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Essays in the Economics of Inequality

Moser, Christian Alexander

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Essays in the Economics of Inequality

Christian Alexander Moser

A Dissertation
Presented to the Faculty
of Princeton University
in Candidacy for the Degree
of Doctor of Philosophy

Recommended for Acceptance
by the Department of
Economics
Advisers: Mikhail Golosov and Nobuhiro Kiyotaki

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Abstract

What are the drivers of recent trends in earnings inequality? What role have economic policies played in this evolution? How should optimal policy relating to inequality and redistribution be designed? Finding answers to these questions is central to improving economic welfare in a world that is experiencing rapid shifts in the distribution of income. This collection of essays contributes to our understanding of the determinants of earnings inequality and the design of optimal redistributive policies.

Chapter 1, co-authored with Jorge Alvarez and Niklas Engbom, uses administrative matched employer-employee data to decompose a decline in earnings inequality in Brazil from 1996 to 2012. Almost half of the overall decline is due to a fall in firm-specific pay differences, while around one quarter of the decline stems from falling pay differences due to unobserved worker characteristics. A large share of the decline is attributable to changes in firms' wage setting policies.

Chapter 2, co-authored with Niklas Engbom, quantifies the contribution of a rise in the minimum wage to Brazil's inequality decline in an equilibrium search model with heterogeneity in worker ability and firm productivity. Monopsonistic competition among firms for workers leads to spillover effects of the minimum wage on higher earnings ranks. We estimate the model using indirect inference and find that the rise in the minimum wage explains 70 percent of the decline in the variance of log earnings.

Chapter 3 examines the interaction between financial frictions, barriers to entry, and firms' extensive margin decisions in a dynamic occupational choice model. Inefficient firm liquidations lead to a substantial welfare loss. Government subsidies to the least productive entrepreneurs can have sizable effects on aggregate productivity. Efficiency implications of such interventions depends crucially on the persistence of the stochastic process underlying entrepreneurs' productivity evolution.

Chapter 4, co-authored with Pedro Olea de Souza e Silva, analyzes retirement savings policies in a model of optimal taxation with unobservable differences in earnings ability and heterogeneity in time preferences. A government with redistributive and paternalistic motives optimally offers low income individuals a one-size-fits-all savings instrument while offering high earners a set of savings policies.

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I believe Economics truly is a "social" science in that our field flourishes through active exchange among ourselves. Studying Economics at Princeton has been a special experience because the people that one could interact with here shared one mindset: that we will be better economists if we hold each other to higher standards. This dissertation represents but a snapshot of where I stand on this continuous path.

I would not have traveled far without the generous guidance from many people. Mikhail Golosov, my co-adviser, has instilled in me the ambition to ever improve on myself. He has been stern when I would benefit from such nudges, and has shown true empathy in critical times. I will be ever grateful that, though never fully understand why, he picked me up when I was in a confused state. Nobuhiro Kiyotaki, also my co-adviser, has taught me to reduce the complexity of the world around us. He provided generous guidance as my first mentor at Princeton. His patient support has carried me a long way. Mark Aguiar has been a refreshing force on my committee. I have benefitted greatly from his encouragement and strive to emulate his humility. Richard Rogerson has been a committee member who steered me in the right direction when I was drifting. My thinking sharpened as a result of our frequent conversations and email exchanges. Were I to append them, they would form a dissertation in themselves.

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I am especially thankful to my family, in particular my mother, Petra Lattner. She has always put her children first and enabled us to pursue an education, which she had been denied. Growing up with two brothers, I learned about the benefits of competition from an early age. The strength of family ties, I believe, reveals itself in times of need.

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To my mother

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Chapter 1

Firms and the Decline in Earnings Inequality in Brazil

1.1 Introduction

Brazil has experienced a large reduction in earnings inequality since the mid-1990s. This came after decades of Brazil being infamously known as the most unequal country in Latin America, which itself ranked among the most unequal regions in the world. While the decline of earnings inequality in Brazil resembles other Latin American economies' experience during this period,[1] it stands in stark contrast to that of the U.S. and many developed countries, which saw inequality steadily increasing over the past two decades.[2] To investigate the sources of this decline, in this paper we decompose Brazil's earnings inequality evolution into changes in firm and worker characteristics on the one hand, and the returns to such characteristics on the other.

[1] See Lopez and Perry (2008) and Tsounta and Osueke (2014).

[2] Using administrative data, Kopczuk, Saez and Song (2010) document in detail earnings inequality trends for the U.S., while chapter 8 of Atkinson and Bourguignon (2015) discusses inequality trends in middle- and high-income countries.

Guided by recent research, which suggests that firms are an important determinant of earnings dispersion in a number of high-income countries,[3] we decompose the sources of Brazil's inequality decline by exploiting a large administrative linked employer-employee dataset containing information on over one billion job spells between 1988 and 2012. By linking individual workers to their employers and tracking both over time, we are able to separately identify the contributions of firm- and worker-specific factors towards worker's pay (Abowd, Kramarz and Margolis, 1999). Applying this econometric framework repeatedly within overlapping subperiods, we decompose the overall inequality decline into changes between firms and between workers. We, we investigate the link between firm performance and the firm component of pay by linking our first stage estimation results to another confidential dataset containing detailed information on the characteristics of hundreds of thousands of Brazilian firms between 1996 and 2012.

We uncover three main results. First, firms played an important role in the decline in earnings inequality in Brazil over this period, explaining 45 percent of the fall in the variance of log earnings between 1996 and 2012. Compression in worker fixed effects explains an additional 24 percent of the decline, with the remaining part being attributed a decline in the covariance between worker and firm fixed effects and the residual. As worker heterogeneity is most important in levels, the compression in firm-specific pay components contributed more than proportionately towards Brazil's inequality decline.

Second, changes in the link between firm performance and pay accounts for a significant fraction of the compression in the firm component of workers' earnings. We first show that a substantial share of the variation in the firm component of pay is explained by differences in observable firm characteristics, with more productive firms paying more. Moreover, more than half of the decline in the firm component

[3]See Abowd, Kramarz and Margolis (1999); Card, Heining and Kline (2013); Barth, Davis and Bryson (2014); Bloom et al. (2015), and Mueller and Ouimet (2015).

is accounted for by observable firm characteristics. All of this decline is driven by a weakening pass-through from productivity to pay, and none is due to firms becoming more similar in observable characteristics. A weaker link between observable firm characteristics and worker pay thus explains 25 percent of the overall fall in the variance of log earnings over this period.

Third, a decline in the return to measures of ability such as experience and education explains a sizable share of the fall in the variance of the worker component of pay. In levels, age, education and occupation explain around 40 percent of the variance of the worker component of pay. However, we observe no compression in the underlying distributions of such characteristics over time. Instead, the inequality decline is driven by a rapid fall in the returns to observable measures of worker ability, especially the returns to education. We find that lower returns to education explain 13 percent and lower returns to age explain three percent of the overall fall in the variance of log earnings over this period.

This decomposition of the sources of Brazil's earnings inequality decline informs our understanding of various commonly proposed stories explaining the decline. On the worker side, our results do not support a widely-held belief that changes in educational attainment accounted for a significant share of Brazil's inequality evolution over the period.[4] While educational attainment increased over this period, we find that all else equal this resulted in a small net *increase* in inequality. We reach similar conclusions regarding the implications of changes in the age structure of the workforce. On the firm side, a reading of the existing literature would suggest that trade dynamics and the productivity evolution during this period could have been important drivers behind changes to the earnings distribution.[5] Yet, in line with U.S. trends, we find that also the Brazilian productivity distribution grew more dispersed over this period.

[4]See for example Barros et al. (2010).

[5]See for example Dunne et al. (2004) and Faggio, Salvanes and Van Reenen (2010), who conclude that parts of the increase in earnings inequality in the U.S. can be explained by widening dispersion in the firm productivity distribution.

Our findings thus pose a challenge to candidate explanations for the decline in earnings inequality over this period. Our results suggest that a theory of the inequality decline needs to be consistent with the following three facts: (i) firm-level changes explain a significant share of initial inequality levels and an even larger share of its decline; (ii) a weaker pass-through from firm productivity to worker pay was a key driver behind the declining dispersion in the firm component of pay; and (iii) a lower return to worker ability explains a significant share of the decline in the worker component of pay. We conclude that changes in pay policies rather than in worker and firm fundamentals played an important role in the decline in inequality in Brazil during this period.

Related literature. With the current project we contribute to three broad strands of the literature. First, we provide a decomposition of earnings into worker and firm heterogeneity in a developing economy. With the increasing availability of large, administrative matched employer-employee datasets, a recent literature has started to examine the role of firms in wage determination. The first paper to make use of such large, linked employer-employee datasets to jointly study the role of worker unobservables and firms for pay is Abowd, Kramarz, and Margolis (1999, henceforth AKM), who study the role of firm and worker heterogeneity for wage inequality in France. They find an important role for firms in generating earnings inequality. Similar conclusions using the same methodology have been reached among others for the state of Washington in the U.S. (Abowd, Creecy and Kramarz, 2002), Denmark (Bagger, Sorensen and Vejlin, 2013), Austria (Gruetter and Lalive, 2009), and Germany (Card, Heining and Kline, 2013). The last paper is close to our methodology of applying the AKM framework in overlapping subperiods to study changes in wage determinants over time. They find that increasing dispersion in the firm-specific component of pay contributed significantly to rising earnings inequality in West Germany. Bloom

et al. (2015) highlight firms as an important driver behind the increase in U.S. labor earnings inequality since 1980, but do not employ the AKM methodology to control for sorting of highly-paid workers into high-paying firms.

Second, we highlight the sources of changes in the worker and firm component of inequality by studying the role of changes in observable characteristics versus changes in the return to such characteristics. AKM study the link between firm observable characteristics and the estimated firm effects, but do not focus on changes over time to this relationship. Menezes-Filho, Muendler and Ramey (2008) study the link between firm characteristics and wages in Brazil in the cross-section of linked data on worker earnings and firm characteristics in Brazil's manufacturing and mining sectors. Bagger, Jesper and Holloway (2014) investigate the role of labor misallocation in driving the positive correlation between labor productivity and wages at the firm using Danish data. Card, Cardoso and Kline (2015) study the degree of rent-sharing in Portugal with a particular emphasis on gender differences in profit participation and the allocation of workers across firms.

Third, we add to the literature on the sources of the decline in inequality in Brazil by providing a comprehensive decomposition of the decline. Many previous papers have studied the role of specific, isolated mechanisms in Brazil's inequality decline over the last two decades. For example, Ulyssea (2014) considers the role of worker flows between the informal and formal sectors. In related work, de Araujo (2014) studies the role of labor adjustment costs in propagating wage inequality in a frictional search framework. Dix-Carneiro and Kovak (2015) analyze the long-lasting impact of industry-specific tariff cuts in the presence of wage-equalizing migration. Barros et al. (2010) use Brazilian household data to study inequality trends since 1977 and decompose the decline in labor earnings inequality in Brazil since 1990. Given their data and method, those authors conclude that the inequality decline was in equal shares driven by education reform and labor market integration. Medeiros, Souza

and de Castro (2014) use administrative tax return data to study the evolution of top income inequality in Brazil from 2006–2012, but they cannot distinguish between the role played by worker versus firm characteristics during that period. Using linked employer-employee data, Lopes De Melo (2013) conducts a static decomposition of earnings inequality levels in Brazil's formal sector into components due to firms and workers. Helpman et al. (2013) use the same worker-level data to show that a significant share of overall wage inequality is due to between-firm differences and that Brazil's trade liberalization starting in the late 1980s led to increasing between-firm earnings inequality.

Outline. The rest of the paper is structured as follows: Section 1.2 provides an overview of the main institutional changes and macroeconomic trends affecting Brazilian labor markets from 1988 to 2012. Section 1.3 summarizes the administrative datasets used in our empirical analysis and discusses sample selection and variable definitions. Section 1.4 provides descriptive statistics on trends in earnings inequality in Brazil during this time. Section 1.5 introduces the empirical framework we use to decompose the variance of log earnings into a worker and firm effect as well as the subsequent regressions we run to link these estimates to worker and firm fundamentals. Section 1.6 presents our main empirical results as well as checks on the validity of our empirical framework. Finally, Section 1.7 summarizes our key findings and concludes.

1.2 Institutions and macroeconomic trends in Brazil

During our period of study, Brazil resumed democratic elections (1989), ended a decade of hyperinflation (1994), and inaugurated two decades of sustained economic growth—between 1996 and 2012 real gross domestic product grew by 2.3 percent per year on average. In this section, we discuss some of the institutional changes that

could have have affected inequality during this period, including labor regulation, trade liberalization and social policy.

Brazil had a highly regulated labor market before reforms started in the late 1980s. For instance, since 1965, a national Wage Adjustment Law mandated yearly wage increases for all workers in the economy and dismissal costs were high. After the transition to civil rule and the signing of a new constitution in 1988, flexibility in labor markets was further affected by firing penalties and an increased power of labor unions, which gather about a quarter of employed formal workers in Brazil.[6] The Wage Adjustment Law was finally abandoned in 1995, introducing a period of greater flexibility and less regulated wage-setting practices. Further legislation in 1997–1998 eased restrictions on temporary contracts and lowered dismissal barriers. Subsequently, formal employment increased by around five percent and unemployment fell from 10 percent in 2000 to around six percent in 2011 (World Bank, 2015). The overall labor participation rate has remained stable at 73–75 percent over this period.[7]

Hyperinflation also encouraged the adoption of automatic wage adjustment practices. From 1980 to 1989, yearly inflation averaged 355 percent, which was followed by a yearly average of 1,667 percent between 1990 and 1994 (World Bank, 2015). As a result, wage indexation to the minimum wage became the norm, with labor payments being adjusted first annually and then on a monthly basis proportionately to the previous period's realized inflation rate. In 1994, hyperinflation finally subsided with the introduction of the "Real Plan". This ambitious stabilization program introduced a gradual float of the local currency, tightened monetary and fiscal policy, and lowered inflation below two-digits.

[6]Bioto and Marcelino (2011) argue that there has been an uptake in labor strike activity in Brazil since the year 2000.

[7]Labor force as a percentage of total population aged 15–64, from OECD Employment and Labor Market Statistics.

In parallel to monetary stabilization, Brazil also undertook trade liberalization reforms during this period. Starting with initially high import tariffs that had substituted import bans from the previous decade, a series of trade liberalization bills in the late 1980s eliminated selected tariffs and eradicated quantitative import controls. When social democrat Fernando Henrique Cardoso became president in 1995, he strengthened this agenda with a reduction of tariff and non-tariff trade barriers to one tenth of their levels in 1987 (Pavcnik et al., 2004). The opening up to trade over the last 25 years has been frequently cited as a major contributor to the country's growth in total factor productivity (TFP)(Ferreira and Rossi, 2003; Ferreira, Leite and Wai-Poi, 2007; Moreira, 2004; Muendler, 2004; Córdova and Moreira, 2003). In addition, Helpman et al. (2013) argue that trade reforms contributed to the rise in income inequality seen in the late 1980s and early 1990s, and later to the start of the decline in wage dispersion in 1995.

Health, education and other social programs began expanding during the late 1990s, a trend that strengthened once the left-wing Workers' Party ascended to power in 2003. It doubled social expenditure as a fraction of GDP and, although it remains less than one percent, it is often portrayed as an important contributor to the reduction in household income inequality.[8] The reach of the public cash transfer program, Bolsa Família, increased to cover 11 million families in 2006, which comprised nearly 25 percent of the total population (Barros et al., 2010). Education spending increased reaching 5.5 percent of GDP in 2009 (compared to 3.5 percent in 2000 and 5.7 percent among G20). As we discuss in Section 1.4 this is reflected in a rapidly rising share of the labor force with a high school degree. Moreover, the quality of education relative to other countries, as measured by the international PISA scores, has also improved, with Brazil having the greatest increase in mathematics among 65 countries since 2003 (OECD, 2012).

[8]Using household data, Barros et al. (2010) estimate that social programs accounted for about 20 percent of the decline in household income inequality.

The Worker's Party complemented social policies with minimum wage increases above the previous upward trend. Within their first year in office, they established a 20 percent increase in 2003 and continued to implement yearly increases averaging over 10 percent during the next 10 years. As a result, the minimum to median wage in Brazil increased from around 34 percent in 1996—similar to U.S. levels—to over 50 percent, which is close to the level in France. Engbom and Moser (2015) argue that this large increase in the minimum wage can explain a significant fraction of the reduction in earnings inequality in Brazil over the 1996–2012 period, while being consistent with the other facts we document in the current paper.

With this brief overview of recent developments in Brazil, we turn to a discussion of the data we use to decompose the decline in inequality experienced in Brazil over the past two decades.

1.3 Data

Our analysis uses two confidential administrative datasets from Brazil: the *Relação Anual de Informações (RAIS)* contains earnings and demographic characteristics of workers as reported by employers, and the *Pesquisa Industrial Anual Empresa (PIA)* contains detailed information on revenues and costs of large firms in Brazil's mining and manufacturing sectors. To make the reader familiar with these confidential data, we briefly discuss their collection, coverage, variable definitions, and sample selection.

1.3.1 Description of linked employer-employee data (RAIS)

Collection and coverage. The RAIS data contains linked employer-employee records that are constructed from a mandatory survey filled annually by all registered firms in Brazil and administered by the Brazilian Ministry of Labor and Employment *(Ministério do Trabalho e Emprego*, or *MTE*). Data collection was initiated in 1986

within a broad set of regions, reaching complete coverage of all employees at formal establishments of the Brazilian economy in 1994.[9] Fines are levied on late, incomplete, or inaccurate reports, and as a result many businesses hire a specialized accountant to help with the completion of the survey. In addition, MTE conducts frequent checks on establishments across the country to verify the accuracy of information reported in RAIS, particularly with regards to earnings, which are checked to adhere to the minimum wage legislation.[10]

The RAIS contains an anonymized, time-invariant person identifier for each worker, which allows us to follow individuals over time. It also contains anonymized time-invariant establishment and firm IDs that we use to link multiple workers to their employers and follow those over time. Although it would be possible to conduct part of our analysis at the establishment instead of firm level, this paper focuses on firms for three reasons. First, to the extent that there is substantial variation in pay across establishments within firms, our firm-level analysis provides a lower bound on the importance of workers' place of employment.[11] Second, we think that many of the factors that could give rise to employer-specific components of pay including corporate culture, company leadership, etc., act at the firm level. Additionally many regulations targeting pay policies differ as a function of firm-level employment, not establishment-level employment. Third, we will later use data on firm characteristics such as financial performance that are not available at the establishment-level.

[9]Because registration with the central tax authorities is necessary and sufficient for a firm to be surveyed, the RAIS covers only workers in Brazil's formal sector. Complementing our analysis with data from the Brazilian household survey *Pesquisa Nacional por Amostra de Domicílios* (*PNAD*), we find that the formal sector employment share among male workers of age 18–64 grew from 64 to 74 percent between 1996 and 2012. Differential inequality trends between formal and informal sector workers are discussed at more length in Engbom and Moser (2015).

[10]In addition to being fined, non-compliant firms are added to a "Black List of Slave Work Employers," made available publicly under law Decree No. 540/2004. A recent version of the list dated March 2015 is available from Brazilian television news channel Repórter Brasil at `http://reporterbrasil.org.br/documentos/lista_06_03_2015.pdf`.

[11]As we will show later, however, the explanatory power of our model incorporating firm and person effects is high, leaving little variation to be explained by separate establishment level effects.

Variable definitions. For each firm at which a worker was employed during the year, the RAIS contains information on the start and end date of the employment relationship, the amount the worker was paid and a broad set of worker and job characteristics. Reported earnings are gross and include regular salary payments, holiday bonuses, performance-based and commission bonuses, tips, and profit-sharing agreements. Although this is a broad measure of earnings, it does not contain other sources of income such as capital income or in-kind transfers. We divide total earnings from an employment relationship in a given year by the duration of the job spell.[12] This accounts to some extent for labor supply. As hours worked only exists for some years, we do not use this to construct a measure of per hour pay. Instead, to limit the impact of unmeasured labor supply differences, we focus on adult males.[13]

We define a consistent age variable by calculating the year of birth for any observation, and then setting an individual's year of birth as the modal implied value and finally reconstructing age in each year using this imputed year of birth.[14] Because age is only reported in bins prior to 2002, we code all subsequent years into the same age bins (18–24, 25–29, 30–39, 40–49, and 50–64 years old).

We define a consistent measure of years of schooling by first setting it to its modal value within a year in case of multiple job spells in a year and then ensuring that the years of schooling are non-decreasing across years. Subsequently, we define four education groups based on attained degree implied by the reported number of years of schooling and the education system in Brazil (primary school, middle school, high school, and college).

The data also contain information on detailed occupation classification of the job and detailed sector classification of the employer establishment. Both the indus-

[12] That is, if an employment relationship is reported as active for seven months during the year, we divide total earnings reported for that employment relationship for that year by seven.

[13] In the years for which we have data on hours, we find relatively little variation in hours, with most adult males reporting 44 hours of work a week.

[14] We use age instead of experience throughout our analysis; results are similar using age plus six minus years of education as a measure of experience.

try and occupation classification systems underwent a significant change during the period we study. For occupations, we use the pre-2003 classification (*Classificação Brasileira de Ocupações*, or *CBO*) at the one-digit level. We also use two-digit sectoral classifications (*Classificação Nacional de Atividades Econômicas*, or *CNAE*) according to the pre-2003 period. We make occupations and sectors reported for 2003–2012 consistent with the older CBO and CNAE classifications by using conversion tables provided by IBGE. In order to achieve a high level of consistency between the old and the new classification schemes, we cannot go less coarse than one digit occupation and two digit sector, but we believe that for the purpose of this paper this restriction is not of major importance.

Our firm size measure is the number of full-time equivalent workers during the reference year. Importantly, we calculate this prior to making any sample restrictions so that it reflects to the greatest extent possible the total amount of labor used by the firm during the year. We calculate it as the total number of worker-months employed by the firm during the year divided by 12.

Sample selection. We exclude observations with either firm IDs or worker IDs reported as invalid as well as data points with missing earnings, dates of employment, educational attainment or age. Together, these cleaning procedures drop less than one percent of the original population, indicative of the high quality of the administrative dataset. Subsequently, to limit the computational complexity associated with estimating our model, we restrict attention to one observation per worker-year. We impose this restriction by choosing the highest-paying among all longest employment spells in any given year. As the average number of jobs held during the year is 1.2 and there is not trend in this, we do not believe that loosening this restriction would meaningfully affect our results.

Finally, we restrict attention to adult male workers of age 18–64. We make this restriction as a trade-off between our results being comparable to a large part of the literature focusing on prime age males on the one hand and to obtain as complete as possible coverage of the changes in the Brazilian wage structure over the period on the other.[15] We have tried alternative sample restrictions, including focusing on only prime age males of age 18–49, as well as both male and female adult workers.

Descriptive statistics. Table 2.2 provides key summary statistics for the RAIS data for six subperiods of five years each with one year overlap between adjacent periods, namely 1988–1992, 1992–1996, 1996–2000, 2000–2004, 2004–2008, and 2008–2012. Since our analysis focuses on adult males and adult males working for large manufacturing and mining firms, we provide a brief comparison of these subpopulations to the overall population of formal sector employees. As we will be primarily concerned with the later four subperiods during which inequality declined markedly and for which we have firm level data, we focus our discussion on these periods.

Panel A shows statistics for the overall formal sector work force in Brazil and Panel B for the subpopulation of adult males. Adult males are consistently about 0.3–0.4 years older than the population average. They also have 0.78 years of schooling less than the overall sample in the 1996–2000 subperiod; this gradually drops to 0.65 years in the last subperiod. Finally, adult males earn about eight to nine log points more than the overall population, but the variance of log earnings is very similar to the overall population.

Panel C presents statistics on the subpopulation of adult males working at large mining and manufacturing firms. Adult males in the PIA subpopulation are about 0.8 years younger than all adult males in the 1996–2000 subperiod, which gradually

[15]The restriction to only male workers has the advantage of avoiding issues with changing patterns of female labor supply and labor market discrimination. In a separate ongoing research project, we investigate the degree to which firm-level average pay and profit sharing differs between male and female employees and how the gender pay gap has evolved over time.

increases to 1.3 years younger in the last subperiod. They are similar to all adult males in terms of education. The PIA sample of adult males earned on average 27 log points more than all adult males in the 1996–2000 subperiod; this declined to only a 19 log point premium in the last subperiod. Finally, they display a two log point higher standard deviation of log earnings in the 1996–2000 period, which increases to four log points in the last subperiod.

Table 1.1: RAIS summary statistics

			Earnings		Age		Schooling	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	# Worker-years	# Unique workers	Mean	St.d.	Mean	St.d.	Mean	St.d.
Panel A. All Formal Sector Workers (RAIS)								
1988–1992	165.5	41.9	1.10	0.86	31.91	11.47	7.65	4.45
1992–1996	162.1	43.4	1.18	0.86	33.20	11.32	8.08	4.41
1996–2000	174.6	47.0	1.19	0.84	33.68	11.27	8.60	4.27
2000–2004	202.7	52.7	1.00	0.80	34.02	11.33	9.49	4.05
2004–2008	254.2	62.7	0.81	0.74	34.26	11.48	10.25	3.78
2008–2012	326.5	76.2	0.71	0.71	34.55	11.66	10.78	3.52
Panel B. Adult Male Workers								
1988–1992	86.5	25.5	1.24	0.87	33.26	10.82	7.04	4.30
1992–1996	87.3	26.4	1.29	0.87	33.88	10.79	7.37	4.27
1996–2000	92.7	28.8	1.27	0.85	33.97	10.78	7.82	4.16
2000–2004	105.3	32.5	1.07	0.80	34.14	10.92	8.70	4.00
2004–2008	126.9	37.3	0.88	0.75	34.44	11.11	9.50	3.79
2008–2012	154.2	43.9	0.80	0.72	34.91	11.34	10.13	3.59
Panel C. Adult Male Workers at Large Manufacturing and Mining Firms (PIA)								
1996–2000	16.6	6.3	1.54	0.87	33.20	10.07	7.83	4.05
2000–2004	18.0	6.8	1.29	0.85	33.04	10.20	8.75	3.91
2004–2008	23.2	8.5	1.09	0.80	33.11	10.45	9.41	3.77
2008–2012	26.9	9.9	0.99	0.76	33.60	10.70	10.04	3.60

Notes: The number of worker-years and number of unique workers are reported in millions. Statistics on earnings are in log multiples of the current minimum wage, schooling is in years. Panel A includes all workers in the RAIS dataset. Panel B includes male workers that are between 18 and 64 years old. Panel C includes male workers age 18–64 working at large manufacturing and mining firms included in the PIA firm characteristics data. Means are computed by period. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a subperiod.

1.3.2 Description of firm characteristics data (PIA)

Collection and coverage. The PIA data contain information on firm financial characteristics from 1996–2012. The dataset is constructed by the Brazilian National Statistical Institute (*Instituto Brasileiro de Geografia e Estatística*, or *IBGE*) based on annual firm surveys in the manufacturing and mining sector. This survey is mandatory for all firms with either more than 30 employees or above a revenue threshold as well as for an annual random sample of smaller firms.[16] As with RAIS, completion of the survey is mandatory and non-compliance is subject to a fine by national authorities. Each firm has a unique, anonymized identifier, which we use to link firm characteristics data from PIA data to worker-level outcomes in the RAIS data.

Variable definitions. The PIA dataset includes a breakdown of operational and non-operational revenues, costs, investment and capital sales, number of employees and payroll. All nominal values are converted to real values using the CPI index provided by the IBGE. Instead of the measure of firm size in the PIA, we prefer our measure of full-time-equivalent employees constructed from the RAIS as it accounts for workers only employed during part of the year. We define operational costs as the cost of raw materials, intermediate inputs, electricity and other utilities, and net revenues as the gross sales value due to operational and non-operational firm activities net of any returns, cancellations, and corrected for changes in inventory.[17]. We finally construct value added as the difference between net revenues and intermediate inputs, and value added per worker as value added divided by full-time equivalent workers. This is our main measure of firm productivity. We have also constructed alternative measures of firm productivity by cleaning value added per worker off industry-year

[16]The revenue threshold for inclusion in the deterministic survey has grown over the years, standing at USD300,000 in 2012.

[17]We have explored alternative revenue definitions such as only restricting attention to operational revenues or excluding certain types of non-operational revenues. Such robustness checks yield very similar results to what we report below.

effects and some measures of worker skill. In our main analysis, we focus on "raw" value added per worker and present results containing these alternative measures in the Appendix.

Our productivity measure differs from the commonly used total factor productivity (TFP) (Bartelsman, Haltiwanger and Scarpetta, 2009, 2013) since it does not control for capital intensity. A major reason for this is that we do not have data on capital, only on investment. To construct a measure of the capital stock, we would need to assume a depreciation rate to be able to impute capital using reported investment. We would also need to impute capital in 1996 since we do not have data prior to that, as well as for any firm that enters the PIA population. We have constructed such a measure of the capital stock using an assumed annual depreciation rate of five percent and using data on the aggregate capital stock at the subsector level.[18] However, the multiple imputations required to obtain capital as well as the fact that the investment data is incomplete for many firms lead us to prefer value added per worker as our measure of firm productivity.[19]

Sample selection. The PIA firm survey spans the universe of large firms (as defined above) in Brazil's manufacturing and mining sectors in addition to a random sample of smaller firms. Because parts of our analysis make use of the panel dimension on the firm side and to avoid issues with excessive sample attrition related to our later estimation procedure, we focus our analysis on the deterministic set of relatively large firms.

[18]Each new firm starts with an initial capital equal to its current net investment plus a share of total capital in its subsector. The shares are given by taking the share of capital at a firm to be proportional to the share of total net revenues assuming a firm-level production function of the form $y = Ak^{\alpha}$ for $\alpha = 1/3$. Firms entering the PIA at a later year are initiated by applying the same method to get those firms' capital stock proportional to scaled firm revenues relative to the subsector total.

[19]In addition, several bargaining models of the labor market have in common that workers and capital owners split the surplus from production, and value added per worker is arguably the best measure of that surplus. Thus, to the extent that such models well describe Brazilian labor markets, value added per worker is an important metric.

Descriptive statistics. Table 1.2 shows key summary statistics on firms during the four periods for which we have firm financial data: 1996–2000, 2000–2004, 2004–2008, and 2008–2012. All results are weighted by the number of full-time equivalent workers employed by the firm. The number of firms in the PIA increased by 57 percent between the first and the last period. The average firm size increased by 32 percent and average real value added per worker grow by 27 percent. There is significant dispersion in both log firm size and log value added per worker across firms, with the standard deviation of the former being close to two and that of the latter exceeding one. Furthermore, there is no evidence of convergence in either measure. The standard deviation of firm size monotonically increases whereas the standard deviation of value added per worker first increases rapidly, then falls again in the last subperiod. To the extent that firm characteristics matter for employees' labor remuneration, these results suggest that the decline in earnings inequality in Brazil cannot be explained by declining dispersion in these characteristics over time.

Table 1.2: PIA summary statistics

			Firm size		Value added p.w.	
	(1)	(2)	(3)	(4)	(5)	(6)
	# Firm-years	# Unique firms	Mean	St.d.	Mean	St.d.
1996–2000	142.4	51.1	6.34	1.80	11.15	1.13
2000–2004	168.7	59.9	6.26	1.85	11.19	1.32
2004–2008	202.4	73.0	6.49	1.96	11.22	1.34
2008–2012	230.2	80.2	6.62	2.05	11.30	1.31

Note: The number of firm-years and number of unique firms are reported in thousands Firm size is the log number of full-time equivalent employees. Value added per worker is the log of real value added per worker. Means and standard deviations are weighted by the number of full-time employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a subperiod.

1.4 Inequality trends in Brazil from 1988–2012

In this section, we first document Brazil's rapid decline in earnings inequality. We then demonstrate that the decline in inequality in Brazil occurred throughout a large part of the earnings distribution. Subsequently, we present results from a series of Mincer regressions, which provides a first look at possible factors behind the decline. Finally, we provide some suggestive evidence of firms being an important source of inequality as well as a factor behind the decline in inequality in Brazil.

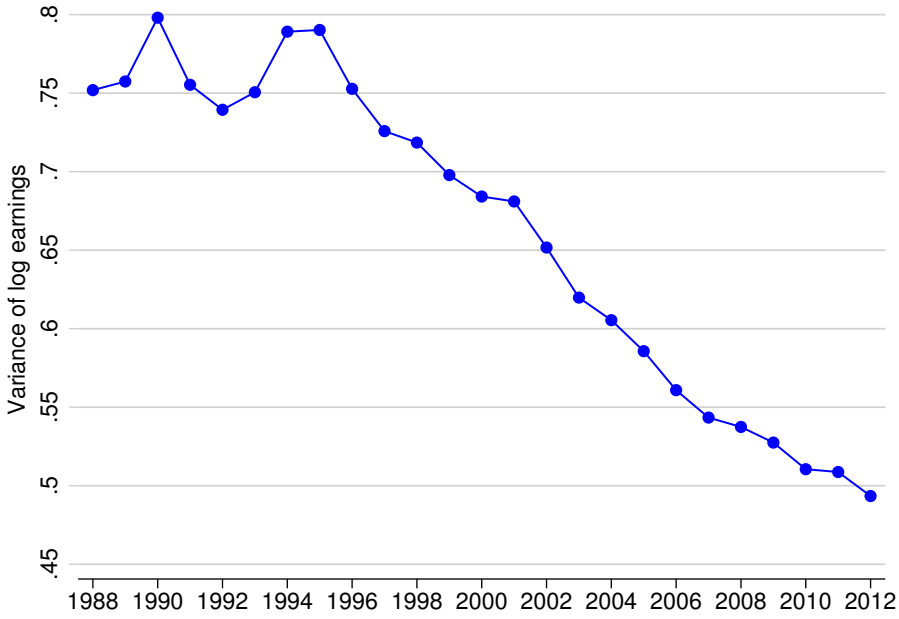
1.4.1 The evolution of earnings inequality

Starting from high initial levels, Brazil experienced a rapid and steady declined in earnings inequality from 1995 onwards. The inequality decline followed years of stable or slightly increasing inequality between the late 1980s and the mid-1990s. While the inequality decline is apparent in many measures, Figure 1.1 plots the variance of log earnings, which is a common inequality measure in the labor literature. Between 1996 and 2012, the variance of log earnings in Brazil declined by 26 log points or 35 percent, from 0.75 to 0.49 over the period. To put this decline into context, the U.S. saw an increase in the same measure of 22 log points from 1968–2006.

1.4.2 Compression in different parts of the earnings distribution

Figure 1.2 plots the log percentile ratios of earnings, normalized to zero in 1996. Two striking facts emerge from the graph: First, there was widespread compression in the distribution of earnings—inequality declined past the 75th percentile. Second, the amount of compression gradually declines as one moves further up in the distribution. For instance, whereas the log 90–50 percentile ratio falls by 20 log points, the log 50–

Figure 1.1: Variance of log earnings in Brazil, 1988–2012



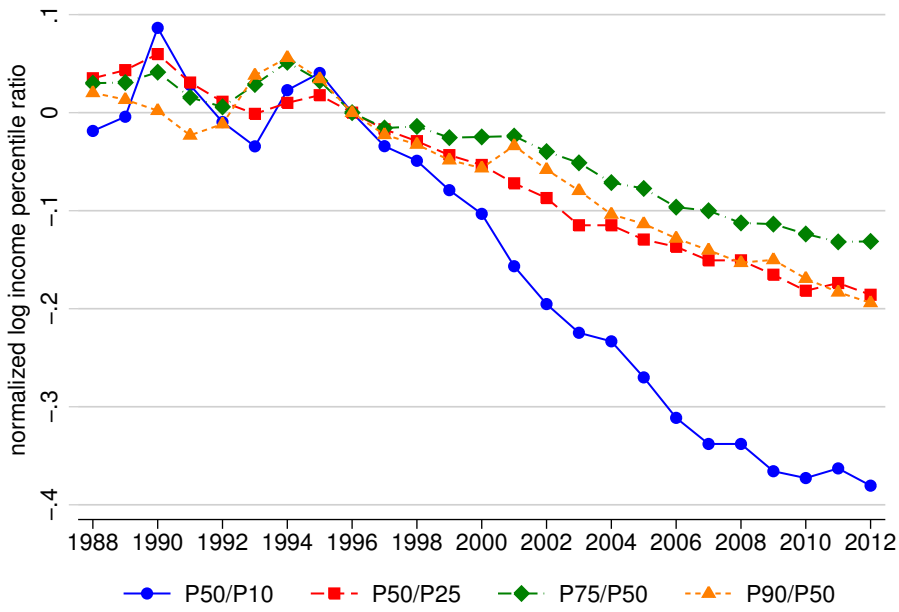
10 ratio falls by a remarkable 35 log points. Similarly, compression in the log 50–25 percentile ratio exceeds compression in the log 75–50 ratio.[20]

1.4.3 The (un-)importance of worker observables

One candidate explanation for the decline in earnings inequality is increasing educational attainment. As can be seen in the left panel of Figure 1.3, the fraction of the Brazilian formal sector workforce with a high school degree rose rapidly during this time, while the fraction with primary school fell sharply (the fraction with a middle school degree and a college degree remained relatively flat). There were also important changes to the premia associated with a higher degree, as can be seen in the right pane of Figure 1.3 . In particular, the premia associated with a middle school

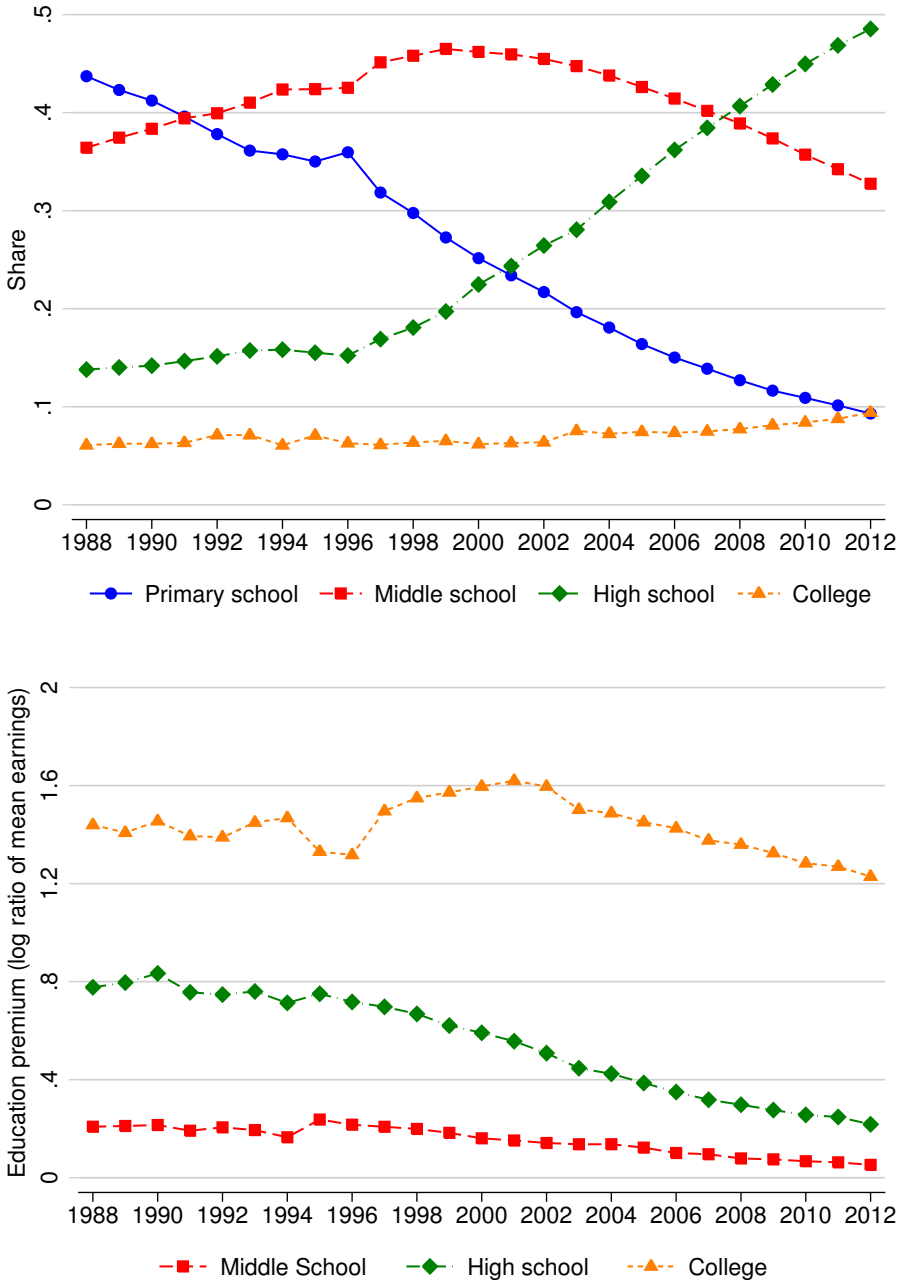
[20]For ease of interpretation and to show that there was significant real wage growth at all percentiles of the income distribution over the period for which such data exist, Appendix A plots the real percentile earnings levels evolution from 1996–2012, normalized to zero in 1996.

Figure 1.2: Log percentile ratios of the earnings distribution in Brazil (1996 = 0)



and high school degree relative to the lowest education group fell rapidly over the past 20 years.

Figure 1.3: Educational attainment (top) and education premia (bottom)



To provide a first look at whether changes in observable worker characteristics were an important driver of earnings inequality in Brazil over this period, we run a series of Mincer regressions. In particular, we regress log earnings of individual i in

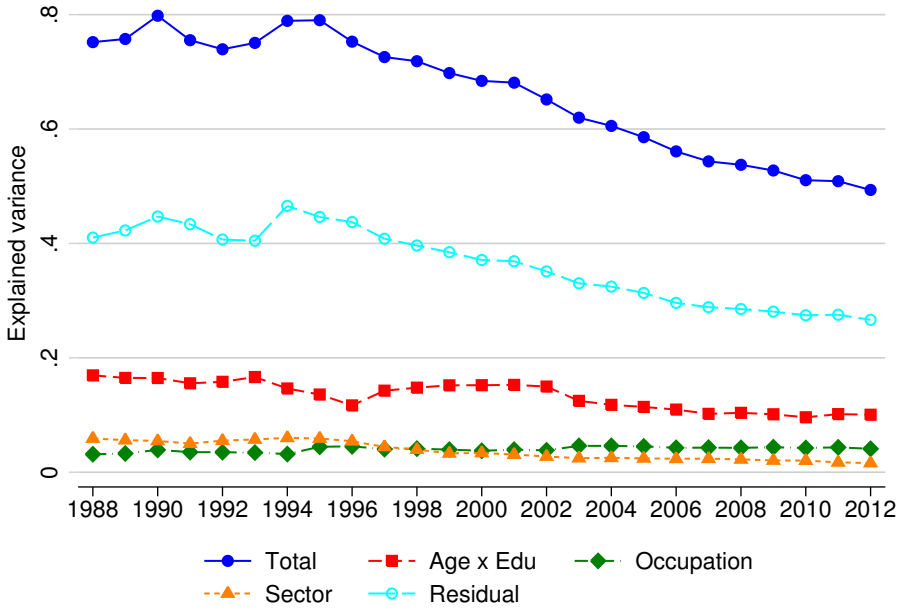
year t, denoted y_{it}, on five age group dummies interacted with nine education group dummies, two digit occupation dummies and two digit sector dummies,

$$\log(y_{it}) = age_{it} \times edu_{it} + occ_{it} + sec_{it} + \varepsilon_{it}$$

Note that all explanatory variables are allowed to vary freely by year. Based on this regression, we calculate the predicted value due to each component and report the variance of these predicted values.

Figure 1.4 plots the results. In levels, worker observables jointly explain about 45 percent of the overall variance in log earnings. This does not change much over time, and hence worker observables explain close to 45 percent of also the fall in inequality. Decomposing this, age and education account for roughly 20 percent of the variance of log earnings and 27 percent of the decline. Inequality between occupations increases relative to overall inequality from four to eight percent of total variance (and in fact it also grows slightly in absolute terms). The fraction of total inequality explained by differences in means across sectors falls from eight to three percent—this accounts for 12 percent of the overall decline in variance. Finally, covariances between the explanatory variables increase slightly in importance from 11 percent to 14 percent of total variance and explain four percent of the overall fall in inequality. We conclude that even when controlling for detailed worker characteristics, more than half of the level of inequality as well as its decline is residual in nature.

Figure 1.4: Variance decomposition from Mincer regressions



1.4.4 The evolution of earnings inequality between and within firms

As a first step towards understanding the role of firms for earnings inequality, we investigate the variance of earnings within and between firms. To this end, we define between-firm inequality as the variance of the average log earnings at the firm across firms (weighted by firm size) and within-firm inequality as the variance of the difference between workers' log earnings and the average log earnings at their firm. Based on these definitions, one could imagine two hypothetical polar extremes. First, average earnings could be identical across firms so that overall earnings inequality is completely due to variance in earnings within firms. In this case, a firm is just a microcosm of the overall economy. Second, all workers could earn the same wage within the firm so that inequality arises entirely due to differences in earnings across firms. In reality, the question is which channel is quantitatively most important.

Formally, we follow Bloom et al. (2015) in letting y_{ijt} denote earnings of worker i employed by firm j in year t, then:

$$y_{ijt} = \underbrace{\overline{y_t}}_{\text{economy average}} + \underbrace{\left(\overline{y}_t^j - \overline{y_t}\right)}_{\text{employer deviaion}} + \underbrace{\left(y_{ijt} - \overline{y}_t^j\right)}_{\text{worker deviation}}$$

Re-arranging and taking variances on both sides we get

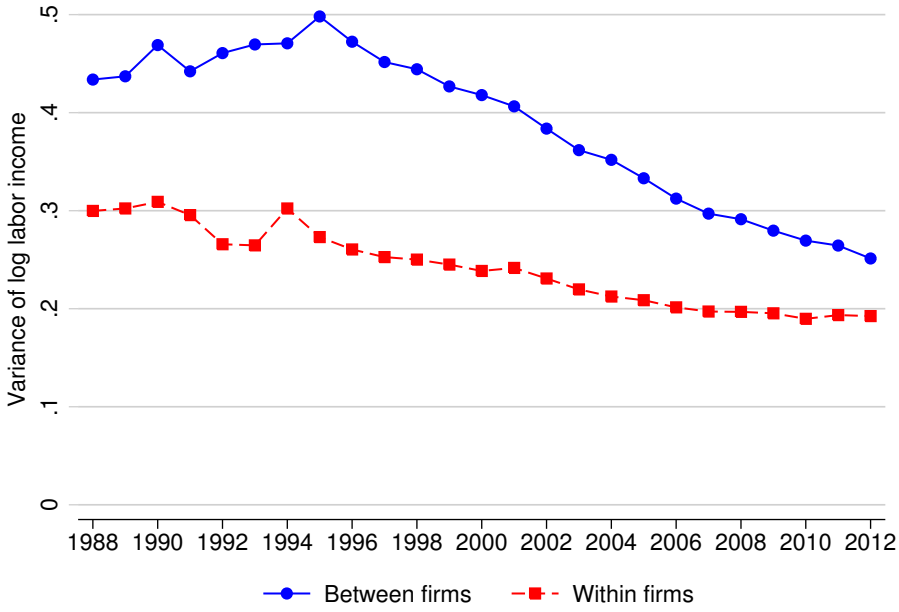
$$Var\left(y_{ijt} - \overline{y_t}\right) = Var\left(\overline{y}_t^j - \overline{y_t}\right) + Var\left(y_{ijt} - \overline{y}_t^j\right) + \underbrace{2Cov\left(\overline{y}_t^j - \overline{y_t}, y_{ijt} - \overline{y}_t^j\right)}_{=0}$$

where the last term is zero by construction. Simplifying, we have our decomposition of the overall variance of earnings into between- and within-firms inequality:

$$\underbrace{Var\left(y_{ijt}\right)}_{\text{overall}} = \underbrace{Var\left(\overline{y}_t^j\right)}_{\text{between firms}} + \underbrace{\overline{Var\left(y_{ijt} \middle| i \in j\right)}}_{\text{within firms}}$$

Figure 1.5 plots this decomposition over time in Brazil. We note two insights: Firstly, there is significant variability in earnings within firms, but an even greater amount of earnings inequality across firms. Secondly, although both measures of inequality fell during this time, the decline was particularly pronounced across firms: inequality across firms declined by 25 log points or 45 percent between 1988 and 2012, whereas within-firm inequality dropped by 10 log points or 33 percent.

Figure 1.5: Variance of log earnings within and between firms



Another way of illustrating the importance of firm is to compare earnings growth of different workers with average earnings growth at their employers. Making use of the link between workers and firms in the matched employer-employee data (RAIS), one can ask how much of the earnings growth accruing to a certain income group was mediated by rises in average pay at firms employing workers from that income group.[21]

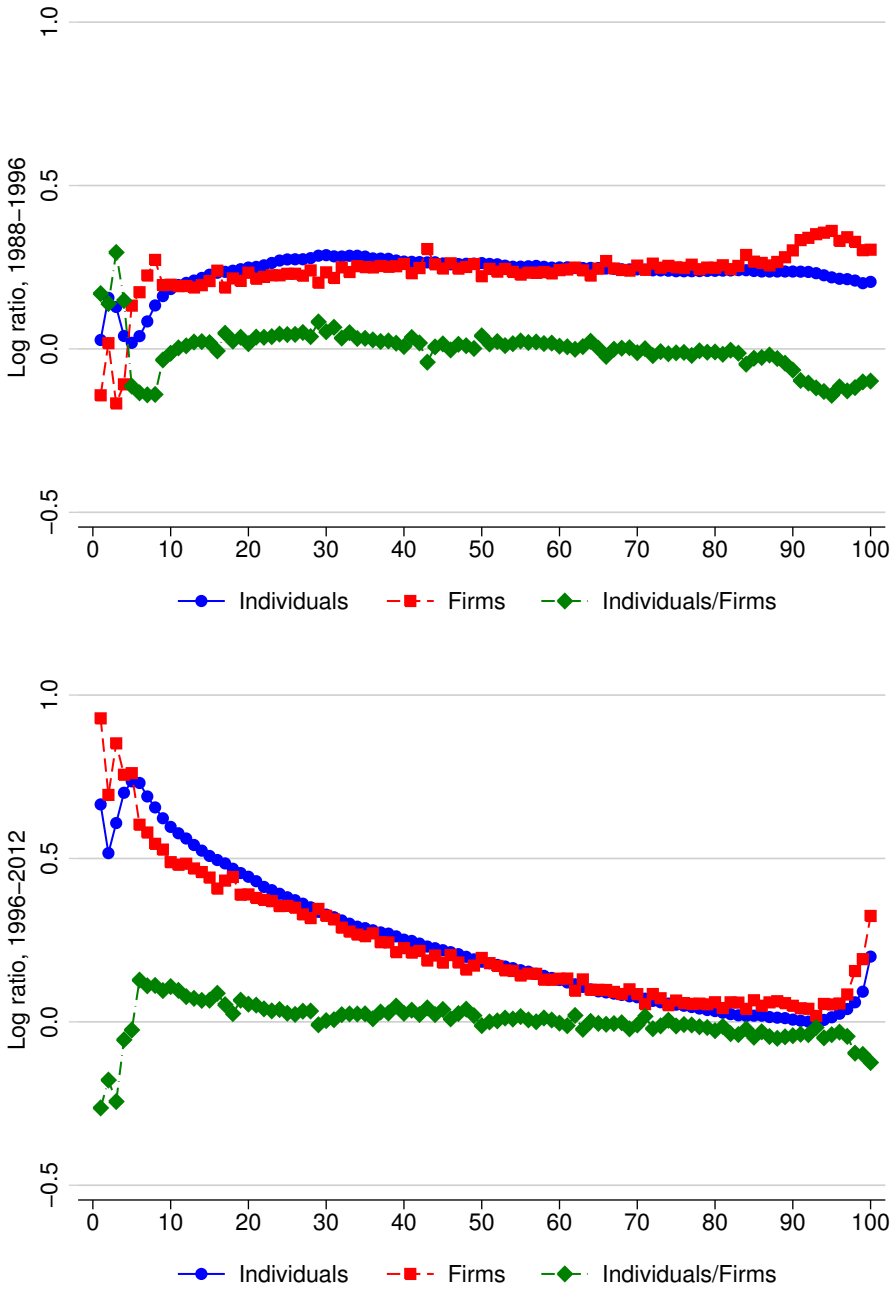
The results of this exercise are shown in Figure 1.6, first for the period 1988–1996, when earnings inequality remained roughly constant, and then for 1996–2012, when earnings inequality declined rapidly. Average earnings growth (solid blue line with circles) was relatively evenly distributed throughout the earnings distribution between 1988 and 1996, and firm average pay (solid red line with squares) grew equally in line with the growth rate of wages. The period from 1996–2012, on the other hand, was marked by a rapid catch-up of the lowest earnings groups, which in

[21]Similar calculations are presented for the U.S. in Barth, Davis and Bryson (2014) and Bloom et al. (2015).

turn is almost entirely explained by the growth of firm average earnings among those groups. Throughout both periods, there were no significant changes in within-firm earnings inequality (solid green line with diamonds).

Although informative, however, these type of decompositions of raw earnings cannot necessarily be interpreted as firms differing fundamentally in the way they compensate their workers. The reason is that some firms could hire workers who always get paid more regardless of where they work (maybe because they are more productive, have a higher bargaining power, etc). In this case differences in pay across firms would arise as a result of recruitment policies and not pay policies. The next section formalizes our approach to identifying the importance of firm pay policies for earnings inequality.

Figure 1.6: Individual labor earnings growth between and within firms, 1988–1996 (top) and 1996–2012 (bottom)



1.5 Empirical framework

For a long time, economists have recognized that worker observables fail to explain a large fraction of the variance of earnings. As we showed above, this is true also for Brazil—even with detailed occupation and sector controls, Mincer regressions explain less than half of the overall variation in earnings in Brazil. Furthermore, a recent literature has argued that there are important differences across firms in terms of pay (Abowd, Kramarz and Margolis, 1999), and we presented some evidence above suggesting that firms might be an important determinant of earnings also in Brazil. Motivated by these insights, we estimate two-way fixed effects econometric models controlling for both unobserved worker and firm heterogeneity. To be able to speak to changes over time in the components of inequality, we estimate our model separately in six sub-periods covering 1988–1992, 1992–1996, 1996–2000, 2000–2004, 2004–2008, and 2008–2012, respectively. Subsequently, we correlate the estimated firm and worker effects with observed characteristics of firms and workers in order to investigate what led to changes in the firm and worker component of pay over time.

1.5.1 Introducing the two-way fixed effects model

In order to identify the two-way fixed effects framework pioneered by Abowd, Kramarz and Margolis (1999), one needs to observe a panel of workers with the ability to link multiple workers to the same firm. Our data satisfy these requirements. Within each subperiod, we observe a large number I of workers for up to five years while working at J firms for a total of N worker-years. Let $J(i,t)$ give the employer of worker i in year t. We assume that the log earnings of individual i in year t within a period time window p, denoted y_{it}, can be written as the sum of a worker effect, α_i^p, a firm effect, $\alpha_{J(i,t)}^p$, a time dummy, γ_t, and an error term, ε_{it}. Although our current paper does not provide a microfoundation to this reduced form empirical model, in subsequent work

we show how it can be rationalized in a frictional labor market with firm productive heterogeneity and worker ability differences (Engbom and Moser, 2015).

Our specification does not control in this first stage for observable worker and firm characteristics. Instead, we correlate the estimated fixed effects with observable characteristics of workers and firms in a second stage of our analysis. We prefer this specification to avoid identifying these effects off changes within workers and firms during the limited time frame of each subperiod. With regards to age, we additionally notice that unrestricted age controls would be perfectly collinear with person effects and the time dummies.[22] Similarly, we also correlate worker observables, including age, education, and occupation, with the estimated worker effect in a second stage. We argue that any error due to growing earnings with age is likely to be of second order importance within our five year subperiods. We thus estimate

$$\log y_{it} = \alpha_i^p + \alpha_{J(i,t)}^p + \gamma_t + \varepsilon_{it}$$

for $t \in p = \{t_1, \ldots, t_5\}$ and where α_i^p denotes the individual fixed effect of worker i in period p, $\alpha_{J(i,t)}^p$ denotes the firm effect of the employer of worker i at year t, γ_t is a year dummy, and ε_{it} is an error term that satisfies the strict exogeneity condition $\mathbb{E}\left[\varepsilon_{it} \middle| i, t, J\left(i,t\right)\right] = 0$.[23]

As shown by Abowd, Kramarz and Margolis (1999), worker and firm effects can only be separately identified within a set of firms and workers connected through the mobility of workers. Table 1.3 presents summary statistics on the largest set of workers in each subperiod—this covers 97–98 percent of all workers in each subperiod. Given that it covers such a large fraction of all adult males, it is not surprising that this subpopulation looks very similar to the overall population in all observable

[22] Although restrictions could be imposed to address this collinearity problem, we note that for instance the popular restriction advocated by Deaton (1997) requires many years to be well identified.

[23] We later discuss in greater detail the empirical validity of the assumption on the error term.

dimensions. Thus the restriction to the largest connected set imposed in the rest of our analysis appears to be inoccuous.

Table 1.3: Summary statistics on workers in largest connected set (as a fraction of adult males)

			Earnings		Age		Schooling	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	# Worker-years	# Unique workers	Mean	St.d.	Mean	St.d.	Mean	St.d.
1988–1992	85.4 (99%)	25.1 (98%)	1.3	0.9	33.3	10.8	7.0	4.3
1992–1996	85.6 (98%)	25.8 (98%)	1.3	0.9	33.9	10.8	7.4	4.3
1996–2000	90.2 (97%)	27.9 (97%)	1.3	0.8	34.0	10.8	7.8	4.2
2000–2004	102.1 (97%)	31.4 (97%)	1.1	0.8	34.2	10.9	8.7	4.0
2004–2008	123.7 (97%)	36.2 (97%)	0.9	0.8	34.4	11.1	9.5	3.8
2008–2012	151.0 (98%)	42.8 (98%)	0.8	0.7	34.9	11.3	10.1	3.6

Notes: We report in parentheses the proportion of the reported statistics relative to the group of adult males listed in Table 2.2. Earnings are in log multiples of the minimum wage, schooling is years of education.

As identification critically derives from workers switching between firms, Table 1.4 presents statistics on the fraction of switchers in each subperiod. The degree of labor mobility is high in Brazil with more than 30 percent of the population switching firm at some point in the subperiod. The average number of firms worked at during the five years in each subperiod is about 1.5. There is no strong trend in either statistic.

Table 1.4: Frequency of switches, by period

	(1)	(2)	(3)
	# Unique workers	Average # of jobs	% switchers
1988–1992	25.1	1.55	0.36
1992–1996	25.8	1.47	0.32
1996–2000	27.9	1.44	0.31
2000–2004	31.4	1.45	0.31
2004–2008	36.2	1.53	0.36
2008–2012	42.8	1.64	0.40

Note: Number of unique workers in millions. A switcher is defined as a worker who is associated with two or more employers during the period.

The assumption we impose on the error term is often referred to in the literature as that of requiring *exogenous mobility*. As explained by AKM, this rules out dependency of the error term on the worker effect, the firm effect or the time controls. In particular, a worker is not allowed to switch between firms based on the unobserved error term, because if say matches with a particularly poor match effect are more likely to break up, the residual of remaining matches does not have mean zero. Moreover, this assumption rules out assortative matching of the type found in Roy models, since these models emphasize the complementarity in matches between workers and firms, whereas the AKM framework imposes log-additivity between the two.

We investigate whether this assumption is violated in two ways. First, we follow Card, Heining and Kline (2013) in dividing estimated firm effects into quartiles and investigate whether the gain in the firm component of those switching between for instance the first and fourth quartile is similar to the loss of those making the reverse switch. To the extent that the labor market is better characterized by assortative matching as in Roy models, we would expect these to be very different. Second, we examine the distribution of error terms across worker and firm effects quantiles to check for systematic variation, which could be an indication that our log additive model is misspecified.

Based on our estimated equation, we decompose the variance of log earnings within any subperiod into the variance of the worker component, the firm component, the year trend and the residual, as well as the covariance between the worker and the firm component, the worker and year component, and the firm and year component:

$$\begin{aligned} Var\left(\log y_{it}\right) &= Var\left(\alpha_i\right) + Var\left(\alpha_{J(i,t)}\right) + Var\left(\gamma_t\right) + 2Cov\left(\alpha_i, \alpha_{J(i,t)}\right) \\ &\quad + 2Cov\left(\alpha_i, \gamma_t\right) + 2Cov\left(\alpha_{J(i,t)}, \gamma_t\right) + Var\left(\varepsilon_{it}\right) \end{aligned} \tag{1.1}$$

We note that sampling error in the estimated person and firm effects will cause us to overestimate the variance of worker and firm effects and induce a negative bias in the covariance between worker and firm effects. We do not attempt to correct for this. Instead, we assume that this error is constant over time, so that even if the level of our estimated variances is slightly overstated, the changes we document over time are still valid. [24]

1.5.2 Determinants of the estimated firm effects

In the second stage of our empirical investigation, we study how the estimated firm effects relate to observable measures of firm performance available in the PIA survey. In particular, we are interested in understanding what firm characteristics are related to pay, and whether any changes in the distribution of firm effects over time can be explained by underlying changes in firm characteristics or the way the labor market translates those into pay. Since the PIA only covers the set of large manufacturing and mining firms,[25] we are forced to restrict attention to only these firms and workers when linking firm effects to firm characteristics. We implement this by first estimating the AKM model for the universe of firms and then subsequently restricting attention to only large firms.

Consider a given subperiod and let a_j be the estimated firm component of pay, VA_j average log value added per worker during the subperiod and S_j a set of 26 two

[24] To better estimate firm effects, Bonhomme, Lamadon and Manresa (2015) suggest restricting attention to firms whose fixed effect is "well-identified" due to a high number of switchers. In practice, this procedure boils down to restricting attention to workers at firms with at least 10 switchers during the estimation period. With this restriction, we find a slightly more pronounced role for worker effects in explaining both the initial levels and the decline of earnings inequality between 1996–2000 and 2008–2012.

[25] As described in Section 1.3, we restrict attention to the deterministic stratum of PIA containing only large firms. We drop small firms contained in the random stratum to ensure that firms stay in the sample for multiple years for our estimation procedure below.

digit subsector controls. For each subperiod, we regress

$$a_j = \gamma_0 + \gamma_1 \mathrm{VA}_j + \mathrm{S}_j + \varepsilon_j$$

Notice that all regressions are run with sub-period averages of all variables. Additionally, we have considered versions including a range of other firm characteristics, but as these are only marginally important we do not show them here. Based on the above regression, we compute the variance due to value added per worker as

$$Var\left(\hat{a}_j\right) = \left(\hat{\gamma}_1\right)^2 Var\left(\mathrm{VA}_j\right)$$

In a similar vein, we can compute the variance of firm effects due to sub-sector differences.

In order to isolate the importance of a compression in firm fundamentals versus a compression in the pass-through from such fundamentals to pay, we consider two counterfactuals. First, we keep the pass-through from value added per worker to pay, $\hat{\gamma}_1$ constant at the estimated level in 1996–2000 and let the variance of value added per worker change as in the data. Second, we keep the variance of value added per worker at its 1996–2000 level and letting the estimated coefficient $\hat{\gamma}_1$ change as in the data. Similarly, we decompose the change in the variance due to sub-sectors into changes in sub-sector pay premia versus a change in the sectoral composition of the economy. A comparison of the two counterfactuals allows us to address whether a change in the variance of firm pay is explained by changes in underlying firm characteristics or due to a change in the degree of pass-through from firm value added to worker pay.

1.5.3 Determinants of the estimated worker effects

In the second stage of our analysis, we also investigate what factors influence the worker component of pay. To this end, we regress the estimated worker effects, a_i, on

five age bin dummies, four education dummies and ten occupation dummies:

$$a_i = \text{age}_i + \text{edu}_i + \text{occ}_i + \varepsilon_i$$

We report results both with and without occupation controls. We have also examined versions of this regression with age and education interacted as well as including sector controls, but neither of these alternatives changes the estimated results significantly. Based on the above regression estimates, we predict the variance due to each of age, education and occupation.

Subsequently, we decompose the evolution of the variance of the age, education and occupation components into that due to a change in the underlying distribution of such characteristics versus a change in the return to them. That is, we first keep the estimated return to the characteristic of interest constant and change only the underlying distribution to match its evolution in the data. This shows how important changes in the distribution of worker characteristics were for the overall decline. Subsequently, we instead change the return to the characteristic of interest as in the data, while holding the underlying distribution constant. This allows us to evaluate how important changes in the return to age, education and occupation were for the overall decline.

1.6 Results

In this section, we first present the results from our two-way fixed effects model decomposing earnings inequality into a firm and a worker component. Subsequently, we investigate the sources of the firm and worker component of pay. Finally we provide additional results evaluating the assumptions imposed by our econometric model.

1.6.1 AKM decomposition

Table 2.4 presents the variance decomposition based on the estimation results of the AKM model in equation (1.1) for each of the six five-year subperiods between 1988 and 2012. To illustrate the relative importance of the various components of this decomposition over time, Figure 1.7 plots the variance of raw earnings (solid blue line with circles), the variance of estimated worker effects (dashed red line with squares), and the employee-weighted variance of firm effects (dash-dotted green line with diamonds) in each subperiod.

Two important results emerge from our analysis. First, although firm heterogeneity is a non-negligible source of earnings inequality *levels*, worker heterogeneity is the single most important determinant. In the 1996–2000 subperiod, the variance of worker fixed effects makes up 50 percent of the total variance of log earnings. This share increases monotonically to 60 percent by the last subperiod. The variance of firm effects, on the other hand, makes up 23 percent of the variance of log earnings in the 1996–2000 subperiod, decreasing to 15 percent by the last subperiod.

Second, in terms of explaining *time trends*, we observe a more than proportionate fall in the variance of firm effects. Between 1996–2000 and 2008–2012, the variance of firm effects falls from 17 to eight log points whereas the variance of person effects falls from 36 to 31 log points. Additionally, the declining variance of each component is reflected in a lower covariance between the two, with the correlation between worker and firm effects staying fairly constant at around 0.3 throughout the period.[26] Given the large role played by firms in the decline, understanding the drivers of more equal pay across employers over time is an important question which we address in the following section.

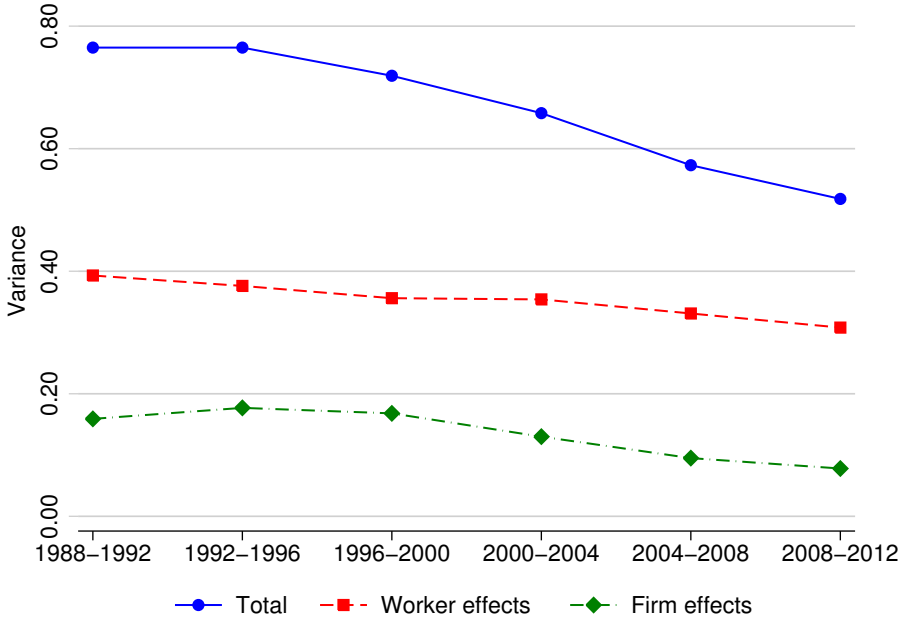
[26]Given that sorting patterns—measured by the correlation between worker and firm effects—remains fairly constant over time, the true reduction in earnings inequality due to firms would be even greater if the covariance term were distributed proportionately between workers and firms.

Table 1.5: AKM variance decomposition results

	(1) 1988–1992	(2) 1992–1996	(3) 1996–2000	(4) 2000–2004	(5) 2004–2008	(6) 2008–2012	(7) Change 1988 to 2012	(8) Change 1996 to 2012
Var. of log earnings	0.77 (100%)	0.77 (100%)	0.72 (100%)	0.66 (100%)	0.57 (100%)	0.52 (100%)	-0.25 (100%)	-0.20 (100%)
Var. of worker effects	0.39 (51%)	0.38 (49.2%)	0.36 (50%)	0.35 (54%)	0.33 (58%)	0.31 (60%)	-0.09 (34%)	-0.05 (24%)
Var. of firm effects	0.16 (21%)	0.18 (23.1%)	0.17 (23%)	0.13 (20%)	0.10 (17%)	0.08 (15%)	-0.08 (33%)	-0.09 (45%)
Var. of year effects	0.02 (2%)	0.01 (2%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	-0.02 (6%)	0.00 (0%)
2×Cov. worker and firm	0.14 (18%)	0.14 (18%)	0.14 (20%)	0.12 (18%)	0.10 (18%)	0.10 (20%)	-0.04 (14%)	-0.04 (22%)
2×Cov. worker and year	-0.01 (-2%)	-0.02 (-2%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.01 (-2%)	0.00 (0%)
2×Cov. firm and year	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0.%)	0.00 (0%)	0.00 (0%)	0.00 (-1%)	0.00 (-1%)
Var. of residual	0.08 (10%)	0.07 (10%)	0.06 (8%)	0.05 (7%)	0.04 (7%)	0.04 (7%)	-0.04 (17%)	-0.02 (10%)
# worker years	85.4	85.6	90.2	102.0	123.7	151.0		
# firm years	0.98	1.04	1.23	1.44	1.75	2.22		
R^2	0.90	0.91	0.92	0.93	0.93	0.93		

Note: Variance decomposition $Var\left(y_{it}\right) = Var\left(a_i\right) + Var\left(a_j\right) + Var\left(\hat{\gamma}_t\right) + Var\left(e_{it}\right) + 2Cov\left(a_i, \hat{\gamma}_t\right) + 2Cov\left(a_j, \hat{\gamma}_t\right) + 2Cov\left(a_j, \hat{\gamma}_t\right)$. Cells contain variance explained by each decomposition element. Share of the total variance explained by each decomposition element is given in parentheses.

Figure 1.7: Variance decomposition from AKM model with firm and worker fixed effects



When studying the link between firm effects and firm performance, we limit attention to the manufacturing and mining sector, for which we have data on firm performance and characteristics.[27] Table 1.6 compares AKM estimates for this subpopulation with the overall population. The overall variance of log earnings is four log points higher in the PIA subpopulation in the 1996–2000 subperiod and falls by 19 log points over the subsequent three subperiods (versus 20 log points in the overall population). The variance of worker effects is three log points higher in the 1996–2000 subperiod and four log points greater in the 2008–2012 subperiod. The variance of firm effects is two log points less in 1996–2000 and falls by eight instead of nine log points. We conclude from this that trends in inequality are similar in the PIA subpopulation as in the overall population.

[27] As noted earlier, we impose the restriction to the PIA subpopulation after estimating the AKM model on the entire population of adult males.

Table 1.6: AKM decomposition results for large manufacturing&mining firms (as fraction of largest connected set)

	(1)	(2)	(3)	(4)	(5)
	1996–2000	2000–2004	2004–2008	2008–2012	Cumulative change from 1996 to 2012
Variance of log earnings (% of pop. estimate)	0.76 (100%)	0.74 (100%)	0.65 (100%)	0.58 (100%)	-0.19 (100%)
Variance of worker effects (% of pop. estimate)	0.39 (51%)	0.39 (53%)	0.38 (58%)	0.35 (61%)	-0.04 (20%)
Variance of firm effects (% of pop. estimate)	0.15 (19%)	0.12 (16%)	0.08 (13%)	0.07 (11%)	-0.08 (43%)
Variance of year effects (% of pop. estimate)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)
2×Cov. worker and firm effects (% of pop. estimate)	0.18 (23%)	0.18 (24%)	0.15 (23%)	0.12 (21%)	-0.06 (30%)
2×Cov. worker and year effects (% of pop. estimate)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)
2×Cov. firm and year effects (% of pop. estimate)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (0%)	0.00 (-1%)
Variance of residual (% of pop. estimate)	0.05 (7%)	0.05 (6%)	0.04 (6%)	0.04 (6%)	-0.02 (10%)
# Worker years (% of pop. estimate)	16.60 (18%)	17.90 (18%)	22.80 (18%)	26.30 (17%)	
# Firm years (% of pop. estimate)	0.11 (3%)	0.13 (3%)	0.16 (3%)	0.18 (2%)	
R^2 (% of pop. estimate)	0.93 (101%)	0.94 (101%)	0.94 (101%)	0.94 (101%)	

Note: Variance decomposition of AKM model estimated using manufacturing firms covered by PIA. Ratio between estimates using manufacturing firms relative to AKM estimates using all sectors given in parentheses. Worker-years, unique workers, firm-years, and unique firms in millions.

1.6.2 The link between firm effects and firm characteristics

Given that a reduction in dispersion of firm effects has been an important element in the decline, a natural question is whether differences in pay at the firm level are related to observable characteristics of the firm and whether changes in such observable characteristics explain the changes we observe in the firm-specific component of pay over time. Using measures of firm performance from the PIA data, Table 1.7 reports results from regressing estimated firm effects on log value added per worker both controlling for and not controlling for two-digit subsectors.[28]

Several features are worth highlighting. First, greater value added per worker is associated with a higher firm component of pay. Second, the amount of dispersion in firm effects explained by firm performance is notable, with an R^2 above 0.6 (around 0.5 without subsector controls). Third, the pass-through from value added per worker to the firm component of pay declines substantially over time. In 1996–2000, a one log point increase in value added per worker was associated with a 0.24 log points increase in estimated firm effects (0.26 without subsector controls); in 2008–2012, the same increase in value added per worker was only associated with a 0.13 log point higher firm effect (0.15 without subsector controls).

Table 1.7: Regression of estimated firm effects on firm productivity

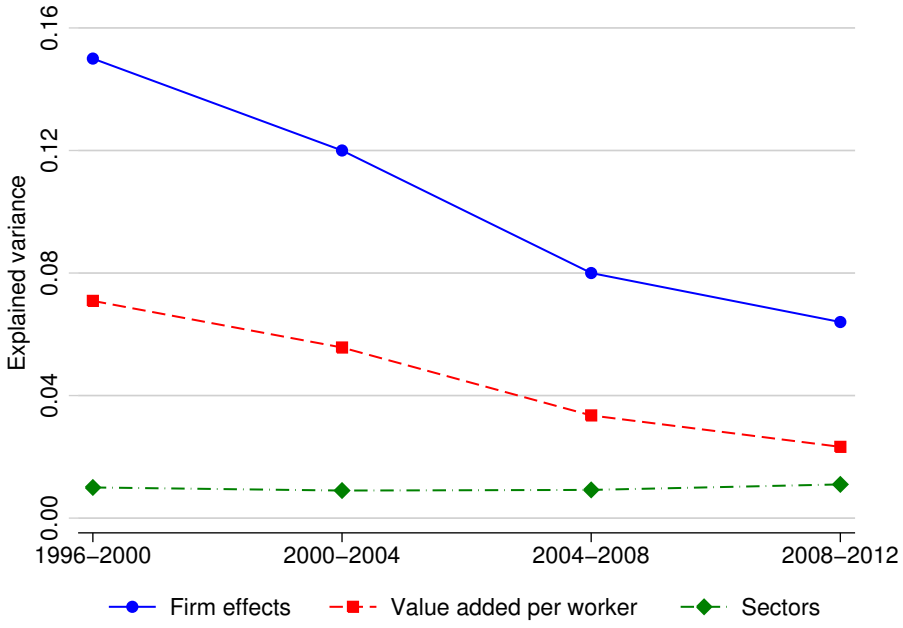
	1996–2000		2000–2004		2004–2008		2008–2012	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Value added p.w.	0.258	0.236	0.201	0.182	0.163	0.143	0.147	0.126
Sector controls	No	Yes	No	Yes	No	Yes	No	Yes
Worker-years	16.6	16.6	17.9	17.9	22.8	22.8	26.3	26.3
R^2	0.583	0.655	0.562	0.650	0.537	0.639	0.483	0.624

Note: Dependent variable is the estimated firm effect a_j. Independent variable is log value added per worker. Number of worker-years in millions.

[28] We find that other firm characteristics only marginally improve the explanatory power of the model. Moreover, the variance of the predicted values due to other firm characteristics is essentially zero, and hence we do not report them here.

Figure 1.8 evaluates the implication of the estimation results in Table 1.7 for the compression in firm pay in Brazil. It plots the overall variance of firm effects (solid blue line with circles), the predicted variance only due to value added per worker (dashed red line with squares), and the predicted variance only due to sectors (dash-dotted green line with diamonds) across periods. Value added per worker explains almost half of the initial variance in firm effects, while sectors initially explain seven percent. Between 1996–2000 and 2008–2012, the variance explained by value added per worker declines by five log points or 25 percent of the overall decline in the variance of log earnings in Brazil over this period. The variance due to sectoral variation, on the other hand, is small and remains constant in levels.[29]

Figure 1.8: Regression of firm effects on productivity and sector controls



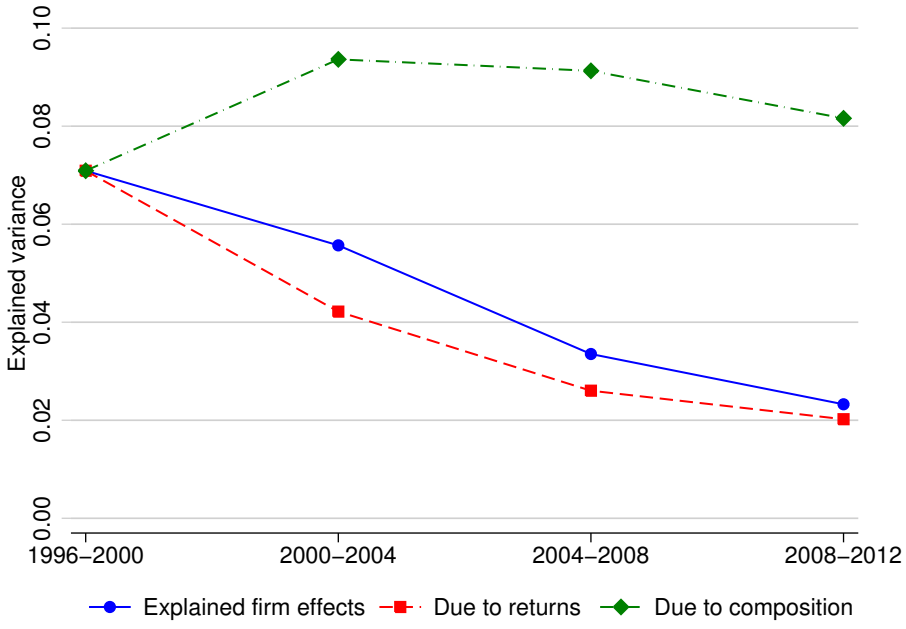
To quantify the importance of changes in the firm productivity distribution versus the pass-through from productivity to pay, we consider the two counterfactual exercises outlined in Section 1.5. Thus, we first hold the pass-through from value

[29]We separately investigated the role of sectors in the overall economy, not just the manufacturing and mining sectors, and find a significant role for variation in firm-level pay within sectors; results are available upon request.

added per worker to the firm component of pay constant at its estimated 1996–2000 value and allow only the variance of value added per worker to change as in the data. Second, we hold the variance of value added per worker fixed and change only the pass-through to match the data.

Figure 1.9 plots the result from this exercise. The predicted variance of firm effects due to value added per worker (solid blue line with circles) declines from seven to two log points, the predicted variance from value added per worker holding pass-through constant (dashed red line with squares) increases from seven to eight log points, and the predicted variance holding the dispersion in value added per worker constant (dash-dotted green line with diamonds) falls slightly more than the total predicted variance. We conclude that, ceteris paribus, a declining pass-through from firm performance to pay contributed significantly to reduced earnings inequality in Brazil during this period.

Figure 1.9: Variance of predicted firm effects, variance predicted by fixed pass-through and variance predicted by fixed productivity distribution



1.6.3 The link between worker effects and worker characteristics

We finally turn to our results regarding the link between observable worker characteristics and the worker component of pay. Table 2.6 shows the result from a regression of the estimated worker effects on age and education with and without occupation controls. We note the following: First, more able workers, as measured by labor market experience or education level, are paid more. For instance, the oldest age group earns on average 50 log point more than the youngest age group conditional on place of work. Secondly, such worker characteristics explain roughly 40 percent of the variance in worker effects, leaving substantial room for unobserved worker heterogeneity. Third, both the age gradient and the return to education declines over time, with a particularly pronounced fall in the latter. For instance, a college degree was associated with a 116 log point higher worker component of pay in 1996–2000 but only a 78 log point higher worker effect in 2008–2012.

To quantify the contribution of the estimation results in Table 2.6 towards the declining dispersion in worker heterogeneity in Brazil, Figure 1.10 plots the overall variance of worker effects (solid blue line with circles), the predicted variance only due to age structure (dash-dotted green line with diamonds), and the predicted variance only due to variation in educational attainment (dashed red line with squares). We do not plot the variance due to occupation as this remains small and steady at around one, two log points over the entire period (in particular, it increases slightly from 0.019 in 1988–1992 to 0.023 in 2008–2012).[30] We find that while education and age explain only around 20–25 percent of total worker heterogeneity, they account for almost half of the decline in the variance of worker effects.

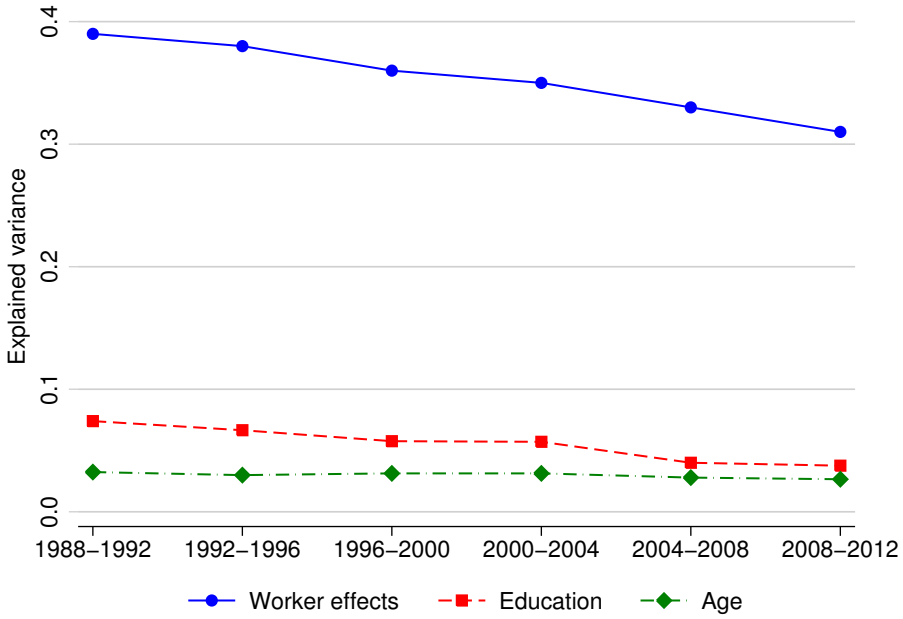
[30] Also not shown, the sum of covariances increases modestly from 0.032 in 1988–1992 to 0.040 in 2008–2012.

Table 1.8: Regression of estimated worker effects on worker characteristics

	1988–1992		1992–1996		1996–2000		2000–2004		2004–2008		2008–2012	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Age groups												
25–29	0.20	0.18	0.18	0.18	0.19	0.18	0.18	0.18	0.16	0.15	0.16	0.15
30–39	0.39	0.35	0.35	0.32	0.35	0.32	0.34	0.32	0.31	0.29	0.30	0.28
40–49	0.52	0.47	0.50	0.46	0.51	0.47	0.49	0.45	0.45	0.41	0.42	0.39
50–64	0.48	0.44	0.47	0.44	0.50	0.46	0.52	0.48	0.52	0.47	0.51	0.47
Education groups												
Middle school	0.21	0.16	0.19	0.15	0.16	0.12	0.14	0.10	0.13	0.08	0.11	0.07
High school	0.61	0.51	0.56	0.46	0.45	0.37	0.37	0.29	0.32	0.21	0.27	0.19
College or more	1.21	1.10	1.19	1.04	1.16	1.00	1.19	1.00	1.15	0.82	1.10	0.78
Occupation controls	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
# Workers	85.4	85.4	85.6	85.6	90.2	90.2	102.0	102.0	123.7	123.7	151.0	151.0
R^2	0.35	0.40	0.34	0.37	0.34	0.38	0.35	0.39	0.36	0.41	0.37	0.44

Note: Dependent variable are estimated worker effects a_i. Number of workers in millions. Education estimates are relative to "less than middle school " category. Age estimates are relative to "age 18–24" category.

Figure 1.10: Regression of worker effects on age and education



We further decompose the decline in the dispersion due to age and education into one due to changes in returns and one due to changes in the composition of the workforce.[31] Figure 1.11 plots in the top (bottom) panel in solid blue with circles the overall compression explained by education (age), in dash-dotted green with diamonds the compression resulting from holding the returns to education (age) fixed, and in dashed red with squares that when holding the underlying education (age) distribution fixed.

Holding returns fixed, changes over this period in the age structure as well as the educational attainment of the workforce resulted in a small net increase in inequality. Given the large increase in educational attainment documented earlier, it might come as a surprise that this did not induce a larger change in the underlying distribution of educational attainment. This is the result of two offsetting forces: on the one hand, the share of the lowest education group shrinks, but on the other more workers obtain

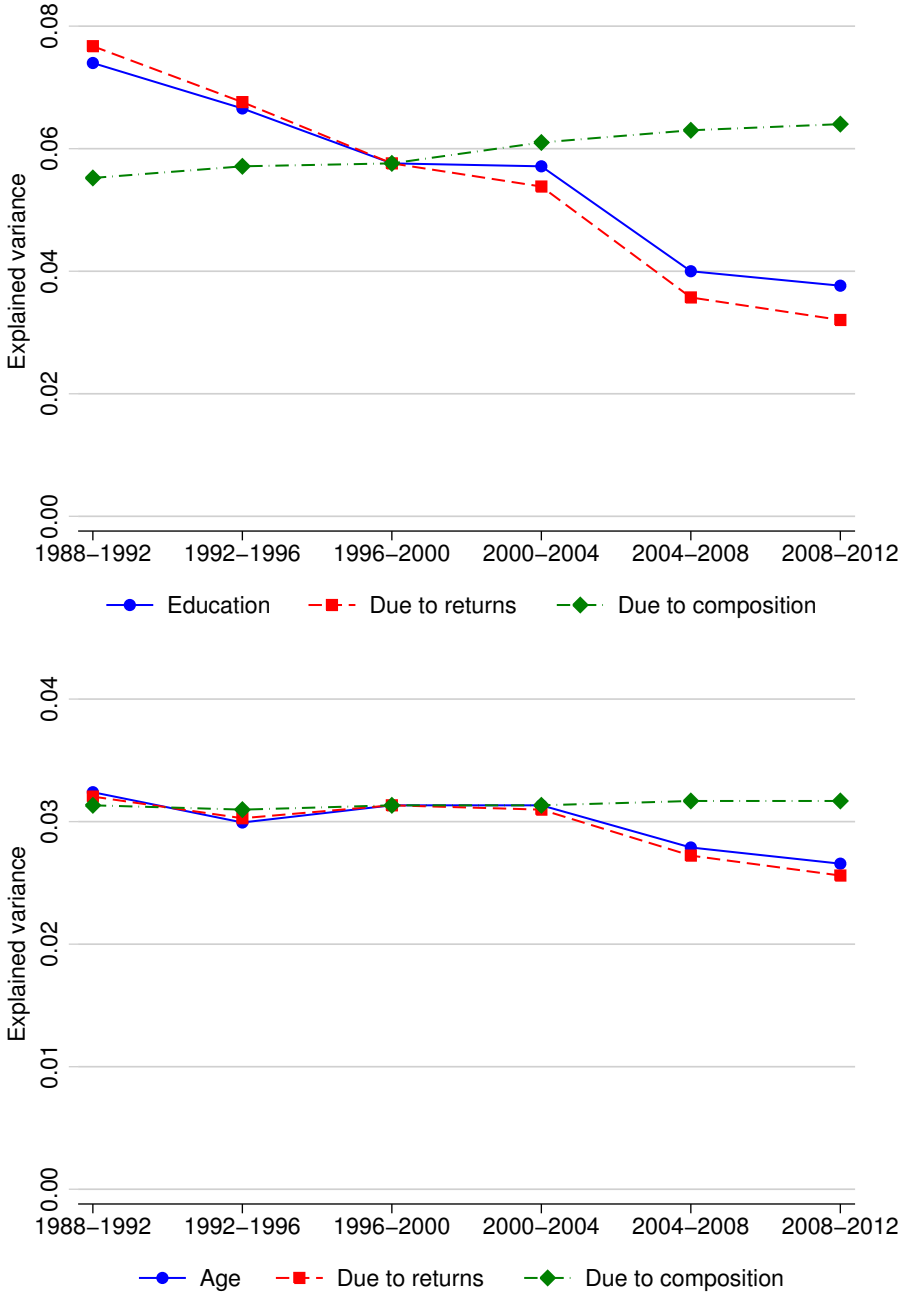
[31]Since the variance due to occupation is small and does not change much over time we do not present a decomposition of this.

high-school or college degrees, which earn more than the average. Altogether, these two effects have small positive net effects on inequality.

At the same time the rapid decline in the return to education—and to a lesser extent age—resulted in significant earnings compression. In fact as can be seen by comparing the solid blue lines with circles to the dashed red line with squares, the compression in returns resulted in an even larger fall in inequality than the overall fall. Holding everything else constant, the fall in the return to education (age) explains 13 (three) percent of the overall decline in the variance of log earnings between 1996–2000 and 2008–2012.[32]

[32]Of course, the compression in the return to education could have partly resulted from changes in educational attainment. The counterfactual of holding the distribution constant while changing returns should be interpreted with this qualification in mind.

Figure 1.11: Decomposition of variance of worker effects predicted by age (top) and education (bottom)



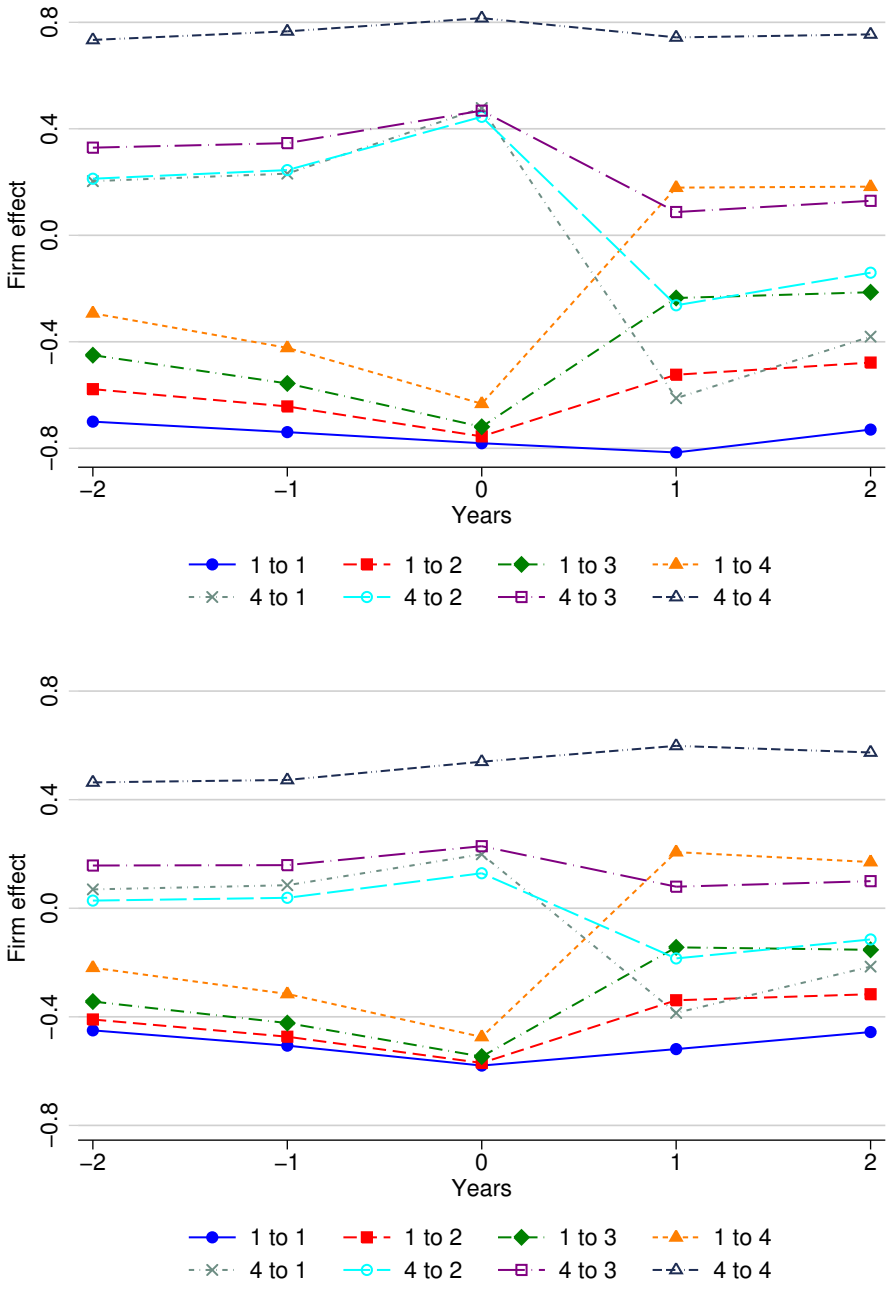
1.6.4 Empirical support for the AKM model

A large part of our analysis has been based on estimation results from repeated applications of the AKM framework. Although the consistently high R^2 coefficient of our model suggests that the model provides a good fit to the data, our estimates can be biased if the residual is correlated with either the firm or worker component of earnings. To investigate this possibility further, we replicate using our Brazilian data two diagnostics exercises proposed by Card, Heining and Kline (2013) for the case of Germany.

Figure 1.12 shows the average firm effect of workers who switch firms up to two years prior to the switch and two years after the switch for the first and last period of our sample. Switchers are classified by the firm effect quartile of the pre and post transition firms. Consistent with the AKM specification, workers that switch from the lowest quartile experience gains in firm effect and workers that switch from the highest quartile experience losses. Additionally, the gains of those switching up are similar to the losses of those making the reverse switch.[33] This suggests that our additive model is consistent with the pattern observed among workers who transition between firms.

[33]For expositional ease we only show switches out of the first and fourth quartile, but other quartiles display similar pattern and are available on request.

Figure 1.12: Average changes in firm effects of workers switching employers, 1996–2000 (top) and 2008–2012 (bottom)

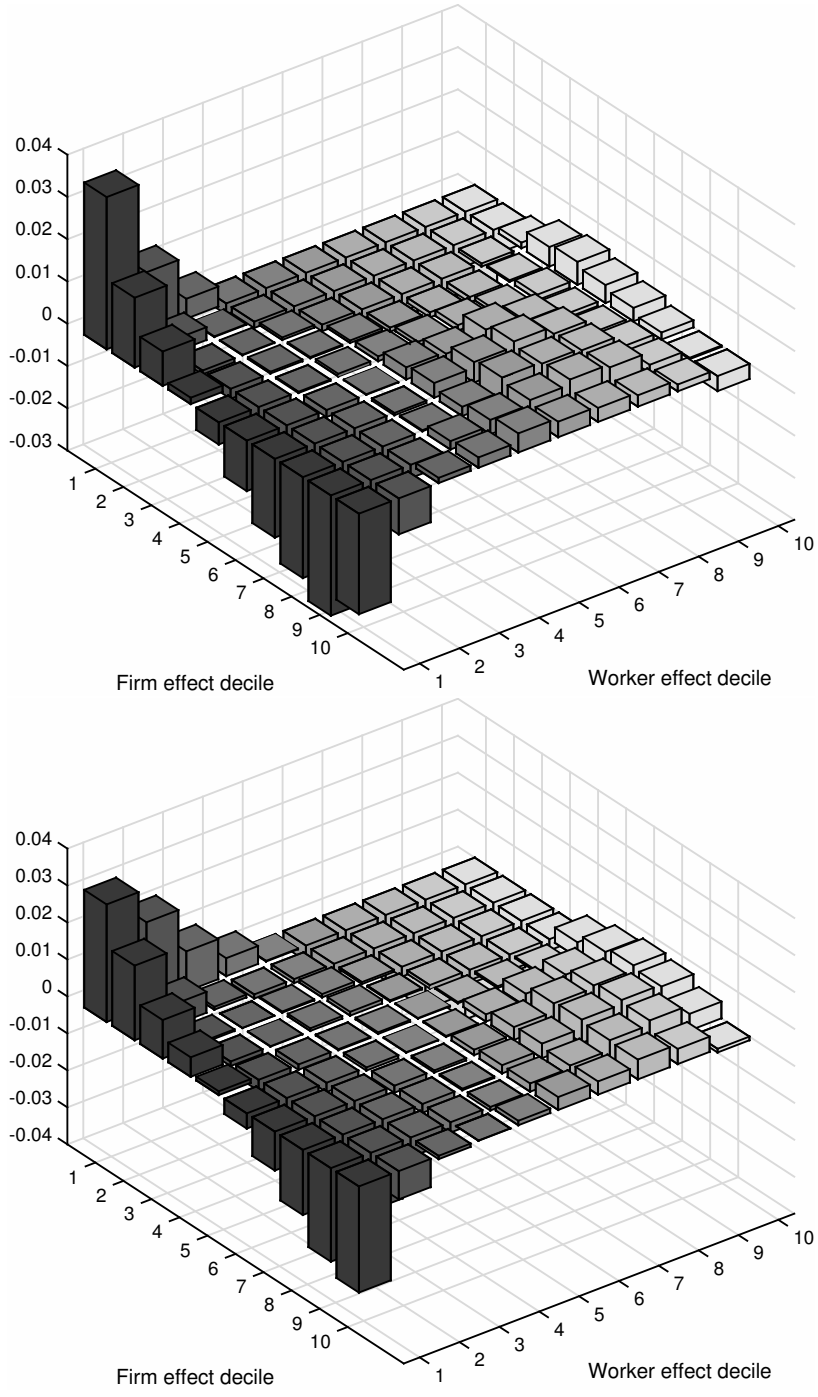


Second, we find little evidence in the data for match effects that are systematically correlated with either person of match effects. Figure 1.13 shows the average

estimated residual by decile of worker and firm effect. There is some evidence of misspecification for the lowest decile of workers in the sense that they display a systematically positive residual while working at the lowest paying firms. However, the magnitude of the error is modest and beyond the lowest decile of worker effects, errors do not exhibit any systematic relationship with firm and worker effects.[34] This reassures us that the log additive assumption is an appropriate characterization of Brazilian wage setting policies.

[34]The systematic variation among the lowest decile of worker effects is due to those individuals having a strongly positive error term when working at the lowest-paying firms, and strongly negative error term when working at the highest-paying firms. In other words, for lowest worker effects individuals the pay gradient across firms is very flat. We argue in Engbom and Moser (2015) that this pattern is consistent with a frictional labor market with a binding minimum wage.

Figure 1.13: AKM residual by firm and worker fixed effect deciles, 1988–1992 (top) and 2008–2012 (bottom)



1.7 Conclusion

In this paper, we document and dissect a large decline in earnings inequality in Brazil between 1996 and 2012. To understand the sources of this decline, we estimate a high-dimensional fixed effects model of earnings controlling for both worker and firm heterogeneity. Our results suggest that firms contributed more than proportionately to the decline. While more productive firms pay more, compression in firm productivity was not a factor behind declining inequality. Instead, we find that a declining pass-through from firm productivity to pay played an important role behind the decline. On the worker side, higher ability workers as measured by educational attainment or experience are paid more, yet also such measures of worker ability show no compression over time. Rather, a significant share of the explained decline in worker heterogeneity is due to rapidly falling returns to measures of worker ability. We conclude that changes in pay policies, rather than changes in firm and worker fundamentals, played a significant role in Brazil's inequality decline.

Our paper suggests a set of facts that a potential theory of the inequality decline in Brazil should be consistent with. Such a theory must generate pay differences between firms for identical workers, and they have to be strongly positively correlated with firm productivity. Moreover, any promising explanation needs to generate a compression in firm pay differences over time, not through compression in firm productivity but through a weaker link between productivity and pay. On the worker side, it should generate compression primarily through declining returns to ability, and not through a compression in underlying ability.

We think that promising candidates behind the decline in inequality in Brazil are changes in the nature of wage setting. In follow-up work Engbom and Moser (2015), we argue that the rapid rise in the minimum wage in Brazil during this period can explain a significant fraction of the decline in inequality, while being consistent with the stylized facts presented in this paper. Other institutional reforms, including changes

in union bargaining structure and labor compensation laws, represent interesting avenues for future research.

Chapter 2

Earnings Inequality and the Minimum Wage: Evidence from Brazil

2.1 Introduction

Earnings inequality has become central to recent debates in academic and policy circles.[1] A majority of respondents to an international survey identified government policies as the most frequently cited reason for prevailing inequality levels.[2] In this context, economists seek answers to two important questions: First, what factors drive the evolution of earnings inequality? Second, to what extent can economic policies affect these trends?

To shed light on these questions, we study Brazil as an economy that experienced a rapid decline in earnings inequality between 1996 and 2012. Starting at high initial

[1] Examples of recent research concerned with earnings inequality include Atkinson and Bourguignon (2015) for an overview of academic work, OECD (2015) for policy issues in a number of developed countries, and IMF (2015); World Bank (2013) for policy relating to emerging markets and developing economies.

[2] See Pew Research Center (2014).

inequality levels, Brazil saw a 26 log points (or 35 percent) fall in the variance of log earnings over this period. By comparison, in the U.S. the same measure increased by six log points (or 12 percent) during those years.[3] Concurrent with Brazil's remarkable inequality decline, the country's minimum wage rose by 119 percent in real terms. Yet, given the ongoing debate in the literature about consequences of the minimum wage, the extent to which these two trends are related is far from clear.[4]

The main contribution of this paper is to quantify the effect of the rise in the minimum wage on Brazil's inequality evolution. To this end, we use administrative matched employer-employee data to review and extend key empirical patterns that we document in detail in Alvarez, Engbom, and Moser (2015). Specifically, the three facts characterizing Brazil's earnings inequality decline that we address in the current paper are: (i) the decline is more pronounced towards the bottom of the distribution; (ii) one quarter of the decline stems from an increase in relative pay at less productive firms; and (iii) another quarter of the decline is attributable to falling pay differences due to worker heterogeneity, largely driven by decreasing returns to education and age. Hence, any candidate explanation for Brazil's inequality evolution needs to generate compression in the earnings distribution driven from the bottom with changes in the returns to firm and worker characteristics playing a prominent role.[5]

To assess the extent to which the rise in the minimum wage can account for these facts, we build a model of frictional wage dispersion based on the canonical search framework by Burdett and Mortensen (1998). Motivated by our empirical findings, we extend this framework in a tractable way to include heterogeneity in both worker ability and firm productivity. The key feature of this environment is that the minimum wage indirectly affects higher parts of the earnings distribution. Because firms com-

[3]Inequality measures are defined over labor income for workers of age 18–64 using the *March Current Population Survey (CPS)* for the U.S.; and the *Relação Anual de Informações Sociais (RAIS)* for Brazil. See Appendix B.1 for details.

[4]See Flinn (2010) for a selective survey of the literature.

[5]In contrast, we show that the underlying distributions of firm and worker characteristics, notably firm productivity and workers' educational attainment, became more dispersed over the period.

pete for workers on wages, higher productivity firms increase their equilibrium wage offers above the new minimum wage in order to poach and retain workers. Therefore, while the minimum wage has a direct impact on the least productive workers and firms in the economy, its effects will slowly fade out towards the top of the earnings distribution. These spillover effects open the door to the minimum wage qualitatively accounting for the three facts on Brazil's inequality decline.

We find that the minimum wage is also quantitatively successful at explaining the documented facts on the inequality evolution. To this end, we estimate key model parameters guiding labor mobility as well as heterogeneity in worker ability and firm productivity using indirect inference on the Brazilian microdata from 1996–2000. We then use the estimated model to simulate the effects of the observed minimum wage increase. The main result of this experiment is that 70 percent of the observed decline in the variance of log earnings are accounted for by the rise in the minimum wage. More than half of this decline is due to indirect effects of the minimum wage. In line with our empirical findings, the model generates significant compression up to the top decile of the earnings distribution. A sizable share of the overall inequality decline is due to a weaker productivity-pay gradient across firms, with the model generating a drop of 4.3 log points in the variance of log earnings due to this channel, relative to 5.0 log points in the data. Furthermore, the model predicts a fall in the dispersion of worker pay heterogeneity explaining an additional 6.2 log points fall in the variance of log earnings, compared to 5.6 log points in the data.[6] Together, these results suggest that the minimum wage was an important driver behind Brazil's inequality decline.

A central feature of the model is the presence of spillover effects of the minimum wage on higher earnings ranks. Their source is the upwards-sloping labor supply curve faced by monopsonistic firms under search frictions, creating a trade-off in wage

[6]In the model, all of the decline in worker-specific pay is due to convergence in the returns to worker types. Also in the data, we find that all of the decline in worker heterogeneity explained by observable worker characteristics (age, education, and occupation) is due to decreasing returns to these characteristics.

setting between firm size and profitability. In equilibrium, more productive firms offer higher wages and workers gradually climb up a job ladder by moving to better-paying employers. Since the rates of poaching and retaining workers depend on a firm's rank in the wage offer distribution within each labor market segment, the minimum wage indirectly affects equilibrium wage posting strategies of all firms in the market. As the competitive pressure in response to a rise in the minimum wage is weaker for firms further up in the productivity distribution, the resulting productivity-pay gradient across firms is lower and earnings are less dispersed. Analogously, since lower ability worker are more likely and more intensely affected, the minimum wage also leads to compression in the relative rents captured by different worker types. We provide empirical evidence for the mechanism by identifying a job ladder across firms in the Brazilian microdata and show that, in line with the model predictions, this job ladder has become flatter as the minimum wage increased over time.

Our model highlights the redistributive effects of the minimum wage. In the presence of search frictions, firms generate monopsony rents because they generally pay workers below their marginal product. Through its direct and indirect effects on firms' wage posting decisions, the minimum wage transfers some of these rents towards workers. While individuals who remain employed at a higher minimum wage benefit unambiguously, not everyone gains from the policy change. The lowest productivity firms stop recruiting from low ability markets or exit altogether for high enough levels of the minimum wage. Similarly, the lowest ability workers first relocate to more productive firms before eventually being forced into unemployment. Evaluating these channels quantitatively, however, we find small displacement effects of the minimum wage for both firms and workers.

As a validation of the minimum wage mechanism, we show that it explains salient sectoral and regional trends in the Brazilian household survey data (*Pesquisa Nacional por Amostra de Domicílios*, or *PNAD*). Exploiting the universal coverage of the survey

data, we document that the earnings inequality declined markedly among formal sector workers in Brazil and to a lesser degree among workers in the informal economy. To the extent that labor regulations like the minimum wage are more strictly enforced in the formal sector, this finding lends additional support to the minimum wage hypothesis. Furthermore, we confirm that sectors and regions that started out at lower average earnings levels experienced more pronounced declines in inequality, in line with the minimum wage having disproportionate effects on those parts of the economy where it is most binding.

We also confirm a key prediction of the model for the effect of the minimum wage on the allocation of workers across firms in the microdata. In the model, the minimum wage renders matches between the lowest productivity firms and the lowest ability workers infeasible because the surplus generated from such matches falls short of the minimum wage. Consequently, the policy induces negative sorting between workers and firms towards the bottom of the firm productivity distribution. In support of this mechanism, we document a negative correlation between firm effect and worker effect ranks estimated using our earlier two-way fixed effects decomposition.[7] Consistent with our model prediction, we show that this negative sorting pattern becomes more pronounced as the minimum wage increases over time.

A general insight emanating from our analysis is that labor market dynamics can propagate effects of policy on the earnings distribution. Our quantitative analysis attributes more than half of the overall decline in the variance of log earnings to spillover effects of the minimum wage. Consequently, only considering the direct effect of a rise in the minimum wage would significantly understate its impact on earnings inequality. In our framework, the effects of the minimum wage are propagated through monopsonistic competition among firms for workers due to on-the-job

[7]Overall, we find significant positive sorting between workers and firms. Speaking to the weakness of the AKM estimation framework pointed out by Bonhomme, Lamadon and Manresa (2015), we verify that the negative correlation at the bottom is robust to restricting our sample to large firms and frequent worker switchers.

mobility.[8] While our analysis focuses on one particular policy and economic environment, we conjecture that similar quantitative results would obtain in a broader class of models featuring spillover effects in wage setting, and considering a range of other labor market policies such as unemployment insurance, employment protection legislation, and non-discrimination laws.

Related literature. Our work relates to three strands of the literature within the broad realm of understanding inequality in labor markets. The first strand is concerned with decomposing the determinants of earnings inequality into components relating to workers, firms, and other factors, and using this decomposition to understand changes in the earnings distribution over time. The seminal work in this area is that of Abowd, Kramarz, and Margolis (1999, henceforth AKM) who propose a two-way fixed effects framework controlling for unobserved worker and firm heterogeneity. They find that firms explain a significant share of earnings inequality levels in French linked employer-employee data (but do not study changes over time).[9] In later work, Card, Heining and Kline (2013) apply the same methodology to Germany and argue that firms explain a quarter to a third of the overall rise in earnings inequality in Germany. Card, Cardoso and Kline (2015) use a static AKM framework to investigate the degree of differential sorting and rent sharing between male and female workers in Portugal. Alvarez, Engbom, and Moser (2015) applies this methodology to understand Brazil's decline in inequality between 1988 and 2012, and find that falling inequality between firms in pay is an important component of this decline. Although not within an AKM framework, Barth, Davis and Bryson (2014) and Song

[8] More precisely, our model features interdependence in firms' wage offers whenever workers receive outside job offers while employed with strictly positive but finite rate (probability less than one) in a continuous (discrete) time setup.

[9] In a later paper, Abowd, Creecy and Kramarz (2002) note that the approximate estimates obtained in AKM, particularly firm effects, were contaminated due to the approximation method used at the time. However, the authors note that their original conclusion, namely that person effects account for most wage variation in France, remains unchanged.

et al. (2015) argue that changes in pay across firms were important in understanding the increase in wage dispersion in the U.S. during the last decades.

Second, our theoretical framework is closely related to the literature using search models to study wage dispersion. While work in this area goes back to at least McCall (1970), a more recent class of search models pioneered by Burdett (1978) and further developed by Burdett and Mortensen (1998) lays the foundation for our analysis of the effects of the minimum wage in a job ladder environment. A rich body of follow-up work has used versions of this model to jointly study wage dispersion and labor dynamics (van den Berg and Ridder, 1998; Bontemps, Robin and van den Berg, 1999, 2000; Mortensen, 2000, 2003; Postel-Vinay and Robin, 2002; Cahuc, Postel-Vinay and Robin, 2006; Jolivet, Postel-Vinay and Robin, 2006; de Araujo, 2014). To this literature we contribute a tractable model of the minimum wage with heterogeneity in both firm productivities and worker abilities, an environment that previous research highlighted as important but difficult to study. In related work, Hornstein, Krusell and Violante (2011) note that several search models struggle to generate the observed amount of wage dispersion in the data. Their argument is that on-the-job search is crucial for these models to generate realistic levels of frictional wage dispersion. Our complementary insight is that also the effects of policy, such as the minimum wage, can be amplified in such models.

We also connect our structural search model to empirical studies of wage determination. Whereas several empirical studies document significant dispersion in pay across firms using the original AKM methodology, few studies have provided a formal justification for this framework.[10] Providing such a microfoundation is important since other papers have stressed that sorting models of labor markets may lead the AKM framework to produce misleading results (Lentz and Mortensen, 2010; Eeckhout and Kircher, 2011; Lopes De Melo, 2013; Bonhomme, Lamadon and Manresa,

[10] A notable recent exception is Burdett, Carillo-Tudela and Coles (2014).

2015). We bridge these two literatures by contributing a tractable model of frictional wage dispersion with heterogeneity in both worker ability and firm productivity that maps directly into the AKM framework. We characterize this mapping and show that the AKM regression framework recovers the underlying structural parameters of the model from the data.

Finally, our focus on the effects of the minimum wage on the earnings distribution complements a long-standing debate in the literature on how the minimum wage affects labor market outcomes. A salient debate in this literature revolves around the employment consequences of the minimum wage (Card and Krueger, 1994; Neumark and Wascher, 1994; Manning, 2005), with mixed findings but pointing in the direction of small negative employment effects. DiNardo, Fortin and Lemieux (1996), Lee (1999), Card and DiNardo (2002) argue that a decline in the federal minimum wage in the U.S. in the 1980's explains a significant amount of the increase in earnings inequality during that time. Going against this previous literature's conclusions, Autor, Katz and Kearney (2008) and Autor, Manning and Smith (2016) argue that nonmarket factors such as the decline in the minimum wage contributed little to the dynamics of U.S. earnings inequality. We contribute to this debate by showing that the predictions of a structural model of the minimum wage are quantitatively consistent with sizable effects throughout large parts of the earnings distribution. Bárány (2016) studies a model with complementarity between skill groups in the production technology and endogenous educational investment. Like in our framework, the minimum wage in that model has spillover effects on higher income groups, but for orthogonal reasons. Harasztosi and Lindner (2015) study an increase in the minimum wage in Hungary in 2001 that is of similar size as the one experienced in Brazil 1996–2012, and find that it pushed up wages with only a small negative impact on employment. In comparison to their work, our focus is on the effects of the minimum wage on earnings inequality. Komatsu and Menezes Filho (2015) argue empirically

that increases in the minimum wage can explain all of the reduction in earnings inequality in Brazil between 2002 and 2013. Complementary to the focus of his paper, our structural model allows us to separately identify the direct and indirect effects of the minimum wage on Brazil's earnings distribution.

Outline. The rest of the paper is structured as follows. Section 2.2 introduces the three datasets we use to study the evolution of earnings inequality and the role of the minimum wage. Section 2.3 presents key facts on the decline of earnings inequality in Brazil building on Alvarez, Engbom, and Moser (2015). Section 2.4 provides an institutional history of the minimum wage in Brazil. In Section 2.5, we describe our structural model of frictional wage dispersion and characterize the effects of a rise in the minimum wage on workers and firms in the economy. Section 2.6 describes our estimation strategy and the main policy experiment identifying the effects of the observed rise in the minimum wage on the earnings distribution. Section 2.7 presents quantitative results on the effect of the minimum wage on earnings inequality, on compression throughout the income distribution, and on the productivity-pay gradient across firms. In Section 2.8, we provide empirical evidence for effects of the minimum wage on the earnings distribution, discuss our modeling assumptions, and discuss welfare implications of the minimum wage in our framework. Finally, Section 2.9 concludes by putting into context the paper's main findings.

2.2 Data

Our analysis combines data from three separate sources: The first dataset are the Brazilian Household surveys *Pesquisa Nacional por Amostra de Domicílios (PNAD)*, which contain a representative sample of households covering all of Brazil, including workers in the formal and informal sectors. Our second data source consists of an administrative linked employer-employee dataset called *Relação Anual de Informações*

(RAIS), containing annual information from 1996–2012 on earnings and demographic characteristics of formal sector workers as reported by employers. The third dataset is the *Pesquisa Industrial Anual Empresa (PIA),* which contains information on the revenue and cost structure of large firms in Brazil's mining and manufacturing sectors from 1996–2012, and which we merge with the worker-level data contained in RAIS. The following subsections describe each of the three datasets in detail.[11]

2.2.1 Household survey data (PNAD)

The PNAD household surveys consist of a double-stratified sampling scheme by region and municipality, interviewing a representative of households in Brazil. The survey asks the household head to respond on behalf of all family members and report a rich set of demographic and employment-related questions. In particular, the survey asks a question about whether the respondent holds a legal work permit. We use the answer to this survey question to identify individuals as working in the formal or in the informal sector. Survey questions regarding income and demographics of the respondent household members are comparable to the U.S. March Current Population Survey (CPS). We keep only observations that satisfy our selection criteria and have non-missing observations for labor income, whose variable definition we harmonize across years.[12]. Table 2.1 presents basic summary statistics on the PNAD data.

2.2.2 Linked employer-employee data (RAIS)

The RAIS is constructed from a mandatory survey filled annually by all formally registered firms in Brazil. The data collection is administered by the Brazilian Ministry of Labor and Employment, which kindly provided the data for the purposes of

[11] Appendix B.1 also contains summary statistics for PNAD, RAIS, and PIA at a period frequency.

[12] Standardized cleaning procedures are adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at `http://www.econ.puc-rio.br/datazoom/english/index.html`.

Table 2.1: PNAD summary statistics

		Log earnings		
	(1)	(2)	(3)	(4)
	# Workers	Mean	Std. dev.	Formal share
1996	60,176	6.81	0.98	0.65
1997	64,204	6.79	1.00	0.64
1998	63,016	6.78	0.97	0.64
1999	64,328	6.72	0.95	0.63
2000	-	-	-	-
2001	70,558	6.68	0.95	0.63
2002	72,273	6.66	0.93	0.63
2003	71,959	6.59	0.93	0.64
2004	75,617	6.61	0.91	0.64
2005	78,064	6.64	0.90	0.65
2006	78,627	6.71	0.89	0.66
2007	76,616	6.76	0.87	0.68
2008	76,571	6.80	0.85	0.69
2009	77,037	6.83	0.84	0.70
2010	-	-	-	-
2011	67,884	6.93	0.80	0.73
2012	69,297	6.98	0.80	0.73

Notes: All statistics are for adult male workers of age 18–64.Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. Means are computed by period. The The standard deviation is calculated by first demeaning variables by yearand then pooling the years within a sub-period. Surveysare not available in years 2000 and 2010.

this research under a confidentiality agreement. Data collection was initiated in 1986 within a nationally representative set of regions, reaching complete coverage of all employees at tax-registered establishments across all sectors of the economy in 1994. It is common practice for businesses to hire a specialized accountant to help with the completion of the RAIS survey to avoid fines levied on late, incomplete, or inaccurate reports. The data contain a unique, completely anonymized, time-invariant person identifier, which allows us to follow workers over time. It also contains unique, completely anonymized time-invariant establishment and firm IDs that we use to link multiple workers to firms and follow firms over time. We follow our previous work in conducting all analyses at the firm-level.

The dataset contains information on average gross monthly labor earnings including regular salary payments, holiday bonuses, performance-based and commission bonuses, tips, and profit-sharing agreements as well as start and end month of the job. The measure of income adjusts for labor supply by dividing annual earnings by the number of months worked at the job. A worker might have multiple spells in a year if he or she switched employer during the year or worked multiple jobs. We restrict attention to a unique observation per worker-year by choosing the highest-paying among all longest employment spells in any given year. In addition, we observe the age, gender, educational level, and occupation[13] of each worker. On the firm side, we also use sub-sector categories from IBGE, the national statistical institute.[14] Our firm size measure is the number of full-time equivalent workers during the reference year.

We exclude individual observations that have either firm IDs or worker IDs reported as invalid as well as data points with missing earnings, dates of employment, educational attainment or age. Together, these basic cleaning procedures drop less than 1% of the original population, indicative of the high quality of the administrative dataset.

Table 2.2 provides key summary statistics for the RAIS data for six periods spanning 1988-92, 1992-96, 1996-2000, 2000-2004, 2004-08, and 2008-12.[15] All numbers reported in the table are for adult male workers of age 18 to 64. We make the selection based on gender and age to be consistent with our previous work.[16] The group of

[13]We use occupations from the pre-2003 Classificação Brasileira de Ocupações (CBO) at the two-digit level.

[14]Both the industry and occupation classification systems changed during the period we study. We use conversion tables provided IBGE to standardize classification between different years and choose categories for both occupations and sectors coarse enough in order to avoid potential biases arising from mechanical changes in the classification system over time.

[15]To calculate the standard deviation, we demean the data by year before we pool the years within a subperiod.

[16]Extensive labor supply decisions correlated with schooling choice or the timing of retirement could bias our estimates if we were to include these population subgroups. For similar reasons, we also exclude women to avoid biases caused by job switching decisions motivated by maternal leaves and other motherhood-related labor market movements.

adult males represents 55% of the total dataset in 2000 and their average earnings, educational attainment, and age are largely representative of the overall population.

Table 2.2: RAIS summary statistics

			Log earnings	
	(1)	(2)	(3)	(4)
	# Workers	# Firms	Mean	Std. dev.
1996	18.05	0.98	1.32	0.87
1997	18.31	1.06	1.32	0.85
1998	18.65	1.12	1.28	0.85
1999	18.54	1.18	1.25	0.84
2000	19.15	1.22	1.20	0.83
2001	20.45	1.30	1.12	0.83
2002	21.22	1.37	1.06	0.81
2003	21.70	1.42	0.99	0.79
2004	22.78	1.48	0.98	0.78
2005	23.96	1.54	0.94	0.77
2006	25.14	1.61	0.86	0.75
2007	26.58	1.68	0.83	0.74
2008	28.45	1.76	0.83	0.73
2009	29.17	1.84	0.80	0.73
2010	31.01	1.95	0.78	0.71
2011	32.38	2.05	0.81	0.71
2012	33.23	2.13	0.78	0.70

Notes: All statistics are for male workers age 18–64.Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adultmale workers. The standard deviation is calculatedby first demeaning variables by year and then poolingthe years within a sub-period.

2.2.3 Firm characteristics data (PIA)

The PIA dataset contains data on firm characteristics from 1996 to 2012. It is constructed from annual surveys filled by firms in the manufacturing and mining sector and collected by the Brazilian Statistics and Geography Institute (*Instituto Brasileiro de Geografia e Estatística*, or *IBGE*), with whom we have signed a confidentiality agreement. This survey is mandatory for all firms with either more than 30 employees or more than $300,000 in revenues. As with RAIS, completion of the survey is mandatory and non-compliance is subject to a fine by national authorities. Each firm

has a unique, anonymized identifier, which we use to link firm characteristics data from PIA data to worker-level outcomes in the RAIS data. Each firm has a unique, completely anonymized identifier which we use to link the PIA dataset with employee data from RAIS.

The PIA dataset includes a breakdown of operational and non-operational revenues, costs, investment and capital sales, number of employees and payroll. All nominal values are converted to real values using the CPI index provided by the IBGE. Instead of the measure of firm size in the PIA, we prefer our measure of full-time-equivalent employees constructed from the RAIS as it accounts for workers only employed during part of the year. We define operational costs as the cost of raw materials, intermediate inputs, electricity and other utilities, and net revenues as the gross sales value due to operational and non-operational firm activities net of any returns, cancellations, and corrected for changes in inventory.[17]. We finally construct value added as the difference between net revenues and intermediate inputs, and value added per worker as value added divided by full-time equivalent workers. This is our main measure of firm productivity.[18]

Table 2.3 shows key summary statistics for the RAIS data for the four periods for which we have firm financial data in the PIA: 1996-2000, 2000-2004, 2004-08, and 2008-12. All results are weighted by the number of full-time equivalent workers employed by the firm.

2.3 Facts about Brazil's inequality decline

The goal of the current project is to quantify the contribution of a rise in Brazil's minimum wage towards earnings inequality dynamics during 1996–2012. To provide

[17]We have explored alternative revenue definitions such as only restricting attention to operational revenues or excluding certain types of non-operational revenues. Such robustness checks yield very similar results to what we report below.

[18]We have also constructed alternative measures of firm productivity by cleaning value added per worker off industry-year effects and some measures of worker skill.

Table 2.3: PIA Summary Statistics

		Log revenues per worker		Log value added per worker	
	(1)	(2)	(3)	(4)	(5)
	# Firm-years	Mean	S.d.	Mean	S.d.
1996	21,840	11.83	1.00	11.15	1.08
1997	21,022	11.86	1.03	11.15	1.11
1998	22,096	11.88	1.09	11.17	1.19
1999	22,771	12.01	1.16	11.29	1.27
2000	22,751	12.00	1.19	11.22	1.30
2001	24,920	12.01	1.24	11.23	1.32
2002	26,418	12.02	1.30	11.26	1.39
2003	27,853	11.96	1.31	11.18	1.37
2004	28,708	12.00	1.32	11.21	1.35
2005	30,628	11.94	1.30	11.16	1.33
2006	31,962	11.94	1.28	11.18	1.32
2007	31,808	11.97	1.28	11.21	1.31
2008	33,349	12.01	1.27	11.26	1.30
2009	34,200	12.01	1.23	11.31	1.27
2010	34,650	12.03	1.22	11.32	1.25
2011	36,773	12.06	1.20	11.34	1.23
2012	37,858	12.07	1.18	11.36	1.20

Notes: Sample includes all firms covered by the PIA dataset in the mining and manufacturing sectors. The number of firm-years and number of unique firms are reported in thousands. Allmeans and standard deviations are weighted by the number of employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within asub-period.

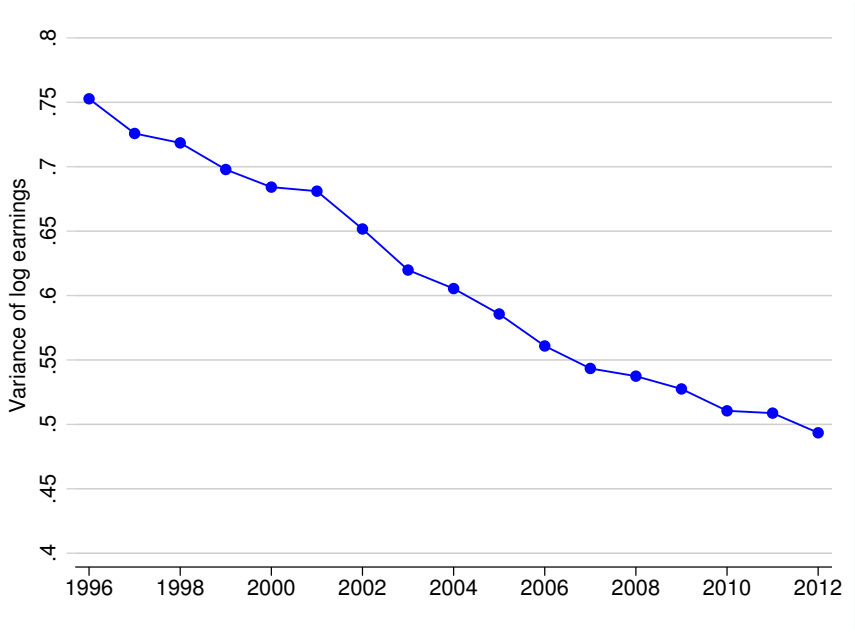
context for our study, we present in this section some key facts on the evolution of earnings inequality in Brazil over this period. All statistics are computed for the population of male formal sector workers of age 18–64 using the RAIS data, but similar trends hold for the overall worker population.[19]

Our motivating observation is that earnings inequality has declined rapidly in Brazil. Figure 2.1 plots the evolution of the variance of log earnings of adult males in the formal sector between 1996 and 2012. The data show a steady decline in the variance of log earnings by 26 log points or 35 percent, from 0.76 to 0.49, over the

[19]In Section 2.8, we explore inequality trends in Brazil's formal versus informal sectors using the PNAD household survey data. See Alvarez, Engbom, and Moser (2015) for a detailed exploration of inequality trends by population subgroups.

period. To put this evolution into context, the variance of log earnings for adult male workers increased by six log points in the U.S. over the same period.[20]

Figure 2.1: Decline in the variance of log earnings in Brazil, 1996–2012



We now present three facts characterizing Brazil's inequality evolution between 1996 and 2012.

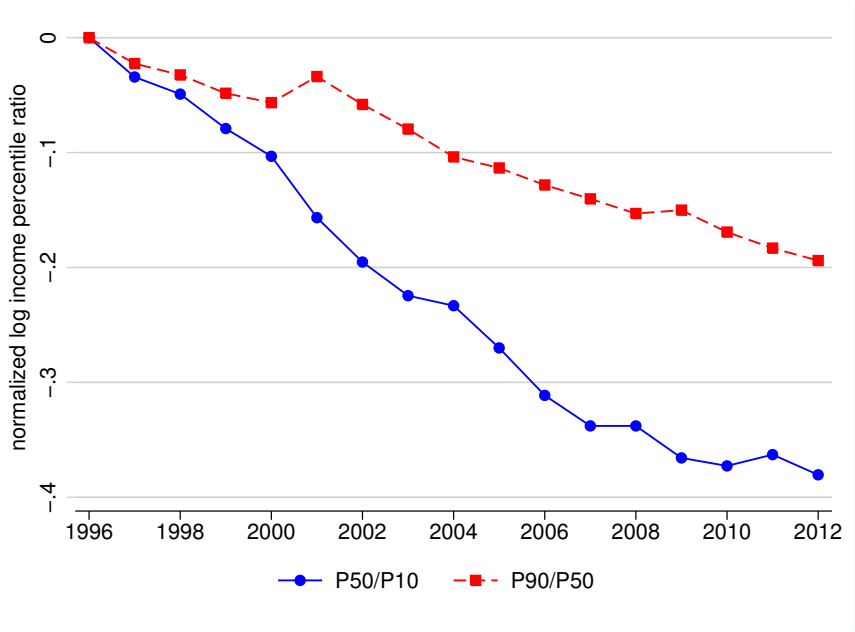
Fact 2.1. *The inequality decline is more pronounced towards the bottom of the earnings distribution.*

While Brazil's inequality decline was characterized by widespread compression throughout the earnings distribution, there was more pronounced catch-up from the bottom. For illustration purposes, we present two particularly prominent measures of top- and bottom-inequality, namely the log 90–50 and log 50–10 percentile ratios of the earnings distribution. Figure 2.2 plots these inequality measures based on the universe of adult male workers in the RAIS data. There was a significant decline in

[20]Calculations for the U.S. are based on the *March Current Population Survey (CPS)*, but evidence from other datasets, including alternative survey data used in Heathcote et al. (2010), or administrative tax returns data from Kopczuk and Saez (2010), point towards similar magnitudes. Thus, by any measure, the decline of inequality in Brazil can be considered large.

both measures, but the log 50–10 percentile ratio declined significantly more over the period. Specifically, the log 50–10 percentile ratio declined by 38 log points while the log 90–50 percentile fell declined by 19 log points at the same time.[21] Hence, while there was rapid compression of the earnings distribution relative to the top percentiles, this catch-up was more pronounced among the lowest earnings groups.

Figure 2.2: Normalized evolution of earnings percentile ratios measuring top and bottom inequality, 1996–2012



Before proceeding with our characterization of Brazil's inequality decline, we present a decomposition of the overall inequality evolution into one component capturing differences in firm pay and another component capturing worker heterogeneity. To motivate this analysis, which was first presented in Alvarez, Engbom, and Moser (2015), we note that most initial earnings inequality and most of the decline are due to dispersion of raw earnings between firms in the data.[22] To distinguish between

[21] Fact B.1 in Appendix B.1 shows that the bottom-driven inequality decline is apparent more broadly using other percentile ratios of the earnings distribution. Specifically, we find that there was compression up to, but not above, the 90th percentile of the earnings distribution. Furthermore, we show that that all percentiles of the earnings distribution experienced rapid real earnings growth over this period.

[22] Details of the between- and within-firm analysis are summarized in Fact B.2 of Appendix B.1.

inherent firm pay differences and the sorting of heterogeneous workers across firms, we follow an estimation methodology pioneered by AKM.[23] Specifically, we estimate a two-way fixed effect regression of log monthly earnings on a large set of worker effects, firm effects and year dummies in five-year sub-periods:

$$\log(y_{ijt}) = \alpha_i^p + \alpha_{J(i,t)}^p + \gamma_t + \varepsilon_{it} \tag{2.1}$$

for $t \in p = \{t_1, \ldots, t_5\}$ and where α_i^p denotes the individual fixed effect of worker i in period p, $\alpha_{J(i,t)}^p$ denotes the firm effect of the employer of worker i at year t, Y_t is a year dummy, and ε_{it} is an error term that satisfies the strict exogeneity condition $\mathbb{E}[\varepsilon_{it}| i, t, J(i,t)] = 0$.

Table 2.4 presents results from the above regression. In particular, we compute and report the variance of the predicted value due to each component from the AKM framework in equation (2.1). The variance of firm effects falls from 17 log points in 1996–2000 to eight log points in 2008–2012, which constitutes 45 percent of the overall inequality decline over the period. Similarly, the variance of worker effects falls from 36 log points in 1996–2000 to 31 log points in 2008–2012, making up 24 percent of the overall decline.

[23] To test the validity of this framework, Figure B.5 in Appendix B.1 plots the changes in estimated firm effects for workers switching firms by quartile of estimated firm effect before and after the switch. Alvarez, Engbom, and Moser (2015) discuss a range of additional checks and conclude that the assumptions underlying AKM holds in Brazil during this time.

Table 2.4: AKM variance decomposition between periods

	(1)	(2)	(3)
	1996-2000	2008-2012	Change
Variance of total earnings	0.72 (100%)	0.52 (100%)	-0.20 (100%)
Variance of worker effects	0.36 (50%)	0.31 (60%)	-0.05 (24%)
Variance of firm effects	0.17 (23%)	0.08 (15%)	-0.09 (45%)
Covariance	0.14 (20%)	0.10 (20%)	-0.04 (22%)
Variance of residual	0.06 (8%)	0.04 (7%)	-0.02 (10%)
# worker years	90.2	151.0	
R^2	0.92	0.93	

Note: Cells contain variance (share) explained by each component. Year dummies are omitted but account for a negligible share of the overall variation. The "Covariance" term reports two times the covariance between worker effects and firm effects from the AKM estimation. Number of worker years is in millions.

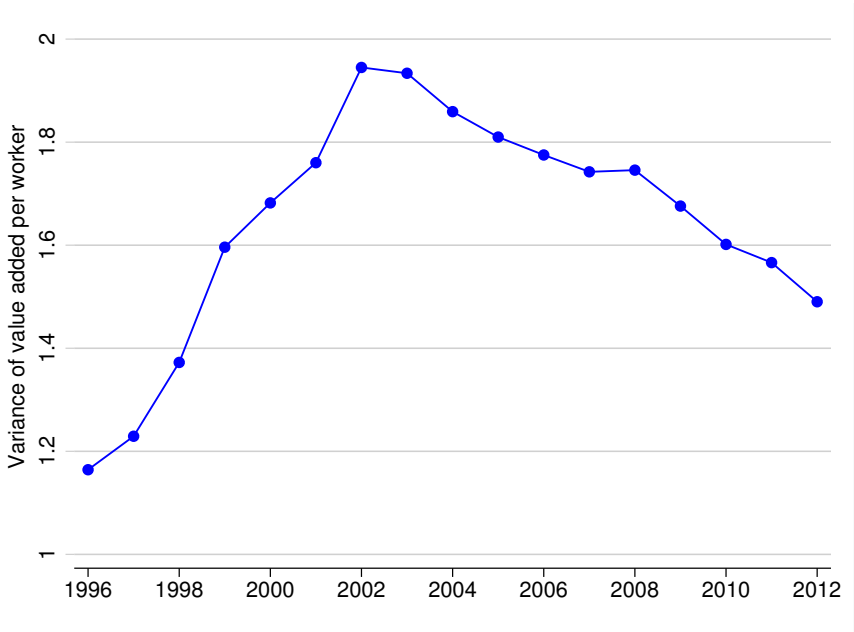
With this statistical decomposition in mind, we now continue our characterization of Brazil's inequality decline.

Fact 2.2. *One quarter of the overall decline in the variance of log earnings is attributable to an increase in relative pay at less productive firms.*

We now move on to investigating the drivers behind the fall in the variance of firm effects, which declined by 17 log points in 1996–2000 to 8 log points in 2008–2012, constituting 45 percent of the overall decline in the variance of log earnings over the period. In explaining the compression in firm-specific pay components, we consider two potential explanations.

The first possibility is that, to the extent that firm characteristics such as productivity matters for pay (Blanchflower, Oswald and Sanfey, 1996; Abowd, Kramarz and Margolis, 1999; Margolis and Salvanes, 2001), firms could have become more equal in such underlying characteristics. Figure 2.3, however, shows that dispersion in worker-weighted firm productivity as measured by value added per worker increased slightly over this period. Qualitatively similar trends are observed for the raw productivity measure shown in the figure and alternative cleaned measures of productivity that control for worker composition and industry, discussed in detail in Appendix B.1. Hence, there is no evidence in favor of a decline in dispersion of underlying firm characteristics.[24]

Figure 2.3: Evolution of dispersion of various productivity measures



The second possibility we investigate is that the degree to which firm productivity passes through to workers' wages could have declined, leading firms with given productivity differences to pay their workers more equally over time. Henceforth, we will refer to the relationship between firm productivity and worker pay as the

[24]We discuss in more detail the trends in other firm characteristics, including firm size and export intensity, in Alvarez, Engbom, and Moser (2015).

"productivity-pay gradient." Using the estimated firm effects from the AKM regression above, we evaluate this possibility by regressing the estimated firm effects on average value added per worker in each five-year subperiod. Consider a given subperiod and let a_j be the estimated firm component of pay and let VA_j denote average log value added per worker during the subperiod. For each subperiod, we regress[25]

$$a_j = \zeta_0 + \zeta_1 \text{VA}_j + \varepsilon_j$$

Notice that all regressions are run with sub-period averages of all variables. Based on the above regression, we compute the variance due to value added per worker as

$$Var\left(\hat{a}_j\right) = \left(\hat{\zeta}_1\right)^2 Var\left(\text{VA}_j\right)$$

Table 2.5 shows that firm productivity explains a significant amount of variation in pay across firms and that the regression coefficient between firm effects and firm productivity dropped from 0.258 to 0.147 over the period we study.[26]

To evaluate the importance of a declining pass through from firm productivity to pay for the overall decline in earnings, we compute the variance of the predicted value from this regression for each subperiod. This fell by five log points, namely from ten to five log points, between 1996–2000 and 2008–2012. As the variance of the underlying productivity distribution did not fall during this time, we conclude that a weakening firm productivity-pay gradient accounts for approximately all of the explained decline in the variance of firm effects, thereby explaining over 25 percent of the decline in the overall variance of log earnings in Brazil over the period.

[25]We have also considered versions including a range of other firm characteristics as well as subsector controls, but since these are not of first order importance we omit the results here.

[26]We do not present standard errors adjusted for the fact that the left hand side of this regression contains estimation error. Yet given the large sample size, we expect such adjustments to still yield strongly statistically significant estimates.

Table 2.5: Regression of firm pay component on productivity

	(1)	(2)	(3)
	1996–2000	2008–2012	Change
Value added p.w.	0.258	0.147	-0.111
Constant	-2.883	-1.599	1.284
Explained variance	0.10	0.05	-0.05
—due to returns			-0.05
—due to composition			0.01
# worker years	16.6	26.3	
R^2	0.583	0.465	

Note: Dependent variable is AKM estimate of firm effect on wages. Independent variable is log value added per worker. Explained variance holds R^2 fixed in 1996–2000. Number of worker years is in millions.

Fact 2.3. *Another quarter is due to declining differences in pay for unobserved worker characteristics.*

As we saw in Table 2.4, another five log points or 24 percent of the decline in the variance of log earnings are due to compression in estimated worker effects in the AKM framework. Analogously to our firm-level analysis, we regress the estimated worker effects, a_i, on five age bin dummies, four education dummies[27]:

$$a_i = \text{age}_i + \text{edu}_i + \varepsilon_i$$

[27]We have also examined versions of this regression with additional occupation controls, as well as with age and education interacted and additional sector controls, but neither of these alternatives changes the estimated results significantly.

Based on the above regression estimates, we predict the variance due to each of age and education. As in our firm-level analysis, we thereby distinguish between changes in the predicted variance due to changes in the fundamental distribution of worker characteristics versus changes in the returns to such characteristics.

Table 2.6 shows the result from this regression of estimated worker effects on age and education. We see that the estimated coefficients on both age and education uniformly declined over time. Furthermore, the explained variance of worker effects attributable to these worker observable characteristics declines by 3.1 log points over the period, all of which is driven by decreasing returns to the characteristics rather than due to changes in the underlying composition of workers.

Table 2.6: Regression of estimated worker effects on worker characteristics

	(1)	(2)	(3)
	1988–1992	2008–2012	Change
Age groups			
25–29	0.20	0.16	-0.04
30–39	0.39	0.30	-0.09
40–49	0.52	0.42	-0.10
50–64	0.48	0.51	-0.03
Education groups			
Middle school	0.21	0.11	-0.10
High school	0.61	0.27	-0.34
College or more	1.21	1.10	-0.11
Explained variance	0.11	0.08	-0.03
—due to returns			-0.03
—due to composition			0.01
# worker years	90.2	151.0	
R^2	0.34	0.37	

Note: Dependent variable is the estimated worker effect a_i. Number of workers in millions. Education estimates are relative to "less than middle school (<7 years)" category. Age estimates are relative to "age 18–24" category. Number of worker years is in millions.

It is important to note that our analysis of the explained decline in worker effects as well as the share explained by lower returns to these characteristics is limited to

observable worker characteristics. It is well known since Mincer (1958) that observable worker characteristics only explain a fraction of the variation in earnings and we confirm this for the Brazilian case in Alvarez, Engbom, and Moser (2015). One may naturally suspect that the returns to unmeasured ability or other pay relevant characteristics, which have been argued to explain a large share of overall earnings dispersion (Abowd, Kramarz and Margolis, 1999; Card, Heining and Kline, 2013; Alvarez, Engbom, and Moser , 2015), have declined in tandem with the returns to observable characteristics such as age and education, which we are able to explicitly investigate above. In this case, our results on the share of the decline in the variance of log earnings due to weaker returns on observable worker characteristics should be interpreted as a lower bound on the true decline explained by returns to both observable and unobservable worker characteristics.

Summary. Summarizing Facts 2.1–2.3 above, Brazil experienced a large decline in earnings inequality between 1996–2012, which was bottom-driven, and a large part of which was due to a weaker productivity-pay gradient across firms as well as a a decline in the returns to worker characteristics. Together, these facts suggest that Brazil's inequality decline was due to changes in pay policies rather than changes in economic fundamentals on either the worker- or the firm-side, and that these changes were particularly salient towards the bottom of the earnings distribution. The goal of the remainder of this paper is to build a model to rationalize these findings.

2.4 The minimum wage in Brazil

The three facts from Section 2.3 that characterize Brazil's earnings inequality decline between 1996 and 2012 highlight the importance of changes in the returns to worker and firm characteristics, rather than changes in their underlying distributions. Our results thus point towards one subset of explanations for Brazil's inequality decline,

which we term changes "wage setting policies." This insight by itself rules out a host of competing explanations for the sharp fall in earnings dispersion[28]. Thus, we are led to search for changes within the set of wage setting policies that can help rationalize Brazil's inequality decline over the period.

In the current paper, we ask whether changes in economic policy can explain the large decline in earnings inequality in general, and the three facts we discussed in Section 2.3 in particular. Our analysis focuses on evaluating the effects of one particular policy, namely the minimum wage. Before proceeding to evaluate its effects on the earnings distribution, we now provide some institutional context and a description of the evolution of the minimum wage in Brazil.

2.4.1 History

The minimum wage in Brazil is primarily a federal institution with only minor adjustments for regional price level differences. It was institutionalized as Decree-Law No. 2162 in 1940 and consolidated in 1943 under new labor laws (*Consolidação das Leis do Trabalho*, or *CLT*).[29] While the minimum wage was initially region-specific and not automatically adjusted to inflation or even legally enforced, it underwent several reforms under different political regimes between the 1940s and 1984, when it was unified across regions.

Leading up to and during Brazil's hyperinflationary period from 1980–1994, the minimum wage was adjusted first annually and later at monthly intervals according to a formula based on realized productivity growth and inflation as well as expected future inflation. Yet, due to forecasting errors in the price level during these turbulent times, the minimum wage lost over a third in real value. Following several failed

[28] Among the set of other explanations, we explore and rule out changes in the worker composition between the formal and informal sectors, regional earnings differences, sectoral composition, and labor mobility patterns. Details of our empirical investigation into each of these candidate explanations are contained in Alvarez, Engbom, and Moser (2015).

[29] The original law was based in parts on Mussolini's Carta del Lavoro in Italy.

stabilization plans, the *Plano Real* in 1994 stabilized the monetary system by pegging the local currency to the U.S. dollar (it was allowed to float again starting in 1999).[30] With the monetary stabilization of 1994 and the new president Fernando Henrique Cardoso of the centrist Brazilian Social Democracy Party taking office in 1995, the minimum wage became a renewed policy focus.

Nowadays, the minimum wage acts as a floor on monthly earnings for every formally registered worker. The Brazilian Ministry of Labor (*Ministério do Trabalho e Emprego,* or *MTE*) heads a compliance unit, which audits businesses through on-site visits and surveying local employees. Yet, according to official reports as well as information from our household and administrative data sources, compliance is less than perfect. While overall compliance is thought to be good in the formal sector, the minimum wage is plausibly less binding in Brazil's sizable informal economy.[31]

2.4.2 Evolution of the minimum wage

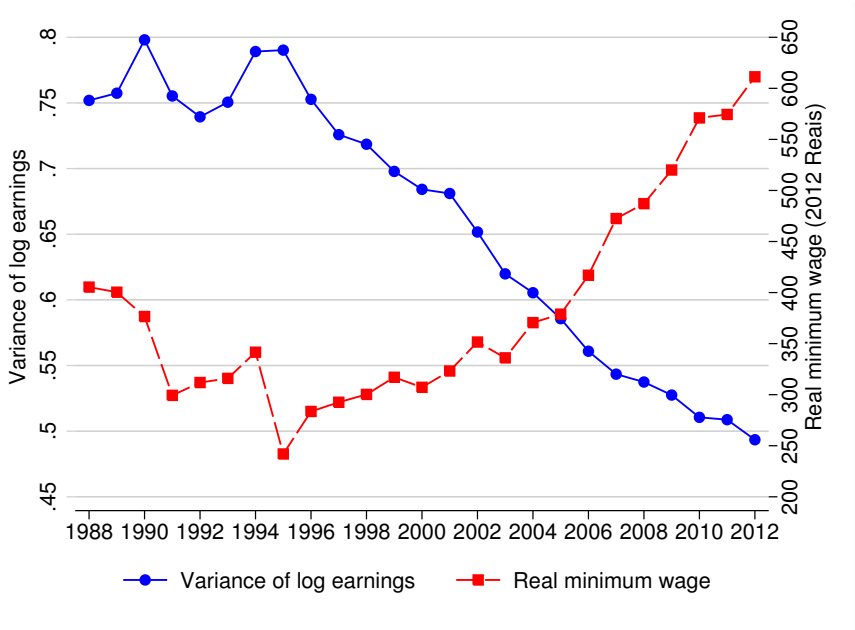
Between 1988 and 1996, the real minimum wage declined and experienced significant volatility as a result of hyperinflation. Then between January 1996 and December 2012, the real minimum wage grew by a total of 119 percent, reaching a value of 622 Brazilian Reais (410 PPP-adjusted U.S. dollars) per month by the end of the period. To put these numbers into context, the minimum wage as a fraction of median earnings increased from around 34 percent in 1996 to 60 percent in 2012. Over the same period, average labor productivity in manufacturing and mining increased by 16.6 percent; hence the ratio of the minimum wage to average labor productivity increased by 56.3 percent over this period.

[30] See Garcia, Guillén and Kehoe (2014) for a comprehensive summary of Brazil's inflation experience and the effects of the various stabilization plans.

[31] We will return to the distinction between the effects of the minimum wage on Brazil's formal and informal sectors in Section 2.8 of the current paper.

Figure 2.4 summarizes the evolution of the variance of log earnings (in blue) and also annual averages of the real minimum wage (in red) between 1988 and 2012.[32] Suggestive of the minimum wage being related to the evolution of earnings inequality, we see that the two time series approximately mirror each other, with the correlation between the two being -0.82 in levels and -0.55 in Hodrick-Prescott (HP) filter cycles over the period.

Figure 2.4: Evolution of earnings inequality versus the minimum wage, 1988–2012



2.4.3 Evaluating effects of the minimum wage through a structural model

While the joint evolution of the minimum wage and earnings inequality between 1988 and 2012 suggest that the two trends might be related, it remains an open question whether their relationship is causal. Importantly, one may note that the direct effect of the minimum wage is bounded above by the fraction of workers between the original

[32]For comparison, Appendix B.1 also shows 3-month running averages of the real minimum wage over the period.

and the new level of the minimum wage.[33] In spite of the large increase in real levels, a back-of-the-envelope calculations shows that these direct effects fall short of explaining the massive decline in earnings inequality over the period, as documented in the beginning of Section 2.3. Furthermore, the direct effects of the minimum wage could hardly speak to either the global compression of the earnings distribution documented in Fact 2.1, nor could they quantitatively explain the documented drop in the productivity-pay gradient across firms as well as the lower returns to worker characteristics noted in Facts 2.2–2.3.

A simplistic view of the minimum wage would thus conclude that its effects are limited to a small population subgroup and its effects have difficulty explaining the three facts from Section 2.3. Hence, in order for the minimum wage to be a promising candidate explanation, its effects need to extend beyond the direct impact on workers earning exactly the minimum wage.

Contrary to this simplistic view, a strand of the labor economics literature has suggested that the minimum wage might lead to spillover effects further up in the earnings distribution. Theories of such indirect effects of the minimum wage go back to at least Burdett and Mortensen, 1998. In a frictional labor market, monopsonistic competition among firms for workers would lead a minimum wage hike to affect wages of workers strictly above the minimum wage. In such an environment, is possible that an increase in the minimum wage ripples through the earnings distribution through such equilibrium effects. Recently, Autor, Manning and Smith (2016) took up this debate empirically in arguing that the magnitude of such spillover effects is indistinguishable from measurement error in the data in the case of the U.S. labor market. To take up this debate and evaluate the importance of such channels for the

[33] An even more critical view would suggest that the share of people affected by the minimum wage is restricted to the share of workers working exactly at the new minimum wage, after the increase. For this to be true, one would need to rationalize disproportionately fast productivity growth among workers with the lowest earnings, which we view as broadly incompatible with widely held beliefs that technical change over this period was characterized as high skill-biased.

case of Brazil, we proceed to build and estimate a structural equilibrium model of the Brazilian labor market. We then proceed to use the estimated model to quantify the degree of spillover effects from minimum wage increases over the period.

2.5 Model

We build an equilibrium search model in the spirit of Burdett and Mortensen (1998) that matches some of the key characteristics of the Brazilian labor market. In particular, our model reproduces the fact that identical workers are paid significantly different depending on where they work. Wage dispersion of this kind arises naturally in our model as a result of labor market frictions impeding the reallocation of workers across firms, thus giving firms monopsony power over workers. Our model also predicts that workers gradually move to better paying employers by climbing a "job ladder," which we show is a prevalent feature of the Brazilian labor market. We use our model to evaluate the impact of an increase in the minimum wage on the earnings distribution.

2.5.1 Environment

Time is continuous and the economy is in steady state. A unit mass of heterogeneous workers and a unit mass of heterogeneous firms meet in a frictional labor market. We describe the two groups of agents in sequence.

Workers. Workers are infinitely-lived, discount a stream of consumption at rate ρ, and differ in their permanent ability level θ. For expositional purposes, we present an economy with a continuum of types $\theta \in \Theta = \left[\theta_0, \overline{\theta}\right]$, each with mass m_θ.[34]

Workers engage in undirected search for jobs in labor markets segmented by worker ability (van den Berg and Ridder, 1998), both from non-employment and while em-

[34] Alternatively, all our results could be stated in terms of an integer number N of worker types.

ployed at a given firm. The assumption that labor markets are separated by worker ability buys us analytical tractability, but we think it also captures a stylized feature of real-world labor markets. For instance, one would expect that a Ph.D. economist will not compete with a high school dropout for the same job, even within firms.

Because of frictions in the labor market, workers do not instantly find a job. Let λ^u denote the probability that a non-employed worker receives a job offer and λ^e the probability that an employed worker receives a job offer. A job offer is an opportunity to work for a wage w drawn from distribution $F_\theta(w)$ with support $[\underline{w}_\theta, \overline{w}_\theta]$. Although a worker treats this distribution as exogenous, it will be determined endogenously in equilibrium through competition among firms for employees. A job is terminated exogenously with probability δ, upon which the worker re-enters non-employment, which gives flow utility b_θ.

Let W_θ denote the value function of a non-employed worker of ability θ and $V_\theta(w)$ the value function of the same worker employed at wage w. These value functions satisfy the Bellman equations

$$\rho W_\theta = b_\theta + \lambda^u \int_{\underline{w}_\theta}^{\overline{w}_\theta} \max\left\{V_\theta(w) - W_\theta, 0\right\} dF_\theta(w)$$

and

$$\rho V_\theta\left(w\right) = w + \lambda^e \int_{w}^{\overline{w}_\theta} \left[V_\theta\left(w'\right) - V_\theta\left(w\right)\right] dF_\theta\left(w'\right) + \delta\left[W_\theta - V_\theta\left(w\right)\right]$$

The value function V_θ is strictly increasing in w, and hence the optimal strategy of a non-employed worker is characterized by a reservation wage ϕ_θ. A non-employed worker accepts all wage offers above ϕ_θ and rejects offers below it. Following Burdett and Mortensen (1998), one can show that the reservation wage ϕ_θ is implicitly defined by

$$\phi_\theta = b_\theta + (\lambda^u - \lambda^e) \int_{\phi_\theta}^{\overline{w}_\theta} \frac{1 - F_\theta(w)}{\rho + \delta + \lambda^e (1 - F_\theta(w))} dw$$

The lowest wage at which a worker of type θ can be employed is thus given by

$$\underline{w}_\theta \left(w^{min} \right) = \max \left\{ \phi_\theta, w^{min} \right\}$$

We refer to $w^{min} > \phi_\theta$ as a binding minimum wage in market θ.

Firms. Firms are characterized by a constant productivity level $p \sim \Gamma(\cdot)$, which is continuous over full support $P = [p_0, \overline{p}] \subseteq \mathbb{R}_{++}$. Firms produce output by combining workers of different ability levels using a linear production technology. Letting l_θ denote the number of employees from market θ, flow output of a firm with productivity p is

$$y\left(p, \{l_\theta\}_{\theta \in \Theta}\right) = p \int_{\theta \in \Theta} \theta l_\theta d\theta$$

A firm attracts workers by posting market-specific wages, w_θ, in the markets it decides to be active in. In equilibrium the wage a firm posts determines the steady state amount of workers it attracts in that market, $l_\theta = l_\theta(w_\theta)$. Its firm size is determined as the sum of workers employed in each market, $l = \int_{\theta \in \Theta} l_\theta d\theta$. In choosing what wage to post, a firm trades off two forces: on the one hand, it attracts and retains a greater mass of workers and consequently produces more output by offering a higher wage. On the other hand, a higher wage reduces its profits per employed worker.

Because workers of different ability are perfect substitutes, the firm maximizes profits in each labor market separately. Conditional on choosing to recruit workers from market θ, the problem faced by a firm with productivity p is to post a wage w_θ to maximize steady-state profits:

$$\max_{w_\theta \geq w^{min}} (p\theta - w_\theta) l_\theta(w_\theta)$$

where the mass of workers employed at the firm, $l_\theta(w_\theta)$, is an equilibrium object that we characterize below.

2.5.2 Equilibrium definition

A key equilibrium object in this economy is the distribution of wages across workers. Let $G_\theta(w,t;w^{min})$ denote the wage distribution in market θ at time t and let $u(t)$ denote the share of workers that are unemployed. Because of on-the-job search, the realized wage distribution in the economy differs from the offer distribution $F_\theta(w;w^{min})$. The following Kolmogorov forward equation describes worker dynamics in each submarket for $w \geq \underline{w}_\theta(w^{min})$:

$$\begin{aligned}\frac{\partial G_\theta\left(w,t;w^{min}\right)}{\partial t} = & - \delta G_\theta\left(w,t;w^{min}\right) \\ & - \lambda^e\left(1-F_\theta\left(w,t;w^{min}\right)\right)G_\theta\left(w,t;w^{min}\right) \\ & + \lambda^u\frac{u(t)}{1-u(t)}F_\theta\left(w,t;w^{min}\right)\end{aligned}$$

In steady state, the unemployment rate is $u = \frac{\delta}{\delta+\lambda^u}$ and the realized wage distribution $G_\theta(w;w^{min})$ and wage offer distribution $F_\theta(w;w^{min})$ are related as

$$G_\theta\left(w;w^{min}\right) = \frac{F_\theta(w;w^{min})}{1+\kappa^e\left(1-F_\theta(w;w^{min})\right)}$$

where $\kappa^e \equiv \frac{\lambda^e}{\delta}$. A direct adaptation of the equilibrium characterization in Burdett and Mortensen (1998) shows that the realized wage distribution $G_\theta(\cdot)$ first-order stochastically dominates the wage offer distribution $F_\theta(\cdot)$, and that both cumulative distribution functions are continuous and strictly increasing. The equilibrium mass of workers employed at a firm that posts wage w is given by

$$l_\theta\left(w;w^{min}\right) = m_\theta(1-u)\frac{dG_\theta(w;w^{min})}{dF_\theta(w;w^{min})}$$

We are now ready to define an equilibrium of our economy.

Definition 2.1. *A* search equilibrium with segmented labor markets *is defined as a set*

$$\left\{w^{min}, \phi_\theta, u_\theta, l_\theta\left(w; w^{min}\right), F_\theta\left(w; w^{min}\right), G_\theta\left(w; w^{min}\right)\right\}$$

for each $p \in [p_0, \overline{p}]$ *and* $\theta \in \Theta = \{\theta_1, \ldots, \theta_N\}$ *such that:*

1. The firm productivity distribution $\Gamma_\theta(p)$ is truncated below at the threshold given by $\underline{p}\left(\theta; w^{min}\right) = \max\left\{\frac{\phi_\theta}{\theta}, \frac{w^{min}}{\theta}, p_0\right\}$.

2. The worker ability distribution $H(\theta)$ is truncated below at $\underline{\theta}\left(w^{min}\right) = \frac{w^{min}}{\overline{p}}$.

3. Workers accept any higher-paid job while employed and any job whose wage exceeds their reservation value while unemployed.

4. Taking as given $F_\theta\left(\theta; w^{min}\right)$ and $G_\theta\left(\theta; w^{min}\right)$, firms choose which markets θ to recruit from and make wage offers $w_\theta\left(p; w^{min}\right)$ to maximize steady-state profits.

5. The aggregate unemployment rate $u = \sum_{\theta\in\Theta} u_\theta m_\theta$ and firm sizes $l(\cdot) = \sum_{\theta\in\Theta} l_\theta\left(\cdot; w^{min}\right) m_\theta$ are consistent with the wage offer distributions $F_\theta(\cdot)$, realized wage distributions $G_\theta(\cdot)$, and labor market frictions $(\delta, \lambda^u, \lambda^e)$.

2.5.3 Equilibrium characterization

Before turning to our main results on the effects of the minimum wage on earnings inequality, we characterize the solution to the general problem of a firm choosing which labor markets to recruit from and what wages $w\left(p, \theta; w^{min}\right)$ to post in each market. Lacking any conclusive data on the type-specific value of unemployment, we assume that the value of unemployment equals output at the lowest productivity

firm, so that[35]

$$\underline{p}\left(\theta; w^{min}\right) = \max\left\{p_0, \frac{w^{min}}{\theta}\right\}.$$

Lemma 2.1. *Firms' optimal wage posting decisions are:*

1. *A firm with productivity p posts wages in all labor markets θ that satisfy*

$$\theta \geq \frac{w^{min}}{p}$$

2. *The unique equilibrium wage offered by a firm with productivity $p \geq \underline{p}\left(\theta; w^{min}\right)$ to workers of ability θ is given by:*

$$w\left(p, \theta; w^{min}\right) = \theta p - \theta \int_{\underline{p}(\theta; w^{min})}^{p} \left[\frac{1 + \kappa^e\left(1 - \Gamma_\theta\left(p; w^{min}\right)\right)}{1 + \kappa^e\left(1 - \Gamma_\theta\left(x; w^{min}\right)\right)}\right]^2 dx \tag{2.2}$$

where

$$\Gamma_\theta(p; w^{min}) = \frac{\Gamma(p) - \Gamma\left[\underline{p}\left(\theta; w^{min}\right)\right]}{1 - \Gamma\left[\underline{p}\left(\theta; w^{min}\right)\right]}$$

The mapping in equation (2.2) is strictly increasing in p.

Proof. All proofs are contained in Appendix B.2. □

Lemma 2.1 extends the equilibrium wage characterization from Burdett and Mortensen (1998) to our model. Part 1 states that firms recruit only from markets in which they can make positive profits. Part 2 shows that the solution to firms'

[35]We note three things with respect to this assumption and the critique raised by Hornstein, Krusell and Violante (2011). Firstly, as noted by these authors, models with on-the-job search are less susceptible to the issues they raise, since it reduces the option value of staying unemployed. Secondly, in our model the option value of remaining unemployed depends on wage dispersion conditional on worker ability, not overall wage variation. We document later that in the data a significant amount of wage dispersion is due to worker heterogeneity. Thirdly, recent papers introducing on-the-job accumulation of human capital show that this produces significant levels of wage dispersion while maintaining realistic assumptions on the value of the outside option (see e.g. Burdett, Carillo-Tudela and Coles, 2014). Although our model does not contain this element, we believe that it could be introduced without changing any of the insights our model provides with regards to the minimum wage.

trade-off between monopsony profits and firm size yields an optimal wage policy as a function of firm productivity p, the labor mobility parameter κ^e, and the offer distribution of other employers in that labor market.

Since the mapping from firm productivity to wages is strictly increasing, the equilibrium wage offer distribution in each market satisfies

$$F_\theta \left[w\left(p, \theta; w^{min}\right) \right] = \Gamma_\theta \left(p\right)$$

2.5.4 Theoretical effects of the minimum wage

Combining our above results, earnings can be written as

$$\begin{aligned} & \log w\left(p, \theta; w^{min}\right) \\ = \; & \log \theta \\ & + \log \left(p - \int_{\max\left\{p_0, \frac{w^{min}}{\theta}\right\}}^{p} \left[\frac{1 - \Gamma\left(\max\left\{p_0, \frac{w^{min}}{\theta}\right\}\right) + \kappa^e \left(1 - \Gamma\left(p\right)\right)}{1 - \Gamma\left(\max\left\{p_0, \frac{w^{min}}{\theta}\right\}\right) + \kappa^e \left(1 - \Gamma\left(x\right)\right)} \right]^2 dx \right) \end{aligned} \quad (2.3)$$

Based on this, the variance of log earnings can be decomposed into two sources: differences in average earnings across θ markets and differences in pay within θ markets. In markets where the minimum wage is not binding, this decomposition is particularly straightforward because the second term in (2.3) is independent of worker ability. As a result, in these markets a firm pays workers of different ability the same multiple of their underlying worker ability, and log earnings are additively separable into a worker and firm effect. Across-markets variance is hence in the non-binding case given by the variance of the underlying distribution of worker ability, and the within variance is identical in each market and determined by the distribution of underlying firm productivities through the mapping from productivity to the firm component of pay.

With a binding minimum wage, the second term in equation (2.3) depends on the minimum wage relative to worker ability. This in general will lead to a minimum wage affecting both the expected value and the variance of the second term, and hence it will have an effect on both inequality across and within markets.[36] We now characterize this in further detail.

First, a minimum wage increases earnings at all firms in affected markets. If the minimum wage is such that it binds in some markets but not all, this will lead to compression between low and high ability workers. Our first proposition states this formally,

Proposition 2.1 (Greater impact at the bottom)**.** *Suppose the minimum wage is initially non-binding. As the minimum wage is gradually raised, it boosts pay of low ability workers relative to high ability workers.*

Proof. See Appendix B.2. □

Secondly, an increase in the minimum wage reduces the return to worker ability within the set of markets where the minimum wage is binding. This also contributes to lower across-market inequality. We state this result formally in Proposition 2.2 under the assumption that firm productivity is uniformly distributed:

Proposition 2.2 (Lower returns to worker ability)**.** *Suppose* $p \sim U(p_0, \overline{p})$*. A minimum wage reduces the worker ability-pay gradient in all markets affected by the minimum wage:*

$$\partial \left[\frac{\partial w\left(p, \theta_i; w^{min}\right)}{\partial \theta} \right] / \partial w^{min} < 0, \quad \forall \theta_i < \frac{w^{min}}{p_0}$$

Proof. See Appendix B.2 □

[36] A minimum wage also cuts off some low productivity firms from some markets and possibly some low ability workers completely. However, it is not clear that the by cutting off the lowest ability individuals and the lowest productivity firms, overall inequality among remaining workers and remaining firms will be reduced, as this depends on the underlying distribution of worker ability and firm productivity. We explore this issue quantitatively in Section 2.8.

Both proposition 2.1 and 2.2 lead to earnings compression across worker ability types, which Fact 2.3 of our empirical documentation established to be a pervasive feature of the overall decline in earnings inequality in Brazil during this time.

Thirdly, a minimum wage reduces within-market inequality in markets affected by the minimum wage. This is because although a minimum wage increases pay at all firms in markets affected by it, it disproportionately increases compensation at lower productivity firms. We again state this formally under the assumption that firm productivity is uniformly distributed:

Proposition 2.3 (Lower productivity-pay gradient)**.** *Suppose* $p \sim U(p_0, \overline{p})$*. A minimum wage reduces the firm productivity-pay gradient in all markets affected by the minimum wage:*

$$\partial \left[\frac{\partial w(p, \theta_i; w^{min})}{\partial p} \right] / \partial w^{min} < 0, \quad \forall \theta_i < \frac{w^{min}}{p_0}$$

Proof. See Appendix B.2. □

Proposition 2.3 states equivalently that the minimum wage leads to a flattening of the job ladder in markets affected by it. Hence it speaks to Fact 2.2 of our empirical documentation, which showed that the firm productivity-pay gradient fell as the minimum was raised in Brazil.

We summarize the effects of an increase in the minimum wage as follows: First, the minimum wage reduces differences in pay between workers of different abilities. Second, earnings inequality falls within markets affected by the minimum wage due to a weaker productivity-pay gradient across firms. As we will see in the quantitative section, these two channels hold more generally in simulations and add up to produce bottom-driven earnings compression that reaches far up in the earnings distribution, in line with our empirical Fact 2.1.

2.6 Estimating the model

In order to quantitatively evaluate the importance of the minimum wage for earnings inequality, we proceed to estimate the model on the initial 1996–2000 subperiod. Subsequently, the next section uses the estimated model to quantify the impact on inequality of a an increase in the minimum wage of the same magnitude as observed in Brazil over this period of time.

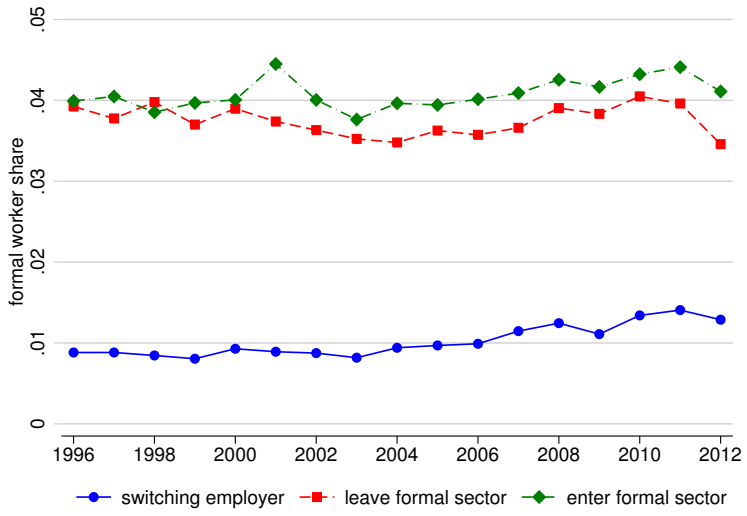
2.6.1 Estimation strategy

First stage. We first use panel information on worker flows together with non-parametric estimates of conditional earnings distributions and estimates of the size of the formal sector labor force in order to estimate the three labor market frictions parameters in our model: the separation rate, δ, the job offer arrival rate from non-employment, λ^u, and the job offer arrival rate from employment, λ^e. As will become clear, these parameters do not depend on the remaining parameters of the model, and hence to simplify the second stage we can pre-calibrate them.

We use a 10% monthly panel from the RAIS to calculate the fraction of entrants, leavers and job-to-job switchers in every year 1996–2012. We also use non-parametric estimates of the overall distribution of firm effects as well as non-parametric estimates of the distribution of firm effects among entrants to the formal sector for each of these years from the RAIS, and data on the relative size of the formal sector labor force among prime age males from the PNAD household survey for each of these years. Since we are unable to distinguish flows from formal sector employment into unemployment, informal employment, or out of the labor force, we can only classify workers as leaving formal sector employment. We label such transitions as employment to non-employment. Figure 2.5 plots the average monthly fraction of formal

sector entrants, leavers and job-to-job switchers in each year 1996–2012. We note that each series remains fairly stable over this period of time in Brazil.

Figure 2.5: Average monthly fraction of leavers, entrants and job-to-job movers from the formal sector, 1996–2012



The separation hazard, δ, can be directly inferred from observed flows out of the formal sector. The offer arrival rate from non-employment, λ^u, can be inferred from the fraction of newly employed workers in the formal sector (%entrants) as well as information of the fraction of the total population of prime age males who work in the formal sector (%formal) through:

$$\lambda^u = \frac{\%\text{formal}}{1 - \%\text{formal}} \times \%\text{entrants}$$

where we imposed our assumption that non-employed workers accept the first job offer they receive.

Finally, the job offer arrival rate on the job, λ^e, cannot be directly inferred from observed job-to-job flows, since an employed worker only accepts offers paying more than his current employer. Our model, however, suggests that by governing the speed

through which workers move up the job ladder, λ^e is intricately linked to the difference between the distribution G and the wage offer distribution F,

$$G_\theta(w) = \frac{F_\theta(w)}{1 + \kappa^e(1 - F_\theta(w))}$$

where $\kappa^e = \lambda^e/\delta$ is the relative probability of getting an offer versus being separated. Although this is based on the distribution of earnings within a θ market, we note that the estimated firm effects perfectly rank firms in each market and that the rank of firms is the relevant notion of the job ladder in our model. Thus we can estimate κ^e non-parametrically using kernel density approximations[37] of the empirical distribution of firm effects, $\hat{\mathcal{G}}(fe)$, as well as the job offer distribution, $\hat{\mathcal{F}}(fe)$, where the latter is approximated using people that just entered the formal sector. The nonparametric estimate of κ^e is then

$$\hat{\kappa}^e = \frac{\hat{\mathcal{F}}(fe) - \hat{\mathcal{G}}(fe)}{\left(1 - \hat{\mathcal{F}}(fe)\right)\hat{\mathcal{G}}(fe)}$$

Using our earlier estimate of δ, we can back out the implied value for λ^e.[38]

Second stage. We estimate the remaining parameters of our model using indirect inference. Thus we solve and simulate the model for a large set of potential parameter values in order to minimize the distance between model generated output and their data equivalents.

The model is estimated fully parametrically

$$\log\theta \sim \mathcal{N}\left(0, \sigma_\theta^2\right), \quad \log p \sim \mathcal{N}\left(0, \sigma_p^2\right)$$

This gives two parameters to estimate: the standard deviation of worker ability, σ_θ, and the standard deviation of firm productivity, σ_p. Additionally, we need an

[37] We use an Epanechnikov kernel with bandwidth 0.04 and 90 bins although we tried alternative kernel, bandwidth, and bin number choices without significant effects on our estimation results.

[38] In Appendix B.3, we discuss alternative ways of estimating κ^e, all producing similar estimates.

estimate of the minimum wage. In order for this to make sense within the model, we need a numeraire of the economy. We chose log median earnings as the numeraire, and express the minimum wage relative to that. Several other choices of numeraire are possible, though, including average earnings or average labor productivity. The former yields very similar results as the median. Average value added per worker is less attractive to us because we only have data on value added for the PIA subsample, and it is plausible that average value added per worker is higher in the subsample of large manufacturing and mining firms, leading us to underestimate the bindingness of the minimum wage (additionally, we worry about measurement error affecting the level of average value added per worker in the data). We do, however, use information on the *growth* in average value added per worker to estimate the growth in real, productivity adjusted minimum wages between 1996–2000 and 2008–2012, which is an important input in our policy experiment in Section 2.7.

The choice of what moments to target is important. In the discussion below of our choice of target moments, the reader should keep in mind that the model is jointly identified and hence it should be viewed as a single moment being particularly informative of one parameter. As we noted earlier, absent a binding minimum wage, log earnings in our model perfectly separates into a worker and a firm component

$$\underbrace{\log w_{ijt}}_{\text{earnings}} = \underbrace{\log \theta_i}_{\text{worker effect}} + \underbrace{\log r(p_j)}_{\text{firm effect}} + \underbrace{\varepsilon_{ijt}}_{\text{error}}$$

where $r(p_j)$ is the firm component of pay as specified in the wage equation (2.3). Thus, underlying worker productivity could be directly inferred from an AKM regression and underlying firm productivity inferred from the estimated firm effect by inverting the mapping between firm productivity and the firm component of pay. Although in the presence of a binding minimum wage log earnings do not perfectly separate into a worker and a firm component, we think that AKM still captures some of the key

dimensions of our model. Hence, we view it provides sufficient information to identify several of the underlying structural parameters of our model. We have conducted several exercises to check the uniqueness of the optimum reached by our estimation algorithm, and it appears unique. We thus use AKM as an auxiliary model and target the variance of estimated AKM worker effects, $Var(\alpha_i)$, for the variance of underlying worker productivity.

From AKM as an auxiliary model we also use the variance of estimated AKM firm effects, $Var(\alpha_j)$, for the variance of underlying firm productivity. Absent a minimum wage, we can find an algebraic expression for how firm productivity maps into pay of workers of that firm. Importantly this is monotonically increasing in firm productivity, implying that the firm component of pay perfectly informs the underlying distribution of firm productivity. In all of our simulations with a binding minimum wage, this monotonicity is preserved and AKM firm effects identify underlying firm productivity.

Our third and final target is the ratio of the log minimum wage to log median earnings, mM. Targeting the minimum wage in our estimated model serves as a normalization as we picked the numeraire to be the expected earnings of a median worker at a median firm by making the parametric assumptions on worker and firm effects above.

We now define the distance criterion for our indirect inference as part of the simulated method of moments. Let S^D denote the statistic of interest, S, in the data and S^M that in the model—formally we estimate the parameters $(\sigma_\theta, \sigma_p, w^{min})$ by assigning the values that minimize the unweighted[39] sum of squared percentage deviations between model moments and data moments:

[39]We opted for uniform weights on the distance criteria because convergence was very smooth and we did not want to build in any ex-ante restrictions on the relative importance of worker effects versus firm effects in the estimation procedure.

$$
\begin{aligned}
\left(\hat{\sigma}_\theta, \hat{\sigma}_p, w^{min}\right) \quad = \quad & \text{argmin}_{\sigma_\theta, \sigma_p, w^{min}} \left\{ \left[\frac{Var\left(\alpha_i^D\right) - Var\left(\alpha_i^M\right)}{Var\left(\alpha_i^D\right)} \right]^2 \right. \\
& + \left[\frac{Var\left(\alpha_j^D\right) - Var\left(\alpha_j^M\right)}{Var\left(\alpha_j^D\right)} \right]^2 \\
& \left. + \left[\frac{mM^D - mM^M}{mM^D} \right]^2 \right\}
\end{aligned}
$$

Further details of the estimation procedure can be found in Appendix B.3.

2.6.2 Parameter estimates and model fit

Before entering the estimation procedure, we fix the discount rate ρ to match an average annual interest rate of 12 percent. Subsequently, we estimate the labor market frictions parameters. We find that δ is 3.8 percent at the monthly level, λ^u is 7.1% and λ^e is 5.3%. From an international perspective, we note that δ is higher than most estimates from the U.S. (and thus also than most European countries). However, our estimate is based on all workers leaving the formal sector, regardless of the destination. If we were able to condition on staying in the labor force, we suspect that our estimate would be lower. λ^u is lower than estimates from European markets (and hence substantially lower than the U.S.). However, it is again using all workers not in formal sector employment, and we suspect that if we were able to calculate the unemployment to employment hazard rate it would be substantially higher. Finally, our estimate of λ^e is on the lower side compared to most European markets (and hence again substantially lower than the U.S.). Our robustness section establishes that our results are not sensitive to a wide range of these underlying labor market friction parameters. Table 2.7 summarizes our estimates:

Table 2.7: Monthly model parameters

Description	Parameter	Value	Target
Discount rate	ρ	0.009	Annual interest rate of 12%
Exogenous separation rate	δ	0.038	Fraction of formal sector leavers
Arrival rate (unemployed)	λ^u	0.071	Fraction of entrants; formal sector size
Arrival rate (employed)	λ^e	0.053	Firm effect and offer distributions; δ

Table 2.8 presents estimates of the variance of underlying worker ability and firm productivity as well as the minimum wage. The model fits the data well. Our estimates imply that heterogeneity in worker ability exceeds variation in firm productivity, but by less than the difference in the variance of the estimated AKM worker and firm effects. The reason is that at the top and the bottom of the firm productivity distribution, little between firm competition for workers imply that an increase in productivity translates to a very small increase in wages.

Table 2.8: Parameter estimates and model fit

Description	Parameter	Estimate	Target moment, 1996–2000	Data	Model
Var(worker ability)	σ_θ^2	0.700	Var(AKM worker effects)	0.347	0.347
Var(firm productivity)	σ_p^2	0.523	Var(AKM firm effects)	0.167	0.168
Minimum wage	w^{min}	0.189	Min.-to-med. log ratio	34%	34%

To investigate whether the underlying parameters are well identified in the model, we evaluated how the difference between model generated moments and the corresponding moments in the data changed when changing one parameter at a time away from the optimum. The distance increases monotonically. Moreover, given the small set of parameters to estimate, we conducted an extensive search for an optimum over a wide grid of values. Although this does not guarantee global uniqueness of the minimum, all the robustness exercises we have conducted indicate that it is.

2.6.3 Policy experiment in the model

We now turn to evaluating the impact on income inequality in the model of an increase in the minimum wage of the same magnitude as that in the data. To do so, we first compute average growth in productivity adjusted real minimum wages between 1996–2000 and 2008–2012. The average real minimum wage (in 2012 values) is 384 Reais in 1996–2000 and 701 in 2008–2012, implying an 60.2 log point growth in real minimum wages. Average log value added per worker grows by a total of 15.4 log points between 1996–2000 and 2008–2012. Thus, we estimate that the real, productivity adjusted minimum wage grew by 44.7 log points between 1996–2000 and 2008–2012. Given this data target, we reestimate our model for the 2008–2012 period by changing the minimum wage to hit a 44.7 log point growth in the minimum wage relative to average log labor productivity, while holding all other parameters fixed at their 1996–2000 estimated values.[40] This implies a hike in the minimum wage from 0.189 to 0.315 or roughly 67 percent. We evaluate the implications for income inequality of imposing this higher minimum wage through the lens of our model.

2.7 Quantitative results

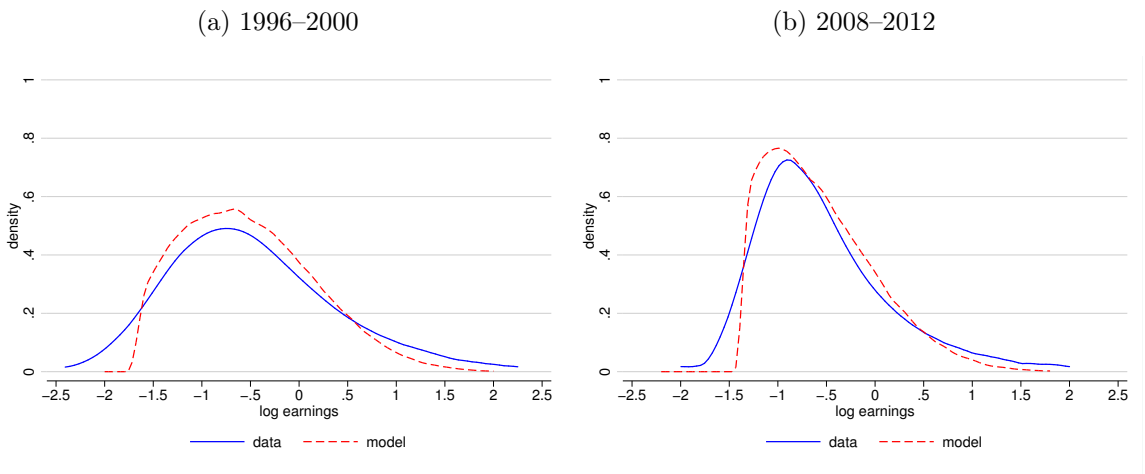
2.7.1 Effect of minimum wage on earnings distribution

In this section, we evaluate the effects of the minimum wage on the earnings distribution in our estimated model. Figure 2.6 shows how the overall distribution of income changes in the model as we increase the minimum wage while holding everything else constant at the 1996–2000 values. We note that the model underestimates the overall variance of wages in the data by 25 log points in the initial period, because we do not

[40]For robustness, we also explored alternative targets for the increase in the minimum wage, including the growth rate of the minimum wage relative to productivity growth in Brazil's services, commerce, and construction sectors (for which we have firm-level productivity data); or relative to growth in aggregate output per capita from national accounts. These alternative targets imply similar increases in the minimum wage and therefore lead to comparable results.

calibrate it to match the error component as well as age, education and year effects in the data. Yet the magnitude of the fall in the overall variance of log earnings is comparable to the data: the variance of log earnings falls by 14.1 log points in the model or 70 percent of the fall in the data.

Figure 2.6: Effect of minimum wage on earnings distribution in model vs. data



The increase in the minimum wage induces a significant compression in both estimated worker and firm effects, as can be seen in Table 2.9. The model generates a 6.2 log point fall in the variance of person effects, a 4.3 log point compression in the variance of firm effects and a 1.8 log point fall in the covariance between them. The corresponding numbers in the data are 5.4 log points, 9.0 log points and 4.4 log points, respectively. Thus like in the data, firm effects account for an outsized share of the inequality decline: the variance of person effects falls by 18 percent and the variance of firm effects falls by 26 percent in the model (versus 16 percent and 54 percent in the data, respectively). Thus the model slightly overpredicts the compression in person effects observed in the data. However, given significant evidence from other countries that technological change over the last two decades has increased the return to ability (so called *skill-biased technical change*), we find it plausible that other forces served to increase the dispersion in person effects over this period in the data. Moreover,

although the model cannot account for the positive covariance between firm and worker effects in the data, it is able to replicate almost 80 percent of the decline in the covariance in the data.

Table 2.9: Variance of earnings in data versus model

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Expl.
Variance of total earnings	0.72	0.46	0.52	0.32	-0.20	-0.14	70%
Variance of worker effects	0.36	0.35	0.31	0.29	-0.05	-0.06	110%
Variance of firm effects	0.17	0.17	0.08	0.13	-0.09	-0.04	48%
Covariance worker-firm	0.14	-0.03	0.10	-0.05	-0.04	-0.02	78%
Variance of residual	0.06	0.00	0.04	0.00	-0.02	0.00	0%

The model does a remarkably good job at generating compression in the earnings distribution up to very high percentiles. As can be seen in Table 2.10 the model predicts consistently more than 64% of compression in the selected percentile ratios, and approximately the full amount higher up in the distribution. For instance, the 50–10 log ratio compresses by 31 log points in the data versus 22 log points in the model (or 71 percent) whereas the log 90-50 ratio compresses by 13 log points in the model versus 12 log points in the data (or 92 percent). Matching the data, the model displays a declining amount of compression in the top of the distribution, as shown by the decreasing marginal increase in the log ratios up to the 90th percentile. For instance, the log 95-50 ratio falls by 2 log points less than the log 90-50 ratio in the model. Hence the model is successful at replicating Fact 2.1 from our empirical section.

Table 2.10: Compression in log percentile ratios of earnings distribution

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
P50-P05	1.06	0.90	0.62	0.62	-0.44	-0.28	64%
P50-P10	0.86	0.77	0.55	0.55	-0.31	-0.22	71%
P50-P25	0.48	0.46	0.33	0.35	-0.15	-0.11	73%
P75-P50	0.60	0.52	0.50	0.44	-0.10	-0.08	80%
P90-P50	1.30	1.01	1.17	0.89	-0.13	-0.12	92%
P95-P50	1.76	1.30	1.65	1.17	-0.11	-0.13	118%

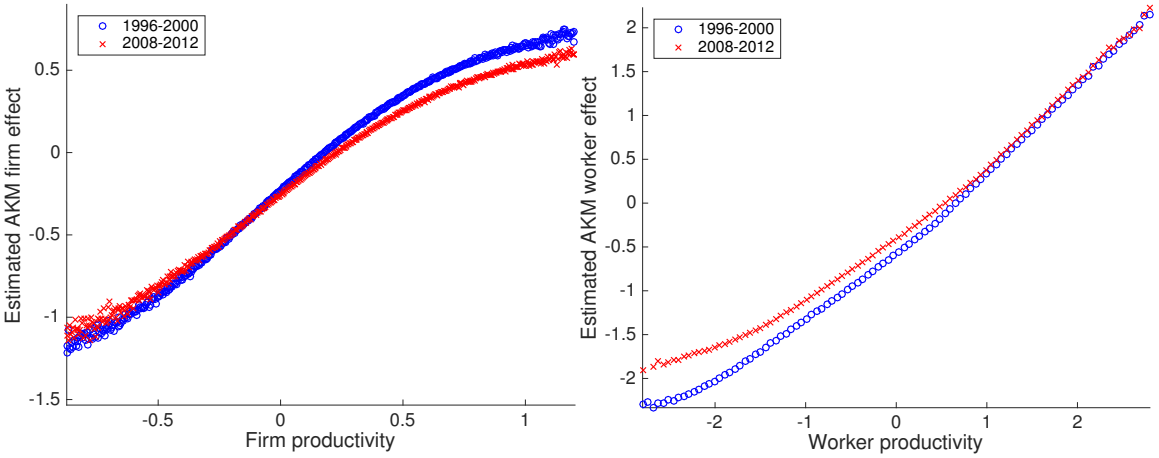
Turning now to a decomposition of the overall inequality decline in our model relative to the data, Figure B.10 in Appendix B shows the model distributions of firm effects and workers effects from the model before and after the minimum wage change. Both distributions experience particularly pronounced compression in their shape at the bottom, consistent with our empirical findings.

The results from inspecting the firm and worker components from the AKM decomposition of earnings in our model is broadly consistent with our empirical findings. As predicted by our theory, the increase in the minimum wage compresses pay across θ markets by disproportionately boosting pay of low productivity workers. This is due to the two channels highlighted in our theoretical section: average firm productivity increases for low productivity workers and average pay increases among continuously active firms. Figure 2.7 demonstrates the quantitative importance of this channel by plotting estimated AKM firm effects from our model simulated data against firm productivity, and AKM worker effects against worker ability in the model for the 1996–2000 and 2008–2012 periods. Average pay clearly compresses across markets,

and this is person effects against underlying worker ability and estimated firm effects against underlying firm productivity in the model for the two subperiods.

Figure 2.7: Gradient effects of the minimum wage

(a) Effect on firm productivity-earnings gradient (b) Effect on worker ability-earnings gradient



Both gradients notably fall. In fact the fall in the gradient explains all of the fall in firm effects in the model, because the minimum wage increase is not high enough to significantly reduce the variance of firm productivity in the model (in fact it increases marginally as a result of reallocation of workers among continuously active firms).

To further investigate this, Table 2.11 presents results from regressing estimated firm effects on firm productivity in the model and in the data. We note that these regressions have a much higher explanatory power in the model relative to the data, likely due to measurement error in productivity in the data. However, the fall in the variance of firm effects attributable to a change in the coefficient is similar in the data and in the model: 5.0 log points versus 4.3 log points.

Thus, we conclude that the model cannot account for the decline in the variance of firm effects not explained by productivity, but that it can explain up to 90 percent of the fall due to a weaker productivity-pay gradient. Furthermore, as can be seen in Table 2.11, all of the decline in the variance of person effects in the model is due to the weaker worker ability-pay gradient, because the magnitude of the minimum wage

increase is not sufficiently large to make some workers unemployable. Although we unfortunately cannot decompose the change in the variance of person effects in the data into a change in underlying characteristics versus returns to these characteristics, we think that a reasonable first pass would be to assume that the underlying distribution of worker abilities did not change much during these 17 years in Brazil (recall that we control for changes in education and age).

Table 2.11: Effect of minimum wage on productivity-earnings gradient

	1996–2000		2008–2012		Change in predicted variance		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
Value added p.w.	0.257	1.088	0.141	0.934	-0.050	-0.043	86%
Constant	-2.883	-0.236	-1.599	-0.254			
# worker years	15.5	0.5	23.9	0.5			
R^2	0.583	0.976	0.465	0.973			

Note: Dependent variable is estimated firm effect from AKM regression, independent variable is average log value added per worker within a subperiod. Number of observation is in millions.

2.7.2 The importance of indirect effects of the minimum wage

In order to evaluate the importance of the equilibrium mechanism that we emphasize in this paper for wage compression, we consider a scenario in which counterfactually there is no impact on earnings above the new minimum wage. To calculate the amount of inequality associated with such a scenario, we hence assume that everyone earning below the new minimum wage gets earnings equal to the minimum wage, but nothing happens to wages above the new threshold.[41] Figure 2.8 illustrates the direct and indirect effects of the minimum wage using actual distributions from our simulation exercise. The variance of log earnings explained by the direct versus indirect effects of the minimum wage are summarized in Table 2.12. More than half of the overall decline

[41] An alternative scenario would have been to simply cut off workers below the new minimum wage, but as this yields similar conclusions we do not report that here.

in the variance of log earnings is explained by the minimum wage raising earnings of workers earning above the new minimum wage. Thus, we conclude that modeling the equilibrium effect of raising the minimum wage is crucial for understanding its impact on inequality.

Figure 2.8: Direct and indirect effects of minimum wage on earnings distribution

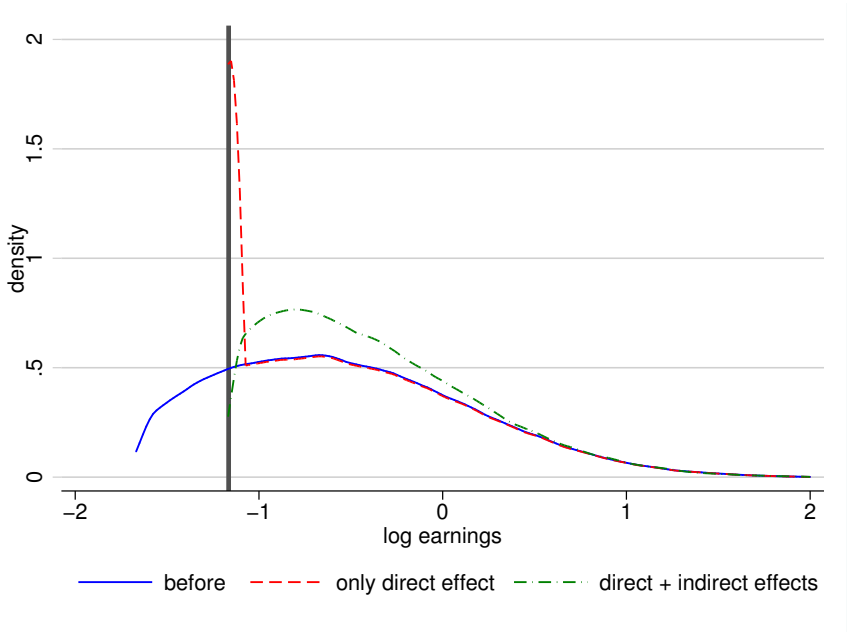


Table 2.12: Direct&indirect effects of minimum wage on variance of log earnings

	Data			Model		
	(1)	(2)	(3)	(4)	(5)	(6)
	Variance of log earnings	Change	% of total change	Variance of log earnings	Change	% of total change
Baseline, 1996–2000	0.721	-	-	0.458	-	-
Only direct effect	0.612	-0.109	54.2%	0.391	-0.067	47.5%
Direct&indirect, 2008–2012	0.520	0.201	100.0%	0.317	-0.141	100.0%

2.7.3 Effects on unemployment

Consistent with empirical observations in Brazil over this time, our model generates very little increase in unemployment.[42] The increase in unemployment predicted by our model is on the order of a tenth of percentage point (from a starting point of 6.9 percent unemployment).

Table 2.13: Effects of the minimum wage on unemployment

	1996–2000		2008–2012		Change		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Data	Model	Data	Model	Data	Model	% Explained
Unemployment rate	0.067	0.067	0.068	0.068	0.001	0.001	79%

It is important to note, though, that our model *has the potential* to generate significant unemployment. We explore this point in more detail in Appendix 2.8.4.

2.7.4 Effects on the firm distribution

Finally, we note that the higher minimum wage leads a set of the least productive firms to exit, thus raising average productivity and the average firm size in the economy. The findings are summarized in Table 2.14. Specifically, while seven percent of all firms are forced to exit because they can no longer operate profitably at the new minimum wage, the re-allocation of workers across remaining firms raises aggregate TFP by four log points and average firm size by two log points. While some of these effects are due to the least productive firms exiting, much of the positive effects on aggregate productivity stem from the re-allocation of workers to more productive firms.

[42]It is important to highlight that a higher minimum wage might affect the job finding rates, which is a channel that is absent from our model. We note, though, that there is little change in job finding rates in Brazil over this period.

Table 2.14: Effects of the minimum wage on firm distribution

	(1) 1996–2000	(2) 2008–2012	(3) Change
Share of active firms	1.00	0.95	-0.05
Average productivity (TFP)	0.79	0.83	0.04
Average number of employees	18.1	18.3	0.2

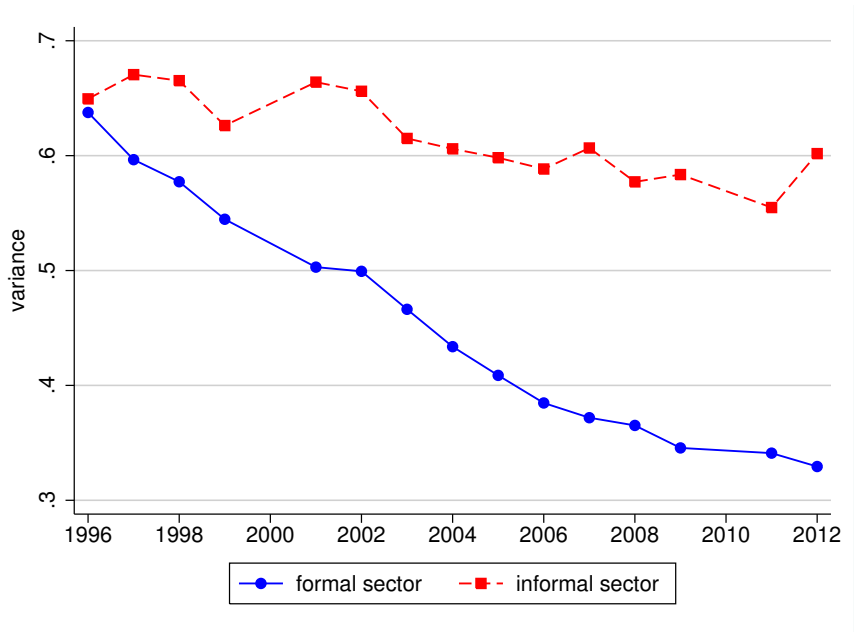
2.8 Discussion

2.8.1 Empirical evidence for effects of the minimum wage on earnings inequality

More pronounced inequality decline among formal sector workers. If the minimum wage played an important role in the decline of earnings inequality in Brazil from 1996–2012, then we would expect the magnitude of the decline to vary with the degree to which the legal minimum wage is enforced. Hence, the inequality decline should be less pronounced in Brazil's informal sector. The latter constitutes a significant share of overall employment but, due to imperfect monitoring of economic activity in Brazil's shadow economy, is less subject to laws and labor regulations. Figure 2.9 confirms this hypothesis in the Brazilian PNAD household data. Our analysis shows that most of the decline in earnings inequality over the period is due to developments in the formal sector.

Greater impact on low-income regions. While the minimum wage is enacted at the federal level, not all parts of the economy are affected equally. For instance, sectors with higher initial average earnings may be less affected by the rise in the minimum wage only with a delay. If the minimum wage was an important driver behind the decline of earnings inequality, we would expect that inequality within geographic regions of the country more affected by the minimum wage would experience more pronounced inequality declines. To pursue this hypothesis, we use the

Figure 2.9: More pronounced decline of earnings inequality in formal sector, 1996–2012

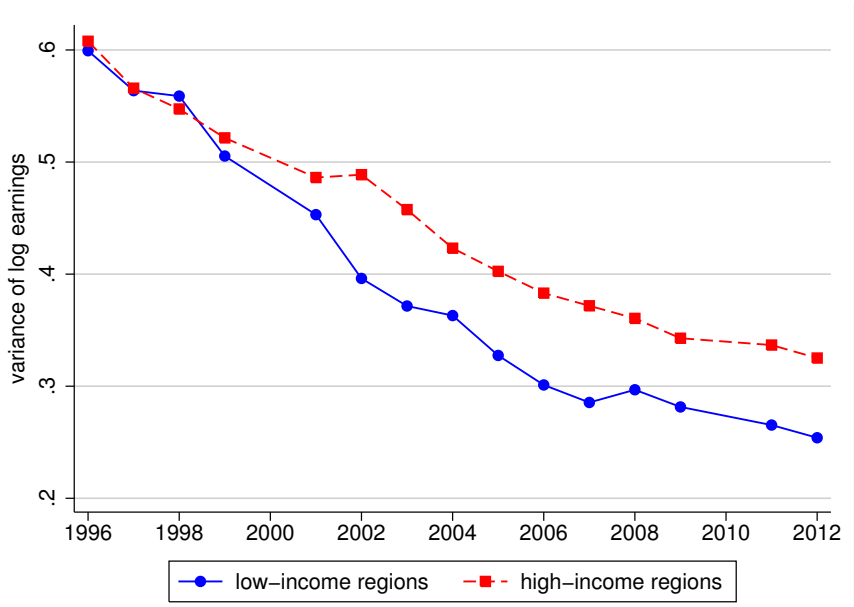


PNAD household survey data[43] and sort the five big geo-economic regions of Brazil (North, Northeast, South, Southeast, Centre-West) into two groups[44] by their average per capita income level over the period. Figure 2.10 plots the variance of log labor earnings in different regions of the country, grouped by two income levels. We see that inequality declined more rapidly within regions that started out at lower average incomes. Specifically, the variance of log earnings was just marginally higher in high-income regions than other parts of the country in 1996, yet by 2012 a 6 log points gap had arisen between the two groups. This is again consistent with a story of the minimum wage affecting these regions differentially.

[43]Our confidentiality agreement with the Brazilian Ministry of Labor does not permit us to disseminate results from analysis using regional identifiers in the administrative RAIS data.

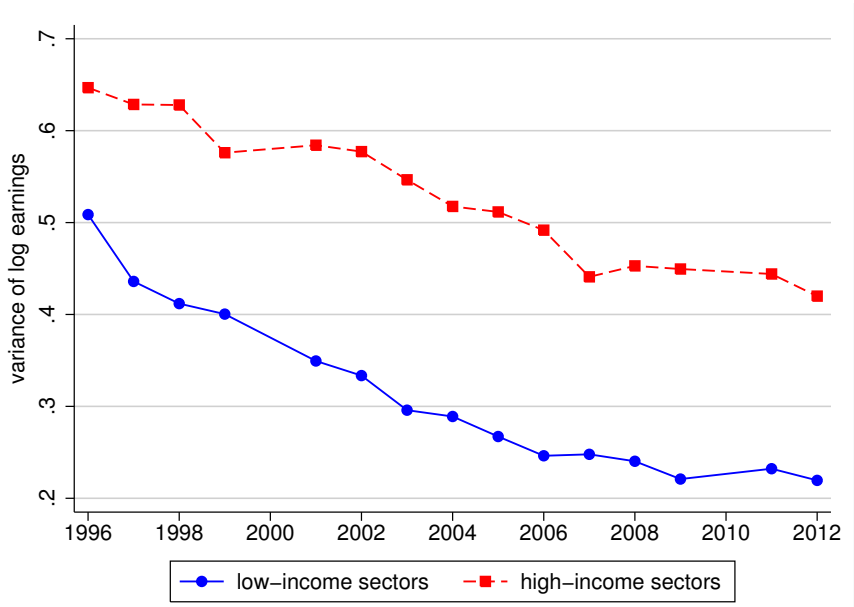
[44]Very similar results obtain when using finer regional units such as states or municipalities.

Figure 2.10: More pronounced decline of earnings inequality in initially low-income regions, 1996–2012



Greater impact on low-income sectors. In a similar vein, using data from the PNAD household surveys, Figure 2.11 plots the variance of log labor earnings for the set of adult males across sectors classified as low- or high-income in 1996. Among others, low-paying sectors include agriculture, services, and commerce; while high-paying sectors include manufacturing, mining, and public administration. Initially, the variance of log earnings was 13 log points higher in the high-income sectors. Yet, by the end of the period that difference had widened to almost 20 log points. The fact that inequality is nowadays significantly lower within low-income sectors and that inequality in those sectors declined more quickly over the period is consistent with stories of the minimum wage affecting those sectors disproportionately more than high-paying sectors.

Figure 2.11: More pronounced decline of earnings inequality in initially low-income sectors, 1996–2012



Summary of empirical evidence. In our earlier empirical analysis, we established that Brazil's inequality decline from 1996–2012 features particularly pronounced compression at the bottom of the earnings distribution, and the U-shaped evolution of the real minimum wage since 1988 mirrored that of earnings inequality. In further support of the minimum wage playing an important role in the evolution of inequality in Brazil over this period, we presented three additional pieces of evidence: First, the inequality decline was more pronounced in Brazil's formal sector relative to the informal sector where labor regulations like the minimum are plausibly harder to enforce. Second, inequality started to decline later in initially higher-paying sectors such as manufacturing and mining, consistent with a rising minimum wage affecting these sectors with a delay. Third, regions that started out at higher average income levels experienced less of a decline in earnings inequality. Together, these facts support the hypothesis of a causal relationship between the rise in the minimum wage and the

decline of earnings inequality over the period, which we implicitly adapted in our theoretical framework.

2.8.2 Job ladder in firm effects

The key ingredient of our model is the ability of workers to receive job offers while currently employed. The possibility of on-the-job mobility circumvents the Diamond (1971) paradox by inducing firms to compete for workers. This competition among firms leads to spill-over effects of an institutional wage floor: some workers will be affected directly and, if the surplus they generate at a given employer is positive net of the new minimum wage, will relocate to the new minimum wage level; but their wages will on average increase further because firms will want to recruit the mass of workers close to the new minimum wage. These ripple effects in wage setting will slowly fade out as we move up the firm productivity distribution. Such competition of firms for workers will result in workers moving up the firm pay ranks during employment spells—a dynamic commonly referred to in the literature as a "job ladder," though in our model such a ladder is really between firms, so we will call it a "firm ladder" henceforth.[45]

Existing work on job ladder models has highlighted their success in capturing key labor market characteristics (Mortensen, 2003). Naturally, testing for the presence of a firm ladder is essential for our proposed mechanism to work and lead to large effects of the minimum wage on the earnings distribution. We present four pieces of evidence in favor of a firm ladder, which we quantify using firm effects from the AKM estimation as the empirical counterpart to our model's firm ladder:

1. The firm effects distribution of stayers first-order stochastically dominates that of previously non-employed workers; see Figure 2.12.

[45]Partly due to data limitations, the previous literature has focused on various alternative manifestations of a job ladder, including firm size and average wages at a firm. We argue that our choice of firm effects is an intuitively appealing counterpart of the job ladder in our context.

2. Job-to-job transitions on are associated with an average increase in firm effects of 5%, equivalent to 5 percentile ranks of the firm effects distribution; see Table 2.15.

3. Workers move up in firm effects more quickly towards the bottom; see Figure 2.13.

4. Worker turnover is lower at employers with higher firm effects.; see Figure 2.14.

Together, these facts support our firm ladder view of the Brazilian labor market.

Figure 2.12: Job ladder fact 1: realized wage distribution FOSDs wage offer distribution, 1996

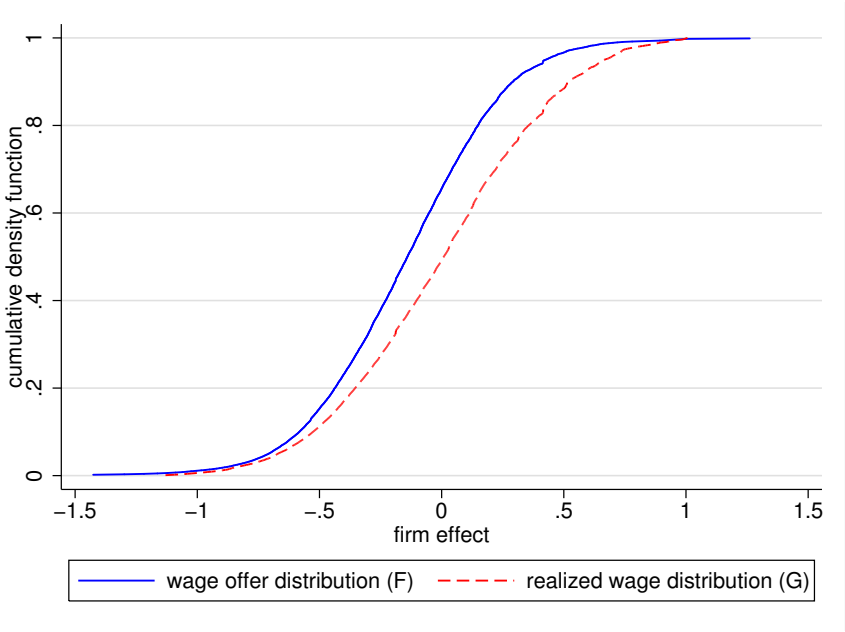


Table 2.15: Job ladder fact 2: Large positive gains in firm effects from switching employers, 1996–2012

Change in firm effect from switching employer	Average value, 1996–2012
Absolute change	3.2
Percentile rank change	6.0

Figure 2.13: Job ladder fact 3: Greater gains in firm effects for workers at lower firm effects employers, 1996

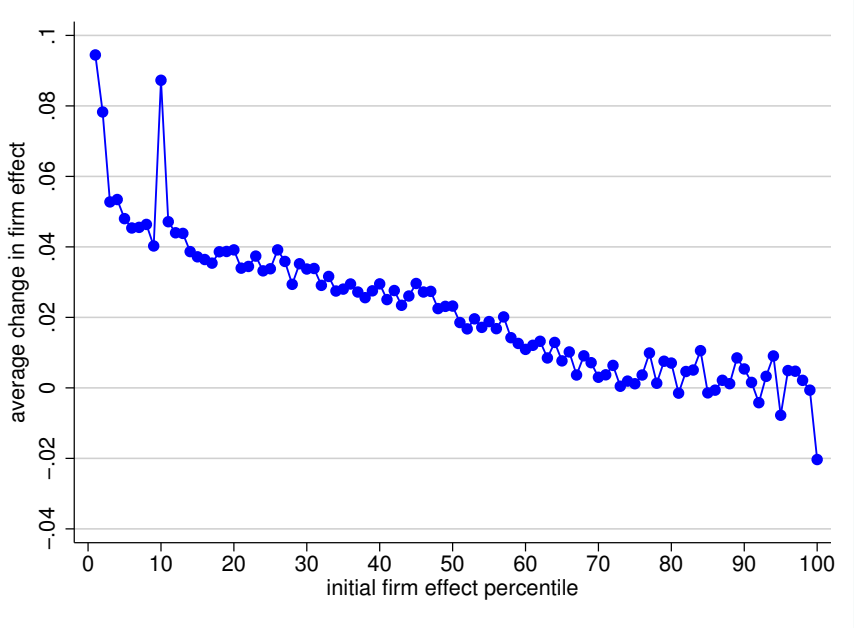
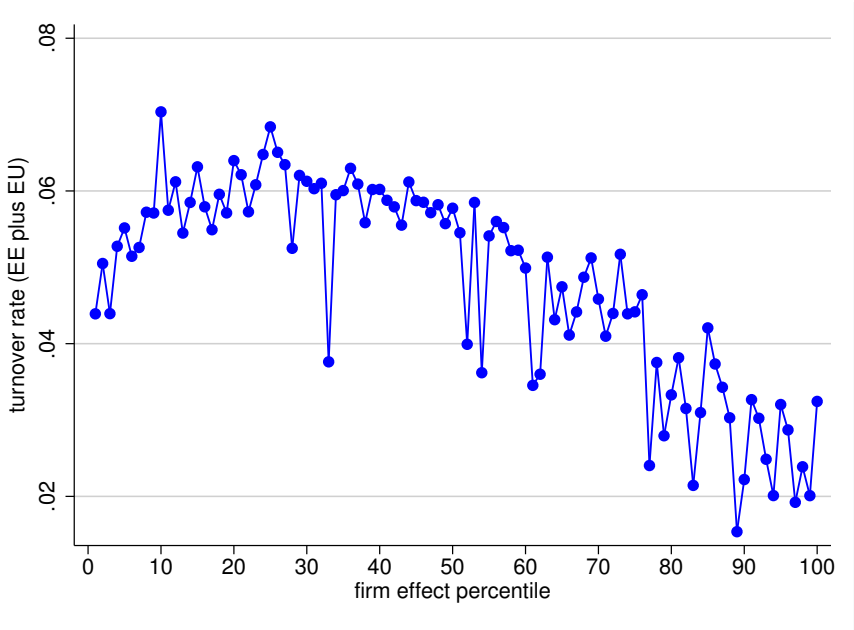


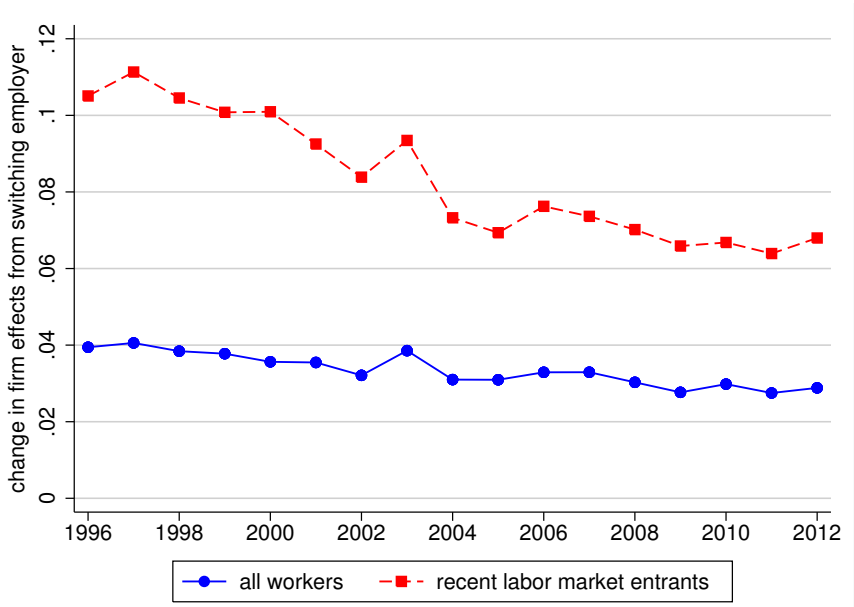
Figure 2.14: Job ladder fact 4: Lower turnover rates at higher fixed effect firms, 1996



While the previous facts lend support to the Brazilian labor market being characterized by a job ladder, a corollary of our theory is that as the minimum wage increases over time the rungs of the job ladder should become compressed. We verify this central prediction of our model by measuring the gains in earnings form switching firms, as given by the difference in estimated AKM firm effects between the source and the target firm, and confirming that these gains are indeed decreasing in magnitude over time.[46] In line with our model predictions, we find strong support for both claims, as shown in Figure 2.15. Specifically, between 1996 and 2012 the average gain in firm effects from switching employers declines by 1.1 log points (or 28 percent; see blue solid line) in the overall worker population and by 3.8 log points (or 36 percent; see red dashed line) among recent labor market entrants.

[46]Furthermore, in line with our model predictions, we verify that the relative gain in firm effects, as measured by the ranks climbed by transitioning, does not decrease over time.

Figure 2.15: Decline of gains in firm effects from switching employers by worker group, 1996–2012



2.8.3 Sorting pattern induced at low productivity firms

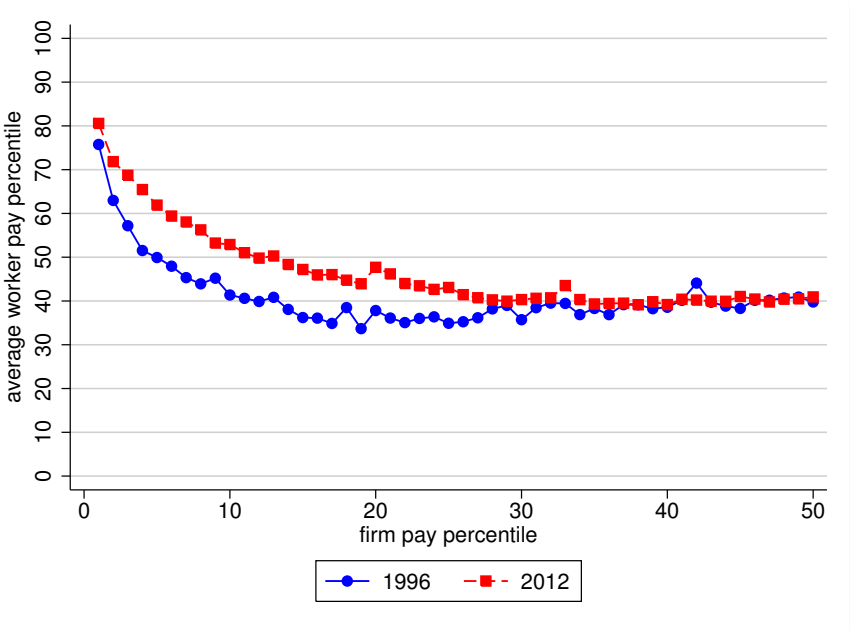
The way we modeled the minimum wage has direct implications for the sorting pattern of heterogeneous workers across firms of different productivity. Specifically, a natural prediction of our model is that matches between low ability workers and low productivity firms eventually become infeasible as the minimum wage gradually increases. In this case, low productivity firms recruit from a subset of labor markets above an ability cutoff satisfying $\theta \geq w^{min}/p$. Consequently, we would expect the average worker quality to be *higher* at low productivity firms. While this is a very straight-forward prediction of our model[47], to the best of our knowledge no previous work has tested for such policy-induced sorting patterns.

To test for changes in the sorting pattern towards the bottom of the firm effects distribution, we first rank worker effects and firm effects within AKM subperiods. We then compute the average worker effects percentile for a given firm effects per-

[47] A similar effect would obtain in a broad class of other models featuring production functions that are log-linear in firm productivity and worker ability.

centile. Figure 2.16 plots the results of this exercise for the bottom half of the firm effects distribution. We confirm the presence of a negative sorting pattern between workers and firms among the lowest-paying employers, and note that this pattern is becoming more pronounced between 1996 and 2012, consistent with the minimum wage becoming more binding over this period. We view this result as corroborating evidence for our specific model mechanism by which the minimum wage affects labor market outcomes in Brazil.[48]

Figure 2.16: Sorting between worker and firm effects, bottom half of firm effects distribution



2.8.4 Discussion of modeling assumptions

While our extended job ladder model is plausibly also going to be successful in these dimensions, the parsimony of the model also begs the question if our specific modeling

[48]Figure B.11 in Figure B.11 in Appendix B shows the sorting pattern of workers across firms for the complete distribution of firm effects, which exhibits positive sorting overall, particularly towards the top of the firm effects distribution. We view the positive sorting pattern as plausibly induced by a mechanism outside of our benchmark model, although were we to allow for heterogeneous job arrival rates, as discussed in Section 2.8.4, then our model would also be able to qualitatively replicate this pattern, even absent other technological adaptations.

choices are necessary to obtain similar qualitative and quantitative results. To this end, we argue that our model is a specific instance of a broad class of model in which a similar economic mechanism leads to spill-over effects of the minimum wage. The key ingredient that unites these models is the competition among firms for workers that arises in the presence of on-the-job arrival of job opportunities, a salient feature of many search models also highlighted by Hornstein, Krusell and Violante (2011). With this in mind, we proceed to discussing how various ingredients featured in our model could be changed without fundamentally changing our conclusions.

Wage posting. Conducting a survey among a representative sample of U.S. workers, Hall and Krueger (2012) present evidence in support of the prevalence of job-to-job mobility in general and of the importance of wage posting in particular. They find that 94 percent of blue-collar workers and two thirds of their overall sample did not negotiate their wage upon entering their last employment spell. Consistent with the wage posting assumption, a large share of these workers reported knowing the exact wage at the prospective employer before making the job transition. They also find that wage bargaining becomes more prevalent among senior employees and "knowledge workers."

While a systematic study of wage setting policies is lacking for the Brazilian case, we think that the wage posting assumption is reasonable for two reasons. First, to the extent that significantly lower education levels in Brazil are associated with jobs that resemble more closely the blue collar jobs found in the U.S., the wage posting assumption appears more appropriate. Second, Brazilian wage contract laws and regulations imposed by central bargaining institutions leave limited scope for individual negotiation of worker pay. For example, Brazilian labor law (*Consolidação das Leis do Trabalho*, or *CLT*) precludes changing individual workers' wages without adjusting accordingly the wages of other workers employed at the same firm.

While the assumption of wage posting appears less restrictive in light of this institutional background, we also believe that a similar mechanism would be at work in models where wages are bargained upon starting the employment relationship, and possibly throughout tenure.[49]In such a model, the minimum wage would have a direct effect on the lowest productivity firms as well as an indirect effect on firms higher up the productivity distribution by boosting workers' outside option in the bargaining game.

Informal sector. Recent work by Meghir, Narita and Robin (2015) explicitly incorporates an informal sector into an otherwise standard job ladder model. Introducing the informal sector into the model has the advantage of being able to speak to worker transitions between the two sectors and hence to competition of firms for workers in the two sectors. While these considerations are of great interest to studying the Brazilian economy, which features a sizable informal sector, we view our abstraction from the informal sector as putting a lower bound on the degree of competition among firms for workers, and hence on the degree of spill-over effects in wage setting across firms. In this sense, our results can be viewed as providing a lower bound on the degree of compression due to the minimum wage.

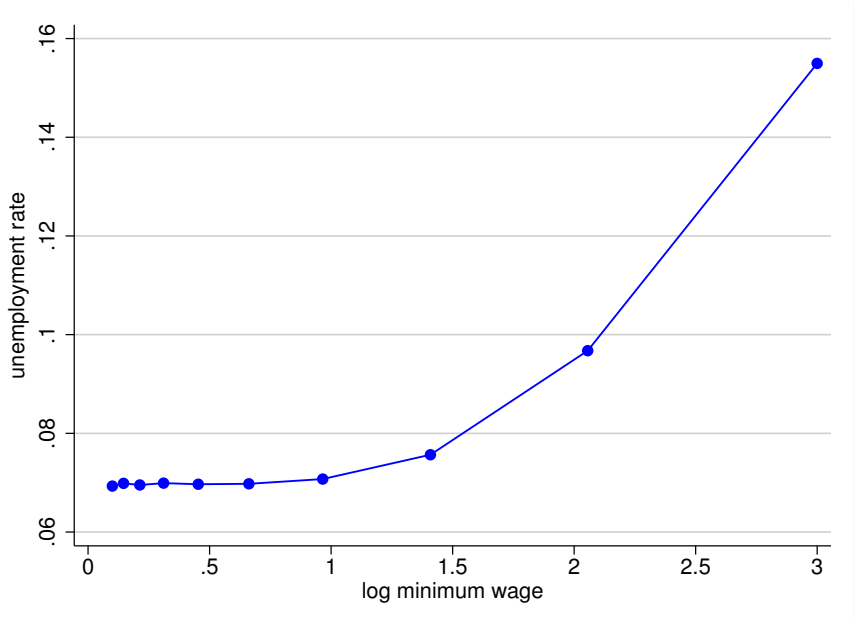
On the other hand, incorporating the informal sector may provide an important way for firms to substitute between workers in the two sectors. For example, one may predict that a large increase in the minimum wage in the presence of a shadow economy that is not subject to such labor market regulations would lead to a sizable shift from formal to informal activity. Contrary to this hypothesis, we find that the informal sector in Brazil shrank over the period we study, comprising 36% of all prime-age male employees in 1996 but only 26% in 2012. Hence, we conclude that

[49]Examples of such model environments have been developed by Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). In their model, a worker's starting wage equals the value of the productive match in their previous employment, or the value of unemployment, respectively.

such additional considerations would not detract from our current analysis, but could be modeled in parallel to our analysis.

Unemployment and endogenous vacancy creation. While our model quantitatively generates little unemployment in response to the observed minimum wage increase, the model does generate significant unemployment for larger minimum wage increases. The mechanism for this is that as the minimum wage continues to increase, an entire labor market segment θ is cut off from work activity as soon as even the most productive firm no longer finds it profitable to recruit from this market, that is as soon as the minimum wage crosses the threshold $w^{min} = \theta\overline{p}$. Figure 2.17 plots the unemployment rate in response to the minimum wage. In the region of the graph corresponding to the 2012 level of the minimum wage, around 0.897 on the horizontal axis, the unemployment rate is little affected by increases in the minimum wage. However, the unemployment rate would more than double and display a steep gradient if the minimum wage were to be increased by an additional 200 log points.

Figure 2.17: Steady-state equilibrium unemployment vs. minimum wage level



A feature absent from our model specification is endogeneity in firms' vacancy creation (Mortensen, 2000, 2003). Allowing firms to respond in their extensive margin recruiting decisions would plausibly lead to a greater unemployment response to the minimum wage. While the empirical evidence on the employment effects of the minimum wage is mixed and pointing towards zero or small positive effects[50], we cannot rule out negative employment effects of the minimum wage in Brazil. Yet, the Brazilian unemployment rate has fallen from 6.5% to 5.5% from 1996 to 2012, the same time period during which also the informal sector shrank, indicating that such effects could not have been of first-order importance.

Allowing search friction parameters to differ by worker type. One may think that the search friction parameters, which we here restrict to be the same for the entire worker population, may differ systematically across worker groups. To allow for this possibility, our model could be readily extended to feature worker type-specific mobility parameters $\{\delta_\theta, \lambda_\theta^u, \lambda_\theta^e\}_{\theta\in\Theta}$. Such a model has important implications for the observed pattern of sorting of worker types across firms, potentially rationalizing the positive sorting pattern emerging from the AKM analysis. We caution, however, that the same estimation procedure by which we identify the current model parameters guiding worker and firm heterogeneity would no longer be unbiased in the AKM estimation. This is because the strict exogeneity condition, $\mathbb{E}\left[\varepsilon_{it}|\, i, t, J\left(i,t\right)\right] = 0$, required for unbiased identification of worker and firm effects in the AKM framework would no longer be satisfied. However, estimating differences in worker flow rates across worker types from the data, the sign and extent of this bias could be estimated in large samples, which we view as a promising extension for future work.

[50]See Card and Krueger (1994), Neumark and Wascher (1994), and a large follow-up literature for an important part of the debate.

2.8.5 Welfare evaluation

Search frictions generate monopsonistic firm rents, which a rise in the minimum wage transfers to workers. While workers who remain employed at the new minimum wage benefit, not everyone gains from the minimum wage increase. The lowest productivity firms stop recruiting low ability workers or exit altogether, while the lowest ability workers are forced into unemployment. In an extension of the model with firm owner-managers, we also account for the loss of rents by monopsony shareholders. Evaluating these channels quantitatively, however, we find small displacement effects of the minimum wage, which are more than offset by allocative efficiency gains from inducing workers to relocate to more productive employers. Nevertheless, the distributional effects of the minimum wage are a significant factor in policy considerations.

Yet other channels through which the minimum wage may effect welfare in the economy are absent from our model. Notably, two channels not present in our benchmark framework would lead counteract the welfare gains of the minimum wage described in our previous analysis.

The first channel is a feedback of the minimum to firms' endogenous vacancy creation. If creating job openings comes at a fixed cost to firms and since the minimum wage will reduce the rent that monopsonist firms can appropriate by posting a job opening, this additional channel would lead to reduced vacancy creation and thus a slow-down in the reallocation of workers across firms in response to the minimum wage increase. An example of such general equilibrium effects on firms' vacancy creation decision is contained in Mortensen (2000) and its application to our framework is left for future exploration.

The second channel is the possibility of pass-through of the minimum wage to consumers through firms' pricing decisions. This channel, which is currently absent from our model to the assumption of a linear production function, would diminish the welfare gains to low-income workers by raising the price of final goods consumption.

However, since firms employ a mix of workers and not all are affected (to the same extent) by the minimum wage, one would expect the pass-through to prices to only partially offset the welfare gains to workers with the lowest level of earnings. On the other hand, the pass-through into consumption prices would pose an added effect on high income workers, reducing their welfare relative to the economy with a lower minimum wage.

2.9 Conclusion

In this paper, we analyzed sources of earnings inequality dynamics in general and the role of the minimum wage specifically. The starting point of our investigation were three key facts about Brazil, which experienced a rapid decline in earnings inequality between 1996 and 2012. Brazil's overall decline in earnings inequality was driven from the bottom. We find that one quarter of this decline stems from a weaker degree of pass-through from firm productivity to wages, and another quarter of the decline is attributable to falling pay differences due to unobserved worker characteristics.

To investigate the contribution of the minimum wage to these facts, we built a search model in the spirit of Burdett and Mortensen (1998), extended with heterogeneous firms and workers. The key feature of the model were spillover effects of the minimum wage due to monopsonistic competition among firms for workers. We characterize the equilibrium of this model and showed that the minimum accounts qualitatively for our documented facts.

Estimating the model on Brazilian microdata, we are also successful in explaining a large share of the overall inequality decline and quantitatively accounting for the three facts. Consistent with the observed compression of earnings, a large share of the inequality decline in our model is due to indirect effects of the minimum wage,

resulting in a lower productivity-pay gradient across firms and lower returns to worker ability.

While the minimum wage may affect many other outcomes of interest (Card and Krueger, 1994; Manning, 2005; Harasztosi and Lindner, 2015), we have focused our analysis on the effects of the minimum wage on the earnings distribution. Although the key mechanism in our model is a general one and relies only on the inter-dependence between firms' wage offers, a key question is to what extent the Brazilian experience carries over to other economies such as the United States, where policy makers currently debate an increase in the minimum wage from 7.25 to 15.00 dollars. Our analysis sheds new light on one aspect of this question and suggests that the effects on earnings inequality will depend crucially on the structural parameters guiding the between-firm competition among firms for employees in those markets. Assessing the strength of this channel for other economies as well as for alternative policies including unemployment insurance, employment protection legislation, and non-discrimination laws would shed further light on the degree to which labor market dynamics can amplify the effects of policy on earnings inequality.

Chapter 3

Firm Dynamics, Financial Frictions, and Industrial Policy

3.1 Introduction

There is significant variation in per-capita income across countries[1] and the largest share of it can be attributed to differences in total factor productivity (TFP).[2] Differences in TFP may stand in for many factors, including differences in the economic environment affecting business operations. For instance, imperfect credit markets may impede entrepreneurship, reducing both aggregate productivity and output. On the one hand, a lack of financing may lead active entrepreneurs to operate below the optimal scale of production or, when individuals foresee this inefficiency, even prevent them from starting a business in the first place. On the other hand, less productive but wealthy entrepreneurs may continue to operate in less than competitive markets that arise with imperfect access to credit. If cost of creating a new business is large,

[1] See Easterly et al. (1993), Jones (1997), Lucas (2004) and Banerjee and Duflo (2005) for a selection of available evidence of large cross-country differences.

[2] This fact is documented in Klenow and Rodríguez-Clare (1997), Hall and Jones (1999), and Hsieh and Klenow (2009), among others.

this misallocation of resources may be further amplified by increasing entrepreneurs' dependence on external financing.

But the story does not end at the individual's decision to enter the market or not. Once active, entrepreneurs may be exposed to business risks and find themselves relatively unproductive for an uncertain amount of time. At that point, individuals face a choice between exercising a relatively attractive outside option, such as working in the labor market, or staying in business in hopes of a quick recovery. The option value of remaining active will be greater the higher the entrepreneur's expectation of future productivity and the higher the barriers to future re-entry. With perfect credit markets, entrepreneurs weigh current losses associated with running a relatively unprofitable business against the expectation of future profits and pick the more attractive option. When access to credit is limited, however, entrepreneurs may decide to liquidate their business prematurely, leading to an inefficiently high rate of firm exits.

In this paper, I first assesses the size of distortions due to entry costs combined with financial frictions and then considers one particular type of industrial policy targeted at mitigating such distortions. I then proceed to calibrate my model to assess the impact of corporate bailouts, or *soft budget constraints*, on a dynamic economy with occupational choice in the presence of entry costs and credit constraints. The focus of the investigation is the role of distortions to existing firms' extensive margins and the effects of government intervention in the form of subsidies to least productive active entrepreneurs. Thus, the analysis is primarily concerned with the *premature exit* of existing firms rather than *barriers to entry* of new entrepreneurs. I will argue that distortions due to premature firm exits can be sizable and that the effects of government intervention depend heavily on the range of underlying parameters.

The spirit of my investigation is to set up the economic environment in favor of finding a positive effect of soft budget constraints. I then look at remaining sources of

inefficiencies due to government intervention and assess which of these are the most significant. I find that the persistence of productivity shocks is a key determinant of the efficiency of an economy with credit frictions and entry costs in general, and of soft budget constraints in particular. My main result is that distortions due to government intervention are largest when the stochastic process guiding firm productivities is persistent.

Credit markets and entry costs are two natural factors affecting firm dynamics. On the one hand, the better the access to credit the more businesses per capita are started on average. Intuitively, running a business is more attractive when operating at the optimal scale and as such better credit markets promote firm entry. On the other hand, higher credit market quality is significantly and negatively related to firm exit rates, which is suggests that financing needs play an important role in shaping firm exit decisions. Firms that have undertaken large investments in the past, potentially related to industry entry, consider this cost as sunk when making their current production decisions but also take into consideration that this investment may be partially lost in case of firm liquidation. Firms with the greatest fixed cost investments that find themselves with easy access to credit markets should, all else equal, be the most reluctant to liquidate their business. A problem arises when firms undergo temporary hardship and simultaneously run into financing constraints. In such a case, information about the prospects of companies may be imperfect and people's perception about the future profitability of the enterprise may be at odds. For example, while General Motors may claim to have prospects of a quick recovery and profitable future business operation, public and private lenders may wonder if their cash injections will help their creditor sustain business in the long-term or whether increased lending just keeps the "zombie" firm alive for a little longer before the next big collapse. Besides informational asymmetries, in such circumstances the government faces what Kornai (1980) termed the soft budget constraint problem:

Ex-post aid may be efficient but to avoid adverse selection problems the government would like to commit ex-ante to not intervening. Such a commitment, of course, is non-credible and as firms foresee this they will make their extensive and intensive margin decisions accordingly. This industrial policy problem, among others, motivates my analysis of the interaction between financial frictions and entry-costs as well as potential government intervention.

The present paper connects to four different strands of the literature. First, scholars of corporate finance have studied firms' optimal liquidity and risk management in conjunction with the soft budget constraint problem that arises either between a firm and a private lender or between a firm and the public sector. Qian and Xu (1998) describe the impact of the soft budget constraint problem on innovation decisions and find that softer budget constraints lead to increased moral hazard on firms' behalf. Since the first studies of soft budget constraints by Kornai (1980), research has focused on the use of finite-horizon models to assess the impact of soft budget constraints as an equilibrium response to liquidity shocks. The first part of my analysis employs a three-period model in the tradition of this literature and translates their stylistic model into a macroeconomic environment. I then extend this literature by formulating the soft budget constraint problem in a full-fledged infinite-horizon growth model. To the best of my knowledge, this is the first paper to incorporate soft budget constraints in an dynamic general equilibrium model with occupational choice.

Second, a vast part of the macroeconomic growth literature has been concerned with identifying potential determinants of cross-country TFP differences. Restuccia and Rogerson (2008) for instance model reduced form policy distortions as productivity wedges in a firm entry-exit model. The role of financial frictions as one concrete distortion in this context has recently been under much debate. For example, TFP differences. Moll (2014) argues that on the one hand transition dynamics to the steady

state are fast when productivity shocks are transient. On the other hand, steady-state losses due to financial frictions can be large when the stochastic process governing productivities has very low persistence. In my work, I also stress the importance of the persistence of the stochastic process governing entrepreneurial talent and argue that, in addition to aggregate losses being sizable, extensive margin distortions due to premature firm liquidations can be sizable when productivities are transient.

Third, there has been an increasing interest in the role of financial frictions in shaping industry specialization and allocation of entrepreneurial talent. For example, Barseghyan and DiCecio (2011) assess the negative relationship between entry costs and firm entry. For an analysis of the interaction between financial frictions and start-up costs in a firm entry-exit model, see Fang (2010) and Asturias et al. (2015). Static period-profit maximization formulations are prevalent in much of the existing literature on occupational choice, including Banerjee and Newman (1993), Cagetti and Nardi (2006), Buera, Kaboski and Shin (2011), and Buera, Moll and Shin (2013). For example, Buera, Kaboski and Shin (2011) conclude: "[with setup costs] financial frictions have an even larger impact on aggregate and sector-level productivity, reflecting the stronger non-convexity that front-loaded setup costs impose". Their calibrated model suggests that extensive margin distortions account for more than half of the distortions to industry TFP in manufacturing but less than one-tenth in the service sector. This partly motivates my own focus on extensive margins though I focus on one particular type of distortions, namely those that affect existing firms' exit margin. Most recently, Cui (2013) embeds capital specificity in a dynamic general equilibrium model with linear production technologies. The delayed capital reallocation that emerges as equilibrium behavior is also a salient feature of my model, though in my context such threshold policies are motivated through sunk costs rather than asset price discounts.

Fourth, there is a renewed interest in the effects of industrial policy and redistributive motives in the context of incomplete markets. See Aghion et al. (2015) for both a theoretical and an empirical study of how competition-friendliness of industrial policies affects productivities and growth prospects. Less work has been done on redistribution with financial frictions, though Cagetti and Nardi (2009) take an interesting approach to the topic in analyzing the effectiveness of estate taxation as a means of redistribution in the presence of incomplete markets. With this paper, I hope to shed further light on issues of redistribution with imperfect incomplete markets as I focus on one particular kind of transfers, namely from the most productive entrepreneurs to the least productive ones.

This paper proceeds to study the interaction between credit constraints and entry costs as well as the effects of soft budget constraints as follows:

Section 3.2 employs a three-period model to illustrate the effects of financial frictions and start-up costs on short-run economic activity. In the context of entry costs and imperfect credit markets, I determine how much of all extensive margin distortions are due to premature firm liquidations. I then investigate the aggregate economic effects of government-provided aid to firms that would otherwise inefficiently liquidate their businesses when such intervention comes at the expense of more productive entrepreneurs. Section 3.3 presents the infinite-horizon analogue of the previous section and describes a stationary equilibrium with government policies. I then simulate the economy for various degrees of government intervention and show that TFP losses are convex in the size of the subsidy to least productive entrepreneurs. Section 3.4 examines how the previous insights depend on industry-specific assumptions on the stochastic process, productivity dispersion, size of fixed costs, the nature and timing of the mechanism by which potential entrepreneurs learn about their ability, the observability of firm characteristics, and the enforceability of credit contracts. Most importantly, I find that the persistence of productivity shocks plays a crucial role in

determining the extent of inefficiencies arising in the context of government intervention. Finally, Section 3.5 concludes and summarizes the paper's main findings.

3.2 Three-period economy

3.2.1 Preferences, technology, government policy, and credit markets

Consider a three-period economy ($t = 0, 1, 2$) with a continuum of size N of individuals described by their current asset holdings, their entrepreneurial talent, and their previous occupational choice.[3]

All agents evaluate consumption streams (c_0, c_1, c_2) by ranking them according to

$$U(c_0, c_1, c_2) = u(c_0) + \beta \mathbb{E}_0 [u(c_1) + \beta u(c_2)]$$

where c_t is consumption at time t, period utility is of the CRRA form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

and $\beta \in (0, 1)$ is an exponential discount factor. For the sake of analytical convenience, I adopt $\sigma = 1$ which yields period log-utility.

Every period, individuals choose between working in the labor market for a competitive wage or alternatively running an enterprise using their entrepreneurial talent. There is no market for entrepreneurial talent and existing businesses are specific to the entrepreneur who started it. Active entrepreneurs pay a per-period fixed cost of operation $\kappa^o \geq 0$ and combine their current entrepreneurial talent z_t with labor input

[3]For example, an individual might set up a business in one period by paying a one-off entry cost, and then continue operation in the next period.

l_t to operate the production technology

$$f(z_t, l_t) = z_t l_t^{\alpha}$$

where $\alpha \in (0,1)$. Note that $f(\cdot)$ exhibits decreasing returns to scale in labor input so that entrepreneurs may make positive profit even in an equilibrium with free firm entry and exit. If a previous wage worker starts a business there is an additional start-up cost $\kappa^s \geq 0$. At time t, let the previous period's occupational choice be denoted $o_{t-1} \in \{E, W\}$ and define

$$\kappa_t \equiv \kappa(o_{t-1}) = \begin{cases} \kappa^o & \text{if } o_{t-1} = E \\ \kappa^o + \kappa^s & \text{if } o_{t-1} = W \end{cases}$$

This implicitly assumes that once an entrepreneur decides to exit the industry, all previous investment is lost and a new entry cost κ^s needs to be paid if the entrepreneur wants to run a business in future periods. While arguably extreme, this assumption could be justified on regulatory grounds or by assuming that the business environment is rapidly changing and thus previously abandoned firms perish quickly. Consider a constant interest rate $r_t = r$ and let the wage level w_t adjust to clear labor markets every period, as in a small open economy.

Furthermore, suppose that initial wealth is distributed $\mu_{a_0}(a_0)$ and individuals enter the economy with a given initial occupation $o_{-1} \in \{E, W\}$ which I assume is distributed $\mu_{o_{-1}}(\cdot)$. I consider $\mu_{o_{-1}}(\cdot)$ to be degenerate at $o_{-1} = W$, so no individual enters the economy with a business already set up. Denote the induced joint distribution of wealth, current productivity, and previous occupation at time $t = 0, 1, 2$ by $\mu(a_t, z_t, o_{t-1})$. Let initial productivities z_0 be equal to z_L for a fraction $\mu_{z_0}(z_L)$ of the population and equal to $z_H > z_L$ for the remaining $1 - \mu_{z_0}(z_L)$ fraction of individuals in the economy. At time t, an individual's next period productivity, z_{t+1}, is

Markovian in current productivity, z_t, and follows a Poisson arrival process described by

$$z_{t+1} \begin{cases} = z_t & \text{with probability } \gamma \\ \sim \mu_z(z_{t+1}) & \text{with probability } 1-\gamma \end{cases}$$

where $\mu_z(\cdot)$ is a stationary probability distribution over $\{z_L, z_H\}$. Thus, γ can be interpreted as a persistence parameter and it follows that $\mathbb{P}(z_{t+1} = z_H | z_t = z_H) > \mathbb{P}(z_{t+1} = z_H | z_t = z_L)$ whenever $\gamma > 0$. While I focus on a binary probability state space for illustration purposes, the model is readily extended to allow for a continuous distribution over a possibly unbounded productivity space.

In order to set the stage for my analysis of premature firm exit, I make some assumptions on productivities and fixed costs in the economy. First, I assume that entrepreneurs indeed face operational risk in the form of productivity shocks.

Assumption 3.1. *There are no absorbing productivity states:* $\gamma < 1$.

Second, I want relatively unconstrained high productivity individuals to optimally start a business in the beginning of the three-period economy.

Assumption 3.2.

$$z_H \left(\frac{\alpha z_H}{w_t}\right)^{\frac{1}{1-\alpha}} - w_t \left(\frac{\alpha z_H}{w_t}\right)^{\frac{1}{1-\alpha}} - \kappa_t > w_t : \quad t = 0$$

Note that with decreasing returns to scale in production, Assumption 3.2 will be satisfied for high enough productivity z_H, even if labor markets are competitive.

Third, I assume that low productivity individuals in the last period prefer to work for a wage over remaining in business with low productivity, even if unconstrained.

Assumption 3.3.

$$w_t \;>\; z_L^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w_t}\right)^{\frac{\alpha}{1-\alpha}} - w_t \left(\frac{z_L \alpha}{w_t}\right)^{\frac{1}{1-\alpha}} - \kappa^o : \quad t = 2$$

Intuitively, Assumption 3.3 is satisfied for low enough productivity z_L relative to the fixed cost of operating a business, κ^o.

In the three-period economy, I focus on the effects of a one-off government intervention in the middle period ($t = 1$) in the form of idiosyncratic taxes on revenues, τ_1, and targeted transfers, ξ_1. In Section 3.2.4, I discuss government's rationale for intervention and its constraints in designing these policies optimally, but for now just take the policy schedule as given. For notational convenience, let $(\tau_1, \xi_1) \equiv (\tau, \xi)$ and otherwise $(\tau_t, \xi_t) \equiv (0, 0)$ for $t \neq 1$. Taking this one-off policy and the market wage rate as given, entrepreneurs solve a static profit maximization problem which in the absence of credit constraints yields optimal unconstrained labor input as a function of current productivity that equates marginal product of labor with the wage rate.

Definition 3.1. *The* unconstrained optimal labor input $l_t^u \equiv l^u(z_t, \tau_t)$ *given current productivity* z_t *and idiosyncratic revenue tax* τ_t *is the level of labor utilization given by*

$$\begin{aligned} l^u(z_t, \tau_t) &= \arg\max_l \left[(1 - \tau_t) z_t l^\alpha - w_t l\right] \\ &= \left(\frac{\alpha(1 - \tau_t) z_t}{w_t}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

In order for an individual to run a business and employ l, both fixed costs κ_t and labor costs $w_t l$ need to be paid upfront, so that entrepreneurs at the beginning of the period need to borrow from a financial institution in order to finance expenses in excess of their own wealth. I assume that all debt is short-term and needs to be repaid at the end of the same period. Limited enforcement of credit contracts gives rise to an endogenous credit constraint as indexed by the fraction of pledgeable post-tax income $\phi \in [0, 1]$. Hence, the parameter ϕ is a measure of the quality of credit markets.[4] In

[4] A simpler form of credit constraint would could be imposed by allowing agents to borrow up to a fraction of $(\lambda - 1)$ of their current wealth every period, where $\lambda \geq 1$ indexes credit market quality. With this formulation, labor input l is bounded from above by the labor utilization constraint

other words, labor in my model takes the role traditionally played by capital in that it requires advance financing, thus acting as "working capital". I explicitly assume that agents have the option of defaulting on their debt but government policy cannot be evaded either way. Suppose for the rest of the exposition that government policy (τ_t, ξ_t) is perfectly enforceable.

This leads to my first result, which is analogous to the finding of Buera, Kaboski and Shin (2011):

Proposition 3.1. *Labor utilization l by an entrepreneur with wealth a_t, productivity z_t, previous occupational choice o_{t-1}, and idiosyncratic tax-subsidy schedule (τ_t, ξ_t) is enforceable if and only if*

$$\begin{aligned} & (1-\tau_t) z_t l^\alpha - (1+r) w_t l - (1+r) \kappa_t + (1+r) a_t + \xi_t \\ \geq \quad & (1-\phi) \left[(1-\tau_t) z_t l^\alpha + \xi_t \right] \end{aligned} \tag{3.1}$$

The highest level of l that is enforceable defines a labor utilization constraint, $\bar{l}_t \equiv \bar{l}(a_t, z_t, \tau_t, \xi_t, o_{t-1}; \phi)$, as a function of current productivity, wealth and the tax-subsidy schedule. Furthermore, $\bar{l}(a_t, z_t, \tau_t, \xi_t, o_{t-1}; \phi)$ is strictly increasing in $z_t, a_t, (1-\tau_t), \xi_t$ and ϕ.

Proof. All proofs can be found in Appendix C.2 of the paper. □

Proposition 3.1 states that incentive compatible debt contracts need to be designed such that an entrepreneur's income after tax and subsidy payments needs to be greater

$\bar{l}(a_t, o_{t-1}; \lambda)$ defined as the highest level of l that satisfies the collateral constraint

$$(1+r) w_t l + (1+r) \kappa_t \leq \lambda a_t$$

It is readily shown that $\bar{l}(a_t, o_{t-1}; \lambda)$ is strictly increasing in a_t and λ but strictly decreasing in κ_t. Note that the main difference between the two types of credit constraints is that the previous one is increasing in $z, (1-\tau), \xi$ while this one depends only an entrepreneur's wealth and costs of business operation. For reasonable parametrization, this constraint is more slack for low-skill entrepreneurs but tighter for high-skill entrepreneurs. Henceforth, I will adopt the previous constraint but posit that my results would not be significantly affected by switching to this alternative specification.

than a fraction $1-\phi$ of business revenues net of taxes and subsidies. In other words, only a fraction ϕ of entrepreneurs' post-transfer revenues can be pledged as collateral and entrepreneurs have the option of defaulting on debt contract in which case they keep the unpledged share of their post-transfer revenues and regain full access to credit markets next period.

If an entrepreneur's credit constraint binds, the optimal unconstrained level of operation will be unattainable. For example, if credit markets are shut down completely ($\phi = 0$) then all wage payments and fixed costs have to be covered by an entrepreneur's wealth $(1+r)\,a$ net of tax payments $\tau z l^{\alpha}$ and subsidies ξ. If, on the other hand, credit markets are perfect ($\phi = 1$) then all entrepreneurs will operate at their optimal unconstrained level of labor utilization and the production decision will be independent of individuals' wealth level. More generally, it will be optimal to employ as many workers as the labor utilization constraint $\bar{l}\,(\cdot)$ permits, subject to staying below the unconstrained optimal level.

Definition 3.2. *The* constrained optimal labor input *is given by*

$$l_t^* \equiv l^*\left(a_t, z_t, \tau_t, \xi_t, o_{t-1}; \phi\right) = \min\left\{\bar{l}\left(a_t, z_t, \tau_t, \xi_t, o_{t-1}; \phi\right), l^u\left(z_t, \tau_t\right)\right\}$$

The next section analyzes the individual's problem recursively.

3.2.2 Recursive formulation

In the simple three-period economy, the individual's problem can be solved by backward induction.

Final period (t=2):

The *value of an individual in the last period* with state vector (a_2, z_2, o_1) is given by

$$v_2(a_2, z_2, o_1) = \max\left\{v_1^E(a_2, z_2, o_1), v_2^W(a_2)\right\}$$

where the *value of being an entrepreneur in the last period* is given by

$$\begin{aligned} v_2^E(a_2, z_2, o_1) &= \max_{c_2, l_2 \geq 0} u(c_2) \\ \text{s.t.} \quad c_2 &\leq z_2 l_2^{\alpha} - (1+r) w_2 l_2 + (1+r) a_1 - (1+r) \kappa(o_1) \\ l_2 &\leq \bar{l}(a_2, z_2, o_1; \phi) \end{aligned}$$

and, analogously, the *value of being a worker in the last period* is given by

$$\begin{aligned} v_2^W(a_1) &= \max_{c_2 \geq 0} \{u(c_2)\} \\ \text{s.t.} \quad c_2 &\leq w_2 + (1+r) a_2 \end{aligned}$$

Since the economy ends after this period, individuals decide on their occupation myopically and consume all their wealth.

Initial and middle periods (t=0,1):

In the middle period only, government may levy an idiosyncratic tax-subsidy schedule, so that $(\tau_0, \xi_0) = (0, 0)$ and $(\tau_1, \xi_1) = (\tau, \xi)$. For now, assume individuals have rational expectations about government intervention in the middle period. The *value of an individual in periods* $t = 0, 1$ with $(a_t, z_t, \tau, \xi, o_{t-1})$ is then given by

$$v_t(a_t, z_t, \tau, \xi, o_{t-1}) = \max\left\{v_t^E(a_t, z_t, \tau, \xi, o_{t-1}), v_t^W(a_t)\right\}$$

where the *value of being an entrepreneur in periods* $t = 0, 1$ is given by

$$\begin{aligned} v_t^E(a_t, z_t, \tau, \xi, o_{t-1}) &= \max_{c_t, a_{t+1}, l_t \geq 0} \{u(c_t) + \beta \mathbb{E}[v_{t+1}(a_{t+1}, z_{t+1}, \tau, \xi, E) | z_t]\} \\ \text{s.t.} \quad c_t + a_{t+1} &\leq (1-\tau) z_t l_t^{\alpha} - (1+r) w_t l_t + (1+r) a_t - (1+r) \kappa(o_{t-1}) + \xi \\ l_t &\leq \bar{l}(a_t, z_t, \tau, \xi, o_{t-1}; \phi) \end{aligned}$$

and, analogously, the *value of being a worker in periods* $t = 0, 1$ is given by

$$\begin{aligned} v_t^W(a_t) &= \max_{c_t, a_{t+1} \geq 0} \{u(c_t) + \beta \mathbb{E}[v_{t+1}(a_{t+1}, z_{t+1}, W) | z_t]\} \\ \text{s.t.} \quad c_t + a_{t+1} &\leq w_t + (1+r) a_t \end{aligned}$$

Note that all continuation values depend on today's occupational choice because of the sunk cost of starting a business.

3.2.3 Barriers to entry and premature exit

Much recent work has shown that credit constraints can generate sizable extensive margin distortions.[5] When credit markets are imperfect, distortions are salient for industries with large fixed costs and in particular if costs are front-loaded as is the case with a one-off entry cost. In my model, I confirm the presence of amplifying interaction effects between financial frictions and start-up costs on extensive margin decisions. In general, occupational choice is distorted in two distinct ways:

1. *Barriers to entry.* With imperfect credit markets, high-productivity workers who would like to become entrepreneurs may be unable to start a business because they are unable to finance the start-up cost when credit constrained. This is the extensive margin distortion examined in Buera (2008), among others.

[5] See for example Buera, Kaboski and Shin (2011) and Midrigan and Xu (2014).

2. *Premature exit.* Low-productivity firms who would like to remain active because they expect a quick recovery and want to avoid paying another entry cost in the future may be forced to exit because they are unable to finance the operating cost under binding credit constraints.

While I acknowledge the importance of barriers to entry, the focus of my investigation lies on the role of premature exit in entrepreneurial dynamics. Towards the end of this section, I show that premature exit may constitute a sizable share of all extensive margin misallocations, in particular if start-up costs are high and upward mobility in productivity draws is probable.[6] In order to better understand firms' extensive margin decisions, I note that both previous workers and previous entrepreneurs follow simple cutoff policies that determine their optimal occupational choice. I proceed to analyze the determinants of individuals' extensive margin decisions.

First, note that if Assumption 3.3 holds then currently unproductive entrepreneurs will always exit for $\kappa^s = 0$. This need no longer be true for $\kappa^s > 0$, even if credit markets are imperfect. This can be seen by noting that $v_t^E(a_t, z_t, W) \leq v_t^E(a_t, z_t, E)$ with strict inequality whenever $\kappa^s > 0$ since new entrepreneurs need to pay a start-up cost. Then if individuals put high enough probability on becoming entrepreneurs in the future, they will take into consideration the prospect of having to pay a new start-up cost upon re-entry. Currently unproductive entrepreneurs may thus want to remain in business because the start-up cost induces a positive option value of remaining active in anticipation of future recovery. Intuitively, if the option value of staying in business is high enough even in bad times, entrepreneurs are willing to tolerate currently low payoffs in anticipation of the future upside. Thus, the multi-period economy captures an important dynamic incentive problem that does not arise in a model of repeated static occupational choice. Said problem arises in the context of credit constraints which may force idle low-productivity entrepreneurs

[6] In the dynamic economy, I generalize these conditions to a joint restriction on γ, z_H and κ^s.

to exit in spite of the expected net present benefits to staying. The key lesson is that individuals' occupational choice is no longer static for $\kappa^s > 0$ due to lagged occupational choice now being a state variable which affects the current period's cost of being an entrepreneur.

In order to formalize this mechanism, I describe individuals' optimal policy functions in the absence of government intervention so that $(\tau, \xi) = (0, 0)$. First, note that individuals with currently high productivity and those with a business already set up are more likely at the margin to become entrepreneurs in the current period.

Lemma 3.1. *Individuals at $t = 0, 1, 2$ adopt an optimal occupational choice policy which takes the form of a reservation wealth level $\underline{a}_t(z_t, o_{t-1}; \phi, \kappa^s)$. This wealth threshold is a possibly unbounded function of their productivity and their previous occupation above which individuals decide to run a firm and below which they decide to work for a wage, and satisfies the following properties:*

1. *The wealth threshold is strictly greater for agents with high productivity,*

$$\underline{a}_t(z_H, o_{t-1}; \phi, \kappa^s) < \underline{a}_t(z_L, o_{t-1}; \phi, \kappa^s)$$

2. *The wealth threshold is weakly greater for previous workers,*

$$\underline{a}_t(z_t, E; \phi, \kappa^s) \leq \underline{a}_t(z_t, W; \phi, \kappa^s)$$

Second, it can be seen that credit constraints distort individuals' extensive margins in a way that makes entrepreneurship less attractive in general. This gives rise to poverty traps[7] which may prevent poor but productive individuals from becoming entrepreneurs.

[7]See Buera (2008) for an analysis of this problem in continuous time but partial equilibrium.

Lemma 3.2. *Absent general equilibrium effects, individuals' wealth threshold is weakly decreasing in the quality of credit markets,*

$$\frac{\partial \underline{a}_t \left(z_t, o_{t-1}; \phi, \kappa^s\right)}{\partial \phi} \leq 0$$

Third, entry costs constitute the second key element in my model leading to extensive margin distortions. With positive start-up costs, $\kappa^s > 0$, only the most productive entrepreneurs will find it worthwhile to start a business. Furthermore, currently active entrepreneurs solve a dynamic, forward-looking stopping time problem with optimal policies depending on both the period payoff and the option value of staying in business in anticipation of future profits.[8] Thus, while positive start-up costs on the one hand deter entry of relatively unproductive entrepreneurs, on the other hand they make it more attractive to remain in business, even for currently unproductive entrepreneurs.

Lemma 3.3. *Absent general equilibrium effects, start-up costs have differential effects on previous workers and entrepreneurs at $t = 0, 1$:*

1. *Previous entrepreneurs' wealth threshold is weakly decreasing in κ^s and strictly decreasing over an initial range,*

$$\frac{\partial \underline{a}_t \left(z_t, E; \phi, \kappa^s\right)}{\partial \kappa^s} \leq 0$$

and there exists some $\bar{\kappa}^s > 0$ such that

$$\left.\frac{\partial \underline{a}_t \left(z_t, E; \phi, \kappa^s\right)}{\partial \kappa^s}\right|_{\kappa^s \in (0, \bar{\kappa}^s)} < 0$$

[8] See Appendix C.1 for a description of the option value of remaining in business as a function of an individual's state variables and fundamental parameters in the economy.

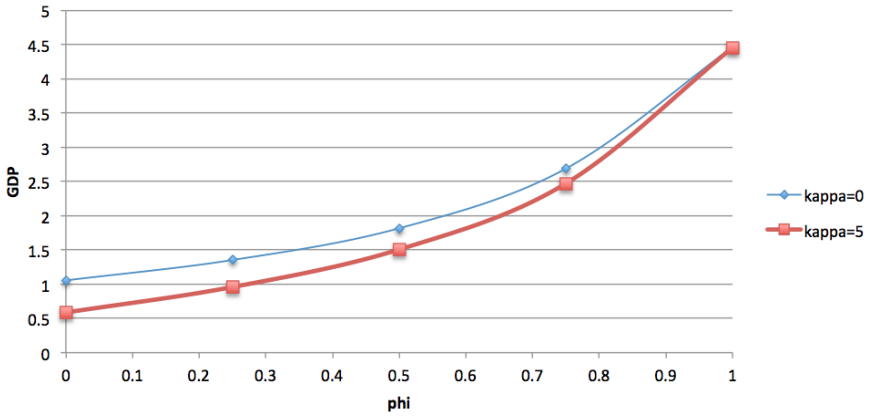
2. *Previous workers' wealth threshold is weakly increasing in κ^s and strictly increasing over an initial range for the most productive individuals,*

$$\frac{\partial \underline{a}_t\left(z_t, W; \phi, \kappa^s\right)}{\partial \kappa^s} \geq 0$$

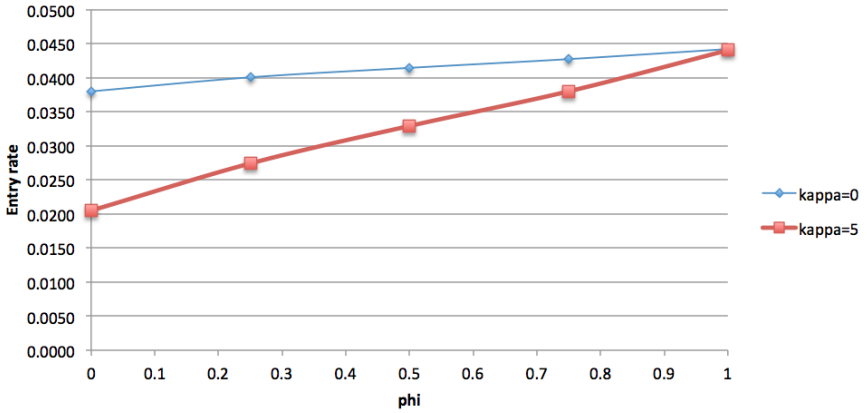
and there exists some $\bar{\bar{\kappa}}^s > 0$ such that

$$\left.\frac{\partial \underline{a}_t\left(z_H, W; \phi, \kappa^s\right)}{\partial \kappa^s}\right|_{\kappa^s \in (0, \bar{\bar{\kappa}}^s)} > 0$$

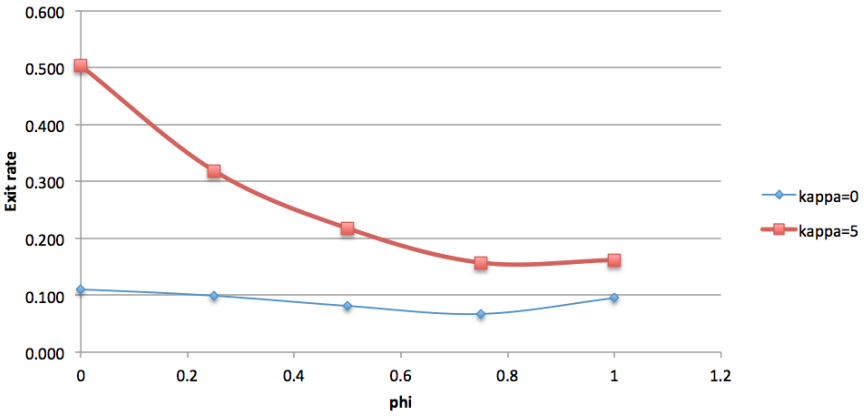
It follows from Lemmas 3.1–3.3 that in the presence of credit market frictions extensive margin distortions for existing firms can be amplified when start-up costs are high. Note, however, that in my three-period model, there is inconclusive theoretical evidence on the joint effect of start-up costs and credit constraints on the number of firm liquidations relative to the unconstrained economy with no start-up costs. This is due to the ambiguous effect that the two forces combined have on extensive margin decisions in the initial period. Figure 3.1 shows the effects on key economic indicators of varying credit market quality for zero and positive entry costs. Interaction effects between ϕ and κ^s can be sizable in simulations. For example, GDP increases by a factor of 4.5 as credit market quality increases from $\phi = 0$ to $\phi = 1$ without entry costs; in the presence of large entry costs, say $\kappa = 5w$, GDP lies uniformly below its previous level and differs by a factor of 9 as credit market quality is varied. Similar comparative statics effects are observed for firm entry and exit rates.



(a) GDP vs. credit market quality (ϕ) at $t = 1$



(b) Entry rate vs. credit market quality (ϕ) at $t = 1$



(c) Exit rate vs. credit market quality (ϕ) at $t = 1$

Figure 3.1: Model simulations for varying credit market quality and entry costs

Before stating my last theoretical result, I restrict attention to the case of high upward mobility in entrepreneurs' productivity levels so that even currently unpro-

ductive individuals have good prospects of finding themselves with high productivity in the future.

Assumption 3.4. *The persistence parameter γ is small relative to z_H.*[9]

Under this assumption, some entrepreneurs, when unconstrained, prefer to remain in business even if current profits are below the outside option of working for a fixed wage in the labor market.

Proposition 3.2. *Under Assumption 3.4, the occupational choice problem is no longer static. In particular, for previous entrepreneurs a wage offer in excess of their potential profits of running a business in the current period is no longer sufficient to induce job switches.*

Combining Lemmas 3.1–3.3 with Proposition 3.2, I conclude that for relatively low persistence of shocks γ and high entry costs κ^s, imperfect credit markets may induce premature firm liquidations of the kind I described in the beginning of this section. For parameterizations consistent with Assumption 3.4, premature firm liquidations may constitute a sizable share of all extensive margin distortions.

In the following section, I explore government intervention designed to mitigate premature firm liquidation.

3.2.4 Government policy

The purpose of this section is to describe the information structure and redistributive tools available to government. A welfare-based approach to such policy evaluation is detailed in Appendix C.1. For the remainder of this section, I assume that government pursues redistribution targeted at mitigating premature firm liquidation. Possible rationales for this sort of intervention are provided in the literature on soft budget constraints as well as the literature on inefficient project liquidation.

[9] More concretely, I require the expected value of tomorrow's productivity conditional on today's state being low productivity, $\mathbb{E}(z_{t+1}|z_t = z_L)$, to be high.

The general spirit of my investigation is to set up the economic environment in favor of finding a positive effect of government intervention of the form I describe. I then look at remaining sources of inefficiencies due to this policy and judge which of these are the most significant. Thus, this section explores the economic consequences of subsidies to the least productive entrepreneurs when government is restricted to using a limited set of redistributive tools, namely a linear revenue tax and targeted transfers.[10]

I start off by defining what information pertaining to particular firms' state vector as well as previous and current actions is public knowledge. For $\mathcal{F}_t \equiv (a_t, z_t, o_{t-1}, o_t, l_t, y_t, c_t, a_{t+1})$, let $\mathcal{F}^t \equiv \cup_{\tau \leq t} \mathcal{F}_\tau$ denote an individual's entire history of states and choice variables, consisting of the cross-sectional union of an individual's period productivity and actions including initial conditions (a_0, z_0, o_{-1}). In general, government policy at time t may depend on any subset of $\mathcal{F}^t$. I want to restrict the information structure as follows:

Assumption 3.5. *Only elements of* $\mathbf{x}_t \equiv (z_t, o_{t-1}, o_t, y_t) \subseteq \mathcal{F}_t$ *are public knowledge.*

In Section 3.4.4, I relax the stark assumption that firms' current productivity is observable so that government will be restricted to offering incentive compatible tax-subsidy schedules as functions of the observable firm characteristics, say $\hat{\mathbf{x}}_t = (o_{t-1}, o_t, l_t)$. For now, government has the ability to offer different tax-subsidy schedules (τ, ξ) to entrepreneurs with distinct $\mathbf{x}_t$.[11]

Next, I restrict the set of redistributive tools available to government. I assume that government may intervene only in the middle period ($t = 1$) and all intervention must be in the form of a linear revenue tax, $\tau : \mathbf{x}_1 \to [0, 1]$, and targeted transfers, $\xi : \mathbf{x}_1 \to \mathbb{R}_+$, to entrepreneurs who decide to stay in business between periods 1 and

[10] In the absence of non-distortionary tax revenue and perfect targeting, it seems natural to think of such policies as having an indirect effect on the intensive margin of current entrepreneurs as well as the entry decisions of previous wage workers.

[11] On the flip side, this still means that entrepreneurs who share the same $\mathbf{x}_1$ are indistinguishable for policy purposes.

2.[12] By focusing on redistribution among entrepreneurs who decide to remain active through linear revenue taxes and targeted transfers, I am effectively examining policy instruments of the form $\tau : \tilde{\mathbf{x}}_1 \to [0,1]$ and $\xi : \tilde{\mathbf{x}}_1 \to \mathbb{R}_+$, where $\tilde{\mathbf{x}}_t \equiv (z_t, E, y_t)$. The latter restriction is important in the light of my focus on mitigating premature exits rather than barriers to entry. Note that I restrict attention to this class of policies by assumption and not by an optimality argument.

Given a policy choice (τ, ξ) and individual's optimal policies, government is required to balance its budget period-by-period. That is, with the one-off intervention in the finite-period model, all subsidy expenditures need to be financed from tax revenues in that period. In general, government needs to raise a fraction $\psi \in [0,1]$ of public funds through distortionary revenue taxes and but can raise the remaining $(1-\psi)$ fraction through non-distortionary lump-sum taxes. Henceforth, I will consider the case of $\psi = 1$. Varying this parameter systematically can be used to isolate distortions to the economy from government intervention.

3.3 Infinite-horizon economy

This section translates the insights from the three-period economy into an infinite-horizon model and focuses on the stationary equilibrium in this environment. In order to connect my findings to the finite-time environment of Section 3.2, I adopt a binary productivity space $\{z_L, z_H\}$. Suppose that given current productivity $z \in \{z_L, z_H\}$, an individual's productivity in the subsequent period, z', is equal to z with probability γ and drawn from a stationary distribution $\mu_z(z')$ with probability $1-\gamma$.[13] As previously, $\gamma \in [0,1]$ can be interpreted as a persistence parameter governing

[12] With a more restricted information structure, in Section 3.4.4, I abstract from revenue taxes and targeted transfers. The efficiency of the kind of redistribution I describe here is adversely affected when government can no longer discriminate based on individuals' productivity levels.

[13] The key insights of this section remain when the model is extended to comprise a richer productivity state space with continuous stationary distribution $\mu_z(z')$. By letting the productivity distribution be Pareto with tail parameter η_z, I can readily connect my model to that used in Buera, Kaboski and Shin (2011) or Buera, Moll and Shin (2013).

the stochastic evolution of entrepreneurial talent. By focusing on a general equilibrium with stationary joint distribution over wealth, productivities, and previous occupational choices, $\mu(a, z, o_{-1})$, a constant wage level $w_t = w$ can be deduced from individuals' optimal policy functions in conjunction with the labor market clearing condition which I describe below. Assume the environment is a small open economy so that the interest rate $r_t = r$ is given exogenously by a country with perfect credit markets. An individual at time 0 has preferences over consumption streams $\{c_t\}_{t\geq 0}$ given by

$$U\left(\{c_t\}_{t\geq 0}\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t is consumption at time t, period utility is of the CRRA form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

and the elasticity of intertemporal substitution parameter is conveniently set to $\sigma = 1$ which yields logarithmic period utility. The production technology and credit constraints are as before and a time-invariant government policy (τ, ξ) which subsidizes low-productivity entrepreneur who remain in business and taxes the revenues of all remaining entrepreneurs is imposed in every period. In this dynamic setting, the government budget need no longer balance period-by-period as the public sector may also borrow funds at interest rate r. Then an individual with state $(a, z, \tau, \xi, o_{-1})$ at the beginning of a period solves the following program:

$$v(a, z, \tau, \xi, o_{-1}) = \max\left\{v^E(a, z, \tau, \xi, o_{-1}), v^W(a)\right\} \tag{3.2}$$

where the *value of being an entrepreneur* is given by

$$\begin{aligned} v^E(a,z,\tau,\xi,o_{-1}) &= \max_{c,a',l\geq 0}\left\{u(c_t)+\beta\mathbb{E}\left[v^E(a',z',\tau,\xi,E)\,|z\right]\right\} \\ \text{s.t.}\quad c+a' &\leq (1-\tau)\,zl^{\alpha}-(1+r)\,wl+(1+r)\,a-(1+r)\,\kappa+\xi \\ l &\leq \bar{l}(a,z,\tau,\xi,o_{-1};\phi) \end{aligned} \tag{3.3}$$

and the *value of being a worker* is given by

$$\begin{aligned} v^W(a) &= \max_{c,a'\geq 0}\left\{u(c)+\beta\mathbb{E}\left[v(a',z',\tau,\xi,W)\,|z\right]\right\} \\ \text{s.t.}\quad c+a' &\leq w+(1+r)\,a \end{aligned} \tag{3.4}$$

Note that all continuation values depend on today's occupational choice because of the sunk cost of starting a business.

3.3.1 Optimal policy functions

Current entrepreneurs solve a dynamic, forward-looking stopping time problem with optimal policies depending on both current profits and the option value of staying in business in anticipation of future business operation. The insight from Section 3.2.3, namely that previous entrepreneurs exit if and only if their wealth level falls below a threshold level $\underline{a}_t(z_t,E;\phi,\kappa^s)$, translates directly into the infinite-horizon environment. Firms effectively face an optimal stopping time problem in which they decide to liquidate their business at the first instance when the discounted option value of staying falls below a certain level. If an entrepreneur with previously high productivity z_H is hit by a bad productivity shock, immediate exit will occur if and only if the entrepreneur's wealth is below that threshold level, that is if and only if $a_t<\underline{a}_t(z_L,E;\phi,\kappa^s)$. On the other hand, wealthy but unproductive entrepreneurs will decide to remain in business in hopes of a quick recovery. Holding the economy's fun-

damental parameters constant, the wealth threshold depends only an entrepreneur's current productivity. Currently unproductive entrepreneurs will keep running down their assets until they cross the threshold value $\underline{a}_t(z_L, E; \phi, \kappa^s)$ or until they receive a good productivity shock—whichever occurs first. Two potential time paths of a representative entrepreneur who has been hit by a bad productivity shock at $t = 0$ are illustrated in Figure 3.2.

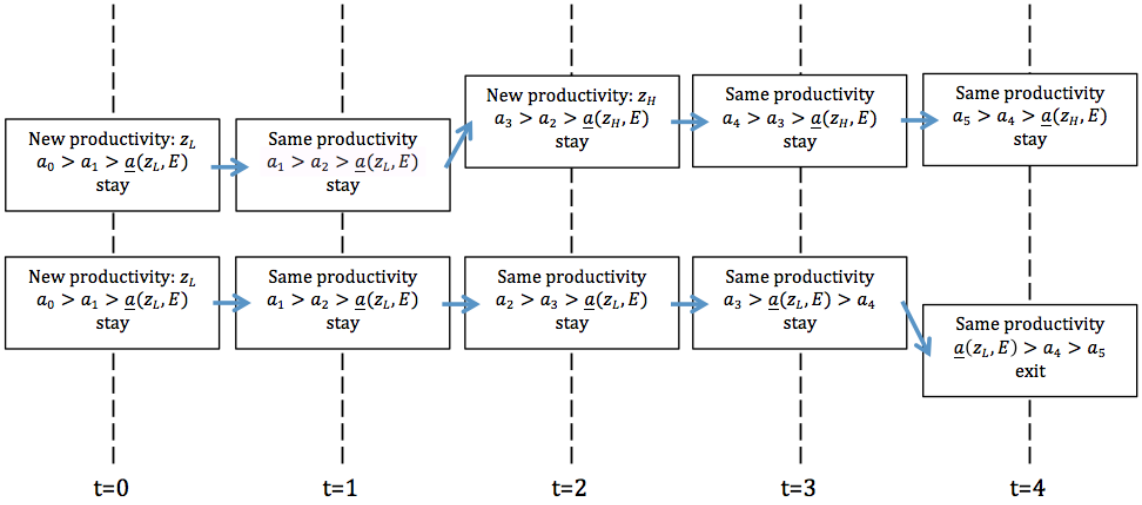
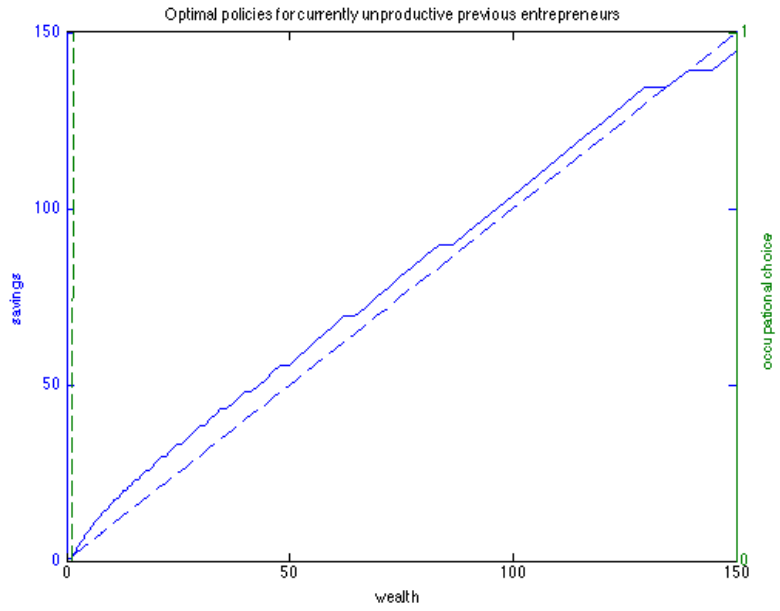
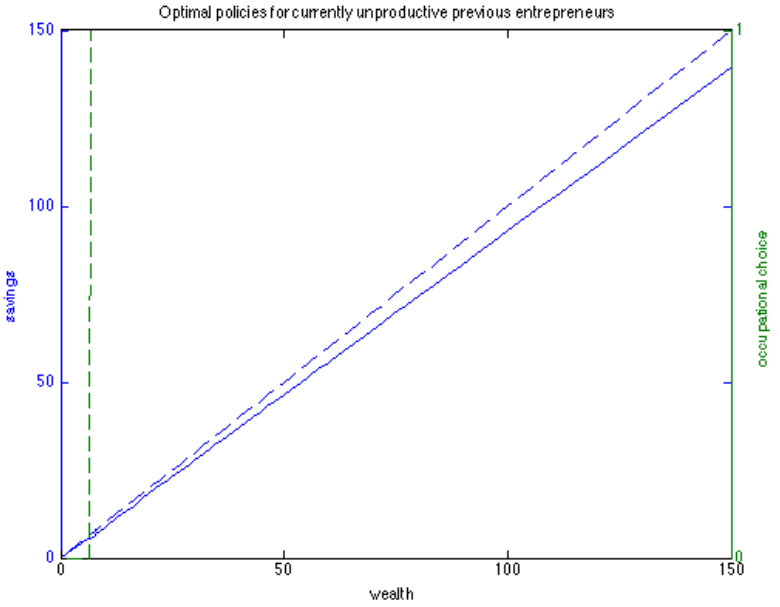


Figure 3.2: Schematic illustration of the optimal stopping time policy

Note that if low-productivity entrepreneurs' wealth falls below the threshold level then it is optimal for them to exit and work for a wage in the labor market. In contrast, high-productivity entrepreneurs accumulate assets until they reach their optimal wealth level which is a function of their productivity and the economy's fundamental parameters. These features of the optimal policy functions are evident in Figure 3.3 which shows the stationary optimal savings policy and occupational choice obtained in numerical simulations. The dashed blue diagonal line is defined as $a' = a$. The dashed green vertical line is the occupational choice cutoff threshold.



(a) Currently productive entrepreneurs



(b) Currently unproductive entrepreneurs

Figure 3.3: Savings and exit policies for currently productive and currently unproductive entrepreneurs

The optimal wealth level has two defining features: First, it allows for entrepreneurs to produce at the unconstrained optimal scale by alleviating credit

constraints sufficiently. Second, the entrepreneur will accumulate precautionary savings in line with Aiyagari (1994) in order to cushion future bad productivity shocks. Note that except for a small region around zero wealth, currently productive entrepreneurs accumulate wealth until the point where their savings policy crosses the diagonal. Currently unproductive entrepreneurs, on the other hand, dis-save and wait for a good productivity shock until their wealth falls reaches a critical threshold below which they decide to exit and become workers.[14]

3.3.2 Stationary equilibrium

For notational convenience, denote an individual's state $\mathbf{x} \equiv (a, z, \tau, \xi, o_{-1})$ and let $y(\mathbf{x}) \equiv z(l^*(\mathbf{x}; \phi))^\alpha$. Then a stationary competitive equilibrium is defined in the usual way:

Definition 3.3. *A* stationary competitive equilibrium *consists of a set of prices* w, r *and rental limits* $\bar{l}(\mathbf{x}; \phi)$*; government policy functions* $\tau(z, o_{-1}, o, y)$*,* $\xi(z, o_{-1}, o)$ *and debt level* B*; individual policy functions* $c(\mathbf{x})$*,* $a'(\mathbf{x})$*,* $l(\mathbf{x})$ *for entrepreneurs,* $c(a), a'(a)$ *for workers, and an occupational choice policy function* $o(\mathbf{x})$*; value functions* $v(\mathbf{x})$*,* $v^E(\mathbf{x})$*,* $v^W(a)$*; and an invariant distribution over wealth, productivities, government policies, and previous occupational choice* $\mu(\mathbf{x})$ *such that:*

1. *(Consumer optimization) Given prices, rental limits, and government policy functions, the value functions solve individuals' problems (3.2)–(3.4) and the policy functions are optimal;*

2. *(Contract enforceability) The set of incentive compatible rental contracts is bounded from above by* $\bar{l}(\mathbf{x}; \phi)$ *which is the supremum of all labor utilization levels that satisfy condition (3.1);*

[14] The reader should recall that this kind of savings behavior and occupational choice are optimal for the parameter range I consider in this paper, in particular for transient bad shocks. As shocks become more persistent then entrepreneurs have less hope of quick recovery and thus exit the industry sooner than depicted above.

3. *(Market clearing) Labor and goods markets clear:*

$$\begin{aligned} \textit{(Labor market)} \quad \int_{\{o(\mathbf{x})=E\}} l(\mathbf{x})\, d\mu(\mathbf{x}) &= \int_{\{o(\mathbf{x})=W\}} d\mu(\mathbf{x}) \\ \textit{(Goods market)} \quad \int c(\mathbf{x})\, d\mu(\mathbf{x}) &= \int_{\{o(\mathbf{x})=E\}} \left[y(\mathbf{x}) - \kappa(o_{-1})\right] d\mu(\mathbf{x}) \end{aligned}$$

4. *(Government budget balance) Government's per-period budget surplus equals interest payments on public debt:*

$$rB = \int_{\{o(\mathbf{x})=E\}} \left[\tau(z, o_{-1}, o, y)\, y(\mathbf{x}) - \xi(z, o_{-1}, o)\right] d\mu(\mathbf{x})$$

5. *(Stationarity) The joint distribution of wealth, productivities, government policies, and previous occupational choice is invariant and described by the fixed point of the following equilibrium mapping:*

$$\begin{aligned} \mu(\mathbf{x}) &= \gamma \int_{\{(\tilde{a},\tilde{z},\tilde{o}_{-1}) | a'(\tilde{a},\tilde{z},\tau,\xi,\tilde{o}_{-1}) \leq a, \tilde{z} \leq z, o(\tilde{a},\tilde{z},\tau,\xi,\tilde{o}_{-1})\}} d\mu(\tilde{a},\tilde{z},\tau,\xi,\tilde{o}_{-1}) \\ &\quad + (1-\gamma)\,\mu_z(z) \int_{\{(\tilde{a},\tilde{o}_{-1}) | a'(\tilde{a},z,\tau,\xi,\tilde{o}_{-1}) \leq a, o(\tilde{a},z,\tau,\xi,\tilde{o}_{-1})\}} d\mu(\tilde{a},z,\tau,\xi,\tilde{o}_{-1}) \end{aligned}$$

3.3.3 Simulations

In this section, I simulate the dynamic economy in stationary equilibrium. For ease of interpretation, I calibrate some of the fundamental parameters of my model as detailed in Table 3.1.

While I have not undertaken a more rigorous calibration of my model, I pick parameter ranges that seem to match key moments from the literature reasonably well. A more unusual choice is the calibration of the persistence parameter γ which I deliberately set below what has been used in parts of the previous literature, calibrating $\gamma = 0.89$ to match an annual establishment exit rate of 0.10. When I adapt

Parameter	Value	Comment
α	0.79	Scale of production from Buera, Moll and Shin (2013)
β	0.92	To match interest rate of 0.04
γ	0.55	Equilibrium entry/exit rate of 0.05
$\mu_z(z_H)$	0.147	Share of productive entrepreneurs of 0.15
σ	1.5	Buera, Kaboski and Shin (2011)
r	0.04	Standard annual net interest rate
κ^o	2	Corresponds to 2-4 times equilibrium wage

Table 3.1: Fundamental parameters

such high persistence in productivities, entrepreneurs' optimal policy functions will be such that entrepreneurs exit immediately when hit by a bad shock so that the option value problem becomes degenerate.

Table 3.2 summarizes steady-state outcomes of the economy with varying degrees of credit market quality and size of fixed costs.

(ϕ, κ^s)	$(1.00, 0)$	$(0.17, 0)$	$(0.17, 10w)$
Wage	0.640	0.600	0.531
Firm entry/exit rate	0.057	0.042	0.011
Entrepreneurs	0.147	0.110	0.028
GDP	2.245	1.682	0.859
TFP	1.003	0.944	0.707
Welfare	14.250	14.248	3.762

Table 3.2: Simulation results with and without financial frictions and entry cost

Both credit market quality and the size of barriers to entry seem to have significant effects on key economic indicators, notably aggregate output and industry TFP.[15] Wages are depressed by over 6% when decreasing ϕ from 1 to 0.17 and by a further 12% when large barriers to entry are in place. Firm entry and exit rates, and the number of active entrepreneurs in the economy all decrease significantly in stationary

[15]I compute aggregate TFP as the residual of an aggregate production function $Y \equiv \int y(\mathbf{x})\, d\mu(\mathbf{x}) = AL^{\alpha}$, where A is total factor productivity and L is total labor utilization in the economy. Then $A = YL^{-\alpha}$.

equilibrium when credit market quality goes down but an even more significant drop is observed when on top of imperfect credit markets there is a high cost of entry. Credit market imperfections and start-up costs combined can account for differences in TFP of roughly 30% across two otherwise identical economies. Following the findings of Moll (2014), I expect these steady-state differences to be even more pronounced for stochastic productivity processes with lower persistence than what I consider in the above simulations.

3.3.4 Dynamic soft budget constraints

This section quantifies the economic long-run effects of government intervention in the form of soft budget constraints, that is subsidies to the least productive entrepreneurs. As a benchmark, I first consider the economy with $\phi = 0.17$ and $\kappa^s = 10w$ without government intervention and then assess how steady state outcomes change for various levels of subsidies to low-productivity entrepreneurs.[16]

In Table 3.3, I show how the key economic indicators vary for different levels of government subsidies.

(ξ, τ)	$(0, 0)$	$(0.25\kappa^o, 0.006)$	$(0.50\kappa^o, 0.012)$	$(0.75\kappa^o, 0.025)$
Wage	0.531	0.527	0.524	0.523
Firm entry/exit rate	0.011	0.009	0.009	0.008
Entrepreneurs	0.028	0.028	0.028	0.028
GDP	0.859	0.861	0.862	0.862
TFP	0.707	0.705	0.702	0.693
Welfare	3.762	3.707	3.652	3.592

Table 3.3: Simulation results with and without government intervention

[16]Note that for subsidies in excess of $\xi = \kappa^o$ my simulated economy approaches a "bad" equilibrium with excessive entrepreneurship among low-productivity individuals and unrealistically high tax rates to finance those tax expenditures. Thus, I restrict my attention to a range of reasonably small subsidy levels.

My simulation results show that as the subsidy to low-productivity entrepreneurs increases from zero, the number of firm exits in steady-state declines significantly. This is intuitively appealing as net positive transfers go to the least productive individuals in the economy who are most likely to exit without government intervention. This change goes hand in hand with an increase of active entrepreneurs in steady-state though both of these effects are smaller in magnitude. Finally, aggregate output in the economy is roughly invariant to policy intervention of the considered magnitude. Consistent with these observations, aggregate TFP in the economy declines at what seems to be an increasing rate as the subsidy level increases. This captures the stylized fact that productivity losses can be large when distortions are positively correlated with individual productivity, as noted in Restuccia and Rogerson (2008). Lastly, welfare effects of even low levels of government intervention can be sizable, with my model generating a loss of about 2% in steady-state welfare when comparing the lowest and the highest levels of subsidies.

To conclude, I note that while the effects of government intervention in the form of soft budget constraints can be partly offsetting—as in aggregate output—even small subsidies can induce sizable distortions to aggregate productivity and in particular to welfare compared to the benchmark economy.

3.4 What determines the efficiency of soft budget constraints?

The insights from all previous sections depend on industry-specific assumptions of the stochastic process, size of fixed costs, the nature and timing of the mechanism by which potential entrepreneurs learn about their ability, and the observability of firms' characteristics. This section analyzes how my quantitative analysis changes as each of these factors is varied. As different industries are associated with different

sets of characteristics along these dimensions, I interpret structural changes that are consistent with such industry differentiation. The key industry distinction I have in mind is between the manufacturing industry and the services industry.

My aim is to analyze how the economy's response to government intervention in the form of soft budget constraints differs along these dimensions in order to infer under which circumstances such intervention is more or less efficient. An alternative interpretation of this section's results is that they provide robustness checks to my previous findings on steady-state outcomes in Section 3.3.3 of the paper.

3.4.1 Stochastic process

In Section 3.2.3, I show that the persistence of shocks plays an important role in shaping individuals' extensive margin decisions through the option value mechanism. Intuitively, for relatively low persistence in productivity shocks, individuals want to remain in business through temporary crises while for relatively high persistence in productivities entrepreneurs who have been hit by a bad shock optimally exit the industry. In this environment, the efficiency of corporate bailouts in the form of subsidies to the least productive entrepreneurs depends crucially on this persistence parameter. On the one hand, if shocks are persistent enough then any subsidies just represent distortionary transfers which tend to decrease the overall productivity in the economy. On the other hand, if shocks are transient then subsidies may serve their purpose to mitigate premature firm exit, thus potentially being less distortionary overall. Table 3.4 shows steady-state outcomes for varying degrees of persistence in productivities while holding all other parameters constant at their previous level, in particular $\kappa^s = 10w$ and $\xi = 0.25\kappa^o$.

I interpret these results as evidence in favor of my hypothesis that the persistence of shocks is negatively related to aggregate productivity in the economy with fixed government intervention. Somewhat surprisingly, TFP remains approximately

	$\gamma = 0.45$	$\gamma = 0.55$	$\gamma = 0.65$
Wage	0.5182	0.5271	0.5374
Firm entry/exit rate	0.0131	0.0092	0.0083
Entrepreneurs	0.0278	0.0280	0.0278
GDP	0.9138	0.8606	0.8034
TFP	0.7071	0.7051	0.7071
Welfare	3.7466	3.7068	3.2901

Table 3.4: Simulation results with varying degrees of persistence of productivity shocks

constant over the same range of persistence parameters. This conundrum is best understood by considering the two mutually offsetting effects that increased persistence in productivities has on steady state outcomes: On the one hand, Moll (2014) among others shows that higher persistence in productivity shocks leads to less distortions in the steady-state because there is more room for entrepreneurs self-financing their ways out of financial constraints. On the other hand, the increased persistence in shocks renders soft budget constraint policies more inefficient as premature exit becomes a less salient distortion the higher the persistence parameter. In combination, these results point to substantial steady-state distortions to aggregate productivity and thus also to output in steady-state.

Finally, note that if productivity transitions are allowed to be asymmetric, so that $\mathbb{P}(z_{t+1} = z_L | z_t = z_L) \neq \mathbb{P}(z_{t+1} = z_H | z_t = z_H)$, then the above discussion generalizes to the conclusion that higher upward mobility for low-productivity entrepreneurs, as measured by $\mathbb{P}(z_{t+1} = z_H | z_t = z_L)$, leads to a larger role for premature firm exits and thus also for government intervention in the form of soft budget constraints.

3.4.2 Size of fixed costs

The second key determinant of the option value of staying in business for a currently unproductive previous entrepreneur is the entry cost. For relatively large barriers to

entry, the option value of remaining in business is higher as future re-entry will be more costly. In an efficient economy, this should lead to reduced firm exit rates in equilibrium. In the presence of financial frictions, however, firm exit rates may be inefficiently high, thus potentially opening up a role for welfare-improving government intervention. Table 3.5 summarizes the steady-state outcomes for varying levels of barriers to entry. Note that for small entry costs, TFP and thus also GDP increase drastically, so I focus on increases in the entry cost.

	$\kappa^s = 10w$	$\kappa^s = 12.5w$	$\kappa^s = 15w$
Wage	0.527	0.511	0.496
Firm entry/exit rate	0.009	0.011	0.011
Entrepreneurs	0.028	0.028	0.028
GDP	0.861	0.972	1.084
TFP	0.705	0.707	0.707
Welfare	3.707	3.409	3.093

Table 3.5: Simulation results with varying levels of entry cost

Notably, aggregate output is increasing in the size of entry costs can and this can likely be attributed to the general equilibrium effects on the wage level which declines by almost 6% over the range of entry costs considered. The same effect, however, leaves most of the economy's agents worse off as over 97% of all individuals work for a wage in the labor market. This helps explain the stark decline in welfare as entry cost increases.

3.4.3 Nature and timing of learning mechanism

Learning about productivities in my model is costless and free of noise. In contrast, Hopenhayn and Rogerson (1993) and Restuccia and Rogerson (2008) describe an environment, in which potential entrants need to pay a fixed cost before learning about their productivity and subsequently decide on optimal intensive and extensive

margins. The distinction is important for firm entry dynamics and potentially affects my analysis in terms of selection effects which I assumed arise in the context of less productive individuals selecting into entrepreneurship when budget constraints are soft. In this sense, my model's learning structure exacerbates potential efficiency losses due to selection effects. While this may help explain the productivity and welfare gap that arises through government intervention, as described in Table 3.3, it should be noted that output is above the undistorted level even with negative selection effects so it can be expected to increase even further if this distortion margin is eliminated.

3.4.4 Observability of firm characteristics

In this section, I discuss the possibility that firm productivities are unobservable. In this case, an incentive-compatible second best allocation can implemented as follows: government offers two tax-subsidy bundles based only on observable firm characteristics, say $\hat{\mathbf{x}}_t = (o_{t-1}, o_t, l_t)$; one bundle consisting of $\tau^H(E, E, l_t) > 0$ and $\xi^H(E, E, l_t) = 0$; and another bundle consisting of $\tau^L(E, E, l_t) > \tau^H(E, E, l_t)$ and $\xi^H(E, E, l_t) > 0$. Intuitively, the two bundles $\left(\tau^H, \xi^H\right)$ and $\left(\tau^L, \xi^L\right)$ are picked such that the second bundle is attractive only to low-productivity entrepreneurs, i.e. the optimal policy follows the traditional mechanism design logic that distortions to the low type relax the incentive compatibility constraint of the high type. While the outcome implemented by such separating contracts is necessarily less efficient than under observable firm productivities, firms can be expected to be separated more efficiently for greater productivity differences $(z_H - z_L)$ as well as for more persistent productivity shocks. To see the latter, note that individuals are more reluctant to remain active with low productivity in the economy with more persistent shocks so that the incentive-compatible contract can separate types more easily. Note that

when labor input is observable, size-dependent policies[17] may emerge as a natural separation mechanism.

3.5 Conclusion

In this paper, I showed that financial frictions in combination with entry costs can create significant extensive margin distortions. Furthermore, government intervention in the form of soft budget constraints—subsidies to the least productive entrepreneurs—can have large effects on aggregate productivity and welfare and the magnitude of the distortions depends crucially on the persistence of the stochastic process underlying entrepreneurs' productivity evolution.

First, I analyze the key comparative statics within a simple three-period model. Next, I formulate an infinite-horizon version of the model and analyze steady-state outcomes under various levels of financial frictions, sizes of barriers to entry, and degrees of government intervention. In a calibrated version of the model, I confirm the presence of large interaction effects between imperfect credit markets and entry costs. I demonstrate that the adverse effects on TFP driven by negative firm selection is quantitatively significant. I find that the persistence of shocks to entrepreneurs' productivities is a key determinant of long-run distortions to the economy due to soft budget constraints. Specifically, I show that distortions due to transfers to the least productive entrepreneurs are increasing in the persistence of productivity shocks.

This result is consistent with the empirical observation of large inefficiencies that were documented in the Japanese economy due to "zombie lending" to unprofitable construction and real estate companies in the early 1990s. Furthermore, my result provides a critical perspective on the efficiency of industrial bailout policies. If recoveries are sluggish then such policies may generate large aggregate distortions.

[17] See also Guner2008a.

Chapter 4

Paternalism vs Redistribution: Designing Retirement Savings Policies with Behavioral Agents

4.1 Introduction

With a budget of more than $700 billion dollars in 2014, the social security survivors and old-age benefits program is the largest U.S. federal government social policy. The public finance literature has suggested this program is a paternalistic policy that aims at helping individuals save for retirement Diamond (1977); Kotlikoff, Spivak and Summers (1982); Feldstein (1985). The large body of evidence for present bias and heterogeneity in present bias reinforces the importance of corrective retirement savings policies, while the rise in labor income inequality in the U.S. has brought redistributive policies to the center of the policy discussion in recent years.[1]An under

[1]See Tanaka, Camerer and Nguyen (2010), Montiel Olea and Strzalecki (2014), Augenblick, Niederle and Sprenger (2015), Jones and Mahajan (2015) and Beshears et al. (2015) for recent findings both on the presence of behavioral biases and its considerable heterogeneity. Piketty and Saez (2003) and Atkinson, Piketty and Saez (2011) document the increase in labor earnings and wealth inequality in the United States.

appreciated fact by both economists and policy makers is that savings policy and redistributive policy are likely to interact in non-trivial ways. For example, a policy that correct savings might reduce incentives for working as behavioral workers cannot allocate consumption the way they think is best, making it harder for the government to redistribute. Thus understanding these interactions is of key importance to the public economics and behavioral economics literature.

In this paper, we develop a two-period model with unobservable heterogeneity in labor earnings ability and in the level of present bias to study the interaction between policies aiming at correcting savings choices and policies aiming at reducing economic inequality.[2] Our main finding concerns the shape of optimal retirement savings policies across different earnings levels when the government has a redistributive motive. At low earnings, policy forces individuals with the same ability but different present bias to save at the same rate. Thus optimal policy is paternalistic and leaves little flexibility for agents in savings decisions. At high earnings, optimal policy is less paternalistic and it offers individuals with the same ability but different present bias more flexibility on their savings choices. As a result, at low earnings optimal policy resembles forced savings through social security and at high earnings there are tailored subsidies or taxes on savings that resemble the availability of multiple individual retirement savings accounts, similar to 401(k)s or IRAs in the United States.

Three key forces in the model interact to generate our main finding. First, concern for present bias leads the government to help people increase their retirement savings. Second, the government uses all available tools to reward hard work so as to improve redistribution. There are two different ways to achieve this in our model: allowing individuals to keep some of their earnings and allowing individuals to indulge to their present bias. Third, as there is unobservable heterogeneity in present bias, the government wants to curtail flexibility on savings. By restricting the menu of savings

[2]In Appendix D.1 we extend our main results to a many period model with hyperbolic preferences.

options available to individuals, the government provides social insurance against the behavioral bias. This is true even if retirement savings are not at the first best level. Because of the interaction of those three forces, there is a push for paternalistic savings policies, whereby the government favors forced savings at low earnings, but at the same time a driving force for more flexibility so that high earnings individuals can indulge to their heterogeneous biases.

As usual in the public economics literature, there are different mechanisms the government can use to implement optimal policy, however we provide an implementation that uses three sets of policy tools. First, the government grants retirees a retirement benefit whose level depends on labor earnings during working life. In addition, young people cannot borrow against their retirement benefits. Second, there is both a regular savings account and multiple special retirement savings accounts. There is a cap on contributions to each special retirement savings account, and the cap depends on the labor earnings level of the person. Lastly, there are taxes on labor earnings and on savings returns. The labor income tax is non-linear in labor earnings and depends on which special retirement savings accounts the person contributes to. Taxes on the regular savings account are linear on savings but depend on earnings. Finally, taxes on special retirement savings accounts are lower than the tax on the regular savings account, and these taxes can also vary by the earnings level.

This implementation is particularly suitable for a comparison with current policy. The set of instruments we choose is very close to the set of instruments actually used by the U.S. government. For instance, retirement benefits directly translate into social security benefits and special retirement accounts resemble defined contribution plans such as a 401(k) account and individual retirement accounts (IRA). To the best of our knowledge, this is the first paper to highlight that a multitude of special retirement accounts can be used to implement optimal retirement savings policies when there is redistribution.

We find that an optimal retirement savings policy offers both social security old-age benefits and 401(k)-like retirement accounts to high earners, and only social security old-age benefits to low earnings individuals. Importantly, 401(k)-like retirement savings accounts are not available at low labor earnings. Indeed, low earnings individuals rely heavily on retirement benefits. High earnings individuals that have less present bias use retirement savings accounts, but high earnings individuals that have severe present bias rely more on social security.

In order to compare current U.S. policy and the normative model prescriptions, we calibrate the distribution of discounting preferences and labor earnings ability in the data. We use current policies and data on wealth at retirement from the Health and Retirement Survey to calibrate for the joint distribution of discount factors and labor earnings. We find a considerable level of heterogeneity in discount factors and a small positive correlation with earnings.[3] These findings are consistent with estimations of heterogeneity in discount factors in the literature and also with a high heterogeneity in present bias found in the behavioral economics literature.[4]

Simulation of optimal policy in our model is complicated because of the government's two-dimensional screening problem. It is well known that results for multidimensional screening problems are hard to prove in closed form, it is also true that the presence of thousands of incentive constraints makes numerical solutions equally difficult (Judd and Su (2006)). To this end, we develop a stable numerical solution algorithm to solve our model. Our algorithm searches for the smallest subset of incentive constraints that are relevant to the global solution of the problem.

[3]In Appendix D.3, we provide an alternative calibration with heterogeneity in present-bias in an incomplete markets life-cycle model with many periods and hyperbolic preferences. The level of heterogeneity we find in discounting of retirement savings is comparable to the one obtained in the benchmark calibration. Therefore, for the sake of not introducing a more complicated model only for the calibration section, in the main text we present only the calibration of the two-period model.

[4]See Alan and Browning (2010) and Alan, Browning and Ejrnaes (2014) for heterogeneity in discount factors. Montiel Olea and Strzalecki (2014), Augenblick, Niederle and Sprenger (2015) and Beshears et al. (2015) using experimental evidence find also considerable heterogeneity in present-bias.

Using calibrated parameters, one still needs to fix government preferences to solve for optimal policy. The government has a discount factor and a redistributive motive. The government discounts retirement consumption of individuals using its discount factor (paternalism) and assigns welfare weights to different individuals depending on their earnings ability (redistribution). We use two alternative procedures to choose the government preferences. First, we develop a benchmark procedure in which we choose government preferences by approximating the normative model and the calibrated model allocation of consumption and earnings under current policy. This follows the approach in Heathcote, Storesletten and Violante (2014) and guarantees that the level of redistribution in the benchmark normative model we choose is similar to the one calibrated for the U.S. economy. In this benchmark, the government has a discount factor that is in the intermediary range of calibrated discount factors for individuals, so that there are both individuals that save too little and individuals that save too much from the government's perspective. Furthermore, the government redistributive motive is considerably less progressive than utilitarian. This is consistent with the findings in the literature.[5] However, this procedure does not guarantee a perfect approximation, accordingly we find that there is a gain available to the government of 18% in consumption equivalent terms at the benchmark normative model.

Moreover, we find in the benchmark normative model a striking difference between optimal retirement benefits and current social security benefits in the United States. We find that social security benefits are considerably lower than retirement benefits prescribed by the benchmark normative model, particularly so at high earnings. As the government is less progressive than utilitarian in the benchmark normative model, there is an important concern about the savings choices of higher earnings individuals. Therefore, the government finds it optimal to force a higher minimum level of savings at high earnings.

[5]See Heathcote, Storesletten and Violante (2014) and Heathcote and Tsujiyama (2015).

Motivated by the findings in our benchmark, in a second procedure we select government preferences by approximating the schedule of retirement benefits in the normative model to the current social security schedule in the United States. We find that this alternative procedure yields a planner with a redistributive motive that is more progressive than utilitarian and has a slightly smaller discount factor level than in the benchmark. As a result of the considerable difference in the redistributive motive, in this procedure we find there are gains of 63% in consumption equivalent terms available to the government. These two exercises highlight that there is a sub-optimal mix of redistributive and savings policies in the current U.S. system according to our normative model. On one hand, in the first exercise we find the level of redistribution is largely inconsistent with the level of retirement benefits. On the other hand, in the second exercise we find that the level of retirement benefits is inconsistent with the current level of redistribution.

Our model can be generalized to explore government policies beyond savings. In section 4.5 we extend the approach in Mullainathan, Schwartzstein and Congdon (2012) and present a general model in which agents and the government disagree about the gains and costs of a generic action. The sources of disagreement can arise because of behavioral bias (e.g., present-bias), an externalities (e.g., pollution policy) or because of political economy considerations as in Aguiar and Amador (2011). Importantly, there is heterogeneity in this disagreement, so that some agents agree with the government and others disagree. Furthermore, there is inequality in labor earnings ability so that redistribution is possibly a concern for the government. We demonstrate that as long as the government has a redistributive motive in addition to preferences for a particular action, optimal policy for the government will offer more flexibility in actions of high earnings individuals.

We explore two applications of this general model: drug policy and fuel efficiency policy. Of course, the drug policy debate is very complex and controversial. Our

model focuses on the paternalistic nature of the policy and on its redistributive implications, with a caveat that it abstracts from other important considerations in this debate. Our findings imply that a paternalistic government will effectively make drug consumption prohibitive for low earnings individuals. However, if the possibility of consuming drugs pleases high earnings individuals enough to improve their willingness to work, then a government with a redistributive motive would set in place a policy in which drug consumption is allowed at a high cost (e.g., using prescriptions). While redistribution demands more flexibility, if the government has no redistributive motive, we show that the government would be paternalistic toward all agents and would implement a prohibition on drug consumption.

In the second application we consider policies fostering the purchase of energy efficient vehicles. In the model, some people do internalize the cost of the extra pollution caused by an inefficient vehicles, but others do not internalize those costs and have preferences for less energy efficient vehicles. If there is no redistribution, optimal policy involves a strict regulation of the level of energy efficiency so that vehicles purchased by all agents would be energy efficient. However, again, if there is redistribution, the government is going to allow higher earnings individuals to purchase less efficient vehicles as long as that improves their willingness to work and contribute toward redistribution. In this case, policy can be implemented with income tax rebates on the purchase of vehicles depending on their energy efficiency level. At low earnings, there is a considerable tax rebate from the purchase of an efficient vehicle, but at high earnings, tax rebates are less generous. Therefore, there is progressivity in the schedule, a feature resembled by actual policy. Currently, there are tax rebates on federal income taxes for electric and plug-in hybrid cars in the United States. Therefore these rebates inherit the progressivity of the income tax schedule as the same rebate has different effects on total taxes paid by different taxpayers.

Moreover, our extension highlights that this paper contributes not only to the public finance literature on savings policies, but also to the intersection of the public finance and behavioral economics literature. Indeed, there are two strands of research in this intersection that closely relate to this paper. The first strand has considered how tax policy can be used to ameliorate behavioral biases. The second strand has analyzed the provision of commitment devices for people with time inconsistent preferences.

There is a growing literature on optimal taxation with behavioral agents that focuses on optimal linear taxation without redistribution. Building upon the framework by Laibson (1997), starting with O'Donoghue and Rabin (2003), Gruber and Kőszegi (2004) and O'Donoghue and Rabin (2006), this literature considers how linear taxes can be used to either prevent over consumption of some goods (e.g., fossil fuels, drugs) or to foster consumption of other goods (e.g., retirement savings). Recently, Farhi and Gabaix (2015) find that if heterogeneity in behavioral biases is sufficiently high, quantity restrictions on consumption of the "behavioral" good for all agents fare better than linear taxes. Relative to these contributions, we consider the general optimal policy problem without a restriction to particular policy tools, and we show that optimal policy effectively restricts quantities at low earnings, but allows for a limited amount of flexibility on consumption at high earnings.

A few papers consider the taxation of behavioral agents with redistribution. Lockwood and Taubinsky (2015) allow for non-linear labor earnings taxes and a linear tax on the good whose demand is affected by a behavioral bias. Conversely, we allow for a general tax structure both on labor earnings and on the good affected by a behavioral bias. Finally, Farhi and Werning (2010) consider a model with a single level of present-bias.[6] We show that the introduction of heterogeneity in present bias allow

[6]In their interpretation of the model disagreement is on the Pareto weight given to the future generation. However, this is isomorphic to present bias in a two-period model.

us to characterize the level of flexibility that policies allow on retirement savings for agents at different earnings levels.

There is also a relatively small literature that has analyzed the optimal provision of commitment and flexibility in savings choices without redistribution. Amador, Werning and Angeletos (2006) develop a model in which there is heterogeneity in taste shocks toward future consumption but agents have a uniform level of present bias. They find that optimal policy is implemented by a minimum savings rule. In their model, there is both a value to offering flexibility and offering commitment even without redistribution. Recently, Galperti (2015) extends this approach to a more general setting. In contrast to those contributions, in our model the value of flexibility is endogenous and it depends on how much flexibility can be used to improve redistribution. Indeed, the main insight on this paper arises from the endogenous variation of the value to flexibility across different earnings levels.

In concurrent work, Yu (2015) finds in a model with hyperbolic preferences that redistribution can be greatly enhanced by the planner. In fact, in his setup he finds that the first best can be implemented under certain conditions. The most important difference between his model and ours is on the timing that the social planner can obtain reports from agents. In his model, agents meet with the planner twice. Both before actually taking work and consumption decisions, and at the time they take those decisions. This is particularly important because agents have different preferences at those points in time and the same information set, thus the planner can extract informational rents from agents as emphasized in Cremer and McLean (1988). In contrast, in our two period model the planner only meets with agents when they actually have the behavioral bias, so that full surplus extraction is not possible. In Appendix D.2, we explore a multi-period version of our model with hyperbolic preference shocks and labor income shocks and show that the planner finds it optimal to offer more flexibility after high income shocks than after low income shocks.

Finally, we also broadly relate to two other strands of the literature. First, we relate to a large literature on the taxation of savings. Atkinson and Stiglitz (1972) show that without behavioral biases and uncertainty on earnings ability, the government should not distort savings decisions for redistribution purposes. Saez (2002) and more recently Diamond and Spinnewijn (2011) and Golosov et al. (2013) point out how a correlation between discount factors and labor earnings ability can break this result in models without a misalignment of individuals' and government's preferences. Second, we relate to a small literature on taxation with multi-dimensional characteristics. Armstrong and Rochet (1999) emphasize that the lack of a general toolbox for the analysis of contracting problems with multi-dimensional heterogeneity hindered their usage in applications.[7] When there is a misalignment in preferences, we are able to prove that allocations allow for an increasing level of flexibility when comparing low and high earnings levels, even without general tools to characterize the full problem's solution. Furthermore, we provide a numerical algorithm to fully solve the problem.

We organize this paper as follows. Section 4.2 develops a benchmark model for the joint analysis of paternalistic and redistributive policies. Then Section 4.3 shows that tax instruments similar to those already used in the U.S. can implement optimal retirement savings policies when there is a concern about behavioral biases in savings.. Section 4.4 quantitatively evaluates the benchmark model and shows that from the normative model perspective there is an inconsistency between current social security benefits and current redistribution policy in the United States. Section 4.5 shows that our main findings have broad implications for policies aiming at behavioral biases in

[7]See McAfee and McMillan (1988), Armstrong (1996) and Rochet and Choné (1998) for important contributions to the mechanism design literature with multi-dimensional types. However, there are important exceptions in public finance. Kleven, Kreiner and Saez (2009) analyzes the taxation of couples and find that a higher earnings by a secondary earner generally reduce optimal taxes on the primary earner. Rothschild and Scheuer (2013) characterizes optimal income taxes when there is heterogeneity in earnings skills across different occupations.

other contexts as well as policies aiming at ameliorating externalities. Finally section 4.6 concludes the paper.

4.2 Optimal policy analysis

This section presents our benchmark model for the analysis of redistribution policies when there is a concern by the government about individuals undersaving for retirement. We develop a two-period model with unobservable heterogeneity in preferences toward retirement consumption and earnings ability. Following the approach in the optimal taxation literature, we first characterize properties of economic resource allocations that are efficient to the government. Then, we find our main insight about government policy when there is a redistributive motive and the government disagrees with agents' preferences.

There is a unit mass of agents that live for two-periods and that differ in two unobservable attributes: earnings ability and preferences toward retirement consumption. We denote earnings ability by $\theta \in \Theta = \{\theta_1, \ldots, \theta_N\}$, where $\theta_N > \cdots > \theta_1 > 0$. The government discounts retirement consumption by δ, and agents discount retirement consumption by $\beta\delta$ where $\beta \in B = \{\beta_1, \ldots, \beta_M\}$ and $0 < \beta_1 < \cdots < \beta_M = 1$. We understand β as a placeholder for the disagreement between the government and the agent. We denote by (θ, β) an agent's type and denote its distribution by π which we assume has full support. The agents utility index at time of decision is

$$U(c_1, c_2, y; \theta, \beta) = u(c_1) - \frac{1}{\theta} v(y) + \beta\delta u(c_2)$$

where $u'(0) = +\infty$, $u''(\cdot) < 0$, $v(0) = 0$, $v'(0) = 0$, $v'(y) > 0$ for $y > 0$ and $v''(y) > 0$ for $y > 0$. Our choice of language for the disagreement between the government and the agents reflects the apparent impossibility to interpret β as present-bias in a two-period model. However, this is actually inconsequential as we can understand the

government preference as the ex-ante preference of agents before they take decisions in the initial period.[8] Finally there is a storage technology with gross rate of return R for shifting resources into the second period.

We assume that the government does not observe agents' types, but it observes consumption levels and labor earnings, and it can choose the rules of the game to be played among agents. These rules can take an arbitrary form, but an important result in mechanism design theory, the Revelation Principle, guarantees that it is sufficient to consider a game in which the government asks agents their type, agents find it in their best interest to report their true type and the government assigns agents a resource allocation depending on their answer. We call this assignment rule and allocation and denote it by $\{c_1(\theta,\beta), c_2(\theta,\beta), y(\theta,\beta)\}_{(\theta,\beta)\in\Theta\times B}$. In the next section we show that the resulting government choice of rules is equivalent to a market economy with a sensible set of government policies in use. However, in this section our focus will be on this abstract resource allocation problem as it actually sheds light into the appropriate set of policy instruments that can implement optimal policy.

When choosing an allocation the government has to make sure it is consistent with the information and technological constraints of the economy. An allocation is said *incentive compatible* if

$$U(c_1(\theta,\beta), c_2(\theta,\beta), y(\theta,\beta);\theta,\beta) \geq U(c_1(\theta',\beta'), c_2(\theta',\beta'), y(\theta',\beta');\theta,\beta) \quad (4.1)$$

for all $(\theta,\beta),(\theta',\beta') \in \Theta \times B$. If an allocation is incentive compatible, it is consistent with the unobservability of agents' types by the government. An allocation is

[8] Amador, Werning and Angeletos (2006) argue in a similar two-period problem that the planner takes decisions before the initial period and that as a consequence the model indeed has three periods, even though agents only take decisions in two different periods. We show our results extend to a T-period model with behavioral biases in appendix D.1.

consistent with the technology in the economy if it satisfies the resource constraint:

$$\sum_{(\theta,\beta)\in\Theta\times B} \pi(\theta,\beta)\left[y(\theta,\beta) - c_1(\theta,\beta) - \frac{c_2(\theta,\beta)}{R}\right] \geq 0 \tag{4.2}$$

so that earnings produced by workers can sustain their life-time consumption at the aggregate level.

The government evaluates individual payoffs according to

$$V(\theta,\beta) = u(c_1(\theta,\beta)) + \delta u(c_2(\theta,\beta)) - \frac{1}{\theta} v(y(\theta,\beta))$$

Therefore the government objective can be written as

$$W\left(\{c_1(\theta,\beta), c_2(\theta,\beta), y(\theta,\beta)\}_{(\theta,\beta)\in\Theta\times B}\right) = \sum_{(\theta,\beta)\in\Theta\times B} \pi(\theta,\beta)\lambda(\theta)V(\theta,\beta) \tag{4.3}$$

where $\lambda(\theta) \geq 0$ are Pareto weights the government assigns to agents. We assume weights depend only on earnings ability of the agent, not on its disagreement with the government. This is consistent with the government believing β is a bias that it ought to correct.[9] Without loss of generality, Pareto weights are normalized so that $\sum_{(\theta,\beta)\in\Theta\times B} \pi(\theta,\beta)\lambda(\theta) = 1$. The government problem is then to choose an allocation that maximizes (4.3) subject to both incentive constraints in (4.1) and the resource constraint in (4.2).

4.2.1 Example with 2×2 types

Before we state our theorems, it is useful to understand the government's problem in a simple example that highlights the key forces in the model. Assume there are two

[9]Using the language of Laibson (1997), we can think of two separate agents: self at period-0 and self at period-1. Self at period-1 has a present bias in saving for retirement ($\beta < 1$) and she takes all consumption and labor decisions. Conversely, self at period-0 has no present bias, but she cannot take period 1 decisions by herself. In this behavioral interpretation, the government preference is in agreement with preferences of self at period-0.

levels of disagreement and two levels of earnings ability. For simplicity, let $R\delta = 1$, $\beta \in \{\beta_L, 1\}$ with $\beta_L < 1$, $\theta \in \{\theta_L, \theta_H\}$ where $\theta_H > \theta_L = 0$, and assume the government is utilitarian, i.e., $\lambda(\theta_L) = \lambda(\theta_H) = 1$.

Bunching at low earnings – In the first best allocation without informational frictions and at a fixed ability type θ, the government bunches agents across their disagreement type β. As V is strictly concave, the government can always improve welfare by bunching the agents with a different disagreement level but the same ability level. In the second best with informational frictions, this same argument holds for low earnings ability agents. However, with unobservable characteristics it is necessary to satisfy incentive constraints. Since agents with low earnings ability cannot work as $\theta_L = 0$, the relevant incentive constraints are the ones preventing deviations of high ability types into the allocations of low ability types:

$$u(c_1(\theta_H, \beta)) - \frac{1}{\theta_H} v(y_1(\theta_H, \beta)) + \beta\delta u(c_2(\theta_H, \beta)) \geq u(c_1(\theta_L, \beta')) + \beta\delta u(c_2(\theta_L, \beta')) \tag{4.4}$$

for $\beta, \beta' \in \{\beta_L, 1\}$. Since incentive constraint (4.4) is linear in utility levels $u(c_1(\theta_L, \beta'))$ and $u(c_2(\theta_L, \beta'))$, then a convex combination of those utility levels satisfy incentive compatibility

$$u(c_1(\theta_H, \beta)) - \frac{1}{\theta_H} v(y_1(\theta_H, \beta)) + \beta\delta u(c_2(\theta_H, \beta)) \geq \bar{u}_1(\theta_L) + \beta\delta \bar{u}_2(\theta_L) \tag{4.5}$$

where $\bar{u}_t(\theta_L) = \sum_{\beta'} \pi(\beta'|\theta_L) u(c_1(\theta_L, \beta'))$. Therefore, it is incentive compatible for the government to allocate $\bar{c}_t(\theta_L) = u^{-1}(\bar{u}_t(\theta_L))$ for low ability agents. Further, the government objective does not change. However, since the utility index u is strictly concave, it follows that

$$\bar{c}_1(\theta_L) + \frac{1}{R}\bar{c}_2(\theta_L) < \sum_{\beta'} \pi(\beta'|\theta_L) \left[c_1(\theta_L, \beta') + \frac{1}{R} c_2(\theta_L, \beta') \right]$$

if the allocations of low ability agents are not all equal. We conclude the government bunches low ability agents. If that was not the case, the government would perturb the allocation as we propose and it would use the extra resources available to strictly increase its objective.

Separation at high earnings – Start with an allocation that bunches high ability agents, so that they receive the same consumption levels in both periods and produce the same labor earnings during working life. We show it is possible to improve the government's objective by changing such an allocation. There are two different cases to consider.

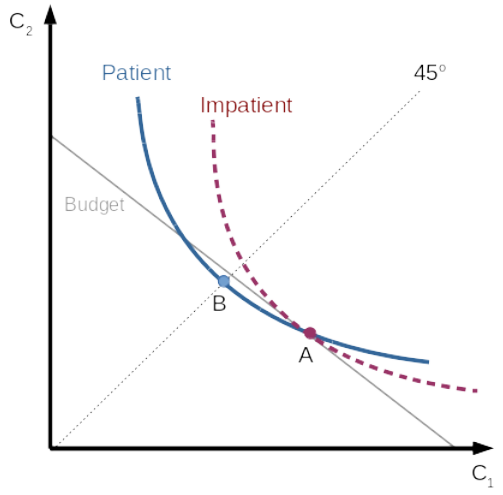
In the first case, there is bunching of high ability agents into an allocation with a higher consumption level in the initial period, that is, $c_1(\theta_H) > c_2(\theta_H)$. In Figure 4.1 we illustrate indifference curves on consumption allocations of individuals with the same level of labor earnings. At point A, we have $c_1(\theta_H) > c_2(\theta_H)$. At this point, the indifference curve of an impatient agent is steeper as a higher change in period two consumption is required by this agent to compensate for the same change in period one consumption.[10] While continuing to offer allocation A, the government can target patient agents by offering allocation B. However, since points on the 45^0 line minimize the cost of providing the same level of utility to patient agents ($R\delta = 1$), allocation B has a strictly lower cost in terms of resources. This is illustrated in the figure by allocation B being inside the budget of resources used to obtain allocation A. Therefore the government can obtain the same objective value while spending less, a contradiction.

[10]The marginal rate of substitution between consumption in periods one and two is given by

$$\frac{\beta\delta u'(c_2)}{u'(c_1)}$$

which is monotone in β. Therefore there is single crossing in preferences conditional on a fixed level of labor earnings.

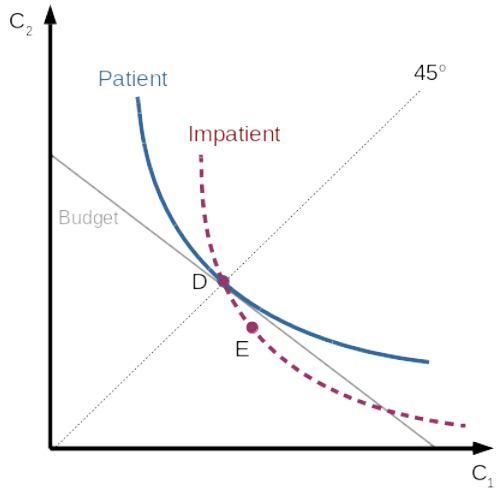
Figure 4.1: Incentives with bunching.



In the second case, there is bunching of high ability agents into an allocation in which period two consumption is weakly higher that period one consumption, i.e., $c_2(\theta_H) \geq c_1(\theta_H)$. This is illustrated in Figure 4.2 by point D. In this case, while keeping the availability of allocation D for patient agents, the government can target impatient agents by offering allocation E. While impatient agents are indifferent between allocations D and E, allocation E has a lower resource cost and it reduces the payoff of impatient agents from the government perspective (the government indifference curve coincides with that of that of the patient agent). Therefore, in this case, we need to show that the government gains in transferring those extra resources to low ability agents are sufficiently high so as to compensate for the loss with impatient high ability agents. But since ability is not observable and $y(\theta_H) > 0 = y(\theta_L)$, then from incentive compatibility it must be the case that either $c_1(\theta_H) > c_1(\theta_L)$ or $c_2(\theta_H) > c_2(\theta_L)$ or both. Therefore, for a utilitarian planner, the marginal gain in transferring resources to low ability agents will surpass the marginal loss of high ability impatient agents as long as E is sufficiently close to D. Finally, we conclude that the government finds it optimal to separate high ability agents.

Distortions at low ability – We already know that low ability agents are bunched. Now, assume that point A in Figure 4.1 represents low ability agents consumption

Figure 4.2: Separation at high ability



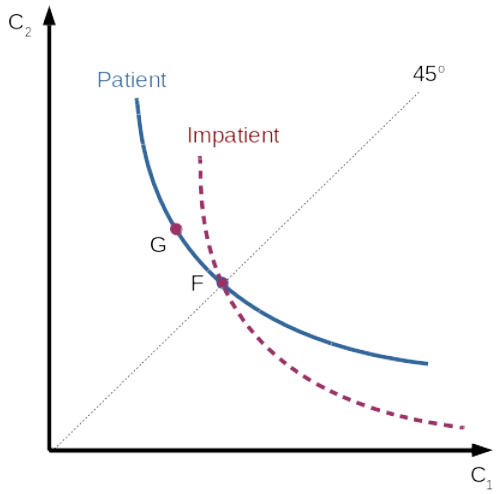
allocation while the indifference curves represented are for high ability agents deviating into that allocation. Then it is possible to keep the government's goal fixed and save resources by offering point B instead to low ability agents. Therefore, we conclude $c_2(\theta_L) \geq c_1(\theta_L)$. Again, in Figure 4.3, we illustrate the consumption allocation of low ability agents shile the indifference curves of high ability agents. We see that if there is perfect consumption smoothing, represented by point F, then it is always possible to relax the incentive constraint of impatient high ability agents by perturbing the allocation in the direction of allocation G. Patient agents are made indifferent by the perturbation, but it strictly relaxes the incentive constraint of high ability impatient agents. It follows that at the solution to the government's problem we have $c_2(\theta_L) > c_1(\theta_L)$ as we will see that the incentive constraint of high ability impatient agents is binding.

Distortion at high ability and patient – If the government offers an allocation with $c_2(\theta_H, \beta_H) < c_1(\theta_H, \beta_H)$ for high ability patient agents, represented by point A in Figure 4.1, it is always possible to offer instead point B and save resources while keeping the government's objective constant. Thus $c_2(\theta_H, \beta_H) \geq c_1(\theta_H, \beta_H)$. Furthermore, the government would offer $c_2(\theta_H, \beta_H) > c_1(\theta_H, \beta_H)$ if and only if the incentive constraint of high ability impatient agents is binding with respect to the

allocation of high ability patient agents. Next we show this is never the case in this example and therefore we conclude $c_2(\theta_H, \beta_H) = c_1(\theta_H, \beta_H)$. Therefore, the government finds it optimal not to distort the consumption allocation of patient agents.

Distortion at high ability and impatient – It immediately follow from separation and the distortion for high ability and patient agents that $c_1(\theta_H, \beta_L) > c_2(\theta_H, \beta_L)$. Therefore the government allows impatient high ability agents to consume proportionally less at retirement than what perfect consumption smoothing would predict.

Figure 4.3: Over consumption at retirement of low ability



Now we illustrate the pattern in which incentive constraints of high ability agents bind. At least one incentive constraint from high ability agents to low ability agents must bind as consumption is not equalized across agents with different ability.

High ability and impatient incentives – First, let's consider high ability impatient agents. Assume by way of contradiction that the incentive constraint of type (θ_H, β_L) is strictly slack with respect to the low ability agents' allocation. In this case we have $c_1(\theta_L) = c_2(\theta_L)$ as there is no gain from not providing perfect consumption smoothing to low ability agents. We have already shown that $c_2(\theta_H, \beta_H) \geq c_1(\theta_H, \beta_H)$, therefore

$c_2(\theta_H, \beta_H) \geq c_2(\theta_L)$. Then a binding incentive constraint of type (θ_H, β_H) implies

$$u(c_1(\theta_H, \beta_H)) - \frac{1}{\theta_H} v(y_1(\theta_H, \beta_H)) + \beta_L \delta u(c_2(\theta_H, \beta_H)) \leq u(c_1(\theta_L)) + \beta_L \delta u(c_2(\theta_L))$$

and all incentive constraint of type (θ_H, β_L) are strictly slack, a contradiction. Therefore it must be the case that the incentive constraint of type (θ_H, β_L) is binding with respect to the allocation of low ability agents.

Second, the high ability impatient agent incentive constraint is slack with respect to the allocation of the high ability patient agent. Assume by way of contradiction that is not the case. Since the incentive constraint of high ability impatient agents deviating into low ability is binding, then we have

$$u(c_1(\theta_H, \beta_H)) - \frac{1}{\theta_H} v(y_1(\theta_H, \beta_H)) + \beta_L \delta u(c_2(\theta_H, \beta_H)) = u(c_1(\theta_L)) + \beta_L \delta u(c_2(\theta_L))$$

At the solution to the planner's problem we must have $c_2(\theta_H, \beta_H) > c_2(\theta_L)$.[11] Therefore we conclude

$$u(c_1(\theta_H, \beta_H)) - \frac{1}{\theta_H} v(y_1(\theta_H, \beta_H)) + \delta u(c_2(\theta_H, \beta_H)) > u(c_1(\theta_L)) + \delta u(c_2(\theta_L))$$

But since high ability agents are separated, we also must have $c_2(\theta_H, \beta_H) > c_2(\theta_H, \beta_L)$ and therefore all incentive constraint of high ability patient agents are strictly slack. But then, the government could improve redistribution in the economy, a contradiction. We conclude that the high ability impatient agent has a slack incentive constraint with respect to the allocation of the high ability patient agent.

[11] Assume by way of contradiction that $c_2(\theta_L) \geq c_2(\theta_H, \beta_H)$. Since there is separation of high ability agents, then $c_2(\theta_L) \geq c_2(\theta_H, \beta_H) > c_2(\theta_H, \beta_L)$. Therefore, the utilitarian government would like to redistribute period two consumption from low ability agents to high ability agents. This is always incentive compatible as low ability agents cannot work. We then obtain a contradiction and conclude that $c_2(\theta_H, \beta_H) > c_2(\theta_L)$.

High ability and patient incentives – Finally, for patient high ability agents the pattern in which incentives constraints bind is not straightforward. In particular, it will depend on several parameters such as the level of β_L and the redistributive motive. On one hand if $\beta_L \approx 1$, we have at the solution to the planner's problem $c_2(\theta_H, \beta_L) > c_2(\theta_L)$ so that

$$u(c_1(\theta_H, \beta_L)) - \frac{1}{\theta_H} v(y_1(\theta_H, \beta_L)) + \delta u(c_2(\theta_H, \beta_L)) > u(c_1(\theta_L)) + \delta u(c_2(\theta_L))$$

where we used our last result that the incentive constraint of (θ_H, β_L) is binding with respect to θ_L. In this case the patient high ability agent has a slack incentive constraint with respect to low ability agents. On the other hand, if $\beta_L \approx 0$ and $\lambda(\theta_H) \approx 0$, then $c_2(\theta_H, \beta_L) \approx 0 < c_2(\theta_L)$, and the incentive constraint of type (θ_H, β_H) with respect to low ability types will be binding. With many types, the characterization of which incentive constraints bind becomes an intractable problem. However, it is still possible to provide key implications for optimal policy even without this characterization.

4.2.2 Main results

Now we turn back into our benchmark model with heterogeneity in disagreement and earnings ability. In the problem with two-dimensional heterogeneity and many types, the major difficulty is the characterization of the pattern in which incentive constraints bind. In a problem with one dimensional heterogeneity, such as in Mirrlees (1971), under fairly general assumptions only incentive constraints of types close to each other can bind. That is not true in problems with multi-dimensional heterogeneity as pointed out by Armstrong (1996) and Rochet and Choné (1998). However, we are still able to characterize key properties of the solution to the government's problem

at the bottom and top levels of earnings ability.[12] In Theorem 4.1 we obtain the main result in this paper and then in Theorem 4.2 we characterize distortions at top and bottom earnings at the solution to the planner's problem.

Theorem 4.1. *Assume $\lambda(\theta)$ is weakly decreasing in θ. Fix θ_N and $\{\beta_2, \ldots, \beta_M\}$ then there exists $\underline{\theta}_1 > 0$, $\bar{\theta}_{N-1} < \theta_N$ such that at the solution to the government's problem:*

1. *if $\underline{\theta}_1 > \theta_1 > 0$, then agents with types in $\{(\theta_1, \beta) : \beta \in B\}$ receive the same allocation, independently of their type β:*

$$c_t(\theta_1, \beta) = \bar{c}_t(\theta_1)$$
$$y(\theta_1, \beta) = \bar{y}(\theta_1)$$

   for $t = 1, 2$ and all $\beta \in B$;

2. *if $\theta_1 < \cdots < \theta_{N-1} \leq \bar{\theta}_{N-1}$, then agents with types in $\{(\theta_N, \beta) : \beta \in B\}$ do not receive the same allocation.*

Proof. See Appendix. □

On one hand, the government finds it optimal to offer different allocations for high earnings ability agents with type θ_N. Since agents preferences disagree with one another, they would like to receive different allocations. Therefore, the government can actually extract some extra resources from them by offering different allocations that agents find more attractive. Since Pareto weights are decreasing, the government would like to redistribute toward lower earnings ability individuals, hence extracting those extra resources has a positive marginal value. As a result the government will be willing to allow for different allocations at high earnings as to obtain extra resources from those agents.

[12] In section 4.4 we find through numerical simulations that those properties are not particular to the end-points of the distribution of types, but that they are in fact a force present at all earnings levels.

On the other hand, the government finds it optimal to bunch the lowest earnings agents along the β dimension. It is not in the interest of the government to extract resources at low earnings ability. Therefore, the planner will find it optimal to bunch agents, effectively providing them full insurance against their preference heterogeneity β-types. In an interpretation of β as reflecting behavioral biases, that we explore fully in appendix D.1, the government here is insuring agents against the possibility they face time inconsistencies into the future.

Even though the government problem is a two-dimensional screening problem, we are able to obtain important insights about the optimal allocation. The assumption of a paternalistic government reduces the dimensionality of the objective function and allows for a partial characterization of optimal policy. It is clear that the government wants to bunch agents across the β dimension, therefore it is easier to understand how incentive constraints bind at the lowest and highest earnings levels.

Now we define wedges on decisions that represent distortions at the solution to the government's problem. The intertemporal wedge

$$\tau^{I}(\theta,\beta) = 1 - \frac{u'(c_1(\theta,\beta))}{R\beta\delta u'(c_2(\theta,\beta))}$$

allow us to understand distortions on intertemporal consumption decisions. A positive intertemporal wedge is alike a tax on future consumption so agents facing it are consuming relatively little in the future. We can also define the government's intertemporal wedge

$$\tau^{PI}(\theta,\beta) = 1 - \frac{u'(c_1(\theta,\beta))}{R\delta u'(c_2(\theta,\beta))}$$

which uses the government's preferences (i.e., $\beta = 1$). The government's intertemporal wedge measures distortions on agents intertemporal decisions from the government's perspective. For example, if an agent faces no intertemporal wedge but has disagreement with the government $\beta < 1$, then the government's intertemporal wedge

is positive for that agent. In the next result we characterize the sign of intertemporal wedges at the solution to the government's problem.

Theorem 4.2. *Assume $\lambda(\theta)$ is weakly decreasing. Fix $\{\theta_2, \ldots, \theta_{N-1}\}$ and $\{\beta_2, \ldots, \beta_M\}$ then there exists $\underline{\theta}_1 > 0$ and $\bar{\theta}_{N-1} < \theta_N$ such that if $\theta_1 \leq \underline{\theta}_1$, $\theta_1 < \cdots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ at the solution to the government's problem*

1. *for types with lowest earnings ability:*

   (a) *intertemporal wedge is negative: $\tau^I(\theta_1, \beta) < 0$ for all $\beta < 1$*

   (b) *planner's intertemporal wedge is weakly negative: $\tau^{PI}(\theta_1, \beta) \leq 0$ for all β*

2. *for types with higher earnings ability:*

   (a) *agents that agree with the government ($\beta_M = 1$) face weakly negative intertemporal wedges $\tau^I(\theta_N, \beta_M) = \tau^{PI}(\theta_N, \beta_M) \leq 0$*

   (b) *agent that disagrees the most with the government ($\beta_1 < 1$) face a positive government's intertemporal wedge $\tau^{PI}(\theta_N, \beta_1) > 0$*

Proof. See the appendix. □

In Theorem 4.2 we find that the intertemporal wedge of the lowest earnings ability individuals is negative for agents that disagree with the government. Further, from the planner's point of view it shows those agents weakly over-consume at retirement. In our numerical simulations we find a strictly positive government intertemporal wedge. By forcing low earnings ability individuals to consume relatively more at retirement than would be optimal in the first best, the government makes their allocation particularly undesirable for high earnings ability individuals that disagree with government preferences ($\beta < 1$). Therefore, the government relaxes their incentive constraints by doing so. But if the optimal allocation was to force all low earnings

individuals into consuming at the first best marginal rate of substitution (a zero government's wedge), then the marginal cost to the planner of forcing them to consume relatively more at retirement is of second order. However, the gain from relaxing incentive constraints is of first order because of redistribution.

Finally Theorem 4.2 shows a dispersion in distortions at high earnings levels. For agents that agree with the government, there is no gain in under-consuming at retirement (in numerical simulations we find it is not optimal to over-consume at retirement as well). Distorting savings in this case generates both a lower government's objective and also reduces incentives for those agents to work. However, for individuals that disagree with the government ($\beta_1 < 1$) and also have high earnings, it is worth to the planner to allow them to consume in the way they like in exchange for extracting some more resources from those individuals in order to improve redistribution. As a result those agents consume too little at retirement relative to their working life consumption.

4.2.3 Comparison to most related results in the literature

In order to better understand the forces at work in our model, it is useful to consider two important particular cases. The first one is the classical result obtained by Atkinson and Stiglitz (1972). It shows that without disagreement, the government does not find it optimal to distort intertemporal consumption decisions for any redistributive motive. The second result is from Farhi and Werning (2010) and it considers the case with disagreement but no heterogeneity in the level of disagreement across agents. They show that there is a distortion in intertemporal consumption decisions and that this distortion is monotonic on earnings.

It is easy to understand the intuition behind the result in Atkinson and Stiglitz (1972) using our 2×2 types example. If $\beta = 1$ for all agents, there are only two types

of agents with $\theta \in \{0, \theta_H\}$. The only incentive constraint in this problem is given by

$$u\left(c_1\left(\theta_H\right)\right) + \delta u\left(c_2\left(\theta_H\right)\right) - \frac{1}{\theta_H} v\left(y\left(\theta_H\right)\right) \geq u\left(c_1\left(0\right)\right) + \delta u\left(c_2\left(0\right)\right)$$

Notice that both types of agents and the government agree on how to value consumption over the life-time. Then at the optimum, it must be the planner is minimizing resource expenditures with consumption of each agent:

$$\min_{c_1, c_2} \left\{ c_1 + \frac{1}{R} c_2 \right\}$$
$$s.t. u\left(c_1\right) + \delta u\left(c_2\right) = \bar{U}\left(\theta\right)$$

where $\bar{U}\left(\theta\right) = u\left(c_1\left(\theta\right)\right) + \delta u\left(c_2\left(\theta\right)\right)$. As only $\bar{U}\left(\theta\right)$ matters for the incentive constraint, not the consumption levels separately. But if we take the first order conditions of the problem above we obtain

$$u'\left(c_1\left(\theta\right)\right) = R\delta u'\left(c_1\left(\theta\right)\right)$$

and therefore the government should not distort agent's intertemporal decisions.

In order to understand the intuition behind the result in Farhi and Werning (2010), consider the example when $\beta_L = \beta_H = \beta < 1$ and $\theta \in \{0, \theta_H\}$. The incentive constraint is given by

$$u\left(c_1\left(\theta_H\right)\right) + \beta\delta u\left(c_2\left(\theta_H\right)\right) - \frac{1}{\theta_H} v\left(y\left(\theta_H\right)\right) \geq u\left(c_1\left(0\right)\right) + \beta\delta u\left(c_2\left(0\right)\right)$$

Assume by way of contradiction that there are distortions to agents intertemporal decisions at the solution to the government's problem. Since $\lambda\left(0\right) = \lambda\left(\theta_H\right)$ and $y\left(\theta_H\right) > 0$, then either $c_1\left(\theta_H\right) > c_1\left(0\right)$ or $c_2\left(\theta_H\right) > c_2\left(0\right)$. Therefore the government wants to transfer more resources to agents with type $\theta = 0$, as a result the incentive

constraint is binding and relaxing it would strictly improve the government's objective. However, under our assumption of no distortion, there are two different ways to relax the incentive constraint. On one hand, the government can increase $c_2(0)$ in exchange for a lower $c_1(0)$ while keeping the amount of resources used by agents with type $\theta = 0$ constant. This implies a second order welfare loss to the planner, but strictly relaxes the incentive constraint of type θ_H as $\beta < 1$. On the other hand, the government can increase $c_1(\theta_H)$ in exchange for a reduction of $c_2(\theta_H)$ while keeping resource usage constant. This perturbation will lead to a second order loss in the government's objective, but will strictly relax the incentive constraint. Finally, the government will optimally use both types of distortions and as a result agents with type θ_H under-consume in period two and agents with type $\theta = 0$ over-consume in period two.

The forces in Atkinson and Stiglitz (1972) and in Farhi and Werning (2010) are present in our model. As we can inspect in Theorem 4.2, the intertemporal distortions display a monotonic shape as in Farhi and Werning (2010). Also high earnings ability individuals that agree with the government ($\beta = 1$) are not distorted.

However, our results rely on a key third force into our model: heterogeneity in disagreement. With heterogeneity in disagreement, now the government also needs to worry about dispersion in allocations of agents with the same level of earnings ability. The interaction of this third force with redistribution then implies that the planner will be particularly worried about dispersion of allocations across low earnings ability agents, but not as much by the dispersion on allocations across high earnings ability agents. Thus, we find that the government will not only distort agents with disagreement as in Farhi and Werning (2010) while not distorting some agents without disagreement as in Atkinson and Stiglitz (1972), but it will also bunch them across disagreement types at low earnings ability.

4.3 Decentralization of optimal retirement savings policies

In this section we provide a decentralization of the solution to the government's problem as a competitive equilibrium in a market economy in which the government uses four sets of policy tools. All of those instruments are very similar to policy tools currently used in the U.S., so we are able to connect our results in the previous section with actual policy.

The first is an old-age benefit which is non-linear on labor earnings and which agents cannot borrow against during working life. Our previous results indicate that agents with a large level of disagreement (present bias) would like to borrow against those resources at period one, however as that would reduce the government objective it enforces by law that retirement benefits cannot be used as a guarantee on loans.[13]

The second set of policy tools includes multiple retirement savings accounts. Those accounts have caps on contributions and they have tax advantages as compared to a regular savings account. Both the caps and taxes on those account depend on the earnings level of individuals in the decentralization. In the U.S. 401(k) accounts are characterized by a differential income tax treatment, the possibility of employer matching contributions and a cap on contributions.

The third set of policy tools used by the government includes linear subsidies or taxes on both savings through retirement savings accounts and savings at the free market rate on bank accounts. Finally, the fourth policy tool used by the government is a non-linear labor earnings tax that depends on the set of retirement savings account used by the agent. This last property is present as well in the current U.S. tax code as contributions to 401(k) accounts or IRA accounts have a differential tax treatment.

[13]In the U.S., the law prohibits the usage of social security benefits as collateral on loans.

Agents can save in a regular savings account with a gross rate of return $(1-\tau_{M-1}(y))R$ and in $m=1,\ldots,M-2$ retirement savings accounts with a rate of return $(1-\tau_m(y))R$ and contribution caps $\bar{a}_m(y)$. Both rates of return and caps depend on the earnings level y of the agent. We have $0 \geq \tau_{m+1}(y) \geq \tau_m(y)$ so that agents that use account $m+1$ also use accounts $1,\ldots,m$. Agents owe income taxes $T_m(y)$ to the government while young. We denote by $T_{M-1}(y)$ labor income taxes on the agent that saves on all retirement savings accounts and on the regular savings account, and by $T_0(y)$ the income tax of agents that do not save on top of their social security contribution. At retirement, agents receive a retirement benefit $b(y)$ that varies with the labor earnings level during working life. The agent's problem is

$$\begin{aligned}
&\max_{c_1,c_2,y,M_0} u(c_1) - \theta v(y) + \beta\delta u(c_2) \\
\text{s.t.} \quad & c_1 + a_s + \sum_{m=1}^{M_0} a_{r,m} = y - T_{M_0}(y) \\
& c_2 = b(y) + Ra_s + R\sum_{m=1}^{M_0}(1-\tau_m(y))\,a_{r,m} \\
& a_s \geq 0 \\
& 0 \leq a_{r,m} \leq \bar{a}_m(y) \\
& M_0 \in \{1,\ldots,M-2\}
\end{aligned}$$

We say a government policy $\left(\{\bar{a}_m(\cdot)\}_{m=1}^{M-2}, \{\tau_m(\cdot)\}_{m=1}^{M-1}, \{T_m(\cdot)\}_{m=0}^{M-1}, b(\cdot)\right)$ is feasible if agents' equilibrium choices given the pension scheme satisfy the economy-wide market clearing condition

$$\sum_{(\theta,\beta)\in\Theta\times B} \pi(\theta,\beta)\left[y(\theta,\beta) - c_1(\theta,\beta) - \frac{c_2(\theta,\beta)}{R}\right] = 0$$

It is straightforward that the solution to the government's problem can be decentralized by a government policy as just described.

Corollary 4.1. *Assume $\lambda(\theta)$ is weakly decreasing in θ and $\lambda(\theta_N) > 0$. Fix θ_N and $\{\beta_2, \ldots, \beta_M\}$, then there exists $\bar{\theta}_1 > 0$, $\bar{\theta}_{N-1} < \theta_N$ and $\bar{\beta}_1 > 0$ such that if $\theta_1 < \bar{\theta}_1$, $\theta_1 < \cdots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ and $\beta_1 < \bar{\beta}_1$, then the solution to the government's problem can be decentralized with a government policy as above and in which:*

1. *agents with types in $\{(\theta_1, \beta) : \beta \in B\}$ do not have access to retirement savings accounts and only receive social security benefits when old;*

2. *agents with types in $\{(\theta_N, \beta) : \beta \in B\}$ have access to retirement savings accounts and some save on top of social security contributions.*

In Corollary 4.1 we show that a pension system with social security benefits and multiple retirement savings accounts in which low earnings individuals do not have access to the retirement savings accounts can be actually optimal for a paternalistic planner. At first, the nature of the pension system in the U.S. where high earners get disproportional incentives on savings for retirement through 401(k) retirement accounts might not sound efficient in an environment where redistribution is important. However, we find the somewhat surprising result that this system shares some characteristics of an efficient paternalistic policy.

4.4 Quantitative exercise

In this section we evaluate current redistributive and retirement savings policies in the U.S. through the lenses of our normative model. We first develop a two-period model with actual policy tools to calibrate for preference heterogeneity present in the data. Then we find planner preferences for redistribution and δ, the government's discount factor, such that we obtain a normative model that is comparable to actual policy in terms of both the levels of redistribution and savings distortions observed. By taking this parsimonious approach, we minimize the chances actual policy is inefficient from

a normative perspective. Finally we compare the results of our normative model to actual policy.

4.4.1 Two-period calibration of current policy

First we calibrate for heterogeneity in discount factors using a two-period model of retirement savings decisions under current government policies. We use data from the University of Michigan Health and Retirement Study (HRS) as provided in Engen, Engen and Engen (2005) on the ratio of wealth at retirement to life-time earnings of households.[14] This statistic is informative about how people save controlling for their life-time labor earnings, the essential heterogeneity we would like to uncover. Therefore it is natural to calibrate the model to the shape of the cross sectional distribution of this statistic in the data.

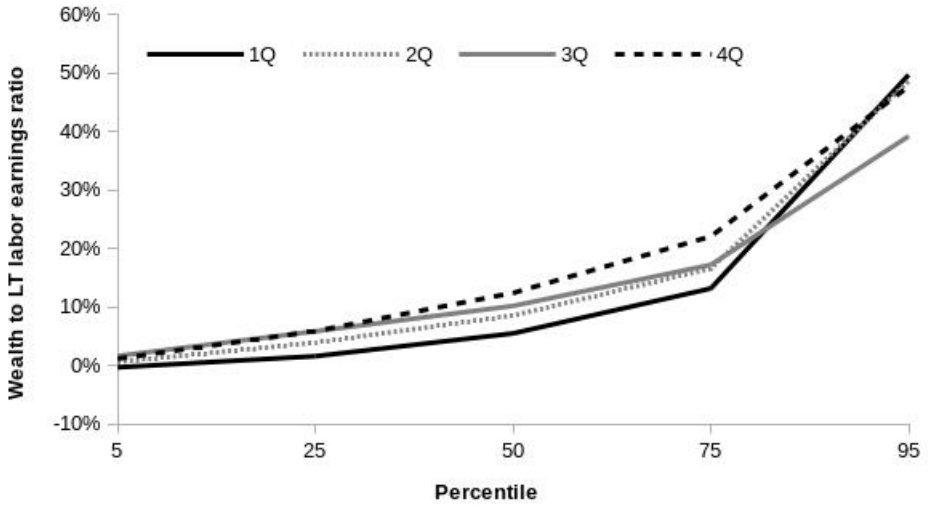
There is considerable heterogeneity in retirement savings rates in the data and a small positive correlation with earnings. In Figure 4.4 we display the data on ratios of wealth at retirement to lifetime earnings that we use to match the model. Each line in the figure displays the percentiles of the distribution of the ratio of wealth at retirement to lifetime earnings within a lifetime earnings quantile. We see that at the different quantiles on lifetime earnings, the statistic under study has a very similar distribution although it is slightly larger at higher life-time earnings quantiles. This points to a small correlation between savings rates and labor earnings in the data.

In our calibration, we parameterize households preferences by

$$U\left(c_1, c_2, y; \theta, \beta\right) = \frac{c_1^{1-1/\sigma} - 1}{1 - 1/\sigma} - \frac{\left(\frac{y}{\theta}\right)^{1+1/\gamma}}{1 + 1/\gamma} + \psi \frac{c_2^{1-1/\sigma}}{1 - 1/\sigma}$$

[14]The wealth measure used in Engen, Engen and Engen (2005) is at the household level and includes net liquid financial assets, half of the principal house equity, other real state equity, business equity, deposits in all types of retirement accounts and estimates of defined contribution pensions benefits (excluding social security). Using the history of current and past reported earnings and estimates from Khitatrakun, Kitamura and Scholz (2000), Engen, Engen and Engen (2005) obtain a measure of lifetime household earnings.

Figure 4.4: Distribution of non-social security wealth at retirement ratio to life-time earnings.



Source: Each line represents the distribution within a different lifetime earnings quantile. Data from Engen, Engen and Engen (2005) using 1992 Health and Retirement Survey sample of households. Non social security household wealth includes all liquid wealth, deposits on retirement accounts, estimated defined benefit plans, business equity, other real estate equity and half of primary home value.

where we use a standard level of $\sigma = 0.5$ for the intertemporal elasticity of substitution and a value on $\gamma = 1$ for the Frisch elasticity of labor supply. Clearly, we cannot disentangle β from δ in the two-period model calibration.[15] Hence, we calibrate here for the joint distribution of $\psi = \beta\delta$ which is the discount factor of individuals at period one, and the distribution of θ which is labor earnings ability. In the next sub-section, we will use current policy to inform us about appropriate values for δ, the government's discount factor.[16] Further we assume that the joint distribution can be written as a Gaussian copula between the marginal distributions of ψ and θ with a correlation parameter ω that we calibrate.

[15]Separate identification of β and δ requires data on at least three periods in a model with hyperbolic discounting.

[16]In Appendix D.3 we provide an alternative calibration using a standard incomplete markets life cycle model in which agents have hyperbolic preferences. We find that the heterogeneity in discount factors is similar. As a result, the comparison of optimal policy and current policy has the same qualitative implications using both approaches to calibrate discount factors.

For the marginal distribution of earnings ability θ, we follow Heathcote, Storesletten and Violante (2014) and Heathcote and Tsujiyama (2015) in using the actual observed labor income distribution. Heathcote, Storesletten and Violante (2014) argue for this approach as labor income and earnings are proportional to each other in logs after controlling for consumption in their model. In our model that is not entirely true, because of the complexity introduced by old-age benefits and retirement accounts that depend on earnings levels. However, we keep that assumption as it is a parsimonious first approximation. Finally, we use the percentiles of the labor income distribution across the age profile from the 2013 March CPS as the distribution of θ.[17]

For the marginal distribution of ψ we assume a Beta(a, b) distribution with shape parameters $a > 0$ and $b > 0$.[18] The Beta distribution is an appropriate choice as it does not impose symmetry, it allows for fat tails on both directions (as well as a bell shape) and it is bounded in $[0, 1]$. In order to make ψ comparable to estimates of annualized discount factors in the literature, we report it in terms of an equivalent annualized value.[19]

In modeling the tax-transfer system, we follow Benabou (2000), Benabou (2002), Heathcote, Storesletten and Violante (2014)and Heathcote and Tsujiyama (2015) by using a parsimonious functional form calibrated to the level and progressivity of the net tax schedule that was introduced by Feldstein (1969). In this formulation, we

[17]We compute the percentiles within each age level from 25 to 60 years old for household heads. Then for each percentile we compute the average labor income across the life-cycle.

[18]In the numerical computations this distribution is discretized in 10 grid points with uniform probability.

[19]We assume that working life spans over 40 periods and that retirement lasts for 20 periods, then

$$\psi_{two-period} = \psi_{annual}^{40} \left(\frac{1 - \psi_{annual}^{21}}{1 - \psi_{annual}^{41}} \right)$$

write net transfers T to individuals as function of taxable income Y :

$$T(Y;\lambda,\tau) = Y - \lambda Y^{1-\tau}$$

where taxable income include both labor earnings and taxable asset income. Estimating the functional parameters off PSID data for 2002–2006, Heathcote, Storesletten and Violante (2014) find that $\tau = 0.151$ and $\lambda = 0.836$ provide a good fit of the current U.S. tax and transfer system among households whose head worked more than 260 hours in the previous year. We adopt their estimates in our calibration.

In addition to the tax-transfer system applicable to individuals' working lives, we model the current retirement savings system with a sequence of accounts that are subject to subsidies and income-specific caps: first, we model the social security old-age benefit using the 2014 replacement ratios and the 2014 \$118,500 cap on eligible labor earnings. Second, we model a 401(k) account as allowing voluntary tax-deferred contributions up to \$18,000 plus 50% matching on these contributions through employers.[20] Third, we model an IRA account as allowing for voluntary tax-deferred contributions up to \$5,500. Finally, we include in our model a regular savings account with a real rate of return of 3.44% following Gourinchas and Parker (2002).

The model is able to match the data considerably well when we calibrate for (a, b, ω). The targets for the calibration are the 25, 50 and 75 percentiles of the distribution of wealth at retirement to lifetime earnings ratios at all four quantiles of the lifetime earnings distribution (12 targets in total). We report in Table 4.1 the fit of the model to the data. As we can see in the table the model is not perfectly able to match the data, but it gets the overall shape very well.

Not surprisingly, we find considerable heterogeneity in discount factors. In Table 4.2 we report the calibration results. We find an average discount factor of 0.985 which

[20]This is the most common matching rate in the U.S. ((Engines, 2015))

Table 4.1: Calibration fit by life time earnings quantile.

Statistic\Quantile	Data Q1	Model Q1	Data Q2	Model Q2
W/Y percentile 25%	0.0165	0.0163	0.0398	0.0367
W/Y percentile 50%	0.0554	0.0899	0.086	0.1021
W/Y percentile 75%	0.1322	0.1431	0.1664	0.1712

Statistic\Quantile	Data Q3	Model Q3	Data Q4	Model Q4
W/Y percentile 25%	0.059	0.0451	0.0593	0.653
W/Y percentile 50%	0.1024	0.1083	0.1248	0.1254
W/Y percentile 75%	0.1726	0.1765	0.2211	0.1839

Source: Author's computations. Objective on calibration is to minimize L2-norm between the model and data statistics.

is slightly higher than the average of 0.96 found by Alan, Browning and Ejrnaes (2014). We also find that the calibration points to considerable heterogeneity in discount factors, but with 90% of mass between 0.9 and 1.0 in annualized terms which is the interval considered in Alan, Browning and Ejrnaes (2014). Finally, we find that the Gaussian copula correlation is negative, indicating that discount factors and ability have a slightly negative correlation. However, we do find a positive correlation between discount factors and labor income as endogenously individuals with a higher discount factor are also willing to work harder.

Table 4.2: Calibration results

Parameter	Calibrated Value
a	0.855
b	1.795
ω	-0.107
$\mathbb{E}(\psi_{annual})$	0.985
90th percentile ψ_{annual}	0.999
10th percentile ψ_{annual}	0.905
$Corr(y, \psi)$	0.10

Source: Author's computations. Shape parameters (a, b) where $\psi \sim Beta(a, b)$ and for the correlation parameter ω between ψ and θ. Discount factor ψ is reported in terms of an equivalent annual discount factor where $\psi_{two-period} = \psi_{annual}^{40}\left(\frac{1-\psi_{annual}^{21}}{1-\psi_{annual}^{41}}\right)$.

Our calibration is broadly in line with other estimates in the literature. Alan, Browning and Ejrnaes (2014) find in PSID data a correlation of -0.02 between dis-

count rates and the fixed effect on the labor income process on their estimation. However, they do find a correlation of -0.76 between the slope of the labor income profile and discount rates. In the behavioral economics literature, although Tanaka, Camerer and Nguyen (2010) found no evidence of a correlation between present-bias and income levels in their experimental setting, Mullainathan, Schwartzstein and Congdon (2012) argues that different types of behavioral biases are more prevalent at the poor (e.g., addiction to cigarettes).

In Appendix D.3 we provide an alternative calibration using an incomplete markets life-cycle model with hyperbolic preferences. That model not only allows for a separate identification of β and δ, but also allows for a precautionary savings motive. We find that the calibrated distribution for $\psi = \beta\delta$ is similar in both calibrations. In fact, the heterogeneity in savings incentives (as measured by the effect time-inconsistency has on savings) is very similar on both models. Therefore, it seems that our benchmark calibration indeed produces a sensible calibration of the heterogeneity in discounting that is consistent with a behavioral explanation of the phenomena and parsimonious at the same time.

In fact, estimates in the behavioral literature for present-bias are largely consistent with the values of β we find in the life-cycle calibration. The distribution of β's we find are within the bounds estimated by Montiel Olea and Strzalecki (2014) on the level of present bias in their experimental data. Finally the average point estimate we find is consistent with the range of estimates in the behavioral economics literature. Laibson, Repetto and Tobacman (2007) find an estimate of 0.7 by calibrating a life-cycle model with hyperbolic preferences. More recently, Augenblick, Niederle and Sprenger (2015) finds that on real effort tasks present bias is 0.88 in a laboratory experiment. Jones and Mahajan (2015) find a present bias of 0.34 in a field experiment providing commitment devices incentivizing savings of income tax returns in the United States.

4.4.2 Quantitative approximation to government problem

In this sub-section we compare the current policy in the U.S. with optimal policies arising from the normative model. To make an appropriate comparison, it is necessary to make sure that we choose a redistributive motive and a discount factor for the government that is consistent with the current levels of redistribution and with current savings policies. In that form, any inconsistency we find between current policy and the normative model prescription is not caused by the particular government preferences we choose.

We parameterize the Pareto weights the government assigns to agents by

$$\lambda(\theta) = \frac{\exp\{-\alpha\theta\}}{\sum_{\theta,\beta} \pi(\theta,\beta)\exp\{-\alpha\theta\}}$$

so that the parameter $\alpha \in \mathbb{R}$ represents the government's redistributive motive (again we follow Heathcote and Tsujiyama (2015) in using this functional form). If $\alpha = 0$ the government is utilitarian and if $\alpha > 0$ the government wants more redistribution than a utilitarian government would. The standard finding in the public finance literature is that the current level of income taxes in the U.S. is consistent with $\alpha < 0$ when ignoring savings policies.[21]

Numerical algorithm for solving two-dimensional screening problem

The literature has recognized that finding numerical solutions to two-dimensional screening problems is generally a difficult task. Many of the techniques that facilitate solving one-dimensional optimization problems fail in a multi-dimensional context. More specifically, we are no longer able to rely on the first-order approach, which reduces the number of relevant constraints to a small set of local incentive compatibility constraints. Instead, solving the problem generally requires the use of all incentive

[21] See Heathcote, Storesletten and Violante (2014) and Heathcote and Tsujiyama (2015).

compatibility constraints in the solution routine. Judd and Su (2006) point out that with the large number of incentive constraint there is a failure of the linear independence constraint qualification that leads conventional optimization routines to fail to find lagrange multipliers correctly.

We contribute to this literature by providing a stable and computationally efficient numerical algorithm to solve our problem even when naive solution approaches are unfeasible. Our algorithm reduces the complexity of the two-dimensional screening problem by searching for a subset of incentive constraints that are sufficient to obtain the solution to the global program. We solve a sequence of relaxed problems with high numerical accuracy for small subsets of incentive constraints.[22] If we find that the solution to this problem is globally incentive compatible, the convexity of the problem guarantees us this is the global solution.[23] This allows us to solve problems with a large number of incentive constraints.

The difficulty is then to find a suitable form of selecting subsets of incentive constraints. In fact, the number of subsets of a constraint set with many constraints is extremely large. Therefore, this whole strategy is only feasible if it is straightforward to converge to the correct set of constraints relatively fast.

It turns out a simple heuristic approach works very well in practice. We start with the first best problem with no constraints. Then we measure the relative violation of each incentive constraint in the original problem. We then add to the relaxed problem a fraction of the incentive constraints with the highest relative violation. We do so in a way that the total number of constraints in the problem does not grow too fast. In particular, we add only a fraction of the difference between the number of variables in the maximization problem and the number of constraints used in the last step. If

[22] In our implementation we use IPOPT to solve each relaxed problem numerically.

[23] Writing the problem in terms of utility levels from consumption and disutility from work, it is easy to see the problem is convex.

this difference is very small, we add one constraint at a time.[24] After the initial step, we drop a random selection of slack constraints used in the relaxed problem.

This algorithm is guaranteed to converge with probability one to the set of incentive constraints that solve the global problem. Since the number of constraints is finite, always adding constraints guarantees convergence. Dropping constraints might generate a problem as it can generate cycles in the search for the correct subset of relevant incentive constraints, randomizing this selection allows us to prevent the algorithm from cycling forever (with probability one). In practice we find this algorithm converges relatively quickly (a few hours) in all our exercises with about 250k incentive constraints.[25]

Results

For any given level of α and $\delta \in (0, 1]$, we solve numerically the government's normative problem and compare the resulting allocation with the allocation arising in the calibrated economy with actual policies. In particular we find the values for the planner's preferences that minimize the L2-norm between the two allocations.[26] In doing so, we follow the approach in Heathcote, Storesletten and Violante (2014); Heathcote and Tsujiyama (2015) in order to obtain a sensible level of redistribution and discounting by the government in our optimal policy computations.

We find that $\alpha = -0.60$ and $\delta_{annual} = 0.974$ better approximate the normative model allocation to the allocation in the calibrated model with current policies, and we find there are unexplored gains from the government's perspective according to the normative model.[27] Further, together with our calibration for ψ this implies

[24] In practice, this happens in intermediary steps when the initial set of constraints used is very different from the solution set.

[25] We have solved problems with up to 10 million constraints using this algorithm using the DELLA computer cluster at Princeton University.

[26] As a robustness check we also found planner's preferences minimizing the consumption equivalent welfare gain between the actual policy and the normative model. Results are qualitatively similar.

[27] We find $\delta = 0.22$ and here we report δ_{annual} which is the discount factor in annual terms.

that the disagreement of agents and the planner, $\beta = \frac{\psi}{\delta}$, is on average $\mathbb{E}(\beta) = 1.42$, that is to say, current policy is well rationalized if on average the government believes individuals are forward biased. Indeed, actual policy displays positive savings taxes and that is only consistent with the normative model if there is forward bias.[28] A consumption equivalent gain of 17.5% is available to the policymaker when we compare the allocation in the previously calibrated model and the approximated normative model.[29]

In Figure 4.5a we plot the proportion of retirement consumption ($\frac{c_2/R}{c_1+c_2/R}$) for the computed level of Pareto weights and discounting for the government in the benchmark normative model. We observe that at low earnings, there is little dispersion in retirement consumption whereas at high earnings there is a higher level of dispersion. In Figure 4.5b we plot the proportion of retirement consumption at the optimal allocation for a utilitarian planner with the same level of discounting.

The level of dispersion at low earnings level is very similar to the benchmark normative model, however there is considerably more dispersion at high earnings. A utilitarian planner is considerably more redistributive than the planner in the benchmark normative model, therefore it will offer considerably more flexibility on savings for high earnings individuals as that is a useful tool in improving redistribution. Conversely, the planner in the benchmark normative model cares considerably about the payoff of high earnings individuals and therefore is particularly worried about their retirement consumption rates. As a result, in the benchmark normative model the planner offers an allocation with a much lower dispersion on the proportion of retirement consumption at high earnings.

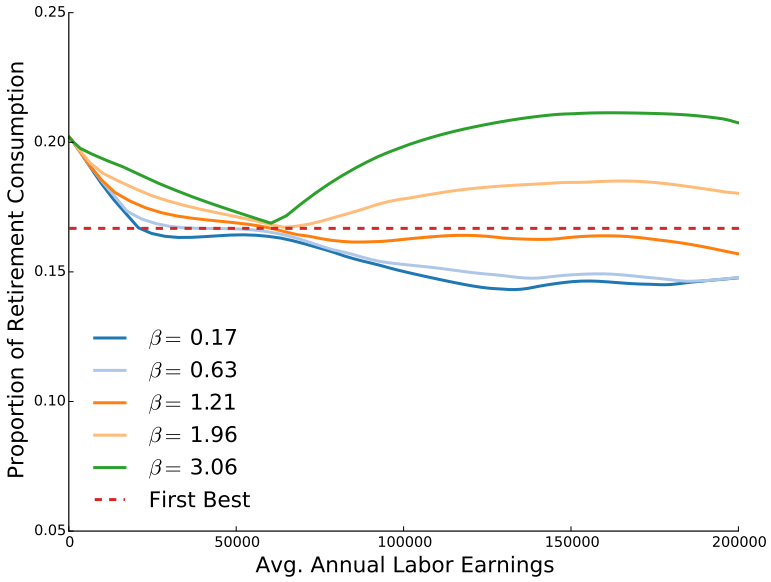
In Figure 4.6 we plot the caps on retirement savings accounts arising from the benchmark normative model. Two accounts are used to a relevant extent and the

[28]Montiel Olea and Strzalecki (2014) indeed find a considerable level of forward bias present in their experiment.

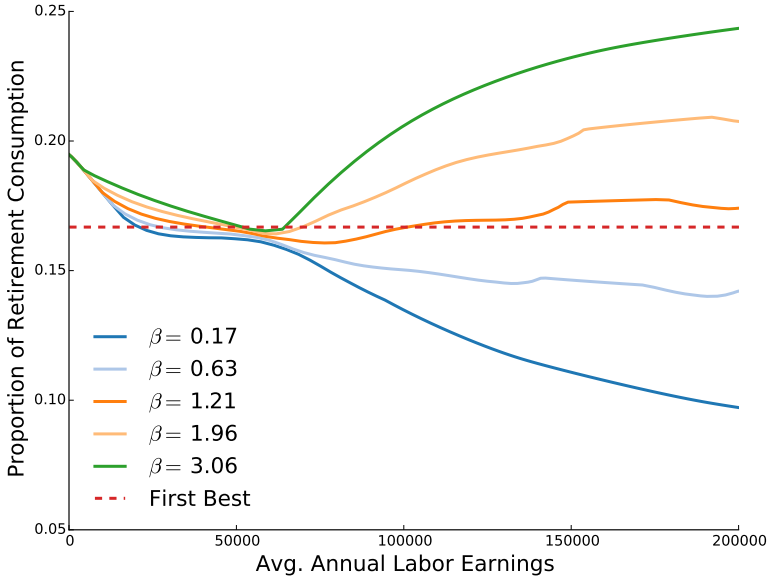
[29]In the robustness check where government gains from optimal policy are minimized, we still find a 13.5% gain available to the policymaker.

Figure 4.5: Proportion of retirement consumption in optimal allocations

(a) Benchmark normative model



(b) Utilitarian planner (same discounting as benchmark)



Source: Author's computations. Proportion of retirement consumption $\frac{c_2/R}{c_1+c_2/R}$ for optimal allocations. On the horizontal axis we have average labor earnings of the individual during its working life and on the vertical axis we have the proportion of retirement consumption at the optimal allocation. Each line in the graph represent the allocation of agents with a different level of β.

Figure 4.6: Caps on retirement savings accounts



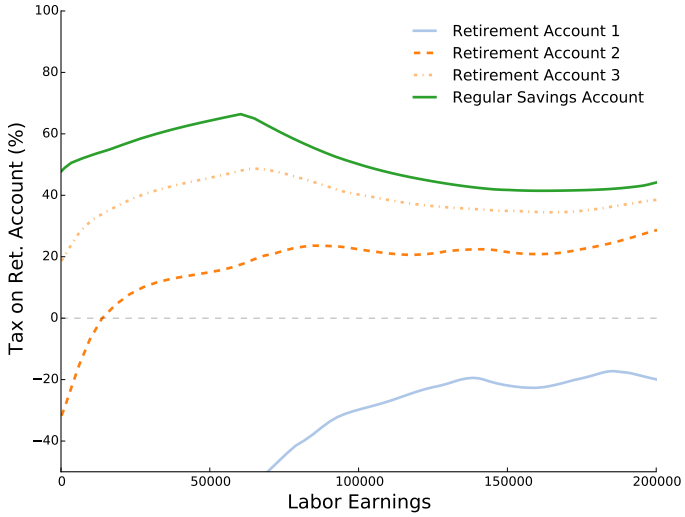
Source: Author's computations. Caps on retirement savings accounts on normative model that better approximate actual policy. On the horizontal axis we have average labor earnings of the individual during its working life and in the vertical axis we have the cap on the contribution to each of the 3 different retirement savings accounts.

caps are well approximated by piecewise-linear functions. Furthermore, low earnings individuals have very little access to these accounts. This points in the direction that optimal policy can be well approximated with simple policy tools.

In Figure 4.7 we plot the optimal levels of taxes (or subsidies if numbers are negative) on savings at the optimal allocation for the different retirement savings accounts. There are high taxes on savings as the planner in the benchmark normative model considers that some individuals are savings too much. There are also considerable tax breaks on the retirement savings accounts. Saving on retirement account 2 entails at least a 10% lower marginal tax rate on the return to savings as compared to savings on the regular savings account.

We now turn into the analysis of social security benefits. In order to understand the difference between current social security benefits and optimal old-age benefits implied by the normative model, it is important to understand how redistribution affects retirement benefits in our model. In Figure 4.8 we compare the current system

Figure 4.7: Tax rates on savings



Source: Author's computations. On the horizontal axis we have average labor earnings of the individual during its working life and in the vertical axis we have the tax rate on savings. Each line represents a different retirement savings account or the regular savings account.

of old-age benefits by earnings level with the normative model implication under the benchmark normative model ($\alpha = -0.6$) and for a utilitarian planner ($\alpha = 0$) with the same levels of δ. We see that a utilitarian government in our normative model would provide benefits with a qualitative shape more in line with the current system.

In order to check whether this wide difference between current retirement benefits and those arising from the normative model, we choose Pareto weights and discounting of the planner to match the current system of retirement benefits. In Table 4.3 we see that the redistribution motive indeed has to be close to utilitarian to explain current social security benefits. But if we take those as the government's preferences, then the available gain for the government of implementing optimal policy would be of around 63%. This highlights the difficulty in reconciling current social security benefits with current redistribution levels in the lenses of our normative model.

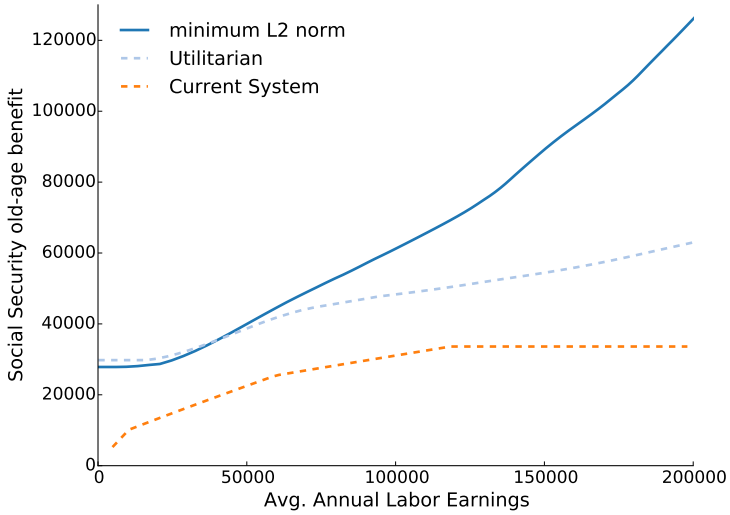


Figure 4.8: Old-age benefits by labor earnings level

Source: Author's computations. Optimal policy for benchmark normative model and utilitarian planner (same discounting level as benchmark model). On the horizontal axis we have average labor earnings of the individual during its working life and in the vertical axis we have the level of social security benefits at retirement. Each line represents the retirement benefits in a different system (current, benchmark normative model (L2 minimum norm) and utilitarian).

Table 4.3: Consumption-equivalent welfare gains

Calibration	annual δ	α	Welfare Gain
Benchmark	0.974	-0.60	17.5%
Fitting SS benefits	0.940	0.15	62.9%

Source: Author's computations. Consumption equivalent welfare gains of moving to optimal policy for the different calibrations of the planner's preferences. The consumption equivalent gain is computed considering a uniform change in consumption during working life and retirement, but no change in labor supply.

Finally, we check how much of the welfare gains available can be obtained with a simple policy change in the current system. We consider the following instruments: tax schedule parameters λ and τ; abolishing the cap on social security benefits and allowing for a multiplier on current benefits by a parameter $\gamma > 0$ at all earnings levels; abolish IRA account and keep only one 401(k) account available with a minimum threshold on earnings for contributions, a cap that is a share of labor earnings and a match rate of 50%. Then using these policy instruments, we maximize the

government's goal (with the same α and δ used in our approximation to current policy).

We find that with these instruments the government (with $\alpha = -0.6$ and $\delta = 0.974$) is able to obtain a sizable consumption equivalent welfare gain of 7%. In this exercise, we find much higher social security benefits ($\gamma = 1.94$) funded by higher income taxes ($\lambda = 0.74$ and $\tau = 0.41$). The minimum income threshold on the 401(k)-like account is \$34,775 average earnings and the cap on matching is 0.6% of labor earnings. As emphasized before, the main source of the gain for this regressive government is the increase in social security benefits to high earnings individuals. Of course, those results are for a particular government preference for redistribution and discounting as opposed to the previous exercise where we argued that there are no government preferences that can explain well both the current level of redistribution and the current level of retirement benefits. However, this final exercise highlights that simple instruments can achieve sizable welfare gains in this economy.

4.5 Extension: Behavioral and Pigouvian policies

Our main insights extend to a variety of behavioral and neoclassical models in which there is disagreement between agents and the government. The key element in the model is that at the time of decision individuals foresee heterogeneous costs (or gains) in their actions that are different than the costs (or gains) the government foresees. This wedge between government's objectives and agent's objectives is a common feature of several economic models.

Mullainathan, Schwartzstein and Congdon (2012) uses an abstract "preference wedge" between the agents and the government to study a variety of models in behavioral economics as well as in classical economics. With behavioral agents a wedge arises from "internalities", which represent inefficient behavior that causes harm to

the agent herself (e.g., present bias).[30]Conversely, in a classical model the wedge is caused by an externality in which the agent cause harm (or benefit) to others. In both cases, agents and the government foresee different costs and gains from agents actions.

In this more general model, our insight implies that optimal policies involve an effective quantity restriction at low earnings whereas high earnings individuals are given more freedom in their choices if there is a redistributive motive. Furthermore, this more abstract setup allow us to show that without redistribution, a quantity restriction implements optimal policy.

There is a continuum of consumers with varying degrees of a generic "preference wedge." Action $a \in [0,1]$ has associated unit cost p and income-equivalent benefit b that depends on a as follows: $b(a) > 0$, $b'(a) > 0$, $b''(a) < 0$. In order to guarantee interior solutions, we also require $b'(0) > 0$ and $b'(1) < p$. Agents act according to their decision utility

$$\mathcal{U}^D(\theta, \alpha) = u(c) + \alpha\varepsilon(a) - \frac{1}{\theta}v(y)$$

where $\alpha\varepsilon(a)$ is a preference wedge and we assume $\varepsilon(a) > 0$, $\varepsilon'(a) > 0$, $\varepsilon''(a) < 0$ and $b(\varepsilon^{-1}(\cdot))$ is weakly concave.

However, experienced utility from consumption c and labor earnings y is given by

$$\mathcal{U}(\theta) = u(c) - \frac{1}{\theta}v(y)$$

where $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $, v''(\cdot) > 0$, $\lim_{y\to 0} v'(y) = 0$. Note that experienced utility depends on the action only through its effect on consumption. For $\alpha = 0$ we have $\mathcal{U}^D = \mathcal{U}$ and there is agreement between the government and the agent. For $\alpha \neq 0$, the agent's private cost (or gain) from action a differs from the social cost (or gain) from his action a. There is unobservable heterogeneity in $(\theta, \alpha) \in$

[30]In fact they show preference wedges arise naturally when agents have incorrect beliefs (over-optimism or over-pessimism) or inattention for example.

$\Theta \times A$ distributed according to π with full support and where $A = \{\alpha_1, \ldots, \alpha_M\}$ and $\Theta = \{\theta_1, \ldots, \theta_N\}$ are finite sets as in our benchmark formulation. An allocation $\{a(\theta, \alpha), c(\theta, \alpha), y(\theta, \alpha)\}_{(\theta,\alpha)\in\Theta\times A}$ is resource compatible if we have

$$\sum_{\theta,\alpha} \pi(\theta, \alpha) [c(\theta, \alpha) + pa(\theta, \alpha) - b(a(\theta, \alpha)) - y(\theta, \alpha)] \leq 0$$

In this model the first best level of the action a does not vary with the agent's type and is independent of redistribution.[31] In fact, at the first best all agent's choose a^* such that

$$b'(a^*) = 1$$

where $a^* \in (0, 1)$ from our previous assumptions.

Laissez-Faire – Without government intervention, agents choose (a, c, y) to maximize $\mathcal{U}^D(\theta, \alpha)$ subject to the budget constraint $c + pa \leq y + b(a)$. In that case, agents with $\alpha \neq 0$ will generally choose $a^{LF}(\theta, \alpha) \neq a^*$ and we have $a^{LF}(\theta, \alpha) \neq a^{LF}(\theta, \alpha')$ for $\alpha \neq \alpha'$.

Quantity restriction - The implementation of a quantity restriction in the laissez-faire economy is efficient to the government that desires no redistribution. In fact, consider a quantity restriction in which the government forces $a = a^*$, but does not implement any redistribution policy so that agent's budget is still given by

$$c + pa^* \leq y + b(a^*)$$

It is then straightforward to show Lemma 4.1.

[31] That was not the case in our benchmark model. In fact, in that model the planner would like an agent that receives more consumption in the initial period to also consume more in the second period.

Lemma 4.1. *(Quantity restriction without redistribution) A quantity restriction of* $a(\theta, \alpha) = a^*$ *such that*

$$b'(a^*) = p$$

is efficient if it is implemented together with no redistribution policy.

Proof. In fact, let Pareto weights be given by $\lambda(\theta) = \frac{1}{u'(c^{LF*}(\theta))}$ where $c^{LF*}(\theta)$ is the level of consumption in the laissez-faire economy with the quantity restriction. □

The model in this section highlights the trade-off between paternalism and redistribution by disentangling the optimal first best level of a for the government from the redistributive motive. In fact, since the government believes preference wedges should be zero, then the first best level of the action a does not depend on the agent's type.

Theorem 4.3. *Fix* θ_N *and assume that* $0 \in A$. *If* $\lambda(\theta)$ *is weakly decreasing in* θ, *then there exists* $\bar{\theta}_1 > 0$ *such that at the solution to the planner's problem*

1. *if* $\theta_1 < \bar{\theta}_1$, *then all agents with types in* $\{(\theta_1, \alpha) : \alpha \in A\}$ *receive the same allocation (quantity restriction)*

2. *if* $\theta_1 < \cdots < \theta_{N-1} \leq \bar{\theta}_N$, *then agents with types in* $\{(\theta_N, \alpha) : \alpha \in A\}$ *do not receive all the same allocation.*

Theorem 4.3 shows that our main insight holds when there is over-consumption and when there is both over-consumption and under-consumption. Even though the signs of the distortions are undetermined in this case (so that Theorem 4.2 does not hold), Theorem 4.1 continues to hold and resulting optimal policies are very strict on low earnings individuals if there is a redistributive motive.

4.5.1 Applications: Environmental and drug policies

We now discuss two concrete applications of this general framework. In the first one, there is heterogeneity in how individuals value externalities generated by pollution of energy inefficient vehicles.[32] Some individuals do care considerably about pollution and buy more energy efficient cars, whereas some other individuals are not particularly concerned with pollution. In the second, the government is concerned about drug usage and wills to reduce its consumption. Again, we assume there is considerable heterogeneity across individuals in preferences toward drug usage at all income levels.

Fuel Efficiency – In the case of energy efficiency, we can think of action $a \in [0,1]$ as the energy efficiency of the vehicle. There is a first best level of energy efficiency $a^* \in (0,1)$. The cost of a vehicle with energy efficiency $a \in (0,1)$ is given by pa. The benefit to society of allocating a vehicle to the agent is given by $b(a)$ in monetary terms which already take into account the cost of pollution. Agents have a preference wedge, in the sense they do not fully internalize the effect of pollution when purchasing a vehicle so that $\alpha < 0$ for some agents (and in general $\alpha \leq 0$).

If there is redistribution in this economy, optimal policy implies that low earnings individuals will purchase all at the same level of high energy efficiency according to the appropriate extension of Theorem 4.2 to this setup.[33] This policy can be implemented with income tax rebates on vehicle purchases that depend both on the level of energy efficiency on the car, but also on the earnings level of individuals. At low earnings levels, individuals receive a very high income tax rebate (or a tax credit) for a highly energy efficient car. At higher earnings levels, the government

[32] Allcott, Mullainathan and Taubinsky (2014) also studies energy policies that deal with externalities.

[33] Since we have $\alpha \leq 0$, that extension implies that at the lowest earnings level we have

$$b'\left(a\left(\theta_L, \alpha\right)\right) < 1$$

allows agents to enjoy energy inefficient vehicles in exchange for lower tax rebates (and hence an improvement in redistribution).

If there is no redistribution in this economy, then Lemma 4.1 implies that $a = a^*$ for all individuals. In this case, the government regulates fuel efficiency directly and forces all agents in the economy to purchase a vehicle with a level a^* of efficiency. We see in this example that the government is willing to trade-off a higher level of externalities from high earnings agents in exchange for an improvement in redistribution.

Drug Policy - In the case of drug policy, the action $a \in [0, 1]$ is drug consumption. The drug has a price of $p > 0$ per unit, and a social monetary benefit $b(a) = 0$ so that from the government's perspective, the optimal level of consumption is $a^* = 0$. However, some individuals disagree with the government and would like to consume drugs so that a $\alpha \geq 0$. Here we assume the government is actually able to control the sale of drugs, which is a rather strong assumption, however we think of it as an important benchmark.

With no redistribution, optimal drug policy is a quantity restriction $a = 0$, however with redistribution drug policy is considerably more heavy handed at low earnings individuals as compared to high earnings individuals. The government can implement optimal policy by offering prescriptions for drug usage whose cost varies with the earnings level of individuals and the quantity of drugs purchased. However, at low earnings levels those prescriptions are prohibitive so that agents are effectively prohibited to use drugs.[34]

4.6 Conclusion

In this paper we develop a normative model of paternalistic policies and show how redistribution plays a key role in shaping those policies. The main insight we find is

[34] Another interpretation of such a policy at low earnings is drug testing of welfare recipients as those with the lowest level of earnings receive transfers.

that optimal policies are very restrictive on the behavior of low earnings individuals, but allow for more flexibility on the behavior of higher earnings individuals. This insight arises from a trade-off between paternalism and redistribution that is present in the model and that varies by earnings levels. Further, this insight implies that optimal retirement savings policies for behavioral agents will involve only social security (forced savings) at low earnings levels, but will involve offering a menu of retirement savings accounts (similar to 401(k) and IRA accounts) to high earners on top of social security benefits.

We show that this insight is very general and hold in behavioral economics models with redistribution. In fact we construct one dynamic extension of the normative model in which agents have hyperbolic preferences and we show our insights hold true. Finally we show this insight holds generally when the government and agents disagree about the cost or gain of an action. This includes both behavioral economics models and also neoclassical models. Our theory implies quantity restrictions on low earnings individuals actions are a more efficient tool than linear taxes in achieving both better decisions from the government's perspective and improving redistribution. However, we show that it is efficient to relax quantity restrictions for high earnings individuals if there is a strong redistributive motive. In fact, by providing more flexibility to high earnings ability individuals, the government creates an extra incentive for them to work hard and actually obtain high earnings in the labor market.

In a quantitative evaluation, we show that actual policy has important differences to efficient policies arising from the normative model. In particular, we find that current social security benefits are consistent with a government that has close to utilitarian preferences, but at the same time the overall system of retirement savings policies and redistribution policies (income taxes and transfers) is better approximated by a more regressive government. This difference relies fundamentally on the heterogeneity in preferences and on two characteristics of the current income tax

code, and the quantitative exercise just highlights those properties. First, there is widespread evidence for heterogeneity in discounting rates and in the level of present-bias. Second, the lack of progressivity in the U.S. tax code is a consensus in the public finance literature. In fact a utilitarian government would implement much higher top income tax rates than the currently implemented in the U.S.(see Saez (2001)). Further, Heathcote and Tsujiyama (2015) find that not only top income taxes are inconsistent with a progressive social planner, but the overall tax structure in the U.S. is considerably more regressive than utilitarian. Finally, the cap on social security benefits for individuals with high earnings implies a considerable level of flexibility on their savings. Those three characteristics of the current economy are inconsistent with the normative model, exactly because a regressive planner is particularly worried about the behavior of high earnings agents and therefore would like to make sure they save on top of the current cap on social security benefits.

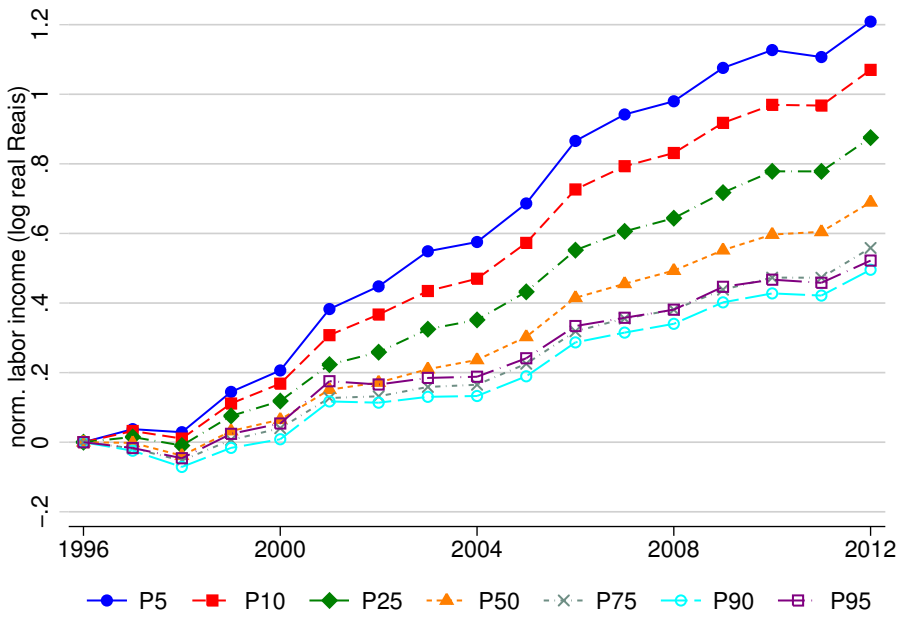
Finally, here we have focused our attention on paternalistic policies arising because of behavioral motives, due to the large body of evidence for present bias in the behavioral economics literature. However, there are important alternative explanations for paternalistic policies that might also play an important role. In particular, Hochman and Rodgers (1969) and Coate (1995) find that altruistic behavior can be used to explain welfare policies. It would be interesting to check if our insight for paternalistic policies arising from behavioral biases would also hold in a setting where paternalistic policies arise for altruistic reasons and a Samaritan's dilemma.

Appendix A

Chapter 1 Supplemental Material

A.1 Empirics

Figure A.1: Real earnings levels evolution



Appendix B

Chapter 2 Supplemental Material

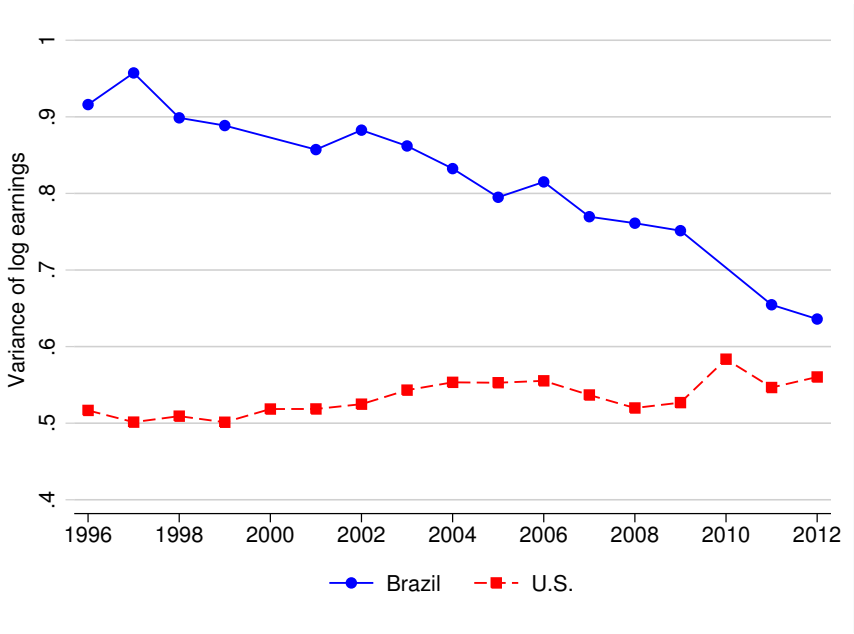
B.1 Empirics

Inequality trends in Brazilian and U.S. household survey data

To put the magnitude of Brazil's inequality decline into context, Figure B.1 plots the evolution of a common inequality measure, the variance of log earnings, from 1996–2012. Data for Brazil come from the largest national household survey, the *Pesquisa Nacional por Amostra de Domicílios (PNAD)*. Data for the U.S. are based on the *March Current Population Survey (CPS)*. In both datasets, earnings inequality is computed over log earnings for male and female labor market participants of age 18–64. The income concept is taken to be labor earnings in the week preceding the survey, and the top and bottom 1% of all observations are dropped to control for outliers.

Figure B.1 shows that while the variance of log earnings in the Brazilian household survey dropped by 27 log points from 1996 to 2012, it rose by six log points in the U.S. household data over the same period. Thus, Brazil's inequality decline is of a relatively large magnitude, both within the Brazilian context and in the comparison with the U.S. experience.

Figure B.1: Evolution of variance of log earnings in Brazil and the U.S., 1996–2012



Dataset descriptions

Table B.1: PNAD summary statistics, by period

		Log earnings		
	(1)	(2)	(3)	(4)
	# Workers	Mean	Std. dev.	Formal share
1996–2000	251,724	6.78	2.75	0.64
2000–2004	290,407	6.63	2.94	0.63
2004–2008	385,495	6.71	2.67	0.67
2008–2012	290,789	6.89	2.00	0.72

Notes: All statistics are for male workers age 18–64 and pooled within 5-year periods. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers.Means are computed by period. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period. Surveys are not available in years 2000 and 2010.

Table B.2: RAIS summary statistics, by period

			Log earnings	
	(1)	(2)	(3)	(4)
	# Worker-years	# Workers	Mean	Std. dev.
1996-2000	92.7	28.8	1.27	0.85
2000-2004	105.3	32.5	1.07	0.80
2004-2008	126.9	37.3	0.88	0.75
2008-2012	154.2	43.9	0.80	0.72

Notes: The number of worker-years and number of unique workers are reported in millions. Statistics on earnings are in multiples of the current minimum wage. All numbers reported are for adult male workers. Means are computed by period.The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period.

Table B.3: PIA summary statistics, by period

			Log revenues per worker		Log value added per worker	
	(1)	(2)	(3)	(4)	(5)	(6)
	# Firm-years	# Unique firms	Mean	S.d.	Mean	S.d.
1996-2000	110,480	34,768	11.85	1.04	11.15	1.13
2000-2004	130,650	40,916	11.98	1.23	11.19	1.32
2004-2008	156,455	48,771	12.02	1.32	11.22	1.34
2008-2012	176,830	55,784	12.06	1.27	11.30	1.31

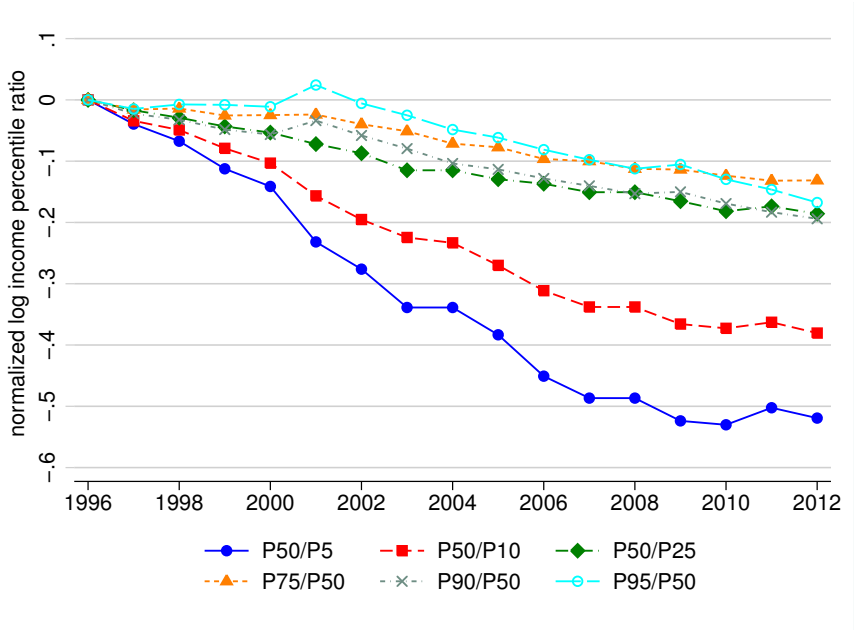
Notes: Population includes all firms covered by the PIA dataset in the mining and manufacturing sectors. All means and standard deviations are weighted by the number of employees. The standard deviation is calculated by first demeaning variables by year and then pooling the years within a sub-period.

Additional facts about Brazil's inequality decline

Fact B.1. *The inequality decline featured compression up to the 90th percentile of the earnings distribution. Yet all parts of the distribution experienced earnings growth between 1996 and 2012.*

Figure B.2 plots the evolution of normalized (to zero in 1996) log percentile ratios, all relative to the median of the earnings distribution. There was pronounced catch-up throughout most of the earnings distribution, but more rapidly between the median and the bottom percentiles, as seen by the drop of the bottom percentile ratios relative those at the top. In fact, we see that above the 90th percentile there was little or no compression, evidenced by the log percentile ratio lines coinciding in the graph.

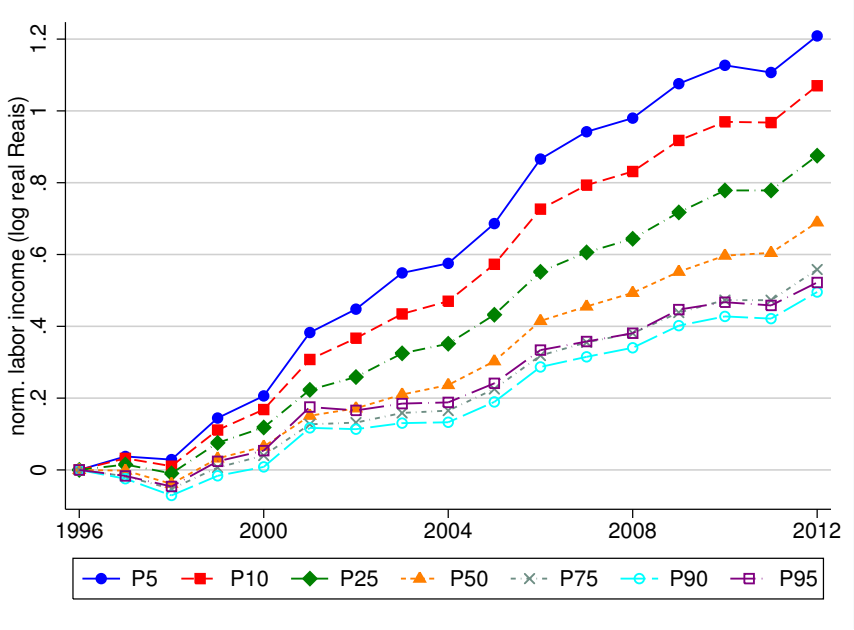
Figure B.2: Normalized evolution of earnings percentile ratios, 1996–2012



While there was compression throughout most of the earnings distribution, all workers experienced rapid earnings growth over the period. The left panel of Figure B.3 plots various percentile ratios of the raw earnings distribution with all ratios being relative to the 90th percentile and normalized to zero in 1996, using the RAIS data. Note that since these are inequality measures (the lower percentile is always

in the denominator), a declining line implies lower inequality or, more concretely, compression relative to the 90th percentile. The figure shows that there was a compression up to the 90th percentile of the earnings distribution, with lower income groups growing monotonically faster in relative terms. The right panel of the figure shows that this compression happened at the same time that all income percentiles experienced growth in real income relative to their 1996 starting point.

Figure B.3: Normalized evolution of earnings percentiles, 1996–2012



Fact B.2. *Most initial earnings inequality and most of its decline are between firms.*

Following a growing literature highlighting the importance of firms in wage setting, we investigate this potential explanation using the employer ID contained in the RAIS data.

Let y_{ijt} denote earnings of worker i employed by firm j in year t, then:

$$y_{ijt} = \underbrace{\overline{y_t}}_{\text{economy average}} + \underbrace{\left(\overline{y}_t^j - \overline{y_t}\right)}_{\text{employer deviaion}} + \underbrace{\left(y_{ijt} - \overline{y}_t^j\right)}_{\text{worker deviation}}$$

Re-arranging and taking variances on both sides we get

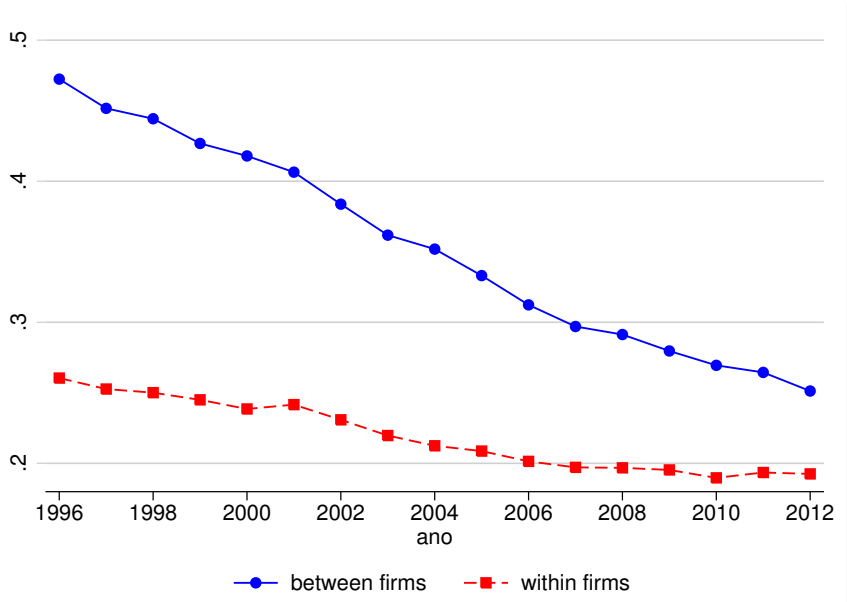
$$Var\left(y_{ijt} - \overline{y_t}\right) = Var\left(\overline{y}_t^j - \overline{y_t}\right) + Var\left(y_{ijt} - \overline{y}_t^j\right) + \underbrace{2Cov\left(\overline{y}_t^j - \overline{y_t}, y_{ijt} - \overline{y}_t^j\right)}_{=0}$$

Simplifying, we have

$$Var\left(y_{ijt}\right) = \underbrace{Var\left(\overline{y}_t^j\right)}_{\text{between firms}} + \underbrace{\overline{Var\left(y_{ijt} \middle| i \in j\right)}}_{\text{within firms}}$$

Figure B.4 plots the results of this decomposition, showing that most initial inequality and most of the decline are in earnings differences across firms.

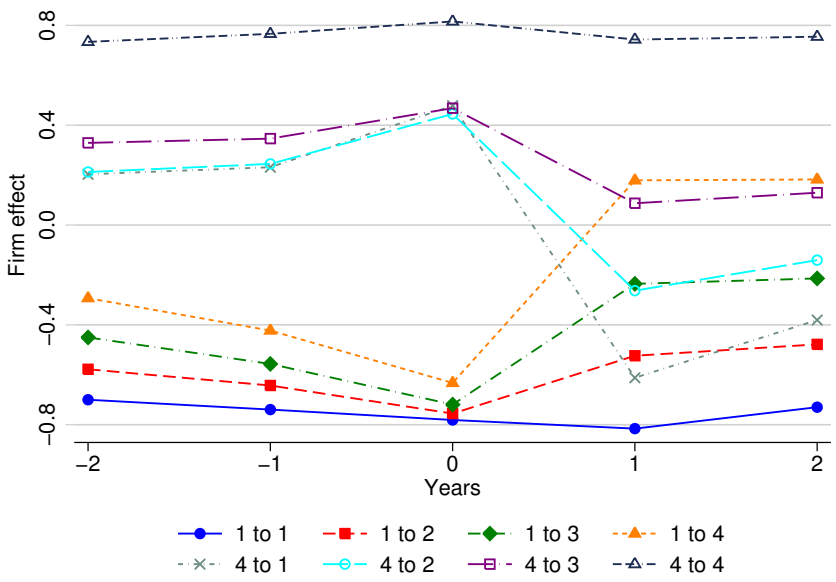
Figure B.4: Between-firm vs. within-firm inequality

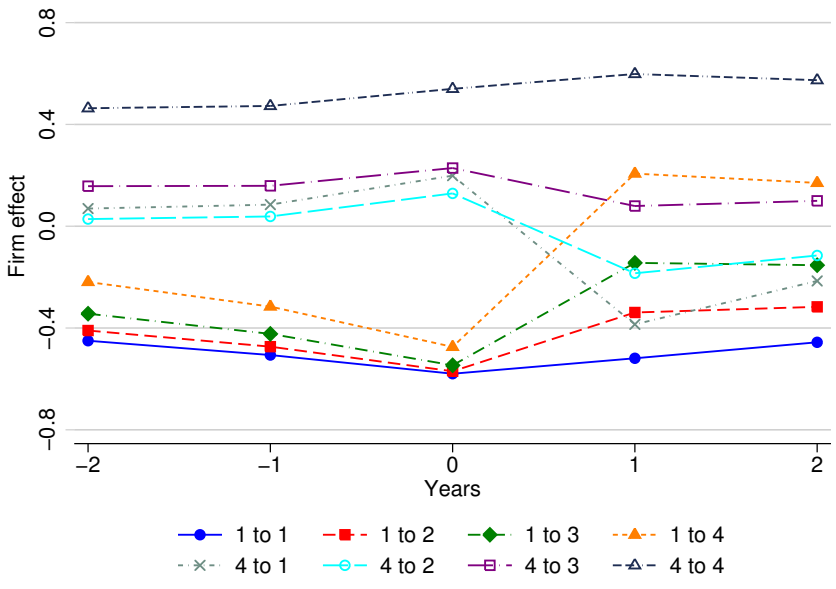


Robustness checks for AKM framework

Figure B.5: Event study graph for switchers between estimated AKM firm effects quartiles

(a) 1996–2000

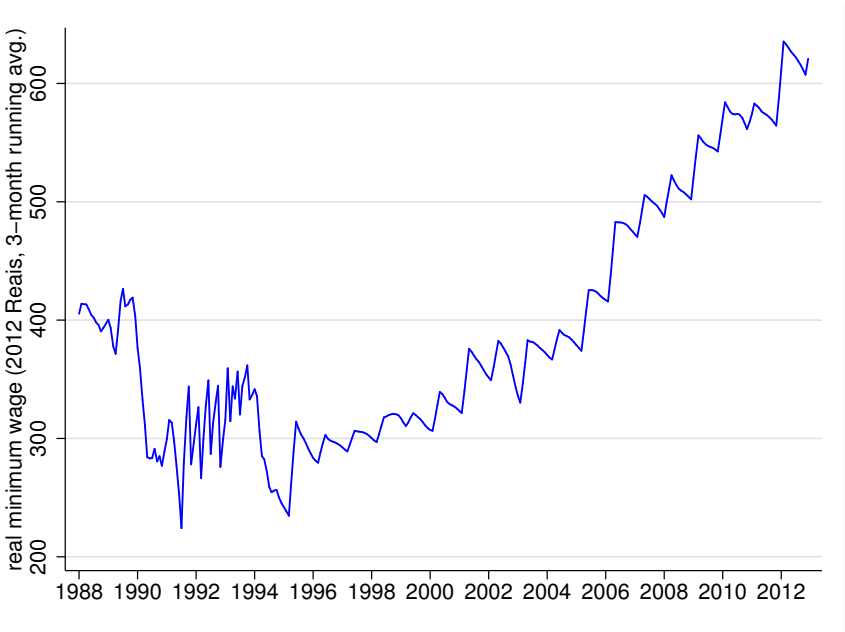




(b) 2008–2012

Evolution of the real minimum wage in Brazil

Figure B.6: Evolution of the real minimum wage in Brazil, 3-month running averages (Source: IPEA)



Distributions of cleaned productivity measures

Fact 2.2 of Section 2.3 argued that there was an increase in productivity dispersion across firms as measured by the variance of employee-weighted value added per worker. But, similar to the reason why we opted for the AKM framework on the wage side, a concern with this statement is that differences in the composition of heterogeneous across firms may hinder inference about underlying firm productivity, which is often regarded as an important pay-relevant firm characteristic (Blanchflower, Oswald and Sanfey, 1996; Abowd, Kramarz and Margolis, 1999; Margolis and Salvanes, 2001). To address this concern, we clean the raw productivity measure in the PIA data, namely reported value added per worker, in three alternative ways: first, controlling for only observable worker demographics including age and education; second,

controlling for worker demographics and the estimated unobservable worker characteristics obtained from the AKM wage regression; and third, controlling for observable demographics and industry (Bartelsman, Haltiwanger and Scarpetta, 2013). The following figures compare the raw productivity measure and its three cleaned versions in the cross-section (Figure B.7) and in the time-series (Figure B.8).

A noteworthy feature of the cross-sectional comparison in Figure B.7 is that the cleaned productivity measures are more concentrated relative to the raw measure, consistent with our previous finding that there is positive sorting of workers across firms along both observable and unobservable dimensions. Furthermore, the various cleaned productivity measures show comparable levels of dispersion and a similar shape overall.

Figure B.8 shows that the time series evolution is also qualitatively the same across all productivity measures: while the various cleaning procedures reduce the overall dispersion, we still find that the variance of each measure is increasing between 1996 and 2012. For example, the increase in the variance of raw productivity is 0.35 log points (or 30 percent) between 1996 and 2012, while the increase in the variance of productivity cleaned by only worker demographics is 0.15 log points (or 23 percent) over the same period.

Figure B.7: Cross-sectional comparison of various productivity measures in 2004

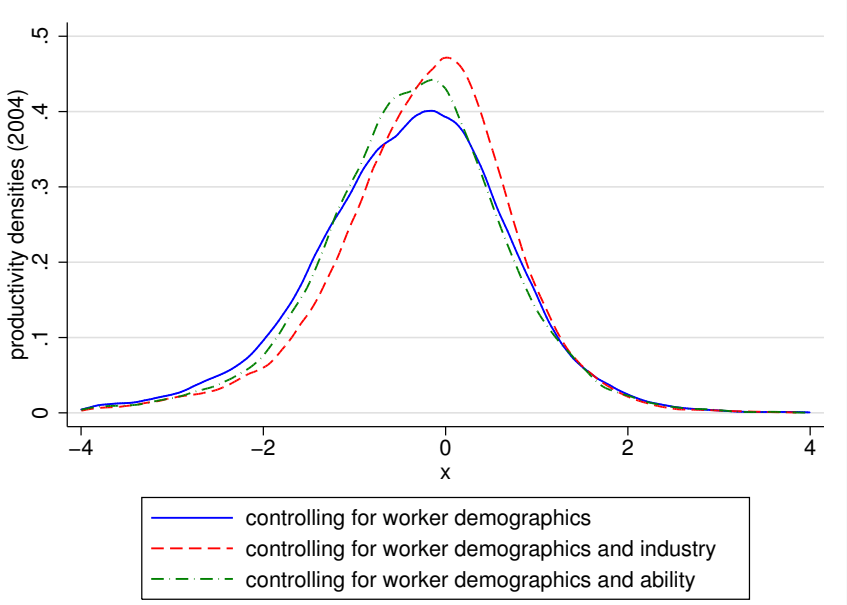
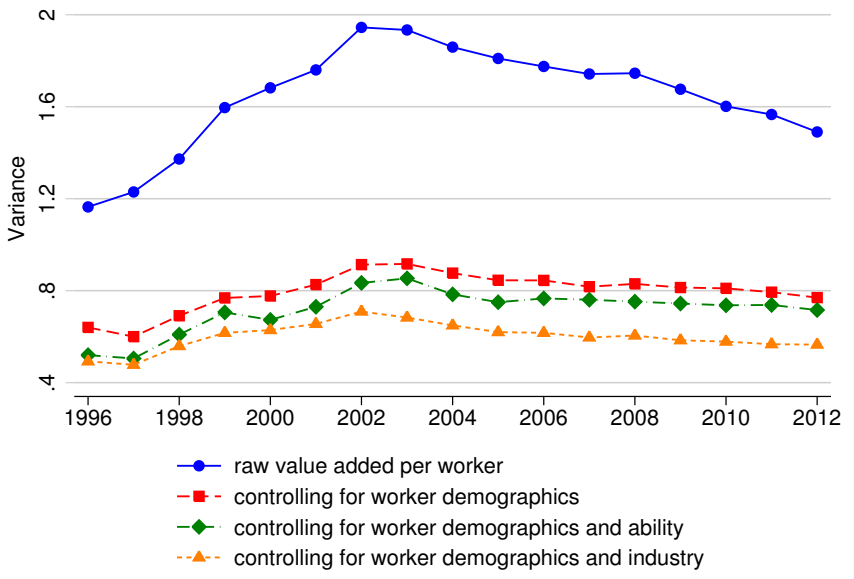


Figure B.8: Time-series comparison of variance of of various productivity measures



B.2 Proofs

Proof of Lemma 2.1

We proceed in order:

1. Because of the minimum wage, workers from markets with $\theta < w^{min}/p$ can not be hired at positive profits by a firm with productivity p. Conversely, since workers from markets with $\theta \geq w^{min}/p$ produce positive profits when working at firm p, that firm will want to attract as many workers as possible from that group.

2. The proof from Burdett and Mortensen (1998) applies to each of our submarkets. The equilibrium wage mapping follows from firms' profit maximization and applying the envelope theorem. The equilibrium wage offered by a firm of productivity p in labor market θ satisfies

$$w\left(p,\theta;w^{min}\right)=\underset{w\geq w^{min}}{\arg\max}\,\frac{p-w}{\left[1+\kappa_e(1-F_\theta(w))\right]^2}$$

By use of the envelope theorem, this implicit relation defines a unique mapping from productivities to wages given by

$$w\left(p,\theta;w^{min}\right)=p-\left[1+\kappa_e(1-F_\theta(p))\right]^2\int_{w^M}^{p}\frac{1}{\left[1+\kappa_e(1-F_\theta(x))\right]^2}dx$$

To prove that $w(p)$ is strictly increasing in p. Specifically, for any two productivity levels $p_1 > p_2$ profit maximization yields:

$$(p_1-w_1)\,l\,(w_1)>(p_1-w_2)\,l\,(w_2)>(p_2-w_2)\,l\,(w_2)>(p_2-w_1)\,l\,(w_1)$$

Subtracting the last from the first term, and subtracting the third from the second term, we get:

$$\begin{aligned}(p_1 - w_1)\, l(w_1) - (p_2 - w_1)\, l(w_1) &> (p_1 - w_2)\, l(w_2) - (p_2 - w_2)\, l(w_2) \\ \Rightarrow \quad l(w_1) &> l(w_2) \\ \Rightarrow \quad w_1 &> w_2\end{aligned}$$

where the last line is a consequence of the fact that $l(\cdot)$ is strictly increasing in w:

$$l(w;\theta) = m_\theta (1 - u_\theta) \frac{dG_\theta(w)}{dF_\theta(w)} = (1 - u_\theta)\, m_\theta \frac{1 + \kappa^e}{\left[1 + \kappa^e (1 - F_\theta(w))\right]^2}$$

Uniqueness of equilibrium in pure strategies and the inverted mapping from wages into productivity follows the proof in Bontemps, Robin and van den Berg (1999, 2000). This concludes the proof of Lemma 2.1.

Proof of Proposition 2.1

Without a binding minimum wage, the piece rate paid by a firm p is given by

$$r(p, \theta; w^{min}) = p - \int_{p_0}^{p} \left[\frac{1 + \kappa^e (1 - \Gamma(p))}{1 + \kappa^e (1 - \Gamma(x))}\right]^2 dx$$

which is independent of θ.

Suppose the minimum wage is binding, then

$$
\begin{aligned}
&\frac{\partial r(p,\theta;w^{min})}{\partial w^{min}} \\
= &\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(\frac{w^{min}}{\theta}\right)\right)}\right]^2 \\
&-\int_{\frac{w^{min}}{\theta}}^{p} 2\frac{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\times\left[-\gamma\left(\frac{w^{min}}{\theta}\right)\frac{1}{\theta}\right] \\
&\times\frac{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)-\left[1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)\right]}{\left[1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)\right]^2}dx \\
= &\frac{1}{\theta}\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(\frac{w^{min}}{\theta}\right)\right)}\right]^2 \\
&+\frac{2\kappa^e\gamma\left(\frac{w^{min}}{\theta}\right)}{\theta}\int_{\frac{w^{min}}{\theta}}^{p}\frac{1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{\left[1-\Gamma\left(\frac{w^{min}}{\theta}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)\right]^3}\left(\Gamma(p)-\Gamma\left(x\right)\right)dx
\end{aligned}
$$

Clearly, both terms in the above expression are positive. This concludes the proof of Proposition 2.1.

Proof of Propositions 2.2 and 2.3

Assume that $p\sim U\left(\underline{p},\overline{p}\right)$. Then we can write the piece rate $\tilde{w}$ offered by a firm with productivity p in market θ as

$$
\begin{aligned}
\frac{\tilde{w}\left(p,\theta;w^{min}\right)}{\theta} &= p-\int_{\underline{p}(\theta;w^{min})}^{p}\left[\frac{1+\kappa^e\left(1-F_\theta\left(p\right)\right)}{1+\kappa^e\left(1-F_\theta\left(x\right)\right)}\right]^2 dx \\
&= p-\int_{\underline{p}(\theta;w^{min})}^{p}\left[\frac{1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\underline{p}(\theta;w^{min})}\right)}{1+\kappa^e\left(\frac{\overline{p}-x}{\overline{p}-\underline{p}(\theta;w^{min})}\right)}\right]^2 dx
\end{aligned}
$$

From here, we consider two cases.

Case 1. $\theta \leq \frac{w^{min}}{\tilde{b}}$ In this first case, for markets affected by the minimum wage, we can write:

$$
\begin{aligned}
&\frac{\tilde{w}\left(p,\theta;w^{min}\right)}{\theta} \\
&= p - \int_{w^{min}/\theta}^{p} \left[\frac{1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-w^{min}/\theta}\right)}{1+\kappa^e\left(\frac{\overline{p}-x}{\overline{p}-w^{min}/\theta}\right)}\right]^2 dx \\
&= p - \left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-w^{min}/\theta}\right)\right]^2 \int_{w^{min}/\theta}^{p} \frac{dx}{\left[1+\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)\overline{p}-\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)x\right]^2} \\
&= p - \left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-w^{min}/\theta}\right)\right]^2 \left\{\frac{1}{\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)\left[1+\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)(\overline{p}-x)\right]}\right\}_{x=w^{min}/\theta}^{p} \\
&= p - \left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-w^{min}/\theta}\right)\right]^2 \\
&\quad \times \left\{\frac{1}{\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)\left[1+\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)(\overline{p}-p)\right]} - \frac{1}{\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)\left[1+\left(\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\right)(\overline{p}-w^{min}/\theta)\right]}\right\} \\
&= p - \left\{\frac{1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-w^{min}/\theta}\right)}{\frac{\kappa^e}{\overline{p}-w^{min}/\theta}} - \frac{\left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-w^{min}/\theta}\right)\right]^2}{\frac{\kappa^e}{\overline{p}-w^{min}/\theta}\left(1+\kappa^e\right)}\right\} \\
&= p - \left(\frac{\overline{p}-\frac{w^{min}}{\theta}+\kappa^e\left(\overline{p}-p\right)}{1+\kappa^e}\right)\left(\frac{p-\frac{w^{min}}{\theta}}{\overline{p}-\frac{w^{min}}{\theta}}\right)
\end{aligned}
$$

Case 2. $\theta \leq \frac{w^{min}}{\tilde{b}}$ In this second case, for markets affected by the minimum wage, we have:

$$
\begin{aligned}
&\frac{\tilde{w}\left(p,\theta;w^{min}\right)}{\theta} \\
&= p - \int_{\tilde{b}}^{p} \left[\frac{1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\tilde{b}}\right)}{1+\kappa^e\left(\frac{\overline{p}-x}{\overline{p}-\tilde{b}}\right)}\right]^2 dx \\
&= p - \left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\tilde{b}}\right)\right]^2 \int_{\tilde{b}}^{p} \frac{dx}{\left[1+\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)\overline{p}-\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)x\right]^2} \\
&= p - \left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\tilde{b}}\right)\right]^2 \left\{\frac{1}{\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)\left[1+\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)(\overline{p}-x)\right]}\right\}_{x=\tilde{b}}^{p} \\
&= p - \left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\tilde{b}}\right)\right]^2 \left\{\frac{1}{\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)\left[1+\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)(\overline{p}-p)\right]} - \frac{1}{\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)\left[1+\left(\frac{\kappa^e}{\overline{p}-\tilde{b}}\right)\left(\overline{p}-\tilde{b}\right)\right]}\right\} \\
&= p - \left\{\frac{1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\tilde{b}}\right)}{\frac{\kappa^e}{\overline{p}-\tilde{b}}} - \frac{\left[1+\kappa^e\left(\frac{\overline{p}-p}{\overline{p}-\tilde{b}}\right)\right]^2}{\frac{\kappa^e}{\overline{p}-\tilde{b}}\left(1+\kappa^e\right)}\right\} \\
&= p - \left(\frac{\overline{p}-\tilde{b}+\kappa^e\left(\overline{p}-p\right)}{1+\kappa^e}\right)\left(\frac{p-\tilde{b}}{\overline{p}-\tilde{b}}\right)
\end{aligned}
$$

Thus, we can write the wages offered at any firm in the economy with a minimum wage as follows:

$$
w\left(p,\theta;w^{min}\right) = \begin{cases} p\theta - \theta\left(\frac{\overline{p}-\frac{w^{min}}{\theta}+\kappa^e(\overline{p}-p)}{1+\kappa^e}\right)\left(\frac{p-\frac{w^{min}}{\theta}}{\overline{p}-\frac{w^{min}}{\theta}}\right) & \text{for } \theta \leq \frac{w^{min}}{\tilde{b}} \\ p\theta - \theta\left(\frac{\overline{p}-\tilde{b}+\kappa^e(\overline{p}-p)}{1+\kappa^e}\right)\left(\frac{p-\tilde{b}}{\overline{p}-\tilde{b}}\right) & \text{otherwise} \end{cases}
$$

Taking derivatives of this expression:

$$
\begin{aligned}
\frac{\partial w\left(p,\theta;w^{min}\right)}{\partial p} &= \begin{cases} \frac{2\theta\kappa^e\left(p-\frac{w^{min}}{\theta}\right)}{(1+\kappa^e)\left(\overline{p}-\frac{w^{min}}{\theta}\right)} > 0 & \text{for } \theta \leq \frac{w^{min}}{\tilde{b}} \\ \frac{2\theta\kappa^e\left(p-\tilde{b}\right)}{(1+\kappa^e)\left(\overline{p}-\tilde{b}\right)} > 0 & \text{otherwise} \end{cases} \\
\partial\left[\frac{\partial w\left(p,\theta;w^{min}\right)}{\partial p}\right]/\partial w^{min} &= \begin{cases} -\frac{2\kappa^e(\overline{p}-p)}{(1+\kappa^e)\left(\overline{p}-\frac{w^{min}}{\theta}\right)^2} < 0 & \text{for } \theta \leq \frac{w^{min}}{\tilde{b}} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

To prove the second part of the proposition, consider two worker types θ_i and θ_j with $\theta_i > \theta_j$ and a firm p active in both markets. Suppose a binding minimum wage is imposed and consider the difference in the firm component of pay between firm p to the two types of workers

$$\begin{aligned}
& r\left(p, \theta_j; w^{min}\right) - r\left(p, \theta_i; w^{min}\right) \\
= & \int_{\frac{w^{min}}{\theta_i}}^{p} \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 dx \\
& - \int_{\frac{w^{min}}{\theta_j}}^{p} \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 dx \\
> & \int_{\frac{w^{min}}{\theta_j}}^{p} \left\{ \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 \right. \\
& \left. - \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 \right\} dx
\end{aligned}$$

It is hence sufficient to show that for $x \in \left[\frac{w^{min}}{\theta_j}, p\right]$:

$$\begin{aligned}
\left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_i}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 & \geq \left[\frac{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(p\right)\right)}{1-\Gamma\left(\frac{w^{min}}{\theta_j}\right)+\kappa^e\left(1-\Gamma\left(x\right)\right)}\right]^2 \\
\iff \quad \Gamma\left(\frac{w^{min}}{\theta_j}\right)\left[\Gamma\left(p\right)-\Gamma(x)\right] & \geq \Gamma\left(\frac{w^{min}}{\theta_i}\right)\left[\Gamma\left(p\right)-\Gamma(x)\right]
\end{aligned}$$

For $x = p$ this inequality is clearly satisfied. For any $x < p$, since by assumption $\theta_i > \theta_j$ it follows that $\Gamma\left(\frac{w^{min}}{\theta_j}\right) \geq \Gamma\left(\frac{w^{min}}{\theta_i}\right)$ by virtue of Γ being a CDF.

To prove the final part of the proposition, note that

$$\mathbb{E}_{\theta_i}\left(p; w^{min}\right) = \mathbb{E}\left(p | \, p \geq \max\left\{\frac{w^{min}}{\theta_i}, p_0\right\}\right)$$

Clearly,

$$\theta_i > \theta_j \Longrightarrow \mathbb{E}\left(p|\, p \geq \max\left\{\frac{w^{min}}{\theta_i}, p_0\right\}\right) \leq \mathbb{E}\left(p|\, p \geq \max\left\{\frac{w^{min}}{\theta_j}, p_0\right\}\right)$$

This concludes the proof of Propositions 2.2 and 2.3.

B.3 Estimation

First stage. In the pre-stage of our estimation procedure, we use panel information on job duration and worker flows together with non-parametric estimates of conditional earnings distributions in order to infer the key labor frictions parameter. This key parameter is $\kappa^e = \lambda^e/\delta$, the ratio of the on-the-job offer arrival rate to the exogenous separation rate. While all later parameters will depend on the estimated degree of search frictions embodied in κ^e, the latter parameter is determined only by the relative ranks of firms and information on worker job mobility. This allows us to separately estimate κ^e before proceeding to the main stage of our estimation procedure.

In connecting κ^e from the model to the data, it turns out that the parameter is over-identified and can thus be estimated off different sets of empirical moments. Following the literature (Jolivet, Postel-Vinay and Robin, 2006), we estimate κ^e in three alternative ways:

1. Using a model prediction of the following linear relationship between the average duration of employment at a given wage and the cumulative distribution of wages up to that point:

$$\overline{d}\,(w) = \frac{1}{\delta}\left(\frac{1}{1+\kappa^e} + \frac{\kappa^e}{1+\kappa^e}G\,(w)\right)$$

By means of a linear regression of $\overline{d}(w)$ on $G(w)$ we can then recover the coefficient of interest as

$$\hat{\kappa}^e_{duration} = \frac{\hat{a}_1}{\hat{a}_0}$$

where hats denote estimates from an ordinary least squares regression and

$$\begin{aligned} a_0 &= \frac{1}{\delta}\frac{1}{1+\kappa^e} \\ a_1 &= \frac{1}{\delta}\frac{\kappa^e}{1+\kappa^e} \end{aligned}$$

2. Purely non-parametrically using the model-implied relationship between the wage offer distribution $F(w)$ and the realized wage distribution $G(w)$. While the latter can be estimated directly using a kernel density approximation[1] of the empirical wage distribution, $\hat{G}(w)$, the former must be inferred from the wage distribution of workers just hired out of unemployment, $F^0(w)$. The nonparametric estimate of the search parameter is then

$$\hat{\kappa}^e_{nonparametric} = \frac{\hat{F}^0(w) - \hat{G}(w)}{\left(1 - \hat{F}^0(w)\right)\hat{G}(w)}$$

3. Using a nonlinear least squares estimate of the relation between nonparametric estimates of the wage distribution of workers just recruited from either unemployment or another firm, $G_m(w)$, relative to the overall realized wage distribution, $G(w)$:

$$\hat{G}_m(w) = \frac{\log\left(1 + \hat{\kappa}^e_{nonlinear}\hat{G}(w)\right)}{\log\left(1 + \hat{\kappa}^e_{nonlinear}\right)}$$

It is worth highlighting that all three estimation strategies above use different dimensions of the RAIS data to identify the key parameter κ^e, including a mix of

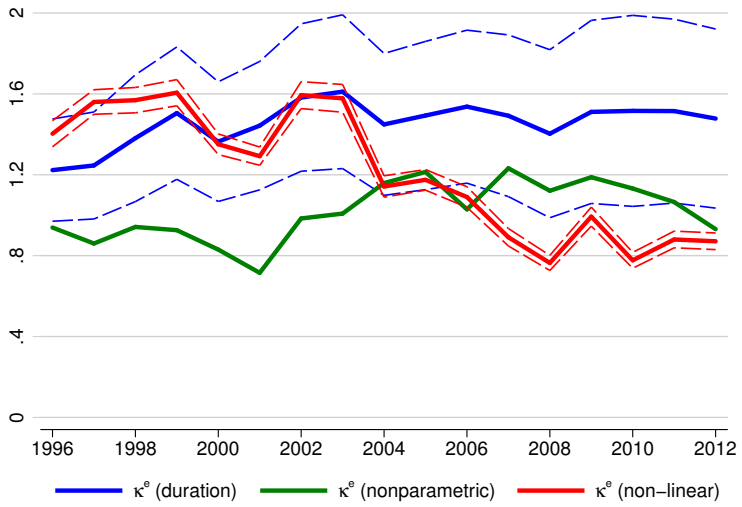
[1] In practice, we use an Epanechnikov kernel with bandwidth 0.04 and 90 bins although we tried alternative kernel, bandwidth, and bin number choices without significant effects on our estimation results.

cross-sectional and longitudinal moments. If the model is well specified these different estimation strategies should yield similar results.

For identification of the key search parameter as well as other basic parameters relating to labor mobility, we use the variables dating workers' dates of accession and separation in the RAIS data in order to convert the dataset to a monthly panel. From this large panel, we draw a 10% random sample of worker IDs, which we use to construct all subsequent labor flow statistics.

Results of the three identification procedures are summarized in Figure B.9.

Figure B.9: Estimation results for key search friction parameter κ^e



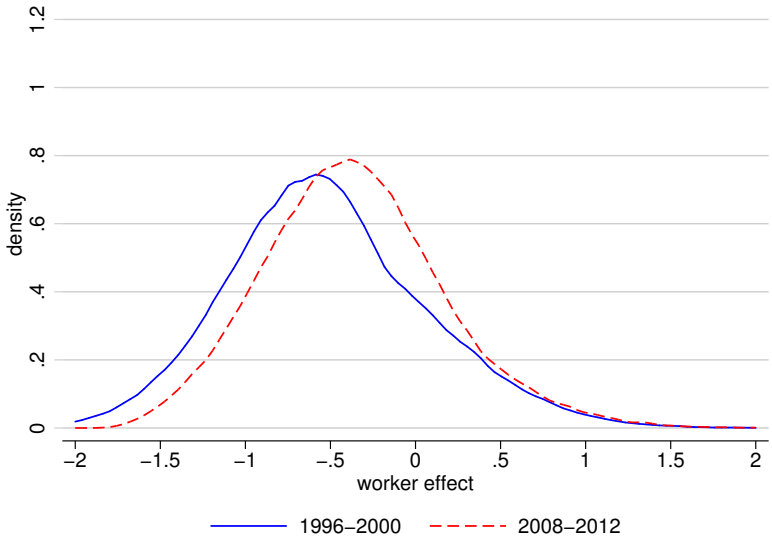
Note: Solid lines denote annual point estimates. Dashed lines denote 95% confidence intervals.

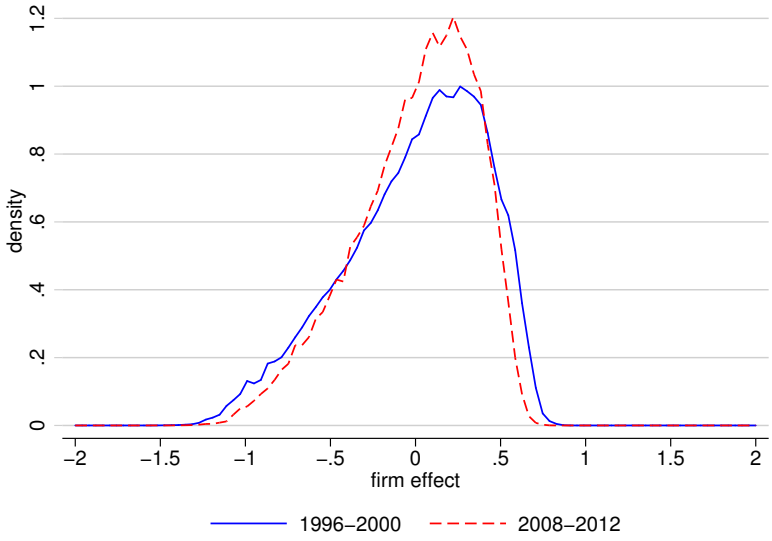
Second stage. In the second stage of our estimation procedure, we take as given the estimate of κ^e from the previous section. We then use a full simulated method of moments procedure to infer distributions of worker ability and firm productivity to match our empirical estimates of worker and firm effect estimates from the AKM decomposition for the period 1996–2000. Remaining details of the estimation procedure are as described in Section 2.6.

B.4 Auxiliary results

Figure B.10: Effect of the minimum wage on AKM estimates in the model

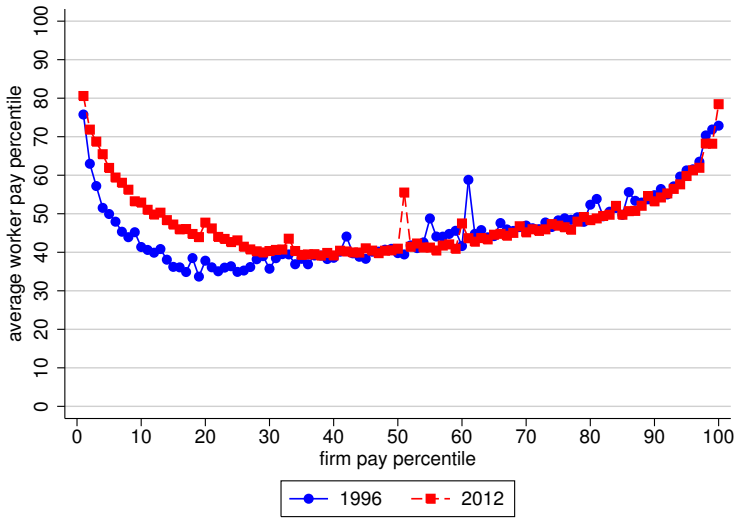
(a) Worker effects





(b) Firm effects

Figure B.11: Sorting pattern between worker and firm effects, complete firm effects distribution



Appendix C

Chapter 3 Supplemental Material

C.1 Theory

Alternative credit constraint specifications

With government intervention, the credit constraint specification depends on assumptions on contract enforceability, though switching between these specifications does not affect my results qualitatively. I here present two alternative constraints. First, if government policies may be evaded, the set of enforceable contracts is defined by the following Proposition.

Proposition C.1. *Labor utilization l by an entrepreneur with wealth a, productivity z and idiosyncratic tax-subsidy schedule (τ, ξ) is enforceable with tax-avoidance if and only if*

$$(1-\tau)\, z l^{\alpha} - (1+r)\, w l - (1+r)\, \kappa + (1+r)\, a + \xi \;\; \geq \;\; (1-\phi)\, z l^{\alpha} \qquad \text{(C.1)}$$

Proof. See Appendix C.2 for all proofs. □

If one assumes that both private lenders and government policies can be evaded, the previous two Lemmas can be combined to yield an even stricter credit constraint

in the sense that the set of enforceable contracts it describes is a subset of those defined in either Proposition 3.1 or Proposition C.1.

Proposition C.2. *Labor utilization l by an entrepreneur with wealth a, productivity z and idiosyncratic tax-subsidy schedule (τ, ξ) is enforceable if and only if*

$$\begin{aligned} & (1-\tau)\, z l^{\alpha} - (1+r)\, w l - (1+r)\, \kappa + (1+r)\, a + \xi \\ \geq \quad & (1-\phi) \max\left\{(1-\tau)\, z l^{\alpha} + \xi, z l^{\alpha}\right\} \end{aligned} \tag{C.2}$$

Option value of staying in business

Let the option value of staying in business between periods 1 and 2 given current wealth and productivity be denoted $\Omega_1(a_1, z_1; \kappa^s)$, which is the difference between the discounted expected continuation values of a previous entrepreneur versus a previous worker, i.e.

$$\Omega_1(a_1, z_1; \kappa^s) \equiv \mathbb{E}\left[v_2\left(a_2^E, z_2, E\right) - v_2\left(a_2^W, z_2, W\right) | z_1\right]$$

where a_2^E and a_2^W are the optimal savings choices of an entrepreneur and a worker, respectively, between periods 1 and 2. If there is no start-up cost, $\kappa^s = 0$, we know that previous period's occupational choice is no longer a relevant state variable so that

$$\Omega_1(a_1, z_1; 0) = 0$$

and we are back to the standard model with entrepreneurs solving a static profit-maximization problem.

To gain further insights into what the option value looks like as a function of the current state, I explore the special case when $a_2 = a_2^E = a_2^W$. Then as the start-up cost becomes arbitrarily high we have that previous wage workers will never start up

a business and thus

$$\begin{aligned}\lim_{\kappa^s \to +\infty} \Omega_1 (a_1, z_1; \kappa^s) &= \mathbb{E} [v_2 (a_2, z_2, E) | z_0] - v_2^W (a_2) \\ &= P (z_1) \left[v_2^W (a_2, z_2, E) - v_2^W (a_2) \right] > 0\end{aligned}$$

where the second line uses the fact that under Assumption (3.3), low-type individuals in the last period prefer to work for a wage over running a business.

For a general start-up cost $0 < \kappa^s < +\infty$, there will be a strictly positive option value to staying in business because

$$\begin{aligned}& \Omega_1 (a_1, z_1; \kappa^s) > 0 \\ \iff \quad & \mathbb{E} [v_2 (a_2, z_2, E) | z_1] > \mathbb{E} [v_2 (a_2, z_2, W) | z_1] \\ \iff \quad & P (z_1) v_2^E (a_2, z_H, E) > P (z_1) \max \left\{ v_2^E (a_2, z_H, W) , v_2^W (a_2) \right\}\end{aligned}$$

where the second follows again from Assumption (3.3). Note that the final inequality holds whenever there is a non-zero probability of an individual having high productivity tomorrow, satisfied by Assumption 3.1, since by construction $v_2^E (a_2, z_H, , E) > v_2^E (a_2, z_H, W)$ whenever $\kappa^s > 0$.

Under the above assumptions we can rewrite the option value as

$$\begin{aligned}& \Omega_1 (a_1, z_1; \kappa^s) \\ = \; & P (z_1) \min \left\{ v_2^E (a_2, z_H, E) - v_2^E (a_2, z_H, W) , v_2^E (a_2, z_H, E) - v_2^W (a_2) \right\} \\ = \; & \begin{cases} P (z_1) \left[v_2^E (a_2, z_H, E) - v_2^E (a_2, z_H, W) \right] & \text{if} \quad a_2 \geq \underline{a}_2 (z_H, W) \\ P (z_1) \left[v_2^E (a_2, z_H, E) - v_2^W (a_2) \right] & \text{if} \quad \underline{a}_2 (z_H, E) < a_2 < \underline{a}_2 (z_H, W) \\ 0 & \text{if} \quad a_2 \leq \underline{a}_2 (z_H, E) \end{cases}\end{aligned}$$

Since the only way that κ^s enters into the option value is in the $v_2^E(a_2, z_H, W)$-term and the latter is strictly decreasing in κ^s, we see immediately that

$$\frac{\partial \Omega_1(a_1, z_1; \kappa^s)}{\partial \kappa^s} \begin{cases} > 0 & \text{if} \quad a_2 \geq \underline{a}_2(z_H, W) \\ = 0 & \text{if} \quad a_2 < \underline{a}_2(z_H, W) \end{cases}$$

Note also that, by construction, $\Omega_1(a_1, z_1; \kappa^s)$ is continuous in, but not necessarily differentiable for all, a_1 at the thresholds $\underline{a}_2(z_H, W)$ and $\underline{a}_1(z_H, E)$. However, the value function need not be differentiable at these points and hence the same qualification applies for $\Omega_1(a_1, z_1; \kappa^s)$. It is then readily seen that

$$\frac{\partial \Omega_1(a_1, z_1; \kappa^s)}{\partial a_1} \begin{cases} < 0 & \text{if} \quad a_2 > \underline{a}_2(z_H, W) \\ > 0 / = 0 / < 0 & \text{for} \quad \underline{a}_2(z_H, E) < a_2 < \underline{a}_2(z_H, W) \\ = 0 & \text{if} \quad a_2 < \underline{a}_2(z_H, E) \end{cases}$$

and

$$\lim_{a_2 \to +\infty} \Omega_1(a_1, z_1; \kappa^s) = 0$$

Figure C.1 graphs the general shape of the option value function for fixed (τ, ξ).

This result is important in understanding the dynamic incentives of a firm contemplating to exit: While remaining in business in spite of low productivity, the entrepreneur will dis-accumulate assets and this in turn affects the option value of staying in business. For levels of next period wealth below the threshold, $a_2 < \underline{a}_2(z_H, E)$, there is no option value to staying in business and thus individuals will always exit the industry in order to work for a wage.

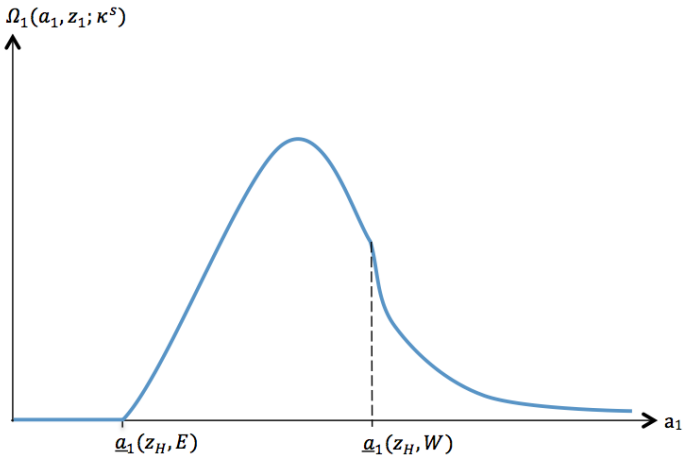


Figure C.1: Option value as a function of wealth carried into the next period.

Welfare evaluation of government intervention

For the main part of my investigation, I explicitly refrain from making normative statements concerning the effects of the type of government intervention I describe in Section 3.2.4. While not providing a full-fledged normative welfare analysis, in the following I describe a welfare criterion by which government may evaluate its policies. I posit that government is benevolent and chooses its policy instruments $(\tau(\mathbf{x}_1), \xi(\mathbf{x}_1))$ so as to maximize utilitarian period welfare with individuals' Pareto weights at time 1 given by $\lambda(a_1, z_1, o_0)$. To be consistent with my above focus on policies that prevent *premature firm exit*, I assume that $\lambda(a_1, z_H, E) \gg \lambda(a_1, z_L, E)$ so that relatively little weight is attached to high-productivity firms in welfare evaluation. Under these assumptions, the welfare-maximization program is given by:

$$\max_{\xi(\mathbf{x}_1), \tau(\mathbf{x}_1)} \left\{ \int_{a_1} \sum_{z_1} \sum_{o_0} \lambda(a_1, z_1, o_0)\, u(c^*(a_1, z_1, o_0, \tau(\mathbf{x}_1), \xi(\mathbf{x}_1)))\mu(a_1, z_1, o_0)\, da_1 \right\}$$
$$\text{s.t.} \int_{a_1} \sum_{z_1} \sum_{o_0} \left[(1 - \tau(\mathbf{x}_1))\, z_1 (l^*)^{\alpha} - \xi(\mathbf{x}_1)\right] \mu(a_1, z_1, o_0)\, da_1 = 0 \tag{C.3}$$

where $l^* = l^*(a_1, z_1, \tau(\mathbf{x}_1), \xi(\mathbf{x}_1), o_0; \phi)$. Alternatively, government may be interested in maximizing aggregate output in the economy period-by-period which is in

line with the assumption of Buera et al. (2012) and Alesina and Tabellini (1990). That is, government in general solves the following static program period-by-period:

$$\max_{\xi(\mathbf{x}_1),\tau(\mathbf{x}_1)} \left\{ \int_{a_1} \sum_{z_1} \sum_{o_0} z_1 \left(l^*\right)^{\alpha} \mu\left(a_1, z_1, o_0\right) da_1 \right\}$$

$$\text{s.t.} \quad \int_{a_1} \sum_{z_1} \sum_{o_0} \left[\left(1-\tau\left(\mathbf{x}_1\right)\right) z_1 \left(l^*\right)^{\alpha} - \xi\left(\mathbf{x}_1\right)\right] \mu\left(a_1, z_1, o_0\right) da_1 \;=\; 0 \qquad \text{(C.4)}$$

where $l^* = l^*\left(a_1, z_1, \tau\left(\mathbf{x}_1\right), \xi\left(\mathbf{x}_1\right), o_0; \phi\right)$. Individuals each period decide between operating a business under the imposed government policy $\left(\tau\left(\mathbf{x}_1\right), \xi\left(\mathbf{x}_1\right)\right)$ or working for a wage in the labor market. With this restriction, we can solve problem (C.3) or alternatively problem (C.4) numerically.

C.2 Proofs

Proof of Proposition 3.1

Proof. This is a variant of the proof in Buera et al. (2011). A given level of labor utilization constraint is enforceable if and only if an entrepreneur with given state vector $\mathbf{x} = (a_t, z_t, \tau_t, \xi_t, o_{t-1})$ weakly prefers to adhere to the debt contract and keep net-of-transfer revenues from production net of pre-financed labor costs, fixed costs, and wealth deposits with the financial institution over defaulting on the debt and keeping a fraction $1-\phi$ of net-of-transfer revenues. Then inequality (3.1) follows immediately. Under standard assumptions on the production function, it is readily shown that the set of labor utilization levels $\mathcal{L}$ that satisfy this inequality is a convex set and $0 \in \mathcal{L}$ whenever $a_t > \kappa_t$. Finally, the comparative statics results follow directly from the functional form of inequality (3.1) and the fact that entrepreneurs optimally choose labor input such that the marginal product of labor lies at or above the marginal cost of an additional labor unit. □

Proof of Lemma 3.1

Proof. It is easy to show that at $t = 2$ there exists a cutoff $a_2(z_2, o_1)$ such that an individual with state (z_2, o_1) will make an occupational choice dependent on the productivity level such that if current productivity is above the threshold then the individual works as an entrepreneur and else the individual becomes a wage worker. Such $\underline{a}_2(z_2, o_1)$ thus solves

$$\begin{aligned} & v_2^E(\underline{a}_2, z_2, o_1) &= v_2^W(\underline{a}_2) \\ \iff \quad & z_2(l_2^*)^\alpha - (1+r) w_2 l_2^* - (1+r)\kappa_2 &= w_2 \end{aligned} \tag{C.5}$$

where the LHS is seen to be strictly increasing and continuous in $\underline{a}_1$, so a unique solution exists by the continuous function theorem and seeing that

$$\lim_{a_t \to 0} \{z_t(l_t^*)^\alpha - (1+r) w_t l_t^* - (1+r)\kappa_t\} = -(1+r)\kappa_t < 0 < w_t$$

by policy design and

$$\lim_{a_t \to +\infty} \{z_t(l_t^*)^\alpha - (1+r) w_t l_t^* - (1+r)\kappa_t\} = z_t l_t^u - (1+r) w_t l_t^u - (1+r)\kappa_t > w_t$$

at time $t = 2$ by Assumption (3.2). Note that by construction of l^*, the LHS of equation (C.5) is weakly increasing—strictly increasing if credit constrained—in z_1 and ϕ strictly decreasing in κ_t. In particular, for large enough κ_t, an individual in period 1 will always become a wage worker, i.e. $\underline{z}_1 \to \infty$ as $\kappa_t \to \infty$. It follows that $\underline{a}_1$ must be weakly decreasing—strictly decreasing if credit constrained—in z_1 and ϕ, strictly increasing in κ_2 for $o_1 = W$, and weakly increasing in κ_2 for $o_1 = E$. Note that there is no option value to staying in business in the last period.

By backward induction we can find cutoffs $\underline{a}_t(z_t, o_{t-1})$ for $t = 0, 1$ which can be shown to follow the same comparative statics as the cutoff $\underline{a}_2(z_2, o_1)$ defined

by equation (C.5). In order to show this, I make use of the initial assumption of the stochastic process governing productivities being Poisson, which implies that $\mathbb{E}\left[z_{t+1} = z_H | z_t = z_H\right] \geq \mathbb{E}\left[z_{t+1} = z_H | z_t = z_L\right]$. The remaining comparative statics results follow by inspection of equation (C.5). □

Proof of Lemma 3.3

Proof. The weak inequalities of part (1) and (2) follow directly from inspection of equality (C.5) which describes occupational indifference. The strict inequalities follow from the fact that entrepreneurs have strict preferences over (1) paying higher start-up costs when considering entry; and (2) facing higher costs of re-entry in the future when considering exit at $t = 1$. □

Proof of Proposition 3.2

Proof. Note that for general $\kappa^s \geq 0$ the threshold $\underline{a}_t$ lies weakly below the productivity level $\tilde{\underline{a}}_t$ which solves

$$\tilde{\underline{a}}_t \left(l_t^*\right)^\alpha - (1+r)\, w_t l_t^* - (1+r)\, \kappa_t = w_t$$

which describes the occupational indifference threshold for the static profit-maximization problem. As noted in Appendix C.1, the occupational choice problem is no longer static with $\kappa^s > 0$. The option value of staying in business between periods t and $t+1$, denoted $\Omega_t\left(a_t, z_t; \kappa^s\right)$, is defined as the difference between the discounted expected continuation values of a previous entrepreneur versus a previous worker, i.e.

$$\Omega_t\left(a_t, z_t; \kappa^s\right) \equiv \mathbb{E}\left[v_t\left(a_t^E, z_t, E\right) - v_t\left(a_t^W z_t, W\right) | z_t\right]$$

where a_t^E and a_t^W are the optimal savings choices of an entrepreneur and a worker, respectively, between periods 0 and 1. Then there is a strictly positive option value to

staying in business whenever there is a non-zero probability of the entrepreneur being productive in the future, which holds by Assumption 3.1. Then we can formulate the individual's occupational choice problem at time t as follows: an individual decides to become an entrepreneur at time t if and only if

$$u\left(c_t^E\right) - u\left(c_t^W\right) + \Omega_t\left(a_t, z_t; \kappa^s\right) \geq 0$$

where c_t^E and c_t^W denote optimal consumption by an entrepreneur and wage worker, respectively, in that period. Thus, for positive values of $\Omega_t\left(a_t, z_t; \kappa^s\right)$, the fact that $c_t^W > c_t^E$ is no longer a sufficient condition for exit. □

Proof of Propositions C.1 and C.2

Proof. These two special cases follow from the proof of Proposition 3.1 and noting that the max-operator is continuous in its arguments. □

Appendix D

Chapter 4 Supplemental Material

D.1 Theory

Extension: life-cycle model with hyperbolic preference shocks

In this section, we present a life-cycle model with stochastic earnings ability and self-control shocks and characterize the efficient dynamic provision of commitment in the model. We show that a trade-off between providing insurance and providing commitment arises when agents face high income shocks but does not arise for agents with low income shocks. As a result commitment is provided for those that face low income shocks, but not so much for those that face high income shocks. This is the exact counterpart of the results obtained in our two-period economy with redistribution.

Our assumption of hyperbolic preferences shocks, instead of stable hyperbolic preferences, allows for a considerable level of tractability as well as changes in behavior over the life-cycle. We effectively sidestep the intricacies studied in detail by Galperti (2015) where out of equilibrium allocations play an important role in the optimal design of commitment devices. In addition, this setup allows for changes in the cross sectional distribution of present bias through the life cycle. Hence it allows, for ex-

ample, for more self-control when individuals are close to the retirement age. Finally, this setup allows for an agent to have both time consistent and time inconsistent behavior during its life-cycle.[1]

The economy is composed of a measure one of agents that have a life-cycle of $T \geq 3$ periods that is composed of working life and retirement.[2] At each period $t = 1, \ldots, T_w$ each agent face an earnings ability shock $\theta_t \in \Theta = \{\theta_1, \ldots, \theta_N\}$ with a transition probability distribution denoted by $\rho_{t+1}(\theta_{t+1}|\theta_t)$ which we allow to vary over the life-cycle and has full support at all periods over Θ for all $\theta_t \in \Theta$. We assume as well that ρ_{t+1} has a stochastic ordering so that higher levels of θ_t imply a distribution that first order stochastically dominates a distribution for lower levels of θ_t. With an abuse of notation we denote the probability distribution over the initial level of earnings ability θ_1 by $\rho_1(\theta_1)$ and assume that it has full support. The period payoff during working life over consumption and obtained earnings is given by

$$U_W(c_t, y_t; \theta_t) = u(c_t) - \frac{1}{\theta_t} v(y_t)$$

where we assume $u' > 0$, $u'' < 0$, $v' > 0$, $v'(0) = 0$, $v'' > 0$ and $v(0) = 0$. At each period $t = T_w + 1, \ldots, T$ the agent is retired so that it only consumes and it's period payoff is given by

$$U_R(c_t) = u(c_t)$$

Below, for ease of notation we write $U_t(c_t, y_t; \theta_t)$ for the period payoff but one should keep in mind that we refer to the two possibilities above.[3] Finally, without loss of generality we order earnings ability shocks by $\theta_1 < \ldots < \theta_N$ so that θ_1 is the earnings

[1] In the traditional example of present bias, the postponement of going to the gym, our setup allows for agents that would go to the gym once in a while without a commitment device in place. This is not allowed by stable hyperbolic preferences as in that setup agents would decide to never go to the gym without help of a commitment device.

[2] Here we implicitly assume that retirement lasts for at least one period.

[3] One might as well think that at retirement $\theta_t = 0$ and therefore $y_t = 0$ and use only the first period payoff for reference.

ability type of an agent that finds it extremely hard to obtain labor earnings whereas θ_N is the earnings ability type of an agent that finds it relatively easy to obtain labor earnings.

Agents in this economy face self-control shocks during their life-cycle. At each period $t = 1, \ldots, T-1$ each agent faces a hyperbolic self-control shock $\beta_t \in B = \{\beta_1, \ldots, \beta_M\}$ which is assumed to be independently distributed both over time and from earnings ability shocks. Without loss of generality we order $\beta_1 < \beta_2 < \ldots < \beta_M$. We allow the probability distribution of self-control shocks at period t,$\gamma_t(\beta_t)$, to vary over the life-cycle but assume it has full support at all periods. We denote the joint type by $h_t = (\beta_t, \theta_t)$ and its distribution at time t by π_t. We follow the usual notation in the literature using a superscript t for the history of types realized until period t, so that $h^t = (h_1, \ldots, h_t) \in H^t$ and with an abuse of notation also denote by π_t the probability distribution over H^t.

Types are unobservable and we rely on the revelation principle to characterize implementable allocations. In Appendix D.2 we show that it is sufficient to consider mechanisms in which at each period the agent report its current period type, and not the whole history of types up to that point.This result is an implication of the assumption that hyperbolic preference shocks are independent over time. An allocation can therefore be written as a sequence of pair of functions $(c_t, y_t) : H^t \to \mathbb{R}^2_+$.

The planner evaluates welfare of an allocation (c, y) according to the period 0 preference of agents in the economy

$$W(c, y) = \sum_{t=1}^{T} \delta^{t-1} \sum_{h^t} \pi_t\left(h^t\right) U_t\left(c_t\left(h^t\right), y_t\left(h^t\right), h_t\right)$$

We therefore interpret this as the problem of an agent at period 0 obtaining the optimal level of insurance over earnings ability shocks and a commitment device to deal with self-control shocks over its life-cycle. The efficient allocation could therefore be

implemented both by the government or by competitive private insurance companies as long as both are able to enforce the contract.

Finally there is a perfect credit market that the planner has access to at a gross rate of return R per period. We say a contract is feasible for the planner if

$$\sum_{t=1}^{T} \frac{1}{R^{t-1}} \sum_{h^t} \pi\left(h^t\right)\left[c_t\left(h^t\right) - y_t\left(h^t\right)\right] \leq 0$$

that is to say that assuming a law of large numbers hold, if all agents in the economy take this contract then the insurance provider is able to fulfill the contract without outside resources. An allocation (c, y) is said to be implementable if it is feasible and incentive compatible. The planner's problem then is

$$\begin{aligned} &\max_{c,y} W(c, y) \\ \text{s.t.} \quad &(c, y) \text{ is implementable} \end{aligned}$$

We now characterize properties of the planner's problem solution, i.e., characteristics of efficient insurance provision and efficient commitment provision. Full-insurance of self-control shocks in a period t implies that agents with lack of self-control ($\beta_t < 1$) and agents with self control ($\beta_t = 1$) are assigned the same allocation conditional on the whole history of earnings ability shocks they reported. Therefore, it's natural to interpret insurance of self-control shocks as the provision of commitment by the planner. Next we show that full commitment is provided under some conditions in our economy.

Theorem D.1. *Given θ_N and $\{\beta_2, \ldots, \beta_M\}$, then*

1. *There is $\bar{\theta}_1 > 0$ such that if $\theta_1 \leq \bar{\theta}_1$ then at the solution to the planner's problem we have that for any fixed $t = 1, \ldots, T-1$ and a fixed history h^{t-1} agents with types in $\{(h^{t-1}, (\theta_1, \beta)) : \beta \in B\}$ are all assigned the same level of consumption*

and earnings in period t and are assigned the same continuation allocation for all future periods:

$$\begin{aligned}
c_t\left(h^{t-1},(\theta_1,\beta)\right) &= c_t\left(h^{t-1},(\theta_1,\beta')\right)\\
y_t\left(h^{t-1},(\theta_1,\beta)\right) &= y_t\left(h^{t-1},(\theta_1,\beta')\right)\\
c_{t+s}\left(h^{t-1},(\theta_1,\beta),(h_{t+1},\ldots,h_{t+s})\right) &= c_{t+s}\left(h^{t-1},(\theta_1,\beta'),(h_{t+1},\ldots,h_{t+s})\right)\\
y_{t+s}\left(h^{t-1},(\theta_1,\beta),(h_{t+1},\ldots,h_{t+s})\right) &= y_{t+s}\left(h^{t-1},(\theta_1,\beta'),(h_{t+1},\ldots,h_{t+s})\right)
\end{aligned}$$

for all $\beta,\beta' \in B$ and for all $(h_{t+1},\ldots,h_{t+s}) \in H_{t+1}\times\cdots\times H_{t+s}$;

2. *there is $\bar{\theta}_{N-1} < \theta_N$ and $\bar{\beta}_1 > 0$ such that if $\theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ and $\beta_1 \leq \bar{\beta}_1$ then at the solution to the planner's problem we have that at all periods $t = 1,\ldots,T-1$ and after all histories h^{t-1} agents with types $\{(h^{t-1},(\theta_N,\beta)) : \beta \in B\}$ are not all assigned the same current allocation and continuation allocations.*

The planner in our economy desires to provide both insurance against earnings ability shocks and commitment against self-control shocks. This result informs us that it is efficient to provide perfect commitment for low earnings agents in our economy but not for high earnings agents. To understand why we need to think about the interaction between the self-control problem and the insurance problem. The lack of self-control that a share of agents in our economy faces imply a demand for flexibility from a period t perspective. These agents with lack of self-control are willing to pay to obtain an allocation that caters to their biased preferences at period t. Without an insurance problem, the planner would not be willing to sell them flexibility and would provide commitment to all agents.[4] However, there is an insurance problem against earnings ability shocks in our environment to which the planner is not able to provide full insurance. Therefore, it is possible to charge high earnings agents for

[4]This is the case if $\theta_t = \theta_0$ for all agents in the economy at all histories.

flexibility and use the proceeds to improve insurance against labor earnings shocks. This transfer of resources in exchange for flexibility is made possible because high earnings individuals are in a relatively better position than lower earnings individuals to pay for flexibility. Therefore, the planner is able to offer this flexibility to high earners without losing the ability to provide commitment for the lowest earnings agents in the economy.

Our second set of results characterize the effects of the available choices on ameliorating (or not) the time inconsistency problem faced by agents. Indeed, so far the results show fundamental differences in the choice sets offered to agents but there was no discussion of how those choices would compare to the case without self-control shocks. In the case of hyperbolic agents, one natural measure for comparison of those choices is the wedge to a efficient time-consistent intertemporal consumption decision. Without self-control shocks ($\beta = 1$), efficient insurance implies that intertemporal choices satisfy the inverse Euler equation[5]

$$\sum_{\theta_{t+1} \in \Theta_{t+1}} \rho_{t+1}\left(\theta_{t+1}|\theta_t\right) \frac{u'\left(c_t\left(\theta^t\right)\right)}{\delta R u'\left(c_{t+1}\left(\theta^t, \theta_{t+1}\right)\right)} = 1$$

As long as this intertemporal condition is satisfied the detrimental effects of time-inconsistency have been completely dealt with. We can define the time inconsistency wedge in our economy for an agent with history h^t as

$$\tau\left(h^t\right) = \sum_{h_{t+1} \in H_{t+1}} \pi_{t+1}\left(h_{t+1}|h^t\right) \frac{u'\left(c_t\left(h^t\right)\right)}{\delta R u'\left(c_{t+1}\left(h^t, h_{t+1}\right)\right)} - 1$$

On one hand, if the time inconsistency wedge is positive it means that consumption into the future is relatively high as compared to current consumption. On the other hand, a negative time inconsistency wedge implies that agents are consuming too

[5]For applications of this result for optimal taxation see Golosov, Kocherlakota and Tsyvinski (2003), Stantcheva (2015) and Golosov, Troshkin and Tsyvinski (2016).

much in the current period as compared to what they consume in the future. If agents face self-control shocks, it's natural to assume that without the availability of commitment devices agents would present a negative time consistency wedge and as a result would have relatively little consumption into the future. The next result extends Theorem 4.2 to this environment.

Theorem D.2. *Given θ_N and $\{\beta_2, \ldots, \beta_M\}$, then*

1. *there is $\bar{\theta}_1 > 0$ such that if $\theta_1 \leq \bar{\theta}_1$ then at the solution to the planner's problem we have that at any fixed $t = 1, \ldots, T-1$ and after any fixed history h^{t-1} agents with types in $\{(h^{t-1}, (\theta_1, \beta)) : \beta \in B\}$ have a weakly positive time inconsistency wedge:*

$$\tau\left(h^{t-1}, (\theta_1, \beta)\right) = \sum_{h_{t+1} \in H_{t+1}} \pi_{t+1}\left(h_{t+1} | \theta_1\right) MRS_{t,t+1}\left(h_{t+1} | \left(\theta_1, \beta\right)\right) - 1 \geq 0$$

   where

$$MRS_{t,t+1}\left(h_{t+1} | \left(\theta_1, \beta\right)\right) = \frac{u'\left(c_t\left(h^{t-1}, (\theta_1, \beta)\right)\right)}{\delta R u'\left(c_{t+1}\left(h^{t-1}, (\theta_1, \beta), (\theta_{t+1}, \beta_{t+1})\right)\right)}$$

2. *there is $\bar{\theta}_{N-1} < \theta_N$ and $\bar{\beta}_1 > 0$ such that if $\theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ and $\beta_1 < \bar{\beta}_1$, then at the solution to the planner's problem we have for any fixed periods $t = 1, \ldots, T-1$ and after any fixed history h^{t-1} agents with types*

   (a) $(h^{t-1}, (\theta_N, \beta_M))$ have a weakly positive time-inconsistency wedge

   (b) $(h^{t-1}, (\theta_N, \beta_1))$ have a negative time inconsistency wedge:

$$\tau\left(h^{t-1}, (\theta_N, \beta_1)\right) < 0$$

D.2 Proofs

Proofs for Section 4.2

It is useful to restate the problem in terms of utility levels:

$$\begin{aligned} u_t(\theta,\beta) &= u(c_t(\theta,\beta)) \quad \text{for } t=1,2 \\ v(\theta,\beta) &= v(y(\theta,\beta)) \end{aligned}$$

and then $c_t(\theta,\beta) = C(u_t(\theta,\beta))$ where $C = u^{-1}$, and $y(\theta,\beta) = Y(v(\theta,\beta))$ where $Y = v^{-1}$. Then the planner's problem becomes

$$\max_{u_1,u_2,v} \sum_{\theta,\beta} \pi(\theta,\beta)\lambda(\theta)\left[u_1(\theta,\beta) - \frac{1}{\theta}v(\theta,\beta) + \delta u_2(\theta,\beta)\right]$$

s.t. for all (θ,β) and (θ',β') :

$$u_1(\theta,\beta) - \frac{1}{\theta}v(\theta,\beta) + \beta\delta u_2(\theta,\beta) \geq u_1(\theta',\beta') - \frac{1}{\theta}v(\theta',\beta') + \beta\delta u_2(\theta',\beta')$$

$$\sum_{\theta,\beta} \pi(\theta,\beta)\left[Y(v(\theta,\beta)) - C(u_1(\theta,\beta)) - \frac{1}{R}C(u_2(\theta,\beta))\right] \geq 0$$

Since $u(\cdot)$ is strictly concave and $v(\cdot)$ is strictly convex, then $C(\cdot)$ is strictly convex and $Y(\cdot)$ is strictly concave, which in turn tells us that the government's problem is a convex problem. This property is used in our numerical solution algorithm, but is not very useful for the proofs below. Also define the payoff provided to individuals by

$$U(\theta,\beta) = u_1(\theta,\beta) - \frac{1}{\theta}v(\theta,\beta) + \beta\delta u_2(\theta,\beta)$$

and the payoff to the government of the individual allocation by

$$V(\theta,\beta) = u_1(\theta,\beta) - \frac{1}{\theta}v(\theta,\beta) + \delta u_2(\theta,\beta)$$

We begin by proving a few Lemmas that are going to be useful in the proofs of the main Theorems. Lemma D.1 shows that if the lowest labor earnings ability is sufficiently low, then agents with lowest earnings ability will have all their incentive constraint with respect to higher ability types strictly slack. Then Corollary D.1 builds upon the proof of Lemma D.1 to show that if the highest labor earnings type is sufficiently high as compared to all other labor earnings ability types, then incentive constraints of all agents with labor ability in $\{\theta_1, \ldots, \theta_{N-1}\}$ are strictly slack with respect to the allocation of types with ability θ_N.

Lemma D.1. *Assume $\lambda(\theta)$ is weakly decreasing. Given $\{\theta_2, \ldots, \theta_N\}$, there is $\bar{\theta}_1 > 0$ such that if $0 < \theta_1 < \bar{\theta}_1$, then at the solution to the government's problem we have*

$$u_1(\theta_1, \beta) - \frac{1}{\theta_1} v(\theta_1, \beta) + \beta\delta u_2(\theta_1, \beta) > u_1(\theta', \beta') - \frac{1}{\theta_1} v(\theta', \beta') + \beta\delta u_2(\theta', \beta')$$

for $\theta' \in \{\theta_2, \ldots, \theta_N\}$.

Proof. In fact, consider $\theta_1 = 0$, then for any $v_1(\theta_1, \beta) > 0$ we would have $V(\theta_1, \beta) = -\infty$. Since π has full support and λ is weakly decreasing, then $\pi(\theta_1, \beta)\lambda(\theta_1) > 0$ and this cannot be optimal for the government. Therefore we have $v(\theta_1, \beta) = 0$ at the solution to the government's problem.

Now fix $\theta' > 0$ and $\beta' \in \{\beta_1, \ldots, \beta_M\}$. Assume by way of contradiction that $\mathrm{v}(\theta', \beta') = 0$. Since $Y'(0) = +\infty$, then this agent can produce enough resources to make all agents strictly better off in the economy, a contradiction.[6] Therefore

[6] In fact consider a perturbation to all agents $\tilde{v}(\theta, \beta) = v(\theta, \beta) + \varepsilon$ and $u_1(\theta, \beta) = u_1(\theta, \beta) + \nu$ for $\varepsilon > 0$ and $\nu > 0$. Since the original allocation is incentive compatible and the perturbation is uniform across all agents, the perturbation is incentive compatible. The change in resources used is given by $dE = \sum \pi(\theta, \beta)[C'(u_1(\theta, \beta))\nu - Y'(v(\theta, \beta))\varepsilon]$. Since $C'(u_1(\theta, \beta)) < \infty$ for all (θ, β), then for any $\varepsilon > 0$ we have $dE = -\infty$ as $Y'(v(\theta', \beta')) = +\infty$. The welfare change is given by $dW = \sum \pi(\theta, \beta)\lambda(\theta)(\nu - \varepsilon) = \nu - \varepsilon$. Therefore, for $\nu > \varepsilon$ and for ν and ε small enough, the perturbation is feasible and increases welfare, a contradiction.

$v(\theta', \beta') > 0$. We finally conclude that

$$u_1(\theta', \beta') - \frac{1}{\theta_1} v(\theta', \beta') + \beta\delta u_2(\theta', \beta') = -\infty$$

so that the result holds for $\theta_1 = 0$. The government's problem satisfy all conditions of the Maximum Theorem as we change θ_1, therefore the solution to the government's problem is continuous in θ_1 and there is $\bar{\theta}_1 > 0$ such that for all $0 \leq \theta_1 < \bar{\theta}_1$ the solution to the planner's problem satisfy

$$u_1(\theta_1, \beta) - \frac{1}{\theta_1} v(\theta_1, \beta) + \beta\delta u_2(\theta_1, \beta) > u_1(\theta', \beta') - \frac{1}{\theta_1} v(\theta', \beta') + \beta\delta u_2(\theta', \beta')$$

for all (θ', β') with $\theta' \in \{\theta_2, \ldots, \theta_N\}$. □

Corollary D.1. *Assume $\lambda(\theta)$ is weakly decreasing. Fix θ_N, then there exists $\bar{\theta}_{N-1} < \theta_N$ such that if $0 < \theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$, then at the solution to the government's problem we have*

$$u_1(\theta', \beta') - \frac{1}{\theta'} v(\theta', \beta') + \beta'\delta u_2(\theta', \beta') > u_1(\theta_N, \beta) - \frac{1}{\theta'} v(\theta_N, \beta) + \beta'\delta u_2(\theta_N, \beta)$$

and $v(\theta_N, \beta) > v(\theta', \beta')$ for all $\theta' \in \{\theta_1, \ldots, \theta_{N-1}\}$ and for all $\beta, \beta' \in \{\beta_1, \ldots, \beta_M\}$.

Proof. We can use again the same strategy of the proof of the previous Lemma. If $\bar{\theta}_{N-1} = 0$, then the result holds as $v(\theta_N, \beta) > 0$ at the solution to the planner's problem. By continuity of the solution to the planner's problem there exists $\bar{\theta}_{N-1}$ that satisfy the conditions of the theorem. □

Lemma D.2. *Assume $\lambda(\theta)$ is weakly decreasing. Fix θ_N and $\{\beta_2, \ldots, \beta_M\}$, then there exists $\bar{\theta}_{N-1} < \theta_N$ such that if $0 < \theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$, then at the solution to the government's problem we have either $u_1(\theta_N, \beta_1) > u_1(\theta, \beta)$ or $u_2(\theta_N, \beta_1) > u_2(\theta, \beta)$ for all $\theta \in \{\theta_1, \ldots, \theta_{N-1}\}$ and all $\beta \in B$.*

Proof. From Corollary D.1 there is $0 < \bar{\theta}'_{N-1} < \theta_N$ such that we have $v(\theta_N, \beta_1) > v(\theta, \beta)$ for $0 < \theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}'_{N-1}$ and all $\beta \in B$. Notice that for $\bar{\theta}''_{N-1} = 0$, we have either $u_1(\theta_N, \beta) > u_1(\theta, \beta)$ or $u_2(\theta_N, \beta) > u_1(\theta, \beta)$ for $\theta = 0$. By the Maximum theorem, the solution to the planner's problem is continuous. Therefore, since the solution to the planner's problem is continuous, there exists $\bar{\theta}'_{N-1} \geq \bar{\theta}_{N-1} > 0$ such that the inequality continues to hold at the solution to the planner's problem for $0 < \theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$. □

Proof of Theorem 4.1

Let's first show part 1 of Theorem 4.1. From Lemma D.1 we know that there exists $\bar{\theta}_1 > 0$ such that $\bar{\theta}_1 < \theta_2$ and so that

$$u_1(\theta_1, \beta) - \frac{1}{\theta_1} v(\theta_1, \beta) + \beta\delta u_2(\theta_1, \beta) > u_1(\theta', \beta') - \frac{1}{\theta_1} v(\theta', \beta') + \beta\delta u_2(\theta', \beta')$$

for all $\beta, \beta' \in \{\beta_1, \ldots, \beta_M\}$ and $\theta' \in \{\theta_2, \ldots, \theta_N\}$. Assume by way of contradiction that types $(\theta_1, \beta) \neq (\theta_1, \beta')$ face a different allocation. Then consider a perturbation

$$\tilde{u}_t(\theta_1, \beta) = \tilde{u}_t(\theta_1) = \sum_{\beta} \left(\frac{\pi(\theta_1, \beta)}{\sum_{\beta'} \pi(\theta_1, \beta')} \right) u_t(\theta_1, \beta)$$
$$\tilde{v}(\theta_1, \beta) = \tilde{v}(\theta_1) = \sum_{\beta} \left(\frac{\pi(\theta_1, \beta)}{\sum_{\beta'} \pi(\theta_1, \beta')} \right) v(\theta_1, \beta)$$

and keep all other allocations the same. Since incentive constraints are linear, they are convex and therefore this perturbation is incentive compatible. But note that since the allocation for (θ_1, β) was initially different from the allocation for (θ_1, β'), and since C is convex and Y is concave, now we have extra resources available in the economy. This is a contradiction as the planner can now improve its objective by distributing those resources uniformly across agents. Therefore, we conclude agents with type in $\{(\theta_1, \beta) | \beta \in \{\beta_1, \ldots, \beta_M\}\}$ are bunched.

Now we turn to part 2 of Theorem 4.1. Assume by way of contradiction that all agents with types in $\{(\theta_N, \beta) \,|\beta \in \{\beta_1, \ldots, \beta_M\}\}$ receive the same allocation denoted by $(u_1(\theta_N), u_2(\theta_N), v(\theta_N))$. This implies that

$$\frac{R\delta C'(u_1(\theta_N))}{C'(u_2(\theta_N))} = \kappa$$

for some fixed $\kappa > 0$. Now consider the perturbation

$$\tilde{u}_1(\theta_N) = u_1(\theta_N) + \delta\varepsilon$$
$$\tilde{u}_2(\theta_N) = u_2(\theta_N) - \varepsilon$$

that keeps the planner's objective constant. Agents with a disagreement level β of discounting will face a payoff change of

$$dU(\theta_N, \beta) = (1 - \beta)\delta\varepsilon$$

so that for $\varepsilon > 0$ this perturbation is incentive compatible. The marginal change in resources used by such a perturbation is given by

$$dE = \left(\sum_{\beta} \pi(\theta_N, \beta)\right)(\kappa - 1)\frac{1}{R}C'(u_2(\theta_N))\varepsilon$$

If $\kappa < 1$, then we reach a contradiction as the perturbation is incentive compatible and generates extra resources for the government. Thus $\kappa \geq 1$.

Now if $\kappa > 1$, consider a perturbation only to the allocation offered to an agent with type (θ_N, β_M) where $\beta_M = 1$:

$$\tilde{u}_1(\theta_N, \beta_M) = u_1(\theta_N) - \delta\varepsilon$$
$$\tilde{u}_2(\theta_N, \beta_M) = u_2(\theta_N) + \varepsilon$$

This perturbation keeps the planner objective constant and is incentive compatible as agents with type $\beta < 1$ find it worsens the allocation for $\varepsilon > 0$. The change in resources used by the planner is given by

$$dE = \frac{1}{R} C' \left(u_2 \left(\theta_N \right) \right) \pi \left(\theta_N, \beta_M \right) \left(1 - \kappa \right) \varepsilon$$

Hence for $\varepsilon > 0$ we have $dE < 0$ as $\kappa > 1$, a contradiction as the government generates extra resource while keeping the objective constant. Therefore we conclude that $\kappa = 1$.

From Lemma D.2 we have that either $u_1 \left(\theta_N \right) > u_1 \left(\theta, \beta \right)$ or $u_2 \left(\theta_N \right) > u_2 \left(\theta, \beta \right)$ for all $\theta \in \{\theta_1, \ldots, \theta_{N-1}\}$ and all $\beta \in \{\beta_1, \ldots, \beta_M\}$. We prove the Theorem in the first case, the second case is analogous. Consider the following perturbation

$$\tilde{u}_1 \left(\theta_N, \beta_1 \right) = u_1 \left(\theta_N \right) + \beta_1 \delta \varepsilon + \nu$$
$$\tilde{u}_2 \left(\theta_N, \beta_1 \right) = u_2 \left(\theta_N \right) - \varepsilon$$

for $\varepsilon > 0$ and $\nu > 0$. For $\left(\theta, \beta \right) \neq \left(\theta_N, \beta_1 \right)$ we set

$$\tilde{u}_1 \left(\theta, \beta \right) = \tilde{u}_1 \left(\theta, \beta \right) + \nu$$

and keep all other allocations constant. It is easy to check this is incentive compatible. The change in resources used by the government is given by

$$dE = \pi \left(\theta_N, \beta_1 \right) \left(\beta_1 - 1 \right) \frac{1}{R} C' \left(u_2 \left(\theta_N \right) \right) \varepsilon + \nu \sum_{\theta, \beta} \pi \left(\theta, \beta \right) C' \left(u_1 \left(\theta, \beta \right) \right)$$

If we set $dE = 0$ we obtain

$$\nu = \pi \left(\theta_N, \beta_1 \right) \left(1 - \beta_1 \right) \delta \frac{C' \left(u_1 \left(\theta_N \right) \right)}{\sum_{\theta, \beta} \pi \left(\theta, \beta \right) C' \left(u_1 \left(\theta, \beta \right) \right)} \varepsilon$$

The government objective change is given by

$$dW = \pi\left(\theta_N, \beta_1\right)\left(1-\beta_1\right)\delta\left[\frac{C'\left(u_1\left(\theta_N\right)\right)}{\sum_{\theta,\beta}\pi\left(\theta,\beta\right)C'\left(u_1\left(\theta,\beta\right)\right)} - \lambda\left(\theta_N\right)\right]\varepsilon$$

Since $\lambda\left(\theta_N\right) \leq 1$ and $u_1\left(\theta_N\right) > u_1\left(\theta,\beta\right)$, we then have $dW > 0$, a contradiction.

Proof of Theorem 4.2

First let's show the results for low earnings ability. By Lemma D.1, given $\{\theta_2, \ldots, \theta_N\}$, there exists $\bar{\theta}_1 > 0$ such that for $\theta_1 < \bar{\theta}_1$ incentive constraints of type (θ_1, β) are strictly slack with respect to agents' (θ', β') allocations for $\theta' > \theta_1$. From Theorem 4.1, agents with type (θ_1, β) receive the same allocation, independent of their β-type. Then we have

$$\frac{R\delta C'\left(u_1\left(\theta_1\right)\right)}{C'\left(u_2\left(\theta_1\right)\right)} = \kappa$$

for some $\kappa > 0$. We want to show that $\kappa < 1$.[7] Assume by way of contradiction that $\kappa > 1$. Then consider the following perturbation in θ_1's allocation

$$\tilde{u_1}\left(\theta_1\right) = u_1\left(\theta_1\right) - \delta\varepsilon$$

$$\tilde{u}_2\left(\theta_1\right) = u_2\left(\theta_1\right) + \varepsilon$$

This keeps the government objective constant and for $\varepsilon > 0$ sufficiently small is incentive compatible because incentive constraints of types θ_1 are slack and agents with $\beta \leq 1$ find it (weakly) worsens the allocation. The marginal change in resources

[7] Remember that $C'\left(\bar{u}\right) = \frac{1}{u'\left(C\left(\bar{u}\right)\right)}$.

used with this perturbation is

$$\begin{aligned} dE &= \left(\sum_{\beta} \pi\left(\theta_1, \beta\right)\right)\left[\frac{1}{R} C'\left(u_2\left(\theta_1\right)\right) - \delta C'\left(u_1\left(\theta_1\right)\right)\right] \varepsilon \\ &= \left(\sum_{\beta} \pi\left(\theta_1, \beta\right)\right)(1-\kappa)\frac{1}{R} C'\left(u_2\left(\theta_1\right)\right) \varepsilon \end{aligned}$$

As $\kappa > 1$ we have for $\varepsilon > 0$ that $dE < 0$, so that the perturbation keeps the government objective constant, it is incentive compatible and it generates extra resources to the government. This is a contradiction as those resources can then be used to improve the government objective. Therefore we proved part 1.(b) of the theorem, part 1.(a) is then a consequence of $\beta < 1$.

Now let's consider the results for high earnings ability agents. First let's show that agent's (θ_N, β_M) where $\beta_M = 1$ face weakly negative intertemporal wedge. Assume by way of contradiction that

$$\frac{R\delta C'\left(u_1\left(\theta_N, \beta_M\right)\right)}{C'\left(u_2\left(\theta_N, \beta_M\right)\right)} > 1$$

Then consider the following perturbation to the allocation of (θ_N, β_M)

$$\begin{aligned} \tilde{u}_1\left(\theta_N, \beta_M\right) &= u_1\left(\theta_N, \beta_M\right) - \delta\varepsilon \\ \tilde{u}_2\left(\theta_N, \beta_M\right) &= u_2\left(\theta_N, \beta_M\right) + \varepsilon \end{aligned}$$

and leave all other allocations constant. This perturbation is incentive compatible as agents with $\beta_M = 1$ are indifferent between the original allocation and this one, and agents with $\beta < 1$ like the original allocation for type (θ_N, β_M) better. But this perturbation generates extra resources at the margin as we have

$$dE = \pi\left(\theta_N, \beta_M\right)\left(1 - \frac{R\delta C'\left(u_1\left(\theta_N, \beta_M\right)\right)}{C'\left(u_2\left(\theta_N, \beta_M\right)\right)}\right)\varepsilon$$

and since for $\varepsilon > 0$ we have $dE < 0$. This is a contradiction as the extra resources can then be used to improve the planner's objective. Therefore we just proved 2.(a). Notice that the proof of 2.(b) is within the proof of Theorem 4.1.

Proofs of results for dynamic model with self-control shock

Types are unobservable and we rely on the revelation principle to characterize implementable allocations. Hence we define an allocation as a pair of functions $(c_t, y_t) : H^1 \times \cdots \times H^{t-1} \times H^t \to \mathbb{R}^2_+$ for each period t that assigns a consumption level and an earnings level for any reported history $h^t \in H^t$ at period t and any past reported history $\hat{r}^{t-1} = (h^1, \ldots, h^{t-1}) \in H^1 \times \cdots \times H^{t-1}$. A strategy for an agent is a sequence of reporting strategies $\sigma_t : H^1 \times \cdots \times H^{t-1} \times H^t \to H^t$. The overall payoff after history h^t, previous reports $\hat{r}^{t-1} = (r^1, \ldots, r^{t-1}) \in H^1 \times \cdots \times H^{t-1}$ and following a strategy $(\sigma_s)_{s=t}^T$ from period t on is given by

$$\begin{aligned} V_t\left(\hat{r}^{t-1}, h^t, (\sigma_s)_{s=t}^T\right) =& U_t\left(c_t\left(\hat{r}^t, \sigma_t\left(\hat{r}^t, h^t\right)\right), y_t\left(\hat{r}^{t-1}, \sigma_t\left(\hat{r}^{t-1}, h^t\right)\right), \theta_t\right) \\ &+ \beta_t \sum_{s=t+1}^{T} \delta^{s-t} \sum_{h^s \succ h^t} \pi_s\left(h^s | \theta_t\right) U_s\left(\hat{c}_s\left(\hat{r}^{s-1}, h^s\right), \hat{y}_s\left(\hat{r}^{s-1}, h^s\right), \theta_s\right) \end{aligned} \tag{D.1}$$

where with an abuse of notation $\hat{c}_s\left(\hat{r}^{s-1}, h^s\right) = c_s\left(\sigma_s\left(\hat{r}^{s-1}, h^s\right)\right)$ and $\hat{y}_s\left(\hat{r}^{s-1}, h^s\right) = y_s\left(\sigma_s\left(\hat{r}^{s-1}, h^s\right)\right)$. So the preference is hyperbolic with a present-bias of β_t at period t.[8]

Agents are sophisticated as they take into account their time inconsistencies into the future. An allocation is said to be incentive compatible if truth-telling is a subgame perfect equilibrium of the game played between the selves at all periods and after all histories of reports and realized types. Hence incentive compatibility requires

[8] We use the symbol $h^s \succ h^t$ to denote continuation histories for $s > t$ that are consistent with h^t.

that after any history of reports $\hat{r}^{t-1} \in H^1 \times \cdots \times H^{t-1}$ and an actually realized type h^{t-1}

$$\sigma_t^{Truth} \in \arg\max_{\sigma_t} V_t\left(\hat{r}^{t-1}, h^t, \left(\sigma_t, \left(\sigma_s^{Truth}\right)_{s=t}^{T}\right)\right)$$

that is to say: taking into account that future selves will consider it optimal to report the truth, reporting the truth at period t after history h^t is optimal for any previous reports $\hat{r}^t$. The revelation principle guarantees us that the outcome of any mechanism can be obtained using the allocations defined above.[9]

Our assumptions of full support over types, the Markovian nature of the stochastic process over types and the planner's objective allow us to further simplify incentive constraints. From the Markovian nature of the problem we have that, conditional on $\hat{r}^{t-1}$, the preferences after any history $\tilde{h}^t \in H^t$ with $h_t = \tilde{h}_t$ have the same ordering as the preferences after history h^t. As we will show below, the planner's objective function is strictly concave so we must have that at an optimal allocation the allocations from period t on of those types of agents coincide. Hence we can write

$$c_{t+s}\left(\hat{r}^{t-1}, h^t, \ldots, h^s\right) = c_{t+s}\left(\hat{r}^{t-1}, h_t, \ldots, h^s\right)$$
$$y_{t+s}\left(\hat{r}^{t-1}, h^t, \ldots, h^s\right) = y_{t+s}\left(\hat{r}^{t-1}, h_t, \ldots, h^s\right)$$

Using this argument recursively for all periods $s > t$ we obtain

$$c_{t+s}\left(\hat{r}^{t-1}, h^t, \ldots, h^s\right) = c_{t+s}\left(\hat{r}_1, \ldots, \hat{r}_{t-1}, h_t, \ldots, h_s\right)$$
$$y_{t+s}\left(\hat{r}^{t-1}, h^t, \ldots, h^s\right) = y_{t+s}\left(\hat{r}_1, \ldots, \hat{r}_{t-1}, h_t, \ldots, h_s\right)$$

[9]Indeed this is a Bayesian game with positive probabilities at all nodes of the game.

where we used that $\hat{r}_1, \ldots, \hat{r}_t$ are optimal reports for an agent with that history of types. Therefore it is without loss of generality that the mechanism requires only reporting of the current period type and not of the full history of types.[10]

The next result is the extension of Lemma D.1 for the dynamic case.

Lemma D.3. *Given $\{\theta_2, \ldots, \theta_N\}$, there is $\bar{\theta}_1 > 0$ such that if $\theta_1 < \bar{\theta}_1$ then at the solution to the planner's problem we have*

$$\underbrace{V_t\left(h^{t-1}, (\beta, \theta_1)\right)}_{\text{truthful report}} \geq \underbrace{V_t\left((\beta', \theta_1)\,|h^{t-1}, (\beta, \theta_1)\right)}_{\text{deviation in } \beta} > \underbrace{V_t\left((\beta', \theta')\,|h^{t-1}, (\beta_t, \theta_1)\right)}_{\text{deviation in } \theta}$$

and $y_t\left(h^{t-1}, (\beta', \theta')\right) > y_t\left(h^{t-1}, (\beta, \theta_1)\right)$ for all $\theta' > \theta_1$, for all $\beta' \neq \beta$, for all h^{t-1} and for all t.

Proof. In fact, if $\theta_1 = 0$, then $y_t\left(h^{t-1}, (\beta, \theta_1)\right) = 0$ for all $\beta \in B$ and all h^{t-1}. For any $\theta' > 0$, then $y_t\left(h^{t-1}, (\beta', \theta')\right) > 0$, therefore

$$V_t\left((\beta', \theta')\,|h^{t-1}, (\beta, \theta_1)\right) = -\infty$$

and this proves the strict inequality. The first inequality is a requirement from incentive compatibility. From continuity of the solution to the planner's problem, it follows that for fixed $\{\theta_2, \theta_3, \ldots, \theta_N\}$, there is $\bar{\theta}_1$ such that for all $\theta_1 \leq \bar{\theta}_1$ we have at the solution to the planner's problem

$$V_t\left(h^{t-1}, (\beta, \theta_1)\right) \geq V_t\left((\beta', \theta_1)\,|h^{t-1}, (\beta, \theta_1)\right) > V_t\left((\beta', \theta')\,|h^{t-1}, (\beta_t, \theta_1)\right)$$

and we proved the result. □

[10] This characterization implies that only equilibrium path allocations are important for incentive compatibility (see Fernandes and Phelan (2000) and Kapicka (2013)). This argument can break down in problems with perfect correlated types. One example is in Battaglini and Lamba (2015). The full support in β_t types is particularly important in the current analysis for this alternative characterization to be valid. If there is no full support in β_t it is possible to design a mechanism in which off equilibrium path allocations relax incentive constraint on the equilibrium path and therefore this simplified characterization does not hold.

Corollary D.2. *Fix θ_N, then there exists $\bar{\theta}_{N-1} < \theta_N$ such that if $0 < \theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$, then at the solution to the government's problem we have*

$$V_t\left(h^{t-1}, (\beta', \theta')\right) > V_t\left((\beta, \theta_N)\,|h^{t-1}, (\beta', \theta')\right)$$

and $v_t\left(h^{t-1}, (\beta, \theta_N)\right) > v_t\left(h^{t-1}, (\beta', \theta')\right)$ for all h^{t-1} , for all $\theta' \in \{\theta_1, \ldots, \theta_{N-1}\}$ and for all $\beta, \beta' \in \{\beta_1, \ldots, \beta_M\}$.

Proof. We can use again the same strategy of the proof of the previous Lemma. If $\bar{\theta}_{N-1} = 0$, then the result holds as $v_t\left(h^{t-1}, (\beta, \theta_N)\right) > 0$ at the solution to the planner's problem. By continuity of the solution to the planner's problem there exists $\bar{\theta}_{N-1}$ that satisfy the conditions of the theorem. □

Lemma D.4. *Fix θ_N and $\{\beta_2, \ldots, \beta_M\}$, then there exists $\bar{\theta}_{N-1} < \theta_N$ and $\bar{\beta}_1$ such that if $0 < \theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ and $\beta_1 < \bar{\beta}_1$, then at the solution to the government's problem we have $u_t\left(h^{t-1}, (\beta_1, \theta_N)\right) > u_t\left(h^{t-1}, (\beta, \theta)\right)$ for all h^{t-1}, for all $\theta \in \{\theta_1, \ldots, \theta_{N-1}\}$, for all $\beta \in \{\beta_1, \ldots, \beta_N\}$ and for all $t = 1, \ldots, T-1$.*

Proof. From Corollary D.2 we have that $v_t\left(h^{t-1}, (\beta_1, \theta_N)\right) > v_t\left(h^{t-1}, (\beta, \theta)\right)$. Note that if $\beta_1 \approx 0$, incentive compatibility requires $u_t\left(h^{t-1}, (\beta_1, \theta_N)\right) > u_t\left(h^{t-1}, (\beta, \theta)\right)$. By the Maximum theorem, the solution to the planner's problem is continuous. Therefore there exists $\bar{\beta}_1 > 0$ such that this inequality continues to be strict for all $\beta_1 < \bar{\beta}_1$. Since $T < \infty$ we can pick a uniform level of $\bar{\beta}_1 > 0$. □

Theorem D.3. ***(Theorem D.1 part 1)*** *Given $\{\theta_2, \ldots, \theta_N\}$, there is $\bar{\theta}_1 > 0$ such that if $\theta_1 \leq \bar{\theta}_1$ then at the solution to the planner's problem we have that at all periods $t = 1, \ldots, T-1$ and after all histories h^{t-1} agents with types in $\{(\theta_1, \beta) : \beta \in B_t\}$ are all assigned the same level of consumption and earnings in period t and are assigned the same continuation allocation for all future periods.*

Proof. Consider the problem in terms of utility levels from consumption and disutility levels from working.[11] Assume by way of contradiction that for a fixed $t < T$ and fixed history $h^{t-1} \in H^{t-1}$ and for $\beta, \beta' \in B_t$ we have

$$u_t\left(h^{t-1}, (\beta, \theta_1)\right) > u_t\left(h^{t-1}, (\beta', \theta_1)\right)$$

at the solution to the planner's problem. Consider now an allocation that is a convex combination between between $(h^{t-1}, (\beta^*, \theta_1))$ allocations for all $\beta^* \in B$ and offer it to all types with low ability θ_1 after history h^{t-1}:

$$\tilde{u}_t\left(h^{t-1}, (\beta^*, \theta_1), h_{t+1}^{t+s}\right) = \sum_{b \in B} \frac{\pi_t\left((b, \theta_1) | h^{t-1}\right)}{\sum_{b' \in B} \pi_t\left((b', \theta_1) | h^{t-1}\right)} u_t\left(h^{t-1}, (b, \theta_1), h_{t+1}^{t+s}\right)$$
$$\tilde{v}_t\left(h^{t-1}, (\beta^*, \theta_1), h_{t+1}^{t+s}\right) = \sum_{b \in B} \frac{\pi_t\left((b, \theta_1) | h^{t-1}\right)}{\sum_{b' \in B} \pi_t\left((b', \theta_1) | h^{t-1}\right)} v_t\left(h^{t-1}, (b, \theta_1), h_{t+1}^{t+s}\right)$$

for all $\beta^* \in B$, where $h_{t+1}^{t+s} = (h_{t+1}, \ldots, h_{t+s})$ We have from Lemma (D.3) that there exists $\bar{\theta}_1$ such that for $\theta_1 < \bar{\theta}_1$

$$V_t\left(h^{t-1}, (\beta, \theta_1)\right) \geq V_t\left((b, \theta_1) | h^{t-1}, (\beta, \theta_1)\right) > V_t\left((\beta'', \theta') | h^{t-1}, (\beta, \theta_1)\right)$$

for all $\theta' > \theta_1$, for all β''. Therefore, incentive compatibility constraints at nodes $(h^{t-1}, (b, \theta_1))$ are satisfied for all $b \in B$. From linearity of the objective function, it follows as well that incentive compatibility of types $(h^{t-1}, (b, \theta))$ are satisfied for all $\theta > \theta_1$ and all $b \in B$. Therefore this perturbation is incentive compatible at period t. Further, notice that the perturbation is such that from the planner's point of view, continuation utility at h^{t-1} is unchanged. Therefore welfare is unchanged and from

[11] That is to say, consider the standard transformation

$$u\left(c_t\left(h^t\right)\right) = u_t\left(h^t\right)$$
$$v\left(y_t\left(h^t\right)\right) = v_t\left(h^t\right)$$

and let $C(u)$ and $Y(v)$ denote the inverses of u and v, respectively.

hyperbolic preferences, all incentive constraints for period $s \leq t-1$ are satisfied. For histories $h^s \succ h^{t-1}$ for $s > t$, note once again that the convex combination does not affect incentives because the objective is linear. However, we then reach a contradiction since $C(u) = u^{-1}(u)$ is a convex function, then the total cost of the new allocation is strictly lower as $u_t(h^{t-1}, (\beta, \theta_1)) > u_t(h^{t-1}, (\beta', \theta_1))$ and π_t has full support (so these types have positive mass). □

Theorem D.4. ***(Theorem D.1 part 2)*** *Given θ_N and $\{\beta_2, \ldots, \beta_M\}$, there is $\bar{\theta}_{N-1} < \theta_N$ and $\bar{\beta}_1 > 0$ such that if $\theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ and $\beta_1 < \bar{\beta}_1$ then at the solution to the planner's problem we have that at all periods $t = 1, \ldots, T-1$ and after all histories h^{t-1} agents with types (θ_N, β) are not bunched all at the same allocation.*

Proof. Assume by way of contradiction that for some h^{t-1} we have that $(h^{t-1}, (\beta_t, \theta_N))$ for all $\beta_t \in B_t$ face the same allocation. Then

$$\mathbb{E}_t \left[\frac{C'\left(u_{t+1}\left(h^{t-1}, (\beta_t, \theta_N), (\beta_{t+1}, \theta_{t+1})\right)\right)}{\delta R_t C'\left(u_t\left(h^{t-1}, (\beta_t, \theta_N)\right)\right)} | \theta_N \right] = \kappa$$

for some constant $\kappa > 0$. I'm going to show this cannot be optimal. Recalling that $\beta_M = 1$, consider the following perturbation for the allocation of type (β_M, θ_N):

$$\tilde{u}_t\left(h^{t-1}, (\beta_M, \theta_N)\right) = u_t\left(h^{t-1}, (\beta_M, \theta_N)\right) - \varepsilon$$
$$\tilde{u}_{t+1}\left(h^{t-1}, (\beta_m, \theta_n), (\beta_{t+1}, \theta_{t+1})\right) = u_{t+1}\left(h^{t-1}, (\beta_M, \theta_N), (\beta_{t+1}, \theta_{t+1})\right) + \frac{1}{\delta}\varepsilon$$

for $\varepsilon > 0$. The welfare of type $(h^{t-1}, (\beta_M, \theta_N))$ is kept constant by such a change. Further, types $(h^{t-1}, (\beta_j, \theta_N))$ for $\beta_j < 1$ dislike the perturbation, so it is incentive compatible. The marginal change in the amount of resources used by type $(h^{t-1}, (\beta_M, \theta_N))$

is

$$\begin{aligned} dE &= -C'\left(u_t\left(h^{t-1},(\beta_M,\theta_N)\right)\right)\varepsilon \\ &\quad + \frac{1}{R_t}\mathbb{E}_t\left[C'\left(u_{t+1}\left(h^{t-1},(\beta_M,\theta_N),(\beta_{t+1},\theta_{t+1})\right)\right)|\theta_N\right]\frac{1}{\delta}\varepsilon \\ &= (\kappa-1)\,C'\left(u_t\left(h^{t-1},(\beta_M,\theta_N)\right)\right)\varepsilon \end{aligned}$$

At the solution to the planner's problem it must be the case $dE \geq 0$. Therefore we conclude $\kappa \geq 1$. Now assume by way of contradiction that $\kappa > 1$. Since $\beta_t \leq 1$ for all $\beta_t \in B_t$, then consider the following perturbation to all types $(h^{t-1},(\beta_t,\theta_N))$:

$$\begin{aligned} \tilde{u}_t\left(h^{t-1},(\beta_t,\theta_N)\right) &= u_t\left(h^{t-1},(\beta_t,\theta_N)\right)+\varepsilon \\ \tilde{u}_{t+1}\left(h^{t-1},(\beta_t,\theta_N),(\beta_{t+1},\theta_{t+1})\right) &= u_{t+1}\left(h^{t-1},(\beta_t,\theta_N),(\beta_{t+1},\theta_{t+1})\right)-\frac{1}{\delta}\varepsilon \end{aligned}$$

Type $(h^{t-1},(\beta_M,\theta_N))$ is indifferent between the original allocation and the perturbed one, types $(h^{t-1},(\beta_t,\theta_N))$ with $\beta_t < 1$ strictly prefer the perturbed allocation for $\varepsilon > 0$. Since no incentive constraint from types $\{\theta_1,\theta_2,\ldots,\theta_{n-1}\}$ are binding upwards, then the perturbation is incentive compatible for $\varepsilon > 0$ small enough. The change in resources used by each type of agent is

$$dE = (1-\kappa)\,C'\left(u_t\left(h^{t-1},(\beta_t,\theta_N)\right)\right)\varepsilon$$

and we reach a contradiction since $\kappa > 1$. Therefore we conclude $\kappa = 1$ and if agents are bunched at θ_N, then the inverse Euler equation holds.

Now we need to show this implies a contradiction. By Lemma D.2, since agent's with the high earnings shock are bunched, we have $u_t\left(h^{t-1},(\beta_t,\theta_N)\right) >$

$u_t\left(h^{t-1},(\beta_t,\theta_j)\right)$ for all $j<N$. Then consider the following perturbation

$$\tilde{u}_t\left(h^{t-1},(\beta_1,\theta_N)\right)=u_t\left(h^{t-1},(\beta_1,\theta_N)\right)+\varepsilon-\nu$$
$$\tilde{u}_{t+1}\left(h^{t-1},(\beta_1,\theta_N),(\beta_{t+1},\theta_{t+1})\right)=u_{t+1}\left(h^{t-1},(\beta_1,\theta_N),(\beta_{t+1},\theta_{t+1})\right)-\frac{1}{\delta\beta_2}\varepsilon$$

From the point of view of β_1 the payoff change is

$$dU\left(\beta_1\right)=\left(1-\frac{\beta_1}{\beta_2}\right)\varepsilon-\nu$$

Since $\beta_1<\beta_2$, then we can choose $\varepsilon>0$ and $\nu>0$ such that $\left(1-\frac{\beta_1}{\beta_2}\right)\varepsilon=\nu$. In this case agent $(h^{t-1},(\beta_1,\theta_N))$ is made indifferent with the perturbation and the original allocation. Further, agents with type $\beta>\beta_1$ dislike the perturbation. Therefore, since other incentive constraints with respect to type $(h^{t-1},(\beta_1,\theta_N))$ are slack, the perturbation is incentive compatible. However, in terms of resources we have

$$\begin{aligned}
dE &= C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)(\varepsilon-\nu)\\
&\quad-\frac{1}{R_t}\mathbb{E}_t\left[C'\left(u_{t+1}\left(h^{t-1},(\beta_M,\theta_N),(\beta_{t+1},\theta_{t+1})\right)\right)|\theta_N\right]\frac{1}{\delta\beta_2}\varepsilon\\
&= C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)\left[\left(1-\frac{\kappa}{\beta_2}\right)\varepsilon-\nu\right]\\
&= C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)\left(\frac{\beta_1}{\beta_2}-\frac{\kappa}{\beta_2}\right)\varepsilon
\end{aligned}$$

Since $\beta_2\leq 1$, then for $\varepsilon>0$ and $\nu>0$ we have $dE<0$ so that the planner saves resources. Those resources can be uniformly distributed across all types in the economy. Since $u_t\left(h^{t-1},(\beta_t,\theta_N)\right)>u_t\left(h^{t-1},(\beta_t,\theta_j)\right)$ for all $j<N$. There exists $\varepsilon>0$ small enough such that welfare improves with this redistribution and we reach a contradiction. □

Proof of Theorem D.2

We first show result 1 in Theorem D.2. Fix a period t and a history h^{t-1}. From Theorem D.1 we have that there exists $\bar{\theta}_1 > 0$ such that all agents with a history in $\{(h^{t-1}, (\beta, \theta_1)) : \beta \in B\}$ are bunched at the same continuation allocation. In particular they all face the same inverse Euler equation distortion which we denote by

$$\kappa = \sum_{(\beta_{t+1}, \theta_{t+1}) \in B \times \Theta} \gamma_{t+1}(\beta_{t+1}) \rho_{t+1}(\theta_{t+1}|\theta_1) \left[\frac{C'(u_{t+1}(h^{t-1}, (\beta, \theta_1), (\beta_{t+1}, \theta_{t+1})))}{\delta R_t C'(u_t(h^{t-1}, (\beta, \theta_1)))} | \theta_N \right]$$

for all $\beta \in B$ and for a constant $\kappa > 0$. We want to show that $\kappa \geq 1$. Assume by way of contradiction that $\kappa > 1$. Then consider the following perturbation

$$\tilde{u}_t\left(h^{t-1}, (\beta, \theta_1)\right) = u_t\left(h^{t-1}, (\beta, \theta_1)\right) - \delta\varepsilon$$
$$\tilde{u}_{t+1}\left(h^{t-1}, (\beta, \theta_1), (\beta_{t+1}, \theta_{t+1})\right) = u_{t+1}\left(h^{t-1}, (\beta, \theta_1), (\beta_{t+1}, \theta_{t+1})\right) + \varepsilon$$

for all $(\beta_{t+1}, \theta_{t+1}) \in B \times \Theta$. This perturbation keeps welfare constant and therefore does not affect incentive compatibility at period $s < t$ (hyperbolic preferences). From Lemma D.3 there exists $\bar{\theta}_1$ such that for $\theta_1 < \bar{\theta}_1$ we have that agents with histories in $\{(h^{t-1}, (\beta, \theta_1)) : \beta \in B\}$ incentives are strictly slack with respect to any other agent in the economy not in this group. Since for $\varepsilon > 0$ agents with $\beta \leq 1$ find this perturbation to weakly decrease the payoff of this allocation, we conclude the perturbation is incentive compatible. The marginal change in usage of resources is given by

$$dE = (\kappa - 1)\delta C'\left(u_t\left(h^{t-1}, (\beta, \theta_1)\right)\right)\varepsilon$$

For $\varepsilon > 0$ and $\kappa < 1$ we then get $dE < 0$, a contradiction as the planner can economize resources while keeping welfare constant. Hence we proved the first part of the theorem.

For part 2(a) of the Theorem, let

$$\kappa_H = \sum_{(\beta_{t+1},\theta_{t+1})\in B\times\Theta} \gamma_{t+1}\left(\beta_{t+1}\right)\rho_{t+1}\left(\theta_{t+1}|\theta_N\right)\frac{C'\left(u_{t+1}\left(h^{t-1},\left(\beta_M,\theta_N\right),\left(\beta_{t+1},\theta_{t+1}\right)\right)\right)}{\delta R_t C'\left(u_t\left(h^{t-1},\left(\beta_M,\theta_N\right)\right)\right)}$$

and assume by way of contradiction that $\kappa_H < 1$. Consider the perturbation

$$\tilde{u}_t\left(h^{t-1},\left(\beta_M,\theta_N\right)\right) = u_t\left(h^{t-1},\left(\beta_M,\theta_N\right)\right) - \delta\varepsilon$$
$$\tilde{u}_{t+1}\left(h^{t-1},\left(\beta_M,\theta_N\right),\left(\beta_{t+1},\theta_{t+1}\right)\right) = u_{t+1}\left(h^{t-1},\left(\beta_M,\theta_N\right),\left(\beta_{t+1},\theta_{t+1}\right)\right) + \varepsilon$$

for all $(\beta_{t+1},\theta_{t+1}) \in B\times\Theta$. Since $\beta_M = 1$, this is clearly incentive compatible for $\varepsilon > 0$. The marginal change in the usage of resource is given by

$$dE = \left(\kappa_H - 1\right)\delta C'\left(u_t\left(h^{t-1},\left(\beta_M,\theta_N\right)\right)\right)\varepsilon$$

therefore if $\kappa_H < 1$ we have that $dE < 0$ for $\varepsilon > 0$. From full support on the distribution of types, the planner can generate strictly positive resources with this incentive compatible perturbation, while keeping welfare constant. This is a contradiction as the extra resources can be used to strictly improve welfare. Thus we proved part 2(a) of the Theorem.

For part 2(b), notice that from Lemma D.4 and from Theorem D.1 we have that there exists $\bar{\theta}_{N-1} < \theta_N$ and $\bar{\beta}_1 > 0$ such that for $\theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_{N-1}$ and $0 < \beta_1 < \bar{\beta}_1$ we have that agents with histories $\{(h^{t-1},(\beta,\theta)) : \beta \in B\, and\, \theta < \theta_N\}$ strictly prefer their own allocation to the allocation of any agent with a history $\{(h^{t-1},(\beta,\theta_N)) : \beta \in B\}$. Further we have $u_t\left(\left(h^{t-1},\left(\beta_1,\theta_N\right)\right)\right) > u_t\left(\left(h^{t-1},\left(\beta,\theta\right)\right)\right)$ for all $\theta < \theta_N$ and all $\beta \in B$. Denote

$$\kappa_H^1 = \sum_{(\beta_{t+1},\theta_{t+1})\in B\times\Theta} \gamma_{t+1}\left(\beta_{t+1}\right)\rho_{t+1}\left(\theta_{t+1}|\theta_N\right)\frac{C'\left(u_{t+1}\left(h^{t-1},\left(\beta_1,\theta_N\right),\left(\beta_{t+1},\theta_{t+1}\right)\right)\right)}{\delta R_t C'\left(u_t\left(h^{t-1},\left(\beta_1,\theta_N\right)\right)\right)}$$

and assume by way of contradiction that $\kappa_H^1 \geq 1$. Consider the following perturbation

$$\begin{aligned}
\tilde{u}_t\left(h^{t-1},(\beta_1,\theta_N)\right) &= u_t\left(h^{t-1},(\beta_1,\theta_N)\right)+\beta_1\delta\varepsilon+\nu \\
\tilde{u}_{t+1}\left(h^{t-1},(\beta_1,\theta_N),(\beta_{t+1},\theta_{t+1})\right) &= u_{t+1}\left(h^{t-1},(\beta_1,\theta_N),(\beta_{t+1},\theta_{t+1})\right)-\varepsilon \\
\tilde{u}_t\left(h^{t-1},(\beta,\theta)\right) &= \tilde{u}_t\left(h^{t-1},(\beta,\theta)\right)+\nu
\end{aligned}$$

for all $(\beta_{t+1},\theta_{t+1}) \in B\times\Theta$ and all $(\beta,\theta)\neq(\beta_1,\theta_N)$. For $\varepsilon>0$ and $\nu>0$, this perturbation is incentive compatible since agents with $\beta\geq\beta_1$ find it (weakly) unattractive. In order to keep welfare unchanged with this perturbation we need

$$dW = \pi\left(\beta_1,\theta_N|h^{t-1}\right)(\beta_1-1)\,\delta\varepsilon+\nu=0$$

so that $\nu=\pi\left(\beta_1,\theta_N|h^{t-1}\right)(1-\beta_1)\,\delta\varepsilon$. The marginal change in resource usage of this perturbation is given by

$$\begin{aligned}
dE =& \pi\left(\beta_1,\theta_N|h^{t-1}\right)C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)\delta\left(\beta_1-\kappa_H^1\right)\varepsilon \\
&+\nu\sum_{(\beta,\theta)}\pi\left(\beta,\theta|h^{t-1}\right)C'\left(u_t\left(h^{t-1},(\beta,\theta)\right)\right) \\
=& A\left(h^{t-1}\right)\left[\left(\frac{\beta_1-\kappa_H^1}{1-\beta_1}\right)+\sum_{(\beta,\theta)}\pi\left(\beta,\theta|h^{t-1}\right)\frac{C'\left(u_t\left(h^{t-1},(\beta,\theta)\right)\right)}{C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)}\right]\varepsilon
\end{aligned}$$

where $A\left(h^{t-1}\right) = \pi\left(\beta_1,\theta_N|h^{t-1}\right)\delta C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)(1-\beta_1) > 0$ Since we have $u_t\left(h^{t-1},(\beta_1,\theta_N)\right) > u_t\left(h^{t-1},(\beta,\theta)\right)$ for $\theta<\theta_N$ and $u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\geq u_t\left(h^{t-1},(\beta,\theta_N)\right)$ for all β, then we have

$$\sum_{(\beta,\theta)}\pi\left(\beta,\theta|h^{t-1}\right)\frac{C'\left(u_t\left(h^{t-1},(\beta,\theta)\right)\right)}{C'\left(u_t\left(h^{t-1},(\beta_1,\theta_N)\right)\right)}<1$$

and since $\beta_1 < 1 \leq \kappa_H^1$ we conclude that $dE < 0$ for $\varepsilon > 0$, a contradiction. Hence we proved 2(b).

Proofs for Section 4.5

In this Appendix we provide a proof for Theorem 4.3. For the first part of the Theorem the proof is analogous to the proof of Theorem 4.1 and relies only on the convexity of the set of incentive constraints when we rewrite the problem in terms of utility levels and the wedge level. For the second part of the Theorem, note that for $\theta_1 < \ldots < \theta_{N-1} \leq \bar{\theta}_N$ we have $v(\theta_N, \alpha) > v(\theta_1)$. Therefore by incentive compatibility it must be the case that

$$u(\theta_N, \alpha) + \alpha\varepsilon(\theta_N, \alpha) > u(\theta_1) + \alpha\varepsilon(\theta_1)$$

In particular, for $\alpha = 0$ we have $u(\theta_N, \alpha) > u(\theta, \alpha')$ for all (θ, α'). Assume by way of contradiction that $u(\theta_N, \alpha) = u(\theta_N)$, $v(\theta_N, \alpha) = v(\theta_N)$ and $a(\theta_N, \alpha) = a(\theta_N)$ at the solution to the planner's problem. Then we have $u(\theta_N) > u(\theta_1)$. Let $\alpha_{max} = \max A \geq 0$. If $b'(a(\theta_N)) > p$, then consider the following change to the allocation:

$$\begin{aligned}
\tilde{a}(\theta_N, \alpha_{max}) &= \alpha(\theta_N) + \nu \\
\tilde{u}(\theta_N, \alpha_{max}) &= u(\theta_N) - \alpha_{max}\nu + \eta \\
\tilde{u}(\theta, \alpha) &= \tilde{u}(\theta, \alpha) + \eta
\end{aligned}$$

for all $(\theta, \alpha) \neq (\theta_N, \alpha_{max})$. For $\nu > 0$ this perturbation is incentive compatible since for $\alpha < \alpha_{max}$ the change in the deviation payoff into the allocation of (θ_N, α_{max}) is $(\alpha - \alpha_{max})\nu < 0$ and the original allocation is incentive compatible. Now the

marginal change in resources used is given by

$$dE = \pi\left(\theta_N, \alpha_{max}\right)\left[-\alpha_{max}\nu C'\left(u\left(\theta_N\right)\right) - \left(\frac{b'\left(\varepsilon^{-1}\left(\varepsilon\left(\theta_N\right)\right)\right)}{\varepsilon'\left(\varepsilon^{-1}\left(\varepsilon\left(\theta_N\right)\right)\right)} - \frac{p}{\varepsilon'\left(\varepsilon^{-1}\left(\varepsilon\left(\theta_N\right)\right)\right)}\right)\nu\right]$$
$$+ \eta \sum_{\theta,\alpha} \pi\left(\theta, \alpha\right) C'\left(u\left(\theta, \alpha\right)\right)$$

If $\alpha_{max} = 0$, then we already get a contradiction since for $\nu > 0$ we can set $\eta > 0$ small enough such that welfare increases. For $\alpha_{max} > 0$, the marginal change in the government's objective is given by

$$d\mathcal{W} = -\pi\left(\theta_N, \alpha_{max}\right)\lambda\left(\theta_N\right)\alpha_{max}\nu + \eta$$

If we set $\eta = \pi\left(\theta_N, \alpha_{max}\right)\lambda\left(\theta_N\right)\alpha_{max}\nu$ so that $d\mathcal{W} = 0$, then the marginal change in resources used is given by

$$\frac{dE}{B} = \nu\left\{\lambda\left(\theta_N\right)\sum_{\theta,\alpha}\pi\left(\theta, \alpha\right)\frac{C'\left(u\left(\theta, \alpha\right)\right)}{C'\left(u\left(\theta_N\right)\right)} - 1\right\}$$
$$- \frac{\nu}{\alpha_{max}C'\left(u\left(\theta_N\right)\right)}\left(\frac{b'\left(\varepsilon^{-1}\left(\varepsilon\left(\theta_N\right)\right)\right)}{\varepsilon'\left(\varepsilon^{-1}\left(\varepsilon\left(\theta_N\right)\right)\right)} - \frac{p}{\varepsilon'\left(\varepsilon^{-1}\left(\varepsilon\left(\theta_N\right)\right)\right)}\right)$$

where $B = \pi\left(\theta_N, \alpha_{max}\right)\alpha_{max}C'\left(u\left(\theta_N\right)\right) > 0$. Since $\lambda\left(\theta_N\right)$ and since $C\left(\cdot\right)$ is convex and $u\left(\theta_N\right) \geq u\left(\theta, \alpha\right)$, then $dE < 0$ for $\nu > 0$, a contradiction. Therefore we must have $b'\left(a\left(\theta_N\right)\right) \leq p$. Again we can make an analogous perturbation for $\alpha_{min} = \min A \leq 0$. Hence we conclude $b'\left(a\left(\theta_N\right)\right) = p$. If $\alpha_{max} > 0$, note that the exact same perturbation above also implies $dE < 0$ since $u\left(\theta_N\right) > u\left(\theta, \alpha\right)$ for $\theta < \theta_N$, a contradiction. The case for $\alpha_{min} < 0$ is analogous. Finally we exhausted all possible cases and reach a contradiction to our initial assumption that all agents in $\{(\theta_N, \alpha) : \alpha \in A\}$ receive the same allocation.

D.3 Calibration

Alternative life-cycle calibration with hyperbolic preferences

In this appendix we build a incomplete markets life-cycle model with hyperbolic discounting. This model allows us to calibrate for β's and δ separately. We compare the heterogeneity generated by this calibration and the benchmark calibration and find it to be very similar so that in terms of savings distortions the two calibrations yield very similar results.

Households start their lives at age 25 and have a working life of up to age 65. At age 65 households retire and live up to age 99 at most. At each period there is a probability s_t of survival until the next period (from 2010 U.S. Life Tables). At each period, households receive endowment shocks e_t that follows

$$\ln e_{i,t+1} = \phi_{t+1} + \rho \ln e_{i,t} + \nu_{i,t+1}$$

where ϕ_{t+1} is the life-cycle component and $\nu_{i,t+1} \sim N(0, \sigma^2)$. We estimate ϕ_{t+1}, ρ and σ^2 from the PSID from 1999-2009. We find $\rho = 0.829$ and $\sigma = 0.42$.[12]

Each period workers can save $a_t \geq 0$ resources into the next period at a gross rate of interest R. Finally workers pay taxes on labor and asset income according to

$$T(y_t, Ra_{t-1}) = \left(t_0 + y_t - t_1 y_t^{1-t_2}\right) + t_a Ra_{t-1}$$

[12]In particular, in order to estimate ϕ_{t+1} as the set of age dummies in a regression of reported labor income on fixed household effects and a set of controls for the business cycle (polinomial on state level unemployment rates) and for demographics (family size, marital status, number of children). Then as a measure of e_t we use fixed effects plus residuals from this regression. Then we estimate an AR(1) with this measure to obtain ρ and σ estimates.

We assume linearity on asset income taxes to make the dynamic model more tractable. Using the Cross National equivalent PSID files from 1999-2009 and using non-linear least squares, we find $t_0 = 2488$, $t_1 = 1.67$, $t_2 = 0.07$ and $t_a = 0.28$.[13]

After age 65, individuals receive no labor income, only asset income and social security benefits $b(y_{65})$. We follow Gourinchas and Parker (2002) and Laibson, Repetto and Tobacman (2007) in assuming that retirement benefits depend only on labor income only in the last period. For $b(\cdot)$ we use the current schedule on social security old-age benefits. Finally for tax purposes only 55% of social security benefits are included into taxable income so we take that into account.

Agent's have hyperbolic preferences (with a constant β present-bias over time) and therefore choices during working life are solutions to

$$U_t(x_t, s_t) = \max u(c_t) + \beta\delta\mathbb{E}_t\left[V_{t+1}(a_{t+1}, y_{t+1})|y_t\right]$$

$$s.t. c_t + a_{t+1} = x_t$$

$$a_{t+1} \geq 0$$

where $u(c) = \frac{c^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}$ and

$$V_t(a_t, s_t) = u(c_t(a_t, y_t(s_t), s_t)) + \delta\mathbb{E}_t\left[V_{t+1}(a_{t+1}(a_t, y_t(s_t), s_t), s_{t+1})\right]$$

$$x_{t+1} = R_{t+1}a_{t+1} + y_{t+1}(s_{t+1}) - T(y_{t+1}(s_{t+1}), R_{t+1}a_{t+1})$$

[13] All variables in 2010 dollars, scaled by family size as

$$size = 1 + 0.7 * other\ adults + 0.5 * children$$

used in Attanazio and Pistaferri (2014). We consider overall household income for households a head within 25-65 years old, that worked more than 260 hours in the year and that made at least \$4000 in labor income. We exclude households with negative asset income and household in the highest percentile of asset income. Finally, we exclude the lowest and highest percentile on TAXSIM estimated taxes paid.

We then obtain the following Euler equation:

$$u'\left(c_t^i\left(x_t^i, s_t^i\right)\right) = (1-\tau_a) R\delta \mathbb{E}_t\left[u'\left(c_{t+1}^i\left(x_{t+1}^i, s_{t+1}^i\right)\right)|s_t^i\right] - (1-\tau_a) R\delta (1-\beta) \mathbb{E}_t\left[MPC_{t+1}^i\left(x_{t+1}, s_{t+1}\right) u'\left(c_{t+1}^i\left(x_{t+1}^i, s_{t+1}^i\right)\right)|s_t^i\right]$$

where

$$MPC_{t+1}^i\left(x_{t+1}, s_{t+1}\right) = \frac{\partial c_{t+1}^i\left(x_{t+1}^i, s_{t+1}^i\right)}{\partial x_{t+1}^i}$$

Finally we assume $\frac{\beta}{2} \sim Beta\left(a, b\right)$ in the population so that we allow both for present bias and future bias.

We match exactly the same percentiles of the distribution of private wealth at retirement to lifetime earnings ratio. In Table D.1 we see the model matches exactly the three statistics from the data. Finally in the table we report statistics on the distribution of β we find as well as on the distribution of lifetime average time inconsistency wedge:

$$\tau_\beta = 1 - (1-\beta) MPC_{t+1}^i\left(x_{t+1}, s_{t+1}\right)$$

This second measure gives us a better sense of how distorted savings decisions are each year on average. Notice that those distortions are relatively small in a yearly basis, but when compounded over the life-cycle a distortion of $\tau_\beta = 0.96$ implies individuals care very little about retirement when young in life as compared with individuals with $\tau_\beta = 1.0$.

Table D.1: Life-Cycle Calibration Results

Statistic	Data	Value
δ		0.9856
$avg\ \beta$		0.80
$std\,\beta$		0.62
avg		0.96
$std\ b$		0.058
$corr\,(b, LT\ Labor\ Earnings)$		-0.01
W/Y percentile 25%	0.043	0.043
W/Y percentile 50%	0.0906	0.0906
W/Y percentile 75%	0.1768	0.1768

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