

**From the holdings of the Department of Special Collections , Princeton University Library,
Princeton, NJ**

These documents are either in the public domain, or can only be used for educational and research purposes (Fair use) as per U.S. Copyright law (text below). By accessing this file, all users agree that their use falls in the public domain or within fair use as defined by the copyright law. We do not charge any permission or usage fees for the publication of images of material in our collections, including those provided by Princeton University Library, via our website, catalog, or directly from staff. For more information regarding copyright, credit, and citation please review the information on the Copyright, Credit and Citation Guidelines page of our website.

<https://library.princeton.edu/special-collections/policies/copyright-credit-and-citation-guidelines>. If you have any questions, including how to order higher-quality images from the collection, contact Special Collections Public Services via our Ask Us! Form. <https://library.princeton.edu/special-collections/ask-us>

Doctoral Dissertations, 2011-2026

Economics

2018

Essays in International Trade

Razhev, Mark

Princeton, NJ : Princeton University

<http://arks.princeton.edu/ark:/88435/dsp01fn1071655>

U.S. Copyright law: The copyright law of the United States (Title 17, §108, United States Code) governs the making of photocopies or other reproductions of copyrighted material. Under certain conditions specified in the law, libraries and archives are authorized to furnish a photocopy or other reproduction. One of these specified conditions is that the photocopy or reproduction is not to be used for any purpose other than private study, scholarship, or research. If a user makes a request for, or later uses, a photocopy or reproduction for purposes in excess of fair use that user may be liable for copyright infringement. This institution reserves the right to refuse to accept a copying order if, in its judgment, fulfillment of the order would involve a violation of copyright law. <https://www.copyright.gov/title17/title17.pdf>

ESSAYS IN INTERNATIONAL TRADE

MARK RAZHEV

A DISSERTATION
PRESENTED TO THE FACULTY
OF PRINCETON UNIVERSITY
IN CANDIDACY FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

RECOMMENDED FOR ACCEPTANCE
BY THE DEPARTMENT OF
ECONOMICS
ADVISER: OLEG ITSKHOKI

JUNE 2018

© Copyright by Mark Razhev, 2018.

All rights reserved.

Abstract

This dissertation consists of three essays exploring specific issues in international trade.

The first chapter studies how disaggregated input-output interactions shape trade and welfare responses to changes in trade costs. I develop a model with a large number of products linked through a general "snakes and spiders" network. The central feature of the model which also makes it highly intractable is endogenous formation of comparative advantage. To overcome this complexity challenge, I obtain a perturbation solution in terms of intuitive summary statistics. I find that, in contrast to the composite intermediate good structure often employed in the literature, imperfect supplier diversification transforms fundamental comparative advantage in two ways. First, as exogenous productivity differences accumulate along supply chains, endogenous variation in relative costs is increasing in the level of trade frictions. Second, the comparative advantage of upstream and downstream industries becomes positively correlated. The first effect generates larger welfare gains from trade and also raises the average product-level import share. The second effect, however, is trade-reducing: the tendency of comparative-advantage industries to source disproportionately from each other increases aggregate home bias.

The second chapter links the growth of trade to structural transformation in the major economies over the last several decades which have seen steadily declining manufacturing employment. I find that, under increasing returns, falling employment in manufacturing amplifies comparative advantage forces relative to trade barriers, specialization deepens to offset losses in scale in the marginal industries, and trade increases. I establish this result in a model based on Dornbusch, Fischer, and Samuelson (1977) to which I add exogenously nontraded services and fixed costs in manufacturing.

The third chapter studies optimal trade policy in the presence of intermediate goods. I consider a multi-industry two-stage production economy where both final and intermediate products are traded and the available policy instruments include all import and export tariffs. Import taxes for intermediate goods are uniform, while other instruments are heterogeneous, responding to variation in model parameters. I find that stronger international production sharing reduces optimal import tariffs but increases optimal export taxes.

Acknowledgements

I am extremely grateful to my adviser Oleg Itskhoki for his guidance and support during the whole development of this work. I would like to thank Gene Grossman, Ezra Oberfield, Steve Redding, and Esteban Rossi-Hansberg for numerous discussions that gave me encouragement and many insights for my research. I would also like to thank my classmates Kevin Lim, David Nagy, Ricardo Reyes-Heroles, and Sharon Traiberman for creating a fantastic research environment. Comments from numerous participants of the trade lunch seminar at Princeton are gratefully acknowledged as well.

As my job market paper, Chapter 1 of this dissertation was presented at The Higher School of Economics, Cambridge and Melbourne Universities, and The University of New South Wales. It was also presented at the New Faces in International Trade conference at The Pennsylvania State University. I would like to thank all seminars participants and especially Jonathan Eton who provided a discussion for my paper.

I would also like to thank Laura Hedden and Sharon Ernst for their extraordinary assistance with organizational matters. Financial support from the International Economics Section at Princeton University is gratefully acknowledged.

Finally, I would like to thank my parents and my wife Anastasia for their love and encouragement.

To my parents.

Contents

Abstract	iii
Acknowledgements	v
List of Tables	xi
List of Figures	xii
1 Input-Output Structure and Trade Elasticity	1
1.1 Introduction	1
1.2 Model	8
1.2.1 Closed Economy	8
1.2.2 Open Economy: Setup and General Relationships	13
1.2.3 Local Analysis Near Free Trade	23
1.3 Neutrality Result	25
1.3.1 I-O Trade and Gravity	26
1.3.2 Connection to the Literature	30
1.3.3 Perturbation	31
1.4 Perturbation Solution	33
1.4.1 Endogenous Relative Cost Heterogeneity	33
1.4.2 Welfare	37
1.4.3 Approximation quality	41
1.5 Conclusion	43

2	Structural Change, Increasing Returns, and the Growth of Trade	45
2.1	Introduction	45
2.2	Planner's Allocation	52
2.2.1	Environment and Planner Optimality	52
2.2.2	Structural Change	57
2.2.3	Simulation Example	58
2.2.4	Trade Costs	60
2.2.5	Discussion	61
2.3	Oligopoly	62
2.3.1	Model Setup	62
2.3.2	Specialization and Structural Change	65
2.3.3	Markups and Redistribution	68
2.4	Conclusion	69
3	Optimal Trade Policy with Intermediate Goods	73
3.1	Introduction	73
3.2	Direct Problem	76
3.2.1	Preferences and Technology	76
3.2.2	Home Economy	78
3.2.3	Foreign Economy	81
3.2.4	Trade Policy	82
3.3	Indirect Approach	88
3.3.1	Import tariffs	91
3.3.2	Export taxes	95
3.3.3	Comparative Statics	97
3.4	Conclusion	99

A Illustrations and Proofs for Chapter 1	100
A.1 Evidence	100
A.2 LLN Lemma	102
A.3 Sales	103
A.4 Proof of Proposition 1.2.1	104
A.5 The Local ACR Result	105
A.6 Local Analysis Near Free Trade	106
A.7 Neutrality Result	110
A.8 Derivation of (1.43) and (1.45)	116
A.9 Derivation of (1.46)	118
A.10 Trace Properties	118
A.11 Proof of Proposition (1.4.1)	120
A.12 Proof of Proposition (1.4.2)	121
B Proofs for Chapter 2	124
B.1 Proofs for Section 2.2	124
B.1.1 Planner's Problem	124
B.1.2 Proof of Proposition 2.2.1	126
B.1.3 Proof of Proposition 2.2.3	128
B.2 Proofs for Section 2.3	129
B.2.1 Proof of Proposition 2.3.1	129
B.2.2 Proof of Proposition 2.3.2	129
B.2.3 Proof of Proposition 2.3.3	130
B.3 A Model with Cournot Competition	133
C Proofs for Chapter 3	137
C.1 Foreign Equilibrium	137
C.2 Proof of Proposition 3.3.3	139

C.3 Proof of Proposition 3.3.4	140
Bibliography	141

List of Tables

1.1	Main Notation for Chapter 1	15
3.1	Domestic Shares	82

List of Figures

1.1	Comparing Production Networks	22
1.2	Welfare and Comparative Advantage	38
1.3	Trade Flows Decomposition	41
1.4	Standard Deviation of Cost Shifters	42
1.5	Approximation with $\rho = 3$	43
1.6	Approximation with $\rho = 5$	44
2.1	Simulation Example	60
2.2	Prices in Home	72
3.1	Variation in Policy Instruments	98
A.1	Histogram Herfindahl Indices	100
A.2	Comparative Advantage Spillovers	101
A.3	Traces	120

Chapter 1

Input-Output Structure and Trade Elasticity

1.1 Introduction

How does the input-output (I-O) structure of the economy affect the response of trade flows and welfare to changes in trade barriers? Once we acknowledge the great complexity of the world's production architecture in which goods are linked through an intricate "snakes and spiders" network, approaching this question is quite challenging even within stylized competitive environments. Thus, in spite of clear microfoundations, models of supply chains struggle with providing a transparent link between outcome variables, such as welfare and trade flows, and model fundamentals which include trade costs and the pattern of I-O interactions. Gaining intuitive insight typically involves either shutting down particular channels (what happens if trade is in final goods only) or semi-informal arguments (when countries exchange both final and intermediate goods, the impact of trade frictions is magnified by multiple border crossing). The lack of understanding of which I-O characteristics are essential in the trade context may be uncomfortable given the growing attention to international

production fragmentation.¹ Related to this, little is known about potential biases arising from various simplifications employed in the literature, such as a composite intermediate good assumption which is a common way to avoid specifying the detailed structure of linkages while introducing trade in inputs.

As identified in this paper, a major source of complications in studying supply chain trade lies not in I-O linkages per se, but in their interplay with comparative advantage (CA). My benchmark theoretical model has two symmetric countries, a large number of products, $i = 1..N$, and features a log-linear relationship between relative marginal costs on the one hand and relative exogenous cost shifters and relative prices² on the other:

$$\log \frac{MC_i^*}{MC_i} = \log \frac{Z_i^*}{Z_i} + \beta \sum_{i'=1}^N \omega_{ii'} \log \frac{P_{i'}^*}{P_{i'}}, \quad (1.1)$$

where β is the input share and $\omega_{ii'}$ is the share of intermediate spending by product i on product i' . As common in multi-industry models, the distribution of relative marginal costs is the central object that determines trade flows and the gains from trade. In two cases – when intermediate inputs are absent ($\beta = 0$) and with a composite, product-invariant intermediate good – heterogeneity in relative marginal costs is shaped directly by fundamental CA (variation in relative exogenous costs) independently of trade barriers. In general, however, the I-O structure interacts with trade costs to transform fundamental CA. Depending on the level of trade integration, the differences in fundamental productivities accumulate along production chains, which makes marginal cost heterogeneity endogenous to trade frictions. My goal is to in-

¹The ongoing surge in vertical trade studies is driven by a variety of factors, starting from a basic empirical regularity that trade flows are dominated by intermediate goods. See Johnson (2014) for a review of recent developments.

²Prices are decoupled from marginal costs because of trade. All goods enter production function as well as preferences as Armington CES bundles aggregating the corresponding domestic and imported varieties. This structure nests the case of perfect substitutes when the Armington elasticity is infinite. Although this latter case activates the extensive margin and excludes two-way trade, its difference in terms of aggregate outcomes is quantitative rather than qualitative.

investigate how this endogenous CA formation depends on the pattern of I-O linkages and how it maps into trade flows and welfare. Given that under general form of linkages functional form assumptions do not bring much or any tractability, overcoming the model’s black box nature is not straightforward. However, the following simple observation allows moving forward.

Although I-O interactions transform CA, they do not create it out of nothing. In particular, in the absence of exogenous technology heterogeneity, there is no relative variation in endogenous marginal costs under any specification of cost shares $\omega_{ii'}$ in (1.1). The model then collapses into (effectively) one-product economy with only intra-industry trade. In fact, this neutrality result applies more broadly to multiple asymmetric countries and non-constant I-O coefficients. Proposition 1.3.1 considers a multi-country multi-product economy in which product-level trade flows satisfy a gravity equation and states that, without fundamental CA forces, the standard constant-elasticity gravity holds at the aggregate level under very general I-O structure. Despite the fact that multiple border crossing makes import prices move more than one-for-one with trade costs, the *relative* prices are only affected by I-O when it interacts with CA.³ For its constructive value, Proposition 1.3.1 suggests the gravity framework as a natural point of departure for studying the effects of CA (as well as other forms of product-level heterogeneity). In parallel to macroeconomics, the gravity system can be viewed as a ”steady state” to which the economy converges in the absence of shocks. To fully exploit this analogy, I adopt the perturbation methodology to modeling cross-section variation in international trade.⁴

³In Section 1.3 I discuss the connection of this result to the existing literature. Most importantly, Yi (2003, 2010) introduced the idea that international production sharing can magnify the responses of trade flows to changes in trade costs. For the role of CA, a special case of Proposition 1.3.1 is present in French (2016) who does not focus on I-O linkages and employs a composite intermediate good structure.

⁴Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) use a similar methodological idea that network interactions can be studied by perturbing a cross-section of homogenous economic units. While the main distinction of my work is in exploring features of the model that are specific to international trade, in Section 1.3 I also discuss technical differences with this paper.

I obtain a second-order approximation for the weighted variance of relative marginal costs as a function of trade barriers and a few summary statistics for the I-O matrix and then show that this object is sufficient to capture second-order departures of trade flows and welfare from the gravity benchmark. My three main results are the following.

First, heterogeneity in relative marginal costs is magnified by I-O interactions and increases with the level of trade frictions. Under free trade, when international production sharing is not constrained, producers in each country face the same input prices and the only source of heterogeneity in relative marginal costs is variation in fundamental productivities. When trade is restricted, firms increasingly rely on domestic intermediate inputs; the differences in fundamental productivities accumulate along production chains, resulting in stronger marginal cost heterogeneity. In other words, due to trade barriers, firms in different countries face different input prices, which contributes to marginal cost variation. This mechanism depends crucially on the extent of supplier diversification in the economy which is captured by the weighted average product-level Herfindahl of cost shares. In case of perfect supplier diversification (for example, under complete I-O network), there is no amplification of exogenous cost heterogeneity because variation in per unit total material costs vanishes for each particular product. At the same time, when products concentrate their intermediate spending on narrow subsets of specialized inputs, trade barriers cause the variance of relative marginal costs to be greater than the variance of relative exogenous costs. This result highlights the importance of the "spider" dimension of production networks.

Second, the combined welfare effects of CA and production sharing are approximated with an additional term in the formula by Arkolakis, Costinot, and Rodriguez-Clare (2012), henceforth ACR. This term is precisely the variance of relative marginal costs multiplied by a measure of trade integration (a function of trade costs and the

Armington elasticity only). Therefore, holding other model primitives fixed, a particular I-O structure generates a higher welfare if and only if it corresponds to stronger marginal cost heterogeneity. Thus, perfect supplier diversification gives the lowest gains from trade. This suggests that imposing a composite intermediate good assumption and ignoring the disaggregated nature of I-O linkages will tend to underestimate the gains from trade.

Third, the effects of CA and production sharing on trade flows are more nuanced. Marginal cost heterogeneity affects trade flows by two channels. One is the average product-level import share and the other is CA spillovers expressed as a covariance between upstream and downstream industries' performance. The second channel arises from an accounting decomposition and intuitively means that products sourcing from CA industries tend to produce more compared to autarky. Importantly, this second channel (CA spillovers) decreases trade flows for a given level of trade barriers or, in other words, amplifies the home bias.⁵ In a second-order approximation, the average import share is proportional to the variance of relative marginal costs, while the strength of CA spillovers is proportional to the derivative of this variance with respect to trade costs. Analyzing the behavior of these two channels, I find that, compared to the complete network, the role of imperfect supplier diversification in a general I-O structure with disaggregated interactions is to amplify the effect of small trade costs, but to generate relatively more trade when trade costs are high. Although variation in relative marginal costs increases in the level of trade frictions, this happens slowly near free trade, so that the trade-increasing effect of stronger heterogeneity is dominated by the trade-decreasing effect of CA spillovers. When trade costs are

⁵To get a clearer understanding of this effect, it is helpful to compare domestic expenditure by consumers (D^F for final) and firms (D^I for intermediate). Intuitively, D^F is dominated by consumers' purchases of goods in which a country has a comparative advantage, while D^I mostly counts intermediate spending on comparative-advantage goods *by* comparative-advantage industries. This "two-sided selection" increases D^I when comparative advantage in downstream and upstream industries is positively correlated. Interestingly, home bias for intermediate expenditure can be present even under free trade, due to self-sourcing (I-O matrix diagonal) or positive correlation between upstream and downstream exogenous productivities.

high, however, these spillovers decay fast, so that their negative contribution to trade flows becomes weaker relative to the positive contribution of stronger marginal cost heterogeneity.⁶

To summarize, the present paper establishes a theoretical result that imperfect supplier diversification magnifies the gains from trade, while also amplifying the home bias for moderate trade costs. The latter effect occurs due to CA spillovers. What can we say about the extent of supplier diversification and the presence of CA spillovers in the data? Appendix A.1 presents some evidence which is based on the B.E.A. detailed I-O table and trade flows for 2007. Figure A.1 gives the histogram of cost shares Herfindahl indices for 388 industries, $HHI_i = \sum_j \omega_{ij}^2$ where ω_{ij} is the share of product j in intermediate spending of industry i . The average value⁷ of 0.12 indicates that supplier diversification at this level of aggregation is quite far from perfect. For different measures of industries' CA, I calculate the upstream CA for industry i as a linear combination of its suppliers' CA with the weights equal to the corresponding input shares. I find that, first, consistent with nontrivial concentration of intermediate spending, there is substantial variation in upstream CA. Second, there is a positive and statistically significant relationship between upstream and downstream CA. This evidence is consistent with the model's implication that CA spillovers endogenously arise under costly trade.

Literature

Multistage production. My paper relates closely to the multistage production literature originated by Yi (2003, 2010). Subsequent developments include Johnson and Moxnes (2013), Connolly and Yi (2014), Arkolakis and Ramanarayanan (2009), and

⁶Although perturbation approximations in this paper are derived for a finite Armington elasticity, numerically I find qualitatively similar results when domestic and imported goods are perfect substitutes. In particular, for a two-stage production structure similar to Yi (2010), the home bias is magnified (compared to roundabout production Eaton and Kortum, 2002) only locally for moderate trade costs.

⁷This paper shows that the (weighted) average Herfindahl index of supplier diversification is the key summary statistic for the I-O matrix as it captures how fundamental CA is magnified by I-O linkages.

Kim (2013). These models rely on specific, highly stylized patterns of the production network by assuming two-stage production and/or composite intermediate good. While Baldwin and Venables (2013) draw attention to a more general "snakes and spiders" structure, little progress is made in terms of general understanding of supply chain trade because of limited tractability. The perturbation approach of this paper allows overcoming this challenge and obtaining analytical characterizations.

Multisector gravity. Another strand of related research is the multisector gravity literature, e.g. Caliendo and Parro (2015), Costinot and Rodriguez-Clare (2014), Ossa (2015). The focus of this literature is mainly quantitative; the role of international production sharing is only demonstrated by shutting down trade in inputs and comparing the resulting trade and welfare effects. My work is complementary to this line of research. On the one hand, I contribute to the theoretical understanding of the mechanisms operating in these papers, unifying them with the multistage production literature. On the other hand, I focus on highly disaggregated I-O interactions, while in the multisector gravity literature the number of sectors is relatively small. As my benchmark model assumes uncorrelated cost shocks to particular industries, it does not incorporate large industrial clusters and therefore does not, in general, nest the setup of, for example, Caliendo and Parro (2015).

Endogenous formation of CA. The central conceptual idea of my paper, so far underappreciated in the literature, is that variation in relative costs is endogenously determined together with international production sharing. A related argument made in the literature, see Koopman, Wang, and Wei (2014), is that refining the concept of revealed CA needs to control for I-O links. I rather emphasize that the *strength* of CA forces is co-determined with vertical specialization, which, to the best of my knowledge, is a new angle. At the same time, endogenous CA formation with feedback effects is examined in several other contexts, including external economies of scale in Grossman and Rossi-Hansberg (2010); intra-industry heterogeneity interacted with

Heckscher-Ohlin forces in Bernard, Redding, and Schott (2007); directed innovation in Somale (2014).

The rest of the paper is organized as follows. Section 1.2 presents the benchmark two-country model. While several basic results can be obtained analytically, further progress requires using some type of approximation. Section 1.3 establishes a neutrality result which suggests perturbation with respect to CA as a way forward. Section 1.4 then provides a perturbation solution to the model. Concluding remarks are given in Section 1.5.

1.2 Model

The world economy consists of two symmetric countries, Home and Foreign. Different products (or industries) are indexed $i = 1..N$. For most results I will focus on the limit case as $N \rightarrow \infty$. Formally, I consider a sequence of economies with increasing number of products, so all variables are indexed by N which I suppress for brevity as it causes no confusion. I start with describing the closed economy setup which is similar to Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Saleh (2012).

Throughout the paper, I denotes the identity matrix and $\mathbf{1}$ denotes the column vector of ones. For some vector x , D_x denotes the diagonal matrix generated by x . Both $D_x a = D_a x$ and $x \circ a$ are used for element-wise multiplication of vectors x and a .

1.2.1 Closed Economy

Environment

Consumers maximize Cobb-Douglas preferences parameterized with the vector of expenditure shares α :

$$U = \prod_{i=1}^N \left(\frac{c_i}{\alpha_i} \right)^{\alpha_i}, \quad (1.2)$$

with $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$.

Production function is also Cobb-Douglas: the amount q_i of good i is produced using labor l_i and intermediate inputs m_{ij} :

$$q_i = \exp(-z_i) \left(\frac{l_i}{1-\beta} \right)^{1-\beta} \prod_{j=1..N} \left(\frac{m_{ij}}{\beta \omega_{ij}} \right)^{\beta \omega_{ij}}, \quad (1.3)$$

where z_i is a cost shock, $0 \leq \beta < 1$ is the total share of intermediate inputs, and $0 \leq \omega_{ij} \leq 1$ are the shares of particular intermediates. Constant returns to scale are imposed, so $\sum_j \omega_{ij} = 1$ for each i . The matrix Ω of ω_{ij} is referred to as the I-O matrix.⁸

The triple (α, β, Ω) defines the I-O structure of the economy.⁹ Apart from a disaggregation condition, formulated in Assumption 1 below, this I-O structure can be arbitrary. Some goods may be intermediate inputs only ($\alpha_i = 0$), while other ones can only enter final consumption (good j is a pure final good if $\omega_{ij} = 0$ for all i). More generally, the model accommodates any pattern of "snakes and spiders" linkages, nesting, in particular, two-stage production in Yi (2010)¹⁰ and the complete symmetric I-O network with $\omega_{ij} = \frac{1}{N}$ for all i, j .

Consumers inelastically supply L units of labor and competition is perfect in all markets. After normalizing the nominal wage $W = 1$, cost minimization for (1.3) gives a linear relationship between log marginal costs mc_i and log prices p_j :

⁸An alternative convention would be to call $\beta\Omega$ the I-O matrix. Its element (i, j) would then be the share of product j in product i 's total costs rather than material costs. As long as this section restricts attention to a common input share β , it is straightforward to switch between these two interpretations.

⁹Recall that we consider a sequence of economies with increasing number of products, so the full notation has α and Ω indexed by N . That is, consumption shares and I-O matrices form a sequence $\{\alpha^{(N)}, \Omega^{(N)}\}_{N=1}^{\infty}$. The input share β is kept fixed.

¹⁰See Section 1.2.3 below.

$$mc_i = z_i + \beta \sum_j \omega_{ij} p_j, \quad \text{or} \quad mc = z + \beta \Omega p. \quad (1.4)$$

In competitive equilibrium, prices are equal to marginal costs, so

$$mc = p = (I - \beta \Omega)^{-1} z. \quad (1.5)$$

With additional normalization $L = 1$, so that $GDP = WL = 1$, the vector of consumption expenditure is $\alpha \cdot GDP = \alpha$. Let b denote the vector of total expenditure or sales (the two are equal due to constant returns to scale and perfect competition). The sales of industry i consist of final expenditure α_i and intermediate expenditure by other industries:

$$b_i = \alpha_i + \sum_j \beta \omega_{ji} b_j, \quad \text{or} \quad b = \beta \Omega' b + \alpha, \quad (1.6)$$

which implies that

$$b = (I - \beta \Omega')^{-1} \alpha. \quad (1.7)$$

Since total labor income should equal a fraction $(1 - \beta)$ of aggregate sales $Y = \sum_i b_i$, it holds that $WL = (1 - \beta) Y$, and therefore $\sum_i b_i = \frac{1}{1 - \beta}$. This (autarky) sales vector b is often called "the influence vector",¹¹ as it summarizes the importance of particular industries (or, more generally in network analysis, "nodes"). In autarky, b_i equals expenditure on product i relative to GDP, or $(1 - \beta) b_i$ equals the share of industry i in aggregate sales.

As the aggregate (log) price index for (1.2) is equal to $\bar{p} = \sum_i \alpha_i p_i$, it is expressed in terms of cost shocks z simply as¹²

¹¹Usually after normalizing its components to sum up to one. I refer to b as the influence vector without such a normalization.

¹²Combining (1.5) and (1.7), $\alpha' p = \alpha' (I - \beta \Omega)^{-1} z = b' z$. This relationship is derived in Acemoglu et al. (2012) for $\alpha = \frac{1}{N} \mathbf{1}$ and in Baqaee (2015) for a general α .

$$\bar{p} = \alpha' p = b' z. \quad (1.8)$$

Expression (1.8) shows that in the closed economy, for welfare determination, the influence vector b summarizes the entire I-O structure. This will be no longer true with trade, as economies with exactly the same autarky sales shares can behave very differently depending on other characteristics of Ω .

Disaggregation (LLN assumption)

To obtain sharp analytical results, this paper relies on the law of large numbers which can be applied to highly disaggregated economy. Holding the size of the economy fixed (aggregate sales $Y = \sum_i b_i = \frac{1}{1-\beta}$), consider increasing the level of disaggregation by letting the number of products N grow to infinity. To formalize this disaggregation, it is natural to require that the size of the largest industry in the limit shrinks to zero (yet it may decline slowly):

$$\lim_{N \rightarrow \infty} \max_i \{b_i\} = 0. \quad (1.9)$$

By looking at (1.8), this condition says that welfare is less and less affected by shocks to individual products as these products are being defined more and more narrowly: $\frac{\partial \bar{p}}{\partial z_i} = b_i \rightarrow 0$ for all i . Stated in the form (1.9), the disaggregation condition has a simple interpretation, but the actual law of large numbers results will use an equivalent formulation in terms of the Euclidian norm:¹³

$$\lim_{N \rightarrow \infty} \|b\|_2 = \lim_{N \rightarrow \infty} \left(\sum_i b_i^2 \right)^{1/2} = 0. \quad (1.10)$$

Condition (1.9) or (1.10) is a joint restriction on the I-O structure determined by (α, β, Ω) . For technical reasons, an additional restriction on Ω is imposed: if some

¹³In particular, (1.9) implies (1.10) since $\sum_i b_i^2 \leq \max_i \{b_i\} \sum_i b_i = \frac{1}{1-\beta} \max_i \{b_i\} \rightarrow 0$. The converse is also obvious: $\sum_i b_i^2 \geq (\max_i \{b_i\})^2$, so a failure of (1.9) produces a contradiction with (1.10).

vector of (alternative) consumption shares $\tilde{\alpha}$ is diversified in the sense that $\|\tilde{\alpha}\|_2 \rightarrow 0$, the corresponding influence vector is also diversified:

$$\lim_{N \rightarrow \infty} \|\tilde{\alpha}\|_2 = 0 \Rightarrow \lim_{N \rightarrow \infty} \|\tilde{b}\|_2 = 0, \text{ where } \tilde{b} = (I - \beta\Omega')^{-1} \tilde{\alpha}. \quad (1.11)$$

To state a formal assumption, reintroduce for a moment the full notation in which all vectors and matrices that change size are indexed with superscript (N) .

Assumption 1. *The sequence $\{\alpha^{(N)}, \Omega^{(N)}\}_{N=1}^{\infty}$ satisfies:*

- (i) *(1.10) holds for the corresponding influence vectors $\{b^{(N)}\}_{N=1}^{\infty}$.*
- (ii) *For any alternative consumption shares $\{\tilde{\alpha}^{(N)}\}_{N=1}^{\infty}$, the implication (1.11) holds.*

Summary statistics for the I-O structure

In the volatility literature, such as Acemoglu et al. (2012), the influence vector's norm $\|b\|_2$ is the key object that determines how fast the aggregate volatility declines with the level of disaggregation.¹⁴ The present paper is concerned with comparative advantage, which is cross-sectional rather than time-series variation. Under Cobb-Douglas preferences and technology, cross-sectional productivity dispersion is inconsequential (from (1.8), welfare does not depend on the distribution of z_i beyond its first moment), but in the open economy it affects trade flows and the gains from trade. To characterize the open-economy effects of productivity heterogeneity, I now introduce the following list of summary statistics for the I-O structure. Each index takes some product-level statistic and calculates the weighted average across all industries with the weights $(1 - \beta) b_i$. First, the average Herfindahl index of supplier diversification:

$$HHI = (1 - \beta) \sum_i b_i \sum_j \omega_{ij}^2. \quad (1.12)$$

¹⁴Gabaix (2011), not in the I-O context, provides a similar result that aggregate volatility depends on the sales Herfindahl.

This index is the average of product-level input shares Herfindahls $HHI_i = \sum_j \omega_{ij}^2$. It shows, on average, how dispersed intermediate spending of individual producers is. A zero limit value of HHI , which is the case, for example, for complete and symmetric I-O network with all $\omega_{ij} = \frac{1}{N}$, means perfect diversification of suppliers. The prices of input bundles $p_i^M = \sum_j \omega_{ij} p_j$ become deterministic for all industries, so the randomness in the marginal cost mc_i only comes from idiosyncratic cost shock z_i . Note that HHI in (1.12) contrasts to the sales Herfindahl defined as $\sum_i [(1 - \beta) b_i]^2$. While the latter is a macro-level statistic describing concentration of total sales, the former is a micro-level characteristic in the sense that it captures diversification of intermediate spending by individual industries.

Second, two diagonal, or "self-sourcing" indices:

$$SSI = (1 - \beta) \sum_i b_i \omega_{ii} \text{ and } SSI_2 = (1 - \beta) \sum_i b_i \omega_{ii}^2. \quad (1.13)$$

Finally, the "short loop index" which captures the extent of reciprocal sourcing (how often industries supply their immediate suppliers, including themselves):

$$SLI = (1 - \beta) \sum_i b_i \sum_j \omega_{ij} \omega_{ji}. \quad (1.14)$$

1.2.2 Open Economy: Setup and General Relationships

Consider two countries, Home and Foreign, which share the same fundamentals except for the cost shock vectors z and z^* which have a symmetric distribution. The Foreign variables have asterisk notation. Unless otherwise noted, all further results in this section are derived under the assumption that the pairs of cost shocks (z_i, z_i^*) are iid across products (yet z_i may be correlated with z_i^* for the same i) with a pdf $g(z_i, z_i^*) = g(z_i^*, z_i)$ restricted such that $\eta_i = z_i^* - z_i$ has four finite moments. Labor endowments

are $L = L^* = 1$. It will be shown that as $N \rightarrow \infty$ the law of large numbers implied by Assumption 1 allows normalizing the two (equilibrium) wages to one: $W = W^* = 1$.¹⁵

The model incorporates two motives for trade. One is Ricardian cross-industry heterogeneity as the two countries may differ in relative efficiency in producing different goods. Second, the model incorporates intra-industry trade by allowing for product differentiation in the Armington fashion: each product i enters consumption or production as a CES bundle x_i which aggregates the corresponding domestic and imported varieties x_{id} and x_{im} with the elasticity of substitution $\rho > 1$:

$$x_i = \begin{cases} \left(x_{id}^{1-1/\rho} + x_{im}^{1-1/\rho} \right)^{\frac{\rho}{1-\rho}}, & \rho < \infty \\ x_{id} + x_{im}, & \rho = \infty \end{cases} \quad (1.15)$$

While this paper is primarily concerned with the effects of CA across industries, intra-industry trade is introduced for both tractability and realism, as the presence of two-way trade even for narrowly defined products is well-documented. As known in the literature, the Armington formulation of intra-industry trade can be replaced with the one proposed by Eaton and Kortum (2002), henceforth EK. Moreover, it can be easily replaced with a Krugman (1980) monopolistic competition specification, but only without free entry (avoiding corner solutions).¹⁶ Although the Armington formulation involves the simplest notation, the choice of interpretation may depend on the level of aggregation.

Costs, prices, and domestic shares

International trade involves symmetric iceberg costs $\exp(\tau)$, so free trade corresponds to log trade cost $\tau = 0$. Domestic trade is assumed frictionless. The log price index for good i in Home is

¹⁵Specifically, equal wages imply that trade imbalances go to zero in the probability limit.

¹⁶In multi-industry monopolistic competition models, e.g. Romalis (2004), fixed costs generally lead to zero entry in most competitive-disadvantage industries, with the cutoff being endogenous to trade frictions.

$$p_i = \min\{mc_i, mc_i^* + \tau\} \quad \text{for } \rho = \infty \quad (1.16)$$

$$p_i = mc_i + \frac{1}{\rho-1} \log(\lambda_i) \quad \text{for } \rho < \infty,$$

where λ_i is the domestic share for good i . Denoting the difference in log marginal costs $\xi_i = mc_i^* - mc_i$,

$$\lambda_i = \frac{1}{1 + e^{(1-\rho)(\tau+\xi_i)}}. \text{ Similarly, } \lambda_i^* = \frac{1}{1 + e^{(1-\rho)(\tau-\xi_i)}}. \quad (1.17)$$

Table 1.1 summarizes the main notation for this chapter.

Table 1.1: Main Notation for Chapter 1

Variable	Definition
mc_i	log of marginal costs for (1.3)
p_i	log of price index (1.16)
ξ_i	$mc_i^* - mc_i$
η_i	$z_i^* - z_i$
λ_i	domestic share in (1.17)
y_i	sales of industry i

One key model relationship is for the marginal costs. (1.4) and (1.16) imply

$$mc_i = z_i + \beta \sum_j \omega_{ij} p_j, \text{ or } \xi_i = \eta_i + \beta \sum_j \omega_{ij} [p_j^* - p_j]. \quad (1.18)$$

In the matrix notation, also using that $p_j^* - p_j = \xi_j + \frac{1}{\rho-1} \log\left(\frac{\lambda_j^*}{\lambda_j}\right)$,

$$\xi = \eta + \beta \Omega \left[\xi + \frac{1}{\rho-1} \log\left(\frac{\lambda^*}{\lambda}\right) \right], \quad (1.19)$$

where $\log\left(\frac{\lambda^*}{\lambda}\right)$ is element-wise. Expression (1.19) captures how η , the difference in exogenous cost shocks, both directly and indirectly determines ξ , the difference in endogenous marginal costs. Thus, a change in η_j affects ξ_i directly for $i = j$ and also indirectly by changing relative input prices. Differentiating (1.18) with respect to η_j ,¹⁷

$$\frac{d\xi_i}{d\eta_j} = 1_{\{i=j\}} + \beta \sum_{i'} \omega_{ii'} (\lambda_{i'} + \lambda_{i'}^* - 1) \frac{d\xi_{i'}}{d\eta_j}, \quad (1.20)$$

where $0 \leq \lambda_{i'} + \lambda_{i'}^* - 1 \leq 1$ is an inverse measure of trade intensity in industry i' . Under free trade this measure is zero, so indirect effects coming with $\frac{d\xi_{i'}}{d\eta_j}$ do not contribute to $\frac{d\xi_i}{d\eta_j}$. This is also seen immediately from $p_j^* = p_j$ at $\tau = 0$: as production sharing is not constrained under free trade, input prices are the same in the two countries and therefore marginal cost differences only arise from exogenous cost shocks. With costly trade, the sum of domestic shares exceeds one, $\lambda_{i'} + \lambda_{i'}^* - 1 > 0$, and fundamental cost differences η_j propagate downstream (in other words, ξ_i is determined by upstream η_j). In general, keeping track of such CA spillovers which depend on trade costs is a challenging task; Section 1.4 uses a perturbation approximation to obtain a parsimonious characterization of relative marginal costs ξ and relate it to trade flows and welfare.

Sales and trade flows

Recall that autarky sales are given by the influence vector b in (1.7). The open economy sales vectors y and y^* generally differ from b because of specialization. To determine y and y^* , consider the global I-O structure that links different industries in the two countries. For a clearer exposition, reintroduce GDP, C and C^* , which is total consumer expenditure. The global I-O balance takes the form

¹⁷Use that $d\lambda_j = (\rho - 1) \lambda_j (1 - \lambda_j) d\xi_j$ and $d\lambda_j^* = -(\rho - 1) \lambda_j^* (1 - \lambda_j^*) d\xi_j$.

$$\underbrace{\begin{pmatrix} y \\ y^* \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} D_\lambda \alpha \cdot C + D_{1-\lambda^*} \alpha \cdot C^* \\ D_{\lambda^*} \alpha \cdot C^* + D_{1-\lambda} \alpha \cdot C \end{pmatrix}}_{\mathbf{f}} + \underbrace{\begin{bmatrix} D_\lambda \beta \Omega' & D_{1-\lambda^*} \beta \Omega' \\ D_{1-\lambda} \beta \Omega' & D_{\lambda^*} \beta \Omega' \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} y \\ y^* \end{pmatrix}}_{\mathbf{y}}, \quad (1.21)$$

in which $\mathbf{y}$ is the vector of world sales, $\mathbf{f}$ is the vector of world final consumption, and $\mathbf{A}$ is the global I-O matrix. In particular, final consumption of Home-produced goods comes from Home demand $D_\lambda \alpha \cdot C$ and Foreign demand $D_{1-\lambda^*} \alpha \cdot C^*$. To produce y , Home industries need to spend the vector $\beta \Omega' y$ on intermediate inputs, of which $D_\lambda \beta \Omega' y$ goes into demand for Home-produced goods and $D_{1-\lambda} \beta \Omega' y$ into demand for Foreign-produced goods.

Applying $C = WL = C^* = W^* L^* = 1$, which is symmetry plus normalization, Appendix A.3 derives that

$$y + y^* = 2b, \quad (1.22)$$

$$y = b + (I - \beta D_{\lambda+\lambda^*-1} \Omega')^{-1} D_b (\lambda - \lambda^*), \text{ and} \quad (1.23)$$

$$y^* = b + (I - \beta D_{\lambda+\lambda^*-1} \Omega')^{-1} D_b (\lambda^* - \lambda). \quad (1.24)$$

Specific to Cobb-Douglas, the sum of world sales is proportional to the influence vector b regardless of the level of trade costs. Expressions (1.23)-(1.24) show that departures of open-economy sales in each country from their autarky levels b are linked to $(\lambda - \lambda^*)$, which is a measure of cost advantage. (From (1.17), higher domestic shares in Home relative to Foreign correspond to higher marginal cost differences in favor of Home). The diagonal matrix $D_{\lambda+\lambda^*-1}$ captures the (inverse) degree of trade integration: it is zero under free trade ($\tau = 0$ implies $\lambda_i + \lambda_i^* = 1$ for each i) and converges to I as $\tau \rightarrow \infty$. Interestingly, it enters the Leontief-inverse-type

matrix $(I - \beta D_{\lambda+\lambda^*-\mathbf{1}}\Omega')^{-1}$ as if controlling the input share. Section 1.4 shows that this phenomenon is related to the fact that international production sharing affects variation in relative marginal costs.

Aggregate domestic and import expenditure by Home consumers (final consumption) and producers (intermediate consumption) is

$$D^F = \lambda'\alpha; \quad M^F = 1 - D^F; \quad D^I = \lambda'\beta\Omega'y; \quad M^I = \frac{\beta}{1-\beta} - D^I, \quad (1.25)$$

where total final expenditure is $D^F + M^F = WL = 1$ and total intermediate expenditure is $D^I + M^I = \frac{\beta}{1-\beta}$.

Total domestic and import expenditure in Home is

$$D = D^F + D^I = \lambda'\alpha + \lambda'\beta\Omega'y \text{ and } M = M^F + M^I = \frac{1}{1-\beta} - D. \quad (1.26)$$

As an accounting decomposition,¹⁸ domestic expenditure can be represented as

$$D = \sum_i b_i \lambda_i + \beta \sum_i (y_i - b_i) \lambda_i^{Upstream}, \quad (1.27)$$

where $\lambda_i^{Upstream} = \sum_j \omega_{ij} \lambda_j$ is the average domestic share among suppliers of industry i and the term $\sum_i (y_i - b_i) \lambda_i^{Upstream}$ captures the covariance between performance of individual industries and cost advantage of their suppliers. In Section 1.4, this covariance term is shown to be positive for $0 < \tau < \infty$, meaning that products which source from cost-advantage industries (with high domestic shares) tend to sell more relative to autarky. As seen from (1.27), such CA spillovers increase the home bias

¹⁸Write $D^I = \lambda'\beta\Omega'y = \sum_i y_i \left(\beta \sum_j \omega_{ij} \lambda_j \right) = \beta \sum_i (y_i - b_i + b_i) \lambda_i^{Upstream}$. It further equals to $\beta \sum_i (y_i - b_i) \lambda_i^{Upstream} + \beta b' \Omega \lambda = \beta \sum_i (y_i - b_i) \lambda_i^{Upstream} + (b - \alpha)' \lambda$. Combining with $D^F = \lambda'\alpha$, $D = \lambda'b + \beta \sum_i (y_i - b_i) \lambda_i^{Upstream}$.

above the level based on the average domestic share. This reflects the conceptual difference between final and intermediate expenditure. Intuitively, D^F is dominated by consumers' purchases of goods in which Home has comparative advantage, while D^I counts intermediate spending on comparative-advantage goods *by* comparative-advantage industries. This two-sided selection is minimal under free trade because input prices are the same in Home and Foreign. However, as shown in Section 1.2.3, it does not vanish completely in the presence of self-sourcing.

Welfare

Welfare (the real wage) is the inverse of the aggregate price index for consumers which is, in logs,

$$\bar{p} = \alpha' p = b' \left[z + \frac{1}{\rho-1} \log(\lambda) \right]. \quad (1.28)$$

This expression¹⁹ implies that welfare in a trade equilibrium relative to the autarky level, or the gains from trade, is given by a multi-industry ACR formula

$$GfT = \exp \left[\sum_i b_i \frac{1}{1-\rho} \log(\lambda_i) \right] = \prod_i \lambda_i^{\frac{b_i}{1-\rho}}. \quad (1.29)$$

Expression (1.29) holds more generally at the country level in the Cobb-Douglas perfect competition environments.²⁰ This formula is equivalent to expression (28) in Costinot and Rodriguez-Clare (2014), to expression (15) in Caliendo and Parro (2015), and to a related expression in Ossa (2015).²¹

While (1.29), developed in the multisector gravity literature, provides a measuring tool for evaluating the gains from trade, two issues require further exploration. First,

¹⁹Obtained from (1.4) and (1.16): $p = mc + \log \left(\lambda^{\frac{1}{\rho-1}} \right) = z + \beta \Omega p + \log \left(\lambda^{\frac{1}{\rho-1}} \right)$, which means that $p = (I - \beta \Omega)^{-1} \left[z + \log \left(\lambda^{\frac{1}{\rho-1}} \right) \right]$. Multiplying both sides by α' and using the definition of b in (1.7) yields (1.28).

²⁰In particular, with heterogeneity in the elasticities of substitution, trade costs and labor shares across products and for multiple asymmetric countries that may have different I-O matrix coefficients.

²¹A clear connection to the influence vector is less common in the literature; one paper that also emphasizes this link is Albrecht and Tombe (2015).

in terms of theory, (1.29) hides the actual mechanisms through which I-O interactions affect the gains from trade. This is because domestic shares are determined by heterogeneity in production costs which is endogenous to trade costs and the I-O structure. Using a perturbation technique, Section 1.4 demonstrates that the gains from trade are parsimoniously related to heterogeneity in exogenous cost shocks and a few summary statistics for the I-O matrix. Second, applying (1.29) at disaggregated level is problematic with currently available data because production and use data is limited or missing. (Trade data is available at more detailed levels, but trade data alone does not allow calculating domestic expenditure shares.) In Section 1.5 I discuss how to explore variation in disaggregated trade flows without having disaggregated production data in order to give an approximate estimate for the gains from trade.

An interesting feature of this setup is that when trade is either absent or completely frictionless, welfare does not depend on the I-O matrix Ω as long as Assumption 1 (the law of large numbers) is imposed. This is directly seen in (1.28) for the case of autarky when $\lambda_i = 1$ for all i . By Lemma 1A, $\bar{p} = b'z \xrightarrow{p} \frac{1}{1-\beta} E(z_i)$. Under free trade ($\tau = 0$), Home and Foreign industries pay the same prices for intermediate inputs, so (1.18) implies $\xi = \eta$ and λ_i 's, which now only depend on iid η_i 's, are iid as well. Therefore, by the same lemma, $\sum_i b_i \log(\lambda_i)$ also converges to an expectation that is proportional to $\frac{1}{1-\beta}$ but does not depend on Ω .

Appendix A.4 proves the following result that relates local changes in welfare to aggregate trade flows.

Proposition 1.2.1. *In the probability limit, a change in the log price index is equal to total imports (relative to GDP=1)*

$$\frac{d}{d\tau} \bar{p} = M. \quad (1.30)$$

In fact, Appendix A.4 proves a more general result that, keeping the wages fixed, cost minimization and utility maximization alone imply $\frac{d}{d\tau} \left(\frac{1}{2}\bar{p} + \frac{1}{2}\bar{p}^* \right) = \frac{M+M^*}{World\ GDP}$. The role of the law of large numbers is only to ensure that in equilibrium $W = W^* = 1$ for any trade costs, as well as $\bar{p} = \bar{p}^*$ and $M = M^*$.

Proposition 1.2.1 is a symmetric-countries version of the main result in Fan, Lai, and Qi (2014), yet it allows a more general I-O structure.²² Equation (1.30) is used below in assessing how the I-O links can amplify the effects of trade costs, but it can also be of some independent interest. In particular, (1.30) has all trade flows counted equally, independently of the value added content.

We can define the aggregate domestic share $\bar{\lambda} = \frac{D}{Y}$ and the "trade elasticity"

$$\epsilon(\tau) = -\frac{d}{d\tau} \log \left(\frac{1-\bar{\lambda}}{\bar{\lambda}} \right). \quad (1.31)$$

Appendix A.5 shows that (1.30) implies that $\frac{d\bar{p}}{d \log(\bar{\lambda})} = \frac{1}{(1-\beta)\epsilon(\tau)}$, so the ACR interpretation of the trade elasticity holds locally.²³

The fact that welfare in autarky and under free trade does not depend on Ω , combined with Proposition 1.2.1, implies that it is not possible that some I-O matrix $\Omega^{(1)}$ generates uniformly (over τ) more or less trade than another I-O matrix $\Omega^{(2)}$. This result contrasts with the notion that multistage production as in Yi (2010) magnifies the effect of trade costs. The impossibility of such uniform magnification is formally stated as

$$\bar{p}^{Autarky} - \bar{p}^{Free\ Trade} = \int_0^\infty M(\Omega^{(1)}) d\tau = \int_0^\infty M(\Omega^{(2)}) d\tau \text{ for any } \Omega^{(1)}, \Omega^{(2)}. \quad (1.32)$$

²²Fan, Lai, and Qi (2014) only consider multistage production without "spider" linkages. Results of this type also appear in Burstein and Cravino (2012), Atkeson and Burstein (2010), and Allen, Arkolakis, and Takahashi (2014).

²³Brooks and Pujolas (2014) work on generalizing the ACR formula for the cases when log-linear gravity does not hold because of non-homotheticities.

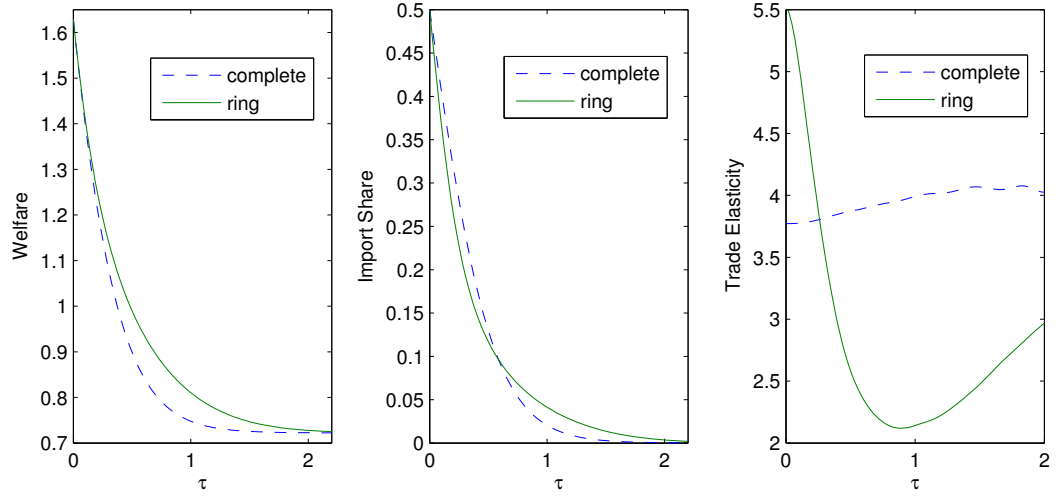


Figure 1.1: Complete symmetric network vs. Ring

This is illustrated in Figure 1.1 which compares the complete and symmetric I-O network ($\omega_{ij} = \frac{1}{N}$) with the ring ($\omega_{N1} = 1$, $\omega_{ij} = 1$ for $i = 1..N-1$, $j = i+1$ and $\omega_{ij} = 0$ otherwise). In both cases, $\alpha = \frac{1}{N}\mathbf{1}$, productivities e^{-z_i} and $e^{-z_i^*}$ and independently Frechet distributed with $\theta = 4$, and the Armington elasticity is $\rho = 15$. The ring generates uniformly higher welfare, which is achieved with lower import shares for small trade costs but with higher import shares when trade costs are large. The area between the two import share lines integrates to zero.²⁴ While the next subsection provides an exact characterization of trade flows near $\tau = 0$, understanding these differentiated welfare and trade responses to changes in trade costs will require a perturbation approximation which would be able to relate the two components of (1.27) to the pattern of I-O linkages. Note also that the elasticity of trade (1.31) is non-monotone for the ring (for the complete network, $\epsilon(\tau)$ would be equal $\theta = 4$ if $\rho = \infty$, which is the EK setup).

²⁴As $\int_0^\infty M(\Omega^{(1)}) d\tau = \int_0^\infty M(\Omega^{(2)}) d\tau$, which is written in terms of import-to-GDP ratios, the same holds for trade shares which are $\frac{M}{Y} = (1 - \beta) M$.

1.2.3 Local Analysis Near Free Trade

Even with Cobb-Douglas preferences and technology and maximum symmetry, the model of general I-O interactions remains extremely complex, which motivates the use of perturbation approach in Section 1.4. However, the model's solution simplifies dramatically when trade is frictionless, so that an exact, rather than approximate, characterization is available. In this subsection I highlight two key results of the local analysis near $\tau = 0$, while all technical discussion is delegated to Appendix A.6.

Free-trade home bias

Given the symmetry, it is not surprising that under free trade consumers spend one half of their income (GDP, which is normalized to one) on domestically produced goods:

$$D^F = \lambda' \alpha \xrightarrow[p]{\quad} \frac{1}{2}. \quad (1.33)$$

For producers, however, the situation is different. Of the total intermediate expenditure $\frac{\beta}{1-\beta}$, firms spend on domestic products

$$D^I = \lambda' \beta \Omega' y \xrightarrow[p]{\quad} \frac{1}{2} \frac{\beta}{1-\beta} [1 + SSI \cdot var(\varepsilon_i)], \quad (1.34)$$

where ε_i , defined in (A.11), is a monotone transformation of η_i . Domestic intermediate expenditure D^I is strictly greater than import of intermediates $M^I = \frac{\beta}{1-\beta} - D^I$ as long as $var(\varepsilon_i) > 0$ (there is comparative advantage) and $SSI > 0$ (industries' spending on their own output²⁵ is nonnegligible). This free-trade home bias arises endogenously from the I-O interactions, as opposed to the one exogenously introduced into preferences in some studies. (See the discussion after (1.27) for intuition.)

²⁵In the open economy, such "self-sourcing" means that industries use CES bundles (1.15) aggregating output of these same industries from the two countries. The effect is present for any $\rho > 1$ and it is the strongest when $\rho = \infty$ as $var(\varepsilon_i)$ attains its maximum value 1.

More generally, $D^I > M^I$ despite $\tau = 0$ occurs under positive correlation between upstream and downstream cost shock differences η_i , of which self-sourcing is a primitive example.²⁶ As I show below, the effects of such correlation can be quite strong with limited supplier diversification (large HHI), so that small trade costs are recovered from high home shares. The issue, however, is that correlation in exogenous cost shocks is hard to discipline within my framework. Nevertheless, one implication of this paper is that firm-level studies of supply chain trade should pay special attention to the upstream-downstream correlation in technology.

Trade elasticity

In Appendix A.6 I consider separately the case $\rho < \infty$ and the case of perfect substitutes $\rho = \infty$. Although this paper focuses on the former, local analysis near free trade provides an opportunity to consider high values of ρ as well as very strong cross-product productivity heterogeneity.

One result that allows making a clear connection to existing literature, is for perfect substitutes when productivity e^{-z_i} is Frechet distributed with the dispersion parameter θ . In the absence of self-sourcing ($SSI = 0$), the relative import spending near free trade satisfies

$$\log \left(\frac{M}{D} \right) \xrightarrow{p} -\theta [1 + \beta^2 (HHI + 2SLI)] \tau + o(\tau). \quad (1.35)$$

The elasticity of trade (the negative of the slope is the above expression) in this case is $\epsilon = \theta [1 + \beta^2 (HHI + 2SLI)]$. Indeed, as previously argued in the literature, the elasticity of trade $\epsilon \geq \theta$ can be amplified by the presence of I-O linkages. Near free trade, this amplification is only determined by supplier diversification and reciprocal sourcing (short loops).

This case of perfect substitutes $\rho = \infty$ is mainly of theoretical interest, since two-way trade is present in the data even at highly disaggregated level. Yet considering

²⁶In principle, negative correlation may induce $D^I < M^I$.

(1.35) for different I-O matrices Ω helps to develop a unified understanding of the existing models. With Cobb-Douglas technology and preferences, the EK model with continuum of goods can be discretized as having $\alpha = \frac{1}{N}\mathbf{1}_N$ and $\omega_{ij} = \frac{1}{N}$. This complete symmetric I-O network has $b = \frac{1}{1-\beta}\frac{1}{N}\mathbf{1}_N$, so $HHI = \frac{1}{N} \rightarrow 0$ and $SLI = \frac{1}{N^2} \rightarrow 0$. The I-O structure in Yi (2010) has two stages of production. Each stage 1 good $i = 1.. \frac{N}{2}$ is used by one and only one stage 2 good $j = \frac{N}{2} + i$. Consumption shares are $\alpha = [\mathbf{0}_{N/2} \ \frac{1}{N/2}\mathbf{1}_{N/2}]'$, as only stage 2 goods are consumed. Also, a composite bundle of stage 2 goods is used by each stage 1 product. The I-O matrix is therefore

$$\Omega^{Yi} = \begin{pmatrix} \mathbf{0}_{\frac{N}{2}, \frac{N}{2}} & \frac{1}{N/2}\mathbf{1}_{\frac{N}{2}, \frac{N}{2}} \\ I_{\frac{N}{2}, \frac{N}{2}} & \mathbf{0}_{\frac{N}{2}, \frac{N}{2}} \end{pmatrix},$$

for which $b = (I - \beta\Omega')^{-1}\alpha = \frac{2}{N}\frac{1}{1-\beta^2}[\beta\mathbf{1}_{N/2} \ \mathbf{1}_{N/2}]'$. While stage 2 goods have perfect supplier diversification, stage 1 goods use only one product each. The average supplier diversification is $HHI = (1 - \beta) \sum_i b_i \sum_j \omega_{ij}^2 = \frac{1}{1+\beta} \left(1 + \frac{\beta}{N/2}\right) \rightarrow \frac{1}{1+\beta} > 0$, which increases the trade elasticity in (1.35) relative to EK. Expression (1.35) also shows that several tractable modifications of Kei-Mu Yi's two-stage model, such as Arkolakis and Ramanarayanan (2009) and Kim (2013), change the original setup dramatically by imposing perfect diversification.²⁷ Yet the role of (1.35) in theoretical understanding of supply chain trade is quite limited. First, the economics behind this result is still not clear. Moreover, it would be wrong to conclude that such amplification holds globally, that is for any trade costs.

1.3 Neutrality Result

This section establishes a neutrality result that, in the absence of CA, the detailed structure of I-O linkages does not matter for trade shares and the gains from trade. I

²⁷In Arkolakis and Ramanarayanan (2009), each stage-two product uses a composite bundle of stage-one goods.

use this finding to argue that the commonly cited "amplification logic" (the impact of trade costs is magnified by multiple border crossing) is incomplete. On the constructive side, the neutrality result suggests the no-CA case as a basis for perturbation, which I explore in Section 1.4.

1.3.1 I-O Trade and Gravity

By looking at equation (1.19), one can guess that $\eta = 0$ implies $\xi = 0$ regardless of Ω . Without variation in relative fundamental productivities, there is no variation in relative marginal costs. In other words, although I-O interactions transform CA, they do not create it out of nothing. This guess is verified by observing that $\xi = 0$ implies $\lambda = \lambda^*$ in (1.17) and therefore $p = p^*$.²⁸ The model then effectively collapses to a one-product Armington economy in which trade flows satisfy a log-linear gravity equation and the welfare gains are given by the ACR formula.

To appreciate this gravity implication, it is worth formulating this proposition for multiple asymmetric countries. I ask what is the maximum level of generality – in particular in terms of the I-O structure – that is consistent with aggregate-level gravity. The detailed description of the environment is given in Appendix A.7. The setup of Section 1.2 is maintained with the following generalizations:

- (i) The world economy consists of countries $k = 1..K$ and products (industries) $i = 1..N$. Bilateral iceberg trade costs are $t_{kk'}$ (uniform across products) and country-industry cost shifters (inverse TFP) are Z_{ik} .
- (ii) Consumer preferences are CES with potentially country-specific utility weights.
- (iii) Industry-specific material bundles are CES.
- (iv) Exogenous trade imbalances are allowed.

²⁸Such a proof by construction also requires showing uniqueness of ξ , which I do in a more general setting of Proposition 1.3.1.

Crucially, the common Armington elasticity ρ is maintained and as well as the common intermediate share β . This setup, in general, features both intra-industry and inter-industry trade. Under the chosen interpretation, the former is driven by Armington product differentiation as in Anderson and van Wincoop (2003), but it can equivalently be related to technology heterogeneity within individual industries as in EK.²⁹ Inter-industry trade arises from asymmetries between countries and generally precludes one from specifying the gravity equation at the aggregate level. The following proposition shows that the root of this gravity failure lies in the presence of CA forces, meaning variation in relative exogenous productivities. In the setting of Appendix A.7, the gravity structure is not affected by I-O linkages as long as relative cost shifters

$$\frac{Z_{ik}/Z_{i'k}}{Z_{ik'}/Z_{i'k'}} \quad (1.36)$$

do not vary across country-industry combinations $(k, k'; i, i')$, which is equivalent to decomposition of Z_{ik} into industry and country fixed effects. Formally, Appendix A.7 proves

Proposition 1.3.1. *If the cost shifters in (A.23) satisfy $Z_{ik} = \tilde{Z}_i \hat{Z}_k$, then there exist a set of importer fixed effects $\{FM_k\}_{k=1}^K$ and a set of exporter fixed effects $\{FX_{k'}\}_{k'=1}^K$ such that equilibrium trade flows $X_{kk'}$ satisfy the log-linear gravity system with trade elasticity $\rho - 1$:*

$$\log X_{kk'} = FM_k + FX_{k'} + (1 - \rho) \log t_{kk'}. \quad (1.37)$$

²⁹It is well known that EK yields the same set of equilibrium equations. In this case, the Frechet dispersion parameter θ plays the role of $\rho - 1$ and heterogeneity in Z_{ik} relates to heterogeneity in location parameters T_{ik} . Caliendo and Parro (2015) and most of the other multisector gravity literature employ the EK setup for modeling intra-sectoral trade, while Ossa (2015) also uses the Armington formulation.

If in addition trade is balanced, the gains from trade for country k are given by the ACR formula

$$\frac{1}{\bar{\lambda}_k^{(1-\rho)(1-\beta)}}, \quad (1.38)$$

where $\bar{\lambda}_k$ is the share of country k 's expenditure spent on domestic goods.

A special case of this result is known in the literature due to French (2015) who considers a multi-industry EK model in which all industries use the same composite intermediate good (same as in final consumption). Imposing such uniformity is a common way to avoid specifying the detailed I-O structure when introducing intermediate goods. Under this assumption, material costs cannot by construction affect variation in relative marginal costs, which rules out any interaction between CA forces and I-O linkages.³⁰

Proposition 1.3.1 states that in the absence of CA the elasticity of trade is completely unaffected by I-O interactions. For any input share β and I-O shares $\omega_{ii'}$,³¹ the value $\rho - 1$ determines the proportional change in trade flows resulting from a given change in trade costs conditional of the fixed effects. The latter qualification, standard in the gravity literature, is equivalently replaced by considering a Head and Ries (2001) type ratio:

$$\rho - 1 = -\frac{\partial \log X_{kk'}}{\partial \log t_{kk'}} = -\frac{d \log \frac{X_{kk'} X_{k'k}}{X_{kk} X_{k'k'}}}{d \log \frac{t_{kk'} t_{k'k}}{t_{kk} t_{k'k'}}}, \quad (1.39)$$

³⁰Although not focusing on I-O interactions, the result in French (2015) is not to be downplayed, however; even without intermediate goods, the gravity structure relies critically on a flat CA pattern. That is, CA forces can only operate within – not between – industries once intra-industry trade is modeled as in EK. In other words, French (2015) emphasizes that gravity is correctly specified at a certain level of aggregation. This is, of course, implicitly understood in multisector models, yet they face a further heterogeneity challenge because they only include a relatively small number of sectors.

³¹To emphasize the generality of production architecture consistent with Proposition 1.3.1, note that an arbitrary pattern of "snakes and spiders" linkages is allowed, nesting, for example, multistage production as in Yi (2010). At the same time, variation in labor shares across industries is assumed away, as it translates into variation in relative costs across countries unless all wages are equal. (In case of symmetric countries, the gravity holds with industry-specific labor shares.)

in which partial differentiation " ∂ " means keeping constant FM_k and $FX_{k'}$ in (1.37), while total differentiation " d " does not restrict the sources of variation in trade flows.³²

The intuition behind this neutrality result is simple. It is true that multiple border crossing makes the prices of traded goods change more than one-for-one with trade costs, and more so for a higher input share β . But the *relative* prices (import vs. domestic) need not be affected by I-O links because all products in all locations use some imported intermediates. Since trade flows depend on relative rather than absolute prices, the possibility of no amplification is not surprising. More specifically, the proof shows that no relative variation in exogenous productivities implies no relative variation in marginal costs. Bilateral trade shares are then uniform across products, hence there are no composition effects and the product-level trade elasticity is unchanged under aggregation.

In terms of welfare gains, the entire effect of the I-O structure is summarized by the share of intermediates β . When firms and not only consumers have access to the world market, the gains from trade are magnified due to compounding effect, which is already present in EK. As long as the share of material inputs in the data is quite large, accounting for tradable intermediates is of first-order importance for estimating the gains from trade.³³

Going back to the benchmark model with two symmetric countries, we have $k \in \{H, F\}$, $\hat{Z}_H = \hat{Z}_F$, and the absence of fundamental CA means $\log \frac{Z_{iF}}{Z_{iH}} = \eta_i = 0$ for all products. It implies that domestic shares in Home and Foreign are equal and uniform across products: $\lambda_i = \lambda_i^* = \lambda_0 = \frac{1}{1+e^{(1-\rho)\tau}}$. Imports are then $M = \frac{1}{1-\beta}(1 - \lambda_0)$, so

³²Note that one needs to be careful when defining the concept of (partial equilibrium) trade elasticity outside the context of log-linear gravity. There are examples in the literature when this concept is well-defined but the second equality in (1.39) does not hold, e.g. translog gravity in Novy (2013). In general, however, using the " ∂ " symbol can be problematic, in particular under I-O interactions. In case of two symmetric countries the elasticity of trade which is *defined* by the last expression in (1.39) happens to (locally) preserve the ACR interpretation.

³³Melitz and Redding (2014) and Samuelson (2001) point out that the gains from trade in inputs may potentially be infinite. In Table 4.1 in Costinot and Rodriguez-Clair (2014) the estimated average gains from trade range from 4.4-15.3% in models without intermediates (including multiple sectors) to 26.9-40% in models with intermediates.

both partial equilibrium and general equilibrium trade elasticities do not depend on the I-O matrix.

1.3.2 Connection to the Literature

From the theory perspective, Proposition 1.3.1 can be viewed as a counterexample for the "conventional wisdom" on the role of international production sharing in magnifying the effects of trade costs. Since Yi (2003), the theoretical understanding of supply chain trade has been centered on "multiple border crossing", a concept which may at first look self-explaining. As tariffs and transportation cost are applied multiple times, production fragmentation should make trade flows more responsive to changes in trade barriers. Despite the awareness that input trade itself is not sufficient to generate such amplification,³⁴ the literature continues to refer to this semi-informal logic, e.g. Johnson and Moxnes (2013), Connolly and Yi (2014). Furthermore, Koopman, Wang, and Wei (2014) and Miroudot, Rouzet, and Spinelli (2013), among others, use international I-O tables to calculate the cumulative burden of tariffs. This is supposed to be informative of the role of supply chains in magnifying the effects of trade costs, yet the general equilibrium interpretation of such numbers is not clear.

If multiple border crossing itself does not necessarily amplify the impact of trade costs, then what is the nature of this amplification? Yi (2010) notes that, unlike the roundabout scheme in EK, the *structure of production* in his model is not invariant to changes in trade barriers. What exactly stands behind this endogenous production structure, however, is not perfectly clear.³⁵ In Section 1.4 I use a perturbation approximation to demonstrate that at the heart of the multistage model lies endogenous relative cost heterogeneity which is present because of imperfect supplier diversifica-

³⁴For example, Yi (2010) mentions that the model in Eaton and Kortum (2002) has no magnification effect despite the presence of I-O linkages. See footnote 14 in his paper.

³⁵For a formal analytical result, Yi (2010) assumes that components are always produced in the region where finished products are consumed, an assumption that is not maintained in the calibration part of the paper. The original model does not admit a closed-form solution, which limits the intuitive understanding of its general equilibrium forces.

tion (each stage-two product uses only one stage-one input). I find that, compared to the EK benchmark, the home bias is only magnified for moderate trade costs.³⁶

1.3.3 Perturbation

The interesting content of Proposition 1.3.1 lies in the fact that a very general form of I-O linkages is allowed. Beyond this, however, all sorts of cross-industry heterogeneity are excluded: no CA, common ρ , β and $t_{kk'}$ across products. It is easy to show that any of that breaks the gravity structure and the ACR formula. And yet the gravity model is too valuable because of its tractability (and significant empirical success) to dismiss it so quickly. Instead, Proposition 1.3.1 can be viewed as to suggest that the effects of product-level heterogeneity may be characterized as deviations from the gravity benchmark. The main methodological contribution of this paper is to formalize and explore this idea with a perturbation approach.

While common in macroeconomics,³⁷ especially in modeling dynamics, perturbation techniques which would go beyond the standard low-dimensional comparative statics exercises are not widespread in international trade. The insight is that such techniques can be very helpful in dealing with cross-sectional heterogeneity which is otherwise difficult to approach because of rich interactions between different industries. The general idea that network interactions can be studied by perturbing a cross-section of homogenous economic units is not new. Methodologically related to my study is the work by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015).³⁸ The

³⁶The perturbation results of Section 1.4 are obtained for a finite Armington elasticity, while the setup in Yi (2010) corresponds to the case of perfect substitutes $\rho = \infty$. This difference, however, is quantitative rather than qualitative.

³⁷Perturbation techniques originated in natural science and were introduced to macroeconomics in Judd and Guu (1997). See Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) for a recent exploration.

³⁸The authors provide first- and second-order Taylor series approximation to a generic aggregate variable determined by network interaction between individual nodes which are subject to idiosyncratic shocks. Technically, my analysis in Section 1.3 is very similar, except that it involves differentiating with a single perturbation parameter rather than calculating all mixed derivatives with respect to idiosyncratic shocks. Using a single perturbation parameter in my case is more con-

novelty of the present paper is to apply perturbation techniques to cross-industry heterogeneity in international trade and to explore the model features specific to supply chain trade.

Once the decomposition of cost shifters Z_{ik} into industry and country fixed effects $\tilde{Z}_i$ and $\hat{Z}_k$ does not hold, there is fundamental CA which interacts with I-O linkages and trade costs to shape endogenous heterogeneity in relative marginal costs which determines trade flows and welfare. Reflecting the great complexity of the real world production architecture, approaching this interplay between CA and I-O is challenging even within a stylized competitive framework. We can represent Z_{ik} in the form

$$\log Z_{ik} = \log \tilde{Z}_i + \log \hat{Z}_k + \delta z_{ik}, \quad (1.40)$$

where z_{ik} is residual variation and δ is the perturbation parameter. The strategy is then to approximate the outcome variables of interest, such as trade flows, welfare or optimal taxes, with Taylor series in δ around $\delta = 0$, which in many circumstances is a simple case. For aggregate outcomes, particularly sharp results can be obtained when the number of industries is large. In this case, assuming that residual cost shocks z_{ik} come from a probability distribution, the law of large numbers allows characterizing the effects of CA parsimoniously with a few moments of z_{ik} and a few summary statistics for the I-O matrix.

venient, especially for higher orders of approximation and under multiple sources of heterogeneity. More substantially, it accommodates easily more general settings which may not fit into Equation (1) in their paper because of nonadditive shocks (which is in case of CES technology) or additional nonlinearity specific to international trade. Finally, while most results in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) are stated in terms of Leontief inverse coefficients, I provide more parsimonious and intuitive summary statistics for the I-O matrix, such as the average level of supplier diversification.

1.4 Perturbation Solution

We can represent the vector of exogenous cost differences η as $\eta = \delta\eta$ and write (1.19) as

$$\xi = \delta\eta + \beta\Omega \left[\xi + \log \left(\frac{\lambda^*}{\lambda} \right)^{\frac{1}{\rho-1}} \right]. \quad (1.41)$$

Scalar δ is the perturbation parameter. The actual model corresponds to $\delta = 1$, while the case $\delta = 0$ is particularly simple, as discussed in the previous section. The strategy is to obtain approximations for various outcome variables of interest as Taylor expansions with respect to δ around $\delta = 0$. From (1.19), disabling CA by setting $\delta = 0$ implies $\xi = 0$ and the log price index (1.28) becomes

$$\bar{p} = \bar{p}^{Autarky} - \frac{1}{1-\beta} \frac{1}{\rho-1} \log(1/\lambda_0), \quad (1.42)$$

where $\lambda_0 = \frac{1}{1+e^{(1-\rho)\tau}}$ is the no-CA home share (common for all products). Total import becomes simply $M_0 = (1 - \lambda_0) / (1 - \beta)$, and the trade elasticity equals $\epsilon = \rho - 1$.

1.4.1 Endogenous Relative Cost Heterogeneity

First, I show that the difference in log marginal costs (1.41) satisfies

$$\xi = (I - \beta(2\lambda_0 - 1)\Omega)^{-1} \eta + o(\delta^2), \quad (1.43)$$

where the next term of the expansion is cubic.³⁹ Note that $\delta = 1$ is used in the first term, as it corresponds to the actual η .⁴⁰ The derivation is provided in Appendix A.8.

Note that the exact expressions for ξ in autarky and under free trade are

$$\xi^A = (I - \beta\Omega)^{-1} \eta \text{ and } \xi^{FT} = \eta. \quad (1.44)$$

Related to (1.23)-(1.24), (1.43) nests (1.44) as if trade costs affect the input share. The case $\tau = 0$, $\lambda_0 = \frac{1}{2}$ corresponds to ξ^{FT} , while $\tau = \infty$, $\lambda_0 = 1$ gives ξ^A . (See production sharing interpretation below.)

Expression (1.43) is immediately informative as it highlights that heterogeneity in production costs is endogenous to trade barriers (which enter λ_0). Let $\widetilde{var}(\xi)$ denote the weighted average variance of ξ_i with the weights given by the autarky sales shares $(1 - \beta) b_i$. Namely, $\widetilde{var}(\xi) = (1 - \beta) \sum_i b_i \xi_i^2 - [(1 - \beta) \sum_i b_i \xi_i]^2$. Appendix A.8 uses (1.43) to derive the second-order approximation to $\widetilde{var}(\xi)$ which, in the probability limit over realizations of η_i , is

$$\widetilde{var}(\xi)^{2nd} = (1 - \beta) \text{tr} \left[\left(I - \tilde{\beta} \Omega' \right)^{-1} D_b \left(I - \tilde{\beta} \Omega \right)^{-1} \right] \sigma_\eta^2, \quad (1.45)$$

where $0 \leq \tilde{\beta} = \beta(2\lambda_0 - 1) \leq \beta$ and σ_η^2 is the variance of η_i .

Under frictionless trade, $\lambda_0 = \frac{1}{2}$ and $\tilde{\beta} = 0$, $\widetilde{var}(\xi)$ is minimal and equals to the variance of exogenous cost shock differences η_i . Intuitively, when international production sharing is not limited by trade barriers, producers in all countries face the same input costs and the only source of heterogeneity in relative marginal costs is the

³⁹Higher order: $\frac{d^3 \xi}{d\delta^3} = 2(\rho - 1)^2 (1 - 2\lambda_0) \lambda_0 (1 - \lambda_0) \beta (I - \beta(2\lambda_0 - 1)\Omega)^{-1} \Omega \left(\frac{d\xi}{d\delta} \right)^3$ and $\frac{d^4 \xi}{d\delta^4} = 0$.

⁴⁰For a useful alternative interpretation, we can represent the true η as $\delta\tilde{\eta}$ imposing a normalization $var(\tilde{\eta}_i) = 1$. The actual model then has $var(\eta_i) = \sigma_\eta^2 = \delta^2$, so the interpretation of the perturbation parameter becomes the standard deviation of exogenous cost differences η_i . The chosen formulation (replacing η with $\delta\eta$, so that the actual model has $\delta = 1$) is convenient in more general cases as it allows dealing with multiple sources of heterogeneity with a single perturbation parameter. It still preserves the interpretation of residual terms of the form $o(\delta^k)$ as $o(\sigma_\eta^k)$.

variation is fundamental productivities. When trade is restricted, firms increasingly rely on domestic intermediate inputs; the differences in fundamental productivities accumulate along production chains,⁴¹ resulting in stronger heterogeneity in marginal costs. Formally,⁴²

$$\frac{\partial}{\partial \tau} \text{tr} \left[\left(I - \tilde{\beta}(\tau) \Omega' \right)^{-1} D_b \left(I - \tilde{\beta}(\tau) \Omega \right)^{-1} \right] \geq 0. \quad (1.46)$$

Expression (1.45) can be used to understand which properties of the I-O structure determine cross-country heterogeneity in production costs (which ultimately determines trade flows and welfare). Appendix A.10 derives several properties of the trace in (1.45). First, its bounds are:

$$1 \leq (1 - \beta) \text{tr} \left[\left(I - \tilde{\beta} \Omega' \right)^{-1} D_b \left(I - \tilde{\beta} \Omega \right)^{-1} \right] \leq \frac{1}{(1 - \tilde{\beta})^2}. \quad (1.47)$$

The upper bound is achieved for the pure diagonal I-O matrix $\Omega = I$, and the lower bound is achieved in the limit as $N \rightarrow \infty$ when $\omega_{ij} = \frac{1}{N}$ for all i, j (which means perfect diversification of suppliers as $N \rightarrow \infty$). Second, one general approximation for this trace is:

$$(1 - \beta) \text{tr} \left[\left(I - \tilde{\beta} \Omega' \right)^{-1} D_b \left(I - \tilde{\beta} \Omega \right)^{-1} \right] \simeq 1 + 2\tilde{\beta} SSI + \tilde{\beta}^2 (HHI + 2SLI), \quad (1.48)$$

which works well for small $\tilde{\beta} = \beta(2\lambda_0 - 1)$ but becomes inaccurate if both λ_0 and β are close to 1 because it ignores higher order (in $\tilde{\beta}$) terms. Finally, if the I-O network

⁴¹To see this accumulation, expand (1.43) as $\xi = \eta + \beta(2\lambda_0 - 1)\Omega\eta + \beta^2(2\lambda_0 - 1)^2\Omega^2\eta + \dots + o(\delta^2)$. Differences in (log) marginal costs ξ_i come from exogenous cost differences η_i themselves, their immediate upstream combinations ${}_j\omega_{ij}\eta_j$, and so on. (The second and subsequent terms represent the expansion of input price differences in (1.19).) These upstream combinations are multiplied by the powers of $(2\lambda_0 - 1) \in [0, 1]$ which is an inverse measure of trade openness.

⁴²See Appendix A.9.

is generated randomly such that productions loops are negligible for large N , there is another approximation that works for any $\tilde{\beta} < 1$:

$$(1 - \beta) \text{tr} \left[\left(I - \tilde{\beta} \Omega' \right)^{-1} D_b \left(I - \tilde{\beta} \Omega \right)^{-1} \right] \simeq (1 - \tilde{\beta}^2 HHI)^{-1}. \quad (1.49)$$

Appendix A.10 does not prove this relationship but demonstrates that it holds in simulations and explains the nature of this result.

These results mean that heterogeneity in marginal costs depends, first of all, on the degree of supplier diversification captured by the average input shares Herfindahl HHI . For a given total input share β and cost shock dispersion σ_η^2 , relying on more suppliers diversifies away the randomness in total input costs, see the intuition after (1.12). Second, the variance in relative marginal costs is affected by production loops, starting from self-sourcing captured with SSI . Think about an I-O structure in which every product has one supplier. In the ring case (no loops shorter than N) the marginal costs in autarky are $mc_1 = z_1 + \beta mc_2 = z_1 + \beta z_2 + \dots$. As z_1 appears again in the summation only multiplied by $\beta^N \rightarrow 0$, the loop structure can be ignored. Each industry's log marginal costs are given by a linear combination of all cost shocks in the economy. The industries effectively spread their intermediate spending across the entire economy, which implies a great deal of diversification. In the presence of (short) loops, there are fewer uncorrelated shocks in the expansion of mc_i , so the variance of marginal costs is larger. This marginal cost dispersion is inconsequential in the closed economy under Cobb-Douglas preferences and technology, but in the open economy it affects the gains from trade.

1.4.2 Welfare

The following proposition, proved in Appendix A.11, gives the second-order approximation for the gains from trade.

Proposition 1.4.1. *The second-order approximation to the change in log price index $g = \bar{p}^{Autarky} - \bar{p}$ equals in the probability limit to*

$$g^{2nd} = \frac{1}{1-\beta} \left[\underbrace{\log \left(\lambda_0^{\frac{1}{1-\rho}} \right)}_{\text{Intra-industry}} + \underbrace{\frac{1}{2} (\rho-1) \lambda_0 (1-\lambda_0) \widetilde{var}(\xi)^{2nd}}_{\text{Inter-industry}} \right]. \quad (1.50)$$

With this proposition we can relate the (log) gains from trade to heterogeneity in relative marginal costs and – armed with (1.45) and its approximations (1.48) and (1.49) – to the primitives of the model. Welfare gains from trade consist of two components. The pure intra-industry part, which in our formulation is due to the Armington love for variety, is the gravity-ACR benchmark that would be the case without cross-industry heterogeneity. The inter-industry component captures the contribution of CA forces. It is represented as the variance of relative marginal costs times $\lambda_0 (1 - \lambda_0)$ which is a measure of trade openness. Both parts are magnified by $\frac{1}{1-\beta}$, reflecting the compounding effect of supply chain trade on the welfare gains.

Looking deeper into the nature of CA forces, variation in relative marginal costs arises from fundamental technology heterogeneity, captured with σ_η^2 in (1.45), that is transformed by the I-O structure and trade barriers. Depending on the degree of supplier diversification (and also the presence of short production loops) on the one hand and the level of trade costs on the other, exogenous heterogeneity is amplified by the possibility that fundamental differences accumulate along production chains:

$$\widetilde{var}(\xi)^{2nd} \geq \sigma_\eta^2. \quad (1.51)$$

The equality in (1.51) is achieved when trade is frictionless and under perfect diversification of suppliers ($HHI = 0$).

Figure 1.2 provides an illustration. In particular, it shows that the welfare ranking of different I-O structures corresponds to the ranking of relative cost heterogeneity. Compared to the complete I-O network, the ring structure has imperfect supplier diversification ($HHI = 1$) and generates stronger heterogeneity which results in a higher welfare for all $\tau > 0$. The diagonal I-O structure ($\Omega = I$), in addition to imperfect diversification, has also extreme self-sourcing ($HHI = SSI = 1$), resulting in very strong amplification of exogenous heterogeneity and therefore even higher welfare.

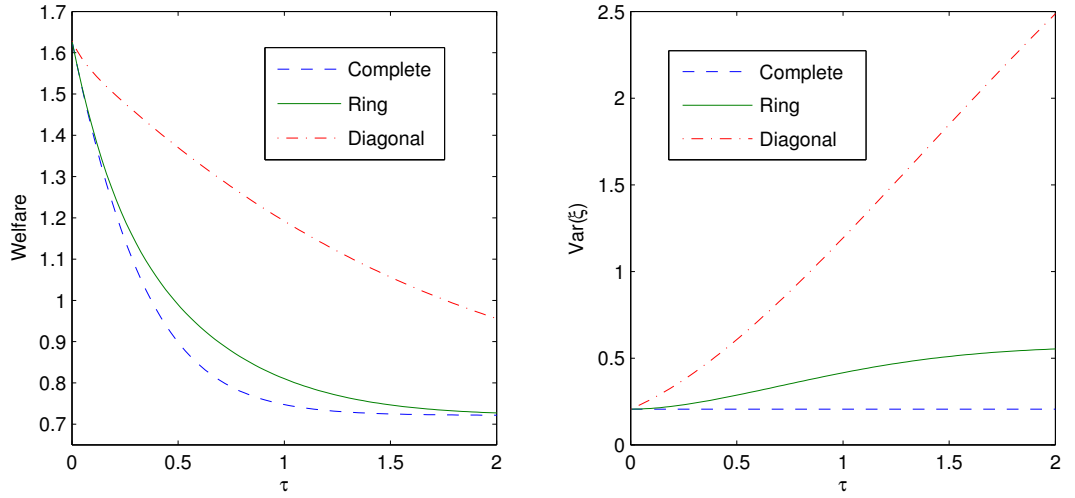


Figure 1.2: Welfare $\exp(-\bar{p})$ and variance of relative marginal costs $\widetilde{\text{var}}(\xi)$ for different I-O networks. The flat dashed line is $\widetilde{\text{var}}(\xi) = \sigma_\eta^2$ for complete network.

Another idea that we can learn from (1.50) and (1.45) is that observed (or recovered) heterogeneity cannot, in general, be considered structural, in particular independent of trade costs. A (misspecified) model with perfect supplier diversification, such as having the complete I-O network, can match a model with granular (industry-level) linkages by adjusting exogenous variation σ_η^2 , but it will fail to predict the effects of changes that affect variation in (endogenous) marginal costs.

Trade flows

By now we understand the welfare part of Figure 1.1. Imperfect supplier diversification in case of the ring I-O structure generates greater variation in relative marginal costs for any $\tau > 0$. The trade part is more nuanced. Note that (1.27) can also be written as

$$M = \sum_i b_i (1 - \lambda_i) - \beta \sum_i (y_i - b_i) \lambda_i^{Upstream}, \quad (1.52)$$

where, again, $\lambda_i^{Upstream} = \sum_j \omega_{ij} \lambda_j$. Total imports are given by a linear combination of industry-level import shares minus a term capturing downstream spillovers of CA.

The following proposition, proved in Appendix A.12, gives a second-order approximation for the two components of total imports (1.52).

Proposition 1.4.2. *Denoting " $\approx$ " the second-order approximation in the probability limit,*

$$\sum_i b_i (1 - \lambda_i) \approx \frac{1}{1-\beta} \left[1 - \lambda_0 + \frac{1}{2} (\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \widetilde{var}(\xi)^{2nd} \right]$$

$$\beta \sum_i (y_i - b_i) \lambda_i^{Upstream} \approx \frac{1}{1-\beta} \frac{1}{2} (\rho - 1) \lambda_0 (1 - \lambda_0) \frac{d\widetilde{var}(\xi)^{2nd}}{d\tau}.$$

To finally explain the trade part of Figure 1.1, we want to understand how supplier diversification differentially impacts trade flows for high and low trade costs.⁴³ Denote $M^{(1)}$ the average import share component $\sum_i b_i (1 - \lambda_i)$ and $M^{(2)}$ the covariance component $\beta \sum_i (y_i - b_i) \lambda_i^{Upstream}$. The complete symmetric network ($\Omega = \frac{1}{N} \mathbf{1}\mathbf{1}'$) is straightforward. The trace involved in $\widetilde{var}(\xi)^{2nd}$ does not depend on trade costs and $\widetilde{var}(\xi)^{2nd} = \sigma_z^2$, so $M^{(1)} = \frac{1}{1-\beta} \left[1 - \lambda_0 + \frac{1}{2} (\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \sigma_z^2 \right]$ and $M^{(2)} = 0$. Because of imperfect diversification, the ring has $\widetilde{var}(\xi)^{2nd} > \sigma_z^2$ (for

⁴³Here we keep σ_z^2 fixed and consider different I-O structures.

$\tau > 0$) and therefore a larger contribution $M^{(1)}$. At the same time, $M^{(2)}$ contributes negatively. Which effect dominates when?

Denote $\Delta M^{(1)}$ the difference between $M^{(1)}$ for a given I-O structure and the complete symmetric network, and similarly $\Delta M^{(2)}$. The last part of Appendix A.12 shows that

$$\lim_{\tau \rightarrow \infty} \frac{\Delta M^{(2)}}{\Delta M^{(1)}} = 0 \quad \text{and} \quad \lim_{\tau \rightarrow 0} \frac{\Delta M^{(1)}}{\Delta M^{(2)}} = 0. \quad (1.53)$$

It means that the covariance between downstream output and upstream cost advantage plays a relatively small role when trade costs are high, but it dominates the direct heterogeneity effect $M^{(1)}$ for low trade costs.

Therefore, compared to the benchmark of complete and symmetric network,⁴⁴ the role of imperfect supplier diversification in a general I-O network with disaggregated interactions is to amplify the effect of small trade costs, but to generate relatively more trade when trade costs are high. Although variation in relative marginal costs increases in the level of trade frictions, this happens slowly near free trade, so that the trade-increasing effect of stronger heterogeneity is dominated by the trade-decreasing effect of downstream CA spillovers. When trade costs are high, however, these spillovers decay fast, so that their negative contribution to trade flows becomes weaker relative to the positive contribution of stronger marginal cost heterogeneity. This is illustrated in Figure 1.3. Note that self-sourcing (diagonal elements of the I-O matrix) plays a special role in determining trade flows. As discussed earlier, even free trade equilibrium exhibits home bias in the sense that import shares are below one half in a two-country economy. The two right panels of Figure 1.3 provide an illustration.

⁴⁴Such stylized roundabout production structure is often assumed in the literature to avoid specifying the detailed I-O network when introducing intermediate goods.

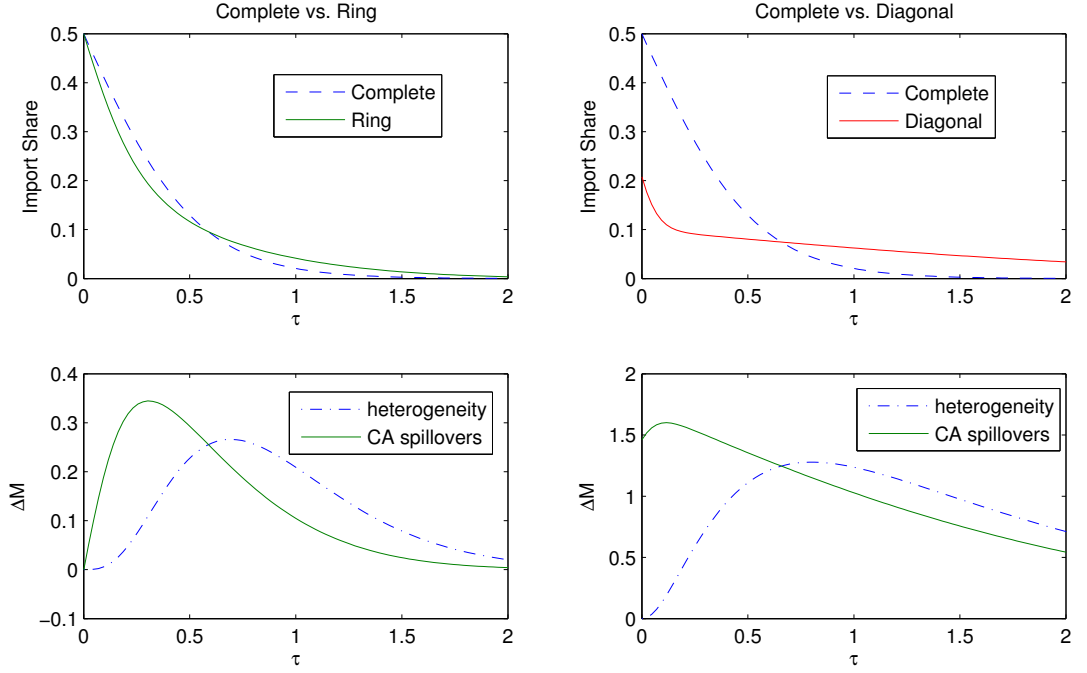


Figure 1.3: Comparing trade flows for different I-O networks.”Heterogeneity” corresponds to differences in average domestic shares $\sum_i b_i^{Comp} \lambda_i^{Comp} - \sum_i b_i^{Ring (Diag)} \lambda_i^{Ring (Diag)}$. ”CA spillovers” correspond to covariance terms $\beta \sum_i (y_i - b_i) \lambda_i^{Upstream}$.

1.4.3 Approximation quality

The perturbation approximations obtained above are quantitatively good when intra-industry trade is sufficiently strong relative to inter-industry trade. This is the case when productivity heterogeneity driven by cost shock variation σ_z^2 is moderate and the Armington elasticity ρ is small. What is the relevant range of σ_z ? From Table 6 in Costinot, Donaldson, and Komunjer (2012), the standard deviation of residual cost shifters (after controlling for country and sector fixed effects) is $\sigma_z = 0.1259$. Given the high level of aggregation in that paper, this number can be viewed as a lower bound for more disaggregated data.⁴⁵ For a reference, Figure 1.4 provides the values of σ_z for Frechet-distributed productivity with the dispersion parameter θ .

⁴⁵A more relevant estimate can be obtained from Hanson and Muendler (2014)

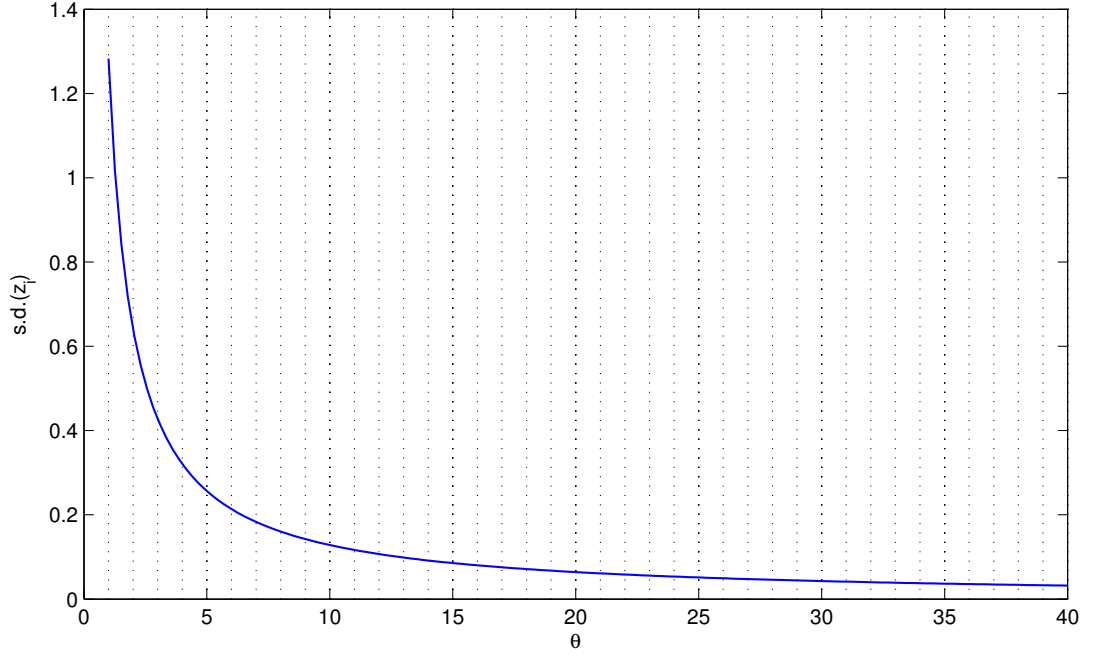


Figure 1.4: Standard deviation of z_i for different values of θ .

In Broda and Weinstein (2006), the median elasticity of substitution among varieties ranges from 2.2 to 3.7, while Feenstra, Luck, Obstfeld, and Russ (2014) find even lower values. At the same time, the distribution of industry-level elasticities has a well-represented right tail, driving the average value to as high as 17.

Figure 1.5 and Figure 1.6 illustrate approximation quality for $\rho = 3$ and $\rho = 5$ for the ring I-O structure. Other parameters are set as $\beta = 0.6$ and $\tau = \log(1.2)$. " η -based approximation" corresponds to the results of Proposition 1.4.1 and Proposition 1.4.2. For higher values of σ_z as well as ρ , approximation quality is improved if the exogenous heterogeneity is expressed in terms ε_i , defined in (A.11), rather than η_i .

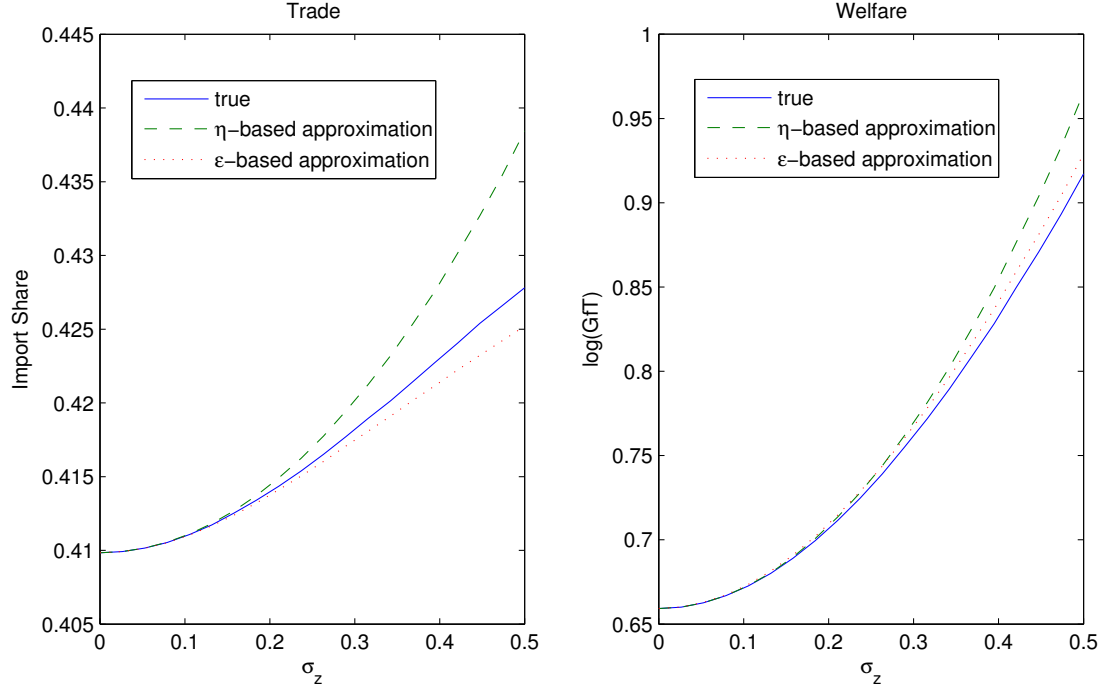


Figure 1.5: $\rho = 3$.

1.5 Conclusion

This paper introduces a perturbation approach to comparative advantage and other forms of cross-industry heterogeneity which allows studying complex input-output networks with a tractable extension of the gravity model. The new method is instrumental in theoretical analysis of global value chains and refining the level of aggregation in face of data limitations. I develop a model that accommodates a nearly arbitrary pattern of "snakes and spiders" linkages between a large number of products. Despite this rich production architecture, trade flows and welfare are related to model fundamentals in a transparent and intuitive way. In particular, I derive summary statistics for the input-output structure needed to characterize the effects of international production sharing. The key feature of the model is endogenous variation in relative costs that is determined simultaneously with vertical specialization. This mechanism primarily depends on average supplier diversification and is also in-

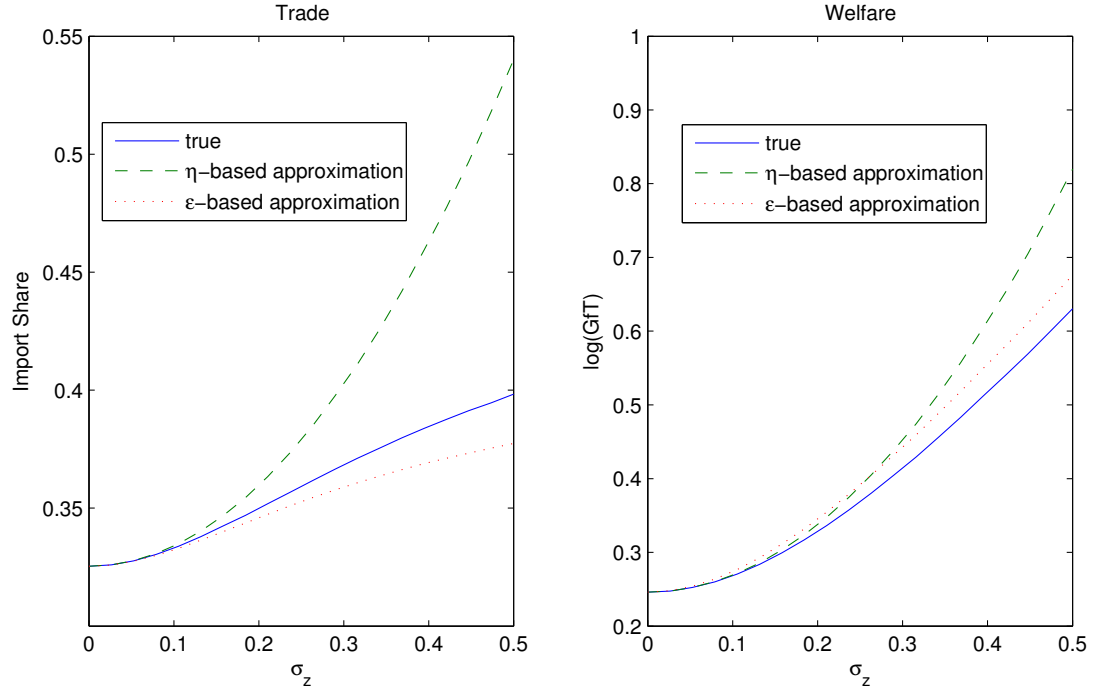


Figure 1.6: $\rho = 5$.

fluenced by short production loops. For this reason, accounting for disaggregated network structure is essential and cannot be replaced with a composite intermediate good assumption often imposed in the literature.

Chapter 2

Structural Change, Increasing Returns, and the Growth of Trade

2.1 Introduction

Since Krugman (1995), the declining GDP share of manufacturing output has been cited as slowing trade-to-GDP growth and thus exacerbating the trade growth puzzle. A common way to formulate this puzzle, e.g. Yi (2003), is that explaining the observed growth in the trade-to-GDP ratio with falling trade barriers implies implausibly high elasticity of substitution between domestic and imported goods.¹ As soon as international trade is mainly in manufactures, the decreasing manufacturing-to-GDP ratio (caused by slower productivity growth in services and/or by nonhomothetic preferences) reduces trade relative to GDP. This paper suggests that there is a force counteracting this effect. Falling manufacturing employment associated with structural transformation in the major economies over the last 30-40 years, combined with

¹Vertical specialization in Yi (2003) allows tariff reduction to account for about a half of trade growth. Overall, skepticism remains about the extent to which falling tariffs explain the recent growth of trade, in particular, by broadening the range of traded goods. Thus, Debaere and Mostashari (2010) find that only from 5% to 12% of the increasing extensive margin of trade is due to tariff reductions.

increasing returns and comparative advantage, can boost trade relative to manufacturing output for the same level of trade costs.

Intuitively, as labor moves away from manufacturing while demand for manufacturing goods is still growing, there becomes “more need for trade”. More specifically, the role of comparative advantage rises relative to transportation costs, and more opportunities for specialization are exploited. This happens precisely because of increasing returns: Specialization deepens to compensate for losses in scale in the marginal industries.

To formalize this argument, I develop a stylized model of structural change based on Dornbusch, Fischer, and Samuelson (1977) (DFS) to which I add exogenously nontraded services and fixed costs in manufacturing industries. Specialization is governed by the usual balance between comparative advantage and trade costs, but now additional resources are required to launch manufacturing products in both countries. This new modified trade-off resolves differently depending on the stage of structural transformation, which is at the heart of the paper. Under complementarity in consumption between services and manufactured goods, faster productivity growth in manufacturing generates endogenous structural change. The resulting decline in manufacturing employment makes it optimal to narrow specialization to save on fixed costs. The range of endogenously nontraded manufactures shrinks, and the countries trade more.

Introducing fixed costs dictates a departure from a competitive environment, but first, to understand the general nature of the underlying forces independently of specific market structure assumptions, I study the planner’s allocation in this economy. Another virtue of the planner’s problem is its high tractability. In the standard DFS setup, under the additional assumption of symmetric countries,² specialization (set

²I assume throughout the paper that countries are similar in all characteristics, including population size and absolute advantage, while comparative advantage in tradables takes the form of labor requirements which are mirror images of each other.

to minimize the cost of living) is solely pinned down by efficiency distribution and trade costs. This is no longer the case in a model with fixed costs and imperfect competition, where specialization is determined by market clearing in a way that involves interaction between the general equilibrium conditions and profits in the marginal industries. Using the planner's problem to abstract from markups greatly simplifies the solution, making the model's key trade-off very intuitive to demonstrate. The optimal specialization pattern maximizes welfare expressed as the ratio of resources left after paying fixed costs to the cost of living index. The incentive to save on fixed costs immediately narrows specialization relative to DFS; structural change moves it further towards the maximum level corresponding to zero trade costs.

Despite lower tractability, analyzing a corresponding decentralized model with imperfect competition is also straightforward. On top of exactly the same fundamentals as in the planner's problem, I consider an oligopolistic market structure where each industry in each country has one potential producer of a good who serves internationally segmented markets competing a la Bertrand.³ If, however, a (less efficient) firm cannot cover the fixed costs, it does not enter leaving both markets to its foreign competitor who is then a monopolist. In general equilibrium, provided that the range of nontraded goods is nondegenerate, to enter is a dominant strategy for the more efficient firm, by which the model avoids the typical complications with multiple equilibria. The marginal firm that decides to enter enjoys effective (adjusted for trade costs that its foreign competitor needs to pay) efficiency advantage just enough to cover the fixed costs. By a general equilibrium effect on wages and sectoral price indexes, structural change tilts the mapping from cost advantage to profitability, such that the marginal firm's markup becomes insufficient to stay in business. Importantly, this effect only operates in an economy with a nontradable sector absorbing an in-

³The model builds on a simplified version of Bernard, Eaton, Jensen, and Kortum (2003). Appendix A.3 solves a similar model under Cournot competition. It gives rise to a richer industry structure which can be of independent interest but is less central to the main argument of this paper.

creasing expenditure share. The precise reason behind that “tilted mapping” is that manufacturing demand rises less than one-for-one with manufacturing productivity because of intersectoral reallocation.

Bertrand competition decentralizes the planner’s allocation not perfectly, but the general pattern of comparative statics holds. Structural transformation forces the weakest comparative advantage industries to shut down, the range of endogenously nontraded goods shrinks, and trade becomes a larger share of manufacturing output for the same level of trade costs. Therefore, structural change provides a new source of extensive margin movements. With an oligopolistic market structure, each entering firm pays fixed costs that are non-negligible relative to the industry size. The role of this industry-level lumpiness of fixed costs is crucial to activate the extensive margin. In contrast, if one were to assume away the discreteness of firms by considering a monopolistic competition framework, this would allow for infinitesimally small industry sizes, which would imply that this mechanism would not be operational.⁴

This theory is falsifiable by certain evidence on industry concentration and the size distribution of firms in manufacturing. If fixed costs (capturing in the model such expenses as managerial effort and product development) do not play a role in determining specialization,⁵ we would observe industries with smaller and smaller firms. As declining employment in manufacturing makes the increasing returns to scale motive more pronounced, the model predicts a thinner left tail of the firm size distribution in tradables. An obvious problem with using this prediction as a test is

⁴It can be shown that placing Krugman (1980) in each DFS industry (similar to Romalis (2004)) gives specialization regions and trade shares that only depend on trade costs, but are unaffected by fixed costs and hence structural transformation. I depart from monopolistic competition because it does not produce interesting results in the context of this paper, but assuming oligopolistic markets is also in accord with ample evidence on the presence of large firms, see Neary (2009) for references.

⁵That is, if discreteness of firms is not essential, or if fixed costs fall substantially over time. Given the lack of estimates of the long-run trends in fixed costs, I follow the standard approach in the literature that treats them as a deep parameter, e.g. Luttmer (2007). The results generalize to the case when fixed costs fall slower than marginal costs if the size of manufacturing sector is sufficiently small; increasing fixed costs would provide an independent source of trade growth (see discussion in Section 2.2.4).

its consistence with the traditional reason for trade growth, that is falling costs of trade. The empirical studies do find that globalization is correlated with elimination of small firms, e.g. Gu, Sawchuk, and Rennison (2003) for the case of Canada, but this is interpreted as confirming the standard trade theories, like Melitz (2003).

What may differentiate between trade liberalization and structural change is the model's implication for markups. The endogenous market structure in manufacturing responds to intersectoral reallocation towards services with unambiguously growing monopolization, which translates into weakly increasing markups. (Markups stay constant if high trade costs do not restrict pricing of the marginal firms in domestic markets.) This is in contrast to the common view of pro-competitive effects of trade (e.g. Melitz and Ottaviano (2008)), but in line with several recent studies that challenge it (Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2012), De Locker, Goldberg, Khandelwal, and Pavcnik (2012)). The present paper emphasizes that, in evaluating pro-competitive effects of trade, it is important to distinguish between different sources of trade growth.

My paper relates to three strands of the literature. First, it contributes to the studies explaining the growth of trade. A well-known worldwide trend is that trade has grown substantially relative to GDP over the last several decades. Meanwhile, tariff reductions have been quite moderate in developed countries – and transportation costs have not fallen that much either (Hummels (1999)) – which turns the amount of trade growth into a major quantitative puzzle.⁶ In an influential paper, Yi (2003) argues that vertical specialization greatly magnifies the impact of tariff reductions, especially when trade costs become small, yet a large part (about a half) of the trade

⁶As cited in Debaere and Mostashari (2010), the implied values of trade elasticity can go from 10 all the way to 30, which is an order of magnitude greater than usually found in the literature, e.g. Simonovska and Waugh (2011). Such high values may look particularly unencouraging in light of Arkolakis et al. (2012), as they imply negligible welfare effects of trade. Although the author's personal view is that trade research should not be guided by temptation to find big gains from trade, this paper makes a step to relax what can be called an unpleasant trade-off between the models' ability to explain trade growth and suggest significant welfare gains from globalization, a point also made by Yi (2003).

growth puzzle still remains.⁷ A natural response to this striking evidence would be to look for alternative sources of trade growth. Thus, Dalton (2009) explores the adoption of just-in-time logistics; Feinberg and Keane (2006) argue for technical change (increasing role of intra-firm intermediates) in the context of multinational corporations. The contribution of this paper is to develop a parsimonious model that introduces structural change in the trade growth debate. For empirical relevance, the main distinction of the mechanism proposed here is that it operates and produces quantitatively significant results with costly trade, while the currently most successful vertical specialization story works best when total trade barriers, both natural and artificial, approach zero.

A second branch of related research is concerned with the interaction between trade and structural change. In the postwar period, the major economies have been facing steadily declining expenditure and employment shares of manufacturing. Moreover, since the early 80s, for example in the U.S., the absolute number of manufacturing jobs has been falling as well. This paper argues that the two historical trends (structural change and growth of trade) have a common source in the form of the growing manufacturing productivity and are linked through increasing returns to scale.⁸ The literature relating trade and structural change has been focusing on one direction: The effect of trade openness on sectoral composition of consumption,

⁷Important motivating evidence for my study also comes from the empirical literature that looks at the relation between extensive margin movements and tariff reductions. Debaere and Mostashari (2010) summarize earlier studies and present new results that the estimated link appears weak. This, in part, challenges the quantitative results of Yi (2003) that rely on extensive margin adjustment. Furthermore, it remains unclear whether natural trade barriers (ignored in the calibration) can be considered small enough, such that trade tariffs approaching zero initiate the nonlinearity effect emphasized by Yi. On the other hand, Bridgman (2012) adds one more stage of production and finds that trade growth is consistent with the fall in directly measured trade costs. The magnitude of trade barriers used in his calibration is, however, much smaller than found, for example, in Anderson and van Wincoop (2004).

⁸For the purposes of this paper, structural change is modeled as a simple Baumol effect, see Ngai and Pissaridis (2007) for a closed economy analysis. Engel's law, formalized by nonhomothetic preferences, is the second important mechanism of structural transformation. Buera and Kobowsky (2009) discuss how well combining the two mechanisms can match the data.

output, and employment (Matsuyama (2009), Yi and Zhang (2010)).⁹ This study investigates a particular aspect of the other direction: The effect of structural change on trade growth relative to GDP. One notable paper that also addresses the question whether structural transformation (due to nonhomothetic preferences) combined with increasing returns (in a monopolistic competition environment) can generate a rising trade-to-GDP ratio is Bergoeing and Kehoe (2003). However, their calibration results are that nonhomothetic preferences cannot account for the observed growth of trade relative to GDP. Bergoeing, Kehoe, Strauss-Kahn, and Yi (2004) reach the same conclusion and argue that vertical specialization as in Yi (2003) is needed to reconcile the growing manufacturing trade with the falling GDP share of manufacturing output.¹⁰ The key difference between my approach and Bergoeing and Kehoe (2003) is that they do not have comparative advantage forces to be augmented by structural transformation. Yi and Zhang (2010) have structural change and comparative advantage but no increasing returns. To my knowledge, the present paper is the first to combine all of these elements. It is the interaction between increasing returns, cross-country technological differences, and transportation costs that stimulates specialization and trade as manufacturing productivity grows and manufacturing employment falls in the process of structural transformation.

Finally, the paper is related to the literature on oligopoly in general equilibrium. One central insight of this literature, e.g. Neary (2009), is that firms are modeled as large relative to the industry but small in the economy, which allows studying strategic interactions in a tractable way. Unlike many papers employing the “continuum-quadratic” preferences, I do not depart from the traditional assumption of CES preferences, which stresses that the modeled effect has primarily a supply-side nature. The model with price competition uses a simplified version of Bernard, Eaton, Jensen,

⁹Sposi (2012) studies how structural transformation in South Korea affected the country’s composition of import and export.

¹⁰The explanation of trade growth suggested in this paper is complementary with Yi (2003).

and Kortum (2003) and extends it by adding fixed costs. The model with quantity competition developed in Appendix A.3 dates back to Brander and Krugman (1983) and is based on a simplified version of Atkeson and Burstein (2008). The equilibrium market structure is endogenous, such that the mode of competition responds to structural change in the economy.¹¹

The rest of the paper is organized as follows. Section 2.2 explores a planner's problem in which specialization and the trade pattern evolves in response to structural change. Section 2.3 then shows how similar effects arise in the market economy with oligopolistic competition. Section 2.4 concludes and discusses broader implications of the paper.

2.2 Planner's Allocation

Consider the Dornbusch-Fischer-Samuelson (1977) model of Ricardian trade between two countries with transportation costs which create a region of goods that are endogenously nontraded. I modify this framework by adding exogenously nontraded services and fixed costs of developing manufactured products. Introducing fixed costs also requires to modify the market environment. In this section, however, I sidestep this issue by looking at the planner's allocation.

2.2.1 Environment and Planner Optimality

A two-country world is populated with identical agents with the following preferences:

$$U(S, M) = (\alpha S^{1-1/\eta} + (1 - \alpha) M^{1-1/\eta})^{\eta/(\eta-1)}, \quad 0 < \alpha < 1, 0 \leq \eta < 1, \quad (2.1)$$

¹¹A number of previous studies have looked how competition mode is affected by trade liberalization, see Chapter 4 in Etro (2009) for a review.

where S is consumption of services and M is aggregate consumption of manufactures:

$$M = \left(\int_0^1 q_i^{1-1/\sigma} di \right)^{\sigma/(\sigma-1)}, \quad \sigma > 0. \quad (2.2)$$

The condition $\eta < 1$ ensures that the direction of employment reallocation during structural transformation caused by a faster productivity growth in manufacturing is from manufacturing to services (Ngai and Pissaridis (2007)).

The two countries, Home and Foreign, share the same fundamentals except unit costs in manufacturing. Asterisk notation is used for Foreign. Agents inelastically supply $L = L^*$ units of labor. Services are exogenously nontraded, while manufactures can be traded with iceberg costs determined by $\tau > 1$: τ units of a good need to be shipped from one country to supply one unit of this good in the other.

Services are produced with a constant returns to scale technology: $L_s = z_s S$, where L_s is labor input and z_s is unit labor requirement ($z_s = z_s^*$).

Operating each industry in each country requires fixed costs $f = f^*$ in terms of labor. I assume the maximum possible symmetry, so the two countries have unit labor requirements in manufacturing which are mirror images of each other. The marginal costs in Home and Foreign are introduced as functions of the industry indicator i in the following way:

$$c_i = z_m \tilde{c}_i \text{ and } c_i^* = z_m \tilde{c}_i^*, \quad \tilde{c}_i^* = \tilde{c}_{1-i}, \quad i \in [0, 1], \quad (2.3)$$

where z_m is the inverse of the common manufacturing productivity and goods are ordered such that the Home's comparative advantage is declining, or $\tilde{c}_i$, assumed to be a differentiable function of i , is strictly increasing: $\tilde{c}_i' > 0$. The Home marginal cost in industry i is therefore equal to the Foreign marginal cost in industry $1 - i$.

There is a global central planner who maximizes the utility of the representative agent. Appendix B.1.1 provides a formal treatment of the planner's problem. Ac-

according to the pattern of comparative advantage, Home has industries in the range $i \in [0, \lambda]$ and Foreign in the range $i \in [1 - \lambda, 1]$, where $\lambda \leq 1$. I focus on the case when parameters are restricted such that it is optimal to produce all goods in the world economy, so $\lambda \geq \frac{1}{2}$.¹² In parallel to the standard DFS setup, goods in the region $[1 - \lambda, \lambda]$ are endogenously nontraded, as the technologies are relatively similar and comparative advantage does not overcome trade costs.

Optimal consumption of each good in Home satisfies

$$q_i = M \left(\frac{c_i}{P} \right)^{-\sigma} \text{ for } i \in [0, \lambda] \text{ and } q_i = M \left(\frac{\tau c_i^*}{P} \right)^{-\sigma} \text{ for } i \in [\lambda, 1], \quad (2.4)$$

where the manufacturing cost index P is given by

$$P = \left(\int_0^\lambda c_i^{1-\sigma} di + \int_\lambda^1 (\tau c_i^*)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (2.5)$$

The labor resource constraint in Home is

$$L_s + \int_0^\lambda q_i c_i di + \int_0^{1-\lambda} \tau q_i^* c_i di + \lambda f = L, \quad (2.6)$$

in which labor is used to pay the fixed costs (λf) and to produce services and manufacturing goods. (2.6) reflects that goods in the range $i \in [0, 1 - \lambda]$ are produced both for domestic consumption and for export (q_i^* denotes consumption in Foreign).

As shown in Appendix B.1.1, the planner's problem of setting the optimal specialization cutoff λ can be expressed as:

$$\max_{\lambda \in [\frac{1}{2}, 1]} U(z_s, z_m, \lambda) = \frac{L - \lambda f}{(\alpha^\eta z_s^{1-\eta} + (1 - \alpha)^\eta P^{1-\eta})^{\frac{1}{1-\eta}}}, \quad (2.7)$$

¹²This restriction is mild, unless the elasticity of substitution σ is very high. It means that the fixed cost f is low enough, other things equal, or, as we will see, that the relative service productivity z_m/z_s is separated enough from zero.

where the numerator is resources left after paying the fixed costs, and the denominator is the cost of living index that depends on z_s and (through P) on z_m and λ .

With zero fixed costs, the problem (2.7) collapses to minimizing the manufacturing cost index, and the value of z_m relative to z_s plays no role. That is, with constant returns to scale in manufacturing, specialization is solely determined by the pattern of comparative advantage and trade costs, which corresponds to the original DFS. More specifically, the cutoff λ minimizes $P(z_m, \lambda)$ given by (2.5).

$$\frac{\partial}{\partial \lambda} P(z_m, \lambda) = P^\sigma \frac{c_\lambda^{1-\sigma} - (\tau c_\lambda^*)^{1-\sigma}}{1-\sigma}. \quad (2.8)$$

Setting $\frac{\partial}{\partial \lambda} P = 0$ implies $c_\lambda = \tau c_\lambda^*$, so the difference in efficiency is exactly balanced by trade costs (z_m only affects the level of P , not its composition).

With $f > 0$, keeping manufacturing industries in each country is costly. There is now a trade-off between minimizing the cost of living and maximizing resources left after paying the fixed costs. As shown in Appendix B.1.1, taking the first order condition for (2.7) gives

$$\frac{\partial}{\partial \lambda} U(z_s, z_m, \lambda) = 0 \Leftrightarrow \frac{\partial}{\partial \lambda} P(z_m, \lambda) = -\frac{f}{M} < 0. \quad (2.9)$$

Therefore, the first order condition implies that $\frac{c_\lambda^{1-\sigma} - (\tau c_\lambda^*)^{1-\sigma}}{1-\sigma} < 0$, or $c_\lambda < \tau c_\lambda^*$. This means that the fixed costs immediately narrow specialization regions (since c_i is increasing, the cutoff λ is lower than in the original DFS). Intuitively, there are potential opportunities for specialization as long as production costs in the two countries are not equal, but not all of them are realized because trade is costly. With the fixed costs, additional opportunities for specialization are exploited, beyond what is implied by the DFS. But this is just comparing two different environments with and without fixed costs. The main message of this exercise is that the presence of fixed

costs affects comparative statics with respect to relative productivity in manufacturing z_s/z_m which generates structural change.

Before proceeding to characterize the effects of structural change, we need to compute a key statistic of the model which is the trade share, the ratio of manufacturing exports to output. To make a link to the data, it should be calculated in values. The following interpretation of the planner's problem allows us to do so. Observe that the role of the central planner is limited to set the optimal value of λ and pay the fixed costs. Conditional on that, everything else can be determined by the competitive equilibrium in which prices are equal to marginal costs. So P can in fact be interpreted as the manufacturing price index (with wages being normalized to one), and PM is the value size of manufacturing.¹³ Appendix B.1.2 proves the following

Proposition 2.2.1. *The trade share, given by*

$$\frac{\int_0^{1-\lambda} \tau q_i^* c_i di}{PM} = \left(1 + \frac{\int_0^{\lambda} \tilde{c}_i^{1-\sigma} di}{\int_0^{1-\lambda} (\tau \tilde{c}_i)^{1-\sigma} di} \right)^{-1}, \quad (2.10)$$

is decreasing in λ .

This is very straightforward, meaning that trade becomes a larger fraction of manufacturing as specialization narrows. Importantly, this proposition implies that productivity growth and general equilibrium conditions affect the trade share only by shifting the specialization cutoff λ . This property that, for a given elasticity σ , trade costs τ , and efficiency distribution, λ is a sufficient statistic for the trade share holds throughout the paper.¹⁴

¹³This market equilibrium analogy is meant to be purely illustrative. It is problematic to define trade flows in values for the planner's problem as it does not involve any prices. However, we can still use the shadow prices in the planner's allocation which are proportional to the competitive prices.

¹⁴For log preferences ($\sigma = 1$), this share is just $1 - \lambda$, or λ is the share of the country's trade with itself, which is consistent with the notation in Arkolakis et al. (2012).

2.2.2 Structural Change

Appendix B.1.2 proves the following proposition which, along with its market economy counterpart in the next section, is the central result of this paper.

Proposition 2.2.2. *For any initial allocation with $\lambda \in (\frac{1}{2}, 1)$, a decrease in z_m/z_s narrows specialization (λ falls).*

Proposition 2.2.2 characterizes the response of specialization to structural change. As productivity in manufacturing increases relative to productivity in services (z_m/z_s falls), labor moves away from manufacturing, putting more emphasis on the incentive to save on fixed costs. This forces the countries to deepen specialization by dropping the weakest comparative advantage industries. The region of endogenously nontraded goods shrinks, and, from Proposition 2.2.1, trade becomes a larger share of manufacturing output and consumption. In other words, structural change amplifies the natural comparative advantage relative to trade barriers, as fixed costs become a larger share of employment in manufacturing.

It is important to emphasize that all ingredients of the model are crucial for this result. Without increasing returns in the form of fixed costs,¹⁵ specialization is similar to DFS and does not depend on relative productivity. With zero trade costs, the countries completely specialize ($\lambda = \frac{1}{2}$) for all z_m/z_s . Finally, in an economy with only a manufacturing sector, changes in z_m have no impact on specialization, despite the fact that they affect the ratio of marginal costs to fixed costs.¹⁶ In all these cases, the trade-to-manufacturing ratio does not change as the countries undergo structural transformation.

¹⁵This nature of increasing returns is important. It can be shown that in a similar DFS-based model with an alternative form of increasing returns, namely multiplicative external economies as in Grossman and Rossi-Hansberg (2010), structural change does not generate growth in the trade-to-manufacturing ratio, as specialization regions are scale-independent.

¹⁶This can be seen directly from (2.7). When $\alpha = 0$, so there are no services in the economy, U is proportional to (the inverse of) z_m and it does not influence the choice of λ . Changes in f , however, still affect specialization.

The next proposition looks at the extreme case when manufacturing productivity grows unboundedly relative to productivity in services.

Proposition 2.2.3. *There exists a strictly positive level of z_m/z_s below which the countries completely specialize.*

The proof is given in Appendix B.1.3. Low enough z_m/z_s implies $\lambda = \frac{1}{2}$ if the assumption that all goods are produced is satisfied (if not, which is the case when $\sigma > 1$ and z_m/z_s is close to zero, then $\lambda < \frac{1}{2}$, which also means complete specialization). This result is more than merely a technical detail, as it says that structural change potentially leads to the maximum possible specialization despite the remaining trade barriers. The trade share, though, still remains below one half if the elasticity of substitution σ is greater than one.

2.2.3 Simulation Example

Consider the economy described in this section with $L = 100$, $f = 2$, $\alpha = 0.9$, $\eta = 0.3$, $\sigma = 4$, $\tau = 1.4$ and $\tilde{c}_i = \exp(\gamma i)$ with $\gamma = 0.5$.¹⁷ Consider a starting point $z_s = 2$, $z_m = 1$. In the optimal allocation services have 72% employment share and 63% value share. The specialization cutoff is $\lambda = 0.7$, and the trade share is 16.9%. The fixed costs are 5% of total employment in manufacturing.

Now introduce sector-biased productivity growth at the rate 1.03 in manufacturing and 1.01 in services. When z_m and z_s fall at these rates during 50 periods (z_s/z_m increases by factor 2.67), λ becomes 0.61 and the trade share becomes 21.1%. Services employment is now 83%, services value share is 82%, and the fixed costs become 7.3%

¹⁷The value $\eta = 0.3$ corresponds to the preferred value in Ngai and Pissaridis (2007) who also consider an alternative $\eta = 0.1$ (the latter generates a stronger decline in manufacturing employment, which would increase the trade share in my model at a higher rate). The value $\gamma = 0.5$ is set more arbitrarily. Eaton and Kortum (2002), for a different distribution of productivity, prefer the value of dispersion parameter $\theta = 8.28$ which implies a standard deviation in efficiency of 15 percent. In this model, the efficiency $\exp(-\gamma i)$ has the standard deviation $\left(\frac{\gamma}{2} \frac{1+\exp(-\gamma)}{1-\exp(-\gamma)} - 1\right)^{0.5}$, which equals 0.15 for $\gamma = 0.52$. Simonovska and Waugh (2011) argue for a lower value of θ implying more variability.

of employment in manufacturing. The four percentage points increase in the trade share is quite substantial relative to the initial level.¹⁸

Figure 2.1 illustrates the resulting path of trade shares and also considers alternative parameter values. If the path of trade shares becomes flat, that means that λ achieves its lower bound $\frac{1}{2}$ (maximum specialization) and there is no further trade growth. In part A, a lower elasticity of substitution corresponds to a higher initial trade share and its faster growth. This is in fact remarkable, as explaining trade growth with falling tariffs always favors a higher elasticity. Part B shows that less variation in efficiency (lower γ) decreases the initial trade share but makes it grow faster in response to structural change. Part C shows that lower trade costs increase the initial trade share but reduce its responsiveness to structural change. Finally, part D illustrates that higher fixed costs both shift the path of trade shares upwards and make it steeper (unless λ becomes one half).

Structural change has a bigger impact on the trade share when the elasticity of substitution is lower, the pattern of comparative advantage is flatter, or the fixed costs are higher. The main requirement is, however, that trade costs are positive and quite large (at least above 20%). In this model, trade costs matter little for structural transformation. Changing τ from 2 to 1.2 increases the services employment share from 71.5% to 72.5% in the starting period and from 82.7% to 83.6% in the final period. This suggests that the effect of trade on structural change is small, but the effect of structural change on trade can be substantial provided that trade costs are nontrivial.¹⁹

¹⁸In this exercise I abstract from population growth. I target the 30% drop in the absolute number of manufacturing jobs between 1980 and 2005 in the U.S. which happened despite growing labor force. As calculated in Yi (2003), the U.S. manufacturing trade share increased from 14% in 1980 to 27% in 2000. The differentiated productivity growth in manufacturing and services of 3% vs. 1% is stronger than estimated in Ngai and Pissaridis (2007) (2% vs. 1%). And, of course, there are only 25 years between 1980 and 2005, rather than 50 years. To generate a sufficient structural change with a narrower gap in productivity growth, I would need to reduce η or extend the model to include nonhomothetic preferences.

¹⁹The former result in this model can be due to similarity of countries and may change in a richer cross-section that exhibits heterogeneity in, for example, sector-level comparative advantage.

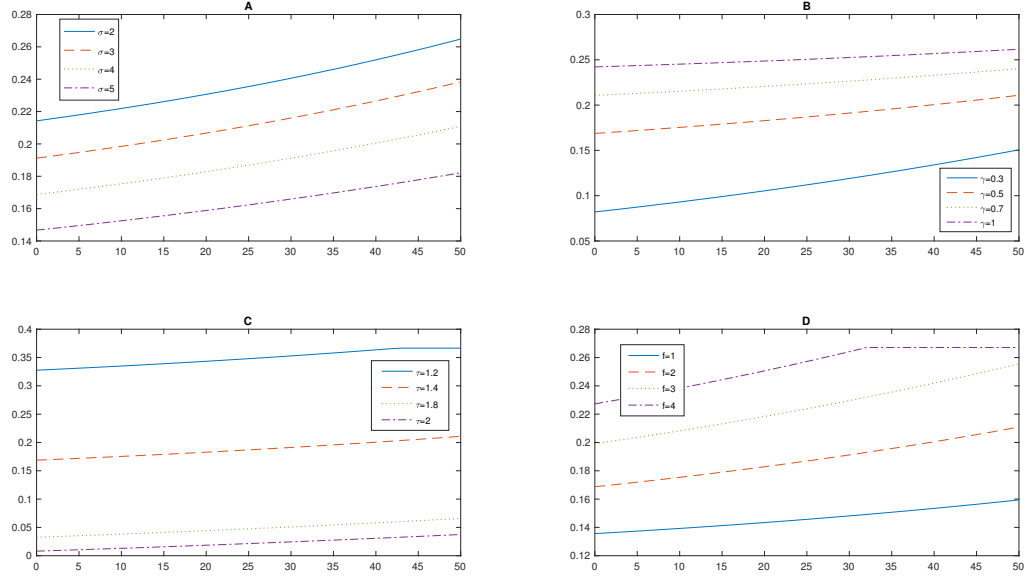


Figure 2.1: Comparative statics of the trade share in z_m/z_s for different parameter values. Benchmark: $\sigma = 4$, $\tau = 1.4$, $\gamma = 0.5$, $f = 2$.

2.2.4 Trade Costs

I now briefly discuss the role of trade costs in the model. In some sense, structural transformation acts as a substitute for trade liberalization: It increases the range of traded goods, up to the maximum level corresponding to frictionless trade, while zero trade costs leave no role for structural change to affect the trade share. The interaction between structural change and trade liberalization is, however, more complicated. As in DFS, a fall in trade costs leads to a higher trade share in the model presented. But lower trade costs have a similar effect as productivity improvements in manufacturing. By this, there arises a trade-induced structural change which (similar to Proposition 2.2.2) acts to amplify the effect of reduction in tariffs or transportation costs.²⁰

Matsuyama (2009) emphasizes that the global pattern of manufacturing decline realizes differently country by country. Teignier (2012) finds that trade has played an important role in structural transformation in particular countries. Reyes-Heroles (2016) shows that trade liberalization can have a bigger impact on structural change when trade is unbalanced.

²⁰One can do the following numerical exercise to shed some light on the nature of this interaction effect. Keeping z_s and z_m fixed, simulate a path of trade shares that results from a declining path of

2.2.5 Discussion

To summarize, this section establishes the presence of a fundamental force that increases trade specialization in response to declining manufacturing employment. So far this result relies on the planner's allocation, but in the next section I show that this force operates in the market economy as well. Structural change is found to be a new, not related to trade liberalization, source of trade growth.

The key assumption made to get these results is the presence of fixed costs. I follow the literature that, in a variety of settings, treat them as a deep parameter. In Luttmer (2007), a combination of constant fixed costs and marginal costs that are falling over time due to technological progress produces a distribution of firm sizes that is consistent with the data. In industrial organization, it is typical to assume that fixed costs are a random draw from a time-invariant distribution. With the interpretation that fixed costs capture such expenses as product development and managerial effort, they are likely to have a nature quite different from marginal costs.²¹ Given the lack of estimates of the long-run trends in fixed costs, I assume them constant, which seems to be a reasonable benchmark.²²

The results generalize to the case when fixed costs fall slower than marginal costs, but under the additional assumption that the size of manufacturing sector is sufficiently small. Increasing fixed costs would provide an independent source of trade growth.

τ . Then compare this path to a counterfactual one that is computed freezing total employment in manufacturing at the initial level. The former path is always above, but the difference is small for reasonable parameter values. This demonstrates that trade in the model affects structural change only moderately, which is in contrast to a much stronger effect in the opposite direction.

²¹A similar argument can be made about fixed costs of exporting. Lincoln and McCallum (2011) test a hypothesis that these costs have declined and reject it.

²²Klenow, Li, and Bollard (2013) argue that technological progress mainly takes the form of declining marginal costs.

2.3 Oligopoly

The main goal of this section is to demonstrate that the key result that structural change increases trade is not an artifact of the planner's formulation but holds in the market economy as well. On top of exactly the same fundamentals as in the planner's problem, I consider an oligopolistic market structure where each industry in each country has one potential producer of a good that serves internationally segmented markets competing a la Bertrand. If, however, a (less efficient) firm cannot cover the fixed costs, it does not enter leaving both markets to its foreign competitor who is then a monopolist. This imposes an additional restriction $\sigma > 1$, which is the only change in the primitives relative to Section 2.2. The market structure (competition mode by industry) is endogenous and will respond to structural change.

2.3.1 Model Setup

Consider the Home country. From the preferences (2.1)-(2.2), the relative expenditure on services and manufacturing goods is

$$\frac{z_s S}{PM} = \left(\frac{\alpha}{1-\alpha}\right)^\eta \left(\frac{z_s}{P}\right)^{1-\eta}, \quad (2.11)$$

where the price of competitively produced services is given by unit labor cost z_s (wages are normalized to one),²³ and the manufacturing price index $P = \left(\int_0^1 p_i^{1-\sigma} di\right)^{1/(1-\sigma)}$ aggregates the goods prices p_i , $i \in [0, 1]$, faced by Home consumers. The demand for each good i is

$$q_i = M \left(\frac{p_i}{P}\right)^{-\sigma}. \quad (2.12)$$

Similar to (2.3), firms in Home and Foreign have marginal costs

²³We normalize the wage to one in one country through the choice of the numeraire. Using country symmetry, the wages in both countries are equal to one.

$$c_i = z_m \tilde{c}_i \text{ and } c_i^* = z_m \tilde{c}_i^*, \quad \tilde{c}_i^* = \tilde{c}_{1-i}. \quad (2.13)$$

The simplest way to describe competition and pricing in manufacturing is to present it as an extension of a simplified BEJK setup. Suppose that in each industry in each country there is one potential producer that may enter by paying the fixed costs f (for the beginning put $f = 0$). Once a firm exists, it can export with iceberg transportation costs $\tau > 1$. The markets in Home and Foreign are segmented, which results in variable markups. The competition is Bertrand, so all markets are served by only one producer that can offer the lowest price. Industries in the range $i \in [0, \lambda^0]$, where Home has comparative advantage, are served by Home firms and get prices $p_i = \min\{\frac{\sigma}{\sigma-1}c_i, \tau c_i^*\}$. Industries in the range $i \in [\lambda^0, 1]$ are served by Foreign firms, and the prices are $p_i = \min\{c_i, \frac{\sigma}{\sigma-1}\tau c_i^*\}$. A typical price profile that arises is presented in Figure 2.2.A. (The picture depends on σ , τ , as well as on the pattern of comparative advantage.) The specialization cutoff λ^0 (the abscissa of point B) is the same as in DFS, yet prices are higher because of positive markups. The goods in the range $i \in [1 - \lambda^0, \lambda^0]$ are endogenously nontraded.

Now add (small) fixed costs $f > 0$. To avoid losses, Home firms in the range $i \in [\lambda, 1]$ do not enter, where $\lambda < \lambda^0$. The marginal Home firm in the industry $i = \lambda$ that decides to enter enjoys effective (adjusted for trade costs that its foreign competitor needs to pay) efficiency advantage just enough to cover the fixed costs. Its gross profit is given by

$$\Pi_\lambda = (p_\lambda - c_\lambda) M \left(\frac{p_\lambda}{P} \right)^{-\sigma} = f. \quad (2.14)$$

Foreign firms in the range $i \in [\lambda, 1]$ are now monopolists not constrained by competition from Home and always set $p_i = \frac{\sigma}{\sigma-1}\tau c_i^*$. Similarly, all Home firms become unconstrained for $i \in [0, 1 - \lambda]$ and set $p_i = \frac{\sigma}{\sigma-1}c_i$. For $i \in [1 - \lambda, \lambda]$, where two

firms enter (to serve their domestic markets only), competition is Bertrand and the prices are as in BEJK (above). Figure 2.2.B shows a typical price profile in Home. (A similar discontinuity as BF , which is where the new λ is, can arise for $i = 1 - \lambda$.)

I focus on the case when the range of nontraded goods, which corresponds in this setup to entry of both firms, is nondegenerate. Although this condition is quite reasonable and unrestrictive (empirically, it relates to trade shares far below one half), in this setting it is powerful. For the market structure that arises in general equilibrium, the fact that the firms in the industry $i = \frac{1}{2}$ stay in business in both countries means that firms with this or better cost advantage can cover the fixed costs from serving the domestic market only independently of the presence of the competitor. Therefore, to enter is a dominant strategy for the more efficient firm, by which the model avoids typical complications with multiple equilibria.²⁴ In what follows, although the fixed costs f stay the same, the specialization cutoff λ will move in response to structural change generated by sector-biased productivity growth.

The manufacturing price index summarizing this market structure is

$$P = \left(\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1} c_i \right)^{1-\sigma} di + \int_{1-\lambda}^{\lambda} \left(\min \left\{ \frac{\sigma}{\sigma-1} c_i, \tau c_i^* \right\} \right)^{1-\sigma} di + \int_{\lambda}^1 \left(\frac{\sigma}{\sigma-1} \tau c_i^* \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (2.15)$$

One can guess that a lower λ corresponds to a higher level of (2.15). The later part of this section will rely on this fact, but it should be formulated in a different way, because (2.15) also depends on z_m which affects λ in equilibrium. Observe that from (2.13) all prices in manufacturing are scaled by z_m . Introduce $\tilde{x}$ for each variable x such that $x = z_m \tilde{x}$ and thus $\tilde{P} = \frac{1}{z_m} P$ is the constant-productivity price index

²⁴To be clear, multiplicity is possible in the model, but if we start from one unique equilibrium (adding small enough fixed costs to BEJK with nonflat cost advantage and positive trade costs), structural change preserves uniqueness as long as the region of nontraded goods does not shrink completely.

in manufacturing. Appendix B.2.1 proves a result analogous to (2.8)-(2.9) which is stated in

Proposition 2.3.1. *For any equilibrium with $\lambda \in (\frac{1}{2}, 1)$, $\frac{\partial}{\partial \lambda} \tilde{P} < 0$.*

As highlighted in Appendix B.2.1, the inequality $\tilde{P}'_\lambda < 0$ in Proposition 2.3.1 is necessarily strict, because switching suppliers from a domestic oligopolist to a foreign monopolist in the industry $i = \lambda$ in Home strictly increases $\tilde{p}_\lambda$ both due to a (weakly) higher markup and a fall in efficiency adjusted by trade costs (in equilibrium with $f > 0$, $\tilde{c}_\lambda < \tau \tilde{c}_\lambda^*$). The second effect is present in the planner's allocation as well (this fall in sourcing efficiency is optimal, as it is compensated by saving on the fixed costs).

Similar to Section 2.2, the trade share can be computed conditional on λ only, independently of the other general equilibrium conditions.

Proposition 2.3.2. *The trade share, given by*

$$\frac{\int_0^{1-\lambda} p_i^* q_i^* di}{PM} = \left(1 + \tau^{\sigma-1} + \frac{\int_\lambda^1 \left(\min\left\{ \frac{\sigma}{\sigma-1} \tilde{c}_i, \tau \tilde{c}_i^* \right\} \right)^{1-\sigma} di}{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1} \tau \tilde{c}_i \right)^{1-\sigma} di} \right)^{-1}, \quad (2.16)$$

is decreasing in λ .

The proof is in Appendix B.2.2.

2.3.2 Specialization and Structural Change

Let us now turn to the central part of the model which is the determination of the specialization cutoff λ . By Walras law, the specialization cutoff is equivalently pinned down by goods or labor market clearing taking into account (2.14). Applying the tilde notation, write it as

$$z_m M \tilde{P}^\sigma (\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma} = f. \quad (2.17)$$

At this point we can already obtain some preliminary intuition. Equation (2.17) implies a mapping from a firm's cost advantage to profitability. The expression $(\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma}$, which is proportional to profit earned by the marginal Home firm λ , is decreasing in λ as the firm's efficiency deteriorates (proved in Appendix B.2.3). By Proposition 2.3.1, $\tilde{P}$ is decreasing in λ as well. Consider an improvement in manufacturing productivity relative to productivity in services, a decrease in z_m/z_s , which generates structural change. Because of the induced intersectoral reallocation towards services, manufacturing demand rises less than one-for-one with manufacturing productivity, so $z_m M$ falls. Then, to satisfy (2.17), λ must fall. Therefore, structural change tilts the mapping from cost advantage to profitability, such that the level of competitiveness required to stay in business goes up.

Appendix B.2.3 explores the labor market clearing condition to formalize this reasoning and prove

Proposition 2.3.3. *For any initial equilibrium with $\lambda \in (\frac{1}{2}, 1)$, a decrease in z_m/z_s narrows specialization (λ falls).*

The proof in Appendix B.2.3 is conceptually intense, as it employs virtually all model's assumptions and intermediate results, but makes no explicit use of $z_m M$ which is substituted for. Going back to the zero cutoff profit condition (2.17), a fall in z_m/z_s leads to a lower value of λ and (equivalently) higher value of $\tilde{P}^\sigma (\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma}$. Therefore, the proof establishes that $z_m M$ is in fact falling in z_s/z_m , confirming our intuition that structural change makes manufacturing demand grow less than one-for-

one with manufacturing productivity.²⁵ Yet another version of (2.14), reintroducing wages W , is

$$(\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma} = \frac{f}{z_m M} \left(z_m \frac{W}{P} \right)^\sigma, \quad (2.18)$$

which highlights how the interplay between general equilibrium conditions (right hand side) and partial equilibrium conditions in the marginal industry (left hand side) determines specialization. In an economy with only a manufacturing sector, the real demand M and the real wage $\frac{W}{P}$ grow one-for-one with productivity $1/z_m$ (CES preferences ensure that specialization regions and markups stay constant). In an economy featuring structural change, slower productivity growth in services is compensated by additional employment coming from manufacturing. As a result, the demand for manufacturing goods lags behind, thus raising the required level of competitiveness needed to cover the fixed costs. In other words, the fact that services absorb an increasing expenditure and employment share leads to stronger selection among manufacturing firms. This effect is reminiscent of Melitz (2003), but here it takes the form of inter-industry reallocation,²⁶ and also the nature of the exogenous shocks is different (sector-biased productivity growth rather than trade liberalization).

Together with Proposition 2.3.2, Proposition 2.3.3 implies that, similar to the planner's allocation, structural change increases the trade share in the market economy with oligopolistic competition.

²⁵Although this makes intuitive sense from the beginning, it was necessary to show that the new features of the model (fixed costs and market structure that is endogenous to structural change) do not reverse this argument by adjustment of markups and saving resources on fixed costs as the weakest comparative advantage industries shut down.

²⁶Intra-industry reallocation cannot be addressed with a model based on BEJK, even by adding more potential producers, but for the same reasons as in the paper structural change can affect selection within industries as well. Appendix A.3 shows that firms with different efficiency levels can coexist in the same market when competition is Cournot. Another way to incorporate within-industry heterogeneity is by considering differentiated varieties. Although combining inter- and intra-industry reallocation is of a big theoretical and empirical interest, this is left for future research. The simple framework developed in this paper best serves the purpose to introduce structural change as a new source of trade growth.

Extrapolating this trend for decades ahead gives a sharp (and somewhat extreme) prediction of steadily increasing share of exporters. The less efficient manufacturing firms continue to operate and serve domestic markets as long as trade costs provide enough protection. While increasing trade flows, structural change reduces the role of trade costs relative to natural comparative advantage, so this protection becomes insufficient to cover the fixed costs. Only firms that can potentially become exporters are guaranteed to survive globalization. Those firms in the tradable sector that in principle cannot expand their sales by capturing new markets will eventually die off as structural transformation proceeds to relocate expenditure and employment to services.

2.3.3 Markups and Redistribution

The main difference between the planner and the market economy comes from the endogenous response of the market structure. Structural transformation in the model is unambiguously associated with growing monopolization. This translates into weakly increasing markups (staying constant if high trade costs do not restrict pricing of the marginal firms in domestic markets), which means that the manufacturing price index becomes weakly higher relative to its cost index analog from the planner’s problem. As long as an increase in markups (other things equal) is equivalent to a productivity drop, this comparison indicates that, in a model with oligopolistic competition, the endogenous response of the market structure damps down the effects of structural change. Nevertheless, the direction of the effect on the trade share is not reversed, so trade growth can happen simultaneously with increasing markups. This is in contrast to the pro-competitive effect of trade found in many models studying the impact of trade liberalization. An implication of this paper is that the term “pro-competitive effects of trade” may be misleading, as it is important to distinguish between different sources of trade growth.

One more result, not emphasized in this draft, concerns the redistribution effect of structural change. In general, the model has an ambiguous prediction on income redistribution between workers and entrepreneurs (if we were to extend the model and associate profits with entrepreneurial income). On the one hand, increasing markups raise profits. Moreover, it is easy to show that the fact that the range of industries, or the measure of firms, declines does not itself reduce total (net or gross) profits, because domestic firms in the industries to the right of $i = 1 - \lambda$ become exporters and get additional profits that outweigh the lost profits of the (less efficient) firms to the left of $i = \lambda$. On the other hand, there is a common decline in manufacturing profitability caused by reallocation to services. Whether workers or entrepreneurs gain more from sector-biased technological progress depends on the parameters of the model.

2.4 Conclusion

This paper shows that the interaction between comparative advantage, trade costs, and increasing returns stimulates specialization and trade as manufacturing productivity grows and manufacturing employment falls in the process of structural transformation. I establish this theoretical result by introducing fixed costs in an otherwise canonical Ricardian trade model extended to include a nontradable sector. Structural change amplifies comparative advantage forces relative to trade barriers, thus raising the manufacturing trade share for the same level of trade costs. This fact helps to explain the trade growth puzzle and, in particular, the weak relation of the extensive margin of trade to tariff reductions observed in the data.

The key feature of the model that accounts for these findings is the presence of fixed costs that are paid at the industry level, or firms are modeled as discrete, large relative to the industry. Exploring this feature produces a set of new theoret-

ical results: i) The trade share varies for a constant level of trade costs, and trade specialization can achieve its maximum level with positive or even high trade costs; ii) The trade-induced structural change amplifies the effect of tariff reduction; iii) Inter-sectoral reallocation towards nontradables induces inter-industry reallocation in tradables towards the industries with stronger comparative advantage; iv) Structural change raises the required level of competitiveness, which leads to stronger inter-industry selection and increases the average productivity in tradables; v) Structural change increases industry concentration and markups; vi) Structural change may have redistributive effects, making either workers or entrepreneurs gain more from sector-biased productivity growth.

The simulation exercise suggests that the simple framework developed in the paper can already be of some empirical interest. Structural change can produce quantitatively significant effects on trade when trade costs are nontrivial. This new explanation of trade growth complements vertical specialization which works best when trade becomes nearly frictionless (in contrast, the effect of structural change on trade vanishes when trade costs approach zero).

There remain a number of additional questions that can be studied within the framework presented. What are the model's implications for the gains from trade, how do they change in the process of structural transformation, do the new features introduced in the model require a modification of Arkolakis, Costinot, and Rodriguez-Clare (2012)? What are the effects of asymmetries between countries, which can include differences in population size, absolute advantage in particular sectors, or different stages of structural transformation? What are the redistributive effects of structural change in a model with imperfect competition and endogenous markups? What are the policy implications of the oligopoly model and how are they affected by structural change?

Meanwhile, using the model for sharper empirical investigations would require major extensions of the setup. For example, as discussed in Bernard, Jensen, Redding, and Schott (2007), intra-industry reallocation is found in the data to be more important than inter-industry reallocation. Inability to address the former and link it to structural change is an obvious limitation of this study. Nevertheless, I would emphasize the broader implications of the paper. In particular, the central idea that structural transformation induces inter-industry reallocation by raising the required level of competitiveness can be applied to intra-industry reallocation as well. This paper thus offers a mechanism that can potentially be useful in other settings in macroeconomics and international trade.

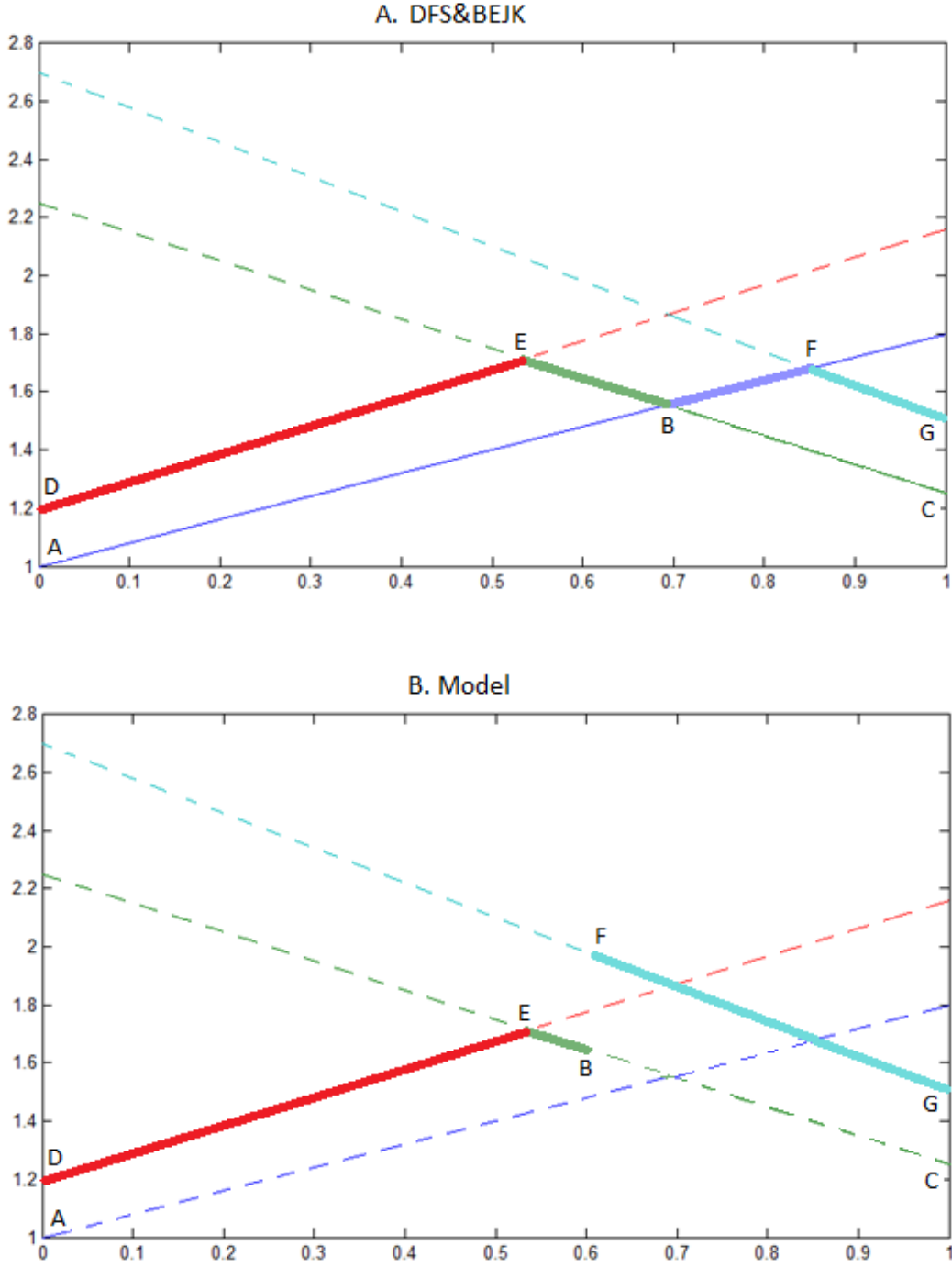


Figure 2.2: Prices by industry in Home. Parameters: $\sigma = 6$, $\tau = 1.25$, $c_i = 1 + 0.8i$. Part A: Prices in DFS are given by the line ABC in which $AB = c_i$ and $BC = \tau c_i^*$; prices in BEJK are $DEBFG$ where $DE = \frac{\sigma}{\sigma-1}c_i$, $EB = \tau c_i^*$, $BF = c_i$, and $FG = \frac{\sigma}{\sigma-1}\tau c_i^*$. Part B: Prices in the model are $DEBFG$.

Chapter 3

Optimal Trade Policy with Intermediate Goods

3.1 Introduction

The growth of international production sharing puts supply chain trade at the center of policy discussion.¹ Yet the theory of international trade is not ready to accommodate this shift, as argued in Blanchard, Bown, and Johnson (2016), henceforth BBJ. These authors offer one of the first systematic studies of the role of supply chains in trade policy. The present paper attempts to make further progress in this direction on the theory side.

I study optimal unilateral import tariffs and export taxes for a two-stage production structure based on Yi (2003, 2010). While quite specific and not entirely realistic, this model environment allows incorporating supply chain considerations in a fully structural way.² In terms of methodology, I follow Costinot, Donaldson, Vogel,

¹See, for example, Baldwin (2012).

²In the theory part of the paper, BBJ abstract from the actual input-output linkages and postulate a production function that depends directly on domestic and foreign value added. Although such specification may result from some kind of approximation, its validity for policy exercises is unclear.

and Werning (2015) who solve for optimal trade policy indirectly by formulating the problem in terms of allocations.

I consider a two-country world in which sophisticated Home maximizes domestic utility using a full set of trade taxes while Foreign is passive. Even for this simple two-stage production structure, there are rich interactions between upstream and downstream decisions. While Section 3.2 describes these interactions in some detail, the direct approach to characterizing optimal trade instruments yields little progress.

Section 3.3 then considers an alternative formulation of the policy problem in the spirit of Costinot et al. (2015). I rely on the fact that using the full set of trade taxes (both final and intermediate, import and export) provides enough flexibility to target all marginal rates of substitution in the world economy, so that the problem admits an equivalent "primal" formulation in terms of allocations. Section 3.3 delivers the main results of the paper.

For **import tariffs**, I find that a uniform rate is applied to foreign value content while domestic content is subsidized. In the model, intermediate goods only use labor in their production, so imported intermediates embody purely foreign labor and have a common tariff despite heterogeneity in comparative advantage, trade costs or trade elasticities. This finding mirrors the uniformity result for import tariffs in models without intermediate goods, provided in Opp (2010) and Costinot et al. (2015). It stems from the fact that, in a large class of neoclassical trade models, countries effectively exchange primary factors, in this case labor. The role of import tariffs is then to tilt the relative labor price. With factor mobility across industries within countries, this mechanism operates at the economy level, so in distorting the common (across industries) wages no heterogeneity in import tariffs arises. Introducing trade in intermediate inputs does not change the core of this logic because all products are eventually produced from labor, but the uniformity result is modified in two ways. First, as final products incorporate labor services from both countries, a uniform

import tariff is applied to foreign content only. Stronger international production sharing reduces the incentives to impose import protection for final goods, because some part of import spending returns to the country in the form of export revenue from intermediate goods. Although in a different framework, BBJ provide a similar result that import tariffs for final goods are decreasing in domestic value content. The theoretical result of the present paper also highlights the role of intermediate export taxes. Second, unlike in Costinot et al. (2015), this uniformity of import tariffs is no longer robust to excluding export taxes from the set of policy variables.

The determination of **export taxes** is related to the problem of a multiproduct monopolist. One major distinction for intermediate export taxes is that the Home government must also take into account productivity losses in Foreign due to higher material costs which eventually hurt Home consumers. Because of this complication, there is no elegant analytical expression for intermediate export taxes. For final export taxes, however, the optimality condition takes a particularly simple form: the standard monopoly markup is discounted by the level of the corresponding final import tariff. Heterogeneity in final export taxes is explained by variation in market power and the strength of production sharing captured by final import tariffs. While the first factor is very intuitive and is already present in Costinot et al. (2015), the second one is specific to supply chain trade. Higher export taxes induce consumer expenditure switching in Foreign, which increases Foreign final production and hence demand for intermediate goods, including import demand. The latter is beneficial for Home as it increases Foreign demand for Home labor and also export tax revenue from intermediate goods. The strength of this channel varies with the share of Home inputs in Foreign final production and the level of intermediate export taxes in a way that the level of final import tariffs is the single summary statistic. This result challenges the view that international production sharing should result in a more liberal trade policy. As a thought exercise, consider a decrease in intermediate trade

costs that reduces intermediate domestic shares as countries source more inputs from their trade partners. The lower final import tariffs may then be offset by higher final export taxes, so that the impact on the overall trade flows is ambiguous.

The rest of the paper is structured as follows. Section 3.2 of the paper introduces the environment and discusses the role of final and intermediate trade taxes. Section 3.3 applies the primal approach to the government's problem and establishes the main analytical results. Section 3.3 also outlines several comparative statics results from simulation exercises. Section 3.4 concludes.

3.2 Direct Problem

This section introduces the environment and discusses the direct approach to solving for optimal policy.

3.2.1 Preferences and Technology

The world economy consists of two countries, Home and Foreign. Labor is the only primary factor of production. It is supplied inelastically and its endowments are denoted with L and L^* (the Foreign variables have asterisk notation). Different products are indexed by $i = 1..N$. Each good is produced in two stages: stage one uses labor only while stage two uses labor and stage one output. Throughout the paper, "stage one" is used interchangeably with "intermediate" or "upstream" and "stage two" with "final" or "downstream". Only final products are consumed and preferences, similar in both countries, are given by the log function

$$U = \sum_{i=1}^N \alpha_i \log C_i, \tag{3.1}$$

with expenditure shares $\alpha_i > 0$, $\sum_i \alpha_i = 1$.

The technology for intermediate goods is linear in labor inputs l_i^I with unit requirements Z_i^I :

$$q_i^I = \frac{1}{Z_i^I} l_i^I. \quad (3.2)$$

The technology for final goods is Cobb-Douglas constant returns to scale in labor inputs l_i^F and intermediate inputs X_i with inverse productivity Z_i^F :

$$q_i^F = \frac{1}{Z_i^F} \left(\frac{l_i^F}{1 - \beta_i} \right)^{1 - \beta_i} \left(\frac{X_i}{\beta_i} \right)^{\beta_i}, \quad (3.3)$$

where $0 \leq \beta_i < 1$ is the intermediate input share. It is the same for both countries but can vary across industries.

For international trade, the standard iceberg trade cost specification is adopted: τ_i^I (τ_i^F) units of an intermediate (final) product must be shipped from one country in order for one unit to arrive in the other country. Trade costs are assumed to be symmetric for expositional simplicity, although this assumption does not affect the results. Domestic trade costs are absent.

Our description of the environment ends with the Armington product differentiation assumption. First, each final good i enters consumption (3.1) as a CES bundle C_i which aggregates the corresponding domestic and imported varieties c_{id} and c_{im} with the elasticity of substitution $\rho_i^F > 1$:

$$C_i = \left(c_{id}^{\frac{\rho_i^F - 1}{\rho_i^F}} + c_{im}^{\frac{\rho_i^F - 1}{\rho_i^F}} \right)^{\frac{\rho_i^F}{\rho_i^F - 1}}. \quad (3.4)$$

Second, each intermediate good i enters production (3.3) as a CES bundle X_i which aggregates the corresponding domestic and imported varieties x_{id} and x_{im} with the elasticity of substitution $\rho_i^I > 1$:

$$X_i = \left(x_{id}^{\frac{\rho_i^I - 1}{\rho_i^I}} + x_{im}^{\frac{\rho_i^I - 1}{\rho_i^I}} \right)^{\frac{\rho_i^I}{\rho_i^I - 1}}. \quad (3.5)$$

The parameters ρ_i^F and ρ_i^I are the same in Home and Foreign but can vary across industries. Introducing the Armington love for variety is a departure from the original two-stage specification in Yi (2003, 2010) which only features Ricardian technological heterogeneity. This is primarily driven by tractability concerns³ but is also justified on empirical grounds as strong intra-industry trade is present for both final and intermediate goods.

3.2.2 Home Economy

Consumers and firms in both countries behave competitively, so all prices of domestically traded goods are equal to marginal costs. There is a government in Home that sets trade taxes and redistributes the revenue to Home consumers. I first characterize consumption and production decisions conditional on policy instruments. Import taxes for final and intermediate products are introduced as coefficients and denoted with tm_i^F and tm_i^I . Thus, in case of final goods, Home consumers face a price that is $(tm_i^F - 1) \cdot 100\%$ higher than $\tau_i^F MC_i^{F*}$ which is Foreign marginal costs multiplied by the iceberg trade costs. Export taxes (also coefficients) are denoted tx_i^F and tx_i^I . For final goods, Home firms receive a fraction $\frac{1}{tx_i^F}$ of a price paid by Foreign consumers. In other words, with competitive pricing Home firms charge Foreign consumers $tx_i^F \tau_i^F MC_i^F$, or export prices are $(tx_i^F - 1) \cdot 100\%$ higher than $\tau_i^F MC_i^F$.

Marginal costs

With W denoting the Home wage and according to (3.2) and (3.3), stage-one marginal costs are

³A finite Armington elasticity generates smooth reactions of trade flows to changes in relative costs. There is always positive production and two-way trade for all products and the problem is well-behaved even with a finite number of industries.

$$MC_i^I = Z_i^I W \quad (3.6)$$

and stage-two marginal costs are

$$MC_i^F = Z_i^F W^{1-\beta_i} (P_{Mi})^{\beta_i}, \quad (3.7)$$

where P_{Mi} is the price index for material input in (3.3). With the CES specification (3.5), it is equal to

$$P_{Mi} = \left((MC_i^I)^{1-\rho_i^I} + (tm_i^I \tau_i^I MC_i^{I*})^{1-\rho_i^I} \right)^{\frac{1}{1-\rho_i^I}}. \quad (3.8)$$

Note that stage-two Home firms import intermediates paying $tm_i^I \tau_i^I$ times Foreign marginal costs, so upstream import tariffs (just as trade costs) make downstream Home firms less productive.

Consumer demand

The nominal income of Home consumers consists of labor income WL and tax revenue R which is specified later. Maximizing (3.1), the amount $\alpha_i (WL + R)$ is spent on final product i . Under the Armington preferences (3.4) we find that a fraction λ_i^F of this expenditure is allocated to the domestic variety and $1 - \lambda_i^F$ to the imported variety, with

$$\lambda_i^F = \frac{(MC_i^F)^{1-\rho_i^F}}{(MC_i^F)^{1-\rho_i^F} + (tm_i^F \tau_i^F MC_i^{F*})^{1-\rho_i^F}}. \quad (3.9)$$

The quantities demanded by Home consumers are then

$$c_{id} = \frac{\alpha_i \lambda_i^F (WL + R)}{MC_i^F} \text{ and } c_{im} = \frac{\alpha_i (1 - \lambda_i^F) (WL + R)}{tm_i^F \tau_i^F MC_i^{F*}}. \quad (3.10)$$

Firm demand

The final production of industry i in Home satisfies domestic demand c_{id} and Foreign import demand c_{im}^* adjusted by trade costs τ_i^F . Therefore, the revenue of final producers in Home, denoted y_i^F , is

$$y_i^F = MC_i^F \cdot (c_{id} + \tau_i^F c_{im}^*). \quad (3.11)$$

According to (3.3), final producers in industry i spend $(1 - \beta_i) y_i^F$ on labor, so they employ

$$l_i^F = \frac{(1 - \beta_i) y_i^F}{W} \text{ workers.} \quad (3.12)$$

Also according to (3.3), the amount $\beta_i y_i^F$ is spent on intermediate goods and there is again a split of this expenditure between domestic and imported varieties in (3.4) with the domestic share given by

$$\lambda_i^I = \frac{(MC_i^I)^{1-\rho_i^I}}{(MC_i^I)^{1-\rho_i^I} + (tm_i^I \tau_i^I MC_i^{I*})^{1-\rho_i^I}}. \quad (3.13)$$

Therefore, the quantities demanded by intermediate producers are

$$x_{id} = \frac{\lambda_i^I \beta_i y_i^F}{MC_i^I} \text{ and } x_{im} = \frac{(1 - \lambda_i^I) \beta_i y_i^F}{tm_i^I \tau_i^I MC_i^{I*}}. \quad (3.14)$$

Next, the revenue of intermediate producers, y_i^I , comes from serving domestic demand x_{id} and Foreign import demand x_{im}^* adjusted by trade costs τ_i^I :

$$y_i^I = MC_i^I \cdot (x_{id} + \tau_i^I x_{im}^*). \quad (3.15)$$

This amount is entirely spent on labor, so intermediate producers in industry i employ

$$l_i^I = \frac{y_i^I}{W} \text{ workers.} \quad (3.16)$$

Tax revenue

As Home consumers spend $(1 - \lambda_i^F) \alpha_i (WL + R)$ on importing final product i , Foreign firms get a fraction $\frac{1}{tm_i^F}$ of that amount. That means that the rest, a fraction $\frac{tm_i^F - 1}{tm_i^F}$, is received by the Home government. Similarly, Foreign consumers purchase the value $(1 - \lambda_i^{F*}) \alpha_i W^* L^*$ of Home goods, and it is split between Home firms and the government with the shares $\frac{1}{tx_i^F}$ and $\frac{tx_i^F - 1}{tx_i^F}$. The tax revenue arising from intermediate trade is analogous, with consumer spending replaced by firm expenditure on materials $\beta_i y_i^F$ and $\beta_i y_i^{F*}$ (only downstream producers use intermediates). Combining the pieces, the total tax revenue is

$$\begin{aligned}
 R = & \sum_i \frac{tm_i^F - 1}{tm_i^F} (1 - \lambda_i^F) \alpha_i (WL + R) + \sum_i \frac{tm_i^I - 1}{tm_i^I} (1 - \lambda_i^I) \beta_i y_i^F + \quad (3.17) \\
 & + \sum_i \frac{tx_i^F - 1}{tx_i^F} (1 - \lambda_i^{F*}) \alpha_i W^* L^* + \sum_i \frac{tx_i^I - 1}{tx_i^I} (1 - \lambda_i^{I*}) \beta_i y_i^{F*}.
 \end{aligned}$$

Market clearing

Note that goods market clearing is already incorporated in (3.11) and (3.15). The condition for labor market clearing is

$$\sum_i (l_i^I + l_i^F) = L. \quad (3.18)$$

Note also that trade balance is implied by market clearing and the fact that Home consumers balance their budget spending $WL + R$.

3.2.3 Foreign Economy

All equilibrium relationships for Foreign are exactly similar. For completeness, I provide them in Part I of Appendix C.1 without detailed explanation.

3.2.4 Trade Policy

Domestic shares

Before formulating the policy problem, it is convenient to minimize the involved notation. In this section, the policy problem has $4 \times N + 2$ variables which are trade taxes $(tm_i^F, tm_i^I, tx_i^F, tx_i^I)_{i=1}^N$ and wages W and W^* , as the planner takes into account the general equilibrium effects on wages. (In fact, as discussed below, both wages can be normalized as long as the complete set of export and import taxes is available.) All production and consumption decisions can be expressed as explicit functions of these variables, so in this formulation the problem only has two constraints which are Home and Foreign labor market clearing conditions. The problem's presentation simplifies significantly once we recognize that both consumer welfare and firm labor demand can be expressed through the domestic shares together with wages and taxes.

Table 3.1 lists all domestic shares as explicit functions of trade taxes and wages with pluses and minuses indicating the signs of the corresponding partial derivatives. It simply summarizes the equations (3.6)-(3.9) and (3.13) for Home as well as (C.1)-(C.4) and (C.8) for Foreign.

Table 3.1: Domestic shares as functions of wages and policy instruments

Variable	Notation
Intermediate domestic share in Home	$\lambda_i^I \left(\begin{smallmatrix} tm_i^I, & W, & W^* \\ + & - & + \end{smallmatrix} \right)$
Intermediate domestic share in Foreign	$\lambda_i^{I*} \left(\begin{smallmatrix} tx_i^I, & W, & W^* \\ + & + & - \end{smallmatrix} \right)$
Final domestic share in Home	$\lambda_i^F \left(\begin{smallmatrix} tm_i^F, & W, & W^*, & tm_i^I, & tx_i^I \\ + & - & + & - & + \end{smallmatrix} \right)$
Final domestic share in Foreign	$\lambda_i^{F*} \left(\begin{smallmatrix} tx_i^F, & W, & W^*, & tm_i^I, & tx_i^I \\ + & + & - & + & - \end{smallmatrix} \right)$

Naturally, intermediate import tariffs tm_i^I reduce Home import of materials, so λ_i^I is increasing in tm_i^I . The impact of intermediate tariffs also propagates downstream. As the material price index (3.8) can be expressed as $P_{Mi} = Z_i^I W(\lambda_i^I)^{\frac{1}{\rho_i^I - 1}}$ and therefore Home marginal costs increase in tm_i^I , Home loses competitiveness in final goods, so consumers switch to Foreign products: λ_i^F is decreasing and λ_i^{F*} is increasing in tm_i^I .

The direct impact of intermediate export taxes tx_i^I is to reduce Foreign imports in downstream: λ_i^{I*} is increasing. In addition to this, as a consequence of higher material costs, Foreign final firms become less competitive, so λ_i^F is increasing and λ_i^{F*} is decreasing in tx_i^I .

The effect of final trade taxes tm_i^F and tx_i^F is simply to reduce final imports in Home and Foreign respectively: λ_i^F is increasing in tm_i^F and λ_i^{F*} is increasing in tx_i^F .

Finally, higher labor prices reduce competitiveness in both upstream and downstream, so all λ 's are decreasing in domestic wages and increasing in trade partners' wages.

Objective function

As the Home government sets trade taxes to maximize consumer utility, we first combine some of the above relationships to solve for equilibrium welfare. The log price index for utility (2.1) equals (up to a constant depending on α 's)

$$\log P = \sum_i \alpha_i \log P_i, \quad (3.19)$$

with the industry-level price indices consistent with the Armington aggregator (3.4)

$$P_i = \left((MC_i^F)^{1-\rho_i^F} + (tm_i^F \tau_i^F MC_i^{F*})^{1-\rho_i^F} \right)^{\frac{1}{1-\rho_i^F}}. \quad (3.20)$$

The objective function for the government can therefore be written as

$$U = \log(WL + R) - \sum_i \alpha_i \log P_i. \quad (3.21)$$

Connecting to the domestic shares, we can substitute $P_{Mi} = Z_i^I W (\lambda_i^I)^{\frac{1}{\rho_i^I - 1}}$ into downstream marginal costs (3.7) and then use $P_i = MC_i^F \cdot (\lambda_i^F)^{\frac{1}{\rho_i^F - 1}}$ to write (3.20) as

$$P_i = W \cdot Z_i^F (Z_i^I)^{\beta_i} \cdot (\lambda_i^I)^{\frac{\beta_i}{\rho_i^I - 1}} \cdot (\lambda_i^F)^{\frac{1}{\rho_i^F - 1}}. \quad (3.22)$$

From this expression we can see the partial equilibrium effect of trade taxes on consumer prices. Import tariffs for both final and intermediate goods increase the price index (3.22). Note that the impact of upstream import tariffs is mitigated by consumer switching to Foreign final goods ($tm_i^I \uparrow \rightarrow \lambda_i^I \uparrow, \lambda_i^F \downarrow$), but this is not enough to overcome the direct effect of higher material costs.⁴ In addition to this, intermediate export taxes also increase P_i by raising final Foreign costs ($tx_i^I \uparrow \rightarrow MC_i^{F*} \uparrow \rightarrow \lambda_i^F \uparrow$).

Constraints

First, we obtain firm revenue expressed in terms of domestic shares. Combining (3.11) and (3.10) as well as the corresponding equations for Foreign, the revenue of Home final firms is

$$y_i^F = \lambda_i^F \alpha_i (WL + R) + \frac{1}{tx_i^F} (1 - \lambda_i^{F*}) \alpha_i W^* L^*, \quad (3.23)$$

and for Foreign final producers it is

$$y_i^{F*} = \lambda_i^{F*} \alpha_i W^* L^* + \frac{1}{tm_i^F} (1 - \lambda_i^F) \alpha_i (WL + R). \quad (3.24)$$

⁴Formally, tm_i^I enters downstream domestic shares by shifting final Home marginal costs by the factor $(\lambda_i^I)^{\frac{\beta_i}{\rho_i^I - 1}}$. Denoting this expression ξ , the log of P_i in (3.22) is equal up to a constant to $\log \xi - \frac{1}{\rho_i^F - 1} \log \left(1 + (\gamma \xi)^{\rho_i^F - 1} \right)$, where γ is another constant. The derivative of this expression with respect to ξ is positive. In economic terms, this simply means that the price of the Armington bundle increases with the price of domestic variety as long as some amount of it is still consumed.

For intermediate producers, combining (3.15) and (3.14) yields

$$y_i^I = \lambda_i^I \beta_i y_i^F + \frac{1}{tx_i^I} (1 - \lambda_i^{I*}) \beta_i y_i^{F*} \text{ and} \quad (3.25)$$

and for Foreign final producers we have

$$y_i^{I*} = \lambda_i^{I*} \beta_i y_i^{F*} + \frac{1}{tm_i^I} (1 - \lambda_i^I) \beta_i y_i^F. \quad (3.26)$$

We can now combine these four expressions with labor demands (3.12) and (3.16) to write the labor market clearing conditions as

$$\sum_i (y_i^I + (1 - \beta_i) y_i^F) = WL \text{ and } \sum_i (y_i^{I*} + (1 - \beta_i) y_i^{F*}) = W^* L^*. \quad (3.27)$$

As for the tax revenue that enters both the objective function (3.21) and the above labor market clearing conditions, it is also expressed through the domestic shares as we already see in (3.17). To have an explicit expression, we just solve this equation which is linear in R .⁵

Without taking into account the tax revenue effect,⁶ final import tariffs increase firm revenue (and hence labor demand) in Home, decrease downstream firm revenue in Foreign and increase upstream firm revenue in Foreign. As seen from (3.23)-(3.26), $tm_i^F \uparrow \rightarrow \lambda_i^F \uparrow \rightarrow y_i^F \uparrow$, $y_i^{F*} \downarrow \rightarrow y_i^I \uparrow$, $y_i^{I*} \uparrow$. (Increasing Home final production raises Home intermediate demand for both Home-produced and Foreign-produced inputs.) In the case of intermediate import tariffs, due to increasing material costs in Home we have $tm_i^I \uparrow \rightarrow \lambda_i^F \downarrow$, $\lambda_i^{F*} \uparrow \rightarrow y_i^F \downarrow$, $y_i^{F*} \uparrow$. The impact on intermediate and total labor demand is ambiguous, however. On the one hand, tm_i^I increases λ_i^I and hence y_i^I for

⁵As an explicit function of trade taxes and wages, R looks the same as the right hand side of (3.17) without R itself adjusted by some multiplier. This is due to the income-expenditure loop as tax revenue returned to households generates new expenditure and new tax revenue.

⁶A way to think about it is to consider a large number of industries so that each one has a small share in the total tax revenue.

a given value of downstream production. But strong enough consumer switching in final goods may cause Home final output to fall so much that upstream demand for labor falls as well.⁷

Final export taxes increase labor demand in Foreign, reduce downstream labor demand in Home and increase upstream labor demand in Home: $tx_i^F \uparrow \rightarrow \lambda_i^{F*} \uparrow \rightarrow y_i^F \downarrow, y_i^{F*} \uparrow \rightarrow y_i^I \uparrow, y_i^{I*} \uparrow$. Intermediate export taxes increase Foreign production of intermediates for a given level of final demand. But, again, consumer switching due to higher final costs in Foreign decreases final labor demand in Foreign and may as well reduce its total labor demand.

Normalization

As usual, one nominal variable can be normalized by choosing the numeraire and we put $W^* = 1$. However, there is an additional degree of freedom because once a given W and $(tm_i^F, tm_i^I, tx_i^F, tx_i^I)_{i=1}^N$ satisfy (3.27), other combinations of these variables can be constructed that also satisfy (3.27) such that the value of (3.21) stays the same. This is because the actual terms of trade vary with the relative wage adjusted by trade taxes. Let us double the Home wage and import tariffs and simultaneously reduce export taxes in half. As a result, none of the domestic shares changes because they depend on relative prices of domestic vs. imported varieties. For intermediate goods, for example, MC_i^I moves proportionally with W , MC_i^{I*} is unaffected, and so the ratio $\frac{tm_i^I \tau_i^I MC_i^{I*}}{MC_i^I}$ remains constant, giving the same λ_i^I . For Foreign, λ_i^{I*} is also unaffected as the ratio $\frac{MC_i^{I*}}{tx_i^I \tau_i^I MC_i^I}$ stays constant (higher MC_i^I is offset by lower tx_i^I). Once the domestic shares are not affected, it is straightforward to see that labor demands and real income remain constant as well. For this reason we put $W = 1$ as an additional normalization. Note that we can only do so provided that the government is equipped with the complete set of trade taxes.

Policy problem

⁷This final expenditure switching effect is particularly strong if consumers view domestic and imported varieties as close substitutes, ρ_i^F is high.

With the chosen normalization $W = 1$ and $W^* = 1$, the Home government sets trade taxes $(tm_i^F, tm_i^I, tx_i^F, tx_i^I)_{i=1}^N$ to maximize equilibrium consumer utility (3.21) subject to the two labor market clearing conditions (3.27).⁸

The task of understanding the nature of optimal trade taxes and their variation across industries is complicated for two reasons. One is the presence of general equilibrium interactions as trade taxes in particular industries affect the economy-level labor constraints, both by changing the domestic shares and affecting the tax revenue. The other reason, specific to the present paper, is the upstream-downstream interactions in this economy. As discussed above, the effects of intermediate taxes propagate forward by the cost mechanism, and the effects of final taxes propagate backward by affecting intermediate demand.

We may try to avoid complications arising because of the tax revenue by changing the set of policy variables. The government may treat R as an independent policy variable, together with $(tm_i^F, tm_i^I, tx_i^F, tx_i^I)_{i=1}^N$, and maximize (3.21) subject to the same constraints (3.27). No additional constraints are needed because, by Walras law-type intuition, labor market clearing (3.27) implies (3.17) which is essentially the government budget balance.⁹ Having more policy variables is potentially helpful for obtaining partial equilibrium intuition because we are able to read optimality relationships conditional on certain (in particular, general equilibrium) variables. But in the presence of forward and backward linkages this extension of policy variables does not result in a major simplification.

This technical complication motivates using the primal approach in the next section. We would like to condition optimality equations not only on general equilibrium variables but also, when possible, on upstream and downstream ones. In this case,

⁸By now we have demonstrated that all endogenous variables that appear in (2.7) and (3.27) are explicit functions of trade taxes, so there are only two constraints in the policy problem in this section.

⁹The argument, of course, relies on the fact that firm revenues in (3.27) incorporate goods market clearing and consumer budget balance.

without explaining the level of trade taxes, we may be able to understand their variation across industries and also the role of supply chains.

3.3 Indirect Approach

Inspired by Costinot et al. (2015), this section explores an alternative approach to the government's problem which is based on solving for optimal allocations rather than optimal values of policy instruments. We rely on the fact that using the full set of trade taxes $(tm_i^F, tm_i^I, tx_i^F, tx_i^I)_{i=1}^N$ provides enough flexibility to target all marginal rates of substitution in the economy, so that the policy problem admits an equivalent "primal" formulation in terms of allocations. More precisely, the Home government directly controls domestic allocations and sets export prices while taking into account Foreign equilibrium relationships.¹⁰ For both intermediate and final goods, we will be able to determine the relative use of domestic vs. imported varieties in the planner's allocation and relate it to the relative marginal costs in a decentralized equilibrium. For final goods, for example, the equilibrium condition

$$\left(\frac{c_{id}}{c_{im}}\right)^{-1/\rho_i^F} = \left(\frac{\lambda_i^F}{1 - \lambda_i^F}\right)^{\frac{1}{1-\rho_i^F}} = \frac{1}{tm_i^F} \frac{MC_i^F}{MC_i^{F*} \tau_i^F} \quad (3.28)$$

allows to infer tariffs tm_i^F from the knowledge of $\frac{c_{id}}{c_{im}}$ and $\frac{MC_i^F}{MC_i^{F*} \tau_i^F}$. Note that our focus is still on the government's problem of setting optimal trade taxes, so we only use the primal formulation to get key intermediate results and then return to the decentralized formulation of Section 3.2.¹¹

¹⁰Perhaps a better term for this formulation would be "semiprimal approach". In principle, the problem can be set in terms of allocations only, so that export prices are also recovered through wedges rather than solved for directly. I prefer the mixed setup of this section as it involves less algebra.

¹¹In particular, alternative implementations of the optimal allocation (such as involving production and consumption taxes) are not discussed.

We adopt a normalization by setting the Foreign wage $W^* = 1$. When connecting to a decentralized equilibrium, the Home wage is also normalized as $W = 1$. The Home government directly controls production and consumption decisions in Home, setting the variables $(c_{id}, c_{im}, x_{id}, x_{im}, X_i, l_i^I, l_i^F)$. In addition to this, it sets export prices at which Foreign firms and consumers purchase Home-produced goods. These prices, for intermediate and final products, are denoted Px_i^I and Px_i^F . The planner's problem has therefore $9 \times N$ controls, while the Foreign variables that appear in the constraints are functions of these controls.

Foreign equilibrium

Part II of Appendix Appendix C.1 derives the relevant Foreign variables as functions of the controls. First, the final and intermediate domestic shares are

$$\lambda_i^{I*}(Px_i^I) = \frac{(Z_i^{I*})^{1-\rho_i^I}}{(Z_i^{I*})^{1-\rho_i^I} + (Px_i^I)^{1-\rho_i^I}} \text{ and} \quad (3.29)$$

$$\lambda_i^{F*}(Px_i^I, Px_i^F) = \frac{\left(Z_i^{F*} (Z_i^{I*})^{\beta_i} (\lambda_i^{I*})^{\frac{\beta_i}{\rho_i^I-1}} \right)^{1-\rho_i^F}}{\left(Z_i^{F*} (Z_i^{I*})^{\beta_i} (\lambda_i^{I*})^{\frac{\beta_i}{\rho_i^I-1}} \right)^{1-\rho_i^F} + (Px_i^F)^{1-\rho_i^F}}. \quad (3.30)$$

The sales of final and intermediate producers are

$$y_i^{F*}(Px_i^I, Px_i^F, c_{im}) = \alpha_i \lambda_i^{F*} L^* + \underbrace{Z_i^{F*} (Z_i^{I*})^{\beta_i} (\lambda_i^{I*})^{\frac{\beta_i}{\rho_i^I-1}}}_{MC_i^{F*}} \tau_i^F c_{im} \text{ and} \quad (3.31)$$

$$y_i^{I*}(Px_i^I, Px_i^F, c_{im}, x_{im}) = \lambda_i^{I*} \beta_i y_i^{F*} + Z_i^{I*} \tau_i^I x_{im}. \quad (3.32)$$

The import quantities demanded by consumers and intermediate producers are

$$c_{im}^* (Px_i^I, Px_i^F) = \frac{\alpha_i (1 - \lambda_i^{F*}) L^*}{Px_i^F} \text{ and } x_{im}^* (Px_i^I, Px_i^F, c_{im}) = \frac{(1 - \lambda_i^{I*}) \beta_i y_i^{F*}}{Px_i^I}. \quad (3.33)$$

Finally, employment is equal or proportional to firm sales:

$$l_i^{I*} (Px_i^I, Px_i^F, c_{im}, x_{im}) = y_i^{I*} \text{ and } l_i^{F*} (Px_i^I, Px_i^F, c_{im}) = (1 - \beta_i) y_i^{F*}. \quad (3.34)$$

Home policy problem

The Home government maximizes domestic utility subject to technological and resource constraints:

$$\max_{c_{id}, c_{im}, x_{id}, x_{im}, X_i, l_i^I, l_i^F, Px_i^I, Px_i^F} U = \sum_i \alpha_i \log \left(c_{id}^{1-1/\rho_i^F} + c_{im}^{1-1/\rho_i^F} \right)^{\frac{\rho_i^F}{1-\rho_i^F}} \text{ subject to } \quad (3.35)$$

$$\text{Final Home output: } c_{id} + \tau_i^F c_{im}^* (Px_i^I, Px_i^F) = \frac{1}{Z_i^F} \left(\frac{l_i^F}{1 - \beta_i} \right)^{1-\beta_i} \left(\frac{X_i}{\beta_i} \right)^{\beta_i} \quad |\mu_i^F \quad (3.36)$$

$$\text{Materials Armington: } X_i = \left(x_{id}^{1-1/\rho_i^I} + x_{im}^{1-1/\rho_i^I} \right)^{\frac{\rho_i^I}{1-\rho_i^I}} \quad |\chi_i \quad (3.37)$$

$$\text{Intermediate Home output: } x_{id} + \tau_i^I x_{im}^* (Px_i^I, Px_i^F, c_{im}) = \frac{1}{Z_i^I} l_i^I \quad |\mu_i^I \quad (3.38)$$

$$\text{Home labor constraint: } \sum_i (l_i^I + l_i^F) = L \quad |\bar{\mu} \quad (3.39)$$

Foreign labor constraint:
$$\sum_i [l_i^{I*} (Px_i^I, Px_i^F, c_{im}, x_{im}) + l_i^{F*} (Px_i^I, Px_i^F, c_{im})] = L^* |\bar{\mu}^*| \quad (3.40)$$

For simplicity, I introduce all constraints as equalities.¹² Note also that we keep the aggregate materials input X_i as a separate control and not put x_{id} and x_{im} directly into (3.36). This is slightly more convenient in terms of exposition.

After forming the Lagrangian as

$$\begin{aligned} \Lambda = & \sum_i \alpha_i \log \left(c_{id}^{1-1/\rho_i^F} + c_{im}^{1-1/\rho_i^F} \right)^{\frac{\rho_i^F}{1-\rho_i^F}} + \sum_i \chi_i \left[\left(x_{id}^{1-1/\rho_i^I} + x_{im}^{1-1/\rho_i^I} \right)^{\frac{\rho_i^I}{1-\rho_i^I}} - X_i \right] \\ & + \sum_i \mu_i^F \left[\frac{1}{Z_i^F} \left(\frac{l_i^F}{1-\beta_i} \right)^{1-\beta_i} \left(\frac{X_i}{\beta_i} \right)^{\beta_i} - c_{id} - \tau_i^F c_{im}^* \right] + \sum_i \mu_i^I \left[\frac{1}{Z_i^I} l_i^I - x_{id} - \tau_i^I x_{im}^* \right] \\ & + \bar{\mu} \left[L - \sum_i (l_i^I + l_i^F) \right] + \bar{\mu}^* \left[L^* - \sum_i (l_i^{I*} + l_i^{F*}) \right] \end{aligned} \quad (3.41)$$

we are ready to characterize optimal policy.

3.3.1 Import tariffs

Intermediate goods

By differentiating (3.41) with respect to domestic and imported inputs x_{id} and x_{im} and taking the ratio, we arrive at

$$\left(\frac{x_{id}}{x_{im}} \right)^{-1/\rho_i^I} = \frac{\mu_i^I}{\bar{\mu}^* Z_i^{I*} \tau_i^I} = \frac{\bar{\mu}}{\bar{\mu}^*} \frac{Z_i^I}{Z_i^{I*} \tau_i^I}. \quad (3.42)$$

The relative quantity of intermediate domestic variety is inversely related to the relative labor costs $\frac{Z_i^I}{Z_i^{I*} \tau_i^I}$ adjusted by a common (across industries) factor $\frac{\bar{\mu}}{\bar{\mu}^*}$ which is the relative shadow price of labor. This expression is particularly simple because only labor is used to produce intermediate goods, so the relative costs of providing

¹²In principle, the Home planner might not use all Home labor or produce more goods than used in consumption or production, but that would never be optimal.

x_{id} and x_{im} are directly linked to exogenous labor requirements. From (3.42) we immediately establish the pattern of variation in intermediate import tariffs. As $\left(\frac{x_{id}}{x_{im}}\right)^{-1/\rho_i^I} = \left(\frac{\lambda_i^I}{1-\lambda_i^I}\right)^{\frac{1}{1-\rho_i^I}} = \frac{MC_i^I}{tm_i^I \tau_i^I MC_i^{I*}}$ in a market equilibrium and the upstream marginal costs are simply unit labor requirements,¹³ we conclude that

$$\frac{\bar{\mu}}{\bar{\mu}^*} \frac{Z_i^I}{Z_i^{I*} \tau_i^I} = \frac{Z_i^I}{tm_i^I Z_i^{I*} \tau_i^I} \Rightarrow tm_i^I = \frac{\bar{\mu}^*}{\bar{\mu}} \text{ for all } i = 1..N. \quad (3.43)$$

While this condition is not informative of the *level* of import tariffs, it gives a sharp result about the *variation* across industries. Namely, import tariffs for intermediate goods are uniform despite the presence of heterogeneity in labor costs, trade costs, or elasticities of substitution. Thus we have proven the following

Proposition 3.3.1. *Optimal import tariffs for intermediate goods are uniform: $tm_i^I = tm^I$ for all $i = 1..N$.*

This result is similar to Costinot et al. (2015). At the heart of it is the fact that countries effectively exchange labor while trading goods and the role of import tariffs is to tilt the relative price of labor. With factor mobility across industries within countries, this mechanism operates at the economy level, so in distorting the common (across industries) wages no heterogeneity in import tariffs arises. Introducing trade in intermediate inputs does not change the core of this logic because all products are eventually produced from labor. However, as we will see below, the uniformity result generalizes so that a common rate is applied to the Foreign labor content rather than the entire product value. From now on we suppress the industry subscript in tm_i^I .

Final goods

We proceed by differentiating (3.41) with respect to domestic and imported final products c_{id} and c_{im} . This is more complicated compared to intermediate goods because Home import c_{im} affects not only downstream Foreign employment l_i^{F*} but

¹³Now we also apply our normalization $W^* = W = 1$.

also its upstream employment l_i^{I*} , and it also enters the intermediate Home output constraint (3.38). The latter is because Home import demand for final goods affects Foreign intermediate import demand. While the first order condition for c_{id} is simply $\frac{\partial U}{\partial c_{id}} = \mu_i^F$, for c_{im} we have

$$\frac{\partial U}{\partial c_{im}} = \bar{\mu}^* \left[\frac{\partial l_i^{F*}}{\partial c_{im}} + \frac{\partial l_i^{I*}}{\partial c_{im}} \right] + \mu_i^I \tau_i^I \frac{\partial x_{im}^*}{\partial c_{im}}. \quad (3.44)$$

This difference is due to the fact that the Home government must explicitly take into account the Foreign supply chain structure, namely upstream propagation of demand. In contrast, maximization with respect to domestic final consumption c_{id} is conditional on the use of intermediate inputs in Home. Of course, the choices c_{im} and intermediate inputs in Home must still be consistent as the constraint (3.36) requires, but this linkage is accounted for by the Lagrange multiplier μ_i^F . From (3.31)-(3.32) and (3.34), $\frac{\partial l_i^{F*}}{\partial c_{im}} = (1 - \beta_i) MC_i^{F*} \tau_i^F$ and $\frac{\partial l_i^{I*}}{\partial c_{im}} = \lambda_i^{I*} \beta_i MC_i^{F*} \tau_i^F$. Also, $\frac{\partial x_{im}^*}{\partial c_{im}} = \frac{(1 - \lambda_i^{I*}) \beta_i}{Px_i^I} \frac{\partial y_i^{F*}}{\partial c_{im}} = \frac{(1 - \lambda_i^{I*}) \beta_i}{Px_i^I} MC_i^{F*} \tau_i^F$. Combining the pieces, (3.44) becomes

$$\frac{\partial U}{\partial c_{im}} = \bar{\mu}^* MC_i^{F*} \tau_i^F \left[1 - \beta_i (1 - \lambda_i^{I*}) \left(1 - \frac{\mu_i^I \tau_i^I}{\bar{\mu}^* Px_i^I} \right) \right]. \quad (3.45)$$

We further observe that $\mu_i^I = \bar{\mu} Z_i^I$ and use the connection between the direct and indirect formulations of the government's problem.¹⁴ The ratio $\frac{\mu_i^I \tau_i^I}{\bar{\mu}^* Px_i^I}$ in (3.45) can be expressed as $\frac{\bar{\mu}}{\bar{\mu}^*} \frac{Z_i^I \tau_i^I}{Px_i^I} = \frac{1}{tm^I tx_i^I}$, after which the ratio $\frac{\partial U}{\partial c_{id}} / \frac{\partial U}{\partial c_{im}}$ is

$$\left(\frac{c_{id}}{c_{im}} \right)^{-1/\rho_i^F} = \left(\frac{\lambda_i^F}{1 - \lambda_i^F} \right)^{\frac{1}{1 - \rho_i^F}} = \frac{1}{tm^I - \beta_i (1 - \lambda_i^{I*}) \left(tm^I - \frac{1}{tx_i^I} \right)} \frac{MC_i^F}{MC_i^{F*} \tau_i^F}. \quad (3.46)$$

¹⁴In particular, the export price Px_i^I is equal to $MC_i^I \tau_i^I$ times the tax coefficient tx_i^I , or $Px_i^I = Z_i^I \tau_i^I tx_i^I$. Also, with $W = 1$ the optimal value of μ_i^F is related to decentralized marginal costs as $\mu_i^F = \bar{\mu} MC_i^F$.

This expression determines final import tariffs tm_i^F as the wedge

$$tm^I - \beta_i (1 - \lambda_i^{I*}) \left(tm^I - \frac{1}{tx_i^I} \right).$$

Expressing this wedge as $(1 - \beta_i + \beta_i \lambda_i^{I*}) tm^I + \beta_i (1 - \lambda_i^{I*}) \frac{1}{tx_i^I}$ completes the proof of the following

Proposition 3.3.2. *Optimal import tariffs for final goods satisfy*

$$tm_i^F = (1 - \beta_i + \beta_i \lambda_i^{I*}) tm^I + \beta_i (1 - \lambda_i^{I*}) \frac{1}{tx_i^I}. \quad (3.47)$$

According to this proposition, variation in final import tariffs arises when Foreign final industries differentially use Home-produced intermediates, or due to variation in intermediate export taxes tx_i^I . The term $(1 - \beta_i + \beta_i \lambda_i^{I*})$ represents the Foreign labor content. A "dollar" amount of final import contains $(1 - \beta_i)$ "dollars" the Foreign downstream labor and $\beta_i \lambda_i^{I*}$ "dollars" of Foreign upstream labor. The domestic (Home) content is $\beta_i (1 - \lambda_i^{I*})$. The intuition here is that Home wants to encourage Foreign to import more intermediates,¹⁵ and in order to do so more final imports are allowed to reach Home consumers. Observe that a "dollar" spent by Home on importing final goods from Foreign increases Foreign final sales by one "dollar", intermediate spending by β_i "dollars" and Home exports by $\beta_i (1 - \lambda_i^{I*})$ "dollars", of which $\frac{1}{tx_i^I} \beta_i (1 - \lambda_i^{I*})$ is received by Home producers and $\frac{tx_i^I - 1}{tx_i^I} \beta_i (1 - \lambda_i^{I*})$ by the Home government. In a sense, Home compensates Foreign consumers for the export taxes they effectively pay. This logic appears more straightforward under an alternative scaling of nominal variables. If instead of setting $W = W^* = 1$ we put $\widetilde{tm^I} = 1$ and $\frac{\widetilde{W^*}}{\widetilde{W}} = tm^I$, we have $\widetilde{tx_i^I} = tx_i^I tm^I$ and (3.47) becomes $\widetilde{tm_i^F} = 1 - \beta_i (1 - \lambda_i^{I*}) \frac{\widetilde{tx_i^I} - 1}{\widetilde{tx_i^I}}$.

¹⁵Stronger import demand for Home intermediates is beneficial for two reasons. First, it translates into higher Foreign demand for Home labor, which is a general equilibrium terms-of-trade channel. (One unit of Home labor can be effectively exchanged for more units of Foreign labor.) Second, it increases Home opportunities to raise export tax revenue.

For positive intermediate export tax rates, $\widetilde{tx}_i^I > 1$, Home subsidizes Foreign final imports by returning Foreign consumers the revenue generated from intermediate exports.

3.3.2 Export taxes

We may relate the problem of setting export taxes to a more familiar price-setting problem of a multiproduct monopolist. Observe that the terms of (3.41) involving Px_i^I and Px_i^F are

$$\widetilde{\Lambda} = -\bar{\mu}^* \sum_i (l_i^{F*} + l_i^{I*}) - \sum_i \mu_i^F \tau_i^F c_{im}^* - \sum_i \mu_i^I \tau_i^I x_{im}^*. \quad (3.48)$$

Appendix C.2 proves Proposition 3.3.3 stating that Foreign employment l_i^{F*} and l_i^{I*} in (3.48) can be expressed through Home export revenue, and therefore $\widetilde{\Lambda}$ resembles a monopolist profit function.

Proposition 3.3.3. *The equation (3.48) can be expressed as*

$$\widetilde{\Lambda} = \sum_i (\bar{\mu}^* Px_i^F - \mu_i^F \tau_i^F) c_{im}^* + \sum_i (\bar{\mu}^* Px_i^I - \mu_i^I \tau_i^I) x_{im}^* - \sum_i \bar{\mu}^* MC_i^{F*} \tau_i^F c_{im} + \text{const}, \quad (3.49)$$

where the final constant term does not depend on the export prices.

The first two terms in (3.49) are analogous to profit of a multiproduct monopolist. For final goods, for example, selling one unit of product i to Foreign generates $\bar{\mu}^* Px_i^F$ in terms of revenue while the costs of providing it are $\mu_i^F \tau_i^F$. The third term, however, is unique to our international production sharing setup. The Home government must take into account that intermediate export prices Px_i^I affect Foreign final marginal costs MC_i^{F*} and thus total labor costs of providing Home final imports c_{im} . Final export prices Px_i^F do not affect Foreign production costs, which allows a relatively simple characterization.

Differentiating (3.49) with respect to Px_i^F gives a standard-looking monopolist first-order condition

$$\bar{\mu}^* c_{im}^* + (\bar{\mu}^* Px_i^F - \mu_i^F \tau_i^F) \frac{\partial c_{im}^*}{\partial Px_i^F} + (\bar{\mu}^* Px_i^I - \mu_i^I \tau_i^I) \frac{\partial x_{im}^*}{\partial Px_i^F} = 0. \quad (3.50)$$

The last term in (3.50) signifies the presence of production linkages. The Home government needs to take into account the positive cross-price elasticity, as higher final export prices Px_i^F induce consumer switching in Foreign toward domestic varieties, which boosts the demand for imported intermediates x_{im}^* . We can define the elasticity of final Foreign import demand as

$$\varepsilon_i^F = -\frac{\partial \log c_{im}^*}{\partial \log Px_i^F}. \quad (3.51)$$

Similar to the variable markups literature that works with nested CES preferences, for example Amiti, Itskhoki, and Konings (2014), this elasticity can be decomposed as $\varepsilon_i^F = (1 - \lambda_i^{F*}) \cdot 1 + \lambda_i^{F*} \rho_i^F$. The industry-level elasticity of substitution ρ_i^F is combined with the Cobb-Douglas elasticity of substitution 1 with the weights depending on $(1 - \lambda_i^{F*})$ which is the share of Home product in the Foreign market.

Appendix C.2 proves the following

Proposition 3.3.4. *Optimal export taxes for final goods satisfy*

$$tx_i^F = \frac{\varepsilon_i^F}{\varepsilon_i^F - 1} \frac{1}{tm_i^F} = \frac{\varepsilon_i^F}{\varepsilon_i^F - 1} \frac{1}{tm^I - \beta_i (1 - \lambda_i^{I*}) \left(tm^I - \frac{1}{tx_i^I} \right)}. \quad (3.52)$$

Two factors determine variation in final export taxes across industries. First, a higher Home market share in the Foreign market and hence a lower import demand elasticity ε_i^F implies a higher markup $\frac{\varepsilon_i^F}{\varepsilon_i^F - 1}$. These market shares are higher for stronger Home comparative advantage in either upstream or downstream, or in the case of lower trade costs τ_i^F . Second, as already mentioned, there is a consumer

expenditure switching effect. We have $Px_i^F \uparrow \rightarrow \lambda_i^{F*} \uparrow \rightarrow y_i^{F*} \uparrow \rightarrow x_{im}^* \uparrow$: the Foreign demand for intermediate imports is increasing in final export prices. The strength of this channel varies with $\beta_i (1 - \lambda_i^{I*})$, the share of Home-produced inputs in Foreign final output, and the exact expression is such that tm_i^F is the single summary statistic. Therefore, the optimal levels of export taxes tx_i^F are discounted by the values of import tariffs tm_i^F . Conditional on the demand elasticity ε_i^F , lower tariffs for final goods correspond to higher final export taxes. Such lower tariffs can arise, for example, for industries with larger intermediate shares β_i , which is a source of variation generally independent of ε_i^F .

Intermediate export prices Px_i^I are more complicated because of the impact on Foreign final marginal costs. [to be completed]

3.3.3 Comparative Statics

What is the role of comparative advantage for optimal trade policy in the presence of intermediate goods? To shed some light on this question, I perform the following numerical exercise. All parameters of the model other than cost shifters are set common across industries. The relative upstream and downstream cost shifters $\frac{Z_i^{I*}}{Z_i^I}$ and $\frac{Z_i^{F*}}{Z_i^F}$ are set on a rectangular grid. Therefore, for a given level of upstream (Home) cost advantage $\frac{Z_i^{I*}}{Z_i^I}$, we can look at variation of optimal trade taxes in response to variation in downstream cost advantage $\frac{Z_i^{F*}}{Z_i^F}$. (Not only the ratio $\frac{Z_i^{I*}}{Z_i^I}$ but also both variables are kept constant.) Similarly, we can look at the role of $\frac{Z_i^{I*}}{Z_i^I}$ keeping $\frac{Z_i^{F*}}{Z_i^F}$ fixed. Figure 3.1 presents the results which are typical for various levels of model parameters.

Similar to Costinot et al. (2015), export taxes increase monotonically in the level of (Home) comparative advantage. In our two-stage production setup, that applies to both final and intermediate export taxes, on the one hand, and, on the other hand, both upstream and downstream comparative advantage. Intuitively, stronger

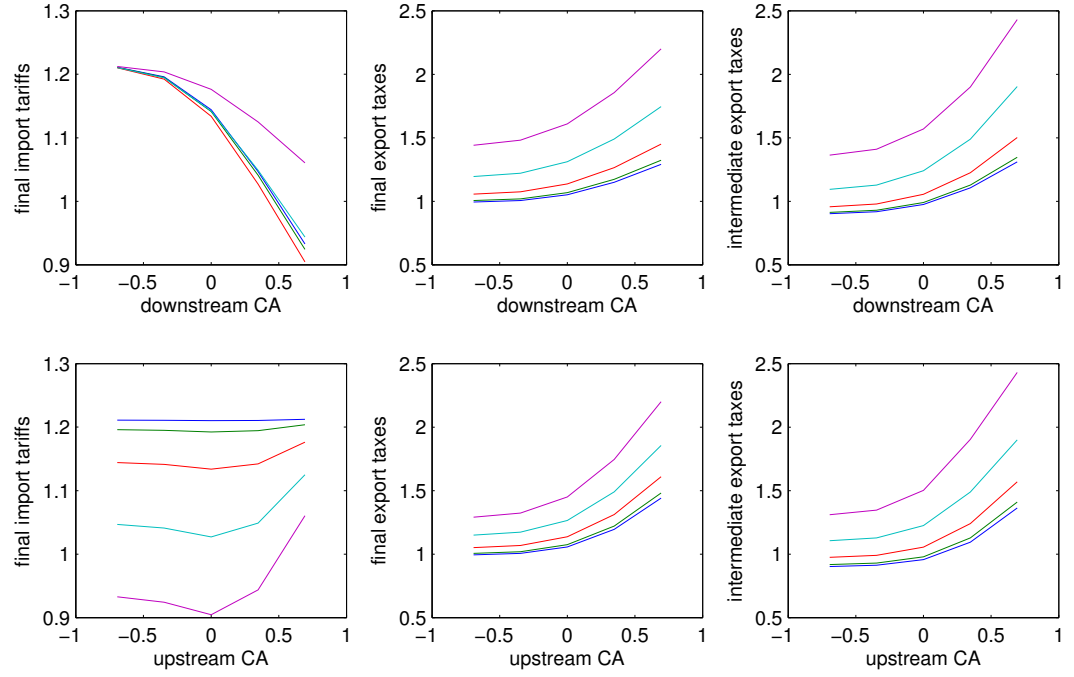


Figure 3.1: Variation in policy instruments in response to variation in downstream and upstream comparative advantage.

comparative advantage corresponds to larger market power and higher monopoly markups.

Final import tariffs are declining in downstream comparative advantage. From $tm_i^F = tm^I - \beta_i (1 - \lambda_i^{I*}) \left(tm^I - \frac{1}{tx_i^I} \right)$, $\frac{Z_i^{F*}}{Z_i^F}$ only affects tm_i^F via intermediate export taxes tx_i^I , both directly and indirectly through λ_i^{I*} . It appears that the negative direct effect always dominates the positive indirect effect, so final import tariffs decline in downstream comparative advantage. At the same time, final import tariffs are generally non-monotone in upstream comparative advantage. We have $\frac{Z_i^{I*}}{Z_i^I} \uparrow \rightarrow \lambda_i^{I*} \downarrow$, $tx_i^I \uparrow$, and the overall effect is ambiguous.

3.4 Conclusion

This paper applies the primal approach to optimal trade policy in a two-stage production environment. It finds that intermediate import tariffs are uniform while final import tariffs decline with domestic labor content. Final export taxes are determined by monopoly markups and discounted by the values of final import tariffs, due to the consumer expenditure switching effect. In general, stronger international production sharing reduces the incentives to impose import protection but increases the incentives to tax exports.

Considering more general input-output structures, the formula for import tariffs (3.47) generalizes easily, but export taxes become increasingly difficult to characterize. Unfortunately, this does not mean that we may abstract from export taxes, because that would break the results for import tariffs.

Appendix A

Illustrations and Proofs for Chapter 1

A.1 Evidence

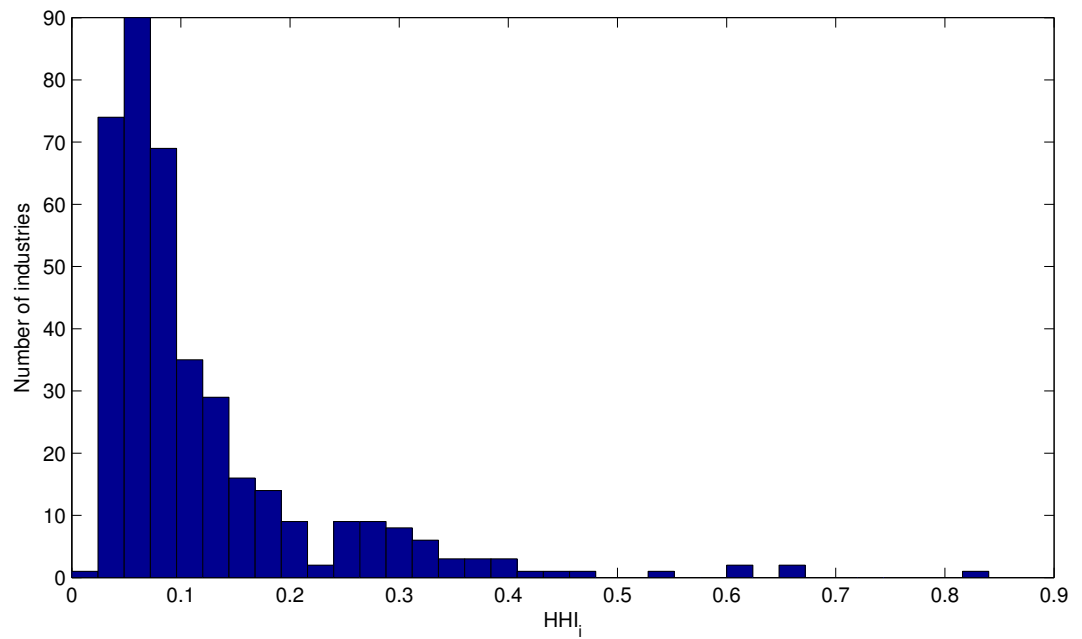


Figure A.1: Histogram of industry-level cost shares Herfindahl indices.

Figure A.2. Based on the B.E.A. detailed I-O table and trade flows for 2007, I calculate a measure of industry i 's CA as $\frac{\text{Export}_i - \text{Import}_i}{\text{Export}_i + \text{Import}_i}$ and industry i 's upstream CA as a linear combination of suppliers' CA with the weights equal to the corresponding input shares. Taking the ratio of net exports to total trade flows is supposed to net out variation attributable to heterogeneity in trade costs. The result is that there is nontrivial variation in upstream CA (which is a case against perfect diversification) and it has predictive power for downstream CA. Total 333 observations after deleting industries with $\text{Export}_i = \text{Import}_i = 0$. Correlation coefficient 0.305 and $R^2 = 0.106$. Similar results if CA is measured as the difference between actual and auratky sales shares which corresponds to specialization.

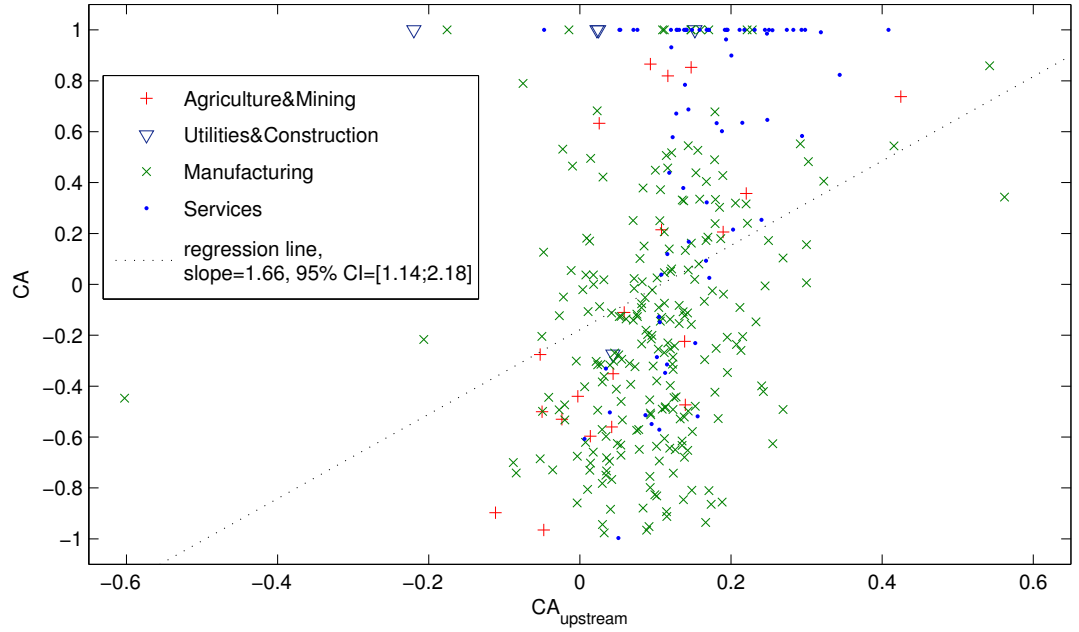


Figure A.2: CA spillovers.

A.2 LLN Lemma

Let $\{A^{(N)}\}_{N=1}^{\infty}$ be a sequence of quadratic deterministic $N \times N$ matrices with entries $a_{ij}^{(N)}$. Let $u_i^{(N)}$, $1 \leq i \leq N \leq \infty$, be iid random variables that form a sequence of random vectors $\{u^{(N)} = (u_1^{(N)} \dots u_N^{(N)})'\}_{N=1}^{\infty}$. To simplify notation, suppress from now on the "N" superscript in $A^{(N)}$ and $u^{(N)}$. Assume that each component of u has four finite moments — μ_1, μ_2, μ_3 , and μ_4 — and $\mu_1 = 0$.

Lemma 1. *If $\lim_{N \rightarrow \infty} \text{tr}(A'A) = 0$, then $u'Au \xrightarrow{p} \mu_2 \text{tr}(A)$.*

Proof:

As u_i are iid, $E[u'Au] = \sum_{ij} a_{ij} E[u_i u_j] = \sum_i a_{ii} E[u_i^2] = \mu_2 \text{tr}(A)$. For symmetric $\tilde{A}$, a textbook result¹ for the variance of quadratic form is that

$$\text{Var}[u'\tilde{A}u] = 2\mu_2^2 \text{tr}(\tilde{A}^2) + (\mu_4 - 3\mu_2^2) \sum_i \tilde{a}_{ii}^2. \quad (\text{A.1})$$

In general nonsymmetric case, $\tilde{A}$ needs to be replaced with $\frac{1}{2}(A + A')$, which gives

$$\text{Var}[u'Au] = \mu_2^2 \text{tr}(A^2 + A'A) + (\mu_4 - 3\mu_2^2) \sum_i a_{ii}^2. \quad (\text{A.2})$$

We want to show that $\text{tr}(A'A) \rightarrow 0$ implies $\text{Var}[u'Au] \rightarrow 0$. Obviously, $0 \leq \sum_i a_{ii}^2 \leq \sum_{ij} a_{ij}^2 = \text{tr}(A'A) \rightarrow 0$, so we only need to show that $\text{tr}(A^2) \rightarrow 0$. This is indeed the case since $|\text{tr}(A^2)| \leq \text{tr}(A'A)$ holds because of the Cauchy-Schwarz inequality.²

Now as $E[u'Au] = \mu_2 \text{tr}(A)$ and $\text{Var}[u'Au] \rightarrow 0$, $u'Au \xrightarrow{p} \mu_2 \text{tr}(A)$. ■

Lemma 1 is the main LLN tool of this paper. Several proofs also use a simpler result for linear forms which is well known. I provide it below for completeness.

Let $\{a^{(N)}\}_{N=1}^{\infty}$ be a sequence of deterministic $N \times 1$ vectors whose components sum up to a constant $\bar{a}$: $\sum_{i=1}^N a_i^{(N)} = \bar{a}$ for all N . Random variables $u_i^{(N)}$, $1 \leq i \leq N \leq \infty$,

¹See Seber and Lee (2002).

²In the space of $N \times N$ real matrices, $\langle A_1, A_2 \rangle = \text{tr}(A_1 A_2')$ is an inner product with induced norm $\sqrt{\text{tr}(A A')}$. Therefore, $(\text{tr}(A_1 A_2'))^2 \leq \text{tr}(A_1 A_1') \cdot \text{tr}(A_2 A_2')$. Taking $A_1 = A$ and $A_2 = A'$, we have $(\text{tr}(A A'))^2 \leq \text{tr}(A A') \cdot \text{tr}(A' A) = (\text{tr}(A' A))^2$, or $|\text{tr}(A^2)| \leq \text{tr}(A' A)$.

are defined as before. Assume that each component of u has two finite moments, μ_1 and μ_2 .

Lemma 1A. *If $\lim_{N \rightarrow \infty} \|a\|_2 = 0$, then $a'u \xrightarrow{p} \bar{a}\mu_1$.*

Proof:

First, $E[a'u] = \sum_i a_i E[u_i] = \bar{a}\mu_1$. Second, as u_i are iid, $Var[a'u] = \sum_i a_i^2 Var[u_i] = \mu_2 \cdot \|a\|_2^2 \rightarrow 0$. The two imply that $a'u \xrightarrow{p} \bar{a}\mu_1$. ■

A.3 Sales

From (1.21) with $C = 1$ and $C^* = 1$, the two sales vectors are determined by the system

$$\begin{aligned} y &= D_\lambda (\alpha + \beta \Omega' y) + D_{\mathbf{1}-\lambda^*} (\alpha + \beta \Omega' y^*) \\ y^* &= D_{\lambda^*} (\alpha + \beta \Omega' y^*) + D_{\mathbf{1}-\lambda} (\alpha + \beta \Omega' y) \end{aligned} \quad (\text{A.3})$$

Taking the sum gives $y + y^* = 2\alpha + \beta \Omega' (y + y^*)$, or $y + y^* = (I - \beta \Omega')^{-1} 2\alpha = 2b$, according to (1.7) which defines the autarky sales vector b . Substituting $y^* = 2b - y$ into the first equation in (A.3) yields

$$y = (I - \beta D_{\lambda+\lambda^*-\mathbf{1}} \Omega')^{-1} [D_\alpha (\lambda + \lambda^* - \mathbf{1}) + 2D_b (\mathbf{1} - \lambda^*)], \quad (\text{A.4})$$

which further implies that $y - y^* = (I - \beta D_{\lambda+\lambda^*-\mathbf{1}} \Omega')^{-1} 2D_b (\lambda - \lambda^*)$. Summing it with $y + y^* = 2b$ results in

$$y = b + (I - \beta D_{\lambda+\lambda^*-\mathbf{1}} \Omega')^{-1} D_b (\lambda - \lambda^*). \quad (\text{A.5})$$

Value-added trade flows, such as defined in Johnson and Noguera (2012), are obtained from (1.21) by setting $C = 0$ or $C^* = 0$, solving for the corresponding sales

y and y^* and then summing the involved value added which (by industry) is $(1 - \beta)y$ and $(1 - \beta)y^*$.

A.4 Proof of Proposition 1.2.1

Although the shortest way to prove this proposition is by applying the envelope theorem to the planner's problem, I provide a direct proof which has useful intermediate results and, more generally, demonstrates the mechanics of the model. From (1.28),

$$\bar{p} + \bar{p}^* = b' \left[z + z^* + \log \left(\lambda^{\frac{1}{\rho-1}} \right) + \log \left(\lambda^{*\frac{1}{\rho-1}} \right) \right]. \quad (\text{A.6})$$

For some variable x_i and $\lambda_i = 1 / (1 + e^{(1-\rho)x_i})$, we have

$$d\lambda_i = (\rho - 1) e^{(1-\rho)x_i} / (1 + e^{(1-\rho)x_i})^2 dx_i, \text{ so}$$

$$d\lambda_i = (\rho - 1) \lambda_i (1 - \lambda_i) dx_i, \quad (\text{A.7})$$

which is used repeatedly in proofs. Then, differentiating (A.6),

$$\begin{aligned} d(\bar{p} + \bar{p}^*) &= \sum_i b_i [(1 - \lambda_i) (d\tau + d\xi_i) + (1 - \lambda_i^*) (d\tau - d\xi_i)] \\ &= \sum_i b_i [(2 - \lambda_i - \lambda_i^*) d\tau + (\lambda_i^* - \lambda_i) d\xi_i] = \frac{2d\tau}{1-\beta} - b'(\lambda + \lambda^*) d\tau - (\lambda - \lambda^*)' D_b d\xi. \end{aligned} \quad (\text{A.8})$$

From (1.19),

$$d\xi = \beta\Omega [(\lambda + \lambda^* - \mathbf{1}) \circ d\xi + (\lambda - \lambda^*) d\tau], \text{ or}$$

$$d\xi = (I - \beta\Omega D_{\lambda+\lambda^*-\mathbf{1}})^{-1} \beta\Omega (\lambda - \lambda^*) d\tau. \quad (\text{A.9})$$

Together (A.8) and (A.9) imply

$$\frac{d(\bar{p} + \bar{p}^*)}{d\tau} = \frac{2}{1-\beta} - b'(\lambda + \lambda^*) - (\lambda - \lambda^*)' D_b (I - \beta\Omega D_{\lambda+\lambda^*-\mathbf{1}})^{-1} \beta\Omega (\lambda - \lambda^*). \quad (\text{A.10})$$

Denote $B = (I - \beta D_{\lambda+\lambda^*-\mathbf{1}} \Omega')^{-1}$. For trade flows, we have

$$\begin{aligned} M + M^* &= \frac{2}{1-\beta} - \alpha'(\lambda + \lambda^*) - \lambda' \beta \Omega' y - \lambda^{*'} \beta \Omega' y^* \\ &= \frac{2}{1-\beta} - \alpha'(\lambda + \lambda^*) - \lambda' \beta \Omega' B [D_\alpha (\lambda + \lambda^* - \mathbf{1}) + 2D_b (\mathbf{1} - \lambda^*)] - \end{aligned}$$

$$-\lambda^{*'} \beta \Omega'^{-1} [D_\alpha (\lambda + \lambda^* - \mathbf{1}) + 2D_b (\mathbf{1} - \lambda)] = \frac{2}{1-\beta} - b'(\lambda + \lambda^*) - (\lambda - \lambda^*)' \beta \Omega' B D_b (\lambda - \lambda^*)$$

$$= \frac{d(\bar{p} + \bar{p}^*)}{d\tau}.$$

A.5 The Local ACR Result

We have

$$\frac{d}{d\tau} \bar{p} = M = \frac{1}{1-\beta} - D = \frac{1-\bar{\lambda}}{1-\beta}.$$

Also,

$$\epsilon(\tau) = -\frac{d}{d\tau} \log [(1-\bar{\lambda})/\bar{\lambda}] = \frac{1}{\bar{\lambda}(1-\bar{\lambda})} \frac{d\bar{\lambda}}{d\tau}.$$

Therefore,

$$\frac{d\bar{p}}{d \log(\bar{\lambda})} = \frac{\frac{1-\bar{\lambda}}{1-\beta} d\tau}{d\bar{\lambda}/\bar{\lambda}} = \frac{1}{(1-\beta)} \left(\frac{1}{\bar{\lambda}(1-\bar{\lambda})} d\bar{\lambda} \right)^{-1} = \frac{1}{(1-\beta) \epsilon(\tau)}.$$

A.6 Local Analysis Near Free Trade

Under free trade, i.e. $\tau = 0$, we have $\xi = \eta$: Home and Foreign industries face the same prices for intermediate inputs, so the difference in marginal costs only arises from the difference in cost shocks z_i and z_i^* . Another major simplification is that $\lambda_i + \lambda_i^* = 1$ for all i . It is convenient to introduce the following transformation of η_i :

$$\varepsilon_i = (\lambda_i - \lambda_i^*)|_{\tau=0} = \frac{1 - e^{(1-\rho)\eta_i}}{1 + e^{(1-\rho)\eta_i}} = \tanh\left(\frac{\rho-1}{2}\eta_i\right). \quad (\text{A.11})$$

Several properties of ε_i are useful:

- (i) $\varepsilon_i \in [-1, 1]$. $\varepsilon_i \in \{-1, 1\}$ if $\rho = \infty$.
- (ii) $\mathbf{E}[\varepsilon_i] = \mathbf{E}[\varepsilon_i^3] = 0$ (due to the symmetry).
- (iii) $\mathbf{E}[\varepsilon_i \varepsilon_j] = 0 \ \forall i \neq j$.

Denote $\mu_2 = \mathbf{E}[\varepsilon_i^2]$ and $\mu_4 = \mathbf{E}[\varepsilon_i^4]$.

With this definition, domestic shares at $\tau = 0$ are

$$\lambda = \frac{1}{2}(\mathbf{1} + \varepsilon) \quad \text{and} \quad \lambda^* = \frac{1}{2}(\mathbf{1} - \varepsilon), \quad (\text{A.12})$$

and sales are

$$y = D_b(\mathbf{1} + \varepsilon) \quad \text{and} \quad y^* = D_b(\mathbf{1} - \varepsilon). \quad (\text{A.13})$$

First of all, these simplifications allow showing that labor market clearing holds asymptotically for $W = W^* = 1$ in the sense that $p \lim L_D = L$ and $p \lim L_D^* = L^*$ where L_D and L_D^* denote aggregate labor demand:

$$L_D = \frac{1}{W} (1 - \beta) \mathbf{1}' y \text{ and } L_D^* = \frac{1}{W^*} (1 - \beta) \mathbf{1}' y^* . \quad (\text{A.14})$$

In fact, $L_D + L_D^* = L + L^* = 2$ holds as identity (simply from that fact that $y + y^* = 2b$), so it is sufficient to demonstrate that $p \lim (L_D - L_D^*) = 0$.

From Appendix A.3, $y - y^* = (I - \beta D_{\lambda + \lambda^* - \mathbf{1}} \Omega')^{-1} 2D_b (\lambda - \lambda^*)$. Therefore,

$$L_D - L_D^* = 2(1 - \beta) \mathbf{1}' (I - \beta D_{\lambda + \lambda^* - \mathbf{1}} \Omega')^{-1} D_b (\lambda - \lambda^*) . \quad (\text{A.15})$$

Using (A.12), it reduces to

$$L_D - L_D^* = 2(1 - \beta) b' \varepsilon \xrightarrow[p]{} 0, \quad (\text{A.16})$$

which is by Lemma 1A.

Free-Trade Home Bias

Expenditure on domestically produced goods:

$$D^F = \lambda' \alpha = \frac{1}{2} + \sum_i \alpha_i \varepsilon_i \xrightarrow[p]{} \frac{1}{2},$$

which is again by Lemma 1A. (An intermediate step is to show that $\|b\|_2 \rightarrow 0$ implies $\|\alpha\|_2 \rightarrow 0$. This follows from $b = (I - \beta \Omega')^{-1} \alpha \geq \alpha$.)

For producers

$$D^I = \lambda' \beta \Omega' y = \frac{1}{2} (\mathbf{1} + \varepsilon)' \beta \Omega' D_b (\mathbf{1} + \varepsilon) \xrightarrow[p]{} \frac{1}{2} \frac{\beta}{1 - \beta} (1 + SSI \cdot \mu_2), \quad (\text{A.17})$$

which is obtained by expanding

$$D^I = \frac{1}{2} (\mathbf{1} + \varepsilon)' \beta \Omega' D_b (\mathbf{1} + \varepsilon) = \frac{1}{2} [\mathbf{1}' \beta \Omega' D_b \mathbf{1} + \mathbf{1}' \beta \Omega' D_b \varepsilon + \varepsilon' \beta \Omega' D_b \mathbf{1} + \varepsilon' \beta \Omega' D_b \varepsilon]$$

The first term is $\mathbf{1}'\beta\Omega'D_b\mathbf{1} = \frac{\beta}{1-\beta}$. The second and the third have probability limit zero by Lemma 1A. For the last term, we have $\text{tr}(\Omega'D_bD_b\Omega) \leq \mathbf{1}'\Omega'D_bD_b\Omega\mathbf{1} = \|b\|_2^2 \rightarrow 0$, then by Lemma 1 $\varepsilon'\beta\Omega'D_b\varepsilon \xrightarrow[p]{p} \text{tr}(\beta\Omega'D_b)\mu_2 = \frac{\beta}{1-\beta}SSI \cdot \mu_2$.

Trade Elasticity

I consider separately the case $\rho < \infty$ and the case of perfect substitutes $\rho = \infty$.

Proposition A.6.1. *For $\rho < \infty$, at $\tau = 0$*

$$\frac{dD}{d\tau} \xrightarrow[p]{p} \frac{\rho-1}{4} \frac{1}{1-\beta} \left[1 - \mu_2 + \beta^2 (\mu_2 (1 - \mu_2) [HHI + 2SLI] - (\mu_4 - \mu_2^2) 3SSI_2) \right]. \quad (\text{A.18})$$

The proof is provided below at the end.

In the absence of self-sourcing, $SSI = 0$, the case of perfect substitutes can be characterized by taking the limit in (A.18). Denote $f(\cdot)$ the pdf of η_i . Under a weak regularity condition on the distribution of cost shocks we have

$$\lim_{\rho \rightarrow \infty} (\rho - 1) (1 - \mu_2) = \lim_{\rho \rightarrow \infty} (\rho - 1) \mu_2 (1 - \mu_2) = 4f(0), \text{ and}$$

$$\frac{dD}{d\tau} \xrightarrow[p]{p} \frac{1}{1-\beta} f(0) \left[1 + \beta^2 (HHI + 2SLI) \right]. \quad (\text{A.19})$$

Once we know the levels and the derivatives of trade flows, we can calculate the trade elasticity. Note that $\log \left(\frac{1+\kappa_1 x + o(x)}{a - \kappa_1 x + o(x)} \right) = -\log(a) + \frac{a+1}{a} \kappa_1 x + o(x)$. Thus, for example, for $SSI = 0$ (no free-trade home bias) and $\rho = \infty$,

$$\log \left(\frac{M}{D} \right) \xrightarrow[p]{p} -4f(0) \left[1 + \beta^2 (HHI + 2SLI) \right] \tau + o(\tau). \quad (\text{A.20})$$

If productivities $\exp(-z_i)$ and $\exp(-z_i^*)$ are independently Frechet distributed, $4f(0)$ equals to the dispersion parameter θ . The elasticity of trade in this case is

$$\epsilon = \theta \left[1 + \beta^2 (HHI + 2SLI) \right].$$

Proof of Proposition A.6.1

Let $\dot{x}$ denote $\frac{dx}{d\tau}$ and $\ddot{x}$ denote $\frac{d^2x}{d\tau^2}$. World domestic spending is $D + D^* = \frac{2}{1-\beta} - M + M^*$. As shown in Appendix A.4,

$$\dot{\xi} = (I - \beta \Omega D_{\lambda+\lambda^*-1})^{-1} \beta \Omega (\lambda - \lambda^*) \text{ and } D + D^* = b' (\lambda + \lambda^*) + \dot{\xi}' D_b (\lambda - \lambda^*).$$

From the same Appendix, applying $\lambda = \frac{1}{2} (1 + \varepsilon)$ and $\lambda^* = \frac{1}{2} (1 - \varepsilon)$, we have $\dot{\lambda} = \frac{\rho-1}{4} D_{1-\varepsilon^2} (1 + \beta \Omega \varepsilon)$ and $\dot{\lambda}^* = \frac{\rho-1}{4} D_{1-\varepsilon^2} (1 - \beta \Omega \varepsilon)$. Therefore,

$$\dot{\lambda} + \dot{\lambda}^* = \frac{\rho-1}{2} (1 - \varepsilon^2) \text{ and } \dot{\lambda} - \dot{\lambda}^* = \frac{\rho-1}{2} D_{1-\varepsilon^2} \beta \Omega \varepsilon.$$

$$\ddot{\xi} = \beta^2 (\rho - 1) \Omega D_{1-\varepsilon^2} \Omega \varepsilon.$$

$$\dot{D} + \dot{D}^* = b' \left(\dot{\lambda} + \dot{\lambda}^* \right) + \ddot{\xi}' D_b (\lambda - \lambda^*) + \dot{\xi}' D_b \left(\dot{\lambda} - \dot{\lambda}^* \right)$$

$$= \frac{\rho-1}{2} b' (1 - \varepsilon^2) + \varepsilon' D_b \beta^2 (\rho - 1) \Omega D_{1-\varepsilon^2} \Omega \varepsilon + \varepsilon' \beta \Omega' D_b \frac{\rho-1}{2} D_{1-\varepsilon^2} \beta \Omega \varepsilon. \quad (\text{A.21})$$

The first term in (A.21) is $\frac{\rho-1}{2} \sum_i b_i (1 - \varepsilon_i^2) \xrightarrow{p} \frac{\rho-1}{2} \frac{1}{1-\beta} (1 - \mu_2)$. For the second term, expand

$$(1 - \beta) \varepsilon' D_b \Omega D_{1-\varepsilon^2} \Omega \varepsilon = (1 - \beta) \sum_i (1 - \varepsilon_i^2) \left(\sum_j \omega_{ij} \varepsilon_j \right) \left(\sum_k \omega_{ki} \varepsilon_k b_k \right),$$

getting $SLI \cdot \mu_2 (1 - \mu_2) - SSI_2 [\mu_4 - \mu_2^2]$. For the third term in (A.21),

$$(1 - \beta) \varepsilon' \Omega' D_b D_{1-\varepsilon^2} \Omega \varepsilon = (1 - \beta) \sum_i b_i (1 - \varepsilon_i^2) \left[\sum_j \omega_{ij} \varepsilon_j \right]^2,$$

which gives $HHI \cdot \mu_2 (1 - \mu_2) - SSI_2 [\mu_4 - \mu_2^2]$.

Now we can compute derivatives of domestic expenditure evaluated at $\tau = 0$.

$$\begin{aligned} \dot{D} + \dot{D}^* &\xrightarrow{p} \frac{\rho-1}{2} \frac{1}{1-\beta} (1 - \mu_2) + \\ &+ \frac{\beta^2(\rho-1)}{1-\beta} (SLI \cdot \mu_2 (1 - \mu_2) - SSI_2 [\mu_4 - \mu_2^2]) + \\ &+ \frac{\beta^2(\rho-1)}{2(1-\beta)} (HHI \cdot \mu_2 (1 - \mu_2) - SSI_2 [\mu_4 - \mu_2^2]), \text{ so} \end{aligned}$$

$$\dot{D} + \dot{D}^* \xrightarrow{p} \frac{\rho-1}{2} \frac{1}{1-\beta} [1 - \mu_2 + \beta^2 (\mu_2 (1 - \mu_2) [HHI + 2SLI] - (\mu_4 - \mu_2^2) 3SSI_2)],$$

which completes the proof (LLN details to be provided). ■

A.7 Neutrality Result

Consider the world economy consisting of countries $k = 1..K$ and products (industries) $i = 1..N$. Consumers maximize CES preferences with the elasticity of substitution σ :

$$U_k = \left(\sum_{i=1}^N \alpha_{ik}^{\frac{1}{\sigma}} c_{ik}^{1-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (\text{A.22})$$

where $\alpha_{ik} \geq 0$ are utility weights (potentially country-specific), $\sum_i \alpha_{ik} = 1$ for each k .³

Technology is Cobb-Douglas over labor and industry-specific composite intermediate input:

$$q_{ik} = \frac{1}{Z_{ik}} \left(\frac{L_{ik}}{1 - \beta} \right)^{1-\beta} \left(\frac{M_{ik}}{\beta} \right)^{\beta}, \quad (\text{A.23})$$

with cost shifters Z_{ik} , input shares $0 \leq \beta < 1$ (uniform across products and countries), and materials being CES bundles with industry-specific elasticities of substitution γ_i :

$$M_{ik} = \left(\sum_{i'=1}^N \omega_{ii'}^{\frac{1}{\gamma_i}} m_{ii'k}^{1-\frac{1}{\gamma_i}} \right)^{\frac{\gamma_i}{\gamma_i-1}}, \quad (\text{A.24})$$

in which the weights $\omega_{ii'} \geq 0$, $\sum_{i'} \omega_{ii'} = 1$ for each i , are common across countries.

For each product i , both consumption and intermediate inputs are Armington CES bundles aggregating country-specific varieties with the elasticity of substitution $1 < \rho < \infty$:

$$x_{ik} = \left(\sum_{k'=1}^K x_{ikk'}^{1-\frac{1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (\text{A.25})$$

where x_{ik} stands for either consumption of product i or its use as an intermediate input in country k and $x_{ikk'}$ denotes the involved amount of variety coming from country k' .

Trade is subject to iceberg costs $t_{kk'} \geq 1$ (for importer country k and exporter country k') which are uniform across all goods.

Consumers in country k inelastically supply L_k units of labor. Competitive equilibrium is defined in the usual way: a set of prices and allocations such that consumers

³This normalization allows considering a countable set of products ($N = \infty$), as the level of real consumption U_k remains bounded and positive. Similar applies to technology in (A.24).

maximize utility (subject to the standard budget constraint potentially allowing exogenous trade imbalances), producers maximize profit, and all markets clear.

Proof of Proposition 1.3.1

Our goal is to show that any competitive equilibrium, once it exists, has trade flows satisfying log-linear gravity. For some given wages $w_k > 0$, consider cost minimization by producers. Denote μ_{ik} the equilibrium marginal cost of producing good i in country k . That is the unit cost associated with (A.23)-(A.25) when producers optimally combine labor and materials from different industries and source countries. The key to the proof is showing that no relative variation in exogenous cost shifters implies no relative variation in (endogenously determined) marginal costs:

$$Z_{ik} = \tilde{Z}_i \hat{Z}_k \Rightarrow \mu_{ik} = \tilde{\mu}_i \hat{\mu}_k. \quad (\text{A.26})$$

We will first establish that (conditional on wages) there can be only one positive solution to μ_{ik} and then will use $Z_{ik} = \tilde{Z}_i \hat{Z}_k$ to construct μ_{ik} in the form $\tilde{\mu}_i \hat{\mu}_k$.

Step 1. Uniqueness of μ_{ik} .

From the Armington aggregator (A.25), the price index for intermediate input i in country k is equal to

$$P_{ik} = \left(\sum_{k'} (t_{kk'} \mu_{ik'})^{1-\rho} \right)^{1/(1-\rho)}. \quad (\text{A.27})$$

From (A.23) and (A.24), the marginal cost of producing good i in country k is

$$\mu_{ik} = Z_{ik} w_k^{1-\beta} (P_{ik}^M)^\beta, \quad (\text{A.28})$$

where the price index for materials (i.e. composite intermediate input) is

$$P_{ik}^M = \left(\sum_{i'} \omega_{ii'} P_{i'k}^{1-\gamma_i} \right)^{1/(1-\gamma_i)}. \quad (\text{A.29})$$

Combining (A.27)-(A.29),

$$\mu_{ik} = Z_{ik} w_k^{1-\beta} \left[\sum_{i'} \omega_{ii'} \left(\sum_{k'} (t_{kk'} \mu_{i'k'})^{1-\rho} \right)^{\frac{1-\gamma_i}{1-\rho}} \right]^{\beta/(1-\gamma_i)}. \quad (\text{A.30})$$

For μ the matrix of μ_{ik} define $f : \mathbb{R}_{++}^{NK} \rightarrow \mathbb{R}^{NK}$ such that $f_{ik}(\mu) = g_{ik}(\mu) - h_{ik}(\mu)$ where $g_{ik}(\mu) = Z_{ik} w_k^{1-\beta} \left[\sum_{i'} \omega_{ii'} \left(\sum_{k'} (t_{kk'} \mu_{i'k'})^{1-\rho} \right)^{\frac{1-\gamma_i}{1-\rho}} \right]^{\beta/(1-\gamma_i)}$ and $h_{ik}(\mu) = \mu_{ik}$. Note that $h_{ik}(\mu)$ is homogenous of degree one and $g_{ik}(\mu)$ is homogenous of degree $\beta < 1$. We have that $f(\mu)$ satisfies gross substitution: $\frac{\partial f_{ik}(\mu)}{\partial \mu_{i'k'}} > 0$ for $i \neq i', k \neq k'$. Then Theorem 2 from Allen, Arkolakis, and Li (2015)⁴ implies that there is at most one solution to $f(\mu) = 0$. Therefore, (A.30) has at most one positive solution.

Step 2. Constructing $\tilde{\mu}_i$ and $\hat{\mu}_k$.

Let's construct $\tilde{\mu}_i$ and $\hat{\mu}_k$ such that $\mu_{ik} = \tilde{\mu}_i \hat{\mu}_k$ indeed holds. The price index (A.27) is

$$\begin{aligned} P_{ik} &= \left(\sum_{k'} (t_{kk'} \mu_{ik'})^{1-\rho} \right)^{1/(1-\rho)} = \left(\sum_{k'} (t_{kk'} \tilde{\mu}_i \hat{\mu}_{k'})^{1-\rho} \right)^{1/(1-\rho)} \\ &= \tilde{\mu}_i \underbrace{\left(\sum_{k'} (t_{kk'} \hat{\mu}_{k'})^{1-\rho} \right)^{1/(1-\rho)}}_{\bar{\mu}_k \text{ (access to suppliers)}}. \end{aligned} \quad (\text{A.31})$$

Then the price index for materials (A.29) is

$$\begin{aligned} P_{ik}^M &= \left(\sum_{i'} \omega_{ii'} P_{i'k}^{1-\gamma_i} \right)^{1/(1-\gamma_i)} = \left(\sum_{i'} \omega_{ii'} (\tilde{\mu}_{i'} \bar{\mu}_k)^{1-\gamma_i} \right)^{1/(1-\gamma_i)} \\ &= \bar{\mu}_k \underbrace{\left(\sum_{i'} \omega_{ii'} \tilde{\mu}_{i'}^{1-\gamma_i} \right)^{1/(1-\gamma_i)}}_{\check{\mu}_i \text{ (material requirement)}}. \end{aligned} \quad (\text{A.32})$$

Now (A.30) can be written as

⁴July 2015 draft.

$$\mu_{ik} = Z_{ik} w_k^{1-\beta} (P_{ik}^M)^\beta = \tilde{Z}_i \hat{Z}_k w_k^{1-\beta} (\bar{\mu}_k \check{\mu}_i)^\beta = \underbrace{\tilde{Z}_i \check{\mu}_i^\beta}_{\tilde{\mu}_i} \cdot \underbrace{\hat{Z}_k w_k^{1-\beta} \bar{\mu}_k^\beta}_{\hat{\mu}_k}, \quad (\text{A.33})$$

which means that industry and country "fixed effects" $\tilde{\mu}_i$ and $\hat{\mu}_k$ are given by

$$\tilde{\mu}_i = \tilde{Z}_i \check{\mu}_i^\beta = \tilde{Z}_i \left(\sum_{i'} \omega_{ii'} \tilde{\mu}_{i'}^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)} \quad \text{and} \quad (\text{A.34})$$

$$\hat{\mu}_k = \hat{Z}_k w_k^{1-\beta} \bar{\mu}_k^\beta = \hat{Z}_k w_k^{1-\beta} \left(\sum_{k'} (t_{kk'} \hat{\mu}_{k'})^{1-\rho} \right)^{\beta/(1-\rho)}. \quad (\text{A.35})$$

Existence of solutions to (A.34)-(A.35) follows from Lemma 1 in Allen, Arkolakis, and Li (2015). Note that it is sufficient to prove existence for (A.34) because (A.35) can be viewed as its special case with $\gamma_i = \rho$. The next argument follows closely Allen, Arkolakis, and Li (2015) in their proof of Theorem 1 in Alvarez and Lucas (2007).

Define $f : \mathbb{R}_{++}^N \rightarrow \mathbb{R}^N$ such that $f_i(\tilde{\mu}) = \tilde{Z}_i \left(\sum_{i'} \omega_{ii'} \tilde{\mu}_{i'}^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)} - \tilde{\mu}_i$ and $F : \mathbb{R}_{++}^{N+1} \rightarrow \mathbb{R}^N$ such that $F_i(\tilde{\mu}', \tilde{\mu}_i) = \tilde{Z}_i \left(\sum_{i'} \omega_{ii'} (\tilde{\mu}_{i'})^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)} - \tilde{\mu}_i$.⁵ For this function F , we have the three conditions of Lemma 1 in Allen, Arkolakis, and Li (2015) satisfied:

- (i) For all $\tilde{\mu}' \in \mathbb{R}_{++}^N$, there exists $\tilde{\mu}_i$ such that $F_i(\tilde{\mu}', \tilde{\mu}_i) = 0$. (Just set $\tilde{\mu}_i = \tilde{Z}_i \left(\sum_{i'} \omega_{ii'} (\tilde{\mu}_{i'})^{1-\gamma_i} \right)^{\beta/(1-\gamma_i)}$.)
- (ii) $\frac{\partial F_i(\tilde{\mu}', \tilde{\mu}_i)}{\partial \tilde{\mu}_i} \frac{\partial F_i(\tilde{\mu}', \tilde{\mu}_i)}{\partial \tilde{\mu}_j'} < 0$ for all j . (Obvious since $\frac{\partial F_i(\tilde{\mu}', \tilde{\mu}_i)}{\partial \tilde{\mu}_i} = -1$ and $\frac{\partial F_i(\tilde{\mu}', \tilde{\mu}_i)}{\partial \tilde{\mu}_j'} > 0$.)
- (iii) There exists $\tilde{\mu}'$ such that for $\tilde{\mu}_i$ defined in $F_i(s\tilde{\mu}', \tilde{\mu}_i) = 0$, $\tilde{\mu}_i = o(s)$. (From $F_i(s\tilde{\mu}', \tilde{\mu}_i) = 0$, $\tilde{\mu}_i \propto s^\beta$.)

Once (i)-(iii) hold, a solution to $f(\tilde{\mu}) = 0$ exists, so (A.34)-(A.35) have positive solutions. Thus we have shown that, given wages and cost shifters $Z_{ik} = \tilde{Z}_i \hat{Z}_k$, there exist equilibrium marginal costs in the form $\mu_{ik} = \tilde{\mu}_i \hat{\mu}_k$. Combining this result with

⁵A note on notation: $\tilde{\mu}$ and $\tilde{\mu}'$ denote two vectors. No matrix transpose appears in this proof.

previously established uniqueness⁶ of μ_{ik} , we conclude that the implication (A.26) indeed holds.

Step 3. Trade shares and gravity.

Denote X_{ik} the total expenditure of country k on product i and $X_{ikk'}$ its expenditure on product i coming from country k' . Using (A.31), we obtain a strong result that trade shares $\pi_{ikk'} = X_{ikk'}/X_{ik}$, which are common for producers and consumers, are not product-specific:

$$\pi_{ikk'} = \left(\frac{t_{kk'} \mu_{ik'}}{P_{ik}} \right)^{1-\rho} = \left(\frac{t_{kk'} \tilde{\mu}_i \hat{\mu}_{k'}}{\tilde{\mu}_i \bar{\mu}_k} \right)^{1-\rho} = \left(\frac{t_{kk'} \hat{\mu}_{k'}}{\bar{\mu}_k} \right)^{1-\rho}. \quad (\text{A.36})$$

The proof is completed by aggregating $X_{ikk'}$ over i :

$$X_{kk'} = \sum_i X_{ikk'} = \sum_i \pi_{ikk'} X_{ik} = \left(\sum_i X_{ik} \right) \left(\frac{t_{kk'} \hat{\mu}_{k'}}{\bar{\mu}_k} \right)^{1-\rho}. \quad (\text{A.37})$$

Therefore, the importer fixed effect is

$FM_k = \log(X_k \bar{\mu}_k^{\rho-1})$, where $X_k = \sum_i X_{ik}$ is the total expenditure of country k , and the exporter fixed effect is $FX_{k'} = \log(\hat{\mu}_{k'}^{1-\rho})$.

Step 4. Gains from trade.

Denote λ_k the home share of country k , which is π_{ikk} in (A.36). Combining $\hat{\mu}_k = \hat{Z}_k w_k^{1-\beta} \bar{\mu}_k^\beta$ and $\lambda_k = \left(\frac{t_{kk} \hat{\mu}_k}{\bar{\mu}_k} \right)^{1-\rho}$, we can express the "access to suppliers" index $\bar{\mu}_k$ as

$$\bar{\mu}_k = w_k \left(t_{kk} \hat{Z}_k \lambda_k^{\frac{1}{\rho-1}} \right)^{\frac{1}{1-\beta}}. \quad (\text{A.38})$$

The price index for aggregate final consumption in country k is

$$P_k = \left(\sum_i \alpha_{ik} P_{ik}^{1-\sigma} \right)^{1/(1-\sigma)} = \left(\sum_i \alpha_{ik} (\tilde{\mu}_i \bar{\mu}_k)^{1-\sigma} \right)^{1/(1-\sigma)}$$

⁶Step 1 is needed because uniqueness for (A.34)-(A.35) alone does not rule out the logical possibility of having marginal costs not in the form $\mu_{ik} = \tilde{\mu}_i \hat{\mu}_k$.

$$= \underbrace{\bar{\mu}_k \left(\sum_i \alpha_{ik} \tilde{\mu}_i^{1-\sigma} \right)^{1/(1-\sigma)}}_{\Gamma_k} = w_k \lambda_k^{\frac{1}{(\rho-1)(1-\beta)}} \left(t_{kk} \hat{Z}_k \right)^{\frac{1}{1-\beta}} \Gamma_k. \quad (\text{A.39})$$

From this expression, the real wage w_k/P_k is proportional to $\lambda_k^{\frac{1}{(\rho-1)(1-\beta)}}$ which summarizes the effect of trade (Γ_k is exogenous). Once trade is balanced, welfare in this setup is given by the real wage. Hence the relative loss in welfare from moving to autarky with $\lambda_k = 1$ (which is changing $t_{kk'}$, $k' \neq k$, from the current levels to infinity) is given by $\lambda_k^{\frac{1}{(\rho-1)(1-\beta)}}$. ■

A.8 Derivation of (1.43) and (1.45)

Totally differentiating $\xi = \delta\eta + \beta\Omega \left[\xi + \log \left(\frac{\lambda^*}{\lambda} \right)^{\frac{1}{\rho-1}} \right]$,

$$d\xi = \eta d\delta + \beta\Omega \left[d\xi + \frac{1}{\rho-1} \left(\frac{d\lambda^*}{\lambda^*} - \frac{d\lambda}{\lambda} \right) \right],$$

which after applying (A.7) becomes

$$\frac{d\xi}{d\delta} = \eta + \beta\Omega \left[\frac{d\xi}{d\delta} + (1 - \lambda^*) \circ \frac{-d\xi}{d\delta} - (1 - \lambda) \circ \frac{d\xi}{d\delta} \right]. \quad (\text{A.40})$$

Evaluating (A.40) at $\delta = 0$ with $\lambda_i = \lambda_i^* = \lambda_0$ gives

$$\frac{d\xi}{d\delta} = \eta + \beta(2\lambda_0 - 1)\Omega \frac{d\xi}{d\delta}, \text{ or } \frac{d\xi}{d\delta} = (I - \beta(2\lambda_0 - 1)\Omega)^{-1} \eta.$$

Differentiating (A.40) again with respect to δ produces

$$\frac{d^2\xi}{d\delta^2} = \beta\Omega \left[\frac{d^2\xi}{d\delta^2} \circ (\lambda + \lambda^* - 1) + (\rho - 1) \left(\frac{d\xi}{d\delta} \right)^2 \circ (\lambda \circ (1 - \lambda) - \lambda^* \circ (1 - \lambda^*)) \right],$$

from which $\frac{d^2\xi_i}{d\delta^2} = 0$ at $\delta = 0$.

Derivation of (1.45) ■

$$\widetilde{var}(\xi) = (1 - \beta) \sum_i b_i \xi_i^2 - \left[(1 - \beta) \sum_i b_i \xi_i \right]^2 = (1 - \beta) \xi' D_b \xi - [(1 - \beta) b' \xi]^2.$$

$$\frac{d\widetilde{var}(\xi)}{d\delta} = (1 - \beta) 2 \left(\frac{d\xi}{d\delta} \right)' D_b \xi - 2(1 - \beta)^2 b' \xi \cdot b' \frac{d\xi}{d\delta}, \quad (\text{A.41})$$

which evaluated at $\delta = 0$ (so $\xi = 0$) is 0. Differentiating (A.41) again wrt δ and evaluating at $\delta = 0$: (using that $\frac{d^2\xi}{d\delta^2} = 0$)

$$\frac{d^2\widetilde{var}(\xi)}{d\delta^2} = (1 - \beta) 2 \left(\frac{d\xi}{d\delta} \right)' D_b \frac{d\xi}{d\delta} - 2(1 - \beta)^2 \left(b' \frac{d\xi}{d\delta} \right)^2. \quad (\text{A.42})$$

Denote $B = (I - \beta(2\lambda_0 - 1)\Omega)^{-1}$. The second term in (A.41) involves $b' B \eta = \eta' B' b$. Note first that $B' b \leq (I - \beta\Omega')^{-1} b$ because $\beta(2\lambda_0 - 1) \leq \beta$. Consider $\tilde{\alpha} = (1 - \beta) b$ for which $\|\tilde{\alpha}\|_2 \rightarrow 0$ by part (i) of Assumption 1. Part (ii) of Assumption 1 implies that $\|(I - \beta\Omega')^{-1} \tilde{\alpha}\|_2 \rightarrow 0$, so $\|B' b\|_2 \rightarrow 0$. Therefore, Lemma 1A applies to $b' B \eta$ and $b' B \eta \xrightarrow{p} 0$.

The first term in (A.41) is $(1 - \beta) 2\eta' B' D_b B \eta$. First,

$$\text{tr}[B' D_b B] \leq \text{tr}\left[(I - \beta\Omega')^{-1} D_b (I - \beta\Omega)^{-1}\right]$$

because $\beta(2\lambda_0 - 1) \leq \beta$. As shown below, for $A_1 = (I - \beta\Omega')^{-1} D_b (I - \beta\Omega)^{-1}$, $\text{tr}(A_1' A_1) \rightarrow 0$ under Assumption 1, so Lemma 1 applies to $\eta' B' D_b B \eta$ and $\eta' B' D_b B \eta \xrightarrow{p} \text{tr}(B' D_b B) \sigma_\eta^2$. The probability limit of the second-order approximation to $\widetilde{var}(\xi)$ is therefore

$$\widetilde{var}(\xi)^{2nd} = (1 - \beta) \text{tr}[B' D_b B] \sigma_\eta^2.$$

Proof that for $A_1 = (I - \beta\Omega')^{-1} D_b (I - \beta\Omega)^{-1}$, $\text{tr}(A_1' A_1) \rightarrow 0$ as $N \rightarrow \infty$. Observe that for any non-negative matrix its trace is less of equal to the sum of its elements. Also using that $\Omega \mathbf{1} = \mathbf{1}$, so that $(I - \beta\Omega)^{-1} \mathbf{1} = \frac{1}{1-\beta} \mathbf{1}$, we have

$$\begin{aligned}
tr(A'_1 A_1) &\leq \mathbf{1}' (I - \beta \Omega')^{-1} D_b (I - \beta \Omega)^{-1} (I - \beta \Omega')^{-1} D_b (I - \beta \Omega)^{-1} \mathbf{1} \\
&= \frac{1}{(1-\beta)^2} b' (I - \beta \Omega)^{-1} (I - \beta \Omega')^{-1} b \leq const \cdot \tilde{\alpha}' (I - \beta \Omega)^{-1} (I - \beta \Omega')^{-1} \tilde{\alpha} \\
&= const \cdot \tilde{b}' \tilde{b} = const \cdot \left\| \tilde{b} \right\|_2^2 \rightarrow 0.
\end{aligned}$$

The second line replaces b with $\tilde{\alpha} \propto b$ ("alternative consumption shares" in part (ii) of Assumption 1 for which $\|\tilde{\alpha}\|_2 \rightarrow 0$ because $\|b\|_2 \rightarrow 0$).

A.9 Derivation of (1.46)

Denote $B = \left(I - \tilde{\beta}(\tau) \Omega \right)^{-1} \cdot \frac{d}{d\tau} tr [B' D_b B] = tr \left[\frac{dB'}{d\tau} D_b B + B' D_b \frac{dB}{d\tau} \right] = 2 tr \left[B' D_b \frac{dB}{d\tau} \right]$. Using that $\frac{dA^{-1}}{d\tau} = -A^{-1} \frac{dA}{d\tau} A^{-1}$, $tr \left[B' D_b \frac{dB}{d\tau} \right] = \frac{d\tilde{\beta}(\tau)}{d\tau} tr [B' D_b B \Omega B] \geq 0$, as $\tilde{\beta}(\tau) = \beta(2\lambda_0 - 1)$ is increasing in τ and $tr [B' D_b B \Omega B]$ is non-negative.

A.10 Trace Properties

Bounds. First, from the fact that, for any non-negative matrix A , $tr(A) \leq \mathbf{1}' A \mathbf{1}$ and from expanding $\left(I - \tilde{\beta} \Omega \right)^{-1}$ the bounds on the trace in (1.45) are obtained:

$$1 \leq (1 - \beta) tr \left[\left(I - \tilde{\beta} \Omega' \right)^{-1} D_b \left(I - \tilde{\beta} \Omega \right)^{-1} \right] \leq \frac{1}{(1 - \tilde{\beta})^2}.$$

The upper bound is achieved for the pure diagonal I-O matrix $\Omega = I$, and the lower bound is achieved⁷ in the limit as $N \rightarrow \infty$ when $\omega_{ij} = \frac{1}{N}$ for all i, j .

⁷For $\Omega = \frac{1}{N} \mathbf{1}\mathbf{1}'$, $\left(I - \tilde{\beta} \Omega \right)^{-1} = I + \frac{\tilde{\beta}}{1-\tilde{\beta}} \frac{1}{N} \mathbf{1}\mathbf{1}'$. The trace in (1.47) is then $\frac{1}{1-\tilde{\beta}}$ plus terms proportional to $\frac{1}{N}$.

General approximation for this trace is obtained by expanding $(I - \tilde{\beta}\Omega)^{-1}$ as the sum $\sum_k \tilde{\beta}^k \Omega^k$, so that

$$(1 - \beta) \operatorname{tr} \left[(I - \tilde{\beta}\Omega')^{-1} D_b (I - \tilde{\beta}\Omega)^{-1} \right] = (1 - \beta) \sum_{l,m} \beta^{l+m} \operatorname{tr}(\Omega^l D_b \Omega^m).$$

Ignoring the terms containing $\tilde{\beta}^3$ and higher powers, we get $1 + 2\tilde{\beta}SSI + \tilde{\beta}^2(HHI + 2SLI)$.

Random networks. In simulations I find that when the I-O structure is generated as a random graph in which each industry has some number (random or deterministic) of randomly chosen suppliers, it appears that $\operatorname{tr}(\Omega^l D_b \Omega^m) \approx 0$ for $l \neq m$. This reflects the fact that production loops tend to be very weak in such randomly generated networks as the number of nodes grows large. More surprisingly, it appears that $\operatorname{tr}(\Omega^m D_b \Omega^m) \approx HHI^m$, provided that product-level input shares Herfindahls $HHI_i = \sum_j \omega_{ij}^2$ and b_i are uncorrelated. This is satisfied when suppliers for different industries are assigned independently. (Across industries $i = 1..N$, the a priori probability of becoming a supplier for a given industry i' can vary, but it should be unrelated to HHI_i). Therefore, another trace approximation which works well for randomly generated networks in which loops tend to be negligible is

$$(1 - \beta) \operatorname{tr} \left[(I - \tilde{\beta}\Omega')^{-1} D_b (I - \tilde{\beta}\Omega)^{-1} \right] \simeq 1 + \tilde{\beta}^2 HHI + .. = (1 - \tilde{\beta}^2 HHI)^{-1}. \quad (\text{A.43})$$

Figure A.3 illustrates that this approximation becomes near perfect for large N . For a fixed $\beta = 0.5$, it considers several I-O specifications which differ in the degree of supplier diversification but suppliers are chosen randomly and independently for each industry. The true value is $(1 - \beta) \operatorname{tr} \left[(I - \tilde{\beta}\Omega')^{-1} D_b (I - \tilde{\beta}\Omega)^{-1} \right]$, while the formula is $(1 - \tilde{\beta}^2 HHI)^{-1}$.

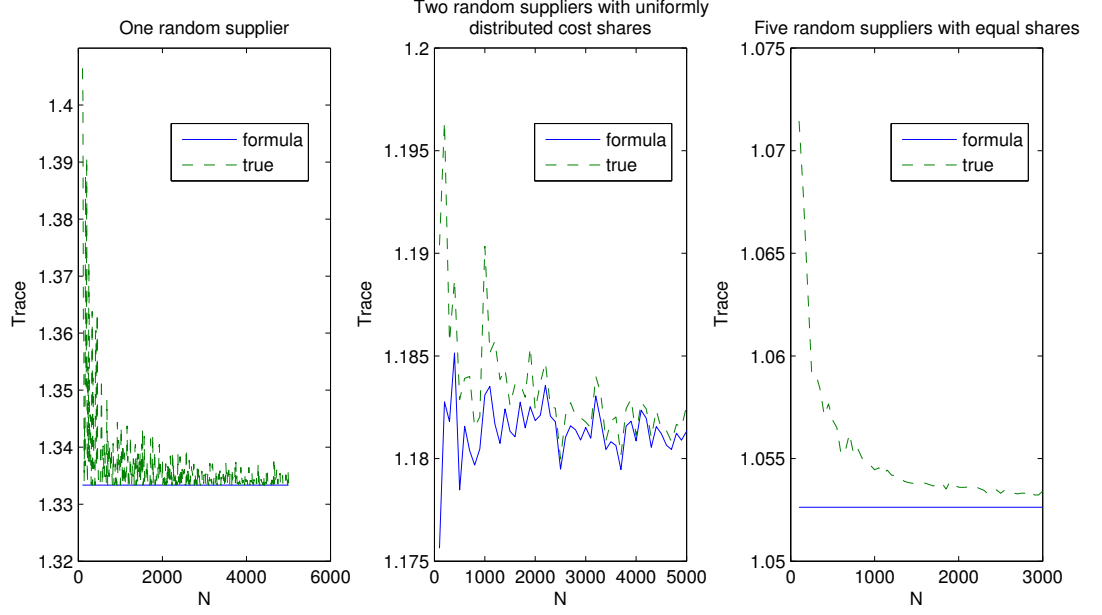


Figure A.3: Approximation quality of formula (A.43).

A.11 Proof of Proposition (1.4.1)

We have the gains from trade $g = \bar{p}^{Autarky} - \bar{p} = -b' \log \left(\lambda^{\frac{1}{\rho-1}} \right)$. At $\delta = 0$, $g = \frac{1}{1-\beta} \log \left(\lambda_0^{\frac{1}{1-\rho}} \right)$.

Let $\dot{x}$ denote $\frac{dx}{d\delta}$ and $\ddot{x}$ denote $\frac{d^2x}{d\delta^2}$ for generic variable x . Recall (1.19), so $\dot{\lambda}_i = (\rho - 1) \lambda_i (1 - \lambda_i) \dot{\xi}_i$ and

$$\dot{g} = - \sum_i b_i (1 - \lambda_i) \dot{\xi}_i. \quad (\text{A.44})$$

Evaluated at $\delta = 0$, it is $(1 - \lambda_0) b' \dot{\xi} = (1 - \lambda_0) b' \left(I - \tilde{\beta} \Omega \right)^{-1} \eta$. As argued after (A.42), $b' \left(I - \tilde{\beta} \Omega \right)^{-1} \eta \xrightarrow{p} 0$.

Differentiating (A.44) again,

$$\ddot{g} = \sum_i b_i (\rho - 1) \lambda_i (1 - \lambda_i) \left(\dot{\xi}_i \right)^2 - \sum_i b_i (1 - \lambda_i) \ddot{\xi}_i. \quad (\text{A.45})$$

Evaluating at $\delta = 0$ (with $\ddot{\xi}_i = 0$),

$$\ddot{g} = (\rho - 1) \lambda_0 (1 - \lambda_0) \left(\dot{\xi} \right)' D_b \dot{\xi}.$$

As shown after (A.42), $\left(\dot{\xi} \right)' D_b \dot{\xi} \xrightarrow{p} \text{tr} (B' D_b B) \sigma_\eta^2$ with $B = \left(I - \widetilde{\beta} \Omega \right)^{-1}$. In other words, the probability limit of the second-order approximation to g is

$$g^{2nd} = \frac{1}{1 - \beta} \left[\log \left(\lambda_0^{\frac{1}{1-\rho}} \right) + \frac{1}{2} (\rho - 1) \lambda_0 (1 - \lambda_0) \widetilde{\text{var}} (\xi)^{2nd} \right].$$

A.12 Proof of Proposition (1.4.2)

Using that $\frac{d\lambda_0}{d\tau} = (\rho - 1) \lambda_0 (1 - \lambda_0)$,

$$M^{2nd} = -\frac{d}{d\tau} g^{2nd} =$$

$$-\frac{-(1 - \lambda_0) + \frac{1}{2} (\rho - 1)^2 (1 - 2\lambda_0) \lambda_0 (1 - \lambda_0) \widetilde{\text{var}} (\xi)^{2nd} + \frac{1}{2} (\rho - 1) \lambda_0 (1 - \lambda_0) \frac{d\widetilde{\text{var}} (\xi)^{2nd}}{d\tau}}{1 - \beta}.$$

Now consider $b'(\mathbf{1} - \lambda)$. Its first-order approximation is $-(\rho - 1) \sum_i b_i \lambda_i (1 - \lambda_i) \dot{\xi}_i$ and its second-order approximation (evaluated at $\delta = 0$) is

$$-(\rho - 1)^2 \sum_i b_i (1 - 2\lambda_i) \lambda_i (1 - \lambda_i) \left(\dot{\xi}_i \right)^2 - (\rho - 1) \sum_i b_i \lambda_i (1 - \lambda_i) \ddot{\xi}_i =$$

$$(\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \sum_i b_i \left(\dot{\xi}_i \right)^2.$$

Comparing to Appendix (1.4.1), the second-order approximation for $b'(\mathbf{1} - \lambda)$ is

$$\frac{1 - \lambda_0}{1 - \beta} + \frac{1}{1 - \beta} \frac{1}{2} (\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \widetilde{\text{var}} (\xi)^{2nd}.$$

Comparing $M^{(1)}$ and $M^{(2)}$

Let $\widetilde{var}(\xi)^{2nd}$ be for a given I-O network that has non-zero HHI , while its value for the complete symmetric network is σ_z^2 .

We have $M^{(1)} = \frac{1}{1-\beta} \left[1 - \lambda_0 + \frac{1}{2} (\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \widetilde{var}(\xi)^{2nd} \right]$, so

$$\Delta M^{(1)} = \frac{1}{1-\beta} \frac{1}{2} (\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \left[\widetilde{var}(\xi)^{2nd} - \sigma_z^2 \right].$$

Similarly, $\frac{d\widetilde{var}(\xi)^{2nd}}{d\tau}$ is for a given network, while for the complete network benchmark this value is zero:

$$\Delta M^{(2)} = \frac{1}{1-\beta} \frac{1}{2} (\rho - 1) \lambda_0 (1 - \lambda_0) \frac{d\widetilde{var}(\xi)^{2nd}}{d\tau}.$$

Note that $\frac{d\widetilde{var}(\xi)^{2nd}}{d\tau} = \frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0} \frac{d\lambda_0}{d\tau} = (\rho - 1) \lambda_0 (1 - \lambda_0) \frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}$, so

$$\Delta M^{(2)} = \frac{1}{1-\beta} \frac{1}{2} (\rho - 1)^2 \lambda_0^2 (1 - \lambda_0)^2 \frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}.$$

Now we can see that the covariance term $M^{(2)}$ is very weak for high trade costs, as it is proportional to $(1 - \lambda_0)^2$, and is negligible relative to $M^{(1)}$:

$$\lim_{\tau \rightarrow \infty} \frac{\Delta M^{(2)}}{\Delta M^{(1)}} = \lim_{\tau \rightarrow \infty} \frac{-\lambda_0 (1 - \lambda_0) \frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}}{(2\lambda_0 - 1) \left[\widetilde{var}(\xi)^{2nd} - \sigma_z^2 \right]} = 0, \quad (\text{A.46})$$

because $\frac{d\widetilde{var}(\xi)^{2nd}}{d\lambda_0}$ remains bounded and $\lim_{\tau \rightarrow \infty} \left[\widetilde{var}(\xi)^{2nd} - \sigma_z^2 \right] > 0$.

Near free trade ($\lambda_0 = \frac{1}{2}$), in contrast, the covariance contribution of $M^{(2)}$ dominates, because $\widetilde{var}(\xi)^{2nd}$ in $M^{(1)}$ departs slowly from its free-trade level σ_z^2 . To see this, we can apply approximation (1.48), which is precise for small trade costs:

$$1 + 2\tilde{\beta}SSI + \tilde{\beta}^2(HHI + 2SLI)$$

$$\widetilde{var}(\xi)^{2nd} = \left[1 + 2\tilde{\beta}SSI + \tilde{\beta}^2(HHI + 2SLI) \right] \sigma_z^2 + o(\tilde{\beta}^3),$$

where $\tilde{\beta} = \beta(2\lambda_0 - 1)$.

We have in approximation

$$\Delta M^{(1)} = \frac{1}{1-\beta} \frac{1}{2} (\rho - 1)^2 (2\lambda_0 - 1) \lambda_0 (1 - \lambda_0) \left[2\tilde{\beta}SSI + \tilde{\beta}^2 (HHI + 2SLI) \right] \sigma_z^2, \text{ or}$$

$$\Delta M^{(1)} = \frac{1}{1-\beta} \frac{1}{2} (\rho - 1)^2 \beta (2\lambda_0 - 1)^2 \lambda_0 (1 - \lambda_0) \left[2SSI + \tilde{\beta} (HHI + 2SLI) \right] \sigma_z^2.$$

Also $\frac{d\widehat{var}(\xi)^{2nd}}{d\lambda_0}$ is approximated as

$$\left[2SSI + 2\tilde{\beta} (HHI + 2SLI) \right] \frac{d\tilde{\beta}}{d\lambda_0} \sigma_z^2 = 4\beta \left[SSI + \tilde{\beta} (HHI + 2SLI) \right] \sigma_z^2$$

and then

$$\Delta M^{(2)} = \frac{1}{1-\beta} 2\beta (\rho - 1)^2 \lambda_0^2 (1 - \lambda_0)^2 \left[SSI + \tilde{\beta} (HHI + 2SLI) \right] \sigma_z^2.$$

Consider two cases. If $SSI > 0$, then $\Delta M^{(2)}$ clearly dominates $\Delta M^{(1)}$ at $\tau \rightarrow 0$ because $\Delta M^{(2)}$ remains positive while $\Delta M^{(1)} \rightarrow 0$ as $\lambda_0 \rightarrow \frac{1}{2}$. If $SSI = 0$, then

$$\lim_{\tau \rightarrow 0} \frac{\Delta M^{(1)}}{\Delta M^{(2)}} = \lim_{\tau \rightarrow 0} \frac{\frac{1}{2} (2\lambda_0 - 1)}{2\beta \lambda_0 (1 - \lambda_0)} = 0.$$

Appendix B

Proofs for Chapter 2

B.1 Proofs for Section 2.2

B.1.1 Planner's Problem

Denote Λ^H , Λ^F , and Λ^{HF} the set of industries for which the planners decides to pay fixed costs in Home only, in Foreign only, and in both countries. Consumption is denoted with q_i and production with x_i . If $i \in \Lambda^H$, then $x_i^* = 0$ as production in Foreign is not possible. Similarly, $x_i = 0$ for $i \in \Lambda^F$. For $i \in \Lambda^{HF}$, it is obvious that no trade is optimal since the products are homogenous,¹ so we can write from the beginning $x_i = q_i$, $x_i^* = q_i^*$. When the planner treats the two countries equally, the problem can be expressed as

$$\max_{S, S^*, q_i, q_i^*, \Lambda^H, \Lambda^F, \Lambda^{HF}, x_i, x_i^*, L_s, L_s^*} U(S, M(\{q_i\})) + U(S^*, M^*(\{q_i^*\})) \text{ s.t.}$$

$$S = z_s L_s, S^* = z_s L_s^*, \quad |\mu_S, \mu_S^* \tag{B.1}$$

$$x_i = q_i, x_i^* = q_i^* \text{ for } i \in \Lambda^{HF}, \quad |\mu_i, \mu_i^* \tag{B.2}$$

¹No trade is the only optimal solution under positive trade costs.

$$x_i = q_i + \tau q_i^*, x_i^* = 0 \text{ for } i \in \Lambda^H, |\mu_i, \mu_i^*| \quad (\text{B.3})$$

$$x_i^* = q_i^* + \tau q_i, x_i = 0 \text{ for } i \in \Lambda^F, |\mu_i, \mu_i^*| \quad (\text{B.4})$$

$$L_s + \int_{i \in \Lambda^H \cup \Lambda^{HF}} q_i c_i di + \int_{i \in \Lambda^H} \tau q_i^* c_i di + f \cdot \int_{i \in \Lambda^H \cup \Lambda^{HF}} di = L, |w| \quad (\text{B.5})$$

$$L_s^* + \int_{i \in \Lambda^F \cup \Lambda^{HF}} q_i^* c_i^* di + \int_{i \in \Lambda^F} \tau q_i c_i^* di + f \cdot \int_{i \in \Lambda^F \cup \Lambda^{HF}} di = L, |w^*| \quad (\text{B.6})$$

Utility U and manufacturing subutility M are as given by (2.1) and (2.2). The last two equations are labor resource constraints ($f \cdot \int_{i \in \Lambda^H \cup \Lambda^{HF}} di$ units of Home labor must be spent to open the industries $\Lambda^H \cup \Lambda^{HF}$ in Home).

The problem of optimal assignments of industries to countries is fundamentally similar to DFS. Given that products are ordered such that the Home's comparative advantage is declining ($c'_i > 0$), the regions $\Lambda^H, \Lambda^F, \Lambda^{HF}$ are determined by two cutoffs λ and λ^* , so that Home has industries $i \in [0, \lambda]$ and Foreign has industries $i \in [\lambda^*, 1]$. We have that $\Lambda^H = [0, \lambda^*]$, $\Lambda^F = [\lambda, 1]$, and $\Lambda^{HF} = [\lambda^*, \lambda]$. It can be shown that if a positive mass of industries violates this cutoff rule, consumption of a positive mass of goods in at least one country can be strictly increased without decreasing consumption of anything else.

Furthermore, given the symmetry of the setup, Home and Foreign have equal masses of active industries and therefore $\lambda^* = 1 - \lambda$. I focus on the case when all products are consumed, so each industry is active in at least one country, so $\lambda \in [\frac{1}{2}, 1]$. This restriction is a mild one, unless the elasticity of substitution σ is too high for a given value of f .

First consider optimal resource allocation conditional on λ . The Lagrange multipliers are assigned as indicated in (B.1)-(B.6). Observe that this stage can actually be decentralized by considering a hypothetical economy in which all markets are per-

fect and consumers supply $L - \lambda f$ units of labor. With constant returns to scale and nested CES preferences, we can immediately express welfare as (2.7) simply by applying the standard CES price index formula.

To obtain (2.9), first take the log of U and then use the equilibrium conditions from the stage two competitive economy analogy that $L - \lambda f = z_s S + PM$ (wages are normalized to one) and $\frac{z_s S}{PM} = \left(\frac{\alpha}{1-\alpha}\right)^\eta \left(\frac{z_s}{P}\right)^{1-\eta}$.

$$\log U = \log(L - \lambda f) - \frac{1}{1-\eta} \log(\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}).$$

$$\frac{\partial}{\partial \lambda} \log U = \frac{-f}{L - \lambda f} - \frac{(1-\alpha)^\eta P^{-\eta}}{\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}} \frac{\partial}{\partial \lambda} P = 0 \Rightarrow$$

$$(1-\alpha)^\eta P^{-\eta} \frac{\partial}{\partial \lambda} P = \frac{-f}{L - \lambda f} (\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}) \Rightarrow$$

$$\frac{1}{P} \frac{\partial}{\partial \lambda} P = \frac{-f}{L - \lambda f} \left(\left(\frac{\alpha}{1-\alpha}\right)^\eta \left(\frac{z_s}{P}\right)^{1-\eta} + 1 \right) = \frac{-f}{L - \lambda f} \frac{z_s S + PM}{PM} \Rightarrow$$

$$\frac{\partial}{\partial \lambda} P = -\frac{f}{M}.$$

For values of λ satisfying $\frac{\partial}{\partial \lambda} U(z_s, z_m, \lambda) = 0$, $\frac{\partial^2}{\partial \lambda^2} U(z_s, z_m, \lambda) < 0$, so the first order condition is sufficient for a local interior maximum $\lambda \in (\frac{1}{2}, 1)$. High enough f or low enough z_m/z_s implies either the corner solution $\lambda = \frac{1}{2}$ or $\lambda < \frac{1}{2}$. These two cases are different. The latter is assumed away as abnormal, while the former necessarily arises for any $f > 0$ as z_m/z_s goes to zero even if $\sigma \leq 1$, see the proof in Appendix B.1.3.

B.1.2 Proof of Proposition 2.2.1

Using (2.4), (2.5), and (2.3),

$$\begin{aligned}
\frac{Export}{Manufacturing} &= \frac{\int_0^{1-\lambda} \tau q_i^* c_i di}{PM} = \frac{MP^\sigma \int_0^{1-\lambda} (\tau c_i)^{1-\sigma} di}{PM} = \frac{\int_0^{1-\lambda} (\tau c_i)^{1-\sigma} di}{P^{1-\sigma}} = \\
&= \frac{\int_0^{1-\lambda} (\tau c_i)^{1-\sigma} di}{\int_0^\lambda c_i^{1-\sigma} di + \int_\lambda^1 (\tau c_i^*)^{1-\sigma} di} = \frac{\int_0^{1-\lambda} (\tau c_i)^{1-\sigma} di}{\int_0^\lambda c_i^{1-\sigma} di + \int_0^{1-\lambda} (\tau c_i)^{1-\sigma} di} = \left(1 + \frac{\int_0^\lambda \tilde{c}_i^{1-\sigma} di}{\int_0^{1-\lambda} (\tau \tilde{c}_i)^{1-\sigma} di} \right)^{-1}.
\end{aligned}$$

In this final expression, $\int_0^\lambda \tilde{c}_i^{1-\sigma} di$ is clearly increasing in λ and $\int_0^{1-\lambda} (\tau \tilde{c}_i)^{1-\sigma} di$ is decreasing, hence a lower specialization cutoff corresponds to a higher trade share. ■

Proof of Proposition 2.2.2

It is sufficient to look at the effect of z_m keeping z_s constant. Define $u(z_s, z_m, \lambda) = \log U(z_s, z_m, \lambda)$. Maximizing u is equivalent to maximizing U .

$$u(z_s, z_m, \lambda) = \log(L - \lambda f) - \frac{1}{1-\eta} \log(\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}).$$

From (2.5) and (2.3), the manufacturing cost index can be written as

$$P = z_m \tilde{P} = z_m \left(\int_0^\lambda \tilde{c}_i^{1-\sigma} di + \int_\lambda^1 (\tau \tilde{c}_i^*)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

$$\frac{\partial}{\partial z_m} u = - \frac{(1-\alpha)^\eta \tilde{P}^{1-\eta} z_m^{-\eta}}{\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}}$$

$$\log \left(- \frac{\partial}{\partial z_m} u \right) = \log(1-\alpha)^\eta + (1-\eta) \log \tilde{P} - \eta \log(z_m) - \log \left(\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta z_m^{1-\eta} \tilde{P}^{1-\eta} \right)$$

$$\begin{aligned}
\frac{\partial}{\partial \lambda} \log \left(-\frac{\partial}{\partial z_m} u \right) &= \frac{1-\eta}{\tilde{P}} \frac{\partial}{\partial \lambda} \tilde{P} - \frac{(1-\eta)(1-\alpha)^\eta z_m^{1-\eta} \tilde{P}^{-\eta}}{\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta z_m^{1-\eta} \tilde{P}^{1-\eta}} \frac{\partial}{\partial \lambda} \tilde{P} = \\
&= (1-\eta) z_m \frac{\partial}{\partial \lambda} \tilde{P} \left(\frac{1}{\tilde{P}} - \frac{(1-\alpha)^\eta P^{-\eta}}{\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}} \right) = \\
&= \frac{1-\eta}{P} \frac{\alpha^\eta z_s^{1-\eta}}{\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}} \frac{\partial}{\partial \lambda} P < 0
\end{aligned}$$

which is because of (2.9).

$$\text{sgn} \left(\frac{d\lambda}{dz_m} \right) = \text{sgn} \left(\frac{\partial^2}{\partial z_m \partial \lambda} u(z_s, z_m, \lambda) \right) = -\text{sgn} \left(\frac{\partial}{\partial \lambda} \ln \left(-\frac{\partial}{\partial z} u(z_s, z_m, \lambda) \right) \right) = +.$$

■

B.1.3 Proof of Proposition 2.2.3

As in Appendix B.1.1, now using $P = z_m \tilde{P}$,

$$\frac{\partial}{\partial \lambda} \log U = \frac{-f}{L - \lambda f} - \frac{(1-\alpha)^\eta \tilde{P}^{-\eta}}{\alpha^\eta z_s^{1-\eta} + (1-\alpha)^\eta P^{1-\eta}} z_m^{1-\eta} \frac{\partial}{\partial \lambda} \tilde{P}.$$

With $z_m \rightarrow 0$, $\frac{\partial}{\partial \lambda} \log U$ converges to $\frac{-f}{L - \lambda f} < 0$. Therefore, the planner's problem does not have interior solutions for sufficiently small z_m (keeping z_s constant for simplicity; what matters is the ratio z_m/z_s). But $\log U(z_s, 0, 1) < \log U(z_s, 0, \frac{1}{2})$, so $\lambda = 1$ is not a solution. Therefore, low enough z_m implies setting $\lambda = \frac{1}{2}$.

■

B.2 Proofs for Section 2.3

B.2.1 Proof of Proposition 2.3.1

To save on notation, replace $\tilde{P}$ with P , just keeping z_m constant. From (2.15), $P'_\lambda =$

$$P^\sigma \frac{\left(\min\left\{\frac{\sigma}{\sigma-1}c_{1-\lambda}, \tau c_{1-\lambda}^*\right\}\right)^{1-\sigma} - \left(\frac{\sigma}{\sigma-1}c_{1-\lambda}\right)^{1-\sigma} + \left(\min\left\{\frac{\sigma}{\sigma-1}c_\lambda, \tau c_\lambda^*\right\}\right)^{1-\sigma} - \left(\frac{\sigma}{\sigma-1}\tau c_\lambda^*\right)^{1-\sigma}}{1-\sigma}.$$

As $\min\left\{\frac{\sigma}{\sigma-1}c_{1-\lambda}, \tau c_{1-\lambda}^*\right\} \leq \frac{\sigma}{\sigma-1}c_{1-\lambda}$ and $\min\left\{\frac{\sigma}{\sigma-1}c_\lambda, \tau c_\lambda^*\right\} \leq \tau c_\lambda^* < \frac{\sigma}{\sigma-1}\tau c_\lambda^*$, raising each expression to the power $1-\sigma < 0$ flips the side of the inequalities, which yields $P'_\lambda < 0$. Note that the resulting inequality is strict because switching from oligopoly to monopoly in the industry $i = \lambda$ in Home strictly increases p_λ . ■

B.2.2 Proof of Proposition 2.3.2

Using (2.12), (2.15), and (2.13),

$$\begin{aligned} \frac{Export}{Manufacturing} &= \frac{\int_0^{1-\lambda} p_i^* q_i^* di}{PM} = \frac{MP^\sigma \int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1}\tau c_i\right)^{1-\sigma} di}{PM} = \frac{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1}\tau c_i\right)^{1-\sigma} di}{P^{1-\sigma}} = \\ &= \frac{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1}\tau c_i\right)^{1-\sigma} di}{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1}c_i\right)^{1-\sigma} di + \int_{1-\lambda}^\lambda \left(\min\left\{\frac{\sigma}{\sigma-1}c_i, \tau c_i^*\right\}\right)^{1-\sigma} di + \int_\lambda^1 \left(\frac{\sigma}{\sigma-1}\tau c_i^*\right)^{1-\sigma} di} = \end{aligned}$$

$$\begin{aligned}
&= \frac{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1} \tau c_i\right)^{1-\sigma} di}{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1} c_i\right)^{1-\sigma} di + \int_{1-\lambda}^{\lambda} \left(\min\left\{\frac{\sigma}{\sigma-1} c_i, \tau c_i^*\right\}\right)^{1-\sigma} di + \int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1} \tau c_i\right)^{1-\sigma} di} = \\
&= \left(1 + \tau^{\sigma-1} + \frac{\int_{1-\lambda}^{\lambda} \left(\min\left\{\frac{\sigma}{\sigma-1} \tilde{c}_i, \tau \tilde{c}_i^*\right\}\right)^{1-\sigma} di}{\int_0^{1-\lambda} \left(\frac{\sigma}{\sigma-1} \tau \tilde{c}_i\right)^{1-\sigma} di} \right)^{-1}.
\end{aligned}$$

By a reason similar to that in Appendix B.1.2, this expression is decreasing in λ . ■

B.2.3 Proof of Proposition 2.3.3

First we prove that the “base profit” of the marginal firm $(p_\lambda - c_\lambda) p_\lambda^{-\sigma}$, which is proportional to its actual profit $(p_\lambda - c_\lambda) M \left(\frac{p_\lambda}{P}\right)^{-\sigma}$, falls as the firm’s efficiency deteriorates (λ increases). Consider separately the two cases when price setting is not constrained by the foreign competitor and when it is. Case 1: $\frac{\sigma}{\sigma-1} c_\lambda < \tau c_\lambda^*$, so $p_\lambda = \min\left\{\frac{\sigma}{\sigma-1} c_\lambda, \tau c_\lambda^*\right\} = \frac{\sigma}{\sigma-1} c_\lambda$. Then $(p_\lambda - c_\lambda) p_\lambda^{-\sigma} = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} c_\lambda^{1-\sigma}$ is decreasing, as $c'_\lambda > 0$. Case 2: $\frac{\sigma}{\sigma-1} c_\lambda > \tau c_\lambda^*$, so $p_\lambda = \min\left\{\frac{\sigma}{\sigma-1} c_\lambda, \tau c_\lambda^*\right\} = \tau c_\lambda^*$. Then $(p_\lambda - c_\lambda) p_\lambda^{-\sigma} = (\tau c_\lambda^* - c_\lambda) (\tau c_\lambda^*)^{-\sigma}$. The derivative is

$$(\tau c_\lambda^*)^{-\sigma} \left(-c'_\lambda + c_\lambda^* \left(\sigma \frac{c_\lambda}{c_\lambda^*} + (1-\sigma) \tau \right) \right) < 0,$$

which is implied by $\frac{\sigma}{\sigma-1} c_\lambda > \tau c_\lambda^*$. Considering case 3 $\frac{\sigma}{\sigma-1} c_\lambda = \tau c_\lambda^*$ is redundant because of continuity. Therefore, the “base profit” falls in λ independently of the mode of markup.

Now we can prove the comparative statics result of Proposition 2.3.3. The labor market clearing condition in Home is

$$L_s + L_m^v + \lambda f = L, \quad (\text{B.7})$$

where $L_s = z_s S$ is employment in services and total manufacturing employment $L_m = L_m^v + \lambda f$ consists of labor spending on variable and fixed costs. The first component can be decomposed into three parts:

$$L_m^v = \int_0^{1-\lambda} q_i c_i di + \int_0^{1-\lambda} q_i^* \tau c_i di + \int_{1-\lambda}^{\lambda} q_i c_i di,$$

in which the first two parts are labor spending by Home monopolists serving Home and Foreign markets, and the last part is labor spending by Home oligopolists (facing competition from abroad) serving the domestic market. As soon as demand for manufactures in both countries is proportional to MP^σ , using the equilibrium prices we obtain

$$L_m^v = MP^\sigma \left((1 + \tau^{1-\sigma}) \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} \int_0^{1-\lambda} c_i^{1-\sigma} di + \int_{1-\lambda}^{\lambda} \left(\min \left\{ \frac{\sigma}{\sigma-1} c_i, \tau c_i^* \right\} \right)^{-\sigma} c_i di \right).$$

Using this result and the equilibrium condition $\frac{z_s S}{PM} = \left(\frac{\alpha}{1-\alpha} \right)^\eta \left(\frac{z_s}{P} \right)^{1-\eta}$, we can write (B.7) as

$$MP^\sigma \left(\left(\frac{\alpha}{1-\alpha} \right)^\eta z_s^{1-\eta} P^{\eta-\sigma} + \kappa \int_0^{1-\lambda} c_i^{1-\sigma} di + \int_{1-\lambda}^{\lambda} \left(\min \left\{ \frac{\sigma}{\sigma-1} c_i, \tau c_i^* \right\} \right)^{-\sigma} c_i di \right) + \lambda f = L,$$

where the constant $\kappa = (1 + \tau^{1-\sigma}) \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma}$.

Next use (2.17) to substitute for MP^σ and apply the $x = z_m \tilde{x}$ notation. This yields

$$\left(\frac{\alpha}{1-\alpha}\right)^\eta \left(\frac{z_s}{z_m}\right)^{1-\eta} \tilde{P}^{\eta-\sigma} + \kappa \int_0^{1-\lambda} \tilde{c}_i^{1-\sigma} di + \int_{1-\lambda}^\lambda (\min\{\frac{\sigma}{\sigma-1}\tilde{c}_i, \tau\tilde{c}_i^*\})^{-\sigma} \tilde{c}_i di = \frac{L-\lambda f}{f} (\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma}.$$

Note that z_m/z_s now only appears in one term. The right hand side is decreasing in λ (both $\frac{L-\lambda f}{f}$ and $(\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma}$ are). By Proposition 2.3.1, the manufacturing price index is decreasing in λ , hence $\tilde{P}^{\eta-\sigma}$ is increasing ($\eta < 1 < \sigma$). Next,

$$\frac{\partial}{\partial \lambda} \left(\kappa \int_0^{1-\lambda} c_i^{1-\sigma} di + \int_{1-\lambda}^\lambda (\min\{\frac{\sigma}{\sigma-1}c_i, \tau c_i^*\})^{-\sigma} c_i di \right) > 0 \quad (\text{B.8})$$

would give a sufficient condition that the whole left hand side is increasing in λ . Taking the derivative, (B.8) becomes

$$\begin{aligned} & (\min\{\frac{\sigma}{\sigma-1}c_{1-\lambda}, \tau c_{1-\lambda}^*\})^{-\sigma} c_{1-\lambda} - \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_{1-\lambda}^{1-\sigma} + \\ & + (\min\{\frac{\sigma}{\sigma-1}c_\lambda, \tau c_\lambda^*\})^{-\sigma} c_\lambda - \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (\tau c_{1-\lambda})^{1-\sigma} > 0. \end{aligned}$$

Showing that $(\min\{\frac{\sigma}{\sigma-1}c_{1-\lambda}, \tau c_{1-\lambda}^*\})^{-\sigma} c_{1-\lambda} \geq \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_{1-\lambda}^{1-\sigma}$ is similar to what we have done above for the “base profit”.² Using the mirror symmetry of c_i and c_i^* , we rewrite the remaining condition as $(\min\{\frac{\sigma}{\sigma-1}c_\lambda, \tau c_\lambda^*\})^{-\sigma} c_\lambda > \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (\tau c_\lambda^*)^{1-\sigma}$ and verify that it is always satisfied.³

By now we have shown that the left (right) hand side of the final equilibrium condition

²If $\frac{\sigma}{\sigma-1}c_{1-\lambda} \leq \tau c_{1-\lambda}^*$, it is satisfied as equality. In the opposite case it becomes $(\tau c_{1-\lambda}^*)^{-\sigma} c_{1-\lambda} \geq \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_{1-\lambda}^{1-\sigma}$, which follows from $\frac{\sigma}{\sigma-1}c_{1-\lambda} > \tau c_{1-\lambda}^*$.

³For $\frac{\sigma}{\sigma-1}c_\lambda \geq \tau c_\lambda^*$, this inequality is $(\tau c_\lambda^*)^{-\sigma} c_\lambda > \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (\tau c_\lambda^*)^{1-\sigma}$, or $\frac{\sigma}{\sigma-1}c_\lambda \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} > \tau c_\lambda^*$ (implied by $\frac{\sigma}{\sigma-1}c_\lambda \geq \tau c_\lambda^*$ and $\sigma > 1$). For $\frac{\sigma}{\sigma-1}c_\lambda < \tau c_\lambda^*$, it is $\left(\frac{\sigma}{\sigma-1}c_\lambda\right)^{-\sigma} c_\lambda > \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (\tau c_\lambda^*)^{1-\sigma}$, which is simply $c_\lambda < \tau c_\lambda^*$. The last condition holds as soon as $f > 0$ (specialization regions in the model are narrower than in DFS or BEJK).

$$\left(\frac{\alpha}{1-\alpha}\right)^\eta \left(\frac{z_s}{z_m}\right)^{1-\eta} \tilde{P}^{\eta-\sigma} + \kappa \int_0^{1-\lambda} \tilde{c}_i^{1-\sigma} di + \int_{1-\lambda}^\lambda \left(\min\left\{\frac{\sigma}{\sigma-1}\tilde{c}_i, \tau\tilde{c}_i^*\right\}\right)^{-\sigma} \tilde{c}_i di = \frac{L - \lambda f}{f} (\tilde{p}_\lambda - \tilde{c}_\lambda) \tilde{p}_\lambda^{-\sigma}$$

is increasing (decreasing) in λ . A decline in z_m/z_s shifts the left hand side upwards, yielding a lower value of λ in the intersection, which completes the proof. ■

B.3 A Model with Cournot Competition

This part of the paper formulates a model with quantity competition instead of price competition in Section 2.3 and gives a sketch of the solution.

The technology is the same. For simplicity, assume Leontief preferences: $U(S, M) = \min\{\alpha S, M\}$ and set $z_s = 1$. Markets in Home and Foreign are again segmented; if a firm is the only supplier of good i in a country, it acts in that market as a monopolist;⁴ if there are two suppliers, they engage in Cournot competition.

The partial equilibrium problem of solving a two-firm quantity competition model is standard. With CES preferences, despite cost asymmetry, it is still possible to obtain a closed-form solution given by the following

Proposition B.3.1. *Consider the following partial equilibrium problem. The demand for a good is given by $Q = AP^{-\sigma}$, $\sigma > 1$. There are two firms with constant marginal costs z_1 and z_2 . Then: A) If $\frac{\sigma-1}{\sigma} \leq \frac{z_2}{z_1} \leq \frac{\sigma}{\sigma-1}$, Cournot competition yields the equilibrium price, quantities and profits given by $P = \frac{\sigma}{2\sigma-1} (z_1 + z_2)$, $q_j = A \left(\frac{2\sigma-1}{\sigma}\right)^\sigma \frac{(1-\sigma)z_j + \sigma z_{-j}}{(z_1 + z_2)^{1+\sigma}}$, $\Pi_j = A \left(\frac{2\sigma-1}{\sigma}\right)^\sigma \frac{((1-\sigma)z_j + \sigma z_{-j})^2}{(2\sigma-1)(z_1 + z_2)^{1+\sigma}}$. B) If $\frac{z_2}{z_1} > \frac{\sigma}{\sigma-1}$, only the first firm survives and monopoly equilibrium is described by $P = \frac{\sigma}{\sigma-1} z_1$, $q_1 = A \left(\frac{\sigma}{\sigma-1} z_1\right)^{-\sigma}$, $\Pi_1 = A \left(\frac{\sigma-1}{\sigma}\right)^\sigma \frac{z_1^{1-\sigma}}{\sigma-1}$.*

⁴This assumption again limits the analysis to considering $\sigma > 1$. By having more firms in each industry from each country it can be relaxed, but at costs of more cumbersome derivations.

The proof uses that best-response functions can be written as $P = \frac{\sigma}{\sigma-s_1}z_1 = \frac{\sigma}{\sigma-s_2}z_2$ where $s_1 = 1 - s_2 = \frac{q_1}{q_1+q_2}$.

Firms face marginal costs (2.3) with the same properties as in Section 2.2 (wages are normalized to one). To cover the complete variety of cases, assume now that there is enough variation in comparative advantage, such that $\frac{c_0^*}{c_0} = \frac{c_1}{c_0} > \frac{\sigma}{\sigma-1}\tau$. This will guarantee that structural change affects specialization and trade for arbitrary small fixed costs. (This assumption is not crucial to generate trade growth as a result of structural change.)

Using the above proposition (call it lemma) we can already characterize the trade pattern which would prevail in this economy with zero fixed costs. Home firms decide to enter in the range $i \in [0, \lambda]$ and firms in the range $i \in [0, \mu]$ also export, $\mu \leq \lambda$. Symmetrically, Foreign produces $i \in [1 - \lambda, 1]$ and exports $i \in [1 - \mu, 1]$. For $i \in [0, 1 - \lambda)$ the Home firm has such a strong comparative advantage that the Foreign one cannot engage in Cournot competition even taking into account that c_i is adjusted by τ . That is, in this range $\frac{c_i^*}{c_i} > \frac{\sigma}{\sigma-1}\tau$ and the Home firm is monopolist in both markets. The marginal Foreign firm that exists is determined by $\frac{\sigma}{\sigma-1}\tau c_{1-\lambda} = c_{1-\lambda}^*$. In the range $i \in [1 - \lambda, \mu)$ Home firms are still competitive enough to export, but now they face competition and have lower profits from exporting. The marginal Home exporter is determined by $\tau c_\mu = \frac{\sigma}{\sigma-1}c_\mu^*$.

Depending on demand elasticity and transportation costs, there can be two different configurations in the middle of the interval $[0, 1]$. If $\tau < \frac{\sigma}{\sigma-1}$, that is, transportation costs are lower than monopoly markup, the exporting cutoff $\mu > \frac{1}{2}$ and there is two-way trade in the region $i \in [1 - \mu, \mu]$. In contrast, if $\tau > \frac{\sigma}{\sigma-1}$, $\mu < \frac{1}{2}$ and countries do not trade in the region $i \in [\mu, 1 - \mu]$.

Now add the fixed costs $f > 0$. Unless f is too high, it does not affect the exporting cutoff μ because export decision is made conditional on existence. On the other hand, λ (or $1 - \lambda$) is affected: the marginal (which just decides to enter; it

serves only domestic market) Foreign firm $i = 1 - \lambda$ makes variable profit just enough to cover the fixed costs:

$$\frac{1}{2\sigma - 1} \left(\frac{2\sigma - 1}{\sigma} \right)^\sigma M P^\sigma \frac{((1 - \sigma) c_{1-\lambda}^* + \sigma \tau c_{1-\lambda})^2}{(c_{1-\lambda}^* + \tau c_{1-\lambda})^{1+\sigma}} = f. \quad (\text{B.9})$$

Expression (B.9) is the key condition in this section. It is derived using the above lemma with $z_1 = c_{1-\lambda}^*$, $z_2 = \tau c_{1-\lambda}$ and demand $Q = q_{1-\lambda}^* = M \left(\frac{p_{1-\lambda}^*}{P} \right)^{-\sigma}$. The cutoff λ now depends on general equilibrium conditions. It is convenient to reintroduce wages for a while and rewrite (B.9) with $1f = Wf$ and $c_i = z_m \tilde{c}_i W$:

$$\frac{1}{2\sigma - 1} \left(\frac{2\sigma - 1}{\sigma} \right)^\sigma \frac{((1 - \sigma) \tilde{c}_{1-\lambda}^* + \sigma \tau \tilde{c}_{1-\lambda})^2}{(\tilde{c}_{1-\lambda}^* + \tau \tilde{c}_{1-\lambda})^{1+\sigma}} = \frac{f}{z_m M} \left(z_m \frac{W}{P} \right)^\sigma. \quad (\text{B.10})$$

The left hand side is decreasing in λ (this is equivalent to the fact that the firm's profit in the lemma increases in the firm's costs advantage). To obtain some preliminary intuition, consider first a change, say, a decrease, in L for a constant z_m . If $\left(\frac{W}{P} \right)^\sigma$ increases, stays constant or decreases less than proportionally to M , the left hand side should increase, so λ falls, which means deeper specialization. This demonstrates that the trade pattern is potentially scale-dependent in this setting. Next consider an increase in the common manufacturing productivity (z_m falls). In the economy with only manufacturing sector, as we will see below, both $z_m M$ and $z_m \frac{W}{P}$ are constant because manufacturing output and the real wage change one for one with manufacturing productivity. However, if the economy features structural change, an increase in M is associated with declining manufacturing employment as labor moves away to less productive services. This means that $z_m M$ decreases as z_m falls, so λ falls as well, generating more trade.

Labor market clearing in Home is

$$\alpha M + \int_0^{1-\mu} q_i^m c_i di + \int_{1-\mu}^{\lambda} q_i^o c_i di + \int_0^{1-\lambda} \tau q_i^{*m} c_i di + \int_{1-\lambda}^{\mu} \tau q_i^{*o} c_i di + \lambda f = L, \quad (\text{B.11})$$

where $\int_0^{1-\mu} q_i^m c_i di$ is serving the Home market as monopolist, $\int_{1-\mu}^{\lambda} q_i^o c_i di$ is serving the Home market as oligopolist, $\int_0^{1-\lambda} \tau q_i^{*m} c_i di$ is serving the Foreign market as monopolist, and $\int_{1-\lambda}^{\mu} \tau q_i^{*o} c_i di$ is serving the Foreign market as oligopolist. Each of these four terms has a common factor MP^σ , so rewrite (B.11) as

$$MP^\sigma (\alpha P^{-\sigma} + z_m^{1-\sigma} H(\tau, \sigma, \mu(\tau, \sigma), \lambda)) = L - \lambda f$$

and then substitute MP^σ from (B.9) to get

$$\frac{\alpha}{z_m} \left(\tilde{P}(\tau, \sigma, \mu(\tau, \sigma), \lambda) \right)^{-\sigma} + H(\tau, \sigma, \mu(\tau, \sigma), \lambda) = \frac{L - \lambda f}{f} \frac{1}{2\sigma - 1} \left(\frac{2\sigma - 1}{\sigma} \right)^\sigma \frac{((1 - \sigma)\tilde{c}_{1-\lambda}^* + \sigma\tau\tilde{c}_{1-\lambda})^2}{(\tilde{c}_{1-\lambda}^* + \tau\tilde{c}_{1-\lambda})^{1+\sigma}}$$

where

$$\begin{aligned} \tilde{P} &= \left(\int_0^{1-\mu} \left(\frac{\sigma}{\sigma-1} \tilde{c}_i \right)^{1-\sigma} di + \int_{1-\mu}^{\lambda} \left(\frac{\sigma}{2\sigma-1} (\tilde{c}_i + \tau\tilde{c}_i^*) \right)^{1-\sigma} di + \int_{\lambda}^1 \left(\frac{\sigma}{\sigma-1} \tau\tilde{c}_i^* \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \\ H(\tau, \sigma, \mu(\tau, \sigma), \lambda) &= \left(\frac{\sigma-1}{\sigma} \right)^\sigma \int_0^{1-\mu} \tilde{c}_i^{1-\sigma} di + \left(\frac{2\sigma-1}{\sigma} \right)^\sigma \int_{1-\mu}^{\lambda} \frac{(1-\sigma)\tilde{c}_i + \sigma\tau\tilde{c}_i^*}{(\tilde{c}_i + \tau\tilde{c}_i^*)^{1+\sigma}} \tilde{c}_i di + \\ &\quad + \left(\frac{\sigma-1}{\sigma} \right)^\sigma \int_0^{1-\lambda} (\tau\tilde{c}_i)^{1-\sigma} di + \left(\frac{2\sigma-1}{\sigma} \right)^\sigma \int_{1-\lambda}^{\mu} \frac{(1-\sigma)\tau\tilde{c}_i + \sigma\tau\tilde{c}_i^*}{(\tau\tilde{c}_i + \tau\tilde{c}_i^*)^{1+\sigma}} \tau\tilde{c}_i di. \end{aligned}$$

The comparative statics pattern can be established as in Section 2.3.

Appendix C

Proofs for Chapter 3

C.1 Foreign Equilibrium

Part I

First, upstream and downstream marginal costs are

$$MC_i^{I*} = Z_i^{I*} W^* \text{ and} \quad (\text{C.1})$$

$$MC_i^{F*} = Z_i^{F*} (W^*)^{1-\beta_i} (P_{Mi}^*)^{\beta_i}, \quad (\text{C.2})$$

where the price index for material input is

$$P_{Mi}^* = \left((MC_i^{I*})^{1-\rho_i^I} + (tx_i^I \tau_i^I MC_i^I)^{1-\rho_i^I} \right)^{\frac{1}{1-\rho_i^I}}. \quad (\text{C.3})$$

From (C.3) we observe that intermediate export taxes imposed by Home reduce, tx_i^I , reduce downstream productivity in Foreign.

The Foreign final domestic share is

$$\lambda_i^{F*} = \frac{(MC_i^{F*})^{1-\rho_i^F}}{(MC_i^{F*})^{1-\rho_i^F} + (tx_i^F \tau_i^F MC_i^F)^{1-\rho_i^F}}. \quad (\text{C.4})$$

and demand quantities are

$$c_{id}^* = \frac{\alpha_i \lambda_i^{F*} W^* L^*}{MC_i^{F*}} \text{ and } c_{im}^* = \frac{\alpha_i (1 - \lambda_i^{F*}) W^* L^*}{tx_i^F \tau_i^F MC_i^{F*}}. \quad (\text{C.5})$$

The revenue of final producers, y_i^{F*} , comes from serving domestic demand c_{id}^* and Home import demand c_{im}^* adjusted by trade costs τ_i^F :

$$y_i^{F*} = MC_i^{F*} \cdot (c_{id}^* + \tau_i^F c_{im}^*). \quad (\text{C.6})$$

Final producers in industry i spend $(1 - \beta_i) y_i^{F*}$ on labor and employ

$$l_i^{F*} = \frac{(1 - \beta_i) y_i^{F*}}{W^*}. \quad (\text{C.7})$$

Domestic shares for intermediate expenditure:

$$\lambda_i^{I*} = \frac{(MC_i^{I*})^{1-\rho_i^I}}{(MC_i^{I*})^{1-\rho_i^I} + (tx_i^I \tau_i^I MC_i^I)^{1-\rho_i^I}}. \quad (\text{C.8})$$

Quantities demanded by intermediate producers are

$$x_{id}^* = \frac{\lambda_i^{I*} \beta_i y_i^{F*}}{MC_i^{I*}} \text{ and } x_{im}^* = \frac{(1 - \lambda_i^{I*}) \beta_i y_i^{F*}}{tx_i^I \tau_i^I MC_i^I}. \quad (\text{C.9})$$

Revenue of intermediate producers:

$$y_i^{I*} = MC_i^{I*} \cdot (x_{id}^* + \tau_i^I x_{im}^*). \quad (\text{C.10})$$

Employment in producing intermediates:

$$l_i^{I*} = \frac{y_i^{I*}}{W^*}. \quad (\text{C.11})$$

Labor market clearing:

$$\sum_i (l_i^{I*} + l_i^{F*}) = L^*. \quad (\text{C.12})$$

Part II

The Foreign equilibrium relationships (3.29)-(3.34) are similar to (C.1)-(C.11), except for the following modifications.

- (i) A normalization $W^* = 1$ is imposed.
- (ii) We have Px_i^I instead of $tx_i^I \tau_i^I MC_i^I$ and Px_i^F instead of $tx_i^F \tau_i^F MC_i^F$, the planner directly sets export prices.
- (iii) The final marginal costs MC_i^{F*} are expressed as $Z_i^{F*} (Z_i^{I*})^{\beta_i} (\lambda_i^{I*})^{\frac{\beta_i}{\rho_i^I - 1}}$. See derivation of (3.22) in the main text of the paper.

C.2 Proof of Proposition 3.3.3

We use that a country's production is equal to domestic absorption plus net export. For final goods this accounting identity is

$$y_i^{F*} = \alpha_i L^* + MC_i^{F*} \tau_i^F c_{im} - Px_i^F c_{im}^*$$

and for intermediate goods it is

$$y_i^{I*} = \beta_i y_i^{F*} + MC_i^{I*} \tau_i^I x_{im} - Px_i^I x_{im}^*.$$

Therefore,

$$l_i^{F*} + l_i^{I*} = (1 - \beta_i) y_i^{F*} + y_i^{I*} = (1 - \beta_i) y_i^{F*} + \beta_i y_i^{F*} + MC_i^{I*} \tau_i^I x_{im} - Px_i^I x_{im}^* =$$

$$y_i^{F*} + MC_i^{I*} \tau_i^I x_{im} - Px_i^I x_{im}^* = \alpha_i L^* + MC_i^{F*} \tau_i^F c_{im} - Px_i^F c_{im}^* + MC_i^{I*} \tau_i^I x_{im} - Px_i^I x_{im}^*.$$

Among the terms of the last expression, $\alpha_i L^*$ and $MC_i^{I*} \tau_i^I x_{im}$ are not affected by export prices. By substituting $l_i^{F*} + l_i^{I*}$ in (3.48) with that expression we get (3.49).

■

C.3 Proof of Proposition 3.3.4

We start with the first-order condition

$$\bar{\mu}^* c_{im}^* + (\bar{\mu}^* P x_i^F - \mu_i^F \tau_i^F) \frac{\partial c_{im}^*}{\partial P x_i^F} + (\bar{\mu}^* P x_i^I - \mu_i^I \tau_i^I) \frac{\partial x_{im}^*}{\partial P x_i^F} = 0.$$

Dividing by $\bar{\mu}^* c_{im}^*$ yields

$$1 + \left(1 - \frac{\mu_i^F \tau_i^F}{\bar{\mu}^* P x_i^F}\right) \frac{\partial c_{im}^*}{\partial P x_i^F} \frac{P x_i^F}{c_{im}^*} + \left(1 - \frac{\mu_i^I \tau_i^I}{\bar{\mu}^* P x_i^I}\right) \frac{\partial x_{im}^*}{\partial P x_i^F} \frac{P x_i^I}{c_{im}^*} = 0.$$

As $\mu_i^F = \bar{\mu} MC_i^F$, $\mu_i^I = \bar{\mu} MC_i^I$ and $\frac{\bar{\mu}^*}{\bar{\mu}} = tm^I$, the ratio $\frac{\mu_i^F \tau_i^F}{\bar{\mu}^* P x_i^F}$ is equal to $\frac{1}{tm^I tx_i^F}$ and the ratio $\frac{\mu_i^I \tau_i^I}{\bar{\mu}^* P x_i^I}$ equals to $\frac{1}{tm^I tx_i^I}$. Our equation becomes

$$1 - \left(1 - \frac{1}{tm^I tx_i^F}\right) \varepsilon_i^F + \left(1 - \frac{1}{tm^I tx_i^I}\right) \frac{\partial x_{im}^*}{\partial P x_i^F} \frac{P x_i^I}{c_{im}^*} = 0. \quad (C.13)$$

We next use that $P x_i^I x_{im}^* = (1 - \lambda_i^{I*}) \beta_i y_i^{F*} = (1 - \lambda_i^{I*}) \beta_i (\alpha_i L^* + MC_i^{F*} \tau_i^F c_{im} - P x_i^F c_{im}^*)$, so $\frac{\partial x_{im}^*}{\partial P x_i^F} = -\frac{1}{P x_i^I} (1 - \lambda_i^{I*}) \beta_i \frac{\partial (P x_i^F c_{im}^*)}{\partial P x_i^F} = -(1 - \lambda_i^{I*}) \beta_i (1 - \varepsilon_i^F) \frac{c_{im}^*}{P x_i^I}$. Once we plug in this expression into (C.13), we have

$$1 - \left(1 - \frac{1}{tm^I tx_i^F}\right) \varepsilon_i^F + \left(1 - \frac{1}{tm^I tx_i^I}\right) (1 - \lambda_i^{I*}) \beta_i (\varepsilon_i^F - 1) = 0, \text{ from which}$$

$$tx_i^F = \frac{\varepsilon_i^F}{\varepsilon_i^F - 1} \frac{1}{tm^I - (1 - \lambda_i^{I*}) \beta_i \left(tm^I - \frac{1}{tx_i^I}\right)}.$$

Bibliography

- [1] Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The network origins of aggregate fluctuations." *Econometrica*, 80, 1977–2016.
- [2] Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2015. "Networks, shocks, and systemic risk." Working paper.
- [3] Albrecht, L. and Tombe, T. 2015. Internal Trade, Productivity, and Interconnected Industries: A Quantitative Analysis. Working Paper.
- [4] Allen, T., C. Arkolakis, and X. Li. 2015. On the existence and uniqueness of trade equilibria. Mimeo, Northwestern and Yale Universities.
- [5] Allen, Treb, Costas Arkolakis, and Yuta Takahashi. 2014. Universal Gravity. Working paper.
- [6] Alvarez, Fernando and Robert Jr. Lucas. 2007. General Equilibrium Analysis of the Eaton-Kortum Model of International Trade. *Journal of Monetary Economics*, Vol. 54, No.6, pp. 1726-68.
- [7] Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2014. "Importers, exporters, and exchange rate disconnect." *The American Economic Review* 104.7 1942-1978.
- [8] Anderson, J. E. 1979. A Theoretical Foundation for the Gravity Equation. *American Economic Review*, 69(1), 106-116.
- [9] Anderson, J., van Wincoop, E., 2003. Gravity with Gravitas: A Solution to the Border Puzzle. *American Economic Review* 93, pp. 170-192.
- [10] Anderson, J., van Wincoop, E., 2004. Trade Costs. *Journal of Economic Literature* 42, pp. 691-751.
- [11] Arkolakis C, Costinot A, Donaldson D, Rodriguez-Clare A. 2012. "The Elusive Pro-Competitive Effects of Trade." Working paper.

- [12] Arkolakis, C., A. Costinot, and A. Rodriguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.
- [13] Arkolakis, Costas, and Ananth Ramanarayanan. 2009. Vertical Specialization and International Business Cycle Synchronization. *The Scandinavian Journal of Economics* 111, no. 4 655-680.
- [14] Armington, P. S. 1969. A Theory of Demand for Products Distinguished by Place of Production. *International Monetary Fund Staff Papers*, 16, 159-178.
- [15] Atalay, Enghin. 2014. How important are sectoral shocks? Working paper.
- [16] Atkeson, Andrew G., and Ariel T. Burstein. 2008. "Pricing-to-Market, Trade Costs, and International Relative Prices." *American Economic Review* 98:1998–2031.
- [17] Atkeson, A., and A. Burstein. 2010. Innovation, Firm Dynamics, and International Trade. *Journal of Political Economy*, 118(3), 433-489.
- [18] Aruoba SB, Fernández-Villaverde J, Rubio-Ramírez J. 2006. Comparing solution methods for dynamic equilibrium economies. *J Econ Dyn Control* 30:2477–2508.
- [19] Baldwin, Richard. 2012. "WTO 2.0: Global governance of supply-chain trade." *CEPR Policy Insight* 64.
- [20] Baldwin, Richard, and Javier Lopez-Gonzalez. 2014. Supply-chain Trade: A Portrait of Global Patterns and Several Testable Hypotheses. *The World Economy*.
- [21] Baldwin, Richard, and Anthony J. Venables. 2013. Spiders and snakes: offshoring and agglomeration in the global economy. *Journal of International Economics* 90.2 (2013): 245-254.
- [22] Baqaee, David Rezza. 2015. "Cascading Failures in Production Networks." Working paper.
- [23] Bems, Rudolfs, Robert C. Johnson, and Kei-Mu Yi. 2010. "Demand spillovers and the collapse of trade in the global recession." *IMF Economic review* 58.2: 295-326.

- [24] Bems, R. and Johnson, R. C. 2015. "Demand for value added and value-added exchange rates." Working paper.
- [25] Bergoeing, Raphael, and Kehoe, Timothy J. 2003. "Trade Theory and Trade Facts." Research Department Staff Report 284 Minneapolis: Fed. Reserve Bank.
- [26] Bergoeing, R., T. J. Kehoe, V. Strauss-Kahn, and K.-M. Yi. 2004. "Why Is Manufacturing Trade Rising Even as Manufacturing Output Is Falling?" American Economic Review Papers and Proceedings, 94(2), 134–138.
- [27] Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel S. Kortum. 2003. "Plants and Productivity in International Trade." American Economic Review, 93(4), 1268-1290.
- [28] Bernard Andrew B, Bradford Jensen J, Redding Stephen J, Schott Peter K. 2007. "Firms in International Trade." Journal of Economic Perspectives. 21:105-130.
- [29] Bernard, Andrew, Andreas Moxnes, and Yukiko Saito. 2014. "Production Networks, Geography, and Firm Performance." Working paper.
- [30] Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott. 2007. "Comparative Advantage and Heterogeneous Firms." Review of Economic Studies, 74(1): 31-66.
- [31] Blanchard, Emily J., Chad P. Bown, and Robert C. Johnson. 2016. "Global supply chains and trade policy." No. w21883. National Bureau of Economic Research.
- [32] Brander, J., Krugman, P., 1983. A 'Reciprocal Dumping' model of international trade. Journal of International Economics 15, 313–323.
- [33] Bridgman, Benjamin. 2012. "The Rise of Vertical Specialization Trade." Journal of International Economics, 86(1): 1333–140.
- [34] Broda, C., and Weinstein, D. E. 2006. Globalization and the Gains From Variety. The Quarterly Journal of Economics, 121(2), 541-585.
- [35] Brooks, W. and Pujolas, P. 2014. Nonlinear Gravity. Working Paper.

- [36] Buera, F. J. and J. P. Kaboski. 2009. "Can Traditional Theories of Structural Change Fit The Data?" *Journal of the European Economic Association*, 7, 469–477.
- [37] Burstein, A., and J. Cravino. 2012. Measured aggregate gains from international trade. Discussion paper, National Bureau of Economic Research.
- [38] Caliendo, L. and Parro, F. 2015. Estimates of the trade and welfare effects of NAFTA. *Review of Economic Studies*, 82(1):1-44.
- [39] Chaney, T. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review*, 98(4), 1707-1721.
- [40] Connolly, Michelle P., and Kei-Mu Yi. 2014. How much of South Korea's growth miracle can be explained by trade policy? Working paper.
- [41] Costinot, Arnaud, Dave Donaldson, and Ivana Komunjer. What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas. *Review of Economic Studies*, 2012, 79 (2), 581–608.
- [42] Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Iván Werning. 2015. Comparative Advantage and Optimal Trade Policy. *The Quarterly Journal of Economics*, 130, 659-702.
- [43] Costinot, Arnaud and Andrés Rodríguez-Clare. 2014. "Trade Theory with Numbers: Quantifying the Consequences of Globalization." In Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics*, vol. 4. Amsterdam: Elsevier.
- [44] Dalton, John T. 2009. "Explaining the growth in manufacturing trade", mimeo, University of Minnesota.
- [45] Deardorff, A.V. (2005), Ricardian comparative advantage with intermediate inputs, *North American Journal of Economics and Finance*, 16, 11-34.
- [46] Coughlin, C., 2004. The Increasing Importance of Proximity for Exports from U.S. States. *Federal Reserve Bank of St. Louis Review* 86, pp. 1-18.
- [47] Debaere, Peter, and Shalah Mostashari. 2010. 'Do tariffs matter for the extensive margin of international trade? An empirical analysis.' *Journal of International Economics* 81(2), 163–169.

- [48] De Locker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik. 2012. "Prices, Markups and Trade Reform," Working Paper 17925, National Bureau of Economic Research.
- [49] Dornbusch, Rudiger, Stanley Fischer and Paul Samuelson, 1977, "Trade in a Ricardian Model with a Continuum of Goods." *American Economic Review*.
- [50] Eaton, J. and S. Kortum. 2002. "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.
- [51] Evans, C., 2006. Border Effects and the Availability of Domestic Products Abroad. *Canadian Journal of Economics* 39, pp. 211-246.
- [52] Etro, F., 2009. Endogenous market structures and the macroeconomy. Springer, New York and Berlin.
- [53] Fally, Thibault and Russell Hillbery. 2014. A Coasian Model of International Production Chains. Working paper.
- [54] Fan, H., E. Lai, and H. Qi. 2014. Global Gains from Reduction of Trade Costs. Working paper.
- [55] Feenstra, Robert C., Philip A. Luck, Maurice Obstfeld, and Katheryn N. Russ. 2014. "In Search of the Armington Elasticity." Working paper.
- [56] Feinberg, Susan E., and Michael P. Keane. 2006. "Accounting for the Growth of MNCBased Trade Using a Structural Model of U.S. MNCs," *American Economic Review*, 96, 1515–1558.
- [57] French, Scott. 2015. The Composition of Trade Flows and the Aggregate Effects of Trade Barriers. Working Paper.
- [58] Gabaix, Xavier. 2011. The Granular Origins of Aggregate Fluctuations. *Econometrica* 79 (3): 733-772.
- [59] Goldberg, Penelope, Amit Kumar Khandelwal, Nina Pavcnik, and Petia Topalova. 2010. "Imported Intermediate Inputs and Domestic Product Growth: Evidence from India." *Quarterly Journal of Economics* 125 (4):1727-1767.
- [60] Gordon, Brett R., Avi Goldfarb, and Yang Li. 2013. "Does Price Elasticity Vary with Economic Growth? A Cross-Category Analysis." *Journal of Marketing Research*, Vol. L (Feb.), 4-23.

- [61] Grossman, Gene M. and Esteban Rossi-Hansberg. 2010. "External Economies and International Trade Redux." *Quarterly Journal of Economics* 125: 829-858.
- [62] Grubel, Herbert G. and Peter J. Lloyd. 1971. "The Empirical Measurement of Intra-Industry Trade." *Economic Record*. 47 (4), 494-517.
- [63] Gu, W., G. Sawchuk, and L.W. Rennison. 2003. "The effect of tariff reductions on firm size and firm turnover in Canadian manufacturing." *Review of World Economics*. Vol. 139. No. 3. p. 440–459.
- [64] Halpern, Laszlo, Miklos Koren, and Adam Szeidl. 2011. "Imported Inputs and Productivity." Working Paper.
- [65] Hanson, Gordon H., Nels Lind, and Marc-Andreas Muendler. 2014. "The Dynamics of Comparative Advantage." Working paper.
- [66] Head, K., Mayer, T., 2009. Illusory Border Effects: Distance Mismeasurement Inflates Estimates of Home Bias in Trade. In: *The Gravity Model in International Trade: Advances and Applications*. Editors: Bergeijk and Brakman, Cambridge University Press.
- [67] Head, Keith and Thierry Mayer. 2013. What separates us? Sources of resistance to globalization. *Canadian Journal of Economics* 46(4):1196–1231.
- [68] Head, K., and Mayer, T. 2014. Gravity equations: workhorse, toolkit, and cookbook. In: Gopinath, G., Helpman, E., Rogoff, K. (Eds.), *Handbook of International Economics*, vol. 4. Elsevier, pp. 131-195 (chapter 3).
- [69] Hillberry, R., Hummels, D., 2003. Intranational Home Bias: Some Explanations. *Review of Economics and Statistics* 85, pp. 1089-1092.
- [70] Hillberry, R., Hummels, D., 2008. Trade Responses to Geographic Frictions: A Decomposition Using Micro-Data. *European Economic Review* 52, pp. 527-550.
- [71] Hottman, Colin, Stephen J. Redding, and David E. Weinstein. 2015. "Quantifying the Sources of Firm Heterogeneity." Working paper.
- [72] Hummels, David. 1999. Have International Transportation Costs Declined? Manuscript. Chicago: Univ. Chicago.
- [73] Hummels D., J. Ishii and K-M.Yi. 2001. The nature and growth of vertical specialization in world trade, *Journal of International Economics* 54, 75-96.

- [74] Imbs, J. and Mejean, I. 2014. Elasticity optimism. *American Economic Journal: Macroeconomics*.
- [75] Johnson R.C. 2014. Five Facts about Value-Added Exports and Implications for Macroeconomics and Trade Research. *Journal of Economic Perspectives*, 28(2): 119-42.
- [76] Johnson R.C., Moxnes A. 2013. Technology, trade costs, and the pattern of trade with multistage production. Working paper.
- [77] Johnson, Robert C. and Guillermo Noguera. 2012. "Accounting for Intermediates: Production Sharing and Trade in Value Added." *Journal of International Economics* 86 (2):224-236.
- [78] Jones, Charles I. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics* 3 (2):1-28.
- [79] Judd, K.L., Guu, S.M., 1997. Asymptotic methods for aggregate growth models. *Journal of Economic Dynamics and Control* 21, 1025–1042.
- [80] Klenow, P., Li, H. and Bollard, A. 2013. "Entry Costs Rise with Development." In 2013 Meeting Papers (No. 471). Society for Economic Dynamics.
- [81] Koopman, Robert, Zhi Wang, and Shang-Jin Wei. 2014. Tracing Value-Added and Double Counting in Gross Exports. *American Economic Review*, 104, 459–494.
- [82] Krugman, Paul. 1980. Scale Economies, Product Differentiation, and the Pattern of Trade. *American Economic Review* 70:950–959.
- [83] Krugman, P. 1995. Growing world trade: Causes and consequences. *Brookings Papers on Economic Activity* (1), 327–377.
- [84] Lincoln, William, and Andrew McCallum. 2011. "Entry Costs and Increasing Trade," University of Michigan Working Paper.
- [85] Luttmer, Erzo G. J. 2007. "Selection, Growth, and the Size Distribution of Firms." *Quarterly Journal of Economics*, 122(3) 1103-44.

- [86] Matsuyama, K. 2009. "Structural Change in an Interdependent World: A Global View of Manufacturing Decline," *Journal of the European Economic Association*, 7, 478–486.
- [87] McCallum, J., 1995. National Borders Matter: Canada-U.S. Regional Trade Patterns. *American Economic Review* 85, pp. 615-623.
- [88] Melitz, M. J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 71, 1695–1725.
- [89] Melitz, M. J. and Gianmarco I. P. Ottaviano. 2008. "Market Size, Trade, and Productivity." *The Review of Economic Studies* 75(1): 295–316.
- [90] Melitz, M. J. and S. J. Redding. 2014. "Heterogeneous Firms and Trade." In G. Gopinath, E. Helpman and K. Rogoff (eds.), *The Handbook of International Economics*, vol.4, Elsevier.
- [91] Melitz, M. J. and S. J. Redding. 2014. "Missing Gains from Trade?" *American Economic Review*, 104(5), 317-21.
- [92] Melitz, M. J. and S. J. Redding. 2015. "New Trade Models, New Welfare Implications." *American Economic Review*, 105(3), 1105-46.
- [93] Miroudot, S., D. Rouzet and F. Spinelli. 2013. "Trade Policy Implications of Global Value Chains", *OECD Trade Policy Papers*, No. 161, OECD Publishing. <http://dx.doi.org/10.1787/5k3tpt2t0zs1-en>.
- [94] Neary, J. P. 2009. *International Trade in General Oligopolistic Equilibrium*. Working Paper.
- [95] Ngai, Rachel L. and Christopher Pissaridis. 2007. "Structural Change in a Multi-Sector Model of Growth." *American Economic Review*. 97(1), pp. 429–443.
- [96] Nitsch, V., 2000. National Borders and International Trade: Evidence from the European Union. *Canadian Journal of Economics* 33, pp. 1091-1105.
- [97] Novy, D. 2013. International trade without CES: Estimating translog gravity. *Journal of International Economics*, 89(2), 271-282.
- [98] Opp, Marcus M. 2010. "Tariff wars in the Ricardian Model with a continuum of goods." *Journal of International Economics* 80.2: 212-225.

- [99] Ossa, R. 2015. Why trade matters after all. Working Paper.
- [100] Pierce, Justin R., and Peter K. Schott. 2012. "The surprisingly swift decline of US manufacturing employment." No. w18655. National Bureau of Economic Research.
- [101] Reyes-Heroles, Ricardo. 2016. Globalization and Structural Change in the United States: A Quantitative Assessment. Working Paper.
- [102] Romalis, J., 2004. Factor Proportions and the Structure of Commodity Trade. *American Economic Review* 94, 67-97.
- [103] Samuelson, P.A. 2001. A Ricardo-Sraffa Paradigm Comparing Gains from Trade in Inputs and Finished Goods, *Journal of Economic Literature*, XXXIX(December), 1204-1214.
- [104] Seber, G. and A. Lee. 2002. *Linear Regression Analysis*. John Wiley and Sons.
- [105] Simonovska, I. and M. Waugh. 2011. "The Elasticity of Trade: Estimates & Evidence," Mimeo, New York University.
- [106] Somale, M. 2014. "Comparative Advantage in Innovation and Production. Working paper.
- [107] Sposi, Michael. 2012. "Evolving Comparative Advantage, Structural Change, and the Composition of Trade." Manuscript.
- [108] Teignier, Marc. 2012. "The Role of Trade in Structural Transformation." Manuscript.
- [109] Wei, S., 1996. Intra-National Versus International Trade: How Stubborn are Nations in Global Integration? NBER Working Paper #5531.
- [110] Wolf, H., 2000. Intranational Home Bias in Trade. *Review of Economics and Statistics* 82, pp. 555-563.
- [111] Yi, K. 2003. "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*.
- [112] Yi, K., 2010. Can Multi-Stage Production Explain the Home Bias in Trade? *American Economic Review* 100, pp. 364-393.

- [113] Yi, Kei-Mu and Jing Zhang, "Structural Change in an Open Economy," April 2010. Mimeo, Federal Reserve Bank of Philadelphia and University of Michigan.