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Three Essays on Macroeconomics of the Labor Market

Taschereau-Dumouchel, Mathieu

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Three Essays on Macroeconomics of the Labor Market

Mathieu Taschereau-Dumouchel

A Dissertation
Presented to the Faculty
of Princeton University
in Candidacy for the Degree
of Doctor of Philosophy

Recommended for Acceptance
by the Department of
Economics
Adviser: Professor Nobuhiro Kiyotaki

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Abstract

The three chapters of this thesis investigate the impact of labor market policies on the macroeconomy. The first two chapters look into the impact of labor unions on the economy. The third chapter, coauthored with Edouard Schaal, studies the design of optimal policy in a labor market with adverse selection.

In the first two chapters, I study the impact of unions on wage inequality, output and unemployment. To do so, I propose, in chapter two, a search and matching model of union formation in which unions arise endogenously through a voting process within firms. In a union firm, workers bargain their wages collectively. In a nonunion firm, each worker bargains individually with the firm. Because of this wage setting asymmetry, a union lowers the profit of a firm and compresses the wage distribution of the workers. Furthermore, to prevent unionization, nonunion firms distort their hiring decisions in a way that also lowers the dispersion of wages. Chapter three evaluates empirically the impact of the union threat. After being calibrated on the United States, the model shows that, even though a partial equilibrium estimate would predict a small impact of unions on inequality, removing the threat of unionization increases the variance of wages substantially. Completely outlawing unions increases wage inequality further. Moreover, outlawing unions increases welfare and output, and lowers unemployment. These results suggest that, even with a small membership, unions might have a significant impact on the economy through general equilibrium mechanisms and the way they distort firms' decisions.

In the last chapter, we study the design of optimal policies in a frictional model of the labor market with adverse selection. Heterogeneous, risk-averse agents look for a job in a labor market characterized by an aggregate matching technology. Firms post vacancies but cannot observe each agent's productivity. This paper emphasizes the need to take into account general equilibrium effects of labor market policies and focus on the non-observability of workers' underlying skills as the main information friction.

Our mechanism design approach shows that the constrained optimal allocation can be implemented by policy instruments such as a non-linear tax on wages, a non-constant unemployment insurance and firm subsidies. We calibrate our model to the US economy and characterize the welfare gains from the optimal policy and its effects on output, employment and the wage distribution. Our findings suggest that the optimal policy under a utilitarian government features a negative income tax, a more generous unemployment insurance for low-skilled workers and higher marginal tax rates, which results in a higher participation in the labor market and a lower unemployment rate. This paper also shows that a government with more egalitarian preferences would favor policies with more *European* characteristics: heavier taxation and more generous unemployment insurance, which result in a lower output and slightly higher unemployment rate.

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À Cécile, Yves et Vincent.

Contents

Abstract iii

Acknowledgements v

List of Tables x

List of Figures xi

1 The Union Threat - Model **1**

1.1 Introduction 1

1.1.1 Related literature 3

1.2 The Model 5

1.2.1 Preferences and technology 7

1.2.2 Labor markets 8

1.2.3 Agents 9

1.2.4 Firms 11

1.2.5 Wage setting 14

1.2.6 Voting procedure 21

1.2.7 Steady state equilibrium 23

1.3 Firm's behavior 24

1.3.1 Behavior of an unconstrained firm 26

1.3.2 Fighting the union 30

1.3.3 Impact of technology on unionization 35

1.4 Conclusion 38

1.5 Appendix . . . 38
1.5.1 Proof of lemma 1 . . . 38
1.5.2 Proof of lemma 2 . . . 39
1.5.3 Proof of lemma 3 . . . 43
1.5.4 Proof of lemma 4 . . . 45
1.5.5 Proof of lemma 5 . . . 46
1.5.6 Proof of lemma 6 . . . 47
1.5.7 Proof of lemma 7 . . . 48
1.5.8 Proof of lemma 8 . . . 49
1.5.9 Proof of lemma 9 . . . 50
1.6 Solving the general equilibrium . . . 53

2 The Union Threat - Quantitative exploration 55
2.1 Introduction . . . 55
2.2 Data and calibration . . . 56
2.3 Impact of unions . . . 65
2.4 Conclusion . . . 67

3 Optimal Policy in a Labor Market with Adverse Selection 69
3.1 Introduction . . . 69
3.1.1 Related literature . . . 71
3.2 The Economy . . . 73
3.2.1 Population and Technology . . . 75
3.2.2 Labor market . . . 76
3.2.3 Contracts . . . 77
3.2.4 Worker's problem . . . 77
3.2.5 Firm's problem . . . 79
3.2.6 Government . . . 82

3.2.7 Stationary competitive equilibrium 83
3.3 Designing the optimal mechanism 84
3.3.1 Goals of the planner 84
3.3.2 Limits on the planner 85
3.3.3 Optimal mechanism 86
3.3.4 The planner's problem 88
3.3.5 Solution approach 90
3.3.6 Implementation of the optimal allocation 92
3.3.7 Optimal control 94
3.4 Optimal Policy in the United States 96
3.4.1 Functional forms 96
3.4.2 Calibration 97
3.4.3 Optimal Policy 100
3.4.4 Economic forces at work 104
3.4.5 Impact of policy instruments 106
3.4.6 Increasing the planner's taste for redistribution 110
3.5 Conclusion 113
3.6 Sensitivity analysis 114
3.7 Proofs 117

Bibliography **123**

List of Tables

1.1 The behavior of a firm under the three scenarios. 34

2.1 Parameters taken directly from the data or the literature 57

2.2 The value of the calibrated parameters 62

2.3 Effects of removing the union threat and outlawing unions on the variance of wages, unemployment, output and welfare. 67

3.1 Pre-calibration parameters 100

3.2 Results of the calibration 101

3.3 Optimal policy in the US 101

3.4 Optimal policy under different welfare preferences 110

List of Figures

1.1 Continuum of labor markets 9

1.2 Sequence of events 14

1.3 The hiring decision of an unconstrained firm 30

1.4 The hiring decision of the firm facing a unionization threat 33

1.5 The profits of a firm as a function of its production technology 37

2.1 Skill distribution 59

2.2 Calibrated skill intensities $z_n(s)$ and $z_u(s)$. 63

2.3 Fit of the calibrated model. 64

2.4 Policy exercises: removing the threat of unionization and outlawing unions. 66

3.1 Contract setting game 78

3.2 Empirical and calibrated hourly wage distribution 100

3.3 Comparing the optimal policy with the US policy 103

3.4 Optimal tax and unemployment insurance for $\underline{s}$=0.01,0.1,0.2 keeping $\theta = 0.1$ and $b_0 = 25$ 108

3.5 Optimal tax and unemployment insurance for $\theta = 0.5, 1, 2$ 108

3.6 Optimal policy for $b_0 = 22, 23, 24$ 109

3.7 Increasing the taste for redistribution: optimal policies with $\xi = 1$ and $\xi = 4$ 111

3.8 Optimal policy for $\gamma = 1.8, 1.9, 2$ keeping $\theta = 1$, $b_0 = 25$ and $\underline{s}$=0.01 . 115

3.9 Optimal policy for $\phi = 0.4, 0.45, 0.5$ keeping $\theta = 1$, $b_0 = 25$ and $\underline{s}$=0.01 116

3.10 Optimal policy for $\mu = 0.5, 0.6, 0.7$ keeping $\theta = 0.5$, $b_0 = 25$ and $\underline{s}$=0.01 116

Chapter 1

The Union Threat - Model

1.1 Introduction

What is the impact of unions on the economy? On the one hand, the unionization rate in the United States is at one of its lowest levels in decades. In 2005, only 9% of American private sector workers were unionized.[1] This low union membership limits the scope of collective bargaining agreements to a small fraction of the workforce. At face value, this suggests that unions have a restricted impact on the economy. On the other hand, there is a large empirical literature suggesting that unions are responsible for lowering wage inequality.[2] The bulk of this literature has, however, been limited to measuring partial equilibrium effects only. In particular, it is generally assumed that the union and nonunion wage schedules remain unchanged by a modification of union policies. This approach abstracts completely from the decision process of the firms. For instance, one could think that, if unions were outlawed, the previously unionized firms would demand workers with different characteristics. Also, nonunion firms might be modifying their behavior in response to a *threat* of unionization. Indeed,

[1] Source: Merged Outgoing Rotation Groups of the 2005 Current Population Survey. I define a worker as unionized if this worker is a union member or if he is covered by a union contract.

[2] Card et al. (2004) provides a nice historical survey of that literature together with its own estimates.

even if we observe that a firm is union free, its workers still have the *legal option* to unionize. If unionization lowers profit, a firm might distort its behavior to prevent the formation of a union. A change in union laws would change this threat and therefore affect the behavior of nonunion firms. Finally, partial equilibrium estimates obviously neglect general equilibrium mechanisms that influence unemployment and the way wages are set.

Because of the low variability of union policies across time[3] and the big differences in union laws across countries, it is hard to imagine a non-structural empirical exercise that could identify the global effects of unions on an economy. We therefore need a general equilibrium theory of firms' decision and union formation. This paper proposes such a theory.

The model features risk neutral heterogeneous agents who randomly meet with heterogeneous firms in a labor market characterized by search frictions. Once a firm has hired its new workers, its employees vote on the creation of a union to represent them. If this vote is successful, a union is established and wages are determined through a collective bargaining process between the firm and the union. On the other hand, if the majority of workers votes against unionization, the firm stays union free and each employee bargains his wage individually with the employer. The interaction between the two bargaining structures and the production technology implies that firms have, in general, a higher profit when a union does not represent the workers. The average wage among workers is, however, higher when a union is in charge of the negotiation for all employees. This leads some firms to distort the distribution of the workers they hire in order to prevent unionization. By doing so, they naturally influence the wages of the workers in such a way as to compress the wage distribution.

Several elements distinguish this paper from the existing literature. First, I consider the economy in general equilibrium. In such a setup, the presence of unions

[3]The last major change in union regulations in the US is the Taft-Hartley Act of 1947.

influences the way nonunion wages are set through the aggregate variables of the economy (for instance, the unemployment rate and the expectations that workers have about their future wages). These mechanisms cannot be captured by traditional empirical estimates and might have an important influence on the way unions influence the economy. Second, the threat of potential unionization is featured prominently in the model. This implies that, even in an economy in which no union actually exists, the possibility of unionization alone influences wages, unemployment and output through the distorted behavior of the firms. Third, a wage compression effect of unions arises naturally from the model and its importance is influenced by the state of the economy. Fourth, the ultimate determinant of the union status of a firm is its production technology. In particular, firms with lower labor intensity tend to be more unionized.

1.1.1 Related literature

There is a large empirical literature that evaluates the impact of unions on wage inequality. Freeman (1980) analyzes data from the first half of the 1970s about private sector male workers in the United States and finds that unions are responsible for an important equalizing effect of wages of union workers inside a given sector. This effect is particularly important in manufacturing. I compute the estimator of Freeman (1980) for the private sector of the US in 2005. It suggests that unions are responsible for lowering the variance of log wages by 0.4%.[4]

DiNardo et al. (1996) uses a semiparametric approach to estimate the impact of labor market institutions on the distribution of wages. They find that the decline in

[4]Card et al. (2004) write this estimator as $V - V^N = U\Delta_v + U(1-U)\Delta_w^2$ where V is the observed variance of log wages, V^N is the variance of log wages without unions, U is the unionization rate, Δ_v is the difference in the variance of log union and nonunion wages and Δ_w is the difference between the mean log of union and nonunion wages. For consistency with the calibrated economy, I clean the data by removing agricultural workers as well as workers earning an hourly wage of less than \$5 or more than \$150.

the unionization rate during the 1980s accounts for 10-15 percent of the rise in wage dispersion for men.

Some studies have explicitly accounted for unobserved productivity in their estimates. Among them, Lemieux (1993) and Card (1996) use longitudinal data on Canadian and American workers respectively to evaluate the impact of unions on job switchers. They find that low-skill union workers tend to have higher unobserved productivity than their nonunion counterparts. This implies that the smoothing effect of wages across skill groups that is observed in the raw data is exaggerated when compared to the causal effect of unions. Lemieux (1993) finds that in the late 1980s, unions were responsible for lowering the variance of male wages by 15% in Canada.

Card et al. (2004) provides more recent estimates of the impact of unions on wage inequality. They find that unions were responsible for lowering the variance of log wages of men by 4.5%.[5] Their sample includes private and public sector workers. The same estimate for women is about 2.4%.

All these studies consider what the variance of wages would be if each union worker was paid according to the nonunion wage structure. They do not take into account how unions could affect the structure of wages itself. Indeed, Card et al. (2004) clearly acknowledges this point in their summary of the literature. The model proposed in the current paper explicitly includes these general equilibrium effects.

Two papers propose a model of unions in general equilibrium. Açikgöz and Kaymak (2008) builds a tractable search and matching model of endogenous union formation to estimate the impact of a rising skill premium on the decline of union membership in the United States. They assume that the degree of wage compression is determined by an exogenous parameter that needs to be estimated. In the current paper the compression arises naturally and varies as a function of the aggregate conditions of the economy. Also, their model abstracts from studying the modification

[5]This estimates comes from their model with skill groups.

of nonunion firms' behavior in response to the unionization threat. Finally, my focus extends also to unemployment, output and welfare. Acemoglu et al. (2001) shows that deunionization had an amplifying effect on the rise in wage inequality during the last 25 years of the 20th century. They propose a model of union formation but abstract from the role of the firm in this process, therefore abstracting completely from the threat that unions exert. Other papers modeling unions include, among many, Farber (1978) and Ashenfelter and Johnson (1969).

Hirsch (2004) summarizes the state of current research on the impact of unions on productivity and profitability. He states that "empirical evidence on unions and productivity was rather sketchy in 1984; it remains less than clear-cut today". The research on profitability is more conclusive. According to Hirsch, "evidence points unambiguously to lower profitability among union companies," a feature that arises in the benchmark case of the model I propose.

Finally, Nickell and Layard (1999) finds a correlation between high union density and unemployment in OECD countries between 1983 and 1994 but explains that this correlation is offset by controlling for the level of coordination between unions and firms.

1.2 The Model

The model incorporates six main elements. First, workers are ex-ante heterogeneous in their skill. Second, firms have decreasing returns to scale and hire multiple workers. Third, the model is built along the lines of the search and matching literature (Mortensen and Pissarides, 1994a; Pissarides, 2000a). Fourth, the formation of a union is decided at the level of the firm by a vote cast by the workers.[6] Fifth, if

[6]Modeling unionization as a firm-level process is consistent with evidence presented by Traxler (1994) and Nickell and Layard (1999) that suggest that the coverage of union contracts is mostly at the enterprise level in the US, Canada and the UK.

a union is established, wages are bargained collectively. Otherwise, workers bargain individually with the firm.[7] Sixth, the whole model is in general equilibrium.

The skill heterogeneity interacts with the labor market friction to generate a union wage gap that varies with skill, a feature generally associated with unions (Farber and Saks, 1980). Also, the skill heterogeneity implies that different workers contribute differently to the firm's production. By bargaining individually, a worker's wage is a function of his own characteristics. By bargaining collectively, the union combines the characteristics of all the firm's employees and redistributes what it extracts from the firm among its members. This implies that workers with valuable attributes, for instance those who have a high marginal product, obtain a higher wage by bargaining individually and therefore vote against the unionization of the firm.

The firm is not indifferent between having its workers unionized or not. The decreasing returns to scale interacts with the union status of the firm to influence its profit. When bargaining individually, the firm treats each worker as if it is the marginal one. If negotiations break down, the firm can still produce with the rest of its employees. When bargaining collectively, on the other hand, the union prevents any production from taking place if an agreement on wages is not reached. Because of this threat, the union benefits from the high marginal surplus generated by the infra-marginal workers. This asymmetry of the threats gives a natural disadvantage to a firm that is bargaining collectively with its employees. This creates an incentive to influence the workers' decision in order to avoid unionization. To do so, the firm distorts the distribution of its employees by hiring more high-skill workers and less low-skill workers. This leads to a non-uniform modification of the workers' marginal products and leads to a compression of the range of wages paid by the firm.

[7] Cahuc and Wasmer (2001) builds a search and matching model with firms with decreasing returns to scale in which wages are set through individual bargaining.

1.2.1 Preferences and technology

There is a single good and time is discrete. There are no savings. I focus on steady state equilibria. The economy is populated by a continuum of ex-ante heterogeneous agents, each endowed with a specific type of labor $s \in [0,1]$. I refer to s as the skill. Firms use the different skills for production. The exogenous density of skills in the economy is $N(s)$ with $N(s) > 0$ for all s. An agent's skill is constant over time. Agents live forever. They are risk neutral and maximize a linear utility function

$$\mathcal{U}(c) = \mathrm{E}_0 \sum_{t=0}^{\infty} \gamma^t c_t$$

where c_t denotes consumption in period t and $0 < \gamma < 1$ is the discount factor.

Firms combine the labor provided by workers of different skills to produce goods. To do so, they use heterogeneous production technologies, indexed by $j \in \{1, \ldots, j_{\max}\}$. There is a mass 1 of firms endowed with each technology. A firm of type j employing a (non-normalized) distribution of workers $g(s)$ produces goods according to the production function

$$F_j(L_j(g)) = A_j L_j^{\alpha}(g) = A_j \left\{ \exp\left(\int z_j(s) \log g(s) ds \right) \right\}^{\alpha_j}$$

where $0 < \alpha_j < 1$, $A_j > 0$ and where L_j is a Cobb-Douglas aggregator that describes how firm j combines the different types of labor for production. The function $z_j : [0,1] \to \mathbb{R}_+^*$ represents the relative intensity of skill utilization and is therefore normalized such that $\int z_j(s)\, ds = 1$. The parameter α_j describes the returns to scale of the production function. To avoid cluttering the notation, I omit the subscript j when referring to a single firm. Also, I sometimes write $F(g)$ directly instead of the more cumbersome $F(L(g))$. Notice that, since $z(s) > 0$, the marginal product of a

worker of type s goes to infinity as $g(s) \to 0$. If the cost of hiring is finite, a firm therefore employs workers of every type.

1.2.2 Labor markets

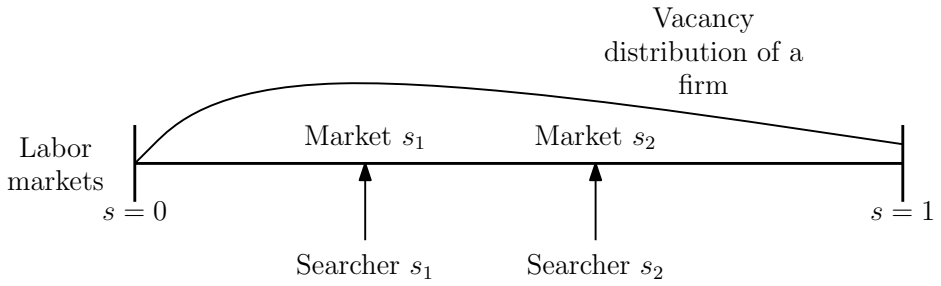
There is a continuum of labor markets in which unemployed agents look for jobs and firms post vacancies. Each vacancy has a cost of κ. Each market is indexed by the skill s of agents searching in it. Agents can only search in the labor market corresponding to their skill.[8] Firms, on the other hand, are free to post a continuum of vacancies that covers all the markets. Figure 1.1 represents this structure. In each market, matches happen randomly at a rate determined by aggregate conditions. If, in a given period, u agents are searching and v vacancies have been posted, $m(u, v)$ matches are made. The matching function $m(\cdot, \cdot)$ is identical across labor markets and is homogenous of degree one. By defining the labor market tightness $\theta \equiv v/u$, the probability that a vacancy is filled in a given period is $q(\theta) \equiv m(u, v)/v$. Similarly, the probability that an unemployed agent finds a job is $\theta q(\theta)$. Notice that q is a strictly decreasing function of θ. Search is free and requires no effort. Every unemployed agent is therefore searching.

All types of firms are posting vacancies in each market. A searching worker can therefore be matched with firms using different technologies and with different union status.

This segmentation of the labor market has two main consequences. First, it allows the firm to control precisely the skill composition of its workforce and, through this channel, influence the unionization vote. Second, it allows me to study the effects of unionization on unemployment rates across skill groups.

[8] In the calibrated model, the unemployment rates across labor markets are decreasing with s while expected wages are increasing with s. Therefore, even if agents were allowed to search in markets with a lower s than their own, they would choose not to do so.

I use the skill index as a way to characterize the heterogeneity of the workers. It is uniquely an index and has no meaning in itself. Later, in the empirical part of the paper, I calibrate the index such that wages are increasing in s and the unemployment rate is decreasing in s. This makes the interpretation of the results more intuitive and explains the name of this index.



Notes: Agents can only search in the market corresponding to their type. Firms can post vacancies in all markets.

Figure 1.1: Continuum of labor markets

1.2.3 Agents

Agents provide labor to firms in exchange for a wage. In each period, an agent is either employed or unemployed. An employed worker loses his job with exogenous probability $\delta > 0$, in which case he goes to the labor market corresponding to his type. With probability $1 - \delta$, the agent remains employed in his current job. Therefore, the lifetime discounted expected utility of a worker of type s who has been matched with a firm of type j and who is earning a wage w is

$$W_e(s, w) = w + \gamma \left[(1 - \delta) W_e(s, w_j(s)) + \delta W_u(s) \right]$$

where $W_u(s)$ is the lifetime utility of being unemployed and $w_j(s)$ is the *equilibrium* wage of a worker of type s provided by a job in firm j. Wages are bargained every period. Therefore the negotiations with the firm are over w only. Both parties consider that $w_j(s)$ is fixed at its equilibrium value.

Every period, an unemployed agent s receives $b_0(s)$ from unemployment benefits and home production. He finds a job with probability $\theta(s)q(\theta(s))$. His lifetime discounted utility is therefore

$$W_u(s) = b_0(s) + \gamma \left\{\theta(s)q(\theta(s))\mathrm{E}(W_e(s,w)) + (1-\theta(s)q(\theta(s)))W_u(s)\right\}$$

where the expectation $\mathrm{E}(W_e(s,w))$ is taken over all the possible wages offered to a searching agent of type s.

An agent will accept to work only if the utility provided by employment exceeds the utility of continuing the search for a job. This never happens in equilibrium. By combining the last two equations we can characterize the utility gain provided by employment:

$$W_e(s,w) - W_u(s) = w + \frac{\gamma(1-\delta)w_j(s) - (1-\gamma)W_u(s)}{1-\gamma(1-\delta)}. \tag{1.1}$$

In equilibrium, the utility provided by a job in firm j is

$$W_e(s,w_j(s)) - W_u(s) = \frac{w_j(s) - (1-\gamma)W_u(s)}{1-\gamma(1-\delta)}.$$

It is useful to define the flow utility of being unemployed by $b(s)$. Therefore,

$$b(s) \equiv (1-\gamma)W_u(s) = \frac{(1-\gamma(1-\delta))b_0(s) + \gamma\theta(s)q(\theta(s))\mathrm{E}(w(s))}{1-\gamma(1-\delta)+\gamma\theta(s)q(\theta(s))}. \tag{1.2}$$

The utility of an unemployed worker takes into account the fact that this worker will spend a part of his time employed in the future. It is therefore a weighted average of $b_0(s)$ and of the wage this agent expects to receive.

To simplify the notation, it is also convenient to define the equilibrium quantity

$$c_j(s) \equiv \frac{b(s) - \gamma(1-\delta)w_j(s)}{1-\gamma(1-\delta)} \tag{1.3}$$

which is the net outside option of the worker. By writing the gain from employment in a firm j at a wage w as

$$W_e(s,w) - W_u(s) = w - c_j(s)$$

we see that $c_j(s)$ is what the worker gets if the bargaining breaks down.

1.2.4 Firms

A firm that employed a distribution of workers g_{-1} during the previous period loses a fraction δ of all of its workers and therefore starts the current period with the distribution $(1-\delta)g_{-1}$. It then posts a schedule of vacancies v to maximize its expected discounted profits. Since the firm is posting a continuum of vacancies in each labor market, a law of large numbers implies that the number of successful matches is deterministic.

Once the new hires have joined the firm, the workers vote on the formation of a union and the firm's optimal behavior will depend on the specifics of the unionization process as well as on how the union and nonunion wages are set. These will be described shortly. For now, it is sufficient to use an abstract function $\int w(s,g)$ to denote the wages that the firm pays as a function of the current workers distribution. Define the current period profit of a firm as $\pi(g) \equiv F(g) - \int w(s,g) \cdot g \, ds$. With this notation, the problem of a firm is

$$\tilde{J}(g_{-1}) = \max_v \pi(g) - \kappa \int v(s)\, ds + \gamma \tilde{J}(g)$$

subject to

$$\begin{cases} g(s) = g_{-1}(s)(1-\delta) + v(s)q(\theta(s)) \\ v(s) \geq 0 \end{cases}$$

where $\tilde{J}(g_{-1})$ is the value function of a firm that ended the previous period with workers g_{-1}. The first constraint is simply the law of motion of the stock of workers. The second constraint states that job separations are exogenous. Firms cannot post negative vacancies.

In a steady state equilibrium in which the aggregate variables remain constant, it is possible to simplify the firm's problem substantially. Suppose that in such an equilibrium, a firm's optimal distribution of workers is given by $g^*(s)$. Such a distribution exists because of the decreasing returns to scale. Two events might move the firm away from $g^*(s)$. First, every period, it loses a fraction of its workers. Second, if one of the wage bargaining sessions breaks down without an agreement, the firm loses additional workers.[9] In both of these cases, the firm has to hire a positive number of workers in the next period to replace those that have been lost. Therefore, $v(s) > 0$ in all markets s such that $g^*(s) > 0$ and $v(s) = 0$ in the other markets. We can therefore substitute v from the law of motion of the workers directly into the objective function. The problem of the firm can be simplified as

$$J\left(\int \frac{g_{-1}}{q(\theta)} ds\right) = \max_{g} \pi(g) - \kappa \int \frac{g - g_{-1}(1-\delta)}{q(\theta)} ds + \gamma J\left(\int \frac{g}{q(\theta)} ds\right) \tag{1.4}$$

where $\int \frac{g_{-1}}{q(\theta)}\, ds$ is a new state variable that represents the value of the stock of workers with which the firm enters the period.[10]

[9] This does not happen in equilibrium but the value function needs to be defined along these paths to correctly characterize the bargaining problems.

[10] Notice that J and $\tilde{J}$ are two different objects but they give the same first order conditions in a steady state equilibrium.

This last value function has two additively separable pieces: one that depends on the distribution of previous period g_{-1} and a second one that depends on the firm's decision in the current period. This implies that, in a steady state, the firm's current period decision is independent of its state variable. The following lemma simplifies the firm's problem.

Lemma 1. *In a steady-state, the firm's dynamic problem can be written as the static optimization*

$$\max_{g} \pi(g) - \kappa(1 - (1 - \delta)\gamma) \int \frac{g}{q(\theta)} \, ds. \tag{1.5}$$

Proof. All proofs are relegated to the appendix. □

This result comes directly from the linearity of the hiring costs, the constant value of θ and the fact that, at the steady state, a firm never wants to downsize in response to a shock.

We now need to describe the wage schedule $w(s, g)$. Figure 1.2 details the sequence of events that occurs once a firm has recruited its new workers. First, the workers vote to decide whether to form a union or not. Then, if a union is established, wages are bargained collectively. The outcome of this bargaining is a wage schedule $w_u(s, g)$ and a profit function $\pi_u(g)$. If the union is rejected, wages are bargained individually. This generates the wage schedule $w_n(s, g)$ and the profit $\pi_n(g)$. Notice that when the vote takes place and when wages are bargained, the distribution of workers g is fixed. Also, when the workers cast their vote, they know exactly what wages they will get if the union is created or not. I first describe the two bargaining procedures and then come back to the voting process.

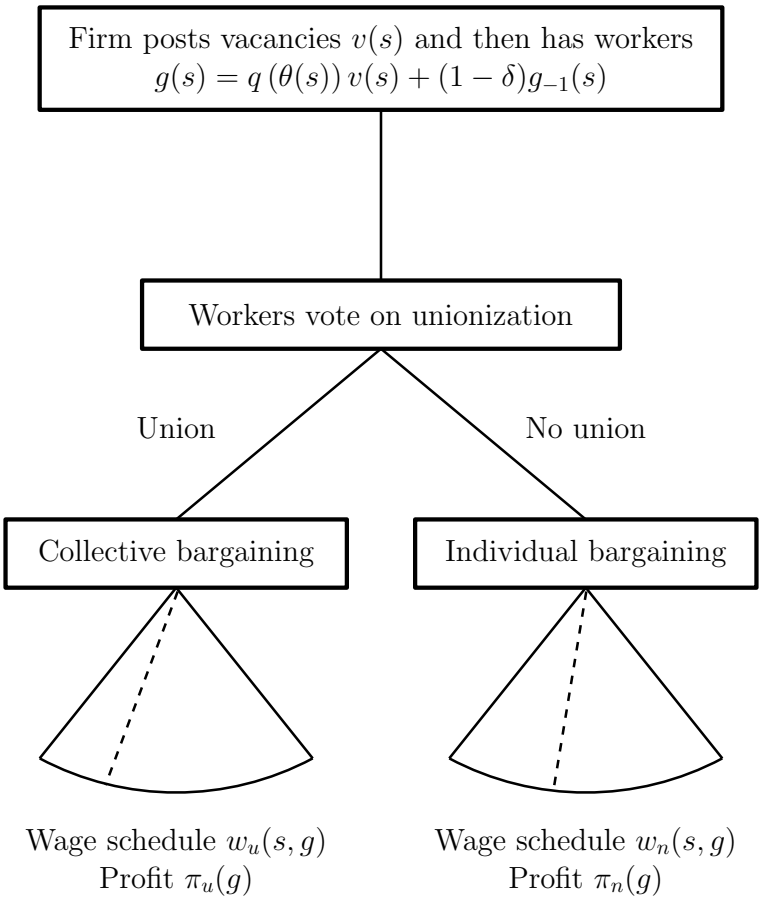


Figure 1.2: Sequence of events

1.2.5 Wage setting

In both a union and a nonunion firm, wages are set using Nash bargaining to share the surplus generated by the match. The surplus that is bargained over is, however, different in both cases. If the firm is unionized and an agreement on wages cannot be reached, the whole workforce quits the firm and no production takes place. In a non unionized firm, if the bargaining with a single worker breaks down, this specific worker goes back to unemployment but the firm can still produce with the other workers. In a nonunion firm, the bargaining therefore takes place over the marginal surplus generated by each worker. In a union firm, the workers and the firm bargain over the total surplus generated by the whole workforce. This asymmetry between

the two surpluses interacts with the decreasing returns of the production function and has important consequences for the firm's profits.

Union bargaining

If the workers vote in favor of unionization, the union is the only group authorized to bargain with the firm. Consider the firm's gain if it reaches an agreement with the union. In a steady-state, the difference in discounted profits for the firm, denoted by Δ^u, is

$$\Delta^u(w) = \left[\pi(g^*, w) + \gamma J\left(\int \frac{g^*}{q(\theta)}\, ds\right)\right] - [\pi(0) + \gamma J(0)] \tag{1.6}$$

where the first term in brackets is discounted profit if an agreement is reached and $\pi(0) + \gamma J(0)$ is the firm's discounted profit if negotiations break down. Notice that in such a scenario, the firm has no worker; it produces nothing and pays no wage. Therefore, the one-period profit $\pi(0)$ is equal to zero. $J(0)$ is the value function of a firm that starts the period with no workers. Because the firm's employment decision is independent of the distribution of its workers, the firm hires back to its steady-state optimal level g^* right away. Therefore,

$$J(0) = \pi(g^*, w^*) - \kappa \int \frac{g^*}{q(\theta)}\, ds + \gamma J\left(g^*\right)$$

where w^* is the equilibrium wage schedule for this firm. This last expression is identical to $J\left(g^*\right)$ except for the fact that the firm hires back all of its workforce in that period and therefore pays a higher vacancy cost. We can rewrite the difference in discounted profit as

$$\Delta^u(w) = \pi(g^*, w) + \gamma J\left(g^*\right) - \gamma\left(\pi(g^*, w^*) - \kappa \int \frac{g^*}{q(\theta)}\, ds + \gamma J\left(g^*\right)\right)$$

But, at the steady state, the firm's value function is

$$J(g^*) = \pi(g^*, w^*) - \kappa\delta \int \frac{g^*}{q(\theta)}\, ds + \gamma J(g^*) \tag{1.7}$$

and therefore the firm's surplus from reaching an agreement is

$$\Delta^u(w) = \pi(g^*, w) + (1-\delta)\gamma\kappa \int \frac{g^*}{q(\theta)}\, ds.$$

The intuition is straightforward. If negotiations breaks down, the firm loses the current period profit π and pays a higher hiring cost tomorrow to compensate for the loss of the fraction $1-\delta$ of its current workforce that would have remained with the firm next period.

We now need to specify the surplus of the union. To do so, an assumption needs to be made about how the workers divide among themselves the rent extracted from the firm. A natural assumption is to have them split that amount by solving a Nash bargaining problem in which each worker has the same bargaining power. In this case, the log of the Nash surplus of the union is given by

$$\int \frac{g(s)}{n} \log(W_e(s, w) - W_u(s))\, ds$$

with $n = \int g(s)\,ds$ and where $W_e(s,w) - W_u(s)$ is given by equation 1.1.[11,12] The union simply aggregates the individual surpluses of each worker.

With this way of sharing the surplus among the workers, the bargaining problem between the firm and the union is simply

$$\max_w \left[\exp\left(\int \frac{g}{n}\log(W_e(s,w) - W_u(s))\,ds\right)\right]^{\beta_u} \times \left[F(g) - \int w\cdot g\,ds + (1-\delta)\kappa\gamma\int \frac{g}{q(\theta)}\,ds\right]^{1-\beta_u}. \tag{1.8}$$

where $0 < \beta_u < 1$ denotes the bargaining power of the union. This coefficient is exogenous to the model and could possibly be influenced by labor market policies.

Lemma 2. *Assume that g is strictly positive on $[0,1]$. Then the following function solves the bargaining problem:*

$$w_u(s,g) - c(s) = \frac{\beta_u}{n}\left(F(g) - \int c\cdot g\,ds + \gamma(1-\delta)\kappa\int \frac{g}{q(\theta)}\,ds\right). \tag{1.9}$$

The solution is unique if the joint surplus of the match is strictly positive at the point w_u.

[11] To see where this equation comes from, consider the discrete case in which there are k different skill groups, each with a weight $\epsilon > 0$, and that g_i workers are of type i. The surplus of a worker of type i, if an agreement is reached at a wage w, is $W_{ei}(w) - W_{ui}$. The log of the joint Nash surplus can be written as

$$\log\left\{(W_{e1} - W_{u1})^{\frac{g_1\epsilon}{n}} \times \cdots \times (W_{ei} - W_{ui})^{\frac{g_i\epsilon}{n}} \times \cdots \times (W_{ek} - W_{uk})^{\frac{g_k\epsilon}{n}}\right\} = \sum_{i=1}^{k}\frac{g_i\epsilon}{n}\log(W_{ei} - W_{ui})$$

where $n = \sum_{i=1}^{k} g_i\epsilon$ is the total number of workers in the firm and where $g_i\epsilon/n$ is the sum of the bargaining power of all the workers of type i. Taking the limit as $k \to \infty$ and $\epsilon \to 0$, we get the log of the union surplus.

[12] Nash's axiomatic theory of bilateral bargaining extends unchanged to a context with numerous players. Krishna and Serrano (1996) provides a strategic approach to multilateral bargaining.

Also, in a union firm with equilibrium distribution of workers g^* and technology j, the equilibrium wage schedule $w_j(s) = w_u(s, g^*)$ is

$$w_u(s, g^*) - b(s) = \frac{1 - \gamma(1-\delta)}{1 - \beta_u\gamma(1-\delta)} \frac{\beta_u}{n^*} \left(F(g^*) - \int b \cdot g^* \, ds + \gamma(1-\delta)\kappa \int \frac{g^*}{q(\theta)} \, ds \right) \tag{1.10}$$

where $n^* = \int g^* ds$ is the optimal size of the firm.

Equation 1.10 implies that, in equilibrium, all the workers are getting the same transfer over their reservation wage $b(s)$. The union is basically mixing together the characteristics of all its members. Therefore, the variance of wages comes from the reservation wage schedule $b(s)$. The macroeconomic conditions, through $b(s)$, have a direct influence on the dispersion of wages in a unionized firm.

It is straightforward to show that the one-period profit of a union firm employing the distribution of workers g is given by

$$\pi_u(g) = (1 - \beta_u)F(g) - (1 - \beta_u) \int c \cdot g \, ds - \beta_u(1-\delta)\kappa\gamma \int \frac{g}{q(\theta)} \, ds. \tag{1.11}$$

Individual bargaining

If the workers vote against unionization, they each bargain individually with the firm. The workers cannot interact with each other. In particular, they cannot create a coalition. Once again, the worker and the firm use Nash bargaining to split the surplus created by the match. These surplus are, however, not identical across all workers. Because of the decreasing returns to scale, the surplus generated by hiring the first worker is higher than the one generated by the marginal worker. To solve this issue I follow the approach of Stole and Zwiebel (1996a,b). They introduce a game in which Nash bargaining is used to split the *marginal* surplus generated by hiring an extra worker. In this setup, the firm negotiates with each of its workers

in turn. If any of the one-on-one negotiations breaks down, wages are renegotiated with all the workers remaining in the firm. When considering the marginal surplus generated by an additional worker, the firm is aware that if the negotiations break down, the other workers might want to rebargain their wages differently.

I show in the appendix (see proof of Lemma 3) that the marginal surplus of the firm from hiring a worker of type s is given by

$$\Delta_n(s,w) = \frac{\partial L}{\partial g(s)}\frac{dF}{dL} - \frac{\partial L}{\partial g(s)}\int \frac{\partial w(s,g(s),L)}{\partial L} g(s)\, ds - \frac{\partial w(s,g(s),L)}{\partial g(s)} g(s) - w(s,g(s),L) + \gamma(1-\delta)\frac{\kappa}{q(\theta(s))}.$$

This equation is fairly intuitive. The first term is the extra output produced by the additional worker. The two following terms represent the marginal effects of the worker on the wages of other members of the workforce. The fourth term is simply the wage paid to the worker and the fifth term is the vacancy costs saved from retaining a fraction $1-\delta$ of today's hire in the next period.

The Nash bargaining implies that the nonunion wage must solve the following equation:

$$\Delta_n(s,w) = \frac{1-\beta_n}{\beta_n}\left(W_e(s,w) - W_u(s)\right). \tag{1.12}$$

Lemma 3. *The wage schedule*

$$w_n(s,g) - c(s) = \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{g(s)}F(g) - \beta_n\ c(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))} \tag{1.13}$$

solves the bargaining problem (equation 1.12) of a firm employing the distribution of workers g.

Also, in a nonunion firm with equilibrium distribution of workers g^* and technology j, the equilibrium wage schedule $w_j(s) = w_n(s, g^*)$ is

$$w_n(s, g^*) - b(s) = \frac{1-\gamma(1-\delta)}{1-\beta_n\gamma(1-\delta)} \left(\frac{\beta_n}{1-(1-\alpha)\beta_n} \frac{\alpha z(s)}{g^*(s)} F(g^*) \right. \\ \left. -\beta_n\, b(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))} \right) \tag{1.14}$$

It follows directly from the wage of nonunion workers that the one-period profit of the firm is

$$\pi_n(g) = \frac{1-\beta_n}{1-(1-\alpha)\beta_n} F(g) - (1-\beta_n) \int c \cdot g \, ds - \beta_n(1-\delta)\kappa\gamma \int \frac{g}{q(\theta)} \, ds. \tag{1.15}$$

Collective vs individual bargaining

The workers and the firm are not indifferent between the two types of bargaining. For a given distribution of workers g, the difference in wages is

$$w_n(s, g) - w_u(s, g) = F(g) \left(\frac{\beta_n}{1-(1-\alpha)\beta_n} \frac{\alpha z(s)}{g(s)} - \frac{\beta_u}{n} \right) - (\beta_n c_j(s) - \beta_u \mathrm{E}_g(c_j)) \\ + \kappa\gamma(1-\delta) \left(\beta_n \frac{1}{q(\theta(s))} - \beta_u \mathrm{E}_g \left(\frac{1}{q(\theta)} \right) \right)$$

where $\mathrm{E}_g(x) = \int x \cdot g \, ds / \int g \, ds$ for any function x. It follows directly that

$$\mathrm{E}_g(w_n) - \mathrm{E}_g(w_u) = \frac{F(g)}{n} \left(\frac{\alpha\beta_n}{1-(1-\alpha)\beta_n} - \beta_u \right) - (\beta_n - \beta_u)\, \mathrm{E}_g(c_j) \\ + \kappa\gamma(1-\delta)\, (\beta_n - \beta_u)\, \mathrm{E}_g \left(\frac{1}{q(\theta)} \right)$$

and, in the case with equal bargaining power ($\beta_n = \beta_u \equiv \beta$):

$$\mathrm{E}_g(w_n) - \mathrm{E}_g(w_u) = -\frac{\beta(1-\beta)(1-\alpha)}{1-(1-\alpha)\beta} \frac{F(g)}{n} < 0.$$

For any distribution g, the workers prefer, on average, to have a unionized firm.

Similarly, with equal bargaining power, the difference in one-period profit is

$$\pi_n(g) - \pi_u(g) = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} F(g) > 0.$$

Notice that, as $\alpha \to 1$, the differences in profits and in average wages go to zero.

In general, the firm prefers to bargain individually while the workers, on average, would rather be represented by a union. This conflict of preferences is a direct consequence of the decreasing returns to scale. When bargaining individually, the firm considers producing with or without the *marginal* worker, who has a relatively small impact on the total production. On the other hand, when the firm bargains with the union, the surplus is a function of *total* production, which includes the relatively high production generated by the infra-marginal workers. By forming a union, the workers can extract a part of these high marginal products, which lowers the firm's profit.

In the calibrated model, even though $\beta_n \neq \beta_u$, firms always prefer to be union free. This is consistent with evidence presented by Kleiner (2001) suggesting that firms generally oppose unions. Bronfenbrenner (1994) also details various tactics used by firms to prevent unionization. Hirsch (2004) summarize the literature on union and profitability and concludes that unions have a negative impact on firms' profits.

1.2.6 Voting procedure

Once a firm has welcomed its new workers, the vote on unionization takes place (see Figure 1.2). The distribution of workers is now fixed and workers are therefore fully aware of the wages they would get in both outcomes of the vote. Workers are rational and vote only to maximize their own individual utility. Each worker has random preferences on the union status of the firm. One can think that some workers have

a negative or positive opinion of unions for reasons that are exogenous to the model. Specifically,

$$\text{Worker } s \text{ votes for a union} \Leftrightarrow w_u(s,g) - w_n(s,g) > \epsilon$$

where ϵ is a logistic random variable drawn independently across all workers. It has mean 0 and scale parameter $1/\rho$.[13]

A law of large numbers applies when aggregating the workers of a given skill. Therefore, a fraction

$$\frac{1}{1+\exp\{-\rho(w_u(s,g) - w_n(s,g))\}}$$

of workers of type s will vote in favor of unionization. By summing up all the voters, we can denote the excess number of workers in favor of unionization by

$$V(g) \equiv \int \frac{g}{1+\exp\{-\rho(w_u(s,g) - w_n(s,g))\}}\, ds - \frac{1}{2}n. \tag{1.16}$$

With that notation, we get the following condition for unionization:

$$\text{Firm is unionized} \Leftrightarrow V(g) > 0. \tag{1.17}$$

which simply states that a firm is unionized if a majority of its workers vote for it.

Notice that even though the preferences are random, the outcome of the vote is fully deterministic. Therefore, at the moment of posting vacancies, the firm knows whether the workers will form a union or not. In fact, the firm is deciding to be unionized or not. Notice also that, as the curvature parameter ρ goes to infinity, the outcome of the vote is decided by the median voter.[14]

[13] The CDF of ϵ is $P(\epsilon < x) = 1/(1+\exp(-\rho x))$.

[14] I use random preferences mainly for numerical purposes. The gradient methods used for the optimization perform much better this way.

The wage equations derived in the last section provide information on what types of workers vote in favor of unionization. Indeed, notice that the union wage (equation 1.9) is a function of the average characteristics of the workforce while the nonunion wage (equation 1.13) is a function of the individual characteristics of a worker. In particular, the union wage depends on the average production $F(g)/n$ while the nonunion wage is a function of the marginal product of each worker

$$\frac{\alpha z(s)}{g(s)} F(g).$$

This implies that a worker with valuable characteristics, for instance a high marginal product, would rather bargain individually with the firm than to share his advantage with the other employees.

1.2.7 Steady state equilibrium

In a steady-state equilibrium, the flows in and out of unemployment in all sub-market s need to be equal:

$$[N(s) - u(s)]\delta = u(s)\theta(s)q(\theta(s)). \tag{1.18}$$

Using the fact that, in the steady state, a firm j posts vacancies $v(s) = \delta g_j(s)/q(\theta(s))$, it is possible to rewrite the last condition as

$$\frac{N(s)\theta(s)q(\theta(s))}{\delta + \theta(s)q(\theta(s))} = \sum_{j=1}^{j_{\max}} g_j(s, \theta). \tag{1.19}$$

Definition 1. *A* stationary competitive equilibrium *in this economy is a reservation wage schedule $b(s)$, a labor market tightness schedule $\theta(s)$, a set of workers distributions $\{g_j\}_{j=1}^{j_{\max}}$ and a set of wage schedules $\{w_j\}_{j=1}^{j_{\max}}$ such that,*

1. *g_j solves the optimization problem of firm j,*

2. *w_j solves the collective bargaining problem (equation 1.9) if firm j is unionized or solves the individual bargaining problem (equation 1.13) if firm j is not unionized,*
3. *$b(s)$ satisfies equation 1.2,*
4. *unemployment is stationary in each labor market: equation 1.19 is satisfied,*
5. *the union status of each firm is consistent with equation 1.17.*

The full general equilibrium in which firms are constrained by the unionization vote cannot be solved analytically. The numerical algorithm that I use in this paper is detailed in appendix 1.6.[15]

1.3 Firm's behavior

Now that we have derived the wage schedules $w_n(s, g)$ and $w_u(s, g)$, we can go back to the firm's problem. Fix a firm with a given technology. In what follows, I omit the index j and I use the subscript $i = \{u, n\}$ to denote this firm in a union or nonunion situation respectively. Remember that at the steady state, the problem of a firm is given by equation 1.5:

$$\max_g \pi_i(g) - \gamma\kappa(1 - (1 - \delta)) \int \frac{g}{q(\theta)}\, ds \tag{1.20}$$

where π_i is given by equation 1.15 if the firm is not unionized (condition 1.17 does not holds) or by equation 1.11 if the firm is unionized (condition 1.17 holds). Notice that when a firm hires its workers, it knows whether a given distribution g will lead to a unionized firm or not. In particular, the firm knows the profit it will get.[16]

[15] The question of the uniqueness of the equilibrium is unresolved analytically. Numerically, the equilibrium always seems unique. See Appendix 1.6 for more details.

[16] In both the union and the nonunion cases, the current period profit π_i is a strictly concave function of g. It is not however clear whether the unionization constraint defines a convex set or not.

Consider a firm that hires its workers at the beginning of a new period, i.e. a firm in the first stage of the sequence of events shown in Figure 1.2. Because of the decreasing returns to scale, the firm generally prefers to be union free. In such a case, profits are given by $\pi_n(g)$ and denote by

$$g_n^*(s) = \underset{g}{\text{argmax}} \ \pi_n(g) - \gamma\kappa(1-(1-\delta)) \int \frac{g}{q(\theta)}\, ds$$

the distribution of workers that blindly maximizes the firms discounted profit in that situation. Given this distribution, the workers can react in two ways. Either, $V(g_n^*) \leq 0$ and they reject the union. Then, $g_n^*(s)$ is actually the solution of the firm's problem. Or, $V(g_n^*) > 0$ and the workers form a union. Such a firm is *constrained* by the unionization vote and, in this case, $g_n^*(s)$ is not a solution to the firm's problem.

A constrained firm can try to fight the union. To do so, it distorts the distribution g_n^* in the least costly way possible to avoid unionization. Denote that distorted distribution by g_n. Therefore,

$$\begin{aligned} g_n = &\underset{g}{\text{argmax}} \ \pi_n(g) - \gamma\kappa(1-(1-\delta)) \int \frac{g}{q(\theta)}\, ds \\ &\text{subject to: } V(g) \leq 0. \end{aligned}$$

Obviously, by imposing the voting constraint on the firm's problem, the discounted profit of the firm goes down. The question is how far down? In particular, if the constraint is important enough, the profit of the firm is higher with the optimal union distribution of workers g_u^*. Such a firm will therefore be unionized. Notice that the firm is *rationally* choosing to be unionized. It is an optimal reaction to the threat imposed by the union.

In the next section, I first consider the firm's behavior when the unionization constraint is not binding and then move to the full constrained problem as a deviation from this case. Solving the full distorted problem of the firm must be done numerically.

1.3.1 Behavior of an unconstrained firm

I consider in this section the decision of a firm j that is not constrained by the unionization vote. In other words, suppose a firm can decide on the outcome of the unionization vote. How would it behave? First, the firm compares the profit it would make in the union and in the nonunion cases and then picks the case providing the highest profit.

The goal of this exercise is to understand the decision process of the firm. I therefore assume that the equilibrium is fixed: $b(s)$, $\theta(s)$ and $c_j(s)$ are fixed.

By combining equations 1.11 and 1.15, we can write the problem of a firm that is not constrained by the unionization vote as:

$$\max_g \Gamma_i F_j(g) - (1-\beta_i) \int c_j \cdot g \, ds - (1-\gamma(1-\delta)(1-\beta_i))\kappa \int \frac{g}{q(\theta)} \, ds$$

where

$$\Gamma_i \equiv \begin{cases} 1-\beta_u & \text{if } i = u \\ \dfrac{1-\beta_n}{1-(1-\alpha)\beta_n} & \text{if } i = n \end{cases}$$

is the share of output retained by the firm. By defining,

$$\mathrm{MC}_i^j(s) \equiv (1-\beta_i)c_j(s) + (1-\gamma(1-\delta)(1-\beta_i))\frac{\kappa}{q(\theta(s))} \tag{1.21}$$

as the marginal cost paid by a firm j to hire a worker s, the firm's optimal hiring decision, g_i^* is given by

$$\mathrm{MC}_i^j(s) = \Gamma_i \frac{\alpha F(g_i^*) z(s)}{g_i^*(s)}. \tag{1.22}$$

Notice that the firm has a different hiring strategy whether it expects its workers to unionized or not. The right-hand side of this last equation is simply the marginal cost of hiring an extra worker of type s, which includes the wage paid to the worker, while the left-hand side is the share of the marginal product of the worker that the firm retains. Notice that MC_i^j depends on the firm's equilibrium wage, on aggregate variables and on the union status of the firm. From equation 1.22, we see that workers who are rare ($\theta(s)$ high) or who have attractive outside options ($b(s)$ high) are expensive to hire ($\mathrm{MC}_i^j(s)$ high) and the firm therefore relies less on them for production ($g_i^*(s)$ small). The equilibrium wage schedule w_j also affects the marginal cost through c_j: a worker who knows he will get a high wage in the next period has more to lose if the bargaining breaks down.

It is straightforward to compare the discounted profit of the firm in both the union and nonunion scenarios:

Lemma 4. *An unconstrained firm j prefers to be union free if and only if*

$$\log\left(\frac{\Gamma_n}{\Gamma_u}\right) \geq \alpha \int z(s) \log\left(\frac{\mathrm{MC}_n^j(s)}{\mathrm{MC}_u^j(s)}\right) ds.$$

The term on the left hand side of this equation is a measure of the relative hiring costs in both the union and nonunion scenarios. The right hand side is a measure of the relative share of output that the firm retains. If $\beta_n = \beta_u$, $\mathrm{MC}_n = \mathrm{MC}_u$ and this condition is automatically satisfied. Also, the firm prefers to be union free when the union is very strong ($\beta_u \to 1$) and it would gladly welcome a union if individual

workers have a strong bargaining power ($\beta_n \to 1$), as the intuition would predict. As we will see, this condition holds for all firms in the calibrated economy.

We now focus on wages and on how workers vote.

Lemma 5. *Assume that the labor market tightness schedule $\theta(s)$ and the outside option schedule $b(s)$ are increasing functions of the skill and that the bargaining powers β_n and β_u are equal. Then, in an unconstrained firm hiring according to g_i^* for $i = \{u, n\}$, the nonunion wage schedule $w_n(s, g_i^*)$ is an increasing function of s.*

The intuition for this lemma is straightforward. The firm hires until the marginal product of a worker is equal to his marginal cost. Under the lemma's assumption, the marginal cost is increasing in s and the result follows since nonunion wages depend directly on the marginal products. This lemma states that for all firms (those that are unionized and non unionized in equilibrium), the nonunion wage they would pay if they hire according to the optimal distribution g_u^* and g_n^* is an increasing function of s.

The following lemma characterizes the vote of the workers:

Lemma 6. *Assume that the labor market tightness schedule $\theta(s)$ and $c_j(s)$ are increasing functions of the skill.*[17] *Under the optimal hiring decision of unconstrained firms g_i^* for $i = \{u, n\}$, the union wage gap $w_n(s, g_i^*) - w_u(s, g_i^*)$ is increasing with s.*

Once again the intuition is straightforward. Since the marginal cost is increasing with skill, the nonunion wage of high-skill workers is higher then the one of low-skill workers. Instead, in a union firm, wages are determined by the average marginal product. This generates a union wage gap that is an increasing function of s.

Lemma 6 is consistent with the findings from Farber and Saks (1980) that the desire to be unionized goes down with the position in the intrafirm earnings distribution. It is also consistent with the large empirical literature suggesting that a union compresses the wage distribution of a firm.

[17] The two schedules θ and c_j are increasing in the calibrated model.

Finally, it is possible to compare the variance of union and nonunion wages in unconstrained firms:

Lemma 7. *Assume that the labor market tightness schedule $\theta(s)$ and $c_j(s)$ are increasing functions of the skill and that the bargaining powers β_n and β_u are equal. Then, for a given firm j,*

$$\mathrm{Var}_{g_n^*}\left(w_n(s, g_n^*)\right) \geq \mathrm{Var}_{g_u^*}\left(w_u(s, g_u^*)\right)$$

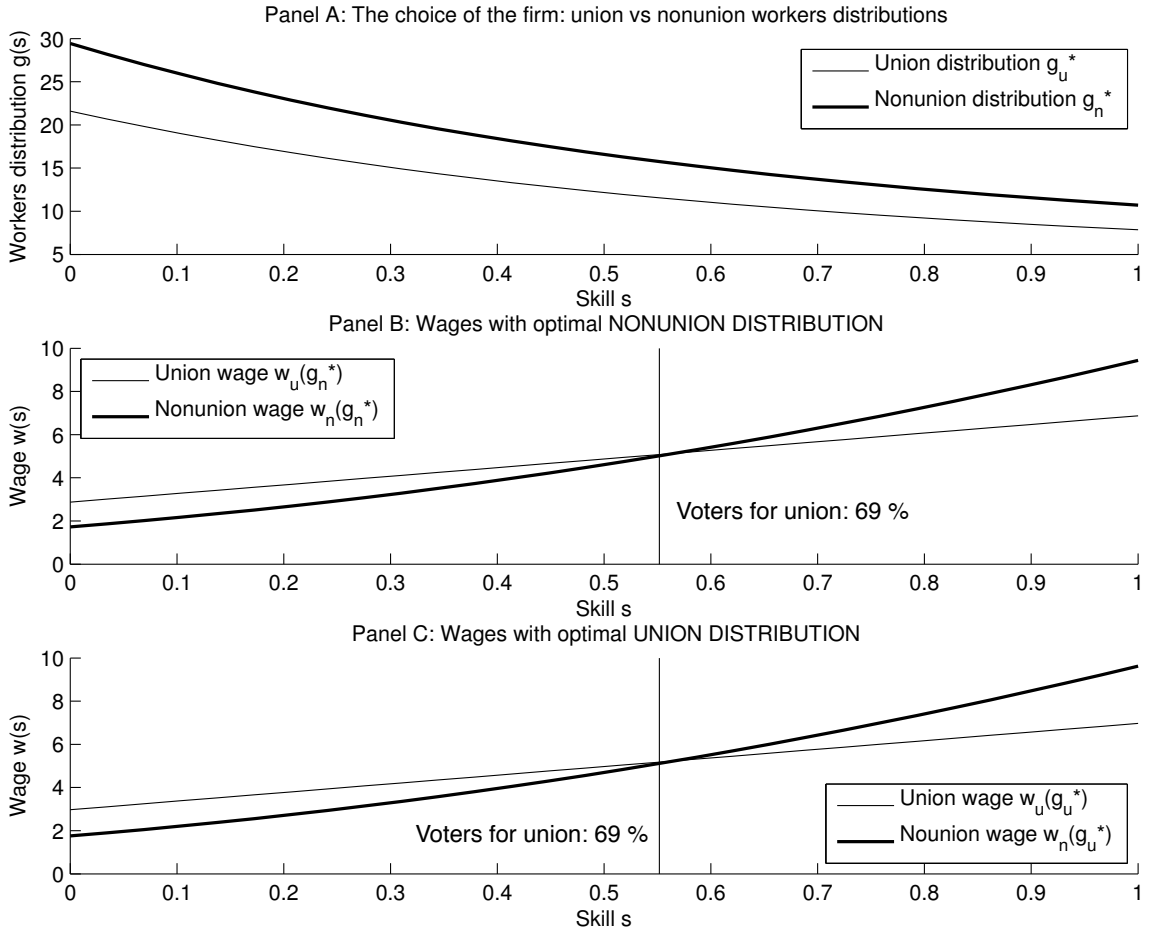
where $\mathrm{Var}_g(x)$ is the traditional variance operator taken with the normalized distribution $g/\int g\,ds$.

This lemma characterizes the variance of wages in firms evolving in a policy environment in which unions are mandatory (unconstrained union) or illegal (unconstrained nonunion).

Figure 1.3 shows the decision process of a firm. The parameters that generate this example are picked to emphasize the various mechanisms. Their magnitudes are not realistic but the mechanisms are the ones present in the calibrated model. Panel A presents the two optimal distributions g_n^* and g_u^*. Panel B and C show the wages that voters are considering when they cast their votes. The vertical lines show the position of the worker who is indifferent between a union and a nonunion firm. There is only one single skill for which this is true. As the lemmas predicted, the wages and the union wage gap are all increasing function of s. Notice that because of this increasing wage gap, the unionization of a firm directly lowers the variance of wages.

In this example, if the firm hires according to g_n^* or g_u^*, the workers will vote in favor of a union. Therefore, g_n^* does not solve the problem of the firm. Notice also that g_n^* is simply a rescaled version of g_u^*. This comes directly from equation 1.22. Also, $g_n^*(s) > g_u^*(s)$ for all $s \in [0, 1]$. This is a direct consequence of the nonunion bargaining. Since the firm bargains individually with each worker over the marginal

surplus, it increases the number of workers to lower this marginal product. This effect was described in Stole and Zwiebel (1996a,b).



Notes: The vertical lines represent the position of the indifferent voter. Parameters of the economy: $\delta = 0.025$, $\gamma = 0.995$, $\beta_n = 3/10$, $\beta_u = 3/10$, $c_j(s) = 1 + 4s$, $\theta(s) = 1 + 9s^2$. Firm characteristics: $A = 10$, $\alpha = 0.8$, $z(s) = (s + 1)/2$.

Figure 1.3: The hiring decision of an unconstrained firm

1.3.2 Fighting the union

Because of the additional profit they generate by bargaining individually, firms generally prefer to be union free. The workers, however, have a strong incentive to form a union: by bargaining collectively, they extract a bigger share of the match surplus. In this section, we consider the case of a firm that is constrained by the unionization vote: $V(g_n^*) \geq 0$.

When a firm is constrained by the vote on unionization, the first order condition given by equation 1.22 is modified to include the impact of the additional worker on the outcome of the vote:

$$\mathrm{MC}_n^j(s) = \Gamma_n \frac{\alpha F(g_n) z(s)}{g_n(s)} - \lambda_n \frac{\partial V(g_n)}{\partial g_n(s)} \tag{1.23}$$

where $\lambda_n > 0$ is the Lagrange multiplier of the voting constraint and where g_n is the optimal distribution for which workers actually reject the union.

When distorting the unconstrained distribution g_n^* the firm takes into consideration three mechanisms:

1. **Fraction of voters for union** By adding more workers at the top of distribution, or by removing workers at the bottom of the distribution, the firm directly lowers the fraction of workers in favor of union.

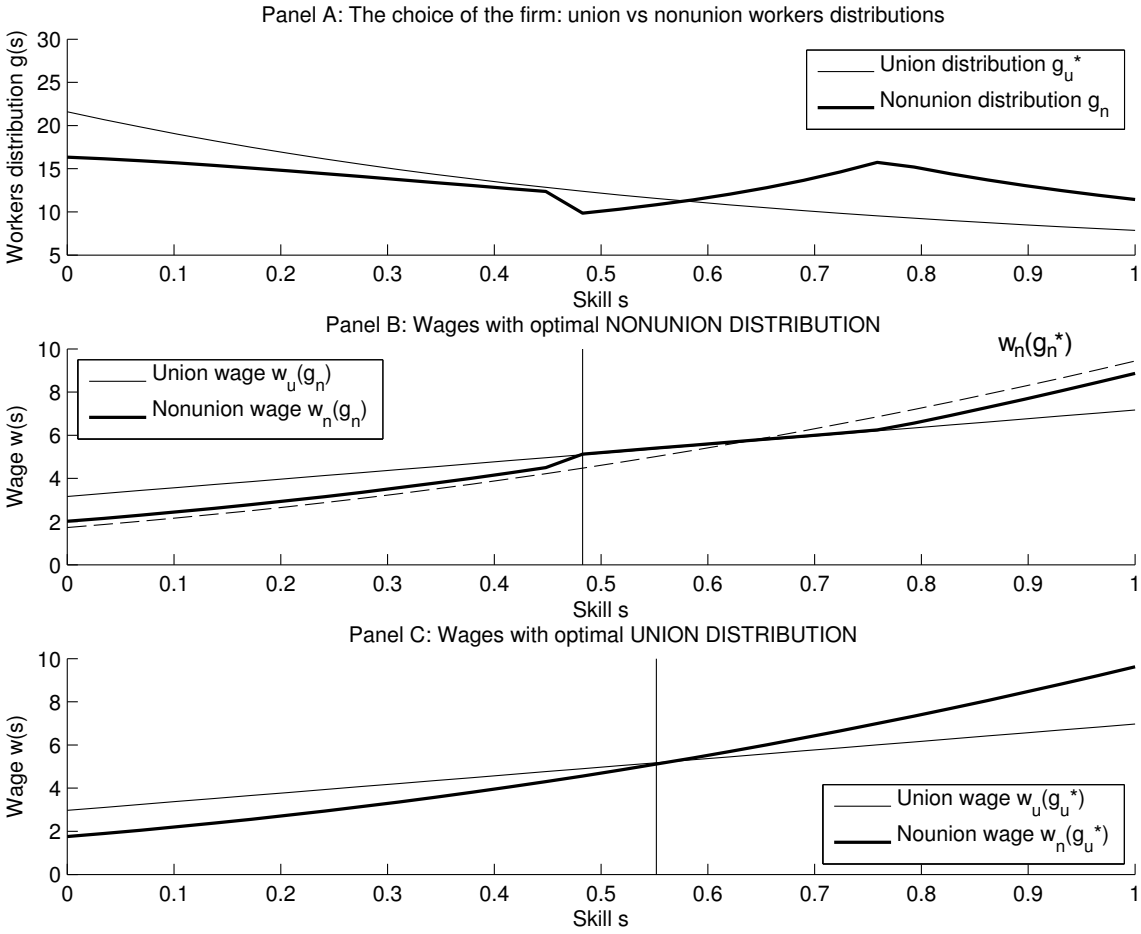
2. **Effect on nonunion wages** By increasing the number of workers of a given skill s the firm lowers the marginal product of these workers which, in turn, lowers their nonunion wage. If the firm increases the number of workers who vote against the union, it needs to make sure that their nonunion wage stays higher than their union wage. Otherwise, these workers will change their vote.

3. **Effect on union wages** While nonunion wages are determined by the marginal products, union wages are determined by the average product. By increasing the number of high-skill workers, for instance, the firm increases the number of high marginal product workers which shifts the union wage upwards. This could make some workers change their vote in favor of unionization. This effect, through the union wages, implies that when the firm wants to increase the numbers of workers against the union it will first do so with the workers of the *lowest skill possible*.

Figure 1.4 shows the decision process of the constrained firm that was represented on figure 1.3. The firm considers the profit it makes in two scenarios: optimal distribution under which the workers will unionize and optimal distribution under which the workers will reject the union. These are featured on Panel A by the thin and thick line respectively. Panel B shows the wages that voters are considering if the firm imposes g_n. The dashed line represents the nonunion wages when there was no unionization constraint (it is the same curve as the thick line of the Panel B of figure 1.3). Panel C shows the union and nonunion wages the workers are voting on when the distribution is g_u^*. Notice that the firm does not have to distort g_u^* here. The workers gladly form a union. In Panel B and C, the vertical lines represent the indifferent voter. On Panel B, the fraction of workers against unionization is 50%.

We can see on Panel A that the distorted distribution g_n has a lower number of low-skill workers and a higher number of high-skill workers then the distribution g_n^* from figure 1.3. The effect of this distortion on wages is clear by looking at Panel B. The constraint lowers the nonunion wage of high-skill workers and increases the wages of low skill workers. By reacting to the fact that the workers *can* unionize, the firm compresses the range of wages it is paying its workers. Notice that this threat effect could be present in an economy in which no firms are unionized but in which the legal system allows the workers to create unions.

We can see on Panel B that the union and nonunion wages of the workers with skill between 0.48 and 0.76 are almost identical. For this range of skills, the behavior of the firm is clearly constrained. The firm would like to hire more of these workers: they vote against the union and their relatively small marginal product has a smaller effect on the union wage schedule than the workers with higher skill. However, if the firm were to hire an additional worker s in this zone, all the workers of type s would change their vote in favor of unionization.



Notes: The vertical lines represent the position of the indifferent voter. Parameters of the economy: $\delta = 0.025$, $\gamma = 0.995$, $\beta_n = 3/10$, $\beta_u = 3/10$, $c_j(s) = 1 + 4s$, $\theta(s) = 1 + 9s^2$. Firm characteristics: $A = 10$, $\alpha = 0.8$, $z(s) = (s+1)/2$.

Figure 1.4: The hiring decision of the firm facing a unionization threat

Table 1.1 compares different characteristics of the firm under the three following scenarios:

1. Unions are mandatory
2. Unions are legal
3. Unions are illegal

First, notice that the firm's profit is highest when unions are illegal and that, when unions are legal, the firm still manages to find a nonunion distribution with higher profit then in the union case. This firm is therefore union free in scenario 2.

Second, the fraction of voters in favor of a union is the same in scenario 1 and 3. In scenario 2, the firm pushes workers to vote against the union until it reaches 50%. Third, the mean of wages is the lowest in scenario 3. This comes from the differences in the bargaining structure. When bargaining individually, the firm is able to retain a higher fraction of the joint surplus. Making unions legal leads to an increase in the mean of wages. Fourth, the variance of wages is the highest when unions are illegal. Allowing the presence of unions brings down the variance. This is the wage compression effect of the unionization threat. Finally, the variance of wage is lowest in scenario 1. The differences in the bargaining structures are such that a union firm has a lower variance than nonunion firms.

Notice finally that, in this example, the unionization threat lowers the *range* of nonunion wages when compared to an unconstrained firm. However, the impact of the threat on the *variance* of wages needs to also take into account the changes in the distribution of workers. In general, the threat lowers the variance but this might be reversed in firms that are extremely constrained by the vote. In these firms, the number of high-skill workers needs to be increased so much that w_n is almost equal to w_u for all the workers rejecting the union. The firm therefore tries to shift the w_u schedule downward by adding to its workforce a large number of workers with very low-skill. This leads to a U-shaped distribution of workers in these firms.

	1. Union mandatory	2. Union legal	3. Union illegal
Union status of the firm	Union	Nonunion	Nonunion
Firm discounted profit	2179	2397	2969
Fraction of voters for union	69%	50%	69%
Mean of wages	4.50	4.88	4.18
Variance of wages	1.37	3.69	4.76

Table 1.1: The behavior of a firm under the three scenarios.

1.3.3 Impact of technology on unionization

Remember that, with equal bargaining powers, a firm always prefers to be union free and that this preference is independent of its technology. Technology has, however, a strong influence on the vote of the workers and, through that channel, on the union status of the firm. The following lemma characterizes how the returns to scale α affect the workers vote.

Lemma 8. *For an unconstrained firm employing a distribution of workers g_n^* (given by equation 1.22):*

$$\frac{d(w_n(s, g_n^*) - w_u(s, g_n^*))}{d\alpha} = \frac{\beta_u}{\alpha^2 \int z(s)/\mathrm{MC}_n^j(s) ds} > 0. \tag{1.24}$$

Also, the fraction of voters in favor of a union, $V(g_n^*)$, is such that

$$\frac{dV(g_n^*)}{d\alpha} = -\frac{\beta_u}{\alpha^2 \left(\int z/\mathrm{MC}_n^j \, ds\right)^2} \int \frac{z}{\mathrm{MC}_n^j} \frac{\exp\{-\rho(w_u - w_n)\}}{(1 + \exp\{-\rho(w_u - w_n)\})^2} \, ds < 0$$

Increasing α increases the gap between w_n and w_u uniformly across skills. All else equal, workers in a firm with a low return to labor α tend to have a bigger advantage to be unionized. The second part of the lemma implies that, as α gets bigger, the share of workers in favor of forming a union goes down. Also, since the gap in wages is smaller, it is easier for the firm to convince the workers to vote against unionization by distorting the distribution of workers. This preference of the workers is consistent with the findings of Hirsch and Berger (1984) that industries that are more capital intensive have a higher share of union workers.

Figure 1.5 shows the firm's decision as a function of its technology. Panel A presents the contour curves of the ratio of nonunion (constrained) profit to union profits as a function of α and $z(s)$. The distributions $z(s)$ used to draw this picture are shown on Panel B, so that a small beta distribution parameter (on the horizontal

axis of Panel A) indicates a distribution z that is skewed to the left (for instance, $d = -0.5$ on panel B). It is clear from this figure that the ratio of profits is an increasing function of the return to scale α. This result holds true in the calibrated economy.

The impact of the skill intensity $z(s)$ on the union status of the firm is less obvious. Two effects are competing. The first one relates to the number of voters, the second one to the average marginal product of the workers and therefore to the union wage schedule. Consider a firm with a skill intensity z highly skewed towards low skill workers (for instance, $d = -0.5$ on panel B of figure 1.5). In such a firm, the median voter has a lower skill than the average voter, which tends to push the firm towards unionization. However, since most workers have low marginal product, the average marginal product is small and so is the union wage that workers would get in case of unionization. These two effects compete with each other. A firm that has a production technology skewed towards high-skill workers has to deal with the exact same two effects: many high-skill workers tend to vote against unionization but their high marginal products pushes union wages up. By looking at figure 1.5, it appears that the voting effect dominates for most technologies. In other words, by moving the median of $z(s)$ to the right, it gets easier and easier for the firm to fight unionization. However, when the median passes a certain point, the wage effect dominates. Union wages get so high that compensating for them becomes harder and harder. The ratio of profits therefore goes down.

Finally, the following lemma shows that the parameter A of the production function has no influence on the union status of a firm or on the wages it pays.

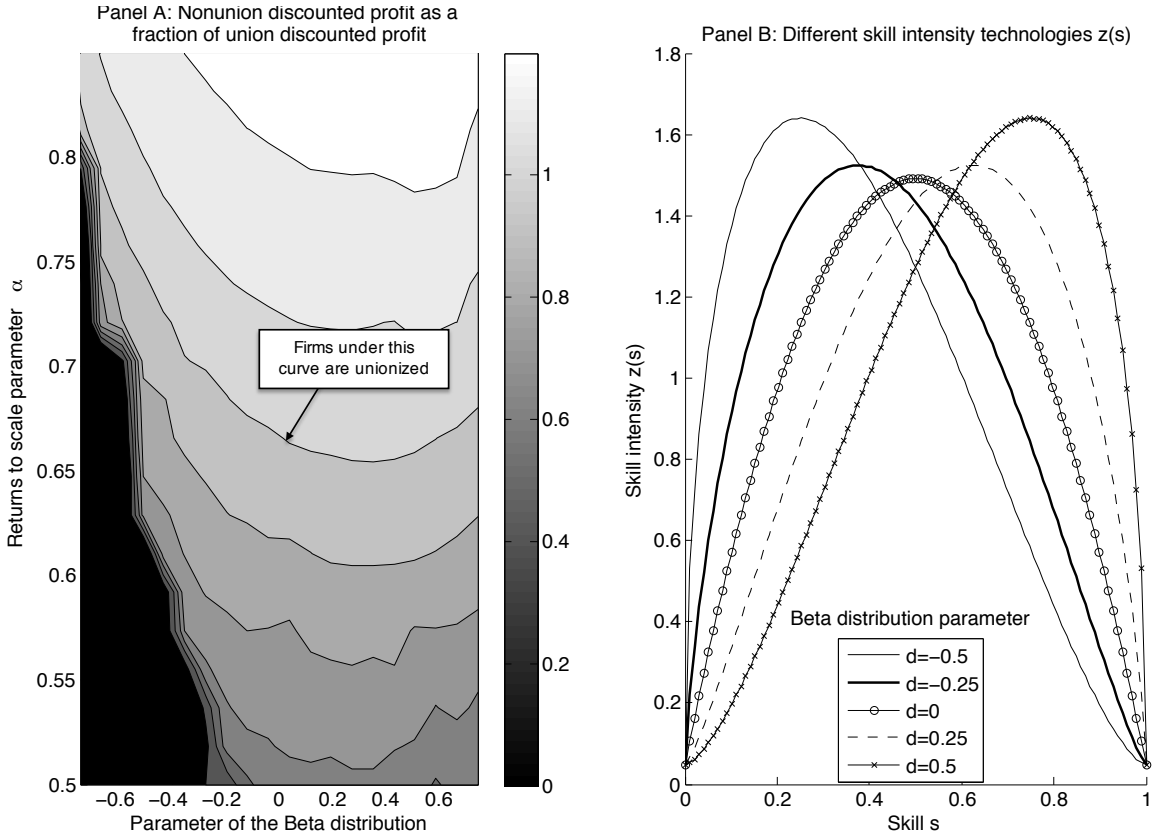
Lemma 9. *Consider two firms, identified by the subscripts 1 and 2, that have identical technologies except for $A_1 \neq A_2$. There exist an equilibrium such that: if g_1 solves the*

problem of firm 1, then

$$g_2 = \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} g_1$$

solves the problem of firm 2. Also, both firms have the same union status and pay the same wages.

This lemma will also be useful in aggregating firms of the same type in the calibration.



Notes: The left panel shows the ratio of constrained nonunion profit to union profit. The lines are contour lines showing where the ratio crosses specific thresholds. The mapping of technology to profits ratio is continuous. The vertical axis is labor share α. The horizontal axis represents different function $z(s)$ taken from the Beta distributions shown in Panel B. The distributions are Beta$(2-d, 2+d)+\epsilon$ where d is the parameter on the horizontal axis and $\epsilon > 0$ is a small number to prevent the distribution from reaching 0 at $s = 0$ and $s = 1$. The parameters of the economy: $\delta = 0.025$, $\gamma = 0.995$, $\beta_n = 3/10$, $\beta_u = 3/10$, $c_j(s) = 1 + 4s$, $\theta(s) = 1 + 9s^2$.

Figure 1.5: The profits of a firm as a function of its production technology

1.4 Conclusion

Empirical estimators of the effects of unions on inequality generally abstract from the decision process of the firms and from general equilibrium mechanisms. In particular, they neglect the possible consequences that the unionization threat exerts on firms. This threat is created by the legal system and therefore may be present even in economies with low union membership.

This paper proposes a general equilibrium theory of firms' decisions and union formation to study the impact of unions on the economy. Workers and firms meet in a labor market characterized by frictions. Each period, the workers of a firm vote to create a union. If a union is created, wages are bargained collectively. Otherwise, each worker bargains his wage individually with the firm. This asymmetry of wage setting mechanisms causes unions to compress the wage distribution inside a firm. Furthermore, by fighting the threat of unionization, firms distort their hiring decisions in a way that also compresses wages.

1.5 Appendix

Here are the proofs from the previous sections.

1.5.1 Proof of lemma 1

Lemma 1. *In a steady-state, the firm's dynamic problem can be written as the static optimization:*

$$\max_{g} \pi(g) - \kappa(1-(1-\delta)\gamma) \int \frac{g}{q(\theta)}\, ds.$$

Proof. At a steady-state, the firm's problem is given by equation 1.4, which we can rewrite

$$J\left(\int \frac{g_{-1}}{q(\theta)}ds\right) = (1-\delta)\kappa \int \frac{g_{-1}}{q(\theta)}\,ds + \max_g \left\{\pi(g) - \kappa \int \frac{g}{q(\theta)}ds + \gamma J\left(\int \frac{g}{q(\theta)}ds\right)\right\}.$$

The term that is maximized is constant with respect to g_{-1}. Denote that constant by B. Then, in particular

$$J\left(\int \frac{g}{q(\theta)}ds\right) = (1-\delta)\kappa \int \frac{g}{q(\theta)}\,ds + B.$$

The firm therefore solves

$$\max_g \pi(g) - \kappa \int \frac{g}{q(\theta)}ds + \gamma\left((1-\delta)\kappa \int \frac{g}{q(\theta)}\,ds + B\right)$$

and the result follows. □

1.5.2 Proof of lemma 2

Lemma 2. *Assume that g is strictly positive on $[0,1]$. Then the following function solves the bargaining problem:*

$$w_u(s,g) - c(s) = \frac{\beta_u}{n}\left(F(g) - \int c\cdot g\,ds + \gamma(1-\delta)\kappa \int \frac{g}{q(\theta)}\,ds\right).$$

The solution is unique if the joint surplus of the match is strictly positive at the point w_u.

Also, in a union firm with equilibrium distribution of workers g^* and technology j, the equilibrium wage schedule $w_j(s) = w_u(s,g^*)$ is

$$w_u(s,g^*) - b(s) = \frac{1-\gamma(1-\delta)}{1-\beta_u\gamma(1-\delta)}\frac{\beta_u}{n^*}\left(F(g^*) - \int b\cdot g^*\,ds + \gamma(1-\delta)\kappa \int \frac{g^*}{q(\theta)}\,ds\right).$$

where $n^* = \int g^* ds$ is the optimal size of the firm.

Proof. To keep a light notation, define

$$\Gamma \equiv F(g) + (1-\delta)\kappa\gamma \int \frac{g}{q(\theta)}\, ds \geq 0.$$

I work in the Lebesgue space $L^2[0,1]$. Two functions are identical if the measure of the set on which they differ is zero. By taking the log of the bargaining problem, we can define the objective function $P(w)$ as

$$P(w) = \beta_u \int \frac{g}{n} \log\left(w - c(s)\right)\, ds + (1-\beta_u) \log\left(\Gamma - \int w \cdot g\, ds\right)$$

and write the collective bargaining problem as

$$\max_w P(w) \tag{1.25}$$

Define the set of admissible functions

$$M = \left\{ w \in L^2[0,1] : w(s) - c(s) \geq 0 \,\forall\, s \in [0,1],\, \Gamma - \int w \cdot g\, ds \geq 0 \right\}.$$

M is the set of wage schedules w which might be agreed upon. For a wage schedule outside of M, some workers are better off unemployed or the firm will have negative surplus.

I first prove four preliminary results that characterize the set M and the function P.

Result 1. *The set of admissible functions M is convex.*

Proof. If M is a singleton then it is convex. If not, take any $w_1, w_2 \in M$ and consider the convex combination $w_a = aw_1 + (1-a)w_2$ with $0 \leq a \leq 1$. Then $w_a(s) \geq c(s)$

for all $s \in [0,1]$ and $\Gamma - \int w_a \cdot g\, ds \geq 0$. Since w_1 and w_2 are in $L^2[0,1]$, w_a is also in $L^2[0,1]$ and therefore M is convex. □

Result 2. *The function P is strictly concave on M.*

Proof. Take any $w_1, w_2 \in M$, $w_1 \neq w_2$ and consider the convex combination $w_a = aw_1 + (1-a)w_2$ with $0 < a < 1$. Since logarithm is a strictly concave function and $g > 0$, we can write

$$\begin{aligned} P(w_a) &= \beta_u \int \frac{g}{n} \log(w_a - c)\, ds + (1-\beta_u) \log\left(\Gamma - \int w_a \cdot g\, ds\right) \\ &> \beta_u \int a\frac{g}{n} \log(w_1 - c)\, ds + (1-\beta_u) a \log\left(\Gamma - \int w_1 \cdot g\, ds\right) \\ &+ \beta_u \int (1-a)\frac{g}{n} \log(w_2 - c)\, ds + (1-\beta_u)(1-a) \log\left(\Gamma - \int w_2 \cdot g\, ds\right) \\ &= aP(w_1) + (1-a)P(w_2). \end{aligned}$$

So P is strictly concave on M.[18] □

Result 3. *The wage function*

$$w_u(s,g) - c(s) = \frac{\beta_u}{n}\left(F(g) - \int c \cdot g\, ds + \gamma(1-\delta)\kappa \int \frac{g}{q(\theta)}\, ds\right)$$

is such that the functional derivative of P at point w_u, $\delta P[w_u]/\delta h$, is zero for every test function h and therefore w_u is a stationary point of P.

Proof. It is straightforward to show that

$$\begin{aligned} \int \delta P(w) \cdot h\, ds &= \left(\frac{d}{d\epsilon} P(w + \epsilon h)\right)\Big|_{\epsilon=0} \\ &= \int g \cdot h \left(\frac{\beta_u}{n(w-c)} - \frac{1-\beta_u}{\Gamma - \int w \cdot g\, ds}\right) ds \end{aligned}$$

[18] It is important here that the measure of the set on which the two functions w_1 and w_2 are different is bigger than zero.

Since $g > 0$ by assumption, the last equation is equal to zero for all h if and only if $w = w_u$. The idea is simply to see how P would vary around a point w if it is distorted in the direction h. If w is an optimum, the change in P should be 0 in all directions. □

Result 4. *If the joint surplus is strictly positive at w_u, then w_u is an interior point of M.*

Proof. The assumption on joint surplus is

$$F(g) - \int c \cdot g \, ds + (1 - \delta)\kappa\gamma \int \frac{g}{q(\theta)} \, ds > 0.$$

This implies that $w_u(s) > c(s)$ for all $s \in [0, 1]$. Furthermore, a simple calculation shows that the firm's surplus is equal to the fraction $(1 - \beta_u)$ of the joint surplus. Therefore,

$$\Gamma - \int w_u \cdot g \, ds > 0$$

and w_u is in the interior of M. □

Putting the pieces together, P is a strictly concave function on the convex set M of the Hilbert space $L^2[0, 1]$. It has the stationary point w_u and w_u is in the interior of M. Therefore, w_u is the unique global maximum of the function P.[19]

The equilibrium wage schedule follows directly by setting $w_u(s, g^*) = w_j$ and using the definition of c given by equation 1.3. □

[19] See Ok (2007) and Luenberger (1997) for an exposition of calculus in a functional space.

1.5.3 Proof of lemma 3

Lemma 3. *The wage schedule*

$$w_n(s,g) - c(s) = \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{g(s)}F(g) - \beta_n\ c(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))}$$

solves the bargaining problem (equation 1.12) of a firm employing the distribution of workers g.

Also, in a nonunion firm with equilibrium distribution of workers g^* and technology j, the equilibrium wage schedule $w_j(s) = w_n(s,g^*)$ is

$$w_n(s,g^*) - b(s) = \frac{1-\gamma(1-\delta)}{1-\beta_n\gamma(1-\delta)}\left(\frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{g(s)}F(g) - \beta_n\ b(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))}\right).$$

Proof. The Stole and Zwiebel (1996a,b) solution to the bargaining problem is the wage function that gives the worker a share β_n of the joint surplus. I discretize the number of skills (each skill has a size ϵ) and the number of agents of a given skill (each agent has a size h). I start by writing the surplus of the firm and of the worker. After some manipulation and using the definition of the bargaining solution, taking the limits has $\epsilon, h \to 0$ will yield equation 1.12.

In the discretized framework, the production function is

$$F(L) = A\,L^{\alpha} = A\exp\left\{\alpha\sum_i \epsilon z_i \log(g_i)\right\}$$

where g_i is the number of workers of type i multiplied by the size of one worker, h.

The bargaining takes place when all vacancies have been posted. When bargaining with a worker, the firm compares two scenarios. Either an agreement is reached, in which case production takes place has planned, or the negotiations break down and

the firm produces without this individual worker. In this last case, the worker departs from the firm and additional vacancies will have to be posted in the next period for the firm to go back to its optimal distribution of workers. In equilibrium, an agreement is always reached. The marginal discounted profit from hiring a worker of type j is

$$\begin{aligned}\Delta_j^n =& F(L) - \sum_i \epsilon w_i(\dots, g_j, \dots) g_i - \left(A \exp\left\{ \alpha \sum_{i \neq j} \epsilon z_i \log(g_i) + \alpha \epsilon z_j \log(g_j - h) \right\} \right. \\ & \left. - \sum_{i \neq j} \epsilon w_i(\dots, g_j - h, \dots) g_i - w_j(\dots, g_j - h, \dots)(g_j - h)\epsilon - h\epsilon\gamma(1-\delta)\frac{\kappa}{q(\theta_j)} \right)\end{aligned}$$

where the notation $w_i(\dots, g_j, \dots)$ denotes the fact that w_i is a function of the whole distribution g. Δ_j^n is simply the difference between current period profits in the case of an agreement and in the case in which negotiations break down. Notice that in the latter case, the firm faces additional hiring costs in the next period. After using a Taylor's expansion on $\log(g_j - h)$ and rearranging, we get

$$\begin{aligned}\Delta_j^n =& A \exp\left(\alpha\epsilon \sum_i z_i \log(g_i) \right) \left(1 - \exp\left\{ -\epsilon\alpha \frac{z_j}{g_j} h + \epsilon\alpha z_j O(h^2) \right\} \right) \\ & - \left(\sum_i \epsilon w_i(\dots, g_j, \dots) g_i - \sum_i \epsilon w_i(\dots, g_j - h, \dots) g_i \right) \\ & - h\epsilon w_j(\dots, g_j - h, \dots) + h\epsilon\gamma(1-\delta)\frac{\kappa}{q(\theta_j)}.\end{aligned}$$

The solution to the Stole and Zwiebel bargaining is the wage function that solves

$$\frac{\beta_n}{1-\beta_n}\Delta_j^n = (W_e(s,w) - W_u(s))\epsilon h$$

where the right hand side is the worker's surplus. By dividing Δ_j^n by h and taking the limit $h \to 0$, we get

$$\lim_{h \to 0} \frac{\Delta_j^n}{h} = \alpha\epsilon \frac{z_j}{g_j} F(L) - \sum_i \epsilon g_i \frac{\partial w_i(\dots, g_j, \dots)}{\partial g_j} - \epsilon w_j(\dots, g_j, \dots) + \epsilon\gamma(1-\delta)\frac{\kappa}{q(\theta_j)}.$$

Because of the symmetry of the production function, the marginal product of a worker j depends only on g_j and on L. Its dependence on the whole distribution g is only through L. We can therefore impose more structure on the wage function and write $w_i(g_i, L)$ instead of $w_i(\dots, g_j, \dots)$. Therefore,

$$\lim_{h\to 0} \frac{\Delta_j^n}{h} = \alpha\epsilon \frac{z_j}{g_j} F(L) - \sum_i \epsilon g_i \frac{\partial w_i(g_i, L)}{\partial L} \frac{\partial L}{\partial g_j} - \epsilon g_j \frac{\partial w_j(g_j, L)}{\partial g_j} - \epsilon w_j(g_j, L) + \epsilon\gamma(1-\delta)\frac{\kappa}{q(\theta_j)}.$$

By dividing the previous expression by ϵ and taking the limit $\epsilon \to 0$, we get

$$\lim_{h,\epsilon\to 0} \frac{\Delta_j^n}{h\epsilon} = \frac{\partial L}{\partial g(s)} \left(\frac{dF}{dL} - \int \frac{\partial w(s, g(s), L)}{\partial L} g(s)\, ds \right) - \frac{\partial w(s, g(s), L)}{\partial g(s)} g(s) - w(s, g(s), L) + \gamma(1-\delta)\frac{\kappa}{q(\theta(s))}.$$

where $\frac{\partial L}{\partial g(s)}$ is a short notation for $z(s)/g(s) \cdot L(g)$. Therefore, the wage function is the solution to the following partial differential equation:

$$\frac{\partial L}{\partial g(s)} \left(\frac{dF}{dL} - \int \frac{\partial w(s, g(s), L)}{\partial L} g(s)\, ds \right) - \frac{\partial w(s, g(s), L)}{\partial g(s)} g(s) - w(s, g(s), L) + \gamma(1-\delta)\frac{\kappa}{q(\theta(s))} = \frac{1-\beta_n}{\beta_n} \left(w(s, g(s), L) - c_j(s) \right)$$

It is straightforward to show that w_n solves this equation. □

1.5.4 Proof of lemma 4

Lemma 4. *An unconstrained firm prefers to be union free if and only if*

$$\log\left(\frac{\Gamma_n}{\Gamma_u}\right) \geq \alpha \int z(s) \log\left(\frac{\mathrm{MC}_n^j(s)}{\mathrm{MC}_u^j(s)}\right) ds.$$

Proof. It is straightforward to show that, using the optimal hiring decision g_i^*, the production of a firm is given by

$$F(g_i^*) = A^{\frac{1}{1-\alpha}} (\alpha\Gamma_i)^{\frac{\alpha}{1-\alpha}} \exp\left\{ \frac{\alpha}{1-\alpha} \int z(s) \log\left(\frac{z(s)}{\mathrm{MC}_i(s)} \right) ds \right\}.$$

The proof follows directly from writing equation 1.5 with the optimal union and nonunion distributions (given by equation 1.22) and by simplifying the inequality. □

1.5.5 Proof of lemma 5

Lemma 5. *Assume that the labor market tightness schedule $\theta(s)$ and the outside option schedule $b(s)$ are increasing functions of the skill and that the bargaining powers β_n and β_u are equal. Then, in an unconstrained firm hiring according to g_i^* for $i = \{u, n\}$, the nonunion wage schedule $w_n(s, g_i^*)$ is an increasing function of s.*

Proof. Consider a firm j that is not unionized in equilibrium. Then by using equation 1.14 and the definition of MC_n^j, we find that the equilibrium wage paid by that firm is

$$w_n(s, g_n^*) = b(s) + \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta(s))} (1 - \gamma(1-\delta)).$$

□

It is then straightforward to show that

$$c_j(s) = \frac{b(s) - \gamma(1-\delta) w_n(s, g_n^*)}{1 - \gamma(1-\delta)} = b - \gamma(1-\delta) \frac{\beta}{1-\beta} \frac{\kappa}{q(\theta(s))}.$$

Then

$$\mathrm{MC}_u^j(s) = \mathrm{MC}_n^j(s) = (1-\beta) b(s) + \frac{\kappa}{q(\theta(s))} (1 - (1-\delta)\gamma)$$

is an increasing function of s. Given the shape of c_j, the wage function can be written as

$$w_n(s, g_i^*) = \frac{\beta}{1-(1-\alpha)\beta}\frac{\mathrm{MC}_i^j(s)}{\Gamma_i} + (1-\beta)b(s)$$

and since $\mathrm{MC}_i^j(s)$ and $b(s)$ are increasing, so is $w_n(s, g_i^*)$.

We now need to show the result for a firm j that is unionized in equilibrium. Equation 1.10 shows that $w_u(s, g_u^*) - b(s)$ is equal to a constant that is independent of s. Denote that constant D; then $w_u(s, g_u^*) - b(s) = D$. Therefore,

$$c_j(s) = \frac{b(s) - \gamma(1-\delta)w_u(s, g_u^*)}{1-\gamma(1-\delta)} = b(s) - \frac{\gamma(1-\delta)D}{1-\gamma(1-\delta)}$$

and we see that c_j is increasing in s. This directly implies that MC_i^j is increasing. We can write the nonunion wage as

$$w_n(s, g_i^*) = \frac{\beta}{1-(1-\alpha)\beta}\frac{\mathrm{MC}_i^j(s)}{\Gamma_i} + (1-\beta)c_j(s) + \beta\gamma(1-\delta)\frac{\kappa}{q(\theta(s))}$$

and since c_j, MC_i^j and θ are increasing, so is $w_n(s, g_i^*)$. We have shown that all firms (those that are unionized and non unionized in equilibrium) pay increasing nonunion wages when they hire according to their optimal distribution g_u^* and g_n^*.

1.5.6 Proof of lemma 6

Lemma 6. *Assume that the labor market tightness schedule $\theta(s)$ and $c_j(s)$ are increasing functions of the skill. Under the optimal hiring decision of unconstrained firms g_i^*, the union wage gap $w_n(s, g_i^*) - w_u(s, g_i^*)$ is increasing with s.*

Proof. With simple algebra, we find that

$$w_n(s, g_n^*) - w_u(s, g_n^*) = \frac{\kappa\beta_n}{q(\theta(s))(1-\beta_n)} - \frac{\beta_u}{\int z(s)/\mathrm{MC}_n^j(s)ds} \times \int \left(\frac{\mathrm{MC}_n^j(s)}{\alpha\Gamma_n} - c_j(s) + (1-\delta)\kappa\gamma\frac{1}{q(\theta(s))} \right) \frac{z(s)}{\mathrm{MC}_n^j(s)}ds$$

such that the variation in union wage premium across skill is coming exclusively from the labor market tightness. If $\theta(s)$ is increasing, the workers with the lowest skill are the ones who are the most likely to vote against unionization in non unionized firms.

Similarly, in a union firm:

$$\begin{aligned} w_n(s, g_u^*) - w_u(s, g_u^*) = {} & \frac{c_j(s)\beta_n^2(1-\alpha)}{1-(1-\alpha)\beta_n} + \frac{\beta_n}{1-(1-\alpha)\beta_n}\left(\frac{1-\gamma(1-\delta)(1-\beta_u)}{1-\beta_u}\frac{\kappa}{q(\theta(s))} \right) \\ & + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))} - \frac{\beta_u}{\int z(s)/\mathrm{MC}_u^j(s)ds} \\ & \times \int \left(\frac{\mathrm{MC}_u^j(s)}{\alpha(1-\beta_u)} - c_j(s) + (1-\delta)\kappa\gamma\frac{1}{q(\theta(s))} \right) \frac{z(s)}{\mathrm{MC}_u^j(s)}ds. \end{aligned}$$

and since c_j is increasing, the result follows. □

1.5.7 Proof of lemma 7

Lemma 7. *Assume that the labor market tightness schedule $\theta(s)$ and $c_j(s)$ are increasing functions of the skill and that the bargaining powers β_n and β_u are equal. Then, for a given firm j,*

$$\mathrm{Var}_{g_n^*}\left(w_n(s, g_n^*)\right) \geq \mathrm{Var}_{g_u^*}\left(w_u(s, g_u^*)\right)$$

where $\mathrm{Var}_g(x)$ is the traditional variance operator taken with the normalized distribution $g/\int g\,ds$.

Proof. With equal bargaining powers and under the optimal hiring decisions given by equation 1.22, the union and nonunion wages are given by

$$w_n(s, g_n^*) = c_j(s) + \frac{\beta}{1-\beta}\frac{\kappa}{q(\theta(s))}$$
$$w_u(s, g_u^*) = c_j(s) + \text{Constant}.$$

Therefore the only source of variability in union wages comes from $c_j(s)$ while the nonunion wage schedule has an additional term coming from the labor market tightness. Under equal bargaining powers, $\text{MC}_u^j(s) = \text{MC}_n^j(s)$ for all s and therefore g_u^* is equal to g_n^* multiplied by a constant. The normalized distribution are therefore identical. We find,

$$\begin{aligned}
\text{Var}_{g_u^*}\left(w_u(s, g_u^*)\right) &= \text{Var}_{g_u^*}\left(c_j(s)\right) = \text{Var}_{g_n^*}\left(c_j(s)\right) \\
\text{Var}_{g_n^*}\left(w_n(s, g_n^*)\right) &= \text{Var}_{g_n^*}\left(c_j(s) + \frac{\beta}{1-\beta}\frac{\kappa}{q(\theta(s))}\right) \\
&= \text{Var}_{g_n^*}\left(c_j(s)\right) + \text{Var}_{g_n^*}\left(\frac{\beta}{1-\beta}\frac{\kappa}{q(\theta(s))}\right) \\
&\quad + 2 \times \text{Cov}_{g_n^*}\left(c_j(s), \frac{\beta}{1-\beta}\frac{\kappa}{q(\theta(s))}\right)
\end{aligned}$$

Since c_j and θ are increasing, the result follows.[20] □

1.5.8 Proof of lemma 8

Lemma 8. *For an unconstrained firm employing a distribution of workers g_n^* (given by equation 1.22):*

$$\frac{d(w_n(s, g_n^*) - w_u(s, g_n^*))}{d\alpha} = \frac{\beta_u}{\alpha^2 \int z(s)/\text{MC}_n^j(s) ds} > 0.$$

[20] See Schmidt (2003) for a proof that the covariance of two increasing functions of a random variable is positive.

Also, the fraction of voters in favor of a union, $V(g_n^*)$, is such that

$$\frac{dV(g_n^*)}{d\alpha} = -\frac{\beta_u}{\alpha^2\left(\int z/\mathrm{MC}_n^j\, ds\right)^2}\int \frac{z}{\mathrm{MC}_n^j}\frac{\exp\{-\rho(w_u - w_n)\}}{\left(1+\exp\{-\rho(w_u-w_n)\}\right)^2}\, ds < 0$$

Proof. With simple algebra, we find that

$$\begin{aligned} w_n(s, g_n^*) - w_u(s, g_n^*) = &\frac{\kappa\beta_n}{q(\theta(s))(1-\beta_n)} - \frac{\beta_u}{\int z(s)/\mathrm{MC}_n^j(s) ds} \\ &\times \int\left(\frac{\mathrm{MC}_n^j(s)}{\alpha\Gamma_n} - c_j(s) + (1-\delta)\kappa\gamma\frac{1}{q(\theta(s))}\right)\frac{z(s)}{\mathrm{MC}_n^j(s)} ds \end{aligned}$$

The first result of the lemma follows directly by taking the derivative of this last equation with respect to α. The second result comes as a consequence of the first result and the definition of $V(g)$ given by equation 1.16. □

1.5.9 Proof of lemma 9

Lemma 9. *Consider two firms, identified by the subscripts 1 and 2, that have identical technologies except for $A_1 \neq A_2$. There exist an equilibrium such that: if g_1 solves the problem of firm 1, then*

$$g_2 = \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} g_1$$

solves the problem of firm 2. Also, both firms have the same union status and pay the same wages.

Proof. Assume first that the equilibrium schedules c_1 and c_2 are identical and denote that schedule by c. I will show this result later in the lemma.

The optimal distribution of workers for firm 2, g_2, is

$$g_2 = \arg\max_g F_2(g) - \int w_2(g)\cdot g\, ds - \gamma\kappa(1-(1-\delta))\int \frac{g}{q(\theta)}\, ds$$

where w_2 is given by

$$w_{n2}(s,g) = \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{g(s)}F_2(g) + (1-\beta_n)c(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))}$$

if the firm is union free and by

$$w_{u2}(s,g) = c(s) + \frac{\beta_u}{n}\left(F_2(g) - \int c\cdot g\,ds + \gamma(1-\delta)\kappa\int\frac{g}{q(\theta)}\,ds\right)$$

otherwise.

The strategy for the proof is to rewrite firm 2's problem as a transformation of firm 1's problem. To do so, rewrite the problem of firm 2 by using the following transformation:

$$g = \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\tilde{g}.$$

Using this notation, the objective function of firm 2 is

$$F_2(g) - \int w_2(s,g)\cdot g\,ds - \gamma\kappa(1-(1-\delta))\int\frac{g}{q(\theta)}\,ds \tag{1.26}$$

$$\begin{aligned}
&= \frac{A_2}{A_1}F_1\left(\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\tilde{g}\right) - \int w_2(s,g)\cdot\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\tilde{g}\,ds\\
&- \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\gamma\kappa(1-(1-\delta))\int\frac{\tilde{g}}{q(\theta)}\,ds\\
&= \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\left(F_1(\tilde{g}) - \int w_2\left(s,\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\tilde{g}\right)\cdot\tilde{g}\,ds - \gamma\kappa(1-(1-\delta))\int\frac{\tilde{g}}{q(\theta)}\,ds\right)
\end{aligned} \tag{1.27}$$

where F_1 and A_1 identify the technology of firm 1. Notice that it is equivalent for firm 2 to pick g to maximize the expression on line 1.26 or to pick $\tilde{g}$ to maximize the expression on line 1.27.

Now, for nonunion wages in firm 2

$$\begin{aligned}
w_{n2}(s,g) &= \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{g(s)}F_2(g) + (1-\beta_n)c(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))} \\
&= \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{(A_2/A_1)^{\frac{1}{1-\alpha}}\,\tilde{g}}\frac{A_2}{A_1}F_1\left((A_2/A_1)^{\frac{1}{1-\alpha}}\,\tilde{g}\right) + (1-\beta_n)c(s) \\
&+ \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))} \\
&= \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\alpha z(s)}{\tilde{g}(s)}F_1(g_1) + (1-\beta_n)c(s) + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta(s))} \\
&= w_{n1}(s,\tilde{g})
\end{aligned}$$

where the last equality denotes the fact that firm 2 using g and firm 1 using $\tilde{g}$ would be paying the same wages if they were union free. Similarly, for union wages in firm 2:

$$\begin{aligned}
w_{u2}(s,g) &= c(s) + \frac{\beta_u}{n}\left(F_2(g) - \int c\cdot g\,ds + \gamma(1-\delta)\kappa\int\frac{g}{q(\theta)}\,ds\right) \\
&= c(s) + \frac{\beta_u}{n}\left(\frac{A_2}{A_1}F_1\left(\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\tilde{g}\right) - \int c\cdot g\,ds + \gamma(1-\delta)\kappa\int\frac{g}{q(\theta)}\,ds\right) \\
&= c(s) + \frac{\beta_u}{\tilde{n}}\left(F_1(\tilde{g}) - \int c\cdot\tilde{g}\,ds + \gamma(1-\delta)\kappa\int\frac{\tilde{g}}{q(\theta)}\,ds\right) \\
&= w_{u1}(s,\tilde{g})
\end{aligned}$$

where $\tilde{n} = \int \tilde{g}\,ds$. Again, the last equality denotes the fact that firm 2 using g and firm 1 using $\tilde{g}$ would be paying the same wages if they are unionized.

We can now rewrite the problem of firm 2:

$$\begin{aligned}
\left(\frac{A_2}{A_1}\right)^{\left(-\frac{1}{1-\alpha}\right)} g_2 = \arg\max_{\tilde{g}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\Bigg(F_1(\tilde{g}) - \int w_2\left(s,\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}\tilde{g}\right)\cdot\tilde{g}\,ds \\
-\gamma\kappa(1-(1-\delta))\int\frac{\tilde{g}}{q(\theta)}\,ds\Bigg)
\end{aligned}$$

where w_2 is given by

$$w_{n2}\left(s, \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} \tilde{g}\right) = \frac{\beta_n}{1-(1-\alpha)\beta_n} \frac{\alpha z(s)}{\tilde{g}(s)} F_1(\tilde{g}) + (1-\beta_n)c(s) + \beta_n \gamma(1-\delta)\frac{\kappa}{q(\theta(s))}$$

if the firm is union free and by

$$w_{u2}\left(s, \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} \tilde{g}\right) = c(s) + \frac{\beta_u}{\tilde{n}}\left(F_1(\tilde{g}) - \int c \cdot \tilde{g}\, ds + \gamma(1-\delta)\kappa \int \frac{\tilde{g}}{q(\theta)}\, ds\right)$$

otherwise.

Because of the link between the wage schedules we can transform the unionization condition for firm 2 such that:

$$\begin{aligned}
&\int \frac{g}{1+\exp(-\rho(w_{u2}(s,g) - w_{n2}(s,g)))}\, ds - \frac{1}{2}n \\
&= \int \frac{g}{1+\exp(-\rho(w_{u1}(s,\tilde{g}) - w_{n1}(s,\tilde{g})))}\, ds - \frac{1}{2}n \\
&= \int \frac{\tilde{g}}{1+\exp(-\rho(w_{u1}(s,\tilde{g}) - w_{n1}(s,\tilde{g})))}\, ds - \frac{1}{2}\tilde{n}.
\end{aligned}$$

We have rewritten the problem of firm 2 as a transformation of the problem of firm 1. Since $\tilde{g} = g_1$ solves the problem of firm 1 by assumption, we get that

$$\left(\frac{A_2}{A_1}\right)^{\left(-\frac{1}{1-\alpha}\right)} g_2 = g_1$$

which is the desired result. Notice that since the two firms have the same union status and are paying the same wages, we find $c_1(s) = c_2(s)$ for every s. □

1.6 Solving the general equilibrium

Here is the algorithm I use to find a general equilibrium of the economy.

Given the parameters of the economy $(\beta_n, \beta_u, \delta, \gamma, \kappa, \rho, \mu, b_0(s), N(s))$ as well as the firms technology $(z_j(s), A_j, \alpha_j)$ for $j \in \{1, \ldots j_{max}\}$, we need to find the aggregate variables $\theta(s)$ and $b(s)$ that sustain this equilibrium.

The algorithm to solve for an equilibrium uses the following strategy:

1. Fix the union status of each type of firm (either union or nonunion).
   (a) Make an initial guess on the aggregate variables: $\theta^0(s)$, $b^0(s)$.
   (b) Given the guess, compute the decision of each firm according to its union/nonunion status.
   (c) From wages and the distribution of hired workers, compute the new $\theta^{i+1}(s)$, then use it to compute the new $b^{i+1}(s)$.
   (d) Measure the distance between (θ^{i+1}, b^{i+1}) and (θ^i, b^i).[21] If the distance is smaller than some criterion $\epsilon > 0$ we found an equilibrium candidate. If not, go back to step 1b using $\theta^{i+1}(s)$ and $b^{i+1}(s)$ as the current guess.
2. Once we have an equilibrium candidate, we need to make sure that firms do not want to deviate from the union status they were assigned at step 1. If no firm wants to deviate, we have an equilibrium. Otherwise, fix the firm union status differently and repeat the steps.

[21] The distance I use is the integral of the square of the differences between the two functions.

Chapter 2

The Union Threat - Quantitative exploration

2.1 Introduction

In this chapter, I calibrate the model of union formation on the private sector of the United States and do an empirical exercise to see how a partial equilibrium estimator would measure the effects of unions on wage inequality. To do so, I give to each union worker the counterfactual wage that he would get if he were working in a nonunion job. This is a procedure that has been used in the empirical literature. This partial equilibrium exercise suggests that, in the calibrated economy, the reallocation of union workers to nonunion jobs would increase the variance of log wages by about 0.4%. I then perform two general equilibrium exercises to evaluate the full impact of unions on the economy. The first one consists in removing the threat of unionization. In other words, nonunion firms do not have to worry about the vote on the formation of a union anymore. Firms that are unionized remain unionized and vice versa. In the new equilibrium, the variance of log wages goes up by 5.7% when compared to the calibrated model. This shows that the threat of unionization alone might have

an important impact on inequality. Welfare also goes up by 1.7%, suggesting that the firms' departure from their optimal hiring decision has a negative impact on the economy as a whole.[1] The second exercise is to eliminate unions completely. All wages are then negotiated on a one-on-one basis with the firms. In this scenario, the variance of log wages goes up by 6% with respect to the calibrated economy. Total production goes up by 2.5% and welfare also increases by 2.2%. The unemployment rate goes down by 2.5 percentage points.

These results suggest that, even with low membership, unions seem to have an impact on wage inequality, output and unemployment through the threat they exert and through general equilibrium mechanisms. Also, this paper shows that partial equilibrium estimates are likely to miss important channels through which unions influence the economy.

2.2 Data and calibration

I calibrate the model on the private sector of the United States in 2005. One period is one month and the unit of all monetary amounts is one thousand dollars. I set the monthly discount rate to $\gamma = 0.996$ and the probability of job destruction to $\delta = 0.027$. For the matching function, I follow Hagedorn and Manovskii (2008) and use $q(\theta) = (1 + \theta^{\mu})^{-1/\mu}$. To estimate μ, I use data from the Job Opening and Labor Turnover Survey (JOLTS) for 2005 together with the probability of job finding from Shimer (2007).[2] The estimate for μ is 1.33. I set the scale parameter of the random

[1] I define welfare as the sum of the utilities of all the agents in the economy.

[2] I take the yearly average of the job finding probability for 2005, its value is 0.4136. Robert Shimer constructed this data. For additional details, please see Shimer (2007) and his webpage. I find the aggregate labor market tightness by using the vacancies and job searchers numbers provided by the JOLTS. I take the average over all months in 2005 of the seasonally adjusted non-farm job opening series together with the seasonally adjusted number of job searchers for part-time and full-time jobs. I find μ by solving $0.4136^{\mu} = \theta^{\mu}(1 + \theta^{\mu})^{-1}$.

preferences for unionization to $\rho = 20$.[3] This is a strong curvature that brings the firm close to the median voter case. Table 2.1 summarizes the parameters.

Parameter	Definition	Value	Source/reason
γ	Discount factor	0.996	5% annual interest rate
δ	Probability of job destruction	0.027	Shimer (2007)
μ	Parameter of the matching function	1.33	JOLTS with Shimer (2007)
ρ	Parameter of preference for union	20	Strong curvature

Table 2.1: Parameters taken directly from the data or the literature

In what follows, all data about individuals is coming from the Merged Outgoing Rotation Groups of the Current Population Survey (CPS) as it is made available by the National Bureau of Economic Research (NBER). Industry data comes from the Bureau of Economic Analysis (BEA).

Calibration strategy

For simplicity, I assume that firms are endowed by two types of technology. In equilibrium, one type of firm is unionized while the other is not. Lemma 9 shows that it is equivalent to change the number of firms of a certain type or the parameter A_j of these firms' technology. We can therefore normalize the number of firms of each type to one and change A_j to adjust their size. I denote the technologies of union and nonunion firms by (A_u, α_u, z_u) and (A_n, α_n, z_n) respectively. The CPS provides information on the union status of the workers as well as the industry they are working in. By using this data together with the BEA data on industry, I set α_u and α_n to match the labor shares. The values of the returns to scale parameters are $\alpha_u = 0.5$ and $\alpha_n = 0.6$.[4]

[3]The bigger ρ is, the closer we get to the median voter scenario. However, with ρ too high, the numerical algorithm struggles.

[4]I pick the α's by hand to have less degrees of freedom in the loss function. Once the model is fully calibrated I compare the calibrated labor shares to the empirical ones. The biggest difference is of 3%.

In what follows, the adjective *empirical* (for instance, the empirical union wage distribution) designates a variable taken directly from the data. These variables are denoted by the superscript *emp*. On the other hand, a *calibrated* variable designates that variable as generated by the calibrated model. The superscript *cal* designates them.

The first step of the calibration is to define a skill index. Then, I use the CPS data to construct the labor market tightness $\theta(s)$ and an the outside option schedule $b(s)$. I then minimize a loss function to find the remaining parameters of the model.

Skill distribution

The skill index is only, well, an index. Throughout this model it is used to characterize the heterogeneity of the agents and to identify variables that are related to them ($\theta(s), b(s)$, N(s), etc.). Nowhere is s appearing alone; s does not mean anything by itself. I first define a skill index from the data and then, when minimizing the loss function, identify the firms' technology using equations from the model. This way, the skill index and the firms technology are consistently determined to make the model match the wage schedules and the distributions of workers.

I use data from the CPS to build the skill index. To do so, I run a regression of the log of normalized monthly *nonunion* wages on two types of variables. The first type includes variables related to each individual. The second type of variables depend on the industry in which the individual works. I then use the predicted variable given by the OLS estimator of the individual characteristics alone as the skill index. Explicitly, denote by w_i the log monthly wage of agent i, who is working in industry $j(i)$. The regression is

$$w_i = \Gamma X_{1,i} + \Psi X_{2,j(i)} + \epsilon_i.$$

and the skill index is therefore given by the predicted values $\hat{s}_i = \hat{\Gamma} X_{1,i}$. The individual characteristics X_1 are *sex*, *age*, *race*, *education* and *occupation* (set of dummy variables). The job related characteristics X_2 are *industry* (dummy variables) and the current *US state* in which the agent lives.[5] I drop from the sample individuals with skill index below the first percentile and above the 99th percentile. I then scale the index using a linear transformation such that s is between 0 and 1. Notice that even though the regression is run only on nonunion workers, the predicted values $\hat{s}_i$ are computed for *all* members of the labor force. Figure 2.1 shows the distribution of $\hat{s}_i$ for the whole sample.

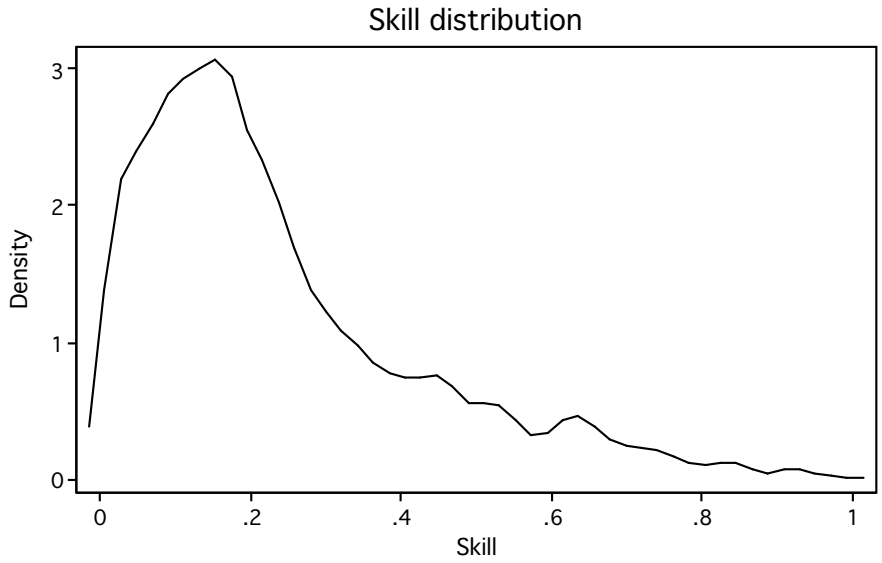


Figure 2.1: Skill distribution

This way of defining the skill distribution has the advantage of making the empirical wages and the empirical labor market tightness increasing with s. This makes the interpretation of the impact of unionization on different workers more intuitive.

[5]Including *US state* as an individual characteristic instead has minimal impact on the distribution. For *industry* and *occupation*, I use the variables generated by the NBER. Both are at the 3-digit level. I clean the sample by removing agricultural workers and individuals with hourly wage higher than 150$ or lower than 5$. I also remove individuals younger than 20 or older than 65 years old.

Labor market tightness and value of outside option

In the United States, unemployment insurance programs are administered by the states. Krueger and Meyer (2002) provides the main characteristics of benefits for some US states in 2000. The replacement ratio is about 50% in every state but the maximum weekly benefits vary considerably. In the model, the variable b_0 also takes into account home production and the value of the extra leisure provided by unemployment, two elements that are harder to quantify. Hall and Milgrom (2008) uses an estimate of the flow value of non-work that is essentially equivalent to a replacement ratio of 71%. I therefore calibrate $b_0(s)$ to be 71% of the mean wage earned by workers of skill s.[6]

It is straightforward to identify the empirical value of some of the aggregate variables. I split the support of the skill distribution in 20 bins of equal sizes and use equation 1.18 together with the observed unemployment rates by bin to compute the labor market tightness θ for workers in each of these bins.[7] Using the mean wages of union and nonunion workers together with the fact that firms hire a fraction δ of their workforce every period, I compute the expected wage of a worker who just found a job. I then use equation 1.2 to compute the outside option b for each of the skill bins. Similarly, by summing the number of agents in each bin, I compute the empirical skill distribution $N^{\text{emp}}(s)$. I also compute $b_0^{\text{emp}}(s)$ in each of the bins.

[6]This estimate takes into consideration the value of the extra leisure associated with unemployment. One might however suspect that the replacement ratio changes with skills. The unemployment insurance programs tend to be more generous with agents earning low incomes. To evaluate the effect of this possible bias, I used data from the Uniform Extracts of the U.S. Census Survey of Income and Program Participation (SIPP) and looked at the unemployment benefits of individuals going from employment to unemployment. The SIPP also provides information on wages and it is therefore possible to build a measure of the replacement ratio as a function of the wage. The fitted replacement ratio of an average worker earning a monthly wage of 1000$ and moving to unemployment is 65%. The ratio then decreases quadratically to reach about 10% for monthly wages of 8000 $. Because high-skill workers stay unemployed for a very short time, their reservation wage is basically the same as when b_0 is taken to be 71% of the mean wage. In Hall and Milgrom (2008), 71% corresponds to the ratio of the value of unemployment on productivity.

[7]One point of the θ schedule departs strongly from the trend. I therefore use a moving average to smooth it.

Loss function

I pick κ, β_n, β_u to minimize the following loss function:

$$\begin{aligned}\text{Loss} = &\int \left(N^{\text{cal}} - N^{\text{emp}}\right)^2 ds + \lambda_b \int \left(b_0^{\text{cal}} - b_0^{\text{emp}}\right)^2 ds \\ &+ \lambda_n \left(\text{LS}_n^{\text{cal}} - \text{LS}_n^{\text{emp}}\right)^2 + \lambda_u \left(\text{LS}_u^{\text{cal}} - \text{LS}_u^{\text{emp}}\right)^2\end{aligned}$$

where LS_n and LS_u are the labor share[8] in the nonunion and union firms respectively and where the λ's are weighting constants picked such that the terms of the loss function have similar magnitudes.

For any κ, β_n, β_u, I use equations 1.22 and 1.13 to identify the firms technology. By using the empirical variables, these equations are

$$\begin{aligned}g_u^{\text{emp}}(s) &= \frac{\alpha_u(1-\beta_u)F_u(g_u^{\text{emp}})z_u(s)}{(1-\beta_u)c_u(g_u^{\text{emp}}, s) + \frac{\kappa}{q(\theta(s))}(1-\gamma(1-\beta_u)(1-\delta))} \\ w_n^{\text{emp}}(s) - b(s) &= \frac{1-\gamma(1-\delta)}{1-\beta_n\gamma(1-\delta)} \\ &\quad \times \left(\frac{\beta_n}{1-(1-\alpha_n)\beta_n}\frac{\alpha_n z_n(s)}{g_n^{\text{emp}}(s)}F_n(g_n^{\text{emp}}) - \beta_n b(s) + (1-\delta)\frac{\beta_n\kappa\gamma}{q(\theta(s))}\right)\end{aligned}$$

where

$$\begin{aligned}c_u(g_u^{\text{emp}}, s) = b(s) &- \frac{\beta_u\gamma(1-\delta)}{1-\beta_u\gamma(1-\delta)}\frac{1}{n^{\text{emp}}} \\ &\times \left(F_u(g_u^{\text{emp}}) - \int b \cdot g_u^{\text{emp}}\, ds + \gamma(1-\delta)\kappa \int \frac{g_u^{\text{emp}}}{q(\theta)}\, ds\right).\end{aligned}$$

I use a fixed point algorithm to find the technologies (A_u, z_u) and (A_n, z_n) from these equations (remember that the integral of z is normalized to one and that F depends

[8]The empirical labor shares are computed by dividing total workers' compensation by value added using the data provided by the Bureau of Economic Analysis. The calibrated labor shares are computed by dividing the wage bill by production minus hiring costs. I also calibrated the model using a loss function without the labor share terms. The calibrated parameters have different values but the policy exercises give very similar results.

on $z(s)$ and A). Notice that by defining (A_u, z_u) this way, the model replicates the distribution of workers in union firms perfectly. That is, $g_u^{\text{emp}} = g_u^{\text{cal}}$.

The idea behind the calibration is straightforward. For any vector of parameters $(\kappa, \beta_n, \beta_u)$, I identify the firms' technology using the model and I can compute the equilibrium. The schedules N^{cal} and b_0^{cal} are those that support this equilibrium. I pick $(\kappa, \beta_n, \beta_u)$ to make N^{cal} and b_0^{cal}, as well as the labor shares, as close as possible to their empirical counterparts.[9] This amounts to calibrating the wage schedules (through b_0) and the distributions of workers (through N).

Table 2.2 shows the parameter values that minimize the loss function.

β_n	β_u	κ
0.29	0.06	0.14

Notes: The units of κ is thousand dollars per month.

Table 2.2: The value of the calibrated parameters

Notice that $\beta_n > \beta_u$. This difference in bargaining powers is necessary to compensate the fact that the decreasing returns provide the unionized workers with more leverage in the negotiations. In the calibrated model, workers always prefer to form a union and the firms need to fight to prevent unionization. Figure 2.2 shows the calibrated technologies $z(\cdot)$ of the firms. We see that nonunion firms are more intensive in high-skill workers. This comes from the fact that, in the data, the distribution of nonunion workers has a fatter tail than the one of union workers.

Figure 2.3 shows how the model fits the distributions of workers and the wage schedules. We can see that the model fits the union workers distribution perfectly. This is a direct consequence of the way the technology of the union firms is identified. The model also fits the nonunion wage schedule and nonunion distribution of workers

[9] I also calibrated the model using the inverse strategy: fix b_0 and $N(s)$ to their observed values and find the vector $(\kappa, \beta_n, \beta_u)$ that makes θ^{cal} and b^{cal}, as well as the labor shares, as close as possible to their empirical counterpart. With this other calibration, the fit is a bit worse and the effects of unions on the economy are very similar qualitatively and quantitatively. This other approach is however more computationally intensive, which limits the size of the skill grid.

quite well. The fit of the union wage schedule is however less precise. This comes from the heavy structure imposed on union wages by equation 1.9. In particular, the shape of w_u is tightly linked to the shape of $b(s)$. Union wages in the calibrated economy are more unequal than in the data. Suggesting that the real equalizing effect of unions might be stronger than the one captured by the calibration. A better fit could be obtained by allowing different workers to have different bargaining powers when they divide what the union extracted from the firm.

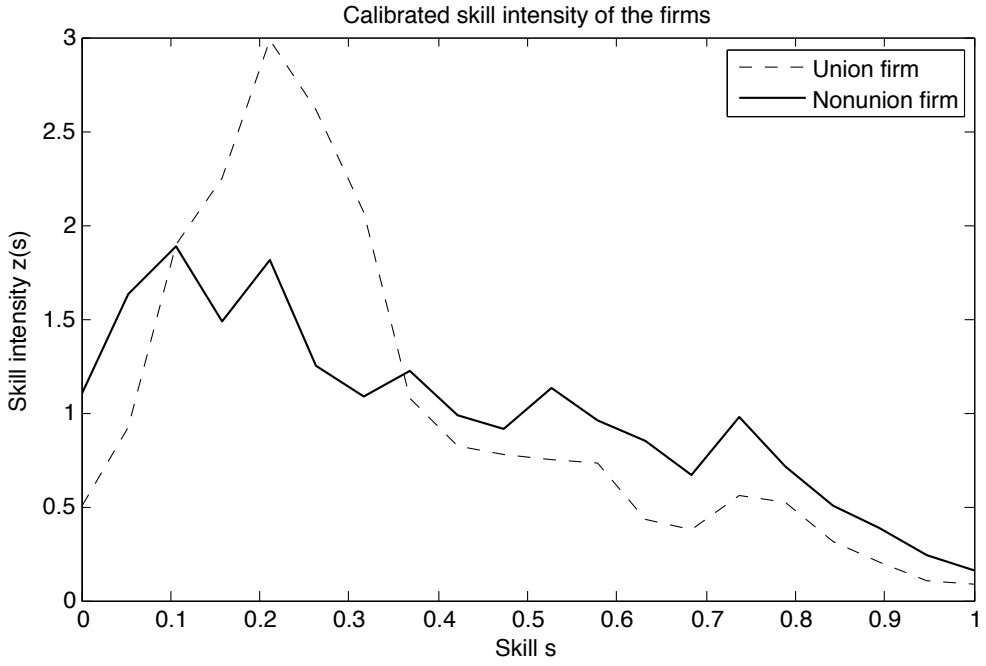


Figure 2.2: Calibrated skill intensities $z_n(s)$ and $z_u(s)$.

Partial equilibrium estimate

With the calibrated model, we can use the partial equilibrium estimator to compute the impact of unions on wage inequality according to a conventional econometric technique. To do so, I compute a new counterfactual wage distribution by giving to each union worker the wage paid to workers of his skill working in a nonunion job. Therefore each union worker s is given his nonunion wage $w_n(s)$. The new

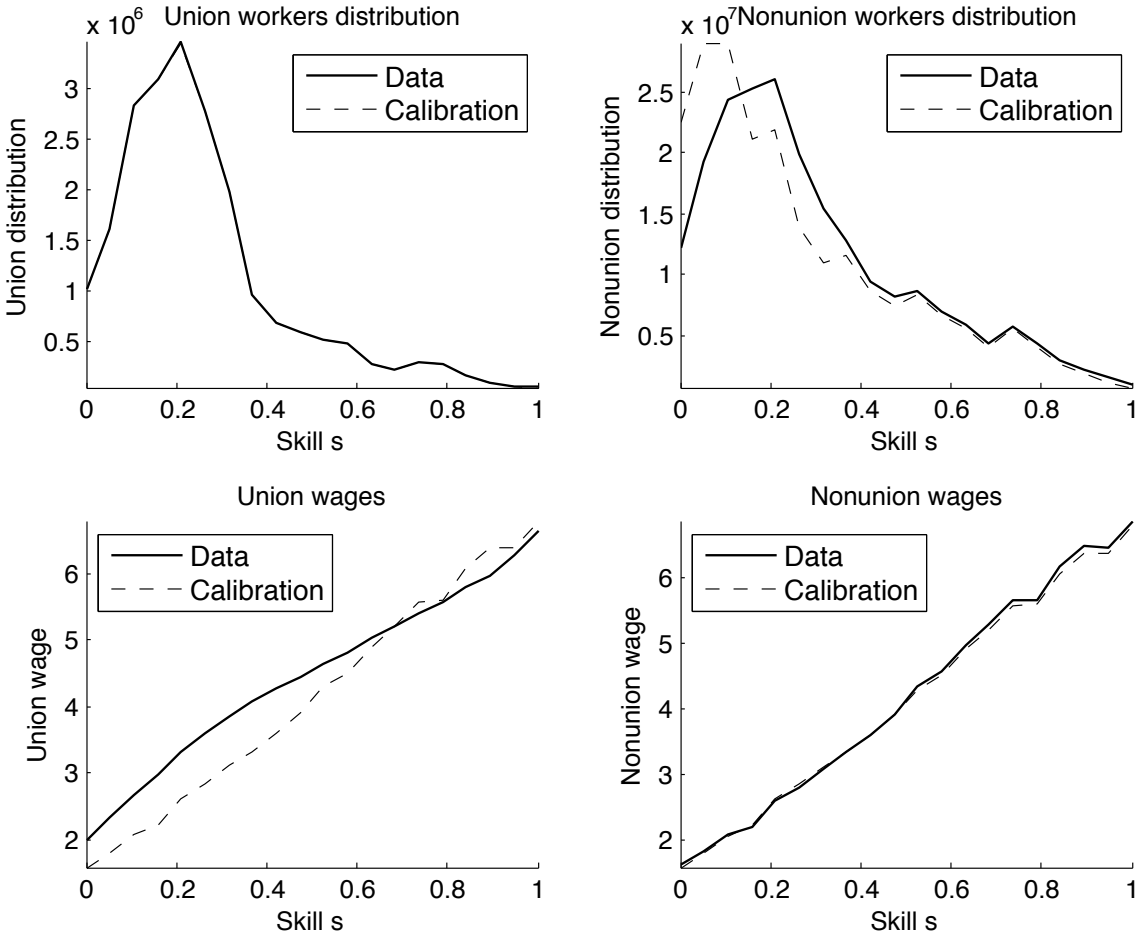


Figure 2.3: Fit of the calibrated model.

wage distribution has a slightly smaller variance. According to this estimate, unions would be responsible for a reduction in the variance of log wages of 0.4%. This implies that the inside-group effect of unions is larger than the between-group effect. In other words, the effect of the smaller variance of union wages is bigger than the effect coming from the difference in means between union and nonunion wages. This estimate is somewhat different from the ones found in the literature. These differences might come from the fact that the present model abstracts completely from the public sector, in which the unionization rate is much higher than in the private sector. In fact, the classical two-sector estimator of Freeman (1980), when applied to the cleaned data set, also finds that unions lower the variance of log wages by 0.4%.

2.3 Impact of unions

I do two comparative statics exercises using the calibrated economy:

1. **Removing the union threat** All union firms stay unionized and all nonunion firms stay union free but the latter do not have to worry about the unionization vote anymore.

2. **Outlawing unions** Unions are completely eliminated from the economy.

In both cases, I compute the new steady state general equilibrium. Figure 2.4 shows the two new equilibria. The top two graphs show the percentage change in union and nonunion wages from the calibrated wage schedules. We see that removing the threat of unionization increases wage inequality by increasing high wages more then low ones. Consider first the modification of the nonunion wage schedule when the union threat is gone. The change comes directly from the reaction of nonunion firms. They change their demand for workers, which leads to higher wages for high-skill workers and lower wages for low-skill workers. This has a direct effect on the value of unemployment $b(s)$, which, in turn, modifies the wages paid by union firms. The impact of the threat removal on wages paid by union firms is purely through a general equilibrium mechanism.

Outlawing unions amplifies the effect on wages further. In this policy exercise, the firms that were previously unionized now bargain wages individually with their workers. This leads to an increase in the slope of the schedule of wages paid by these firms. This, in turn, increases the slope of $b(s)$ which leads to further inequality in wages paid by firms that were previously union free, further amplifying the effect on inequality.

The unemployment rates of all skills go down in both policy exercises. These higher labor market tightnesses have in turn a positive impact on wages. This explains why the total changes in nonunion wages are positive for everyone except for the

workers with very low skill. Without this general equilibrium feedback, the wages of low-skill workers would go down much more when unions are outlawed. If the elasticity of the matching function was different, such that the probability at which vacancies are filled reacted less to a change in labor market tightness, outlawing unions would have a more important negative effect on the wage of low-skill workers.

In the calibrated economy, removing the union threat and outlawing unions has a positive impact on the welfare of almost every agent.[10] The only exception is for workers with very low skills who are better off in an economy with unions.

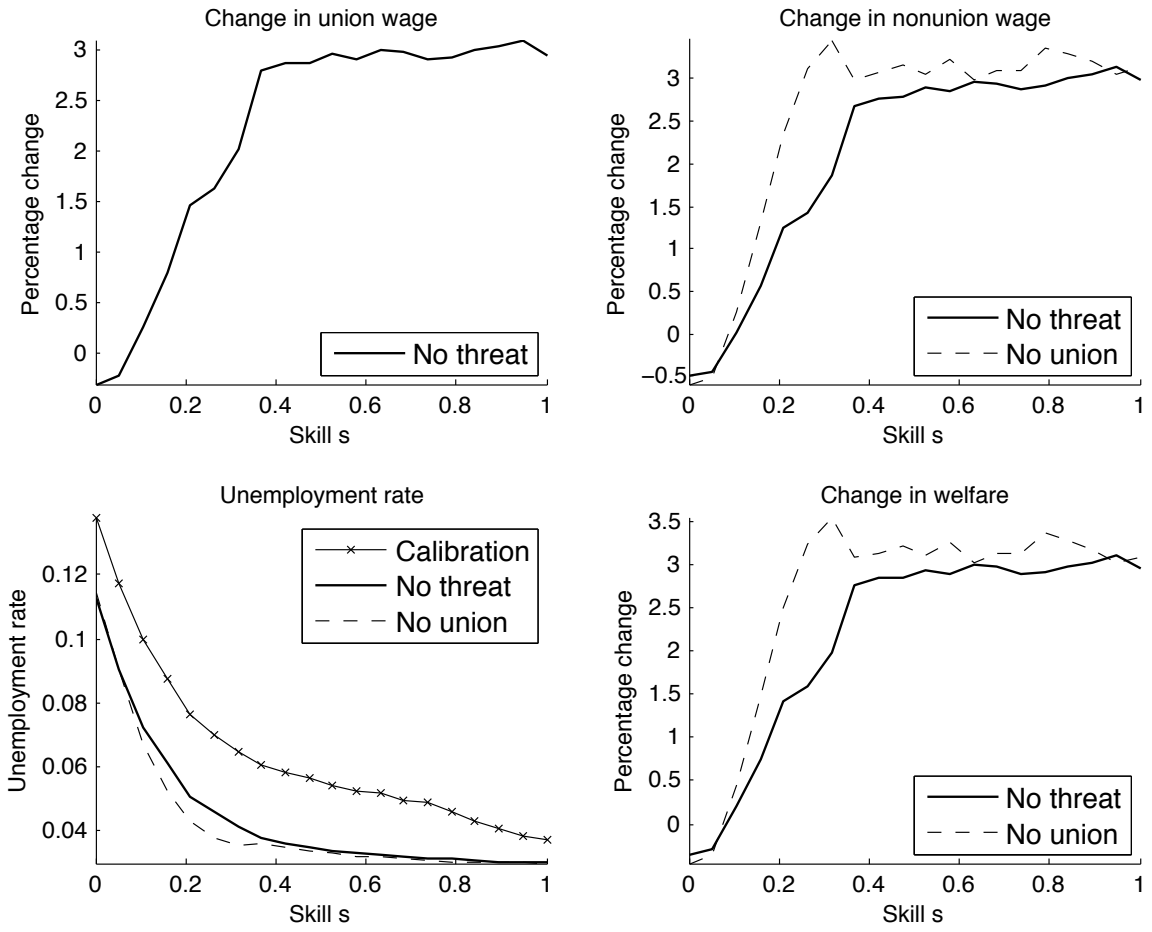


Figure 2.4: Policy exercises: removing the threat of unionization and outlawing unions.

[10]The welfare schedule is computed by summing the welfare of all the agents, employed or unemployed, of a specific skill. In particular, the profits of the firms are not redistributed to the workers. The impact of unions on welfare is similar if profits are redistributed.

Table 2.3 presents the variance of wages, the unemployment rate, total output as well as welfare after removing the union threat and outlawing unions. Removing the threat increases the variance of wages, lowers unemployment and increases output and welfare. These effects are further amplified when unions are outlawed. The variance of log nonunion wages decreases slightly in the last column. This suggests that the variance of nonunion wages is naturally higher in firms producing with technology (α_n, z_n) then in firms producing with technology (α_u, z_u).

Overall, outlawing unions increases total production by 2.5%, welfare by 2.2% and lowers the unemployment rate by 2.5 percentage points. It also increases the variance of log wages by 6%, 15 times more then the partial equilibrium estimate would suggest. This big difference comes directly from the impact of the union threat on nonunion firms and from general equilibrium mechanisms. Importantly, these effects are substantial even if the union membership is small (9% in the calibrated economy).

	Calibration	No union threat	Outlawing unions
Variance log union wages	0.0966	0.1023	-
Variance log nonunion wages	0.1564	0.1648	0.1602
Variance log all wages	0.1510	0.1596	0.1602
Unemployment rate	8.2%	6%	5.7%
Union rate	8.9%	8.6%	0%
Total output ($\times 10^7$)	5.45	5.57	5.59
Welfare ($\times 10^9$)	9.36	9.53	9.58

Table 2.3: Effects of removing the union threat and outlawing unions on the variance of wages, unemployment, output and welfare.

2.4 Conclusion

I calibrate the model of union formation on the United States and show that outlawing unions increases the variance of wages substantially. This increase is much bigger than a partial equilibrium estimate would suggest. Furthermore, outlawing unions

increases welfare and output while lowering unemployment. The welfare gains are more important at the top of skill distribution while workers at the bottom of the distribution are worse off when unions are outlawed.

This paper only deals with the private sector of the economy. Since the public sector is heavily unionized in the United States, it is likely that the counterfactual policy exercises underestimate the full impact of unions.

One possible extension of the model would be to include a government in which the bargaining power of unions is different than in the rest of the economy. Another possible direction for future research would be to allow bargaining at the country level in order to compare the union systems in some European countries with the US system. Also, this theory could be used to study the interaction between the rise in inequality and the strong deunionization that has been observed in the United States during the last decades. In particular, it would be interesting to observe how a change in production technologies or in the skill distribution would impact the unionization rate.

Chapter 3

Optimal Policy in a Labor Market with Adverse Selection

3.1 Introduction

Labor market policies are extremely diverse across countries. A quick look at a survey of these policies (Nickell, 2006) reveals that, in 2000, countries such as France had a minimum wage as high as 60% of the median income while Spain's was at a mere 32%. Scandinavian countries, on the other hand, did not have a legal minimum wage but relied on employer groups and unions to establish a minimum level of earnings for workers. The differences across countries are also important for tax rates. For instance, the 2000 marginal tax rate of a single person with no children earning the average wage was of 21% in New Zealand but of 58% in Germany. The contrast in policies is also significant for union laws, employment protection and unemployment insurance. This leads to two questions. First, what is an optimal labor market policy? What instruments should be used by governments? Second, is it possible to explain the observed discrepancies in terms of governments' and national preferences?

This paper attempts to provide answers to the above questions. We present a search and matching model of the labor market under asymmetric information where heterogeneous, risk-averse agents and firms meet randomly and negotiate the terms of employment. Several important elements distinguish our paper from existing literature. First, the key information friction analyzed in previous studies of optimal unemployment insurance was the inability for governments to monitor job search effort. We choose to abstract from this dimension to focus on the unobservability of workers productivity. Such a friction is especially relevant in a setup where insurance creates disincentives to work, and where skilled workers prefer to shirk at low-productivity jobs to enjoy more generous transfers from the government. Second, our design carefully accounts for general equilibrium effects of labor market policies. Changes in income taxation and unemployment insurance can indeed result in adverse movements in wages, market participation and employment that limit the scope of government intervention. Third, we solve a mechanism design in which workers tell their types to firms and firms report it to the insurance agency. We then provide a way to implement the optimal allocation using simple policy instruments. This method allows us to describe what instruments should be part of an optimal policy without imposing much constraints on the initial problem. Finally, the model's matching and information frictions lead to situations where the economy is in an inefficient state. An additional role for policy is thus to correct these inefficiencies. We therefore support the broader view that labor market policy should be seen not only as a device to promote individual welfare, but also to correct market inefficiencies and fine-tune incentives for agents to choose to work.

After defining the problem of the social planner, we show that the constrained optimal allocation can be implemented in an economy where the wage is observable with simple policy instruments: a non-linear income tax, unemployment insurance based on the previously earned wage and possibly firm subsidies. A minimum wage

can also be used to simplify the tax schedule. Simulations of the model show that a negative income tax for low-productivity workers is often optimal.

We calibrate our model on the US economy and solve for the optimal policy under different welfare criteria, including the utilitarian case. Our baseline optimization in the benchmark case suggests that applying the optimal policy could increase welfare by 17.5% in consumption equivalent, reduce unemployment by 1.3%, and increase labor market participation. In particular, the optimal policy features a close to linear income tax that becomes negative at the lower-end of the income distribution and an unemployment insurance with high replacement ratios. The optimal policy thus achieves a delicate balance between encouraging agents to work, while still supplying high unemployment benefits.

Finally, we investigate how changes in the planner's preferences influence the optimal policy. We show that an increase in the government's taste for redistribution leads to policies that display features similar to those that have been used in Europe during the last decades: higher tax rates and a more generous unemployment insurance. The drawbacks are a slightly higher unemployment level as well as a reduced output level. These findings go against the point of view that generous redistributive policies are suboptimal.

3.1.1 Related literature

This paper is related to previous literature on the optimal design of labor market institution and policy. Our approach is most closely related to Blanchard and Tirole (2008). Their paper examines the joint design of unemployment insurance and employment protection by solving a mechanism design problem in a simple model of the labor market and then providing a way to implement it using unemployment benefits, layoff and payroll taxes. Our paper follows a similar methodology by using a mech-

anism design approach, but focuses on the design of a policy to induce job creation, labor market participation, and efficient production under adverse selection.

Similarly, Mortensen and Pissarides (2002) investigates the effects of taxes and subsidies on labor market variables and characterizes the optimal policy in the labor market. Their paper restricts the set of policy instruments to a linear payroll tax, a job destruction tax and unemployment compensation. Based on a similar model, our paper improves on their approach by solving the optimal mechanism problem, i.e. by first characterizing the optimal allocation and then finding a set of policies that implement it. Furthermore, their model describes an economy with risk neutral agents where the Hosios condition holds (Cf. Hosios (1990)). The equilibrium is therefore efficient. We extend their model by studying an economy with risk-averse heterogeneous workers where the Hosios condition is no longer satisfied and characterize more general constrained Pareto optimal allocations.

Our paper also draws on optimal unemployment insurance literature as in Shavell and Weiss (1979), Wang and Williamson (1996) and Hopenhayn and Nicolini (1997). These articles mostly focus on the moral hazard problem that arise from the inability for the insurer to monitor the job search effort and job performance of the worker. These papers deliver important results on the optimal timing of benefits and their negative relationship with unemployment duration. As these issues have already been studied to a certain extent, we put the search effort dimension aside, but keep the job performance dimension as our model emphasizes the importance of both extensive and intensive margins of labor.

Our study uses to a large extent methods and techniques developed in public finance literature for the analysis of optimal direct taxation under imperfect information. The mechanism design approach that we use draws its inspiration from works such as Mirrlees (1971), Atkinson and Stiglitz (1980), or Stiglitz (1988)) but rejects the assumption of a frictionless labor market. In that sense, this paper is

much closely related to recent studies on optimal taxation of imperfect labor markets. Hungerbühler et al. (2006) uses a similar search model with risk-neutral heterogeneous agents, but focuses on the redistributive aspects of taxation. The Hosios condition is immediately assumed, so that taxation only appears to balance a trade-off between redistribution and efficiency. Their paper develops interesting insights on the optimal tax schedule by modeling explicitly the entry/exit decision of workers and firms in a labor market with frictions and their general equilibrium effects on employment. Our paper considers a similar model, but adds an intensive margin for labor and assumes away the efficiency of the labor market. Hence, direct taxation is not only a tool for redistributive purposes but appears as an efficiency-improving instrument as it enables the government to fine-tune the agents incentives to work.

3.2 The Economy

We consider the problem of a government designing labor market policies in an economy characterized by search frictions and asymmetric information. This government seeks to achieve the right balance between efficiency and welfare. Efficiency related issues arise because of the need for policy to correct labor market frictions. It also concerns the provision of right incentives to elicit participation and work effort from agents. On the welfare point of view, the government aims at providing some insurance against unemployment risk and redistributing wealth across agents.

We build a search and matching model along the lines of Mortensen and Pissarides (1994b) and Pissarides (2000b), where an aggregate matching function determines the creation of matches. We extend the standard model to allow for *ex-ante* heterogeneous workers[1] and describe their decisions on the extensive (whether to work or not) and intensive (how much to work) margins. The latter has been somewhat neglected in

[1]Each worker has a specific productivity that does not evolve over time. We assume away match-specific productivity to keep the dimensionality of the model reasonable.

the job matching literature even though evidence suggests its importance in cross-country differences in total number of hours worked (Rogerson, 2006). It should be noted that agents with different skills respond to changes in policy in radically different ways. As evidenced by Saez (2000), low-productivity workers, being those with the smallest surplus from work, tend to react with their extensive margins, while high-productivity ones rather respond with their intensive margin by changing the time they work. These effects are present in our model and greatly matter for policy design.

In this model, agents need to actively look for a job to receive offers at a hazard rate determined by aggregate labor market conditions. Agents can decide whether to search or not, while firms have to post vacancies to attract job candidates. When a firm and an agent meet, the firm makes a *take-it-or-leave-it* offer to the worker, who can accept or reject it. The offer specifies a wage and an amount of output to be produced.

The policy maker's ability to achieve his objectives is limited by an information asymmetry. Each worker's specific productivity and work effort cannot be observed directly neither by the government nor by the firms. The policy design needs to take into account that agents and firms will try to take advantage of that situation. For example, if unemployment benefits are high for low-skill people or if their entry on the market is subsidized, high-skill workers may accept low paid jobs to obtain high benefits and substitute leisure for work. Such situations limit the government's ability to insure people.

We first define the competitive equilibrium of our economy under taxation. A government provides unemployment benefits $b(w)$ based on the wage the agent received when he was previously employed and a transfer b_0 to agents that have never been employed. Firms receive a constant transfer T when matched with a worker. The government balances its budget by levying a non-linear income tax on workers $\tau(w)$.

He can also impose a minimum wage $\underline{w}$. The following sections will show that this set of instruments is an optimal one. Other instruments are unnecessary or redundant[2].

3.2.1 Population and Technology

There is a unique consumption good. The economy is populated by a continuum of mass 1 of ex-ante heterogeneous agents that differ only in their productivity level s. The cumulative distribution of productivity is $G(\cdot)$, where G is a continuous, increasing function on $\mathbb{R}_+^*$. We denote the corresponding probability density function by $g(\cdot)$. Productivity is constant over time and agents know their own productivity parameter. Agents live forever and time is continuous. If hired by a firm, each agent can provide an amount of labor $h \in [0, \infty)$ to his employer. Agents are assumed risk-averse with life-long preferences

$$\mathcal{U}(c, h) = \int_{t=0}^{\infty} e^{-rt} \left[u\left(c(t)\right) - v\left(h(t)\right) \right] \mathrm{d}t$$

where $u'(\cdot) > 0$, $v'(\cdot) > 0$, $u''(\cdot) < 0$ and $v''(\cdot) > 0$ and where c is the consumption flow.

To study unemployment insurance, we move away from the standard search and matching model by assuming that agents are risk-averse. There is a continuum of identical firms endowed with a production technology f that uses labor as sole input. A firm that employs an agent s working an amount of time h produces $f(sh)$ of the consumption good. We assume that f is concave, strictly increasing, and $f(0) = 0$. The firms are owned by risk-neutral entrepreneurs who only care about profit maximization. Entrepreneurs cannot work and only participate in the labor market as firm owners. The number of firms is not fixed and there is free-entry.

[2]For instance, firing costs and hiring subsidies can be given as lump sum transfer included in other instruments. In a model with endogenous job destruction, firing costs become useful.

3.2.2 Labor market

There is a unique market where firms post vacancies at a cost $\kappa > 0$ per unit of time. Agents decide whether to search for a job or not. Firms do not observe the types of the agents on the market before they meet. In other words, they cannot direct their search to a specific type of worker. Matches occur at a certain Poisson rate given by aggregate labor market conditions. An exogenously given function $m(\cdot, \cdot)$ determines the rate of matching between agents and firms. In a market with U unemployed workers searching for a job and V vacancies, $m(U, V)$ pairs of worker-firm per unit of time meet and decide whether or not to stay together and produce. The function $m(\cdot, \cdot)$ is homogeneous of degree 1.

For convenience, define $\theta \equiv V/U$, the labor market tightness and $q(\theta) \equiv m(U, V)/V$. Therefore, the firm meets a worker at rate $m(U, V)/V = q(\theta)$ and, similarly, the probability rate at which an unemployed worker finds a firm is $m(U, V)/U = \theta q(\theta)$. Jobs are destroyed at an exogenous rate $\delta > 0$, identical across jobs. These assumptions imply that the probability of finding a job is independent of an agent's productivity and the unemployment rate is the same for everyone.

In this economy, frictional unemployment arises because information about job opportunities disseminates slowly, and match creation takes time. As usual in the matching literature, this model is subject to a *congestion* externality. Agents' decisions to participate and search for a job has an adverse effect on the job finding probability of other agents. The more people there are on the job market, the less likely they find a partner. In our model, this externality is further amplified by a *composition* effect due to the heterogeneity of workers. Entry decisions by different groups of agents have a differential impact on the economy, which requires the use of specific policies. This effect is present in our model because search is *random* - not *directed* - and because there is a unique pool of applicants. Although this assumption is a little strong, we choose to maintain it in order to study this composition effect. As

suggested in Ljungqvist and Sargent (2007), the assignment of workers into different pools may have a huge impact on the equilibrium outcome and needs to be studied with care. Keeping this in mind, we choose to focus on the optimal policy in a unique labor market.

3.2.3 Contracts

Upon meeting in the labor market, an agent s and a firm play a three-stage game in which the firm has imperfect information about the worker's real skill level. Figure 3.1 shows the game. At node A, the worker tells a type $\tilde{s}$ to the firm and the firm then makes a *take-it-or-leave-it* offer $\{w(\tilde{s}), y(\tilde{s})\}$ specifying a wage and an amount of goods to be produced. The worker accepts or rejects the offer. We consider stationary Perfect Bayesian Nash equilibria in pure strategies of this game. Workers have complete information, while firms ignore the types of agents and form beliefs $p(\cdot)$ about them. Both players know the structure of the game and firms' information set has to be derived using Bayes rule, wherever it applies.

3.2.4 Worker's problem

A worker of type s decides whether to enter the labor market or not. If he enters and meets a firm, his individual skill being private information, he can pretend to be of a different type than he really is. Given the firm's strategy, he knows that if he declares a type $\tilde{s}$ the offered contract will be the wage $w(\tilde{s})$ and the production of $y(\tilde{s})$. In our setup, an agent s can act as any other agent $s' \in \mathbb{R}_+^*$. However, pretending to be an agent with productivity $s' \gg s$ may require a large work effort.

We define $W_u(s, \tilde{s})$ as the life-long expected utility of an unemployed agent of type s looking for a job and who told his previous employer that he was of type $\tilde{s}$. At the current time, this agent receives $b(w(\tilde{s}))$ as an unemployment compensation from the

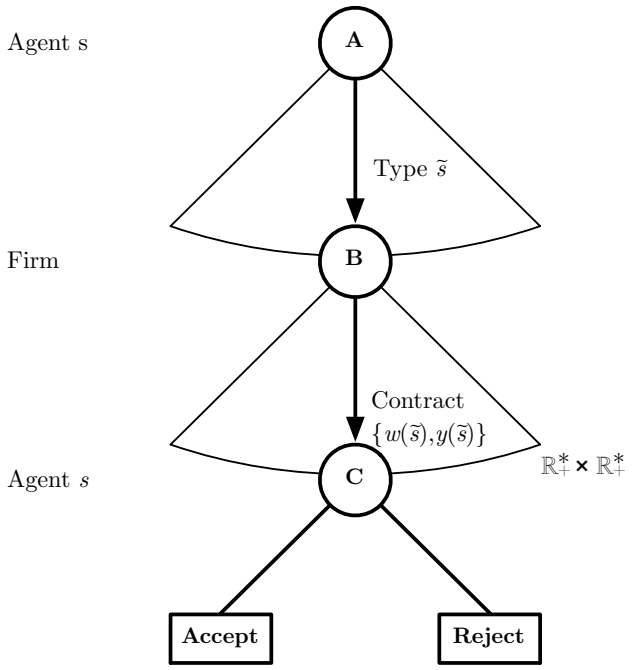


Figure 3.1: Contract setting game

government. His job finding rate is $\theta q(\theta)$. His value function is

$$rW_u(s,\tilde{s}) = u\big(b(w(\tilde{s}))\big) + \theta q(\theta)\left[W_e\left(s\right) - W_u(s,\tilde{s})\right] \tag{3.1}$$

where $W_e\left(s\right)$ is the equilibrium value function of a working agent of type s choosing his optimal strategy. Now, define

$$rW_e(s,\tilde{s}) = u\big(w(\tilde{s}) - \tau(w(\tilde{s}))\big) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \delta\left[W_u(s,\tilde{s}) - W_e(s,\tilde{s})\right]. \tag{3.2}$$

The last equation says that a working agent of type s declaring a type $\tilde{s}$ receives a wage $w(\tilde{s})$ and pays taxes $\tau(w(\tilde{s}))$ in the current period. He loses his job with rate probability δ.

Agent s declares the type $\tilde{s} \in \mathbb{R}_+^*$ that maximizes its expected utility. Define the equilibrium value functions and optimal strategy of agents as follows:

$$\begin{cases} s^*(s) = \arg\max_{\tilde{s}} W_e(s, \tilde{s}) \\ W_e(s) \equiv \max_{\tilde{s}} W_e(s, \tilde{s}) = W_e(s, s^*(s)) \\ W_u(s) \equiv W_u(s, s^*(s)) \end{cases}$$

where $s^*(s)$ is the type declared by an agent of type s. In a truth-telling equilibrium, $s^*(s) = s$ for all s. More explicitly, he solves:

$$\max_{\tilde{s}} \frac{1}{r+\delta} \left[u\big(w(\tilde{s}) - \tau(w(\tilde{s}))\big) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \frac{\delta}{r+\theta q(\theta)} \big(u(b(w(\tilde{s}))) + \theta q(\theta) W_e(s)\big) \right]. \tag{3.3}$$

Some workers, let us call them *inactive*, prefer not to participate in the labor market. They remain unemployed forever and their sole income, b_0, is provided by the government. The corresponding life-long utility is

$$rW_u(s) = u(b_0). \tag{3.4}$$

3.2.5 Firm's problem

The value function of a firm employing an agent who pretends to be of type s[3] is

$$rJ_e(s) = y(s) - w(s) + T + \delta \left[J_u - J_e(s) \right] \tag{3.5}$$

[3]Here it does not matter whether the agent is telling the truth or not. The contract depends only on the declared type.

where $y(s) = f(sh(s))$ is the production generated by a worker of productivity s and J_u is the value function of a firm with a vacant position. More explicitly,

$$rJ_u = -\kappa + q(\theta)\left[\mathrm{E}\left(J_e(s)\right) - J_u\right] \tag{3.6}$$

where κ is the rate cost of posting a vacancy and where $\mathrm{E}(\cdot)$ is the expected value operator over the distribution of workers types *in* the labor market.

We impose a free-entry condition that drives the profit of posting a vacancy to zero, $J_u = 0$. Therefore,

$$\frac{\kappa}{q(\theta)} = \mathrm{E}\left(J_e(s, h(s), w(s))\right). \tag{3.7}$$

Note that the last equation implies that labor market tightness, and therefore the rate probability of being matched, is a function of the expected profits of firms employing workers. Therefore, in an economy with production, firms generate strictly positive profits.

In a Perfect Bayesian Nash equilibrium, taking the agent's strategy as given, the firm updates its belief on the worker's type using Bayes rule, $p(s|\tilde{s}) = p(\tilde{s}|s)p(s)/p(\tilde{s})$, and offers a contract $\{\tilde{w}, \tilde{y}\} \in \mathbb{R}_+^* \times \mathbb{R}_+^*$ that maximizes its profits:

$$\max_{\{\tilde{w},\tilde{y}\}} \mathrm{E}_s \Bigg(\frac{\tilde{y} - \tilde{w} + T}{r+\delta} \times \mathbb{1}\left(\frac{1}{r+\delta}\left[u\left(\tilde{w} - \tau(\tilde{w})\right) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)}\left(u(b(\tilde{w})) + \theta q(\theta) W_e(s)\right)\right] \geq W_u(s)\right) \Big| \tilde{s}\Bigg)$$

where the expression in the indicator function equals 1 when the worker accepts the offer $W_e(s, \tilde{w}, \tilde{y}) \geq W_u(s)$, a decision that is made by the worker at node C. It is important to note that workers may not always reveal their types in such a game and that we would need in principle to consider equilibria with partial pooling. Fortunately, as we will see later, under the assumption of differentiability of the contracts, equilibria have to be truth-telling. We focus on this case from now on.

If the worker reveals his type, the firm's information set reduces to $p(s|\tilde{s}) = \mathbb{1}_{s=\tilde{s}}$. The firm chooses a contract that maximizes its profits subject to the worker's acceptance of the offer:

$$J_e(s) \equiv \max_{\tilde{w},\tilde{y}} \frac{\tilde{y} - \tilde{w}}{r + \delta} \tag{3.8}$$

$$\text{s.t } \frac{1}{r+\delta}\left[u\left(\tilde{w} - \tau(\tilde{w})\right) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r + \theta q(\theta)}\big(u(b(\tilde{w})) + \theta q(\theta) W_e(s)\big)\right] \geq W_u(s). \tag{3.9}$$

This last constraint simply states the worker's willingness to accept the offer. The left part is the present day utility of working under the terms of the offer plus the discounted utility of loosing the job. Note that since the firm always wants a higher production $\tilde{y}$, it is optimal for the firm to offer the worker exactly his alternative valuation and therefore $W_e(s) = W_u(s)$ for all s who reveal their types and therefore $u\big(w(s) - \tau(w(s))\big) - v(h(s)) = u\big(b(w(s))\big)$.

This specific way of setting wages and hours worked is similar to a bargaining setup where the firm has all the bargaining power and receives all the surplus from the match. In this setting, wages are usually set too low to be efficient and tend to induce too much vacancy posting.

The asymmetry of information brings interesting general equilibrium phenomena. Consider for instance an increase in the market tightness $\theta = V/U$ and fix an agent s. This agent could decide to deviate by declaring a type $s' < s$ in which case his utility would be

$$W_e(s, s') = \frac{(r + \theta q(\theta))(u\left(w(s') - \tau(w(s'))\right) - v\left(\frac{f^{-1}(y(s'))}{s}\right)) + \delta u(b(w(s')))}{r(r + \theta q(\theta) + \delta)}.$$

Agent s works a bit less by declaring s' but he also receives lower unemployment payments when he gets fired. An increase in θ changes this trade-off and makes

deviating a better alternative. To prevent a deviation from happening, the firm gives the agent better working conditions, which usually implies a higher wage.

3.2.6 Government

The government maximizes some welfare criterion. It has access to policy tools based on observable variables: employment status of workers, the current wage for employed workers and the previous wage for unemployed ones. This assumption seems indeed reasonable since in practice most governments base their labor market policies on these variables.

The global effect of policy instruments is complicated by the general equilibrium setup. We can however have a sense of the forces at play by looking at the contract setting mechanism. The first relevant equation is the constraint that provides an agent with a utility at least as high as his reservation utility. The second one is the worker's first-order condition, stating the equality of his marginal disutility of labor and his marginal utility of consumption:

$$\frac{1}{r+\delta}\left[u\left(\tilde{w}-\tau(\tilde{w})\right)-v\left(\frac{f^{-1}(\tilde{y})}{s}\right)+\frac{\delta}{r+\theta q(\theta)}\left(u(b(\tilde{w}))+\theta q(\theta)W_e(s)\right)\right]\geq W_u(s) \tag{3.10}$$

$$v'\left(\frac{f^{-1}(y)}{s}\right)\frac{1}{sf'(f^{-1}(y)/s)}=u'(w-\tau(w))(1-\tau'(w))+\frac{\delta}{r+\theta q(\theta)}u'(b(w))b'(w). \tag{3.11}$$

Under the typical policy in this paper[4], the left-hand side of (3.11) is increasing in y and the right-hand side is decreasing in w.

Consider an increase in the *level* of the tax schedule τ. The first effect is to reduce the utility of the worker. To compensate him, the firm has to lower the production and/or to increase the wage. The second effect comes from equation (3.11). A higher

[4]In the optimal policies, $\tau(w)$ is close to linear and $b(w)$ is increasing and concave.

tax increases the right-hand side of the equation while leaving the left-hand side unchanged. To reach an equality, the wage and/or the production have to go up. When combining the two effects, we see that an increase in $\tau(w)$ is likely to lead to a higher wage while the effect on production is uncertain. A similar reasoning tells us that an increase in the *marginal* tax rate lowers the wage and the production. On the other hand, an increase in the level of unemployment benefits b tends to lower wages with an ambiguous effect on production, while an increase in the marginal benefits raises both wages and production.

3.2.7 Stationary competitive equilibrium

Throughout this paper, we focus on stationary equilibrium. Let N be the total number of workers on the labor market (employed and unemployed actively searching). The rate of change over time of the number of unemployed active workers is

$$\dot{U} = (N - U)\delta - U\theta q(\theta) \tag{3.12}$$

where the first term is the number of employed worker loosing their job and the second term the number of searching unemployed agents leaving unemployment. We only consider steady-states of this economy. Therefore $\dot{U} = 0$ and the unemployment rate is

$$\frac{U}{N} = \frac{\delta}{\delta + \theta q(\theta)}. \tag{3.13}$$

We can now define a competitive equilibrium of this economy with government taxation.

Definition 2. *Given taxes $\tau(w)$, unemployment benefits $b(w)$ and b_0, and transfers to firm T, a* stationary competitive equilibrium *in this economy is a strategy for workers $s^*(\cdot)$, a set of contracts offered by firms $\{w(\cdot), y(\cdot)\}$, an equilibrium number of unemployed agents U and a market tightness θ such that,*

1. *equilibrium value functions* $W_e(\cdot), W_u(\cdot), J_e(\cdot)$ *and* J_u *satisfy the system of equations defined by equations (3.1)-(3.7),*
2. *workers' strategies, firms' strategies and information sets* $p(\cdot)$ *form a Perfect Bayesian Nash equilibrium of the wage setting game,*
3. *unemployment is stationary: (3.13) is satisfied,*
4. *the government's budget constraint is balanced.*

3.3 Designing the optimal mechanism

This section introduces the problem of a social planner seeking to maximize some welfare criterion. It characterizes some of its features and shows what policy instruments could be used to implement the optimal allocation in a competitive economy.

3.3.1 Goals of the planner

Three broad objectives guide the planner's decisions. First, he seeks to provide unemployment insurance to ease the shocks created by the labor market frictions. Second, the planner wants to redistribute wealth from high-income agents to low-income ones. Finally, the planner wants to correct the inefficiencies that are built into the model.

There are however limits to insurance and redistribution. The first one is related to incentives. For instance, a generous unemployment insurance program can deter people from entering the labor market. This effect is strongest at the lower end of the skill distribution. Provided that they can secure a comfortable income while unemployed, workers may decide to stop searching for a job and stay unemployed, lowering the efficiency of the policy. Second, insurance and redistribution payments can drive the bargaining power of the workers up by increasing the value of their alternative option, shifting the wage distribution up. This reduces the rent for the firms, which deters them from posting vacancies and finally raises unemployment. Third, these policies need to be funded by increased taxation on workers and firms.

Heavier taxes induce high-skill workers to reduce their job effort and to try to extract transfers from the government by pretending to be of a lower type. Heavier taxes on firms also reduce the rate at which they post vacancies.

Furthermore, the planner tries to correct two inefficiencies. The first one is a *congestion externality* that appears because agents do not internalize the negative impact of their search decision on other agents probability of finding a job. This externality is further magnified by a *composition effect* since agents have ex-ante heterogeneous skills and are all searching in the same pool. The differences in productivity makes the marginal social cost of a low-skill searching worker higher than the one of a high-skill worker. As a consequence, the planner might want to keep low-productivity workers away from the labor market. This results in an *optimal segmentation* rule.

The second inefficiency comes from the wage setting mechanism. Since the firm has all the bargaining power, wages tend to be below their optimal level and do not internalize the search externality (an effect similar to the one found in (Hosios, 1990)). This leads firms to post too many vacancies.

3.3.2 Limits on the planner

Since governments cannot directly observe workers' productivity nor control their individual decision, we endow the social planner with the same information set as the one available to policy makers. The planner's main constraints are therefore of physical and informational nature. Subject to search frictions, he must rely on the signals he receives to allocate resources and provide incentives for agents to comply with his decisions. The social planner is subject to the following limitations:

Matching technology The planner cannot override the search frictions coming from the matching technology. Posting vacancies is costly to the planner, match creation takes time, so workers and firms still receive job offers and candidates at the hazard rate determined by aggregate conditions.

Participation constraint The planner cannot force people into accepting jobs, nor firms into posting vacancies. The entry/exit decisions for both types of agents belongs to them. The optimal allocation will therefore have to satisfy participation constraints.

Incentive constraint The planner cannot observe the underlying productivity parameter of the worker, nor the amount of time spent at work. This limits his ability to provide full insurance to the workers and restricts the set of policy instruments that can be used to implement the optimal policy. In particular, the planner is subject to the way firms and workers negotiate their contracts and choose their optimal reporting strategies. The optimal reporting strategy has to be a Perfect Bayesian Nash equilibrium of the game introduced earlier.

3.3.3 Optimal mechanism

We now introduce the set of transfers over which the social planner optimizes. We focus our attention on steady-states of the economy. In other words, the planner maximizes the steady-state level of welfare. For this reason, we impose time-invariance on the set of transfers we study.

In principle, the social planner would like to design transfers based on all the relevant information. In particular, these transfers could be based on the skill s, employment status (employed/unemployed/inactive) and time spent employed/unemployed. Notice however that, at any date, workers with identical skills and employment status face the same problem. Since there is no dynamic human capital accumulation, nor savings, nor any other element that depends on time, time spent in unemployment does not matter as long as policies (taxes, unemployment insurance) do not vary over time. In addition, these workers enter symmetrically in the social welfare function. We will therefore study symmetrical time-invariant transfers. One could in principle study a more general class of mechanisms with transfers depending on time spent

employed/unemployed. This would however be a much more difficult task, and it would not necessarily improve our findings, as time does not matter for agents unless policy is itself time-dependent.

The social planner solves directly in terms of allocation. Define the optimal transfers to be: a consumption schedule for employed agents $c_e(s)$, consumption of unemployed but active agents $c_u(s)$ and that of the inactive agents c_0, and transfers $t_f(s)$ to firms employing agents of type s. Equilibrium valuations can be redefined in terms of allocations:

$$\begin{cases} rW_e(s,\tilde{s}) = u\left(c_e(\tilde{s})\right) - v\left(\dfrac{f^{-1}(y(\tilde{s}))}{s}\right) + \delta\left[W_u(s,\tilde{s}) - W_e(s,\tilde{s})\right] & (3.14) \\ rW_u(s,\tilde{s}) = u(c_u(\tilde{s})) + \theta q(\theta)\left[W_e\left(s\right) - W_u(s,\tilde{s})\right] & (3.15) \\ rJ_e(s) = f(sh(s)) + t_f(s) + \delta\left[J_u - J_e(s)\right] & (3.16) \\ rJ_u = -\kappa + q(\theta)\left[\mathrm{E}\left(J_e(s)\right) - J_u\right]. & (3.17) \end{cases}$$

Once again, we use the $\tilde{\cdot}$ notation to emphasize variables over which deviations by the agents are possible.

We now describe the type of reporting mechanism we focus on. We study economies where the wage is the signal observed by the government. In the wage-setting process we consider, contracts (w, y) are Nash equilibria of the game introduced earlier. In the planner's setting, contracts do not specify a wage and a production $\{w(\tilde{s}), y(\tilde{s})\}$ but a pair $\{s^R(\tilde{s}), y(\tilde{s})\}$ specifying a type to report and a production objective. The planner optimizes in the space of allocation: $\{c_e(s^R), c_u(s^R), t_f(s^R)\}$. We thus focus on mechanisms where the signal is jointly chosen by the firm-worker pair as a solution of a game similar to the one of figure 3.1. The game is as follows: at the first stage, the worker declares a type $\tilde{s}$, the firm then proposes a production $y(\tilde{s})$ and a signal $s^R(\tilde{s})$ to report to the planner. The

worker accepts or rejects this offer. We consider Perfect Bayesian Nash equilibria in pure strategies of this game.

3.3.4 The planner's problem

The social planner optimally designs transfers $\{c_e(s), c_u(s), c_0, t_f(s)\}$ to adjust the amount of time an agent provides $h(s)$, labor market tightness θ and unemployment rate U/N. To keep track of the agents that participate in the labor market, we define the following indicator function:

$$\chi(s) = \begin{cases} 1 \text{ if agents } s \text{ is employed or unemployed but looking for a job} \\ 0 \text{ if the agent is inactive} \end{cases}$$

The social planner maximizes a sum of a concave transformation of the valuations of workers. This summation is weighted by the respective sizes of employment and unemployment pools. Firms are owned by risk-neutral agents whose utility is valued with coefficient λ by the planner. The social welfare criterion is

$$\begin{aligned} \max_{\substack{c_e(.), c_u(.), c_0, t_f(.), \\ \chi(.), y(\cdot), \theta, U}} & \left(1 - \frac{U}{N}\right) \int \chi(s) \Phi\Big(W_e\big(c_e(s), c_u(s), y(s)\big)\Big) g(s) ds && \text{(employed)} \\ & && \text{(SP)} \\ & + \frac{U}{N} \int \chi(s) \Phi\Big(W_u\big(c_e(s), c_u(s), y(s)\big)\Big) g(s) ds && \text{(unemployed)} \\ & + \int (1 - \chi(s)) \Phi\left(\frac{u(c_0)}{r}\right) g(s) ds && \text{(inactive)} \\ & + \left(1 - \frac{U}{N}\right) \lambda \int \chi(s) J_e(t_f(s)) g(s) ds && \text{(firms)} \end{aligned}$$

where $N = \int \chi(s) g(s) ds$ is the total size of the labor market, and $\Phi(\cdot)$ is an increasing concave function that describes the planner's taste for redistribution. Notice that this

social welfare function encompasses the utilitarian case if $\Phi(\cdot)$ is linear. The first two integrals in SP are the valuations of employed and unemployed workers. The third integral is the utility of agents that stay out of the labor market. The fourth term is the utility of the firms' owners.

The planner's design is subject to the flow condition (3.13), the free entry condition ($J_u = 0$) and to the resource constraint

$$\left(1 - \frac{U}{N}\right) \int \chi(s) c_e(s) g(s) ds + \frac{U}{N} \int \chi(s) c_u(s) g(s) ds + \int (1 - \chi(s)) c_0 g(s) ds \quad \text{(RC)}$$
$$+ \left(1 - \frac{U}{N}\right) \int \chi(s) t_f(s) g(s) ds \leq 0.$$

The following participation constraints also apply. All agents that are participating in the labor market must be willing to search for a job, and stay on the market. Namely,

$$\chi(s) = 1 \Leftrightarrow \begin{cases} W_e(s) \geq W_u(s) \\ W_e(s) \geq \frac{u(c_0)}{r} \\ y(s) + t_f(s) \geq 0 \end{cases} \qquad \text{(PC active)}$$

The top inequality in the set of equations (PC active) implies that, in equilibrium, unemployed agents want to look for a job. The bottom one implies that a firm is willing to hire them. Similarly, inactive agents must be prevented from entering the labor market. Therefore, for all s such that $\chi(s) = 0$ and for all $\tilde{s}$ such that $\chi(\tilde{s}) = 1$,

$$\frac{u(c_0)}{r} \geq \frac{(r + \theta q(\theta))(u(c_e(\tilde{s})) - v(f^{-1}(y(\tilde{s}))/s)) + \delta u(c_u(\tilde{s}))}{r(r + \delta + \theta q(\theta))}. \qquad \text{(PC inactive)}$$

This inequality simply states that no existing job offer can make inactive workers better off.

As stated before, the strategy-belief profile for firms and workers must form a Perfect Bayesian Nash equilibrium of the wage-setting game. Hence, no workers nor

firms can have an incentive to deviate. More precisely, the transfer scheme proposed by the planner $\{c_e(\cdot), c_u(\cdot), c_0, t_f(\cdot)\}$, the optimal strategy for the worker $s^*(s)$, and contracts $\{s^R(\tilde{s}), y(\tilde{s})\}$ have to solve the worker's problem

$$s^*(s) = \underset{\tilde{s} \text{ s.t } \chi(\tilde{s})=1}{\operatorname{argmax}} \frac{1}{r+\delta} \left[u\left(c_e\left(s^R(\tilde{s})\right)\right) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) \right. \tag{3.18}$$

$$\left. + \frac{\delta}{r+\theta q(\theta)} \left(u\Big(c_u\left(s^R(\tilde{s})\right)\Big) + \theta q(\theta) W_e(s) \right) \right]. \tag{3.19}$$

Remember that after reporting a certain type $\tilde{s}$, the worker gets a transfer $c_e(s^R(\tilde{s}))$ during employment and a transfer $c_u(s^R(\tilde{s}))$ while unemployed until he meets with a new firm and starts negotiating again. Therefore, all deviations need to take into account that transfers while unemployed also depend on the currently reported type.

Similarly, the firm's strategy $\{s^R(\tilde{s}), y(\tilde{s})\}$ has to be optimal for the firm, given the worker's strategy and beliefs. Beliefs $p(s|\tilde{s})$ have to be derived using Bayes' rule wherever it applies. In particular, if the worker reveals his type ($s^*(s) = s$ and $p(s|s) = 1$), the firm's problem becomes

$$J_e(s) \equiv \max_{s^R, \tilde{y}} \frac{\tilde{y} + t_f(s^R)}{r+\delta} \tag{3.20}$$

$$\text{s.t } \frac{1}{r+\delta} \left[u\left(c_e(s^R)\right) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)} \left(u(c_u(s^R)) + \theta q(\theta) W_e(s) \right) \right] \geq W_u(s).$$

3.3.5 Solution approach

Solving the social planner's problem as stated above is in general quite complicated, and we must impose additional restrictions in order to deal with the incentive constraints. We substitute workers' and firms' first-order conditions for their incentive constraints. This approach enables us to treat these constraints in a very tractable way. This method is however known since Rogerson (1985) to produce sometimes invalid results. To insure the accuracy of our results, we check in our numerical sim-

ulations that these first-order conditions are indeed sufficient. Our solution strategy nevertheless requires that the optimal transfers $\{c_e(s), c_u(s), c_0, t_f(s)\}$ and firm's production offers $\{y(s)\}$ are differentiable functions of s. This in turn allows us to focus on separating equilibria, as the following lemma shows.

Lemma 10. *If the social planner's optimal transfers $\{c_e(\tilde{s}), c_u(\tilde{s}), c_0, t_f(\tilde{s})\}$ and firm's offered production $y(\tilde{s})$ are differentiable functions of $\tilde{s}$, then it is optimal for the worker to reveal his type.*

Proof. See appendix. □

The rest of the paper assumes from now on that equilibria are fully revealing. Truth-telling allows us to characterize some properties of the constrained optimal allocation.

Lemma 11. *In the constrained optimal allocation:*

(i) If there exists a s such that $\chi(s) = 1$, then $\chi(s') = 1$ for all $s' \geq s$. We define $\underline{s}$ as the smallest s such that $\chi(s) = 1$.

(ii) $W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))} v'\left(\frac{f^{-1}(y(s))}{s}\right) \frac{f^{-1}(y(s))}{s^2}$ and $W_e(s)$ is therefore increasing

(iii) $W_e(s) = W_u(s)$ for all s

(iv) $J_e'(s) = 0$ and therefore, in equilibrium, $J_e(s) = \kappa/q(\theta)$

(v) $W_e(\underline{s}) = u(c_0)/r$

(vi) $c_u(s)$ is increasing

Proof. See appendix. □

The first result tells us that it is optimal for the planner to let all agents above a certain threshold $\underline{s}$ participate in the labor market, while keeping less productive agents out of it. This arises as a consequence of the congestion externality as an

increased participation of low-skill agents does not compensate for the negative impact that they have on the job matching rate of other agents. This also reflects worker's individual rationality: as their wage go to 0, workers are simply better off enjoying leisure and social benefits b_0.

To understand (ii), note that without information asymmetry in this economy, firms would have all the bargaining power, and could drive wages down to the reservation utility $\frac{u(c_0)}{r}$. This does not happen in our setup as agents have their own skill as private information. Firms therefore need to compensate them for possible deviations and need to raise their offers in order to have workers reveal their type. (ii) illustrates this fact as W_e has to be increasing at a rate equal to the marginal utility of deviating. (iii) is a direct consequence that in a truth-telling equilibrium firms can extract all the surplus from workers, under the condition that their incentive constraints are satisfied. Combining these two results, (vi) tells us that unemployment insurance $c_u(s)$ is increasing. (iv) is derived from the Envelope theorem on the firm's problem in a truth-telling equilibrium. Firms choose production and wages optimally for each s. As a result, firms' profit do not vary directly with s and J_e is constant. (v) simply states that the utility of the first type to enter the labor market has to be equal to the reservation utility of receiving social benefits $\frac{u(c_0)}{r}$.

3.3.6 Implementation of the optimal allocation

The previous sections have presented the social planner's problem as a static truth-telling mechanism. It matters in practice to find an easily implementable tax/transfer system that will effectively achieve the optimal allocation. In particular, we will make explicit the way in which policy instruments make agents and firms choose a wage that will reveal their types.

The following proposition shows that the optimal mechanism is implementable in an economy where the wage is observable.

Proposition 1. *If wages are observable, an optimal allocation in this economy is implementable using a non-linear income tax on workers $\tau(w)$, unemployment benefits $b(w)$, a transfer b_0 to inactive agents, and a uniform subsidy to firms T.*

Proof. See appendix. □

This proposition shows that the optimal mechanism can be implemented by simple instruments already used by many governments such as non-linear income taxes on workers and an unemployment insurance system based on the wage previously earned. In principle, the transfers designed by the social planner should depend on all the information available to him, which explains why all transfers are functions of the underlying skills and employment status of workers. This requires in practice the use of some non-linear instruments such as an income tax $\tau(w)$. There is a question whether firms should also be taxed. In our setup, imposing a non-linear tax on firms is however unnecessary as the two tax schedules (on workers and firms) then become redundant. Other tax systems that include taxes on firms may also implement the optimal mechanism.

A minimum wage policy can also be part of an optimal tax system but is not essential as the optimal transfers can shift the general level of wages up or down by themselves. Therefore, a minimum wage policy cannot improve welfare. It can however be used as a device to simplify the optimal tax. Indeed, when expressed as a function of w, the optimal tax may not be defined on the entire real axis. To prevent agents from considering out-of-equilibrium wages, for example below the lowest equilibrium wage, the planner can either impose a large tax, or simply impose a minimum wage equal to this lowest wage.

One may notice that firing costs and hiring subsidies are absent from the optimal policy. This is not a robust feature of the model and is actually a consequence of

exogenous job destruction. In the current setup, employment protection policies like firing costs are redundant with the existing transfers for firms, $t_f(s)$[5].

3.3.7 Optimal control

The optimal allocation being implementable with instruments $\{\tau(\cdot), b(\cdot), b_0, T\}$, it is possible to state the social planner's problem as a function on these instruments. We show in this subsection that the social planner's problem can be cast into an *optimal control* problem. In this new formulation, we treat $W_e(\cdot)$ as the state variable and production $y(\cdot)$ as the control variable. This formulation allows us to use robust resolution techniques to solve the problem.

The optimal control problem is

$$\max_{\theta,\underline{s},b_0,h(\cdot)} \int_{\underline{s}}^{s_{max}} \Phi\left(W_e(s)\right) g(s)ds + \int_0^{\underline{s}} \Phi\left(\frac{u(b_0)}{r}\right) g(s)ds + \lambda \frac{\theta q(\theta)}{\delta + \theta q(\theta)} \frac{\kappa}{q(\theta)} (1 - G(\underline{s})) \tag{3.21}$$

$$\text{s.t} \begin{cases} W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))} v'\left(\frac{f^{-1}(y(s))}{s}\right) \frac{y(s)}{s^2} \\ W_e(\underline{s}) = \frac{u(b_0)}{r} \\ \frac{\theta q(\theta)}{\delta+\theta q(\theta)} \int \chi(s)\tau(s)g(s)ds - \frac{\delta}{\delta+\theta q(\theta)} \int \chi(s)b(s)g(s)ds - G(\underline{s})b_0 - (1 - G(\underline{s})) \frac{\theta q(\theta)}{\delta+\theta q(\theta)} T \geq 0 \end{cases} \tag{3.22}$$

where

$$\begin{cases} b(s) & \equiv u^{-1}(rW_e(s)) \\ \tau(s) & \equiv y(s) - \frac{\kappa(r+\delta)}{q(\theta)} - u^{-1}(rW_e(s) + v\left(\frac{f^{-1}(y(s))}{s}\right). \end{cases}$$

The first term in (3.21) is the welfare contribution of workers participating in the labor market (workers with productivity above $\underline{s}$). The second term is the contribution of inactive workers, while the last one is the firm's expected profits weighted by

[5]The decision to destroy the job or not being exogenous, firing costs cannot influence the firing decisions of firms. It only affects the number of posted vacancies by reducing firms' expected profits. One can show that a tax system with constant transfers to firm is in this case equivalent to any tax system based on firing costs

parameter λ (remember $E[J_e(s)|s \geq \underline{s}] = \frac{\kappa}{q(\theta)}$). Concerning the constraints (3.22), the first equation is the incentive constraint for active workers, the second one combines the incentive constraint for inactive agents and agent $\underline{s}$, while the last one is the planner's resource constraint.

The traditional way of solving this type of problem is to use Pontryagin's maximum principle to derive necessary conditions.[6] To do so, we write the Hamiltonian[7]

$$\begin{aligned}\mathcal{H}(s, W_e, y) = g(s)\Phi(W_e) + \mu(s)\frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))}v'\left(\frac{f^{-1}(y(s))}{s}\right)\frac{f^{-1}(y(s))}{s^2} \\ + \nu g(s)\left[\theta q(\theta)\left(y(s) - u^{-1}\left(rW_e + v\left(\frac{f^{-1}(y(s))}{s}\right)\right)\right) - \delta u^{-1}(rW_e)\right]\end{aligned} \tag{3.23}$$

where $\mu(\cdot)$ is the costate variable of $W_e(\cdot)$ and where ν is the Lagrange multiplier on the resource constraint.

The necessary conditions are

$$\frac{\partial \mathcal{H}}{\partial W_e} = -\mu'(s) \quad \frac{\partial \mathcal{H}}{\partial \mu(s)} = W_e'(s) \quad \frac{\partial \mathcal{H}}{\partial h} = 0 \tag{3.24}$$

and the boundary conditions are

$$\begin{cases} W_e(\underline{s}) = \frac{u(b_0)}{r} \\ \mu(s_{\max}) = 0 \end{cases} \tag{3.25}$$

The first boundary condition comes from lemma 11. The second one is the appropriate transversality condition since $W_e(\cdot)$ is assumed free at s_{max}. Once the optimal control problem is solved for $W_e(\cdot)$ and $y(\cdot)$, finding the optimal $\underline{s}$, b_0 and θ is a simple maximization problem.

[6] A good reference on this type of problem and the use of the appropriate transversality condition is (Fleming and Rishel, 1982)

[7] For simplicity, we include only the terms that are relevant for the partial differential equations system.

3.4 Optimal Policy in the United States

It has now been established that the government can implement the optimal mechanism using an income tax on workers $\tau(w)$, an unemployment benefit schedule $b(w)$, a subsidy to firms T and a transfer to *inactive* agents b_0. In this section, we calibrate the model on the US economy and solve for the optimal policy. We highlight characteristics of the optimal policy and then explain how the various policy instruments influence the allocation.

3.4.1 Functional forms

We use the following functional forms. Worker preferences are given by

$$\mathcal{U}(c_i, h_i) = \int_{t=0}^{\infty} e^{-rt}[u(c_i(t)) - v(h_i(t))]\mathrm{d}t.$$

We use the following CRRA utility function and disutility of labor:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma}, \text{ if } \gamma > 0, \gamma \neq 1 \\ \ln c, \text{ if } \gamma = 1 \end{cases}, \quad v(h) = \rho \frac{h^{1+1/\varphi}}{1+1/\varphi}$$

where $1/\gamma$ is the intertemporal elasticity of substitution for consumption, φ is the Frisch elasticity of labor supply, and $\rho > 0$ is the weight on the disutility of labor.

The production function is

$$f(s \cdot h) = A(s \cdot h)^{\alpha}$$

with $0 < \alpha < 1$, and $A > 0$.

For the labor market variables, we assume that the aggregate matching function is a constant return-to-scale Cobb-Douglas function

$$m(U, V) = MU^{\mu}V^{1-\mu}$$

with $0 < \mu < 1$, and $M > 0$.

Most of the empirical literature has used Pareto and log-normal skill distribution. For our purpose, we choose a log-normal distribution with parameters (μ_s, σ_s^2) as it provides a better fit for the calibration.

In its current form, the model is not identified; a change in the value of A can be achieved by an equivalent and independent change in (μ_s, σ_s^2). Hence, we normalize the average skill in the population to 1. Therefore, $e^{\mu_s + \sigma_s^2/2} = 1$.

3.4.2 Calibration

The idea behind the calibration is as follows: given a set of inputs (parameters, policies and a skill distribution) the model produces a wage distribution and a time-at-work distribution. We therefore try to set these parameters such as to minimize the difference between the calibrated distributions (given by the model) and the empirical ones (observed in the data).

We use a loss function that takes the square of the area between the empirical and the calibrated wage distributions and we add to it the square of the difference between the empirical and calibrated means for time-at-work.[8] We weight these two quantities

[8]The natural thing to do here would have been to also take the difference in the empirical and calibrated density of time-at-work. The distribution of time-at-work is however very messy and properly matching it with the smooth distribution given by the model is impossible.

such that their contributions to the loss are of the same magnitude. Therefore,

$$\begin{aligned}\text{LOSS} = &\int \left[g_{(\text{w calibrated})}(w) - g_{(\text{w empirical})}(w)\right]^2 dw \\ &+ \lambda_{weight} \left[\text{E}(g_{(\text{h calibrated})}) - \text{E}(g_{(\text{h empirical})})\right]^2\end{aligned}$$

where g is the designated distribution, λ_{weight} is the weight given to the second term and E is the operator that gives the mean of the distribution.

The empirical data for the wage and time-at-work distributions comes from the *Current Population Survey* (CPS). To avoid calibrating the model in a turbulent period, we pick the data from January 2005. To estimate the hourly wage distribution, we use the reported weekly earnings and weekly hours actually worked at all jobs. We drop all the reported hourly-wage rates below the Federal minimum wage ($5.15 in January 2005). Also, as their density is insignificant, we drop all observations above $100 per hour. We also take the weekly hours worked data from the CPS for the distribution of h. The time unit is a year. Total income per year is stated in thousands of US dollars.

We use 2005 data from the US *Bureau of Labor Statistics* (BLS) for aggregate labor market variables, such as the unemployment rate, total labor force, and the average duration of unemployment.

According to Petrongolo and Pissarides (2001), estimates for the aggregate matching function elasticity μ range from 0.5 to 0.7. We arbitrarily set $\mu = 0.5$. We also set the risk aversion $\gamma = 2$ and $\varphi = 0.5$.

Data on vacancies in the US have been available since 2000 from the *Job Openings and Turnover Survey* (JOLTS). We use the January 2005 estimate of job openings rate as a proxy for the number of vacancies V.

Table (3.1) summarizes the choice of parameters.

The official federal minimum wage was \$5.15/hour in 2005. We introduce it in the contract setting process as a lower bound on the set of hourly wage rates. The tax schedule is the 2005 federal income tax from the Internal Revenue Service for a single person.

A critical step is the choice of the unemployment compensation schedule b. In our model, $b(w)$ is constant over the unemployment period, which is not the case in the United States. In most states, unemployment compensation is a fraction of previous income received during a predetermined number of weeks. For example, in the state of New York, unemployed workers receive a fraction of 1/26 of their previous quarter income during 26 weeks. This amounts to a flow of 50% of the previous wage for 26 weeks. Since the average duration of unemployment was 19.4 weeks in January 2005, and therefore below 26 weeks, we calibrate b to be 50% of the previous wage.

The life-long utility of the lowest skill entering the labor market is $u(b_0)/r$. We calibrate b_0 to match the expected utility of a worker earning the minimum wage and working the average hours worked in the economy, given aggregate conditions and US policy. The cutoff $\underline{s}$ is then computed to be the first type s entering the market given b_0.

Finally, there is no available data on the rate cost of posting a vacancy κ. Mortensen (1994) calibrates it to be a fraction 0.3 of the production of the highest skilled agent in a given period. We follow Ljungqvist and Sargent (2007) in assuming that κ corresponds to 6 months of the average wage in the economy.

We calibrate the model by minimizing the loss function. Figure 3.2 shows the calibrated and the empirical hourly earnings distributions.

Table 3.2 shows the optimal parameters. From $var(g)$, the variance of the skill density, we can recover the parameters of the log-normal distribution $\mu_s = -0.728$ and $\sigma_s^2 = 1.456$.

Parameter	Origin/Target moment	Empirical moment	Parameter value
γ	Empirical studies		2
φ	Empirical studies		0.5
r	Interest rate		0.02
U	Bureau of Labor Statistics		7,759,000
V	JOLTS	2.9% total empl.	4,067,130
θ	V/U		0.5242
μ	Petrongolo and Pissarides (2001)	[0.5,0.7]	0.5
M	Avg. duration of unemp. $1/M\theta^{1-\mu}$	19.4 weeks	3.7129
δ	Unemployment rate $\delta/(\delta+\theta q(\theta))$	0.052	0.1475
$\underline{w}$	Federal minimum wage		\$5.15/h
κ	6 months of avg. wage	1/2 × \$36,840/yr	18.42

Table 3.1: Pre-calibration parameters

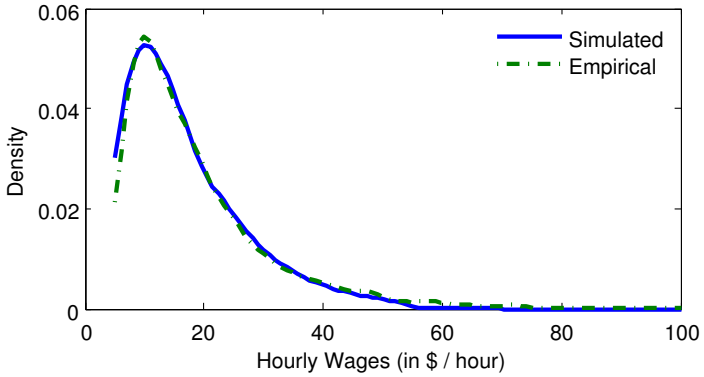


Figure 3.2: Empirical and calibrated hourly wage distribution

3.4.3 Optimal Policy

Using the parameters found in the calibration, we now solve the optimal control problem and find the optimal labor market policy for the US.[9]

Table 3.3 summarizes some of the results. Global welfare goes from -3.11 to -2.7062. This is equivalent to an increase of each agent's consumption by 17.5%. This number is large but it is important to note that, since the government is the only source of insurance in this model, the gain would be smaller if agents had access to savings. Still, the bulk of welfare gains goes to low-skill agents, those that might have difficulties hedging against unemployment by savings.

The planner reduces $\underline{s}$ from 0.047 to 0.001. This shows the inefficiency of minimum wage policies since they prevent low-skill agents from entering the labor market.

[9]The Matlab code used to find the optimal allocation is available at the authors' webpages.

α	A	var(g)	ρ	$\underline{s}$	b_0
0.4431	72.83	3.290	3.016	0.0474	7.246

Table 3.2: Results of the calibration

Instead, the planner prefers a progressive tax schedule to redistribute wealth and to subsidize their entry on the labor market. Also, the unemployment compensation to inactive agents, b_0, is increased by 75%, going from \$7250 to \$12690 per year. The labor market tightness goes from 0.5242 to 0.9575. This results in an unemployment rate of 3.9%, lower than the 5.2% observed in the data for that period.

	$\underline{s}$	b_0	θ	Unemployment rate
Calibrated	0.047	7.246	0.5242	5.2%
Optimal	0.001	12.69	0.9575	3.9 %

Table 3.3: Optimal policy in the US

Figure 3.3 summarizes the economy. Panel (a) shows the before-tax wage, the after-tax wage and the unemployment benefits $b(s)$. Panel (b) presents the welfare $W_e(s)$ of the workers. It is increasing and concave and its precise shape is given by the incentive constraint.

Panel (c) shows the tax schedule. It features a *negative income tax* for the lowest wages. This is a robust feature of the model. A negative tax makes searching for a job more attractive for low-skill agent and therefore tends to lower the number of inactive agent. Furthermore, it reduces the unemployment insurance payments made by the government while increasing global output.

The tax is almost linear for the biggest part of the wage distribution but flattens for higher value. This flattening is a consequence of the higher bound on the skill distribution and the transversality condition. At this point, the incentive constraint is less binding (an increase in $W_e(s_{max})$ does not require a change in other agent's W_e to respect the IC constraint). The planner therefore seeks to minimize the distortionary

effects of the tax schedule by flattening it. The optimal tax schedule is much more progressive than the 2005 US tax schedule.

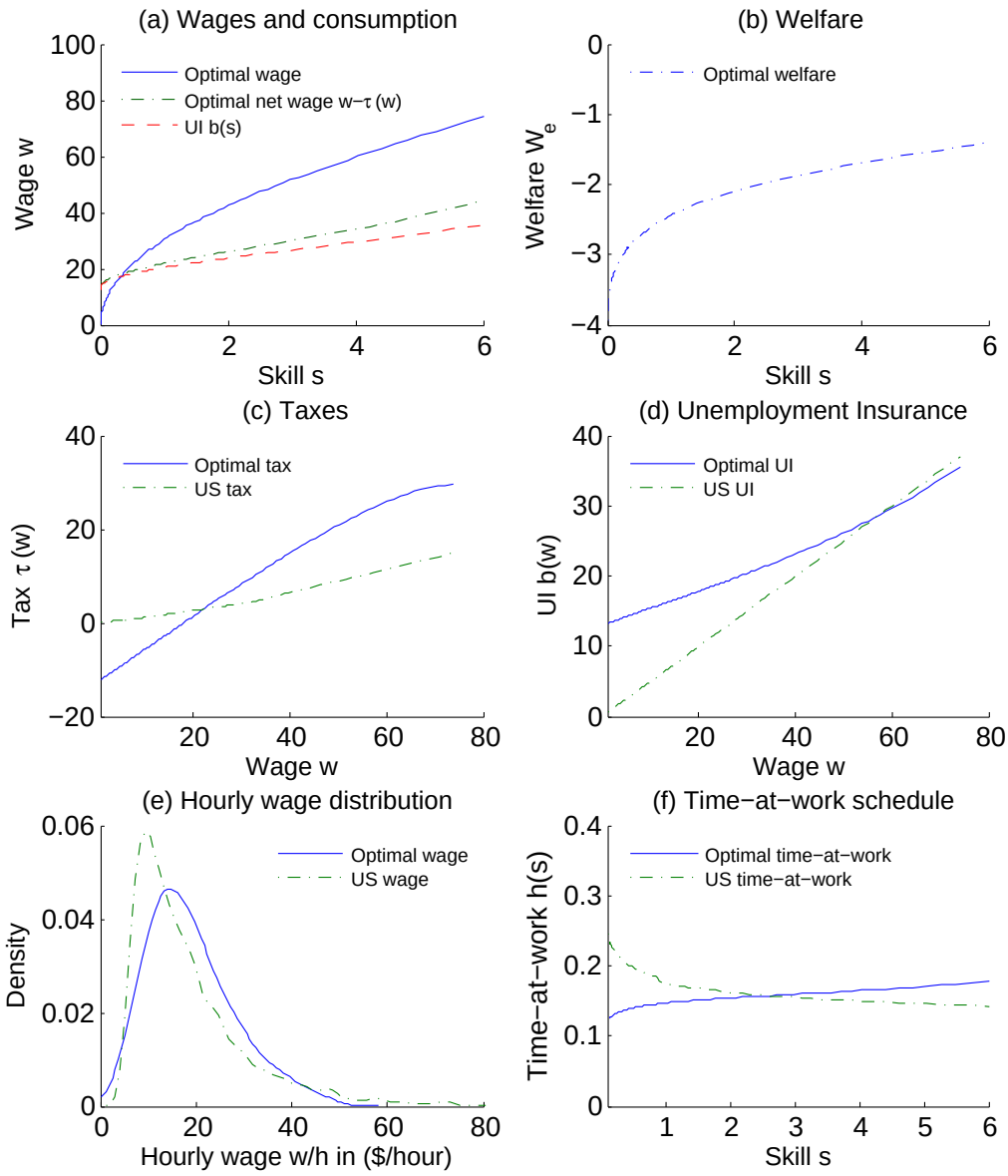


Figure 3.3: Comparing the optimal policy with the US policy

Panel (d) shows the optimal unemployment insurance schedule $b(w)$ and compares it to the one used by the state of New York. Both schedules compensate workers with high wages in a similar way while the optimal one is more generous to workers with low wages. Panel (e) shows the hourly wage distributions. The optimal one is shifted to the right and less skewed than the actual one. Notice also that in the optimal setup many workers earn very low wages. They are however compensated by the negative income tax such that their total consumption is higher.

Panel (f) presents the difference between the optimal time-at-work schedules and the one coming from the calibration. In the optimal case, high-skill workers are incentivized to be more productive. Two phenomena are at play here. The first one is a wealth effect: richer agents substitute consumption for leisure which gives a concave shape to the optimal schedule. The second effect comes from the fact that these agents are highly productive and that the planner wants them to work as much as possible. He aligns the appropriate incentives for that to happen. In facts, the incentive structure manages to change the monotonicity of the curve.

3.4.4 Economic forces at work

The optimal policy presents several robust features. First, $W_e(s)$, $w(s)$ and $b(s)$ are increasing concave functions of s (Figure 3.3). This is a direct consequence of the incentive constraints. Because high productivity workers can deviate and pretend to be low-skilled, the social planner needs to compensate them to reveal their true type. Remember from lemma 11 that the incentive constraint is

$$W_e'(s) = \frac{r + \theta q(\theta)}{r(r + \delta + \theta q(\theta))} v' \left(\frac{f^{-1}(y(s))}{s} \right) \frac{f^{-1}(y(s))}{s^2}.$$

Since the gain of a deviation from the equilibrium is high for low s, W_e is steep at the lower-end of the skill distribution and its slope progressively decreases as the skill increases. This explains the concave shape of the functions.

Also, production y and hours worked h increase with s. Production goes up with s simply because agents are increasingly productive and it becomes less costly for them to produce larger amounts. This is another interesting feature of the optimal policy: it is designed to induce productive workers to reveal their types and to produce more. A more striking element is the fact that working hours also increase with s. This suggests that the optimal tax is set such that the income effect is weak and the marginal rate of substitution between consumption and leisure is distorted so that productive agents actually work more. This can be explained by the following mechanism. Because the disutility of labor is convex, an increase in the hours worked of a low-skill agent s_l induces a large cost for him. However, if an high-skill agent were to claim the same contract, s_l, he would face a relatively lower change in his disutility of labor; the deviation becomes more profitable. Hence, the planner needs to compensate him more to prevent the deviation, which makes increasing the number of hours worked by low-skill agents very costly.

As a consequence of the incentive constraint, the equilibrium consumption and wage schedule are increasing and concave. Wages are however very low for unproductive agents. This comes from the fact that firms only offer contracts that yield positive profits. As production of low-productivity workers goes to 0, wages also have to go to 0. This is harmful in terms of welfare. Therefore, the optimal policy raises the utility of low-skill agents by increasing their unemployment benefits and by subsidizing their entry on the labor market by a *negative income tax*. As a result, the equilibrium tax $\tau(w)$ (Figure 3.3) is quite negative for low wages, and the unemployment benefits $b(w)$ is relatively higher for low-productivity workers.

Notice finally that the optimal tax appears to be close to linear for workers ranging in the middle of the skill distribution. This result is well known in public finance, but is not a robust feature of the optimal policy as it mainly depends on the functions used for the preferences. The curvature of the optimal tax actually changes with the preference parameters, or with other utility functions, as our sensitivity analysis will show.

3.4.5 Impact of policy instruments

Because of the general equilibrium setup, it is a difficult task to isolate precisely the effect of a single policy instrument on the whole economy. In order to get some intuition, we constrain the planner's optimization along some dimensions by fixing certain parameters to see how the optimal policy would adjust. For instance, by fixing $\underline{s}$ and θ to arbitrary values, it is possible to vary b_0 to have an idea of the effects of the benefits to inactive agents on the allocation. We do this exercise for these three parameters $\underline{s}$, θ and b_0.

Extensive margin of the labor supply $\underline{s}$: The extensive margin of labor supply matters for several reasons. As argued in section 3, the social planner wants to induce more people to work to increase production, raise more tax revenue and reduce his spending on welfare transfers. In order to do so, a negative income tax can be used to make people participate more in the labor market. On the other hand, the planner may prefer to limit workers entry on the labor market to ease the *congestion externality*. This can be done by lowering the subsidies at the lower-end of the skill distribution and by raising b_0 to encourage low-skill agents to stay out of the labor market. Figure 3.4 presents the simulated optimal policy for different values of $\underline{s}$, keeping the other parameters at their optimal values. Interestingly, the tax and unemployment insurance change very

little with $\underline{s}$. However, the planner increases the minimum wage observed in this economy. This amounts to put a very high tax on the lowest wages.

Unemployment and market tightness θ: The planner can also affect the unemployment rate U/N by acting on the firm's profits. This comes from the free-entry condition (equation (3.7)) and the steady-state condition (equation (3.13)). An increase in expected profits $\mathrm{E}\left(J_e(s)\right)$ leads to more vacancy posting, a higher labor market tightness θ and thus to a lower unemployment rate U/N. However, this raise in profits must either come from more hours worked or from a smaller wage, both of which lower utility for the agents. To change the level of profits, the social planner can vary the lump-sum transfer T and affect the tax level. Figure 3.5 shows the optimal simulated policy for different values of θ (different unemployment rates). We can see that the social planner prefers to reduce the tax level. As a result, wages go down, while production stays about the same. Profits however go up, increasing the equilibrium level of θ.

Transfers to inactive agents b_0: The transfer b_0 matters not only for the extensive margin, but most importantly because it sets the lowest level of utility in the economy. By raising b_0, workers' utility while inactive goes up. Therefore, to avoid deviations, equilibrium consumption schedules have to increase to compensate. Figure 3.6 shows that increasing b_0 requires an increase in taxes, so that the planner's resource constraint is balanced. As a consequence, production is discouraged and wages go down.

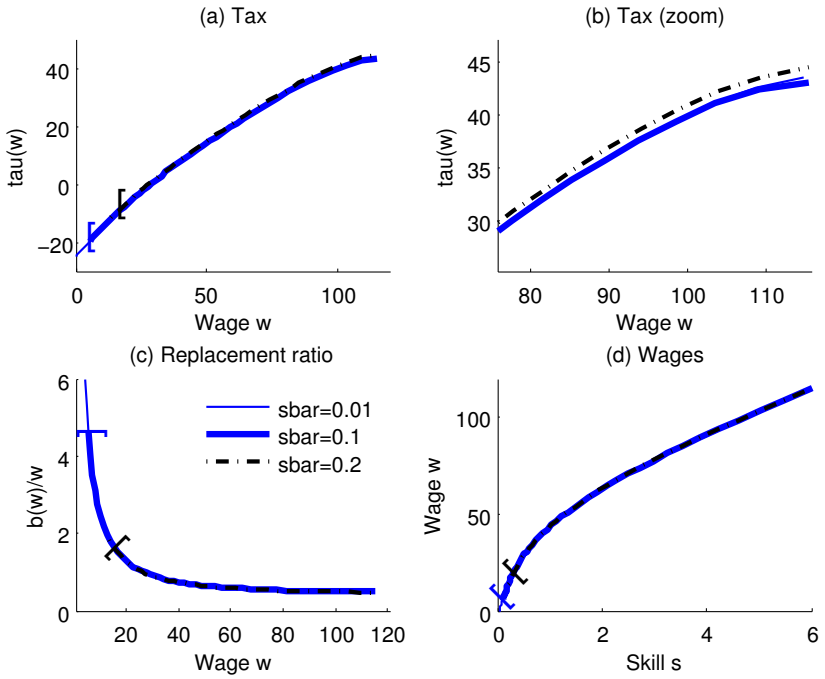


Figure 3.4: Optimal tax and unemployment insurance for $\underline{s} = 0.01, 0.1, 0.2$ keeping $\theta = 0.1$ and $b_0 = 25$

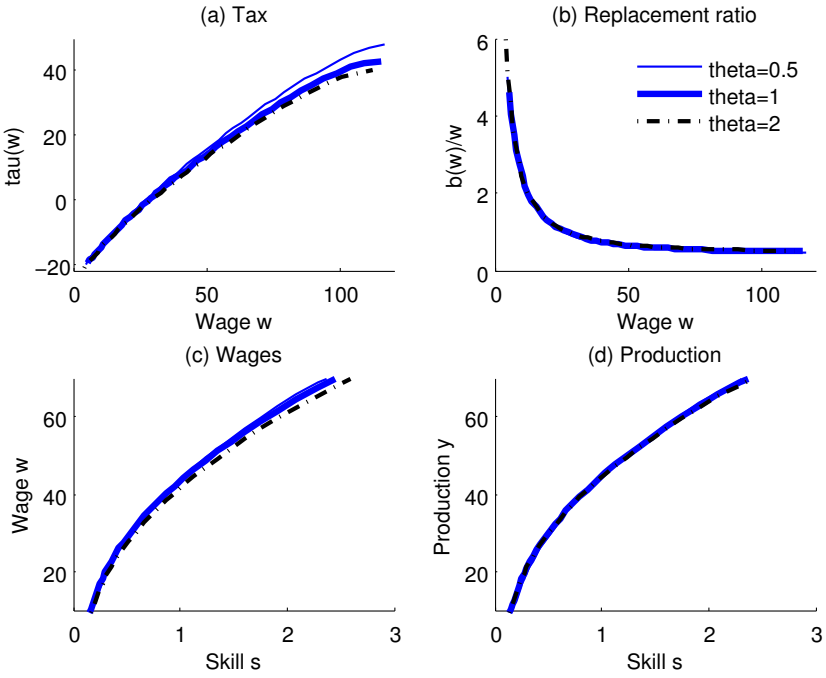


Figure 3.5: Optimal tax and unemployment insurance for $\theta = 0.5, 1, 2$

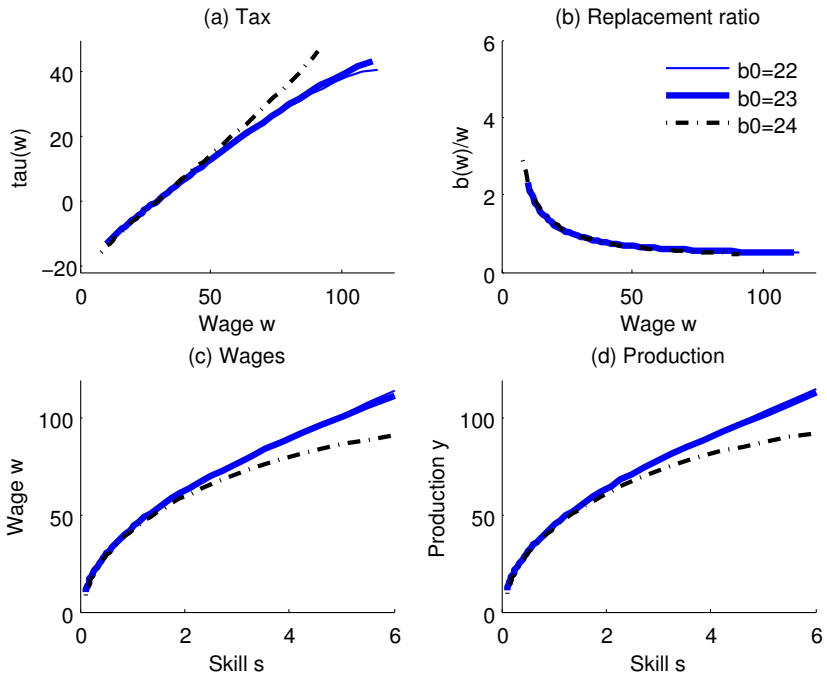


Figure 3.6: Optimal policy for $b_0 = 22, 23, 24$

3.4.6 Increasing the planner's taste for redistribution

We know investigate what happens to the optimal policy if the planner has a higher aversion to inequality. To do so, we increase the concavity of $\Phi(\cdot)$. For computational purposes, we set $\Phi(x) = -(-x)^{\xi}$ and vary ξ to modify the taste for redistribution. The utilitarian case of the last section corresponds to $\xi = 1$. The result of the optimization with $\xi = 4$ are presented in table 3.4 and figure 3.7.

	$\underline{s}$	b_0	θ	Unemployment rate
Calibrated	0.047	7.246	0.5242	5.2%
Optimal ($\xi = 1$)	0.001	12.69	0.9575	3.9 %
Optimal ($\xi = 4$)	0.0014	14.64	0.9170	3.98%

Table 3.4: Optimal policy under different welfare preferences

The planner pushes the skill threshold $\underline{s}$ from 0.001 to 0.0014, a small difference. The planner does not want to limit access to the labor market. The unemployment payments to inactive agents, b_0, goes from \$12690 to \$14640, as expected by increasing the taste for redistribution.

Panel (a) of figure 3.7 shows that high-skill agents are more taxed with $\xi = 4$ while low-skilled ones receive a higher subsidy. The same is true for unemployment benefits, as shown in panel (b). Panel (c) shows the wage schedules. High-skill workers receive a higher wage but are taxed more heavily and work longer hours (panel (d)). Low-skill agents, on the opposite, enjoy much more leisure. Panel (e) show the welfare schedules. Although it is not clearly visible on the graph, the two schedules intersect. As expected, low-skill agents are better off with $\xi = 4$ while high-skilled ones are worse-off. The final panel, (f), shows the wage distributions being shifted to the left with the more redistributive policies.

Notice that the policies observed with $\xi = 4$ share some features with *European* economies: more progressive taxation and generous benefits, which results in fewer hours worked and lower total production. However, the increase in the unemployment

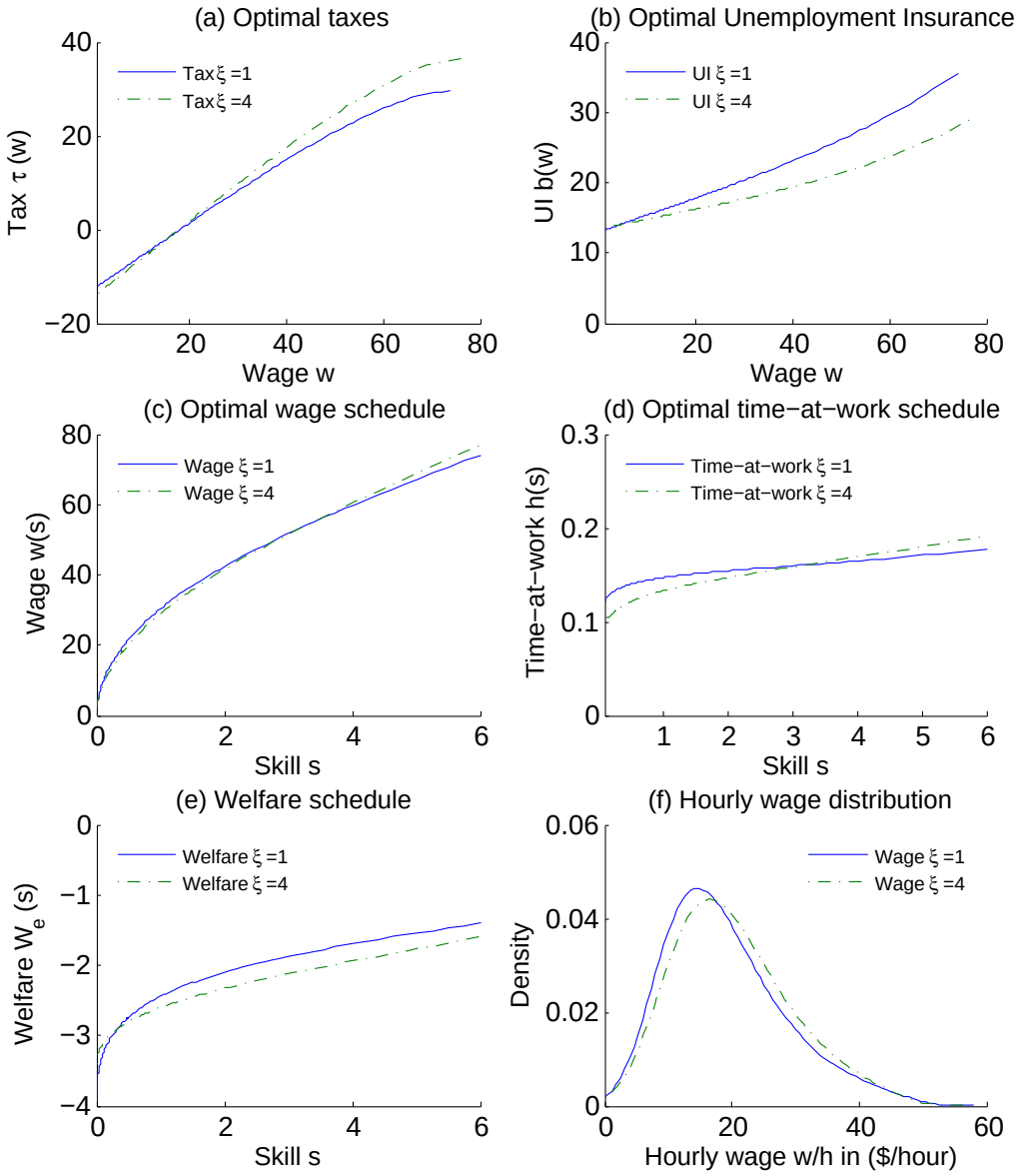


Figure 3.7: Increasing the taste for redistribution: optimal policies with $\xi = 1$ and $\xi = 4$

rate, from 3.9% to 3.98%, is not representative of the observed discrepancies between Europe and the US. Obviously, these numbers represent a comparison of economies under *optimal policies*. There are indeed reasons to believe that observed policies

depart from optimality. Therefore, this last exercise should not be understood as an attempt to account for cross-country differences but instead to emphasize general features that characterize more egalitarian policies. Generous unemployment benefits and heavier taxation should not always be considered sub-optimal. Good policy designs may in fact keep the unemployment level low even with highly redistributive policies.

3.5 Conclusion

This paper has studied the optimal design of policies in a frictional labor market with adverse selection. Heterogeneous workers with unobservable productivity search for jobs in a unique labor market governed by an aggregate matching function. When they meet, firms and workers play a signaling game where workers first reveal a type and firms then offer a contract. We study the optimal mechanism for a general class of welfare functions (including utilitarian) and show that the constrained optimal allocation is implementable in an economy where wages are observable by a non-linear tax on wages, a non-linear unemployment insurance and firm subsidies. Our baseline calibration suggests that switching to the optimal policy would reduce unemployment, increase labor market participation, and increase welfare by 17.5% in consumption equivalent. Our findings also suggest that optimal policies often feature a negative income tax and generous replacement ratios for low-skilled people. Also, increasing the planner's taste for redistribution yields policies that display some *European* features: higher marginal tax rates, more generous benefits and lower hours and production.

The findings in this paper could be extended in several dimensions. First of all, there is no credit market. The government is therefore the only provider of insurance to people. This feature tends to overstate the welfare gains of implementing the optimal policy, and explains why we obtain such a generous unemployment insurance. Also, we have abstracted from possible frictions like imperfect monitoring of search effort, which would also limit the capacity of the government to insure people.

A last extension worth exploring would be to study the full dynamic model and allow for time-dependent policies. However, such an extension would require a tractable dynamic model of the labor market with heterogeneous agents, in which solving for the optimal policy design would become much more complicated. Another interesting dimension could be to introduce human capital and investigate how policies could

influence agents' decisions to look for jobs in industries with higher risks but with better learning opportunities.

3.6 Sensitivity analysis

We proceed to some sensitivity analysis to see how the shape of the optimal policy functions depend on the model's parameters. We perform a similar kind of comparative statics as in the previous section: fixing $\underline{s}$, θ and b_0, we solve for the optimal control problem and present the resulting policy functions. Graphs of the optimal policies are presented in the appendix.

Intertemporal elasticity of substitution $1/\gamma$**:** The optimal policy changes dramatically even for small variations of γ, keeping all other parameters constant (b_0, θ and $\underline{s}$). Figure 3.8 presents the optimal policy for $\gamma = 1.8$, 1.9 and 2. We can see that increasing γ makes our policy functions more concave. Note that γ is also the relative risk aversion of agents in our economy. Also, the tax tends to increase with γ, while production and wages tend to decrease. All these effects come from the marginal utility of low-skill workers. The planner allocates more resources to them and finances these transfers with higher taxation on high-skill agents.

Frisch elasticity of labor supply φ**:** Figure 3.9 presents the optimal policies for $\varphi = 0.4$, 0.45 and 0.5. Results are very similar to the case where γ varies. Part (ii) of lemma 11 states that the slope of W_e is increasing with φ (since $h(\cdot)$ is strictly lower than 1 in all the simulations). This shows that an increase in φ makes deviations more profitable. To prevent low-skill workers from deviating upward, the planner reduces the wage and the production of high-skill agents. This is done through the tax scheme.

Matching elasticity μ: Remember that the matching function is $m(U,V) = MU^{\mu}V^{1-\mu}$. Figure 3.10 shows the optimal policy when the matching elasticity is $\mu = 0.5$, 0.6, and 0.7. The results are a similar to those obtained when θ varies. The explanation is simple: μ only changes the rate at which agents are matched in a steady-state. Raising μ when $\theta = 0.5$ increases the rate at which agents find a job in the economy, while decreasing the matching rate for firms. A higher matching rate for agents increases the slope of $W_e(s)$ and requires the planner to compensate more for possible deviations. Since an increased unemployment insurance can put pressure on the resource constraint, the planner actually reduces the equilibrium production, therefore lowering agents' claims for unemployment benefits. As a consequence, the wage schedule also decreases.

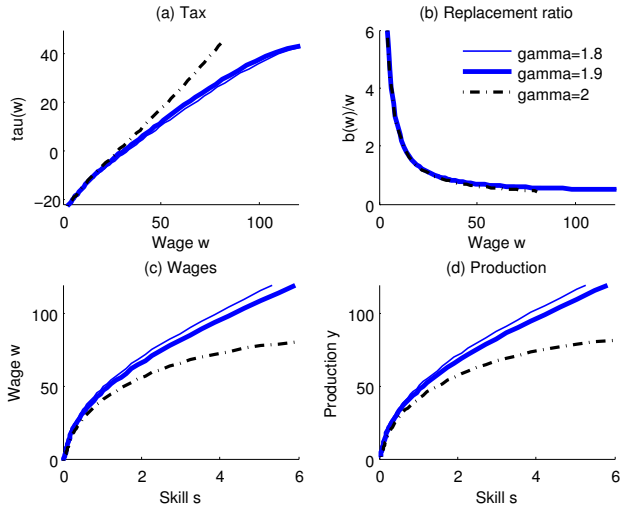


Figure 3.8: Optimal policy for $\gamma = 1.8, 1.9, 2$ keeping $\theta = 1$, $b_0 = 25$ and $\underline{s} = 0.01$

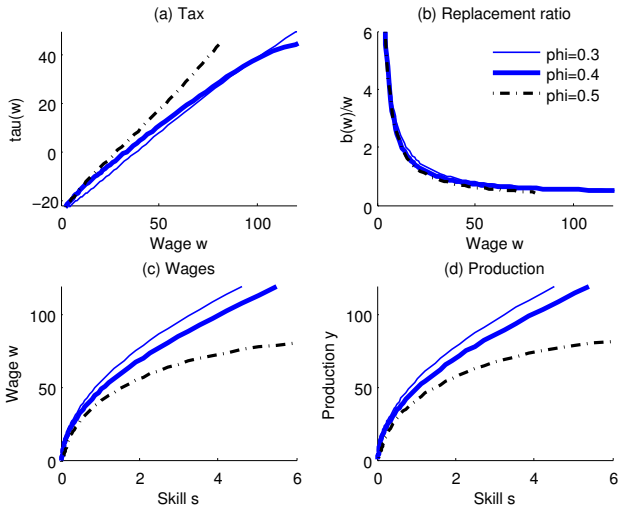


Figure 3.9: Optimal policy for $\phi = 0.4, 0.45, 0.5$ keeping $\theta = 1$, $b_0 = 25$ and $\underline{s} = 0.01$

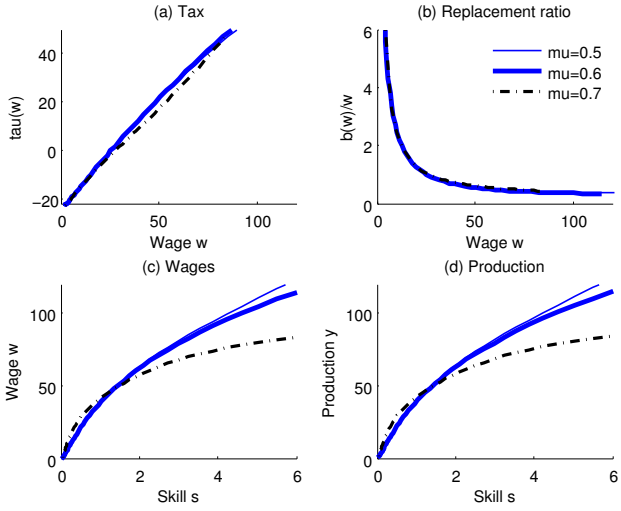


Figure 3.10: Optimal policy for $\mu = 0.5, 0.6, 0.7$ keeping $\theta = 0.5$, $b_0 = 25$ and $\underline{s} = 0.01$

3.7 Proofs

Here are the proofs from the previous sections.

Lemma 10. *If the social planner's optimal transfers* $\{c_e(\tilde{s}), c_u(\tilde{s}), c_0, t_e(\tilde{s}), t_u\}$ *and firm's offered production* $y(\tilde{s})$ *are differentiable functions of* $\tilde{s}$*, then it is optimal for the worker to reveal his type.*

Proof of lemma 10. Taking the first-order condition for a worker of type s willing to work in equation (3.18) yields:

$$u'\big(c_e(s^*)\big)c_e'(s^*) - v'\left(\frac{f^{-1}(y(s^*))}{s}\right)\frac{y'(s^*)}{sf'(f^{-1}(y(s^*)))} + \frac{\delta}{r+\theta q(\theta)}u'(c_u(s^*))c_u'(s^*) = 0$$

The left-hand side of the equation is a strictly increasing function of s. Therefore, this condition can be satisfied by at most one type s. Workers never choose the same contract. □

Lemma 11. *In the second-best optimal allocation:*

(i) If there exists a s *such that* $\chi(s) = 1$*, then* $\chi(s') = 1$ *for all* $s' \geq s$*. We define* $\underline{s}$ *as the smallest* s *such that* $\chi(s) = 1$*.*

(ii) $W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))}v'\left(\frac{f^{-1}(y(s))}{s}\right)\frac{f^{-1}(y(s))}{s^2}$ *and is therefore increasing*

(iii) $W_e(s) = W_u(s)$ *for all* s

(iv) $J_e'(s) = 0$ *and therefore, in equilibrium,* $J_e(s) = \kappa/q(\theta)$

(v) $W_e(\underline{s}) = u(c_0)/r$

(vi) $c_u(s)$ *is increasing*

Proof of lemma 11. (i) Suppose not. Therefore, there is $s' > s$ such that $\chi(s') = 0$ and $\chi(s) = 1$. From equation (PC inactive), we have

$$\begin{aligned} u(c_0) &\geq \frac{(r+\theta q(\theta))(u(c_e(s)) - v(f^{-1}(y(s))/s')) + \delta u(c_u(s))}{r+\delta+\theta q(\theta)} \\ &> \frac{(r+\theta q(\theta))(u(c_e(s)) - v(f^{-1}(y(\tilde{s}))/s)) + \delta u(c_u(s))}{r+\delta+\theta q(\theta)} \\ &= rW_e(s) \end{aligned}$$

But, since $\chi(s) = 1$, equation (PC active) states that $rW_e(s) \geq u(c_0)$, which is a contradiction.

(ii) Recall that

$$\begin{aligned} W_e(s) \equiv \max_{\tilde{s}} \frac{1}{r+\delta} \Bigg[& u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) \\ & + \frac{\delta}{r+\theta q(\theta)} \left(u(c_u(\tilde{s})) + \theta q(\theta) W_e(s)\right) \Bigg] \end{aligned}$$

which can be rewritten

$$W_e(s) = \max_{\tilde{s}} \frac{\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right)\right)(r+\theta q(\theta)) + \delta u(c_u(\tilde{s}))}{r(r+\theta q(\theta)+\delta)}$$

Applying the envelope theorem on this last expression gives the result.

(iii) The result comes directly from the firm's problem. An increase in $\tilde{y}$ yields to an increase in profit and reduces the left-hand side of the agent's participation constraint. Therefore, the constraint binds.

(iv) Remember that in a truth-telling equilibrium, the firm's problem is

$$J_e(s) = \max_{\tilde{y},\tilde{s}} \frac{\tilde{y} + t_f(\tilde{s})}{r+\delta}$$
$$s.t. \frac{1}{r+\delta}\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)}(u(c_u(\tilde{s})) + \theta q(\theta) W_e(s))\right)$$
$$\geq W_u(s)$$

Since $W_e(s) = W_u(s)$, the Lagrangian is

$$\mathcal{L}(s) = \tilde{y} + t_f(\tilde{s}) + \lambda\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) - r\frac{r+\theta q(\theta)+\delta}{r+\theta q(\theta)} W_e(s)\right)$$

The envelope theorem with part *(ii)* yield $J_e' = 0$. In equilibrium, recall from equation (3.7) that

$$\frac{\kappa}{q(\theta)} = \frac{\int_{\underline{s}}^{\infty} J_e(s) g(s) ds}{N} \tag{3.26}$$

Since $J_e(s)$ is constant, the result follows.

(v) Since $\underline{s}$ is the first working agent, we know that $rW_e(\underline{s}) \geq u(c_0)$. Suppose that the inequality is strict and consider the agents of type $(\underline{s} - \epsilon)$ for $\epsilon > 0$ small. Since $\underline{s} - \epsilon < \underline{s}$, $\chi(\underline{s} - \epsilon) = 0$ and

$$u(c_0) \geq \frac{(r+\theta q(\theta))\left(u(c_e(\underline{s})) - v\left(\frac{f^{-1}(y(\underline{s}))}{\underline{s}-\epsilon}\right)\right) + \delta u(c_u(\underline{s}))}{r+\delta+\theta q(\theta)} \equiv \Gamma_{\underline{s}}(\epsilon)$$

Since v is continuous, we can take the limit $\lim_{\epsilon\to 0} \Gamma_{\underline{s}}(\epsilon) = W_e(\underline{s})$. This contradicts the supposition $rW_e(\underline{s}) > u(c_0)$ and completes the proof.

(vi) We have $c_u(s) = u^{-1}(rW_e(s))$ and the result follows since $W_e(\cdot)$ is increasing.

□

Proposition 1. *If wages are observable, an optimal allocation in this economy is implementable using a non-linear income tax on workers $\tau(w)$, unemployment benefits $b(w)$, a transfer b_0 to inactive agents, and in some cases supplemented by a uniform subsidy to firms T.*

Proof of proposition 1. Implementing the equilibrium means we can find an equilibrium wage schedule $w(s)$ and functions $b(w)$, $\tau(w)$ and a subsidy T such that the following holds:

$$\begin{cases} c_e(s) = w(s) - \tau(w(s)) \\ c_u(s) = b(w(s)) \\ t_f(s) = -w(s) + T \end{cases}$$

The first thing to notice is that the level of the firm subsidy T is indeterminate. The existence of such a subsidy is only to prevent wages to be negative in equilibrium. Indeed, $t_f(s)$ can in principle be positive in equilibrium. Therefore, one needs to choose a subsidy high enough so that wages $w(s) = T - t_f(s)$ are always positive. Otherwise, its absolute level does not matter as wages and taxes can simply be adjusted to yield the same allocation. We are going to construct a tax system that implement the mechanism. Pick any $T > \max_s(t_f(s))$.

The only critical part of the proof is to show that the wage can be a sufficient statistic for the skill, so that we can base transfers on the wage with functions $\tau(w)$ and $b(w)$. We are going to show that $t_f(s)$ is different for all s. Assume by contradiction that there exists $s < s'$ such that $t_f(s) = t_f(s')$. Remember that in a truth-telling

equilibrium, the firm solves the following problem:

$$\max_{\tilde{y},\tilde{s}} \frac{\tilde{y} + t_f(\tilde{s})}{r+\delta}$$
$$s.t. \frac{1}{r+\delta}\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)}(u(c_u(\tilde{s})) + \theta q(\theta) W_e(s))\right) \geq W_u(s)$$

Since $t_f(s) = t_f(s')$, the firm picks the type $\tilde{s}$ that gives the highest utility to the worker:

$$\max_{\tilde{s}\in\{s,s'\}} u(c_e(\tilde{s})) + \frac{\delta}{r+\theta q(\theta)} u(c_u(\tilde{s}))$$

as it enables it to increase production. In a truth-telling equilibrium, types s and s' have to be chosen, so the firm must be indifferent between both:

$$u(c_e(s)) + \frac{\delta}{r+\theta q(\theta)} u(c_u(s)) = u(c_e(s')) + \frac{\delta}{r+\theta q(\theta)} u(c_u(s')) \tag{3.27}$$

Now, turn to the worker's problem. Define:

$$V(s,\tilde{s}) = u\big(c_e(\tilde{s})\big) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \frac{\delta}{r+\theta q(\theta)} u(c_u(\tilde{s}))$$

A worker of type s solves the following problem:

$$\max_{\tilde{s}\in\mathbb{R}_+^*} V(s,\tilde{s})$$

Transfers for agents s and s' yield the same expected utility. Thus, workers always choose to declare the type with the smallest production $y(s)$ or $y(s')$. In a separating equilibrium, both contracts must be chosen. This can only happen if workers are indifferent between the two contracts: $y(s) = y(s')$. This will however lead to a contradiction.

Given the differentiability assumption, we can write the following derivative for agent s:

$$\begin{aligned}\frac{\partial}{\partial \tilde{s}} V(s,\tilde{s}) =& u'\big(c_e(\tilde{s})\big)c_e'(\tilde{s}) - v'\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right)\frac{y'(\tilde{s})}{sf'(f^{-1}(y(\tilde{s})))} \\ &+ \frac{\delta}{r+\theta q(\theta)} u'(c_u(\tilde{s}))c_u'(\tilde{s})\end{aligned} \tag{3.28}$$

In a separating equilibrium, agents reveal their true types. Thus: $\frac{\partial}{\partial \tilde{s}} V(s,s) = 0$ and $\frac{\partial}{\partial \tilde{s}} V(s',s') = 0$. Note that (3.28) is a strictly increasing function of s, the true type of the worker. Therefore, given that the first-order condition is satisfied for agent s' at $\tilde{s} = s'$, i.e $\frac{\partial}{\partial \tilde{s}} V(s',s') = 0$, we must have:

$$\frac{\partial}{\partial \tilde{s}} V(s,s') < 0$$

The same function evaluated at s instead of s' is strictly negative. That means: there are contracts just below s' that yield a higher expected utility to agent s. Given his indifference between claiming types s and s', agent s would rather deviate and claim a type just below s' than choose his own contract. This cannot be true in equilibrium. The initial assumption that there exists $s \neq s'$ with $t_f(s) = t_f(s')$ was wrong. We conclude that all wages in equilibrium are different and reveal the worker's type. Thus, set:

$$w(s) = T - t_f(s)$$

It is therefore possible to find functions $\tau(w)$ and $b(w)$ that implement the optimal allocation, i.e:

$$\begin{cases} \tau(w(s)) = w(s) - c_e(s) \\ b(w(s)) = c_u(s) \end{cases}$$

To prevent agents from choosing out-of-equilibrium wages $\tilde{w}$, the government can set a high tax $\tau(\tilde{w}) = 1$. □

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