$

where NET_ISSUE_{it} is firm i 's sale of common and preferred stock minus purchase of common and preferred stock (Item 108–Item 115) in year t scaled by beginning-of-

year book assets ($A_{i,t-1}$, Item 6), and η_t and ξ_i represent year and firm-fixed effects. Here I use the amount of net issuance since both issuance and repurchase are related to the difference in manager and market's valuation of firm asset. In addition, the amount of net issuance is directly related to the value dilution in the manager's adverse selection problem.

$\alpha_{i,t-1}$ is the probability of information asymmetry for firm i in year $t-1$. I expect, $\beta_1 < 0$, that is, NET_ISSUE_{it} be negatively correlated with $\alpha_{i,t-1}$, as information asymmetry give rise to the manager's adverse selection problem which impairs equity financing. This paper focuses on, β_2 , the coefficient for $\alpha \cdot ORDER_INFO_{i,t-1}$, which measures the effect

of the order flow informativeness on alleviating the manager's adverse selection problem. $ORDER_INFO_{i,t-1}$ is the order flow informativeness for firm i in year $t-1$. Both $\alpha_{i,t-1}$ and $ORDER_INFO_{i,t-1}$ are decomposed from the PIN measure. Following the procedure prescribed in Easley, Hvidkjaer, and O'Hara (2002) [18], for each trading day in year $t-1$, I classify all trades between 9:30 A.M. and 4:00 P.M. as either a buyer-initiated trade or a seller-initiated trade using the Lee and Ready (1991) [39] algorithm. I eliminate large size trades (trade size greater than 10,000 shares) and trades coded by TAQ as trading with special conditions. Then I estimate a group of firm-year parameters that PIN is composed of based on the number of buys and sells in each trading day of the year. For reliability, I only include firm-year with more than 60 trading days in the sample. As in some cases the estimates may depend on the initial values for the search algorithm and not yield the global maximum, I run the optimization procedure 243 times for each firm-year using 243 pre-specified sets of initial parameter values (three possible initial values for each of the five parameters). Among the 243 candidate solutions, the algorithm chooses the estimate that does not include any corner solution and, at the same time, maximizes the likelihood function specified in equation (1.6).

Table 1.1: Variable Definition

Variable	Definition
<i>NET_ISSUE</i>	Equity issuance minus equity repurchase
α	Parameter α in the <i>PIN</i> measure per Easley et al. (1996)
<i>PIN</i>	<i>PIN</i> measure per Easley et al. (1996)
<i>ORDER_INFO</i>	μ over $(\varepsilon_b + \varepsilon_s)$; μ , ε_b and ε_s are parameters in the <i>PIN</i> measure per Easley et al. (1996)
Q	Market value of equity plus book value of assets minus book value of equity, scaled by book value of assets
<i>TANG</i>	Tangible asset scaled by book asset
<i>SALES</i>	Net Sales (\$ million)
<i>LOGSALES</i>	Natural Logarithm of Net Sales
<i>PROFIT</i>	Earning before interest and tax over net sales
<i>KZ</i>	KZ score per Kaplan and Zingales (1997)
<i>SIZE</i>	Market capitalization (\$ million)
<i>INSIDER</i>	Number of transactions by insiders scaled by total number of transactions recorded in TAQ
<i>INST_TRADE</i>	Sum of absolute values of quarterly position changes over all institutional traders during the year, scaled by shares outstanding
<i>ANALYST</i>	Number of analysts issuing forecasts or recommendations for the firm
<i>TURNOVER</i>	Annual trading volume over shares outstanding
<i>ID_VOL</i>	The average of squares of the residuals for the regression of daily returns on value-weighted market daily returns
<i>REVERSE</i>	Stock illiquidity measured as inter-day return reversal per Pastor and Stambough (2003)
<i>Amihud_ILLQ</i>	Dollar volume weighted average daily absolute returns over the year
<i>RET</i>	Value-weighted market return adjusted firm return for next three years

I also include a group of firm characteristics as control variables (*Control*): Q , *TANG*, *LOGSALES* and *PROFIT*. Q is the Tobin's q calculated as the market value of equity (price times shares outstanding from CRSP) plus book value of assets minus the book value of equity (Item 6–Item 60), scaled by book assets (Item 6). Q is interpreted as both stock price level and value of investment opportunity. It is expected that firms with more investment opportunity issue more equity. In addition, literature has documented for many times the market timing effect for equity issuance which is characterized by positive correlation between equity financing and stock price level. Therefore, the coefficient γ is expected to be positive, that is,

Table 1.2: Summary Statistics

Variable	Mean	STD	25%	50%	75%	90%	95%
<i>NET_ISSUE</i>	0.053	0.38	-0.005	0.001	0.02	0.09	0.32
α	0.31	0.14	0.23	0.3	0.38	0.47	0.5
<i>PIN</i>	0.2	0.11	0.12	0.18	0.25	0.33	0.39
<i>ORDER_INFO</i>	0.45	0.18	0.31	0.43	0.59	0.7	0.76
<i>Q</i>	2.06	2.47	1.08	1.48	2.26	3.67	5.17
<i>TANG</i>	0.27	0.22	0.09	0.2	0.37	0.62	0.76
<i>SALES</i>	2482.7	1213.6	77.5	307.3	1207.4	4234	9138
<i>PROFIT</i>	0.03	0.24	0.013	0.076	0.13	0.19	0.23
<i>KZ</i>	0.5	2.76	0.04	0.55	1.16	1.77	2.23
<i>SIZE</i>	3256.5	1651.2	82.5	338.7	1346.2	4983.2	11567
<i>INSIDER</i>	0.007	0.018	0.0002	0.0008	0.004	0.014	0.027
<i>INST_TRADE</i>	0.06	0.29	0.019	0.032	0.049	0.08	0.103
<i>ANALYST</i>	6.73	7.21	1	3	7	10	19
<i>TURNOVER</i>	1.53	1.79	0.48	1.01	1.99	3.4	4.59
<i>ID_VOL</i>	0.03	0.02	0.02	0.028	0.04	0.055	0.066
<i>REVERSE</i>	-0.05	0.06	-0.01	-0.005	0.0001	0.003	0.01
<i>Amihud_ILLQ</i>	1.79	20.02	0.002	0.03	0.33	2.43	6.2
<i>RET</i>	0.22	1.57	-0.45	-0.007	0.5	1.32	2.2

NET_ISSUE_{it} be positively correlated with $Q_{i,t-1}$. I include Q here to control for both investment opportunity and the market timing effect. $TANG$ is calculated as the value of net PPEG (Item 8) scaled by book value of total asset. Firms with higher asset tangibility are assumed to be more transparent in their asset value. I include it here to control for the level of information asymmetry. $LOGSALES$ is the natural logarithm of net sales (Item 12). It has also been used in literatures as control for size (Baker and Wurgler (2002) [7], Alti (2006) [2]). I also use market capitalization to further control the effect from firm size in the extended study. $PROFIT$ is calculated as operating profit before depreciation (Item 13) divided by net sales. All these three variables are important firm characteristics related to level of information asymmetry, equity dependence and investment opportunity. I use them to control for the effect that does not come from trading activities.

Table 1 defines all the variables in the test. Table 2 summarizes the summary statistics for all variables for the whole sample of 44,875 observations. The average sample property of our *PIN* estimate is comparable to that reported in Easley, Hvidkjaer and O’Hara (2002) [18]. Specifically, the mean (median) *PIN* in our sample is 0.21 (0.18) with standard deviation of 0.11. In Fig. 1, I plot a time series of the parameter estimates in addition to the *PIN* measure averaged across all stocks in the sample. The arrival rate of noise sell order and noise buy order, ε_b and ε_s , the arrival rate of information based orders, μ , the probability of an information event, α (*PROB_INEVT*), the probability of informed trade, *PIN*, and order flow informativeness, *ORDER_INFO*, are estimated using the likelihood function given in Eq. (1.6). The time-series plot is obtained by calculating each parameter using data for each firm-year and averaging across firms in each year. The parameters ε_b , ε_s and μ , are not stationary. These parameters are related to the trading frequency; hence they are upwards-trending as the number of transactions has increased over the years. In contrast, the estimates of α , *PIN* and *ORDER_INFO* ($\frac{\mu}{\varepsilon_s + \varepsilon_b}$) are stationary over the years, which is consistent with previous literatures.

Table 3 reports the correlation matrix of the information variables and the firm characteristics. Consistent with the decomposition of *PIN*, *PIN* is positively correlated to both α and *ORDER_INFO* with Pearson correlation coefficient 0.15 and 0.69. The correlation between the information variables and other variables is also of interest. I find that both *PIN* and *ORDER_INFO* are negatively correlated with size, asset tangibility and profitability, positively correlated with the Kaplan–Zingales measure of financial constraints. All these correlations are significant with p-value < 0.001. These correlations indicate that firms with higher level of order flow informativeness tend to be smaller, less profitable, less transparent and therefore more equity financing dependent. To prevent that my result mainly comes from firm’s equity dependence, I include these firm characteristics in the regression as control variables. In addition, I further conduct analysis on five quintile subsamples sorted by firms’ level of equity dependence. I use KZ index and firm size measured by market capital value as the proxy for equity dependence.

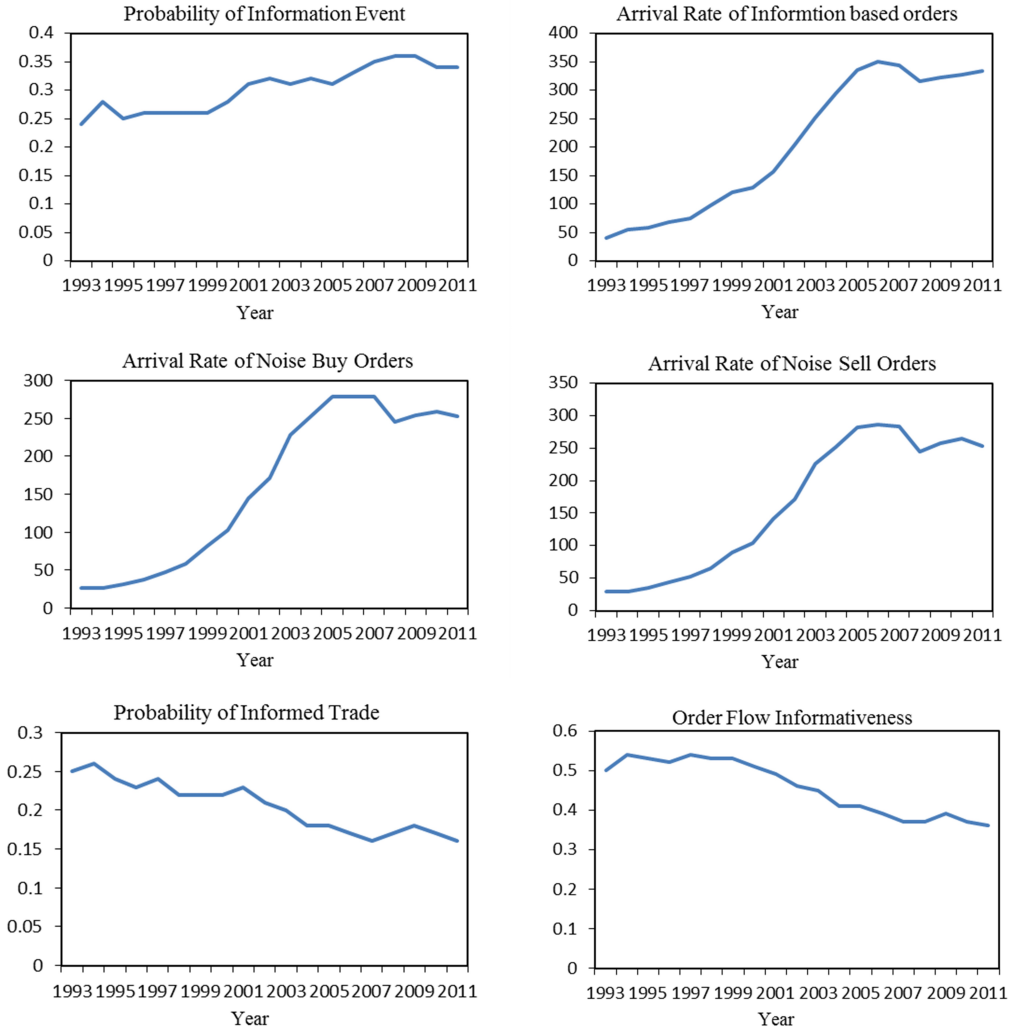


Figure 1.1: Private information trading parameter estimates: this is a plot of the market microstructure model parameters.

Lastly, ε_{it} in Equation (1.9) is the disturbance term that is uncorrelated with the regressors but is allowed to be serially correlated for the same firm. In the estimation, all standard errors are adjusted for arbitrary heteroskedasticity and for error correlations clustered by firm.

1.4.2 Basic test

In this section, I test whether the order flow informativeness can alleviate the manager's adverse selection problem and facilitate equity financing. In addition to the baseline equation,

Table 1.3: Correlation Matrix of Order Flow Informativeness and Firm Characteristics

	α	<i>PIN</i>	<i>ORDER_INFO</i>	<i>Q</i>	<i>TANG</i>	<i>LOGSALES</i>	<i>PROFIT</i>	<i>KZ</i>	<i>SIZE</i>
α	1	0.15	-0.25	0.019	-	0.24	0.07	-	0.04
<i>PIN</i>		1	0.69	-0.02	-0.01	-0.42	-0.08	0.02	-0.17
<i>ORDER_INFO</i>			1	-0.04	-0.02	-0.58	-0.13	0.03	-0.19
<i>Q</i>				1	-0.13	-0.22	-0.28	-0.15	0.08
<i>TANG</i>					1	0.13	0.09	0.06	0.01
<i>LOGSALES</i>						1	0.46	-	0.32
<i>PROFIT</i>							1	-0.02	0.08
<i>KZ</i>								1	-0.05
<i>SIZE</i>									1

Pearson correlation between the information variables and firm characteristic variables.

I also estimate the following Probit model to check the effect from order flow informativeness on equity issuance of high amount.

$$\begin{aligned} \Pr(NET_ISSUE_{it} \geq x) = & \text{Probit}(\beta_1 \cdot \alpha_{i,t-1} + \beta_2 \cdot \alpha_{i,t-1} \cdot ORDER_INFO_{i,t-1} \\ & + \gamma \cdot Q_{i,t-1} + \theta_j \cdot FCharact_{i,j,t-1} + \varepsilon_{it}) \end{aligned}$$

As stated in Section 2, manager's adverse selection problem is more likely to be related to high amount of equity issuance, which motivates the Probit test. x is the threshold level of net equity issuance. Similarly, I expect $\beta_1 < 0$ and $\beta_2 > 0$. Fixed-effects are not included in the Probit model to avoid incidental parameter bias for the nonlinear maximum likelihood model.

Table 4 summarizes the results from estimating Equation (1.9). Column 1 estimates the baseline regression for the net equity financing without firm-fixed effect. Column 2 includes the firm-fixed effect. Finally, Column 3 estimates the Probit model with threshold for net equity issuance at 0.1 which is the 90th percentile of the *NET_ISSUE* in this sample. Results are similar for other threshold levels. Control variables: *Q*, *TANG*, *LOGSALES* and *PROFIT* are included in all three regressions.

Column 1 shows that, without firm-fixed effect, *NET_ISSUE*_{it} is negatively correlated with $\alpha_{i,t-1}$, with the coefficient for $\alpha_{i,t-1}$ estimated at 0.14, significant at less than the 1% level. This result supports the hypothesis that equity financing should be negatively related

to the probability of information asymmetry. I focus on the coefficient for $\alpha \cdot ORDER_INFO$. As shown in Column 1, this coefficient is estimated at 0.2, significant at less than the 1% level. This shows that the negative effect from the probability of information asymmetry α is lower for firms with high level of order flow informativeness. Given that the 25th percentile value of *ORDER_INFO* is 0.31 and median value is 0.43 (Table 2), these estimates indicate that the effect from probability of information asymmetry for a firm with a 25th percentile value of *ORDER_INFO* is -0.078. The effect from probability of information asymmetry will decrease by 0.056 (or 72%), if a firm's *ORDER_INFO* increases from a 25th percentile value to a 75th percentile value of 0.59.

Column 2 repeats the same analysis with firm-fixed effect. It shows that the coefficient for $\alpha_{i,t-1}$ is 0.18 and the coefficient for $\alpha \cdot ORDER_INFO$ is 0.3 (both significant at less than the 1% level). The effect from probability of information asymmetry will change from -0.087 to -0.0035, if a firm's *ORDER_INFO* increases from a 25th percentile value to a 75th percentile value of 0.59. The result is stronger than that without the firm-fixed effect.

Column 3 shows the estimation for the Probit test. The probability of *NET_ISSUE* being greater than or equal to 0.1 is negatively correlated with $\alpha_{i,t-1}$ and positively correlated with $\alpha \cdot ORDER_INFO$ (both significant at less than the 1% level). The result confirms that the probability of information asymmetry and the order flow informativeness take effect for high amount of equity financing. The effect from probability of information asymmetry will change from -0.25 to -0.007, if a firm's *ORDER_INFO* increases from a 25th percentile value to a 75th percentile value of 0.59.

As for the control variables, all columns in Table 2 show that the coefficient estimate for *Q* is significantly positive (at less than the 1% level), confirming the result in the prior literature that firm issues more equity when stock prices are higher. *Q* can also be interpreted as level of investment opportunity. The result is consistent with both interpretation of *Q*. *NET_ISSUE* is reported to be positively related to *TANG* (significant at less than 1% level) in all three columns. This is intuitive as the difference in valuation between the market and the manager is smaller for firms with higher asset tangibility. The coefficients for *LOGSALES*, and *PROFIT* are both negative (at less than 1% level for *PROFIT*) in

Table 1.4: Order Flow Informativeness and Equity Financing

Independent Var.	OLS		PROBIT
	<i>NET_ISSUE</i>		<i>NET_ISSUE</i> $\geq$ 0.1
α	-0.09*** (0.04)	-0.12*** (0.04)	-0.32*** (0.10)
$\alpha \cdot ORDER_INFO$	0.05*** (0.015)	0.07*** (0.02)	0.27*** (0.09)
<i>ORDER_INFO</i>	0.06*** (0.02)	0.09*** (0.04)	0.25** (0.12)
<i>Q</i>	0.04*** (0.005)	0.05*** (0.005)	0.14*** (0.01)
<i>TANG</i>	0.04*** (0.015)	0.08*** (0.02)	0.33*** (0.07)
<i>LOGSALES</i>	-0.006 (0.005)	-0.012 (0.012)	-0.10*** (0.03)
<i>PROFIT</i>	-0.61*** (0.11)	-0.52*** (0.12)	-0.32*** (0.1)
Firm Fixed-effect	No	Yes	No
Year Fixed-effect	Yes	Yes	No
Adjusted/Pseudo- R^2	0.24	0.18	0.25

Definitions of all variables are listed in Table 1. The dependent variable of the Probit Regression is the indicator variable of the net equity issuance being above the threshold 0.1. Year-fixed effects are included in the OLS regressions. Coefficient estimates are shown in bold and their standard errors are displayed right below in the parentheses. Standard errors adjust for both heteroskedasticity and within correlation clustered by firm. Number of observations is 44875 for all columns. *, ** and *** indicate a two-tailed test significance level of less than 10%, 5% and 1%, respectively.

all three columns, which is plausible as large mature firms tend to be less equity financing dependent.

1.4.3 Other tests and robustness checks

In this section, I extend the test by adding more controls as well as analysing the interaction between the order flow informativeness and other important phenomenon related to equity financing. I show the tests without firm-fixed effects. The main result for the extended tests with firm-fixed effects and the Probit test are stronger. However, the interpretation of the results for the interaction between order flow informativeness and other related factors would be more clear without firm-fixed effects.

Sorting by equity financing dependence and size

To further control the effect from equity financing dependence, I assign firm-year observations to quintiles based on their KZ score and estimate equation (1.9) for each quintile. On the basis of parameter estimates from Kaplan Kaplan and Zingales (1997) [37], Baker, Stein, and Wurgler (2003) [6] constructed a four-variable version of the Kaplan–Zingales measure, $KZ4$, as a proxy for firms’ degree of equity dependence. $KZ4$ is calculated as a weighted sum of cash flow (CF_{it} , Item 18 + Item 14), cash dividends (DIV_{it} , Item 19 + Item 21), and cash balances (C_{it} , Item 1), all scaled by lagged assets ($A_{i,t-1}$, Item 6), as well as leverage ratio (LEV_{it} , [Item 9 + Item 34]/[Item 9 + Item 34 + Item 216]):

$$KZ4 = -\frac{1.002CF_{it}}{A_{i,t-1}} - \frac{39.368DIV_{it}}{A_{i,t-1}} - \frac{1.002C_{it}}{A_{i,t-1}} + 3.139LEV_{it} \quad (1.11)$$

Higher values of $KZ4$ indicate that firms are more constrained to equity financing. To assess the sensitivity of our results to firms’ capital constraints, we construct $KZ4$ along the same lines described above and incorporate it into our analysis. We find that $KZ4$ is positively correlated with PIN and $ORDER_INFO$. I follow Baker, Stein, and Wurgler (2003) [6] and assign firm-year observations to quintiles based on their $KZ4$ score.

Table 5 Panel A reports the result of estimating equation (1.9) without firm-fixed effects for each quintile subsample. All the control variables in Table 4 are also included in the analysis. The coefficients of the control variables are similar as in Table 4 for each quintile subsample. I report the coefficient for α and $\alpha \cdot ORDER_INFO$ here.

I find that the coefficient for α stay negative across all five quintile groups, while the coefficient for $\alpha \cdot ORDER_INFO$ stay positive for all groups. They are statistically significant at less than the 1% level for the 4th and 5th quintiles. The lowered significance is expected because of the smaller sample size in each quintile than the full sample. These results indicate that both information friction and external financing dependence play important roles in equity financing, with different firms affected by each factor to a different degree. Panel A also shows that the effect of information asymmetry and order flow informativeness is more statistically significant for firms that are more equity financing dependent. This result

is intuitive that firms with high equity financing needs are more affected by information friction.

I also assess the sensitivity of my results to firm size. As size is negatively correlated with both *ORDER_INFO* and PIN, I need to verify that our main results are not driven merely by size. To check this point, we construct portfolios by size quintiles (where *SIZE* is measured by firms' market equity) and estimate Equation (1.9) for each quintile.

Panel B reports the coefficients for α and $\alpha \cdot \text{ORDER_INFO}$ for each *SIZE* quintile. The coefficient for α stay negative across all five quintile groups, while the coefficient for $\alpha \cdot \text{ORDER_INFO}$ stay positive for all groups. They are statistically significant at less than the 1% level for the 4th and 5th quintiles. Given that the significance is expected to decrease relative to the basic regressions because of reduced sample size in quintiles, these results indicate that the probability of information asymmetry and the order flow informativeness, as captured by α and *ORDER_INFO*, remains an important factor even when size is considered. They also indicate that the effect of the probability of information asymmetry and the order flow informativeness is more significant for bigger firms. The result is consistent with the fact that the original theory of Myers and Majluf (1984) [44] is geared towards mature, low growth-option firms. I perform a series of robustness checks and obtain similar results (not tabulated). These tests include using NYSE quintiles instead of the size quintiles in our sample, forming quintiles using double sorting by capital constraints and size, as well as including *SIZE* and *KZ* directly as control variables in the pooled regression. In all these tests, the sign of the coefficient estimates for α and $\alpha \cdot \text{ORDER_INFO}$ remain consistent with the hypothesis, whereas their significance levels either remain intact or improve.

The business cycle

So far the results show that net equity issuance is negatively correlated to the probability of information asymmetry and positively correlated to the interaction between the probability of information asymmetry and the order flow informativeness, taking other factors not related to trading activities into account. This is consistent with the idea that the private information revealed through trading can alleviate the manager's adverse selection

Table 1.5: Effect of equity financing dependence

Independent Var.	Q1	Q2	Q3	Q4	Q5
Panel A Quintiles formed by <i>KZ4</i>					
α	-0.02 (0.09)	-0.05 (0.05)	-0.07* (0.037)	-0.04*** (0.015)	-0.06*** (0.02)
$\alpha \cdot ORDER_INFO$	0.01 (0.21)	0.03 (0.02)	0.02 (0.04)	0.06*** (0.02)	0.02*** (0.006)
No. of Observations	11454	10267	7466	8046	7642
Adjusted R^2	0.29	0.26	0.31	0.43	0.51
Panel B Quintiles formed by <i>SIZE</i>					
α	-0.24 (0.53)	-0.05 (0.51)	-0.01 (0.06)	-0.05*** (0.02)	-0.09*** (0.03)
$\alpha \cdot ORDER_INFO$	0.47 (0.59)	0.001 (0.09)	0.06 (0.06)	0.07*** (0.03)	0.08*** (0.03)
No. of Observations	5033	7795	9047	10815	12185
Adjusted R^2	0.26	0.26	0.37	0.25	0.13

Definitions of all variables are listed in Table 1. All observations are sorted into five subsamples depending on the quintile in which a firm's *KZ4* score (Panel A) or market size (Panel B) falls into during the previous year. *KZ4* is used to proxy for a firm's financing constraints. OLS Regressions as in Table 4 are estimated in each quintile. Year-fixed effects are included. Shown are the coefficient estimates and their standard errors in the parenthesis. Standard errors adjust for both heteroskedasticity and within correlation clustered by firm. *, ** and *** indicate a two-tailed test significance level of less than 10, 5 and 1%, respectively.

problem in equity financing. There is an important group of literature on equity financing that documents higher equity issuance during hot market (Taggart (1977) [50], Marsh (1982) [42], Jalilvand and Harris (1984) [35], Asquith and Mullins (1986) [5], Jung, Kim et al. (1996) [36], Pagano, Panetta et al. (1998) [45], Hovakimian, Opler et al. (2001) [32], Graham and Harvey (2001) [29]). Besides, investment opportunities also vary a lot over boom and recessions. Therefore, it is of interest to analysis the effect from order flow informativeness for different stages of the business cycles. I refer to the announcement of the NBER Business Cycle Dating Committee to identify the period of recession. My sample covers time period 1993 to 2012 during which the NBER Business Cycle Dating Committee announces: March 2001 as business cycle peak/beginning of last recession, November 2001 as business cycle trough/end of last recession, December 2007 as business cycle peak/beginning of last recession and June 2009 as business cycle trough/end of last recession. I choose the

Table 1.6: Order Flow Informativeness and Equity Financing over the Business Cycles

Independent Var.	Peak	Recession	Other Years
α	-0.03* (0.017)	-0.10*** (0.05)	-0.08*** (0.04)
$\alpha \cdot ORDER_INFO$	0.01* (0.0055)	0.04*** (0.02)	0.05*** (0.015)
No. Obs.	4752	4700	35405
Adjusted R^2	0.25	0.26	0.24

Definitions of all variables are listed in Table 1. All observations are divided into three subsamples according to the stage of the business cycle the observation falls in. The subsample "Peak" covers the data in 2000 and 2007. The subsample "Recession" covers 2001 and 2008. OLS Regressions as in Table 4 are estimated in each subsample. Both year- and firm-fixed effects are included. Shown are the coefficient estimates and their standard errors in the parenthesis. Standard errors adjust for both heteroskedasticity and within correlation clustered by firm. *, ** and *** indicate a two-tailed test significance level of less than 10, 5 and 1%, respectively.

firm-year observations that fall in the years covering more than 6 months of the recession period (2001 and 2008) to form the subsample "Recession", the firm-year observations that fall in the year leading the begin of the recession (year 2000 and 2007) to the subsample "peak", and all the other observations into another subsample.

Table 6 shows the results for each subsample. Columns 1 estimate the baseline regressions for the subsample of observations in the peak years. Columns 2 show the estimation for the baseline regressions for the subsample of observations in the recession years. Columns 3 show the estimation for the baseline regressions for the subsample of observations in all the other years.. All control variables in Table 4 are included and the coefficients are similar as in the previous study. I here report the coefficients for α and $\alpha \cdot ORDER_INFO$.

The coefficients for α and $\alpha \cdot ORDER_INFO$ are consistent with the hypothesis for all stages of business cycles (significant at less than 1% for subsamples other than the peak period). This indicates that the effect from the probability of information asymmetry and the order flow informativeness exists through all stages of business cycle, and is therefore not only a within-firm effect but a cross-sectional effect as well. The effects for α and

$\alpha \cdot ORDER_INFO$ are less statistically significant during the peak of the market. This is intuitive as the problem of information asymmetry is less severe in the hot market.

Controlling for other liquidity measures

According to Easley et al. (1996) [21], PIN is closely related to stock market illiquidity. In addition, Duarte and Young (2009) [17] claims that PIN might be driven by non-information liquidity factors. It is important to control for the effect from market liquidity in this study. I use four measures of market liquidity: share turnover ($TURNOVER$), idiosyncratic return volatility (ID_VOL), dollar volume weighted average daily absolute return ($Amihud_ILLQ$), and the non-information illiquidity developed by Pastor and Stambough ($REVERSE$). $TURNOVER$ is calculated as annual trading volume divided by average shares outstanding over the year. Stocks with higher turnover are considered to be more liquid. Recent study has also shown that high turnover with price soaring and large return volatility can also be a symptom of speculative bubble caused by difference in belief (Scheinkman and Xiong (2003) [49], Hong, Scheinkman, and Xiong (2006) [31]). Idiosyncratic return volatility is the average of squares of the residuals from regressing daily returns on value weighted market returns. It is used as a measure of illiquidity as prices tend to jump around when market is illiquid. Literature has documented negative correlation between financial reporting quality and idiosyncratic return volatility (Rajgopal and Venkatachalam (2011) [48]), which suggests it to be related to information factors. Dollar volume weighted average daily absolute return ($Amihud_ILLQ$) is used as illiquidity measure in literature (Amihud (2002) [4]). It captures market illiquidity by price response to trading volume. The above three measures can be driven by both informational and non-information liquidity factors. Finally, Pastor and Stambough (2003) [46] measures non-information illiquidity by next day price reversal. According to their procedure, I estimate the illiquidity measure ρ_{it} for each firm-year from the following OLS regression:

$$r_{i,d+1,t}^e = a_{i,t} + \phi_{i,t} r_{i,d,t} + \rho_{i,t} \text{sign}(r_{i,d,t}^e) \nu_{i,d,t} + \varepsilon_{i,d+1,t}, \quad d = 1, 2, \dots, D \quad (1.12)$$

Table 1.7: Order Flow Informativeness and Other Liquidity Measures

Independent Var.	<i>PIN</i>	<i>ORDER_INFO</i>	α
<i>TURNOVER</i>	-0.018*** (0.002)	-0.04*** (0.004)	0.017*** (0.002)
<i>ID_VOL</i>	2.67*** (0.41)	5.2*** (0.58)	-1.62** (0.8)
<i>REVERSE</i>	0.017 (0.03)	0.05 (0.07)	-0.034 (0.038)
<i>Amihud_ILLQ</i>	0.0013** (0.0007)	0.0017*** (0.00069)	-0.002 (0.003)
Adjusted R^2	0.18	0.36	0.08

Definitions of all variables are listed in Table 1. Regressions of information variables on *TURNOVER*, *ID_VOL*, *REVERSE* and *Amihud_ILLQ* are estimated. Standard errors are in the parenthesis.

Standard errors are adjusted for both heteroskedasticity and within correlation clustered by firm.

*, ** and *** indicate a two-tailed test significance level of less than 10, 5 and 1%, respectively.

where the quantities are defined as follows: $r_{i,d,t}$ is the stock return for firm i on day d in year t ; $r_{i,d,t}^e = r_{i,d,t} - r_{i,m,t}$ where $r_{i,m,t}$ is the return on the CRSP value weighted market return on day d in year t ; $\nu_{i,d,t}$ is the dollar trading volume for the stock of firm i on day d in year t . The basic idea is that, when the excessive price response to trading volume is mainly due to non-information market illiquidity, the next day price will reverse back, since market expectation does not change. The term $\phi_{i,t}r_{i,d,t}$ is included here to control for the return continuation due to information factors. This mechanism is first studied by Campbell et al. (1993) [1]. Pastor and Stambough (2003) [2] further identified the sensitivity of this illiquidity measure to market wide illiquidity as a liquidity risk factor.

Table 7 reports two dimensional clustered regressions of *PIN*, *ORDER_INFO* and α on *TURNOVER*, *REVERSE*, *ID_VOL* and *Amihud_ILLQ*. *PIN*, *ORDER_INFO* are negatively correlated with *TURNOVER*, and positively correlated with *ID_VOL* and *Amihud_ILLQ* (all are significant at less than 1% level). This is consistent with previous literature that higher level of informed trade is negatively related to market liquidity. However, the coefficient for *REVERSE* is insignificant for all three information variables which confirms the idea that *PIN* and the information variables decomposed from it are not driven by non-information liquidity factors.

Table 1.8: Controlling for other liquidity measures

	Net Equity Issuance			
	<i>TURNOVER</i>	<i>ID_VOL</i>	<i>REVERSE</i>	<i>Amihud_ILLQ</i>
α	-0.10*** (0.03)	-0.07*** (0.026)	-0.11*** (0.03)	-0.09*** (0.04)
$\alpha \cdot ORDER_INFO$	0.06*** (0.025)	0.03*** (0.013)	0.09*** (0.027)	0.05*** (0.015)
<i>TURNOVER</i>	0.002 (0.0015)	–	–	–
<i>ID_VOL</i>	–	-0.14 (0.25)	–	–
<i>REVERSE</i>	–	–	-0.009 (0.015)	–
<i>Amihud_ILLQ</i>	–	–	–	-0.006 (0.006)
Adjusted R^2	0.24	0.24	0.22	0.24

Definitions of all variables are listed in Table 1. OLS Regressions of net equity issuance on α , $\alpha \cdot ORDER_INFO$ and Q and other liquidity measures are estimated. Year-fixed effects are included. Shown are the coefficients for each independent variable and their standard errors in the parenthesis below. Standard errors are adjusted for both heteroskedasticity and within correlation clustered by firm. *, ** and *** indicate a two-tailed test significance level of less than 10, 5 and 1%, respectively.

I include the four liquidity measures to the basic test and show the results in Table 8. Columns 1 and 2 show the effect from *TURNOVER*. Columns 3 and 4 show the results for *ID_VOL* and Columns 7 and 8 report the analysis for *Amihud_ILLQ*. Columns 5 and 6 show the results for *REVERSE*. The coefficients for all four liquidity measures are statistically insignificant. As all four measures contains a high proportion of pre-issuance non-informational liquidity factors, the result indicates that pre-issuance non-informational liquidity factor might not have strong impact on equity financing decisions. Overall, the effect from the probability of information asymmetry and order flow informativeness remain statistically significant after including the liquidity factors into the test.

Insider, institutional trade and analyst coverage

PIN and *ORDER_INFO* are constructed to capture the trading activity of agents that possess information advantage. It is also widely assumed that insider and institutional

Table 1.9: Correlation Matrix of Order Flow Informativeness, Insider, Institutional Trade and Analyst Coverage

	α	<i>PIN</i>	<i>ORDER_INFO</i>	<i>INSIDER</i>	<i>INST_TRADE</i>	<i>ANALYST</i>
α	1	0.15	-0.25	-	-	0.01
<i>PIN</i>		1	0.69	0.017	-	-0.09
<i>ORDER_INFO</i>			1	0.02	0.01	-0.1
<i>INSIDER</i>				1	-	-
<i>INST_TRADE</i>					1	0.05
<i>ANALYST</i>						1

Pearson correlation between the information variables, insider and institutional trade and analyst coverage.

Table 1.10: Insider, Institutional Trade, Analyst Coverage and Equity Financing

	Net Equity Issuance					
	<i>INSIDER</i>		<i>INST_TRADE</i>		<i>ANALYST</i>	
	(1)	(2)	(3)	(4)	(5)	(6)
α	-0.07*** (0.026)	-	-0.07*** (0.026)	-	-0.09*** (0.04)	-
$\alpha \cdot ORDER_INFO$	0.03*** (0.013)	-	0.03*** (0.013)	-	0.05*** (0.015)	-
<i>INSIDER</i>	-0.002 (0.004)	-0.001 (0.003)	-	-	-	-
<i>INST_TRADE</i>	-	-	-0.004 (0.01)	0.004 (0.01)	-	-
<i>ANALYST</i>	-	-			-0.01*** (0.003)	-0.02*** (0.003)
Adjusted R^2	0.27	0.26	0.27	0.26	0.23	0.22

Definitions of all variables are listed in Table 1. Regressions of Net Equity Issuance on α , $\alpha \cdot ORDER_INFO$, Q and insider trade, institutional trade and analyst coverage are estimated. and insider trade, institutional trade and analyst coverage are estimated. Year-fixed effects are included. Shown are the coefficient estimates and their standard errors in the parenthesis. Standard errors are adjusted for both heteroskedasticity and within correlation clustered by firm. *, ** and *** indicate a two-tailed test significance level of less than 10, 5 and 1%, respectively.

traders have more access to inside information than ordinary investors. Therefore, I study the effect from insider and institutional trade on equity financing report the results in Table 9. I also include analyst coverage into the study to check if the coefficients for the private information variables still stays positive when controlling for the public source of information.

I measure the intensity of a firm's insider trading activities (*INSIDER*) as the ratio of insider transactions to the total number of all transactions for a given firm-year as recorded in TAQ. From the summary statistics in Table 2, insider trading only composes

of a very small fraction of the total number of transactions. The intensity of institutional trade (*INST_TRADE*) is calculated by summing up the absolute value of quarterly position change for all institutional holders over a year, scaled by shares outstanding. The analyst coverage (*ANALYST*) is calculated as the nature logarithm of the number of analysts that have issued either an earnings forecast or a stock recommendation for the firm in year $t - 1$.

Table 9 reports the correlation matrix of *PIN*, *ORDER_INFO*, *INSIDER*, *INST_TRADE* and *ANALYST*. *ORDER_INFO* is positively correlated with *INSIDER* and *INST_TRADE* with Pearson correlation coefficient 0.02 and 0.01, which supports using *ORDER_INFO* as proxy for order flow informativeness. The level of the correlation is not high, as not all trading activity from insider and institutional holders arise from information. *ORDER_INFO* is negatively correlated with *ANALYST*. All these correlations are significant with p-value < 0.001 .

Table 10 reports the basic regression including the effect from insider trade, institutional trade or analyst coverage. Columns 1 and 2 of Table 10 show the effect from insider trade. Columns 3 and 4 show the effect from institutional trade. Overall, the effects from *INSIDER* and *INST_TRADE* are not statistically significant. *INSIDER* constitutes only a very small fraction of the trading volume. Informed trade may also come from traders that are very skillful at analyzing publicly available information. Also a lot of institutional trades are triggered by non-information reason such as portfolio rebalance, index tracking, deleverage, & etc. (e.g. Coval and Stafford (2007) [15], Brunnermeier and Pedersen (2009) [10], Adrian and Shin (2014) [1]), therefore *INST_TRADE* cannot distinguish informational trade from other institutional trading activities. Columns 5 and 6 report the analysis for *ANALYST*. It shows that firm with higher analyst coverage issue less equity. The coefficient for α and $\alpha \cdot \text{ORDER_INFO}$ stays consistent with the hypothesis and significant at less than 1% after adding these variables to the regression.

Other specifications

To ensure that the results are not mainly capturing the effect on stock repurchase, I perform the same tests on the sample restricted to observations with only positive net equity

issuance. The result is qualitatively similar (hence not reported). In addition, the results are not qualitatively affected if I exclude the observations from 2007 to 2010, the time period of high frequency trading (Philips (2013) [47], Zhang (2010) [53]).

1.5 conclusion

This paper studies the empirical relation between the amount of information conveyed through order flow and the net equity issuance. Using the measure of order flow informativeness decomposed from the PIN measure, I find strong effect from the amount of information conveyed through order flow on reducing the negative correlation between the information asymmetry and the net equity issuance. This relation is robust to the inclusion of controls for investment opportunity, market timing incentive, non-information liquidity factors, analyst coverage, capital constraint, and firm size, as well as to a variety of different specifications. Overall, my results are consistent with the hypothesis that information transmitted through trading activity alleviates manager's adverse selection problem and facilitates equity financing. The results help establish a better understanding of how informed trading affects firm's equity issuance. Studies on informed trade and cost of capital typically focus on the mechanism through market liquidity. My results reveal another important role of informed trading, namely, signaling the true value of the firm and reducing the information asymmetry between manager and the firm. This factor appears to take effect on firm's financing decisions.

Chapter 2

Central Bank Price Learning and Security Price Informativeness

2.1 Introduction

Aggregating the heterogeneous beliefs possessed by many speculators, prices of financial securities provide useful information about economic fundamentals. It is therefore expected that central banks monitor the market closely, trying to extract new information that will guide their decision process. However, the strategic interaction among speculators may change, when the speculators realize that the security price will influence the monetary policy which will in turn move the economic fundamental that determines their payoff. Economists have long been studying the central bank's impact on price informativeness. Morris and Shin (2005) [43] points out that there is a tension between managing the market expectations and learning from the market expectations. In their paper, it is proved that central bank transparency can reduce the price informativeness as individuals put too much weight on the public information and ignore their own private information.

This paper focuses on the feedback effect from the financial market to the monetary policy makers and its impact on the financial market price informativeness. In the model, the central bank learns from the speculative trading in the financial market about the

economic fundamental, and uses the inferred information to guide its policy decisions. While learning from the market is usually perceived as a positive step helping to improve the decision process of the central bank, our theoretical analysis uncovers a side effect of central bank's learning. The very fact that the central bank learns from the market price turns out to reduce the speculators' incentive to coordinate and reveal their correlated information leading to lower price informativeness.

Turning to the detail of the model, I study a situation where the central bank makes decision about the amount by which to adjust the economic fundamental. The central bank wants to optimize the social welfare which is a quadratic loss function of the distance between the realized economic fundamental and a target value. The central bank is only partially informed about the unadjusted economic fundamental. The financial market contains many small speculators trading a security whose payoff depends on the future economic fundamental. Speculators in the financial market also have pieces of information. By observing the financial security price, the central bank extracts another signal about the economic fundamental. The setting of the model is similar as in Goldstein, Ozdenoren and Yuan (2012) [28] which studies an environment where financial market trading has a feedback effect on the firm's capital provider. While financial security price has a positive feedback effect on the firm's capital provider in their paper, it gets to have a negative feedback effect on the central bank in this model as the central bank wants to adjust the economic fundamental backward towards the target value. This negative feedback effect discourages the speculators from coordinating which is described by the desire of speculators to put more weight on signals that are correlated with other speculators' information.

Another important feature imported from Goldstein, Ozdenoren and Yuan (2012) [28] is the noisy price-elastic supply of the financial security. The market clearing price reflects information about the economic fundamental, as aggregated from speculators' trades. Given the structure of information and trading, the information in the price contains noise from both the noisy supply and the common noise in speculators' information. Without noise in the security supply, the price fully reveals the economic fundamental if speculators put zero

weight on their correlated information, since idiosyncratic signal noises among speculators are canceled out when aggregated. However, as there is noise trading, coordination among speculators is desirable for price informativeness.

Analyzing the weight speculators put on the correlated signal relative to the idiosyncratic signal, I study how central bank's price learning can influence the financial security's price informativeness. Without strategic interactions, this weight should be given by the ratio of precisions between the correlated and the idiosyncratic signals. However, in the equilibrium of this model, the weight is lowered by two sources of strategic substitution. First, the higher the weight put on the common information, the more strongly the information is reflected in the price, hence, the speculators have less incentive to trade on the basis of this information. This effect is the natural outcome of a price mechanism and is weak for high market liquidity. This paper focuses on the second effect which arises due to the negative feedback effect from the price to the central bank's monetary policy decision. In the model, the central bank tries to boost the economy if the expected future fundamental value is lower than the target value and vice versa. When speculators put a higher weight on the correlated information, this information gets to have a stronger effect on the central bank and hence reduce the profit of their position in the security. Then, the incentive of each speculator to put weight on this information is further decreased. Studying the effect of this strategic substitution, the model reaches the result that the price learning of the central bank in making its monetary policy reduces the price informativeness by decreasing the level of coordination among speculators.

I also study the policy implication considering the influence of the central bank's price learning on the market price informativeness. Learning from the market exposes the central bank to a time-inconsistency problem that creates a benefit to commitment. In particular, I show that the central bank can improve the ex-ante effectiveness of its monetary policy by committing to a lower level of central bank intervention than is ex-post optimal. While this generates lower social welfare ex-post, it changes the ex-ante incentive of speculators in

a way that makes them put a higher weight on their correlated information. This, in turn, increases the informativeness of the security price.

The remainder of this paper is organized as follows. In Section 2, I present the model setup and characterize the equilibrium of the model. Section 3 analyzes the determinants of coordination among speculators and its implication for price informativeness in our model. Section 4 gives the policy implication. Section 5 concludes. All proofs are provided in the appendix.

2.2 The Model

2.2.1 The Model Set up

There are two dates $t = 0, 1$. The economy fundamental at time t is described by the state variable θ_t , where $\theta_t \in R$. The future economy state depends both on the current economy fundamental and the central bank's monetary policy, that is, $\theta_1 = \theta_0 + \sigma_\theta \varepsilon_\theta + \Delta\theta$, where ε_θ is normally distributed with mean zero and unit variance. $\Delta\theta$ is the impact from the central bank intervention. Denote $\tau_\theta = \frac{1}{\sigma_\theta^2}$. Let $\tilde{\theta}_1 = \theta_0 + \sigma_\theta \varepsilon_\theta$, which is the future economy state without the central bank intervention. There is one traded security whose payoff is $\beta\theta_1 + \gamma$. Therefore, the payoff of the security is also influenced by the central bank's monetary policy. There is a measure-one continuum of speculators indexed by $i \in [0, 1]$. At $t = 0$, each speculator receives one privately observed signal and one common signal. The common signal is observed by all the speculators. The private signal is $s_i = \tilde{\theta}_1 + \sigma_s \varepsilon_i$, where ε_i is independent of the fundamental noise ε_θ and across all i and normally distributed with mean zero and unit variance. The precision of the private signal is $\tau_s = \frac{1}{\sigma_s^2}$. The common signal is $s_c = \tilde{\theta}_1 + \sigma_c \varepsilon_c$, where ε_c is normally distributed with mean zero and unit variance and is independent of the fundamental noise ε_θ and all the private signal noises. The precision of the common signal is $\tau_c = \frac{1}{\sigma_c^2}$. Denote the information set of speculator i by $\mathcal{F}_i = \{s_c, s_i\}$.

After observing the signals, the speculators trade in the security market. Each speculator can long or short up to one unit of the security. The position of speculator i is denoted by $x(i)$, where $x(i) \in [-1, 1]$. Due to risk neutrality, speculators choose their positions to maximize expected profits. Formally, speculator i chooses $x(i)$ to solve:

$$\max_{x(i) \in [-1, 1]} x(i) E [\beta \theta_1 + \gamma - P | \mathcal{F}_i] \quad (2.1)$$

P is the price of the security. Denote the total speculators' demand of the security by X_s . Note that we assume that speculators do not observe the price when they trade, and hence they submit market orders, as in Kyle (1985) [38]. There is a noisy elastic supply of the security $Q(\xi, P)$. To keep the model tractable, $Q(\xi, P)$ is set to be $1 - 2\Phi(\xi - \alpha P)$, in which $\Phi(\cdot)$ is the standard normal CDF function and ξ is normally distributed with mean zero and variance σ_ξ^2 . Denote $\tau_\xi = \frac{1}{\sigma_\xi^2}$. The parameter α captures the elasticity of the supply curve with respect to the price. It can be interpreted as the liquidity of the market: when α is high, the supply is very elastic with respect to the price, and so large informed demand is easily absorbed in the price without having much of a price impact. The price of the security P is determined by the market clear condition:

$$X_s = Q(\xi, P) = 1 - 2\Phi(\xi - \alpha P) \quad (2.2)$$

Also at $t = 0$, after observing the price P and a private signal, the central bank makes decision on the monetary policy. The central bank's signal is $s_g = \tilde{\theta}_1 + \sigma_g \varepsilon_g$, where ε_g is normally distributed with mean zero and unit variance and is independent of all the other noises. Denote the central bank's information set by $\mathcal{F}_g = \{P, s_g\}$. There is a desirable economy state θ^* . The central bank wants to maximize the social welfare which is described by $-(\theta^* - \theta_1)^2$. Assume that the effect of the monetary policy is linear in the distance between the desirable economy state θ^* and the expected future economy state without central bank intervention $E[\tilde{\theta}_1 | \mathcal{F}_g]$, that is, $\Delta\theta(\lambda) = \lambda \left(\theta^* - E[\tilde{\theta}_1 | \mathcal{F}_g] \right)$ where $\lambda \in [0, 1]$. λ describe the level of central bank intervention. The central bank can commit itself to a

specific level of λ or chooses λ to maximize social welfare ex post. Here I will solve the equilibrium under the first condition and then discuss the effect of both types of monetary policy decision rule.

At $t = 1$, the state variable θ_1 is observed and the payoffs are realized.

2.2.2 Equilibrium

DEFINITION 1: An equilibrium consists of a price function, $P(\tilde{\theta}_1, \varepsilon_c, \xi) : R^3 \rightarrow R$, an monetary policy rule $\Delta\theta(s_g, P) : R^2 \rightarrow R$, speculator's strategy $x(s_i, s_c) : R^2 \rightarrow [-1, 1]$, and the corresponding aggregate demand $X_s(\tilde{\theta}_1, \varepsilon_c) : R^2 \rightarrow [-1, 1]$, such that:

- For speculator i , $x(s_i, s_c) = \arg \max_{x(i) \in [-1, 1]} x(i) E[\beta\theta_1 + \gamma - P | \mathcal{F}_i]$
- The central bank's monetary policy rule is $\Delta\theta(s_g, P) = \lambda(\theta^* - E[\tilde{\theta}_1 | \mathcal{F}_g])$
- The market clearing condition for the security is satisfied:

$$X_s(\tilde{\theta}_1, \varepsilon_c) = \int_0^1 x(s_i, s_c) di = Q(\xi, P)$$

Similar as in Goldstein, Ozdenoren, and Yuan (2012) [28], this paper focuses on the *linear monotone equilibrium* where a speculator longs the security if and only if the linear combination of the signals is above a threshold level.

DEFINITION 2: A *linear monotone equilibrium* is an equilibrium where $x(s_i, s_c) = 1$ if $s_i + ks_c \geq g$ for some constant k and g and $x(s_i, s_c) = -1$ otherwise.

A linear monotone equilibrium is solved in following steps. First, the security price is obtained from the speculators' strategy and the market clearing condition. Second, a signal of the economy state is extracted from the price. Finally, based on the central bank's monetary policy rule and the asset price rule, the speculators' trading strategy is solved.

In a linear monotone equilibrium, the speculators long the security if $s_i + ks_c \geq g$, which is equivalent to $\varepsilon_i \geq \frac{g - ks_c - \tilde{\theta}_1}{\sigma_s}$. Hence, there aggregate long position is $1 - \Phi\left(\frac{g - ks_c - \tilde{\theta}_1}{\sigma_s}\right)$.

Similarly, there aggregate short position is $\Phi\left(\frac{g-ks_c-\tilde{\theta}_1}{\sigma_s}\right)$. The net position from the speculators is $1 - 2\Phi\left(\frac{g-ks_c-\tilde{\theta}_1}{\sigma_s}\right)$. Then by the market clearing condition, the equilibrium price is given by:

$$P = \frac{\sigma_s \xi - g + ks_c + \tilde{\theta}_1}{\alpha \sigma_s} \quad (2.3)$$

$$= \frac{(1+k)\tilde{\theta}_1 + k\sigma_c \varepsilon_c + \sigma_s \xi - g}{\alpha \sigma_s} \quad (2.4)$$

Define the price signal as:

$$z(P) = \frac{\alpha \sigma_s P + g}{1+k} \quad (2.5)$$

$$= \tilde{\theta}_1 + \frac{k}{1+k} \sigma_c \varepsilon_c + \frac{1}{1+k} \sigma_s \xi \quad (2.6)$$

$$= \frac{1}{1+k} \tilde{\theta}_1 + \frac{k}{1+k} s_c + \frac{1}{1+k} \sigma_s \xi \quad (2.7)$$

It contains the same information as in the price P . The precision of the price signal is:

$$\tau_p = \frac{1}{\sigma_p^2} = \frac{(1+k)^2 \tau_c \tau_\xi \tau_s}{k^2 \tau_\xi \tau_s + \tau_c} \quad (2.8)$$

Now the central bank's expectation of the state variable can be derived. Conditional on the price signal $z(P)$ and its own signal s_g , central bank beliefs that $\tilde{\theta}_1$ is normally distributed with mean $\frac{\tau_\theta \theta_0 + \tau_g s_g + \tau_p z(P)}{\tau_\theta + \tau_g + \tau_p}$ and $\frac{1}{\tau_\theta + \tau_g + \tau_p}$. The central bank's monetary policy can be written as:

$$\Delta\theta(\lambda) = \lambda\theta^* - \lambda \frac{\tau_\theta \theta_0 + \tau_g s_g + \tau_p z(P)}{\tau_\theta + \tau_g + \tau_p} \quad (2.9)$$

Given the monetary policy and the price above, the speculator i 's profit from longing one unit of the security can be expressed as:

$$E \left[\beta \left(\tilde{\theta}_1 + \lambda \theta^* - \lambda \frac{\tau_\theta \theta_0 + \tau_g s_g + \tau_p z(P)}{\tau_\theta + \tau_g + \tau_p} \right) + \gamma - \frac{\sigma_s \xi - g + k s_c + \tilde{\theta}_1}{\alpha \sigma_s} \middle| s_i, s_c \right] \quad (2.10)$$

In the best response, the speculators should choose to long the security when the above expression is positive and short otherwise, which can be written into the condition $A(k)s_i + B(k)s_c \geq C(k)$, where $A(k)$, $B(k)$ and $C(k)$ are functions of k . Assume that $A(k)$ is positive (which can be verified in the proof of Propositions), the equilibrium is solved when $B(k) = kA(k)$ which leads to a third-order polynomial. The following proposition is obtained by analyzing the polynomial. All proofs are in the Appendix.

PROPOSITION 1: For a high enough level of price elasticity α , there exists a equilibrium with weight $k^* > 0$ that speculators put on the common signal. This equilibrium is unique when security supply noise or the fundamental noise is large.

The weight k^* that speculators put on the common signal in equilibrium captures the degree of coordination in their trading decisions. In a market with supply noise, an adequate level of coordination among speculators is desirable for price informativeness and social welfare. In the following section, I will analyze how central bank's price learning can influence price informativeness when the level of price elasticity is high.

2.3 The Impact of Central Bank's Price Learning on Price Informativeness

In a world without the strategic interactions, the weight that speculators put on the common signal relative to the private signal would be equal to the ratio of precisions between the signals: $\frac{\tau_c}{\tau_s}$. In this model, the weight that speculators put on the common signal in this model is affected by the strategic substitutes among them.

There are the two sources of strategic substitution. Recall from (2.1), that a speculator's expected profit is $x(i)E[\beta\theta_1 + \gamma - P|\mathcal{F}_i]$. When other speculators put more weight on the common signal, this signal gets to have a stronger effect on the price P , as well as on the central bank's monetary policy $\Delta\theta$ due to the price learning by the central bank.. The first effect pushes the speculator to put a lower weight on the common signal, since relying on the common signal more heavily implies paying a high price when buying and getting a low price when selling. This first effect is the same as the price mechanism in Goldstein, Ozdenoren and Yuan (2012) [28].

However, the second effect is different, in that, it pushes the speculator to put a even lower weight on the common signal. The central bank learns the common signal through price and try to pull back the economy state accordingly which establishes a negative feedback effect, while Goldstein, Ozdenoren and Yuan (2012) [28] get strategic complementary in the second effect as they have positive feedback from the price signal.

Similar as in Goldstein, Ozdenoren and Yuan (2012) [28], in a market of high liquidity, or equivalently high price elasticity α , the first effect is marginal and the second effect plays a key role in the model. Therefore, in the following analysis, I focus on the case where the market liquidity is high, that is, the price elasticity α is high.

To study the effect of the central bank's price learning, it's useful to obtain the benchmark case where there is no feedback effect from price to the central bank's monetary policy. The result is the same as in Goldstein, Ozdenoren and Yuan (2012) [28]. Without the central bank's price learning, the speculator's expected profit is $x(i)E[\beta\tilde{\theta}_1 + \gamma - P|\mathcal{F}_i]$. The equilibrium weight k_{BM} solves the equation:

$$\frac{A_{BM}(k)}{B_{BM}(k)} = \frac{\tau_c}{\tau_s} - \frac{\frac{\sqrt{\tau_s}}{\alpha}k}{\frac{\tau_s}{\tau_f + \tau_s + \tau_c} \left(\frac{\tau_f + 2\tau_g}{\tau_f + \tau_g} - \frac{\sqrt{\tau_s}}{\alpha} \right)} = k \quad (2.11)$$

Then the equilibrium weight that speculators put on the common signal in the case of no central bank's price learning is:

$$k_{BM} = \frac{\left(\left(1 - \frac{\sqrt{\tau_s}}{\alpha} \right) \tau_f + \left(2 - \frac{\sqrt{\tau_s}}{\alpha} \right) \tau_g \right) \tau_c}{\left(\frac{\sqrt{\tau_s}}{\alpha} \right) (\tau_f + \tau_g) (\tau_f + \tau_s + \tau_c) + \left(\left(1 - \frac{\sqrt{\tau_s}}{\alpha} \right) \tau_f + \left(2 - \frac{\sqrt{\tau_s}}{\alpha} \right) \tau_g \right) \tau_s} \quad (2.12)$$

From (2.11) and (2.12), it's obvious that k_{BM} is lower than $\frac{\tau_c}{\tau_s}$ which is the weight speculators put on the common signal when there is no strategic interaction. In addition, k_{BM} converges to $\frac{\tau_c}{\tau_s}$, as the price elasticity α goes to infinity. This is consistent with the effect of the price mechanism discussed above. The following proposition summarizes the relation between the equilibrium weight k^* to the benchmark weight k_{BM} , which characterizes the effect of the price learning by the central bank.

PROPOSITION 2: When there is unique positive equilibrium, the weight k^* that speculators put on their common signal decreases in the level of central bank intervention λ .

Proposition 2 is the outcome of the feedback effect from the financial market to the central bank. If all the other speculators put a high weight on the common signal, an individual speculator will have less incentive to trade on the basis of this information, since the central bank who will adjust the economy fundamental backward according to this information. Given the supply noise, the decrease in the coordination among speculators may impair the price informativeness. The relation between the weight k and the price signal precision is the same as in Goldstein, Ozdenoren and Yuan (2012). The price signal precision τ_p is maximized at $k_{op} = \frac{\tau_c}{\tau_s \tau_\xi}$ with value $\tau_c + \tau_s \tau_\xi$ and is minimized at $k_m = -1$ with value 0. The price signal precision τ_p is increasing in k , for $k \in [-1, k_{op}]$.

This result says that a higher level of coordination among speculators is required to maximize the price informativeness for a higher level of supply noise. When the security supply noise $1/\tau_\xi$ is higher than 1, k_{BM} is lower than k_{op} . Hence, the strategic substitution caused by the central bank's price learning, further lowering the level of coordination among speculators, will push the price informativeness even farther away from optimum. Denote

$\tau_{op} = \tau_c + \tau_s \tau_\xi$, and $\tau_{BM} = \frac{(1+k_{BM})^2 \tau_c \tau_\xi \tau_s}{k_{BM}^2 \tau_\xi \tau_s + \tau_c}$. The following Proposition states this argument formally.

PROPOSITION 3: When there exists unique positive equilibrium and the security supply noise $1/\tau_\xi$ greater than 1, the price informativeness τ_p is lower than that of the benchmark case τ_{BM} .

When the security supply noise $1/\tau_\xi$ is lower than 1, k_{BM} is higher than k_{op} . In this case, a decrease in the coordination level among speculators does not necessarily impair the price informativeness. However, when the central bank's policy rule is aggressive, i.e. $\lambda \rightarrow 1$, the price informativeness can still be decreased by the price learning effect.

PROPOSITION 4: When the price elasticity $\alpha \rightarrow \infty$ and the fundamental precision $\tau_\theta \rightarrow 0$, there exists unique equilibrium with weight $k^* \rightarrow 0$ as the level of central bank intervention $\lambda \rightarrow 1$.

When the fundamental noise is large, the strategic interaction plays a key role. Proposition 4 indicates that the price informativeness is always impaired when the central bank applies aggressive monetary policy rule and fundamental noise is large.

2.4 Policy Implication

In this section, I analyze the optimal policy rule considering the impact of central bank's price learning on security price informativeness. The central bank can commit itself to a specific level of λ or choose λ to maximize social welfare ex-post. The next proposition gives the ex-post optimal monetary policy and the relation between social welfare and price informativeness under the ex-post optimal monetary policy.

PROPOSITION 5: It is always ex-post optimal for the central bank to set $\lambda = 1$. The ex-ante expected social welfare increases in the price informativeness when the central bank chooses λ to be 1.

When making the ex-post policy decision, the central bank takes the price informativeness as given. Without considering its influence on price informativeness, it is always optimal for the central bank to adjust the economy by the full amount of the expected distance from the desirable state. In this case, the precision of the signals received by the central bank is the only factor that influence the social welfare. To see this, consider the extreme case that the central bank knows the exact future economy state, then it can set the future economy state to the desirable state for sure which leads to the highest possible social welfare.

However, in the last section, it is proved that aggressive monetary policy can impair price informativeness. There is obviously trade-off between the benefit of a higher level of central bank intervention and the cost of a lower level of price informativeness. The ex-ante expected social welfare when the central bank commits to a specific $\lambda \in (0, 1)$ can be written as:

$$U(\lambda) = -E \left[\left(\theta^* - \tilde{\theta}_1 - \lambda(\theta^* - E[\tilde{\theta}_1 | \mathcal{F}_g]) \right)^2 \right] \quad (2.13)$$

$$\begin{aligned} &= - \left[(1 - \lambda)^2 \theta^{*2} + E\tilde{\theta}_1^2 + (\lambda^2 - 2\lambda)E[\tilde{\theta}_1 | \mathcal{F}_g]^2 - 2(1 - \lambda)^2 \theta^* E[\tilde{\theta}_1 | \mathcal{F}_g] \right] \\ &= -(1 - \lambda)^2 \left(\frac{1}{\tau_\theta} + (\theta^* - \theta_0)^2 \right) - (1 - (1 - \lambda)^2) \frac{1}{\tau_\theta + \tau_g + \tau_P} \\ &= -(1 - \lambda)^2 E \left[\left(\theta^* - \tilde{\theta}_1 \right)^2 \right] - (1 - (1 - \lambda)^2) \frac{1}{\tau_\theta + \tau_g + \tau_P} \end{aligned} \quad (2.14)$$

From the above expression, the ex-ante expected social welfare is determined by both the ex-ante expected social welfare improved with central bank intervention and the precision of the signals received by the central bank. The first term represents the benefit of a higher level of central bank intervention, while the second term is related to the cost of reducing the price informativeness. When λ is small, the benefit of a higher level of intervention plays a key role. As λ increases, the importance of the precision of signals increases, and it becomes the only influential factor when λ reaches 1. The following proposition analyzes

the desirability of a policy tool in which the central bank commits to a lower level of intervention.

PROPOSITION 6: It is not always ex-ante optimal for the central bank to have $\lambda = 1$.

When the security supply noise is large enough, it is optimal for the central bank to commit to a slight deviation from the ex-post optimal level of intervention when the level of noise trading is high. The reason follows from the envelope theorem, as we know that the cost of a small deviation from the ex-post optimal level of intervention approaches zero, but the benefit from the increased price informativeness is strictly positive. As a result, slight deviation from ex-post optimal level of intervention is always desirable. Of course, large deviations are not desirable. For example, it is never optimal to set λ to be zero. This is because the only advantage to reduce the central bank intervention is the improved price informativeness, but this is no use if the central bank does not take action at all.

Proposition 6 exposes a problem of time inconsistency in the central bank's policy. While the central bank wants to commit ex-ante to a lower level of intervention, once the signals are realized, the central bank would always be tempted to take more aggressive actions. In reality, such commitment can be achieved, as central banks have limited capability in adjusting the economic fundamentals.

2.5 Conclusion

I study strategic interactions among speculators in financial markets and their impact on price informativeness, considering the central bank's price learning in making monetary policy decisions. In addition to the strategic substitution caused by a traditional price mechanism, there is a second source of strategic substitution due to the feedback effect from the price to the central bank. A certain action by speculators changes their final payoff in a way that reduces the incentive of other speculators to take this action, i.e., speculators avoid

to act in the same way as other speculators, which lower the level of coordination among them. This decreases the price informativeness, as speculators' correlated information is buried in the security supply noise. I also analyze the policy implication of this feedback effect on the price informativeness.

Chapter 3

Investment Coordination and Amplified Trading Frenzies

3.1 Introduction

As the global economy is becoming more and more interdependent, it is especially interesting to study how one country's financial market can influence the other country's economy. I study an environment of two regions, each with a capital provider to decide how much capital to invest into the economy. The productivity of each region depends on its own productivity shock and the other region's amount of investment. Higher investment of one region increases the productivity of the other. This can be interpreted as two regions specialising in different positions of the same value chain, and each economy yields high output only when the other region also makes large input of resource. Each capital provider has private information about its own region's productivity shock. They also observe each other's financial market to predict each other's investment decisions. This cross region learning amplifies the feedback effect between the financial market and the real investment.

Each financial market has the same structure as Goldstein, Ozdenoren and Yuan (2013) [28]. It contains many small speculators trading a claim to the economy's output, which increases in the amount of both region's investment. I assume that speculators in

each financial market can only learn about the productivity shock that directly impact its own region. Each individual has access to a private information which is independent across speculators (given the same realized productivity shock) and a common signal which is observed by all speculators. This paper focuses on the weight the speculators put on the common signal in each financial market. The model is closed by a noisy price-elastic supply in the financial market. The endogenous price of each financial market, aggregates information about the productivity of its own region from the speculator's trades. It contains noises from the speculators common signal and the noise trading. The prices provide information for the capital providers to predict its own productivity shock, meanwhile they are also common signals that capital providers can coordinate their investment on. The interaction between speculators from different regions does not present in this model, as the effects from both financial markets are linearly separable. The financial market of each region exhibit similar characteristics.

There are two layers of strategic complementarity in this model. The first is among capital providers. They both put weights on each other's security price for their investment decisions. Prices provide information on the productivity shocks and also facilitates coordination between capital providers' investment decisions. For each region, the price investment sensitivities are increasing in both the precision of the price signal and the level of economic interdependence among the two regions. In addition, the price investment sensitivities are higher than their weighted precision. As the beliefs of the two capital providers reinforce each other, the effect from the economic interdependence on the price investment sensitivities increases nonlinearly and could be enormous. The second strategic complementarity is among speculators. It is represented by the weight the speculators put on their common signals in each financial market. The coordination among capital providers influences the strategic complementarity among speculators through amplified price investment sensitivities. The more the capital providers weight the price signals, the stronger the feedback effect. Therefore the incentive of each speculator to coordinate on their common signal is also increased.

Higher coordination is desirable for price informativeness when the amount of noise trading is high. However, in this setting, there is higher level of coordination when the amount of noise trading is low, which impairs price informativeness. It is due to the fact that the feedback effect between real investment and financial market is stronger with less noise trading. The excess coordination on common signal is referred to as trading frenzy in Goldstein, Ozdenoren and Yuan (2013) [28]. The strategic complementarity among capital providers further amplifies this trading frenzy.

I also study the impacts on investment efficiency from the cross region learning. On the one hand, Cross region learning increases investment efficiency by improving the investment coordination among capital providers of the two regions. On the other hand, cross region learning could decrease the price informativeness by amplified trading frenzies. When the fundamental prior is low, the investment coordination effect dominates. In other words, It could be beneficial to learning from the other region's financial market. In the equilibrium with cross learning, the aggregate investment efficiency increases in price informativeness. High price informativeness helps the capital providers to better predict the its own region's productivity shocks, meanwhile, it also helps with the coordination among capital providers as they both put higher weight on the price signals. Therefore, regulations that enhance the price informativeness of the financial market can have cross regional welfare improvement.

The remainder of this chapter is organized as follows. Section 2 describes and Section 3 solves the model. Section 4 discusses the implication for price informativeness and investment efficiency. Section 5 concludes.

3.2 The Model

3.2.1 The economy

The model studies the feedback effect of Goldstein, Ozdenoren and Yuan (2013) [28] in an environment of two regions, where the regions' productivity depends on each other's investment. Each region $i = 1$ or 2 has a capital provider that decides how much capital

to invest in the economy. There is also a financial market in each region that trades the security on the claim of the cash flow generated from the region's economy. I focus on the secondary market to study the effect from the trading activities on the real economy. Each region i 's financial market is occupied by a mass 1 of informed risk-neutral speculators, respectively. I index speculators for region i by a couple (i, j) , with $j \in [0, 1)$. There are also noise traders in each region i 's financial market.

There are three dates, $t = 0, 1, 2$. At date 0, the speculators trade in their corresponding financial market with their private information, and the security price aggregates their information. At date 1, the capital providers observe the security prices of both their own region and the other region. Having observed all the prices and received their private information, the capital providers decide the amount of capital to invest in the region's economy. All the cash flows are realized at date 2.

3.2.2 Capital Providers and Investment

The two regions have interdependent production technologies. Region i 's production function is $Q^i(I_1, I_2) = F_i G(I_1, I_2)$ where $G(I_1, I_2) = I_1^{q_1} \cdot I_2^{q_2}$ with $q_1, q_2 \in (0, 1)$. The productivity of each region depends on the other region's investment. In the real world, the two regions can be considered as on the same value chain where coordination in production is desired. F_i is the stochastic productivity shock for region i . Denote by f_i the natural log of the productivity shock, and assume that f_1 and f_2 are normal and mutually independent: $f_i \sim N(\bar{f}_i, 1/\tau_{f_i})$, where τ_{f_i} is positive and $i = 1, 2$. I assume decreasing return to scale for each region to achieve a stable solution for the model.

The stochastic productivity shock F_i does not affect the other region i 's cash flow directly. However, due to the interdependence of the economy, the capital provider has an incentive to learn the other region's productivity so as to infer its investment decision. The financial markets provide common signal for the capital providers to coordinate on their investment decisions. I assume no common component for the productivity shocks to emphasize on the effect purely from the coordination needs of the two regions. If there is

common component in the productivity shocks, the capital providers have more incentive to learn from each other's financial market.

At date 1, both capital providers choose the amount of capital I_i simultaneously without observing each other's decision. Capital provider i captures a proportion $\kappa_i \in (0, 1)$ of the output $Q^i(I_1, I_2)$ by providing I_i , which incurs a private quadratic adjustment cost, $C^i(I_i) = \frac{c_i I_i^{q_i+1}}{q_i+1}$. Thus, capital provider i 's problem at $t = 1$ is

$$\max_{I_i} E \left[\kappa_i Q^i(I_1, I_2) - \frac{c_i I_i^{q_i+1}}{q_i+1} \middle| \Gamma_i \right], \quad (3.1)$$

where Γ_i is the information set of capital provider i at $t = 1$. It consists of both regions' security prices P_1 and P_2 , as well as his private signal about the (log)productivity shocks f_i . Specifically, I assume that each capital provider i gets a private noisy and independent signal x_m^i about the region's productivity shock f_i with precision τ_{m_i} , that is, $x_m^i = f_i + \varepsilon_m^i$, with $\varepsilon_m^i \sim N(0, \tau_{m_i}^{-1})$. Therefore, for capital provider i , the information set is $\Gamma_i = \{P_1, P_2, x_m^i\}$. To simplify the information structure, I assume that the capital provider does not receive private signal about the other region's productivity shock. It is realistic that the capital provider has less information about the other region's productivity, especially when it does not have direct impact on his own region's productivity. The result of this model will be intact if I add one noisy signal about the other region's productivity to the capital provider's information set.

The solutions to this maximization problem for region 1 and 2 are

$$I_1 = E \left[\frac{\kappa_1 q_1}{c_1} F_1 I_2^{q_2} \middle| P_1, P_2, x_m^1 \right], \quad (3.2)$$

$$I_2 = E \left[\frac{\kappa_2 q_2}{c_2} F_2 I_1^{q_1} \middle| P_1, P_2, x_m^2 \right]. \quad (3.3)$$

3.2.3 Speculators and Financial Market Trading

At date 0, the remaining cash flow $(1 - \kappa_i)Q^i(I_1, I_2)$, as a security, is traded in a separate competitive financial market for each region i . For region i , denote the price of this security

by P_i . The information structure within each financial market is the same as the single region framework in Goldstein, Ozdenoren and Yuan (2013). In the financial market of region i , each speculator (i, j) has two signals about the region's (log)productivity shock f_i . The first signal, $x_j^i = f_i + \sigma_{\varepsilon_i} \varepsilon_j^i$, is privately observed where ε_j^i is independently normally distributed across speculators and regions with mean zero and unit variance. The precision of this signal is denoted as $\tau_{\varepsilon_i} = 1/\sigma_{\varepsilon_i}^2$. The second signal is $x_c^i = f_i + \sigma_{c_i} \varepsilon_c^i$. This signal is observed by all speculators and ε_c^i is independently and normally distributed with mean zero and unit variance and $\tau_{c_i} = 1/\sigma_{c_i}^2$.

I assume that speculators can only learning about the productivity shock that directly affects their own region. This is consistent with reality that traders have more access to information about their own country. The above information structure is sufficient for studying the effect from capital providers' coordination on the strategic complementary among speculators which is the main focus of this paper.

Each speculator can buy or sell up to a unit of the security. The size of speculator (i, j) 's position is denoted by $d_{ij} \in [-1, 1]$. I assume that the speculators do not observe the price when submitting their orders as in Kyle (1985) [38]. Their problems at $t = 0$ are

$$\max_{d_{ij} \in [-1, 1]} d_{ij} E \left[(1 - \kappa_i) Q^i(I_1, I_2) - P_i | \Gamma_{ij} \right], \quad (3.4)$$

where d_{ij} is speculator (i, j) 's demand. The aggregate demand from the speculators in market i is given by $D_i = \int_0^1 d_{ij} dj$.

To solve the model in closed form, I assume that the noisy supply in financial market i takes the following form:

$$\Delta(\xi_i, P_i) = 1 - 2\Phi(\xi_i - \lambda_i \log P_i),$$

where

$$N(0, \tau_{\xi_i}^{-1}).$$

$\Phi(\cdot)$ denotes the cumulative standard normal distribution function. The exogenous shock ξ_i captures the idiosyncratic noisy supply shock in market i that can be viewed as driven by the noise traders. The noisy supply shocks ξ_i is independent of other shocks in the economy. Meanwhile, λ_i in the noisy supply function captures price elasticity and can be viewed as market liquidity. When λ_i is high, the demand from speculators can be easily absorbed and thus their aggregate demand has little impact on the asset prices.

Finally, in equilibrium, the prices are set to clear the market by equalizing the aggregate speculator demand to the noisy supply in each financial market i :

$$D_i = \Delta(\xi_i, P_i). \quad (3.5)$$

3.3 Equilibrium

3.3.1 Definition of equilibrium

The definition of equilibrium is given as follows.

Definition 4 *An equilibrium consists of a price function for each region i , $P_i(f_i, \xi_i, \varepsilon_c^i) : R^3 \rightarrow R$, an investment policy for each capital provider i , $I_i(P_1, P_2, x_m^i) : R^2 \rightarrow R$, strategies for each speculator (i, j) , $d_{ij}(x_c^i, x_j^i)$ such that:*

- *each speculator (i, j) 's strategy $d_{ij}(x_c^i, x_j^i)$ solves problem 3.4;*
- *each capital provider i 's investment policy $I_i(P_1, P_2, x_m^i)$ solves problem 3.1; and*
- *the market clearing condition for the security 3.5 is satisfied in each financial market.*

I focus on the linear monotone equilibrium where capital providers' investments are log linear in the price signals and their own private signals; and speculator's strategies follows a cutoff rule that depends on linear combinations of the signals each individual receives.

Definition 5 *A linear monotone equilibrium is an equilibrium in which*

- for each speculator (i, j) , $d_{ij}(x_c^i, x_j^i) = 1$ if $x_j^i + \phi_i x_c^i \geq \mu_i$ for constants ϕ_i and μ_i , and $d_{ij}(x_c^i, x_j^i) = -1$ otherwise; and
- for each capital provider i , $I_i(P_1, P_2, x_m^i) = \exp(\rho_{i1} \log P_1 + \rho_{i2} \log P_2 + \rho_{im} x_m^i + \rho_{i0})$ for constants ρ_{i1} , ρ_{i2} , ρ_{im} and ρ_{i0} .

3.3.2 Solving the model

In this section, I explain the main steps for solving the model with cross regional learning. I first derive the security prices from the market clearing conditions and characterize their information content. Base on that, I solve for the capital providers' investment rules. It involves a fixed point problem in solving the capital providers' investment rules, as each region's capital provider's investment rule depends on that of each other. Finally, I solve for individual speculators' trading strategy for each region which involves another fixed point problem.

In a linear monotone equilibrium, given the ϕ_i and μ_i , the aggregate demand from speculators for region i is $D_i = 1 - 2\Phi\left(\frac{\mu_i - (1+\phi_i)f_i - \phi_i\sigma_{c_i}\varepsilon_c^i}{\sigma_{\varepsilon_i}}\right)$. Applying the market clearing condition leads to:

$$P_i = \exp\left(\frac{(1+\phi_i)f_i + \phi_i\sigma_{c_i}\varepsilon_c^i - \mu_i + \sigma_{\varepsilon_i}\xi_i}{\lambda_i\sigma_{\varepsilon_i}}\right) = \exp\left(\frac{f_i + \phi_i x_c^i - \mu_i + \sigma_{\varepsilon_i}\xi_i}{\lambda_i\sigma_{\varepsilon_i}}\right). \quad (3.6)$$

The security price is informationally equivalent to

$$\begin{aligned} x_P^i &\equiv \frac{\mu_i + \lambda_i\sigma_{\varepsilon_i} \log P_i}{1 + \phi_i} = f_i + \frac{\phi_i}{1 + \phi_i}\sigma_{c_i}\varepsilon_c^i + \frac{\sigma_{\varepsilon_i}\xi_i}{1 + \phi_i} \\ &= \frac{f_i}{1 + \phi_i} + \frac{\phi_i}{1 + \phi_i}x_c^i + \frac{\sigma_{\varepsilon_i}\xi_i}{1 + \phi_i}. \end{aligned} \quad (3.7)$$

x_P^i is normally distributed with mean $\bar{f}_i$ and variance $\sigma_{P_i}^2 = \left(\frac{\phi_i}{1+\phi_i}\right)^2 \sigma_{c_i}^2 + \frac{1}{(1+\phi_i)^2} \sigma_{\varepsilon_i}^2 \sigma_{\xi_i}^2$.

Denote the precision of the price signal i as

$$\tau_{P_i} = \frac{1}{\sigma_{P_i}^2} = \frac{(1 + \phi_i)^2 \tau_{\varepsilon_i} \tau_{\xi_i} \tau_{c_i}}{\phi_i^2 \tau_{\varepsilon_i} \tau_{\xi_i} + \tau_{c_i}}. \quad (3.8)$$

ϕ_i is the weight each speculator put on the common signal and it represents the strategic complementary among speculators.

Now I can derive the belief of each capital provider on f_1 and f_2 . Observing P_1, P_2 and x_m^1 , capital provider 1 believes that f_1 is normally distributed with mean $\frac{\tau_{f_1} \bar{f}_1 + \tau_{m_1} x_m^1 + \tau_{P_1} x_P^1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}$ and variance $\frac{1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}$; f_2 is normally distributed with mean $\frac{\tau_{f_1} \bar{f}_2 + \tau_{P_2} x_P^2}{\tau_{f_2} + \tau_{P_2}}$ and variance $\frac{1}{\tau_{f_2} + \tau_{P_2}}$. Similarly capital provider 2 believes that f_2 is normally distributed with mean $\frac{\tau_{f_2} \bar{f}_2 + \tau_{m_2} x_m^2 + \tau_{P_2} x_P^2}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}$ and variance $\frac{1}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}$; f_1 is normally distributed with mean $\frac{\tau_{f_1} \bar{f}_1 + \tau_{P_1} x_P^1}{\tau_{f_1} + \tau_{P_1}}$ and variance $\frac{1}{\tau_{f_1} + \tau_{P_1}}$. In a linear monotone equilibrium, the investment policy for each capital provider can be written as

$$I_1^* = \exp \left(a_{11} x_P^1 + a_{12} x_P^2 + a_{1m} x_m^1 + b_1 \right); \quad (3.9)$$

$$I_2^* = \exp \left(a_{21} x_P^1 + a_{22} x_P^2 + a_{2m} x_m^2 + b_2 \right). \quad (3.10)$$

where a_{in} and b_k are functions of ϕ_i and other model parameters. Using the above equations and the optimal solutions for capital providers (3.2) and (3.3), we obtain the investment policy for each capital provider described in the following lemma.

Lemma 6 *The capital providers' investment policy is characterised by the functions (3.9) and (3.10) with the parameters*

$$a_{11} = \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} + \frac{q_1 q_2}{1 - q_1 q_2} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}}; \quad (3.11)$$

$$a_{22} = \frac{\tau_{P_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} + \frac{q_1 q_2}{1 - q_1 q_2} \frac{\tau_{P_2}}{\tau_{f_2} + \tau_{P_2}}; \quad (3.12)$$

$$a_{12} = \left(\frac{\tau_{P_2}}{\tau_{f_2} + \tau_{P_2}} \right) \left(\frac{q_2}{1 - q_1 q_2} \right); \quad (3.13)$$

$$a_{21} = \left(\frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} \right) \left(\frac{q_1}{1 - q_1 q_2} \right); \quad (3.14)$$

$$a_{1m} = \frac{\tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}; \quad (3.15)$$

$$a_{2m} = \frac{\tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}. \quad (3.16)$$

$$b_1 = (1 - q_1 q_2)^{-1} \left(\begin{aligned} & \frac{\tau_{f_1} \bar{f}_1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} + \frac{q_2 \tau_{f_2} \bar{f}_2}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} + \frac{\tau_{f_1} + \tau_{P_1} + q_1^2 q_2 \tau_{m_1}}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(\tau_{f_1} + \tau_{P_1})} \\ & + \frac{q_2(\tau_{f_2} + \tau_{P_2} + q_2 \tau_{m_2})}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})(\tau_{f_2} + \tau_{P_2})} + \log \frac{\kappa_1 q_1}{c_1} + q_2 \log \frac{\kappa_2 q_2}{c_2} \end{aligned} \right) \quad (3.17)$$

$$b_2 = (1 - q_1 q_2)^{-1} \left(\begin{aligned} & \frac{\tau_{f_2} \bar{f}_2}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} + \frac{q_1 \tau_{f_1} \bar{f}_1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} + \frac{\tau_{f_2} + \tau_{P_2} + q_2^2 q_1 \tau_{m_2}}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})(\tau_{f_2} + \tau_{P_2})} \\ & + \frac{q_1(\tau_{f_1} + \tau_{P_1} + q_1 \tau_{m_1})}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(\tau_{f_1} + \tau_{P_1})} + \log \frac{\kappa_2 q_2}{c_2} + q_1 \log \frac{\kappa_1 q_1}{c_1} \end{aligned} \right) \quad (3.18)$$

Based on the derivation for the price functions, τ_{P_i} is also a function of ϕ_i and other model parameters. a_{in} can be viewed as the investment sensitivity of region i to the corresponding signal. For each region, the investment sensitivity to each price signal is higher than its weighted precision. In Goldstein, Ozdenoren and Yuan (2013) [28], the investment sensitivity to price signal equals to its weighted precision. In this setting, prices serve not only as sources of information but also as the common signals for the two capital providers to coordinate their investment decisions. The higher investment sensitivities to the price signals are driven by the price's role in facilitating the investment coordination. Based on the production technology in this model, the capital provider i wants to learn the productivity shock of its own region from price signal i , which provide incentive for the capital provider $-i$ to put a weight on the same price signal due to economic interdependence. Given that the capital provider $-i$'s investment rule is also relying price i , the capital provider i has even more incentive to weight the price signal.

As expected, the investment sensitivities to price signals are increasing in $q_1 q_2$ which represents the economic cross impact among the two regions. As $q_1 q_2$ approaches 1 from below, the investment sensitivities of each region to the price signals go to infinity. This amplified price investment sensitivity will further give rise to higher level of strategic complementary among speculators in each region which is established and referred to as trading frenzies in Goldstein, Ozdenoren and Yuan (2013) [28]. Denote $\gamma \equiv \frac{q_1 q_2}{1 - q_1 q_2}$ which can be viewed as the effect from the economic interdependence on the price investment sensitivity. γ increases nonlinearly in $q_1 q_2$.

I finally close the model by solving for the speculators' equilibrium trading strategy, characterized by the weight ϕ_i and the constant μ_i .

The decision rule for speculator (i, j) is

$$E \left[(1 - \kappa_i) Q^i(I_1, I_2) - P_i | \Gamma_{ij} \right] > 0 \quad (3.19)$$

in which I_1 and P_i have been characterized by Lemma 1 and (3.6) respectively. It is easy to show that (3.19) can be rewritten as

$$\exp(\alpha_{i0} + \alpha_{i1} x_c^i + \alpha_{i2} x_j^i) - \exp(\beta_{i0} + \beta_{i1} x_c^i + \beta_{i2} x_j^i) > 0. \quad (3.20)$$

where $\alpha_{i0}, \alpha_{i1}, \alpha_{i2}, \beta_{i0}, \beta_{i1}$ and β_{i2} are functions of ϕ_i and other model parameters. The explicit expressions for these coefficients are provided in the proof of Proposition 1 in the Appendix. Let $B_i(\phi_i) = (\alpha_{i2} - \beta_{i2}) / (\alpha_{i1} - \beta_{i1})$. The symmetric equilibrium is solved when $B_i(\phi_i) = \phi_i$. Proposition 7 summarizes the results for the equilibrium. All proofs are in the Appendix.

Proposition 7 *For high enough levels of supply elasticity λ_1 and λ_2 , there exists a linear monotone equilibrium characterized by weights ϕ_1^* and ϕ_2^* that speculators put on the common signals. The equilibrium is unique when the fundamental prior τ_{f1} and τ_{f2} are small enough and the effect from economic interdependence γ is high enough. ϕ_1^* and ϕ_2^* are both increasing in γ and goes to infinity as γ goes to infinity.*

ϕ_1^* and ϕ_2^* characterize the strategic complementarity among speculators in each region's financial market. In the rest part of the paper, I focus on the case of large supply elasticities (λ_1 and λ_2), imprecise fundamental prior (τ_{f_1} and τ_{f_2}) and high level of economic interdependence (γ), for which there exists a unique equilibrium.

3.3.3 Benchmark with no cross region learning

In this section, I derive the benchmark case where the capital provider of each region do not observe the other region's financial market. Without cross region learning, the feedback mechanism between the financial market and the capital provider in each region is the same as in Goldstein, Ozdenoren and Yuan (2013) [28]. They established the framework to study the feedback effect between the financial market and the firm's investment. Their model is illustrated under constant economic return to scale. I derive the model with decreasing return to scale for each region to achieve the stability of the model. The benchmark case helps to understand the cross impact of the financial markets. I still focus on the linear monotone symmetric equilibrium. Denote the region i 's speculators' weight on common signal ϕ'_i . The price function for region i is

$$P_i = \exp \left(\frac{(1 + \phi'_i) f_i + \phi'_i \sigma_{c_i} \varepsilon_c^i - \mu_i + \sigma_{\varepsilon_i} \xi_i}{\lambda_i \sigma_{\varepsilon_i}} \right) = \exp \left(\frac{f_i + \phi'_i x_c^i - \mu_i + \sigma_{\varepsilon_i} \xi_i}{\lambda_i \sigma_{\varepsilon_i}} \right). \quad (3.21)$$

The security price is informationally equivalent to

$$\begin{aligned} x_P^i &\equiv \frac{\mu_i + \lambda_i \sigma_{\varepsilon_i} \log P_i}{1 + \phi'_i} = f_i + \frac{\phi'_i}{1 + \phi'_i} \sigma_{c_i} \varepsilon_c^i + \frac{\sigma_{\varepsilon_i} \xi_i}{1 + \phi'_i} \\ &= \frac{f_i}{1 + \phi'_i} + \frac{\phi'_i}{1 + \phi'_i} x_c^i + \frac{\sigma_{\varepsilon_i} \xi_i}{1 + \phi'_i}. \end{aligned} \quad (3.22)$$

x_P^i is normally distributed with mean $\bar{f}_i$ and variance $\sigma_{P_i}^2 = \left(\frac{\phi'_i}{1 + \phi'_i} \right)^2 \sigma_{c_i}^2 + \frac{1}{(1 + \phi'_i)^2} \sigma_{\varepsilon_i}^2 \sigma_{\xi_i}^2$.

Denote the precision of the price signal i as

$$\tau_{P_i} = \frac{1}{\sigma_{P_i}^2} = \frac{(1 + \phi'_i)^2 \tau_{\varepsilon_i} \tau_{\xi_i} \tau_{c_i}}{\phi_i'^2 \tau_{\varepsilon_i} \tau_{\xi_i} + \tau_{c_i}}. \quad (3.23)$$

The capital providers only observe the price signal of their own region. Therefore, capital provider 1 believes that f_1 is normally distributed with mean $\frac{\tau_{f_1} \bar{f}_1 + \tau_{m_1} x_m^1 + \tau_{P_1} x_P^1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}$ and variance $\frac{1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}$; f_2 is normally distributed with mean $\bar{f}_2$ and variance $\frac{1}{\tau_{f_2}}$. Similarly capital provider 2 believes that f_2 is normally distributed with mean $\frac{\tau_{f_2} \bar{f}_2 + \tau_{m_2} x_m^2 + \tau_{P_2} x_P^2}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}$ and variance $\frac{\tau_{f_2} \bar{f}_2 + \tau_{m_2} x_m^2 + \tau_{P_2} x_P^2}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}$; f_1 is normally distributed with mean $\bar{f}_1$ and variance $\frac{1}{\tau_{f_1}}$. The investment policy for each capital provider can be written as

$$I_1 = \exp(a'_{11} x_P^1 + a'_{1m} x_m^1 + b'_1); \quad (3.24)$$

$$I_2 = \exp(a'_{22} x_P^2 + a'_{2m} x_m^2 + b'_2). \quad (3.25)$$

where a'_{in} and b'_k are functions of ϕ'_i and other model parameters. Using the above equations and the optimal solutions for capital providers (3.24) and (3.25), we obtain the investment policy for each capital provider described in the following lemma.

Lemma 8 *The capital providers' investment policy when there is no cross region learning is characterised by the functions (3.24) and (3.25) with the parameters*

$$a'_{11} = \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}; \quad (3.26)$$

$$a'_{22} = \frac{\tau_{P_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}; \quad (3.27)$$

$$a'_{1m} = \frac{\tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}; \quad (3.28)$$

$$a'_{2m} = \frac{\tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}. \quad (3.29)$$

$$b'_1 = (1 - q_1 q_2)^{-1} \left(\begin{aligned} & \frac{\tau_{f_1} + q_1 q_2 (\tau_{m_1} + \tau_{P_1})}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} \overline{f_1} + q_2 \overline{f_2} + \frac{1 - q_1^2 q_2}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})} \\ & + \frac{q_2(1 - q_2)}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})} + \frac{q_1^2 q_2}{2\tau_{f_1}} + \frac{q_2^2}{2\tau_{f_2}} + \log\left(\frac{\kappa_1 q_1}{c_1}\right) + q_2 \log\left(\frac{\kappa_2 q_2}{c_2}\right) \end{aligned} \right) \quad (3.30)$$

$$b'_2 = (1 - q_1 q_2)^{-1} \left(\begin{aligned} & \frac{\tau_{f_2} + q_1 q_2 (\tau_{m_2} + \tau_{P_2})}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} \overline{f_2} + q_1 \overline{f_1} + \frac{1 - q_2^2 q_1}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})} \\ & + \frac{q_1(1 - q_1)}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})} + \frac{q_2^2 q_1}{2\tau_{f_2}} + \frac{q_1^2}{2\tau_{f_1}} + \log\left(\frac{\kappa_2 q_2}{c_2}\right) + q_1 \log\left(\frac{\kappa_1 q_1}{c_1}\right) \end{aligned} \right) \quad (3.31)$$

Without cross region learning, the capital providers care about the events in the financial market only to the extent that they affect the quality of the information they have when making the investment decision. Therefore, the investment sensitivity to the price signal in each region equals the weighted precision of the price which is lower than that with cross region learning.

The speculators' decision rule for each region i can also be written as

$$\exp(\alpha'_{i0} + \alpha'_{i1} x_c^i + \alpha'_{i2} x_j^i) - \exp(\beta'_{i0} + \beta'_{i1} x_c^i + \beta'_{i2} x_j^i) > 0. \quad (3.32)$$

where α'_{i0} , α'_{i1} , α'_{i2} , β'_{i0} , β'_{i1} and β'_{i2} are functions of ϕ'_i and other model parameters. The explicit expressions for these coefficients are provided in the proof of Proposition 2 in the Appendix. Let $B'_i(\phi'_i) = (\alpha'_{i2} - \beta'_{i2}) / (\alpha'_{i1} - \beta'_{i1})$. The symmetric equilibrium is solved when $B'_i(\phi'_i) = \phi'_i$. Proposition 9 summarizes the results for the equilibrium without cross region learning.

Proposition 9 *For high enough levels of noisy supply elasticity λ_1 and λ_2 , and small enough precision of fundamental priors τ_{f_1} and τ_{f_2} , there exists a unique equilibrium without cross region learning in which speculators put a positive weight $\phi_i'^* > 0$ on the common signal. In particular, $\phi_i'^* < \phi_i^*$, where ϕ_i^* is the speculators' weight on the common signal in the corresponding equilibrium with cross region learning.*

With the feedback effect between the financial market and the capital provider's investment decision, the speculators still have incentive to coordinate on their common signal, as

the payoff from the security is increasing in the amount of investment. With cross region learning, the levels of coordination among speculators in both financial markets are even higher, as prices have higher impacts on both capital providers' investment decisions. The level of strategic complementary among speculators have important implication for price informativeness and investment efficiency.

3.4 Price informativeness and investment efficiency

In this section, I explore the effect that cross region learning have on the investment efficiency. In this model, cross region learning affects each region's investment efficiency through two channels. First, cross region learning provides better prediction of the other region's investment decision in the environment of economic interdependence. This facilitates the investment coordination among regions. Second, cross region learning gives rise to higher levels of strategic complementary among speculators in both financial market which influences the precision of the price signal. I explore the impacts from both channels.

3.4.1 Cross region learning and price informativeness

Price aggregates both private and common information from speculators. Let me first explore the impact from the strategic complementary among speculators on price informativeness. The following lemma describes the relation between the precision of the price signal and the weight that the speculators put on their common signals for each region i .

Lemma 10 *The precision of the price τ_{P_i} is maximized at $\phi_{i,OP} = \tau_{c_i}/\tau_{\xi_i}\tau_{\varepsilon_i}$; More specifically, τ_{P_i} is increasing in ϕ_i for $\phi_i < \phi_{i,OP}$, and decreasing in ϕ_i for $\phi_i > \phi_{i,OP}$.*

Based on the above lemma, the price informativeness can be analyzed by comparing the equilibrium level of speculators' coordination with $\phi_{i,OP}$. The following proposition summarizes the relation between the equilibrium levels of speculators' coordination with and without cross region learning and $\phi_{i,OP}$.

Proposition 11 *For high enough levels of supply elasticity λ_1 and λ_2 , there exists $\bar{\tau}_{\xi_1}$ and $\bar{\tau}_{\xi_2}$ such that*

- for $\tau_{\xi_i} > \bar{\tau}_{\xi_i}$, region i 's equilibrium levels of speculators' coordination with and without cross region learning are both higher than the level of coordination that maximizes the precision of the price for region i ($\phi_{i,OP} < \phi_i'^* < \phi_i^*$) for $i = 1, 2$;
- for $\tau_{\xi_i} < \bar{\tau}_{\xi_i}$, region i 's equilibrium levels of speculators' coordination without cross region learning is lower than the level of coordination that maximizes the precision of the price for region i ($\phi_{i,OP} > \phi_i'^*$); there exists $\bar{\gamma}_i$ such that $\phi_i^* > \phi_{i,OP}$ for all $\gamma > \bar{\gamma}_i$ and $\phi_i^* < \phi_{i,OP}$ for all $\gamma < \bar{\gamma}_i$. for $i = 1, 2$.

Without cross region learning, speculators coordinate too much in markets with less noise trading and coordinate too little in markets with more noise trading. For high level of noise trading and low level of economic dependence, cross region learning improves price informativeness by increasing the level of speculators coordination. For low level of noise trading, the cross region learning further impairs price informativeness due to even higher strategic complementarity among speculators. In addition, the price precision converges to the precision of the common signal as ϕ_i^* goes to infinity. When the precision of the common signal is low enough and the level of economic dependence is high enough, cross region learning always decreases price informativeness from its no cross region learning benchmark.

3.4.2 Cross region learning and investment efficiency

To analyze investment efficiency, I look at the ex ante expected net benefit of investment. The investment efficiency criterion for region i is given by

$$E_0 \left[\max_{I_i} E \left[\kappa_i Q^i(I_1, I_2) - \frac{c_i I_i^{q_i+1}}{q_i + 1} | \Gamma_i \right] \right]. \quad (3.33)$$

The aggregate investment efficiency criterion is given by

$$E_0 \left[\max_{I_1} E \left[\kappa_1 Q^1(I_1, I_2) - \frac{c_1 I_1^{q_1+1}}{q_1+1} | \Gamma_1 \right] + \max_{I_2} E \left[\kappa_2 Q^2(I_1, I_2) - \frac{c_2 I_2^{q_2+1}}{q_2+1} | \Gamma_2 \right] \right]. \quad (3.34)$$

Proposition 12 characterises the optimal level of speculators coordination for investment efficiency which is linked to the accuracy of the price information.

Proposition 12 *The investment efficiency for region i is maximized at $\phi_i = \phi_{i,OP}$ and $\phi_{-i} = \phi_{-i,OP}$ for $i = 1$ and 2 .*

The investment efficiency is maximized when price informativeness for both security prices are maximized. High price informativeness helps the capital providers to better predict the productivity shocks, meanwhile, it also helps with the coordination among capital providers as they both put higher weight on the price signals.

Proposition 13 compares the aggregate investment efficiency between the cases with and without cross region learning

Proposition 13 *for low enough fundamental prior τ_{f_1} and τ_{f_2} the aggregate investment efficiency with cross region learning is higher than that without cross region learning.*

On the one hand, Cross region learning increases investment efficiency by improving the investment coordination among capital providers of the two regions. On the other hand, cross region learning could decrease the price informativeness by enhancing the incentive for the speculators to coordinate. When the fundamental prior is low, the investment coordination effect always dominates.

3.5 Conclusion

I study strategic interaction among capital providers of two regions and its impact on the strategic interaction among speculators in each financial market, when the productivity of each region depends on other region's investment. The financial markets play two roles in the real economy. They provide more information for each capital provider to estimate

the productivity shock that directly impact his own region. More interestingly, they also provide common signals for the capital providers to coordinate their investment decisions. Both of them provide incentive for the capital providers to weight the security price in their investment decision which give rise to the incentive for the speculators in each financial market to coordinate on their common signal. When the economic interdependence increases, the investment coordination needs of the capital providers amplify the price investment sensitivity and therefore the strategic complementary among speculators to extreme high level. The impacts on the price informativeness and investment efficiency are also discussed.

Appendix A

Proofs for Chapter 2

Proof of Proposition 1: The coefficient $A(k)$ and $B(k)$ are given as follows,

$$A(k) = \left(\frac{\tau_s}{\tau_\theta + \tau_s + \tau_c} \right) \left[\beta \left(1 - \lambda \frac{\tau_g + \frac{(1+k)\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}}{\tau_\theta + \tau_g + \frac{(1+k)^2\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}} \right) - \frac{1}{\alpha\sigma_s} \right] \quad (\text{A.1})$$

$$\begin{aligned} B(k) = & \left(\frac{\tau_c}{\tau_\theta + \tau_s + \tau_c} \right) \left[\beta \left(1 - \lambda \frac{\tau_g + \frac{(1+k)\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}}{\tau_\theta + \tau_g + \frac{(1+k)^2\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}} \right) - \frac{1}{\alpha\sigma_s} \right] \\ & - \beta\lambda \frac{\frac{k(1+k)\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}}{\tau_\theta + \tau_g + \frac{(1+k)^2\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}} - \frac{k}{\alpha\sigma_s} \end{aligned} \quad (\text{A.2})$$

Then the equilibrium condition leads to the third-order polynomial $H(k) = ak^3 + bk^2 + ck + d = 0$, where,

$$a = \left(-\frac{\beta\tau_s^2\tau_\xi}{\tau_\theta + \tau_s + \tau_c} \right) (\tau_\theta + (1-\lambda)\tau_g + \tau_c) - \frac{\tau_\theta + \tau_c}{\tau_\theta + \tau_s + \tau_c} \frac{(\tau_\theta + \tau_g + \tau_c)\tau_\xi\tau_s}{\alpha\sigma_s} \quad (\text{A.3})$$

$$b = \left(\frac{\beta \tau_c \tau_\xi \tau_s}{\tau_\theta + \tau_s + \tau_c} \right) \left(\tau_\theta + (1 - \lambda) \tau_g + \tau_c - \left(2 - \lambda - \frac{2}{\alpha \beta \sigma_s} \right) \tau_s - \frac{(\tau_\theta + \tau_g + \tau_c)}{\alpha \beta \sigma_s} \right) - \left(\beta \lambda + \frac{1}{\alpha \sigma_s} \right) \tau_c \tau_\xi \tau_s \quad (\text{A.4})$$

$$c = \left(-\frac{\beta \tau_s \tau_c}{\tau_\theta + \tau_s + \tau_c} \right) [\tau_\theta + (1 - \lambda) \tau_g + (1 - \lambda) \tau_\xi \tau_s - \left((2 - \lambda) - \frac{2}{\alpha \beta \sigma_s} \right) \tau_c \tau_\xi] - \frac{(\tau_\theta + \tau_g) \tau_\xi \tau_s / \tau_c + \tau_\xi \tau_s}{\alpha \beta \sigma_s} - \left(\beta \lambda + \frac{1}{\alpha \sigma_s} \right) \tau_c \tau_\xi \tau_s - \frac{(\tau_\theta + \tau_g) \tau_c}{\alpha \sigma_s} \quad (\text{A.5})$$

$$d = \left(\frac{\beta \tau_c}{\tau_\theta + \tau_s + \tau_c} \right) [(\tau_\theta + (1 - \lambda) \tau_g) \tau_c + (1 - \lambda) \tau_c \tau_\xi \tau_s - \frac{(\tau_\theta + \tau_g) \tau_\xi \tau_s + \tau_c \tau_\xi \tau_s}{\alpha \beta \sigma_s}] \quad (\text{A.6})$$

It is obvious that a is negative, hence, $H(k)$ is negative for k large enough. when $\alpha > \beta \frac{(\tau_\theta + (1 - \lambda) \tau_g + (1 - \lambda) \tau_\xi \tau_s) \tau_c}{(\tau_\theta + \tau_g + \tau_c) \tau_\xi \sqrt{\tau_s}}$, d is positive which leads to positive $H(0)$. Then the polynomial must have a positive root. In addition, $A(k)$ is positive for α large and k positive.

Now we prove the uniqueness of the equilibrium when τ_ξ is small. For τ_ξ small enough, it can be proved that the first order derivative of the polynomial is always negative which is a sufficient condition for the uniqueness. Check the equation $\frac{\partial H(k)}{\partial k} = 0$, which is:

$$3ak^2 + 2bk + c = 0 \quad (\text{A.7})$$

From the expression of a, b and c , we have $a = \nu \tau_\xi$, $b = \mu \tau_\xi$, and $c = c_1 + \rho \tau_\xi$ for some constant ν, μ, c_1 and ρ . In addition, we have $\nu, c_1 < 0$, hence, the discriminant of the above equation $b^2 - 3ac$ is negative for τ_ξ small enough, and there is no solution for $\frac{\partial H(k)}{\partial k} = 0$ and $\frac{\partial H(k)}{\partial k}$ is negative for all k . *QED*.

The uniqueness of equilibrium when α is large and τ_θ small enough is proved in Proposition 4.

Proof of Proposition 2: First, when $H(k)$ has only one root, it crosses $H(k) = 0$ from above only once. In addition,

$$H\left(\frac{\tau_c}{\tau_s}\right) = -\frac{\tau_c^3 \tau_\xi (\tau_\theta + \tau_g + \tau_c)}{\tau_s^2 \alpha \sigma_s} - \left(\beta \lambda + \frac{1}{\alpha \sigma_s}\right) \left(\frac{\tau_c^3 \tau_\xi}{\tau_s} + \tau_c^2 \tau_\xi\right) - \frac{(\tau_\theta + \tau_g) \tau_c^2}{\alpha \sigma_s^3} < 0 \quad (\text{A.8})$$

Therefore, the root $k^* < \frac{\tau_c}{\tau_s}$. On the other hand,

$$\frac{\partial H(k)}{\partial \lambda} = -\frac{\beta}{\tau_\theta + \tau_s + \tau_c} [\tau_\xi \tau_c \tau_s ((\tau_\theta + \tau_c) k + \tau_c) (1 + k) - \tau_g (\tau_\xi \tau_s k^2 + \tau_c) (\tau_s k - \tau_c)] \quad (\text{A.9})$$

$\frac{\partial H(k)}{\partial \lambda} < 0$ for $k < \frac{\tau_c}{\tau_s}$, therefore k^* decreases as λ increases.

Proof of Proposition 3: By Proposition 2 the equilibrium weight is lower than k_{BM} . Then, the price signal precision is lower than that of benchmark case, since it decreases in the weight k and $k^* < k_{BM} < k_{op}$ for τ_ξ lower than 1. *QED*.

Proof of Proposition 4: For $\alpha \rightarrow \infty, \tau_\theta \rightarrow 0$, and $\lambda \rightarrow 1$, the polynomial becomes:

$$H(k) = -\left(\frac{\beta \tau_\xi \tau_c \tau_s^2}{\tau_\theta + \tau_s + \tau_c}\right) (1 + k)^2 k = 0 \quad (\text{A.10})$$

which has two roots at 0 and -1 .

On the other hand,

$$\frac{\partial H(k)}{\partial \tau_\theta} \Big|_{k=0, \tau_\theta=0, \lambda=1} = \frac{\beta \tau_c^2}{\tau_s + \tau_c} > 0 \quad (\text{A.11})$$

$$\frac{\partial H(k)}{\partial \tau_\theta} \Big|_{k=-1, \tau_\theta=0, \lambda=1} = \frac{\beta (\tau_s \tau_\xi + \tau_c) (\tau_s + \tau_c)}{\tau_s + \tau_c} > 0 \quad (\text{A.12})$$

When τ_θ increase slightly from zero, $H(k)$ shifts up near $k = 0$ and -1 , therefore, there is unique positive root k^* . From the proof of Proposition 2, $\frac{\partial H(k)}{\partial \lambda} < 0$ for $k < \frac{\tau_c}{\tau_s}$, hence,

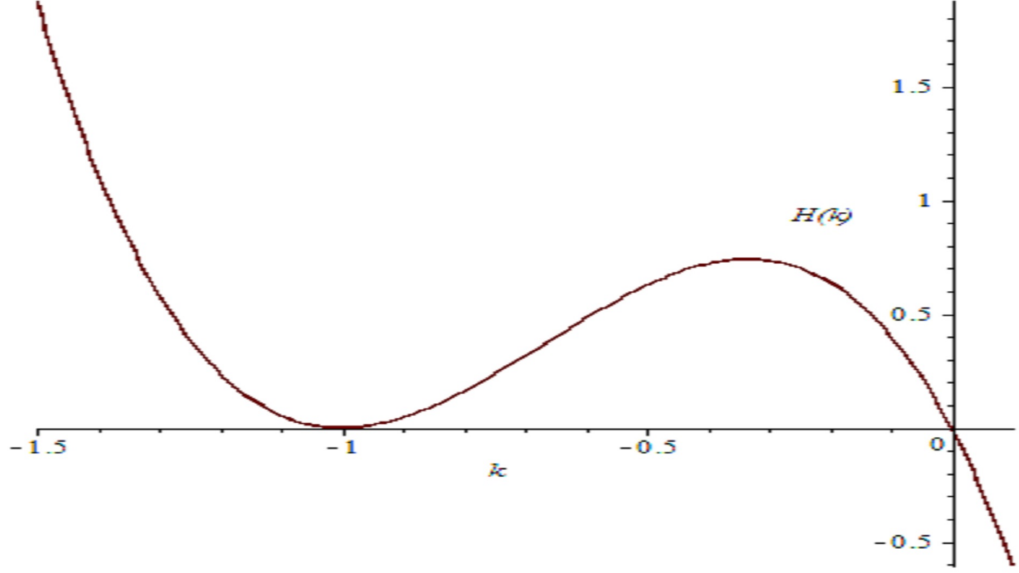


Figure A.1: $H(k)$: for $\alpha \rightarrow \infty$, $\tau_\theta \rightarrow 0$ and $\lambda \rightarrow 1$

$H(k)$ keeps shifting up for $k < \frac{\tau_c}{\tau_s}$ as λ moves from 1 to 0. Then, there is at most one root k^* in the region $[-\infty, \frac{\tau_c}{\tau_s}]$. k^* decreases in λ for $k^* \in [-\infty, \frac{\tau_c}{\tau_s}]$.

It is left to prove that $H(k)$ has no root greater than $\frac{\tau_c}{\tau_s}$. From the proof of Proposition 2, $H(\frac{\tau_c}{\tau_s}) < 0$. When $\alpha \rightarrow \infty$

$$\begin{aligned} \frac{\partial H(k)}{\partial k} \Big|_{k=\frac{\tau_c}{\tau_s}} &= -\frac{\beta\tau_c^2\tau_\xi(\tau_\theta + (1-\lambda)\tau_g + \tau_c + (2-\lambda)\tau_s + 2\lambda)}{\tau_\theta + \tau_s + \tau_c} \\ &\quad - \frac{\beta\tau_c\tau_s(\tau_\theta + (1-\lambda)\tau_g + (1-\lambda)\tau_s\tau_\xi)}{\tau_\theta + \tau_s + \tau_c} \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} \frac{\partial^2 H(k)}{\partial k^2} \Big|_{k=\frac{\tau_c}{\tau_s}} &= -\frac{5\beta\tau_c^2\tau_\xi(\tau_\theta + (1-\lambda)\tau_g + \tau_c)}{\tau_\theta + \tau_s + \tau_c} \\ &\quad - \frac{\beta(2-\lambda)\tau_s^2\tau_\xi\tau_c}{\tau_\theta + \tau_s + \tau_c} - \beta\lambda\tau_s\tau_\xi\tau_c \end{aligned} \quad (\text{A.14})$$

$\frac{\partial H(k)}{\partial k} \Big|_{k=\frac{\tau_c}{\tau_s}} < 0$ and $\frac{\partial^2 H(k)}{\partial k^2} \Big|_{k=\frac{\tau_c}{\tau_s}} < 0$ for α large enough. $H(k)$ keep decreasing in k for all $k > \frac{\tau_c}{\tau_s}$. $H(k)$ has no root greater than $\frac{\tau_c}{\tau_s}$ when α is large enough. *QED*.

Proof of Proposition 5: The central bank solves the following problem in making the monetary policy decision:

$$\max_{\lambda \in [0,1]} -E \left[\left(\theta^* - \tilde{\theta}_1 - \lambda(\theta^* - E[\tilde{\theta}_1 | \mathcal{F}_g]) \right)^2 | \mathcal{F}_g \right] \quad (\text{A.15})$$

The first order condition leads to:

$$2\lambda \left(\theta^* - E[\tilde{\theta}_1 | \mathcal{F}_g] \right)^2 - 2 \left(\theta^* - E[\tilde{\theta}_1 | \mathcal{F}_g] \right)^2 = 0 \quad (\text{A.16})$$

Therefore, it is always ex post optimal for the central bank to set $\lambda = 1$.

Proof of Proposition 6: It can be proved that when there is unique positive equilibrium and $\tau_\xi < 1$, it is not optimal to set $\lambda = 1$. Taking the first order derivative of (2.13) leads to

$$\frac{\partial U}{\partial \lambda} |_{\lambda=1} = \frac{1}{(\tau_\theta + \tau_g + \tau_P)^2} \frac{\partial \tau_P}{\partial k^*} \frac{\partial k^*}{\partial \lambda} \quad (\text{A.17})$$

Taking the first order derivative of (2.8), it is obvious that $\frac{\partial \tau_P}{\partial k^*} > 0$ since $k^* \in [-1, k_{op}]$.

In addition, according to Proposition 2, k^* is decreasing in λ when there is unique positive equilibrium. Then, I have $\frac{\partial U}{\partial \lambda} |_{\lambda=1} < 0$. *QED*.

Appendix B

Proofs for chapter 3

Proof of Proposition 7 and 9

Starting from equation (3.19) and speculators' information updating, the coefficients for equations (3.20) and (3.32) are derived as follows.

$$\begin{aligned}
 \alpha_{10} = & \frac{\left(1 + \frac{q_1 a_{11} + q_2 a_{21}}{1 + \phi_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \tau_{f_1} \bar{f}_1}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \frac{\left(1 + \frac{q_1 a_{11} + q_2 a_{21}}{1 + \phi_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right)^2}{2\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \quad (\text{B.1}) \\
 & + \frac{1}{2\tau_{\varepsilon_1} \tau_{\xi_1}} \left(\frac{q_1 a_{11} + q_2 a_{21}}{1 + \phi_1}\right)^2 + \frac{q_1^2 \tau_{m_1}}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})^2} \\
 & + \left(q_1 a_{11} + q_2 a_{21} + \frac{q_2 \tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}\right) \bar{f}_2 + \frac{\left(q_1 a_{11} + q_2 a_{21} + \frac{q_2 \tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}}\right)^2}{2\tau_{f_2}} \\
 & + \frac{(q_1 a_{11} + q_2 a_{21})^2}{2\tau_{P_2}} + \frac{q_2^2 \tau_{m_2}}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})^2} + q_1 b_1 + q_2 b_2 + \log(1 - \kappa)
 \end{aligned}$$

$$\begin{aligned}
 \alpha_{11} = & \left(1 + \frac{q_1 a_{11} + q_2 a_{21}}{1 + \phi_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\
 = & \left(\frac{\tau_{f_1} + \left(1 + \frac{q_1}{1 + \phi_1}\right) \tau_{P_1} + (1 + q_1) \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\
 & + \frac{q_1 q_2 (1 + q_1)}{(1 - q_1 q_2) (1 + \phi_1)} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \quad (\text{B.2})
 \end{aligned}$$

$$\begin{aligned}
\alpha_{12} &= \left(1 + \frac{q_1 a_{11} + q_2 a_{21}}{1 + \phi_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + (q_1 a_{11} + q_2 a_{21}) \frac{\phi_1}{1 + \phi_1} \\
&= \frac{\tau_{f_1} + \left(1 + \frac{q_1}{1 + \phi_1}\right) \tau_{P_1} + (1 + q_1) \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \frac{\tau_{P_1} q_1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} \frac{\phi_1}{1 + \phi_1} \\
&\quad + \frac{q_1 q_2 (1 + q_1)}{(1 - q_1 q_2) (1 + \phi_1)} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} \left(\frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \frac{\phi_1}{1 + \phi_1} \right) \tag{B.3}
\end{aligned}$$

$$\beta_{10} = \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \left(\frac{\tau_{f_1} \bar{f}_1 + \frac{1}{2} \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} - \mu_1 \right) + \frac{1}{2\lambda_1^2 \tau_{\xi_1}} \tag{B.4}$$

$$\beta_{11} = \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \tag{B.5}$$

$$\beta_{12} = \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \left(\frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \phi_1 \right) \tag{B.6}$$

$$\begin{aligned}
\alpha'_{10} &= \frac{\left(1 + \frac{q_1 a'_{11}}{1 + \phi'_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \tau_{f_1} \bar{f}_1}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \frac{\left(1 + \frac{q_1 a'_{11}}{1 + \phi'_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right)^2}{2\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\
&\quad + \frac{1}{2\tau_{\varepsilon_1} \tau_{\xi_1}} \left(\frac{q_1 a'_{11}}{1 + \phi'_1} \right)^2 + \frac{q_1^2 \tau_{m_1}}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})^2} + \left(q_1 a'_{11} + \frac{q_2 \tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} \right) \bar{f}_2 \\
&\quad + \frac{\left(q_1 a'_{11} + \frac{q_2 \tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} \right)^2}{2\tau_{f_2}} + \frac{(q_1 a'_{11})^2}{2\tau_{P_2}} + \frac{q_2^2 \tau_{m_2}}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})^2} \\
&\quad + q_1 b'_1 + q_2 b'_2 + \log(1 - \kappa) \tag{B.7}
\end{aligned}$$

$$\begin{aligned}
\alpha'_{11} &= \left(1 + \frac{q_1 a'_{11}}{1 + \phi'_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\
&= \left(\frac{\tau_{f_1} + \left(1 + \frac{q_1}{1 + \phi'_1}\right) \tau_{P_1} + (1 + q_1) \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} \right) \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \tag{B.8}
\end{aligned}$$

$$\begin{aligned}
\alpha'_{12} &= \left(1 + \frac{q_1 a'_{11}}{1 + \phi'_1} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \frac{q_1 a'_{11} \phi'_1}{1 + \phi'_1} \\
&= \left(\frac{\tau_{f_1} + \left(1 + \frac{q_1}{1 + \phi'_1}\right) \tau_{P_1} + (1 + q_1) \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\
&\quad + \left(\frac{\tau_{P_1} q_1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}}\right) \frac{\phi'_1}{1 + \phi'_1}
\end{aligned} \tag{B.9}$$

$$\beta'_{10} = \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \left(\frac{\tau_{f_1} \bar{f}_1 + \frac{1}{2} \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} - \mu_1 \right) + \frac{1}{2\lambda_1^2 \tau_{\xi_1}} \tag{B.10}$$

$$\beta'_{11} = \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \tag{B.11}$$

$$\beta'_{12} = \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \left(\frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \phi'_1 \right) \tag{B.12}$$

Note that

$$\alpha'_{11} - \beta'_{11} = \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \left(\frac{\tau_{f_1} + \left(1 + \frac{q_1}{1 + \phi'_1}\right) \tau_{P_1} + (1 + q_1) \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \right) \tag{B.13}$$

$$\begin{aligned}
\alpha_{11} - \beta_{11} &= \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \left(\frac{\tau_{f_1} + \left(1 + \frac{q_1}{1 + \phi_1}\right) \tau_{P_1} + (1 + q_1) \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \right) \\
&\quad + \frac{q_1 q_2 (1 + q_1)}{(1 - q_1 q_2) (1 + \phi_1)} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}}
\end{aligned} \tag{B.14}$$

$$\alpha'_{21} - \beta'_{21} = \frac{\tau_{\varepsilon_2}}{\tau_{f_2} + \tau_{c_2} + \tau_{\varepsilon_2}} \left(\frac{\tau_{f_2} + \left(1 + \frac{q_2}{1 + \phi'_2}\right) \tau_{P_2} + (1 + q_2) \tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} - \frac{\sqrt{\tau_{\varepsilon_2}}}{\lambda_2} \right) \tag{B.15}$$

$$\begin{aligned}\alpha_{21} - \beta_{21} = & \frac{\tau_{\varepsilon_2}}{\tau_{f_2} + \tau_{c_2} + \tau_{\varepsilon_2}} \left(\frac{\tau_{f_2} + \left(1 + \frac{q_2}{1+\phi_2}\right) \tau_{P_2} + (1+q_2) \tau_{m_2}}{\tau_{f_2} + \tau_{m_2} + \tau_{P_2}} - \frac{\sqrt{\tau_{\varepsilon_2}}}{\lambda_2} \right) \\ & + \frac{q_1 q_2 (1+q_2)}{(1-q_1 q_2)(1+\phi_2)} \frac{\tau_{P_2}}{\tau_{f_2} + \tau_{P_2}} \frac{\tau_{\varepsilon_2}}{\tau_{f_2} + \tau_{c_2} + \tau_{\varepsilon_2}}\end{aligned}\quad (\text{B.16})$$

A sufficient condition for $\alpha_{11} - \beta_{11} > 0$ and $\alpha'_{11} - \beta'_{11} > 0$ is $\lambda_1 > \sqrt{\tau_{\varepsilon_1}}$. Similarly A sufficient condition for $\alpha_{21} - \beta_{21} > 0$ and $\alpha'_{21} - \beta'_{21} > 0$ is $\lambda_2 > \sqrt{\tau_{\varepsilon_2}}$. Then we need to solve for $B_i(\phi_i) = (\alpha_{i2} - \beta_{i2}) / (\alpha_{i1} - \beta_{i1}) = \phi_i$ and $B'_i(\phi'_i) = (\alpha'_{i2} - \beta'_{i2}) / (\alpha'_{i1} - \beta'_{i1}) = \phi'_i$ to obtain the equilibrium with and without cross region learning. Since the expression of $B_1(\phi_1)$ does not involve ϕ_2 and the expression of $B_2(\phi_2)$ does not involve ϕ_1 , I can solve for ϕ_1^* and ϕ_2^* separately. And so is for $\phi_1'^*$ and $\phi_2'^*$. Let me first focus on solving ϕ_1^* and $\phi_1'^*$. Substituting and rearranging, $B_1(\phi_1) = \phi_1$ is equivalent to the following equation,

$$\begin{aligned}H_1(\phi_1) = & \left(1 + \frac{\tau_{P_1} q_1}{(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(1+\phi_1)} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}\right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\ & - \frac{\tau_{\varepsilon_1}}{(\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1})} \left(1 + \frac{\tau_{P_1} q_1}{(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(1+\phi_1)} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}\right) \phi_1 \\ & + \frac{\tau_{P_1} q_1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} \frac{\phi_1}{1+\phi_1} - \frac{\sqrt{\tau_{\varepsilon_1}} \phi_1}{\lambda_1} + \frac{q_1 q_2 (1+q_1)}{(1-q_1 q_2)} \frac{(\tau_{c_1} + (\tau_{c_1} + \tau_{f_1}) \phi_1)}{(\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1})(1+\phi_1)} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} \\ = & 0;\end{aligned}\quad (\text{B.17})$$

$B'_1(\phi'_1) = \phi'_1$ is equivalent to the following equation,

$$\begin{aligned}H'_1(\phi'_1) = & \left(1 + \frac{\tau_{P_1} q_1}{(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(1+\phi'_1)} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}\right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\ & - \frac{\tau_{\varepsilon_1}}{(\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1})} \left(1 + \frac{\tau_{P_1} q_1}{(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(1+\phi'_1)} + \frac{q_1 \tau_{m_1}}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}\right) \phi'_1 \\ & + \frac{\tau_{P_1} q_1}{\tau_{f_1} + \tau_{m_1} + \tau_{P_1}} \frac{\phi'_1}{1+\phi'_1} - \frac{\sqrt{\tau_{\varepsilon_1}} \phi'_1}{\lambda_1} \\ = & 0;\end{aligned}\quad (\text{B.18})$$

It is easy to see that

$$H_1(\phi_1) = H'_1(\phi_1) + \Delta_1(\phi_1) \quad (\text{B.19})$$

where

$$\begin{aligned} \Delta_1(\phi_1) &= \frac{q_1 q_2 (1 + q_1)}{(1 - q_1 q_2)} \frac{(\tau_{c_1} + (\tau_{c_1} + \tau_{f_1}) \phi_1)}{(\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}) (1 + \phi_1)} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} \\ &= \frac{\gamma (1 + q_1) (\tau_{c_1} + (\tau_{c_1} + \tau_{f_1}) \phi_1)}{(\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}) (1 + \phi_1)} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} > 0 \end{aligned} \quad (\text{B.20})$$

$H'_1(\phi_1)$ is continuous function of ϕ_1 . For $\lambda_1 > \sqrt{\tau_{\varepsilon_1}}$, $H'_1(\phi_1)$ goes to ∞ as ϕ_1 goes to $-\infty$ and goes to $-\infty$ as ϕ_1 goes to ∞ . $\Delta_1(\phi_1)/\gamma$ is positive and bounded above by $\frac{(1+q_1)(2\tau_{c_1}+\tau_{f_1})}{\tau_{f_1}+\tau_{c_1}+\tau_{\varepsilon_1}}$ and bounded below by $\frac{(1+q_1)\tau_{c_1}}{\tau_{f_1}+\tau_{c_1}+\tau_{\varepsilon_1}} \frac{\min(\tau_{c_1}, \tau_{\varepsilon_1} \tau_{\xi_1})}{\tau_{f_1} + \min(\tau_{c_1}, \tau_{\varepsilon_1} \tau_{\xi_1})}$.

Note that

$$\begin{aligned} H_1(0) &= \left(1 + \frac{(\tau_{\varepsilon_1} \tau_{\xi_1} + \tau_{m_1}) q_1}{\tau_{f_1} + \tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} + \frac{\gamma (1 + q_1) \tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \frac{\tau_{\varepsilon_1} \tau_{\xi_1}}{\tau_{f_1} + \tau_{\varepsilon_1} \tau_{\xi_1}} \\ H'_1(0) &= \left(1 + \frac{(\tau_{\varepsilon_1} \tau_{\xi_1} + \tau_{m_1}) q_1}{\tau_{f_1} + \tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1}} - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1} \right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \end{aligned}$$

For $\lambda_1 > \sqrt{\tau_{\varepsilon_1}}$, $H_1(0)$, $H'_1(0) > 0$, there exists positive root for $H_1(\phi_1)$ and $H'_1(\phi_1)$.

Now I discuss the uniqueness of the equilibrium without cross region learning. Recall that $\frac{(1+\phi_i)^2 \tau_{\varepsilon_i} \tau_{\xi_i} \tau_{c_i}}{\phi_i^2 \tau_{\varepsilon_i} \tau_{\xi_i} + \tau_{c_i}}$. Denote $h_i \equiv \tau_{\varepsilon_i} \tau_{\xi_i}$. Substituting for τ_{P_1} and simplifying, solving $H'_1(\phi'_1) = 0$ is equivalent to finding the root for the following polynomial:

$$\begin{aligned}
0 = & -((\tau_{f_1} + \tau_{m_1} + \tau_{c_1})(\tau_{f_1} + \tau_{\varepsilon_1} + \tau_{c_1}) + q_1 \tau_{\varepsilon_1} \tau_{m_1}) \phi_1^3 \\
& -\tau_{c_1} ((2\tau_{f_1} + 2\tau_{c_1} - q_1 \tau_{m_1} + 2\tau_{\varepsilon_1}) - q_1 (\tau_{f_1} + \tau_{c_1})) \phi_1^2 \\
& -\tau_{c_1} (\tau_{\varepsilon_1} - (q_1 - 1) \tau_{f_1} - (2q_1 - 1) \tau_{c_1}) \phi_1 + \gamma_1 q_1 \tau_{c_1}^2 \\
& -\frac{\tau_{c_1}}{h_1} ((\tau_{\varepsilon_1} \tau_{f_1} + (1 + q_1) \tau_{\varepsilon_1} \tau_{m_1} + \tau_{f_1}^2 + \tau_{c_1} \tau_{f_1} + \tau_{f_1} \tau_{m_1} + \tau_{c_1} \tau_{m_1}) \phi_1 - q_1 \tau_{c_1} \tau_{m_1}) \\
& + \left(1 - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}\right) ((\tau_{f_1} + \tau_{c_1})(\tau_{f_1} + \tau_{m_1} + \tau_{c_1}) \phi_1^3 + \tau_{c_1} (3\tau_{f_1} + 3\tau_{c_1} + \tau_{m_1}) \phi_1^2 \\
& + \tau_{c_1} (\tau_{f_1} + 3\tau_{c_1}) \phi_1 + \tau_{c_1}^2) + \left(1 - \frac{\sqrt{\tau_{\varepsilon_1}}}{\lambda_1}\right) \frac{\tau_{c_1}}{h_1} ((\tau_{f_1} + \tau_{c_1})(\tau_{f_1} + \tau_{m_1}) \phi_1 + \tau_{c_1} (\tau_{f_1} + \tau_{m_1}))
\end{aligned} \tag{B.21}$$

Letting λ_1 goes to infinity and τ_{f_1} goes to zero, the discriminant of the above polynomial is $(\tau_{c_1}/h_1)^3$ times:

$$\begin{aligned}
& - \left(\begin{aligned} & 4(1+q_1)^4 \tau_{c_1}^4 \tau_{m_1} + 4(1+q_1)^3 (4+q_1) \tau_{c_1}^3 \tau_{\varepsilon_1} \tau_{m_1} + 8(1+q_1)^4 \tau_{c_1}^3 \tau_{m_1}^2 \\ & + (1+q_1)^3 (36q_1 + 16) \tau_{c_1}^2 \tau_{\varepsilon_1} \tau_{m_1}^2 + 4(1+q_1)^4 \tau_{c_1}^2 \tau_{m_1}^3 \\ & + (1+q_1)^2 (27(1+q_1)^2 - 18q_1 - 19) \tau_{c_1} \tau_{\varepsilon_1}^2 \tau_{m_1}^2 + 12(1+q_1)^2 (2+q_1) \tau_{c_1}^2 \tau_{\varepsilon_1}^2 \tau_{m_1} \\ & + 4(1+q_1) (3q_1 + 4) \tau_{c_1} \tau_{\varepsilon_1}^3 \tau_{m_1} + 4(1+q_1) \tau_{\varepsilon_1}^4 \tau_{m_1} \end{aligned} \right) h_1^3 \tag{B.22} \\
& - \left(\begin{aligned} & 4(1+q_1)^4 \tau_{c_1}^5 \tau_{m_1} + 4(1+q_1)^3 (4+q_1) \tau_{c_1}^4 \tau_{\varepsilon_1} \tau_{m_1} + 12(1+q_1)^4 \tau_{c_1}^4 \tau_{m_1}^2 \\ & + (1+q_1)^3 (44q_1 - 32) \tau_{c_1}^3 \tau_{\varepsilon_1} \tau_{m_1}^2 + 10(1+q_1)^2 ((1+q_1)(3q_1 + 1) + 3) \tau_{c_1}^2 \tau_{\varepsilon_1}^2 \tau_{m_1}^2 \\ & + (1+q_1)^3 (40q_1 + 16) \tau_{c_1}^2 \tau_{\varepsilon_1} \tau_{m_1}^3 + 4(1+q_1)^4 \tau_{c_1}^2 \tau_{m_1}^4 + (1+q_1)^3 (36q_1 + 16) \tau_{c_1} \tau_{\varepsilon_1}^2 \tau_{m_1}^3 \\ & + 12(1+q_1)^2 (2+q_1) \tau_{c_1}^3 \tau_{\varepsilon_1}^2 \tau_{m_1} + 4(1+q_1) (3q_1 + 4) \tau_{c_1}^2 \tau_{\varepsilon_1}^3 \tau_{m_1} + 12(1+q_1)^4 \tau_{c_1}^3 \tau_{m_1}^3 \\ & + (1+q_1)^2 (32 - 12q_1) \tau_{c_1} \tau_{\varepsilon_1}^3 \tau_{m_1}^2 + 12(1+q_1)^2 \tau_{\varepsilon_1}^4 \tau_{m_1}^2 + 4(1+q_1) \tau_{c_1} \tau_{\varepsilon_1}^4 \tau_{m_1} \end{aligned} \right) h_1^2 \\
& + \left(\begin{aligned} & (1+q_1)^2 (q_1^2 + 20q_1 - 8) \tau_{c_1}^3 \tau_{\varepsilon_1}^2 \tau_{m_1}^2 + (1+q_1)^3 (20q_1 - 16) \tau_{c_1}^2 \tau_{\varepsilon_1}^2 \tau_{m_1}^3 \\ & + (1+q_1)^3 (24q_1 - 16) \tau_{c_1} \tau_{\varepsilon_1}^3 \tau_{m_1}^3 - 8(1+q_1)^4 \tau_{c_1} \tau_{\varepsilon_1}^2 \tau_{m_1}^4 + \\ & (1+q_1)^2 (20q_1 - 16) \tau_{c_1}^2 \tau_{\varepsilon_1}^3 \tau_{m_1}^2 - 12(1+q_1)^3 \tau_{\varepsilon_1}^4 \tau_{m_1}^3 - 8(1+q_1)^2 \tau_{c_1} \tau_{\varepsilon_1}^4 \tau_{m_1}^2 \end{aligned} \right) h_1 \\
& - \left(4(1+q_1)^4 \tau_{\varepsilon_1}^4 \tau_{m_1}^4 + 4(1+q_1)^3 \tau_{c_1} \tau_{\varepsilon_1}^4 \tau_{m_1}^3 \right)
\end{aligned}$$

The coefficient for h_1^2 in (B.22) is strictly negative, so the quadratic part of (B.22) is maximized at

$$h_1 = \frac{\left(\begin{aligned} &(1+q_1)^2 (q_1^2 + 20q_1 - 8) \tau_{c_1}^3 \tau_{\varepsilon_1}^2 \tau_{m_1}^2 + (1+q_1)^3 (20q_1 - 16) \tau_{c_1}^2 \tau_{\varepsilon_1}^2 \tau_{m_1}^3 \\ &+ (1+q_1)^3 (24q_1 - 16) \tau_{c_1} \tau_{\varepsilon_1}^3 \tau_{m_1}^3 - 8(1+q_1)^4 \tau_{c_1} \tau_{\varepsilon_1}^2 \tau_{m_1}^4 \\ &+ (1+q_1)^2 (20q_1 - 16) \tau_{c_1}^2 \tau_{\varepsilon_1}^3 \tau_{m_1}^2 - 12(1+q_1)^3 \tau_{\varepsilon_1}^4 \tau_{m_1}^3 - 8(1+q_1)^2 \tau_{c_1} \tau_{\varepsilon_1}^4 \tau_{m_1}^2 \end{aligned} \right)}{2 \left(\begin{aligned} &4(1+q_1)^4 \tau_{c_1}^5 \tau_{m_1} + 4(1+q_1)^3 (4+q_1) \tau_{c_1}^4 \tau_{\varepsilon_1} \tau_{m_1} + 12(1+q_1)^4 \tau_{c_1}^4 \tau_{m_1}^2 \\ &+ (1+q_1)^3 (44q_1 - 32) \tau_{c_1}^3 \tau_{\varepsilon_1} \tau_{m_1}^2 + 10(1+q_1)^2 ((1+q_1)(3q_1 + 1) + 3) \tau_{c_1}^2 \tau_{\varepsilon_1}^2 \tau_{m_1}^2 \\ &+ (1+q_1)^3 (40q_1 + 16) \tau_{c_1}^2 \tau_{\varepsilon_1} \tau_{m_1}^3 + 4(1+q_1)^4 \tau_{c_1}^2 \tau_{m_1}^4 + 4(1+q_1) \tau_{c_1} \tau_{\varepsilon_1}^4 \tau_{m_1} \\ &+ (1+q_1)^3 (36q_1 + 16) \tau_{c_1} \tau_{\varepsilon_1}^2 \tau_{m_1}^3 + 12(1+q_1)^2 (2+q_1) \tau_{c_1}^3 \tau_{\varepsilon_1}^2 \tau_{m_1} \\ &+ 4(1+q_1) (3q_1 + 4) \tau_{c_1}^2 \tau_{\varepsilon_1}^3 \tau_{m_1} \\ &+ (1+q_1)^2 (32 - 12q_1) \tau_{c_1} \tau_{\varepsilon_1}^3 \tau_{m_1}^2 + 12(1+q_1)^2 \tau_{\varepsilon_1}^4 \tau_{m_1}^2 + 12(1+q_1)^4 \tau_{c_1}^3 \tau_{m_1}^3 \end{aligned} \right)} \quad (\text{B.23})$$

Substituting this back to the quadratic above, the maximized value is

$$\begin{aligned}
& \left(\begin{aligned}
& (1+q_1)^4 ((1+q_1)^4 - 28(1+q_1)^3 + 270(1+q_1)^2 - 972q_1 - 243) \tau_{c_1}^6 \tau_{\varepsilon_1}^4 \tau_{m_1}^4 \\
& + (1+q_1)^5 (-24(1+q_1)^3 + 456(1+q_1)^2 - 2376q_1 - 432) \tau_{c_1}^5 \tau_{\varepsilon_1}^4 \tau_{m_1}^5 \\
& + (1+q_1)^5 (-16(1+q_1)^3 + 688(1+q_1)^2 - 5424q_1 - 673) \tau_{c_1}^4 \tau_{\varepsilon_1}^5 \tau_{m_1}^5 \\
& + (1+q_1)^6 (192(1+q_1)^2 - 1920q_1 - 192) \tau_{c_1}^4 \tau_{\varepsilon_1}^4 \tau_{m_1}^6 + (1+q_1)^6 (256(1+q_1)^2 \\
& - 4096q_1 - 256) \tau_{c_1}^3 \tau_{\varepsilon_1}^5 \tau_{m_1}^6 - 512q_1 (1+q_1)^7 \tau_{c_1}^3 \tau_{\varepsilon_1}^4 \tau_{m_1}^7 + (1+q_1)^6 (96(1+q_1)^2 \\
& - 2656q_1 - 96) \tau_{c_1}^2 \tau_{\varepsilon_1}^6 \tau_{m_1}^6 - 1024q_1 (1+q_1)^7 \tau_{c_1}^2 \tau_{\varepsilon_1}^5 \tau_{m_1}^7 - 512q_1 (1+q_1)^7 \tau_{c_1} \tau_{\varepsilon_1}^6 \tau_{m_1}^7 \\
& + (1+q_1)^4 (-24(1+q_1)^3 + 456(1+q_1)^2 - 2376q_1 - 432) \tau_{c_1}^5 \tau_{\varepsilon_1}^5 \tau_{m_1}^4 \\
& + (1+q_1)^5 (264(1+q_1)^2 - 3952q_1 - 264) \tau_{c_1}^3 \tau_{\varepsilon_1}^6 \tau_{m_1}^5 \\
& + (1+q_1)^6 (-384q_1 - 128) \tau_{c_1} \tau_{\varepsilon_1}^7 \tau_{m_1}^6 \\
& + (1+q_1)^4 (192(1+q_1)^2 - 1920q_1 - 192) \tau_{c_1}^4 \tau_{\varepsilon_1}^6 \tau_{m_1}^4 \\
& + (1+q_1)^5 (-864q_1 - 128) \tau_{c_1}^2 \tau_{\varepsilon_1}^7 \tau_{m_1}^5 - 48(1+q_1)^6 \tau_{\varepsilon_1}^8 \tau_{m_1}^6 \\
& - 512q_1 (1+q_1)^4 \tau_{c_1}^3 \tau_{\varepsilon_1}^7 \tau_{m_1}^4 - 64(1+q_1)^5 \tau_{c_1} \tau_{\varepsilon_1}^8 \tau_{m_1}^5
\end{aligned} \right) \\
& \left. \begin{aligned}
& 4(1+q_1)^4 \tau_{c_1}^5 \tau_{m_1} + 4(1+q_1)^3 (4+q_1) \tau_{c_1}^4 \tau_{\varepsilon_1} \tau_{m_1} + 12(1+q_1)^4 \tau_{c_1}^4 \tau_{m_1}^2 \\
& + (1+q_1)^3 (44q_1 - 32) \tau_{c_1}^3 \tau_{\varepsilon_1} \tau_{m_1}^2 + 12(1+q_1)^4 \tau_{c_1}^3 \tau_{m_1}^3 \\
& + 10(1+q_1)^2 ((1+q_1)(3q_1+1) + 3) \tau_{c_1}^2 \tau_{\varepsilon_1}^2 \tau_{m_1}^2 \\
& + (1+q_1)^3 (40q_1 + 16) \tau_{c_1}^2 \tau_{\varepsilon_1} \tau_{m_1}^3 + 4(1+q_1)^4 \tau_{c_1}^2 \tau_{m_1}^4 \\
& + (1+q_1)^3 (36q_1 + 16) \tau_{c_1} \tau_{\varepsilon_1}^2 \tau_{m_1}^3 \\
& + 12(1+q_1)^2 (2+q_1) \tau_{c_1}^3 \tau_{\varepsilon_1}^2 \tau_{m_1} + 4(1+q_1)(3q_1+4) \tau_{c_1}^2 \tau_{\varepsilon_1}^3 \tau_{m_1} \\
& + (1+q_1)^2 (32 - 12q_1) \tau_{c_1} \tau_{\varepsilon_1}^3 \tau_{m_1}^2 + 12(1+q_1)^2 \tau_{\varepsilon_1}^4 \tau_{m_1}^2 \\
& + 4(1+q_1) \tau_{c_1} \tau_{\varepsilon_1}^4 \tau_{m_1}
\end{aligned} \right) \quad (B.24)
\end{aligned}$$

which is negative for $q_1 \in (0, 1)$. Therefore the discriminant is strictly negative for λ_1 large enough and τ_{f_1} small enough and the polynomial (B.21) has a unique root.

Now I discuss the uniqueness of the equilibrium with cross region learning. When λ_1 goes to infinity and τ_{f_1} goes to zero, I have

$$\Delta_1(\phi_1; \lambda_1 \rightarrow \infty, \tau_{f_1} \rightarrow 0) = \frac{q_1 q_2 (1+q_1)}{(1-q_1 q_2)} \frac{\tau_{c_1}}{\tau_{c_1} + \tau_{\varepsilon_1}}, \quad (B.25)$$

When $\gamma = \frac{q_1 q_2}{1 - q_1 q_2}$ is high enough, all local minimals (if more than one) of $H_1(\phi_1) = H'_1(\phi_1) + \Delta_1(\phi_1)$ are above the x -axis. There is only one positive root for $H(\phi_1)$.

It's straightforward that $\phi_1^* > \phi_1'^*$, as $H_1(\phi_1) > H'_1(\phi_1)$ for all ϕ_1 .

As everything is symmetric, using the same technique, I can prove that for high enough levels of supply elasticity λ_2 , $\phi_2^*, \phi_2'^* > 0$ exists for region 2. $\phi_2'^*$ is unique for small enough τ_{f_2} . ϕ_2^* is unique for small enough τ_{f_2} and high enough γ . $H_i(\phi_i)$ shifts up as γ increases, therefore ϕ_1^* and ϕ_2^* are both increasing in γ .

Proof of Lemma 10 It is straight forward by taking the first order derivative of $\frac{(1+\phi_i)^2 \tau_{\varepsilon_i} \tau_{\xi_i} \tau_{c_i}}{\phi_i^2 \tau_{\varepsilon_i} \tau_{\xi_i} + \tau_{c_i}}$ w.r.t ϕ_i .

Proof of Propostion 11 for λ_1 large enough, $H'_1(\phi_{1,OP})$ is approximately

$$\left(1 + \frac{q_1 (\tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1})}{\tau_{f_1} + \tau_{c_1} + \tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1}}\right) \frac{\tau_{c_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \quad (\text{B.26})$$

$$\begin{aligned} & - \frac{\tau_{c_1}}{\tau_{\varepsilon_1} \tau_{\xi_1}} \left(1 + \frac{q_1 (\tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1})}{\tau_{f_1} + \tau_{c_1} + \tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1}}\right) \frac{\tau_{\varepsilon_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1}} \\ & + \frac{q_1 \tau_{\varepsilon_1} \tau_{\xi_1}}{\tau_{f_1} + \tau_{c_1} + \tau_{m_1} + \tau_{\varepsilon_1} \tau_{\xi_1}} \frac{\tau_{c_1}}{\tau_{c_1} + \tau_{\varepsilon_1} \tau_{\xi_1}} \end{aligned} \quad (\text{B.27})$$

which is negative for τ_{ξ_1} small. Moreover, it eventually increases in τ_{ξ_1} and becomes positive. This means that a cutoff $\bar{\tau}_{\xi_1}$ exists such that $\phi_{1,OP} < \phi_1'^*$ for $\tau_{\xi_1} > \bar{\tau}_{\xi_1}$ and $\phi_{1,OP} > \phi_1'^*$ for $\tau_{\xi_1} < \bar{\tau}_{\xi_1}$. Focusing on the case $\tau_{\xi_1} < \bar{\tau}_{\xi_1}$ and $\phi_{1,OP} > \phi_1'^*$, in which $H'_1(\phi_{1,OP}) < 0$, there is a cutoff $\bar{\gamma}_1$ such that $H_1(\phi_{1,OP}) = H'_1(\phi_{1,OP}) + \frac{\gamma(1+q_1)(\tau_{c_1} + (\tau_{c_1} + \tau_{f_1})\phi_{1,OP})}{(\tau_{f_1} + \tau_{c_1} + \tau_{\varepsilon_1})(1+\phi_{1,OP})} \frac{\tau_{P_1}}{\tau_{f_1} + \tau_{P_1}} > 0$ for all $\gamma > \bar{\gamma}_1$ and $H_1(\phi_{1,OP}) < 0$ for all $\gamma < \bar{\gamma}_1$. The part for $i = 2$ is proved following the same process.

Proof of Proposition 12 Let me first derive the investment efficiency for region 1,

$$E_0 \left[\max_{I_1} E \left[\kappa_1 Q^1(I_1, I_2) - \frac{c_1 I_1^{q_1+1}}{q_1 + 1} |\Gamma_1| \right] \right] = E_0 \left[E \left[\kappa_1 Q^1(I_1^*, I_2^*) - \frac{c_1 I_1^{*q_1+1}}{q_1 + 1} |\Gamma_1| \right] \right]. \quad (\text{B.28})$$

Substituting I_1^* and I_2^* using (3.9) and (3.10) and simplifying, the above expression equals to,

$$\left(\frac{\kappa_1 \kappa_2 q_1 q_2}{c_1 c_2} \right)^{\frac{q_2(1+q_1)}{1-q_1 q_2}} \left(1 - \frac{\kappa_1 q_1}{1+q_1} \right) \exp \left(\begin{aligned} & \bar{f}_1 \left(1 + q_1 + \frac{q_1 q_2 (1+q_1)}{1-q_1 q_2} \left(1 - \frac{\tau_{f_1} \tau_{m_1}}{(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(\tau_{f_1} + \tau_{P_1})} \right) \right) \\ & + \bar{f}_2 \left(q_2 + \frac{q_1 q_2 (1+q_2)}{1-q_1 q_2} \left(1 - \frac{\tau_{f_2} \tau_{m_2}}{(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})(\tau_{f_2} + \tau_{P_2})} \right) \right) \\ & + \frac{1}{2\tau_{f_1}} \left(\frac{1+q_1}{1-q_1 q_2} \right)^2 - \frac{1}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})} \frac{q_1(1-q_2)(1+q_1)}{1-q_1 q_2} \\ & - \frac{1}{2(\tau_{f_1} + \tau_{P_1})} \frac{q_1 q_2 (1+q_1)(2+q_1-q_1 q_2)}{(1-q_1 q_2)^2} \\ & + \frac{1}{2\tau_{f_2}} \frac{(1+q_1)^2 q_2^2}{(1-q_1 q_2)^2} + \frac{1}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})} \frac{q_2(1-q_2)(1+q_1)}{(1-q_1 q_2)} \\ & - \frac{1}{2(\tau_{f_2} + \tau_{P_2})} \frac{q_1 q_2^2 (1+q_1)(1+q_2)}{(1-q_1 q_2)^2} \end{aligned} \right). \quad (\text{B.29})$$

The above expression is increasing in τ_{P_1} and τ_{P_2} .

$$\left(\frac{\kappa_1 \kappa_2 q_1 q_2}{c_1 c_2} \right)^{\frac{q_1(1+q_2)}{1-q_1 q_2}} \left(1 - \frac{\kappa_2 q_2}{1+q_2} \right) \exp \left(\begin{aligned} & \bar{f}_2 \left(1 + q_2 + \frac{q_1 q_2 (1+q_2)}{1-q_1 q_2} \left(1 - \frac{\tau_{f_2} \tau_{m_2}}{(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})(\tau_{f_2} + \tau_{P_2})} \right) \right) \\ & + \bar{f}_1 \left(q_1 + \frac{q_1 q_2 (1+q_1)}{1-q_1 q_2} \left(1 - \frac{\tau_{f_1} \tau_{m_1}}{(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})(\tau_{f_1} + \tau_{P_1})} \right) \right) \\ & + \frac{1}{2\tau_{f_2}} \left(\frac{1+q_2}{1-q_1 q_2} \right)^2 - \frac{1}{2(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})} \frac{q_2(1-q_1)(1+q_2)}{1-q_1 q_2} \\ & - \frac{1}{2(\tau_{f_2} + \tau_{P_2})} \frac{q_1 q_2 (1+q_2)(2+q_2-q_1 q_2)}{(1-q_1 q_2)^2} \\ & + \frac{1}{2\tau_{f_1}} \frac{(1+q_2)^2 q_1^2}{(1-q_1 q_2)^2} + \frac{1}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})} \frac{q_1(1-q_1)(1+q_2)}{(1-q_1 q_2)} \\ & - \frac{1}{2(\tau_{f_1} + \tau_{P_1})} \frac{q_2 q_1^2 (1+q_1)(1+q_2)}{(1-q_1 q_2)^2} \end{aligned} \right). \quad (\text{B.30})$$

The investment efficiency is increasing in τ_{P_1} and τ_{P_2} . The investment efficiency of region 1 and 2 are both maximized at $\phi_i = \phi_{i,OP}$ and $\phi_{-i} = \phi_{-i,OP}$.

Proof of Proposition 13 The investment efficiency without cross region learning for region 1 is

$$\left(\frac{\kappa_1 \kappa_2 q_1 q_2}{c_1 c_2}\right)^{\frac{q_2(1+q_1)}{1-q_1 q_2}} \left(1 - \frac{\kappa_1 q_1}{1+q_1}\right) \exp \left(\begin{aligned} & \bar{f}_1 \frac{1+q_1}{1-q_1 q_2} + \bar{f}_2 \frac{q_2(1+q_1)}{1-q_1 q_2} + \frac{(1+q_1)(1+q_1-q_1 q_2)}{2(1-q_1 q_2) \tau_{f_1}} \\ & - \frac{(1+q_1)^2(1-q_1 q_2) - (1+q_1)(1-q_1^2 q_2)}{2(1-q_1 q_2)(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})} - \frac{(1+q_1)^2 \tau_{f_1}}{2(\tau_{f_1} + \tau_{m_1} + \tau_{P_1})^2} \\ & + \frac{(1+q_1) q_2(1-q_2)}{2(1-q_1 q_2)} \frac{q_2(1-q_2)}{(\tau_{f_2} + \tau_{m_2} + \tau_{P_2})} + \frac{q_2^2(1+q_1)}{2(1-q_1 q_2) \tau_{f_2}} \end{aligned} \right). \quad (\text{B.31})$$

When τ_{f_i} is small enough, the term $\frac{1}{\tau_{f_i}}$ in the investment efficiency equations dominates all the other terms. It's easy to check that the coefficients for $\frac{1}{\tau_{f_1}}$ and $\frac{1}{\tau_{f_2}}$ are larger for the investment efficiency equation with cross region learning. The prove for region 2 is similar.

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